Trade Costs and Multimarket Collusion*

by

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Abstract

This paper explores the implications of reciprocal trade liberalization for implicit collusion and welfare in the context of a homogeneous goods duopoly model with multimarket contact and quantity competition. A key finding of our work is that collusive conduct does not necessarily induce firms to geographically separate markets and eliminate intra-industry trade. When trade costs and discount factors are not too high, efficient cartel agreements that involve the cross hauling of goods are easier to sustain because such agreements entail lower deviation incentives. As a consequence, for a certain range of discount factors, welfare is lower when there are no trade costs than when trade costs are so high that they eliminate trade. This establishes an important sense in which trade liberalization is pro-collusive in the neighborhood of unimpeded (i.e., free) trade. In contrast, reductions in trade barriers are welfare-improving when these barriers are initially high enough to eliminate trade flows in the efficient cartel agreement. The analysis demonstrates how tariffs and transport costs differ in terms of their effects on welfare. The analysis is also extended to consider the possible presence of several firms in each country, more general punishments, and imperfect substitutability in demand.

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I. Introduction

The conventional wisdom on the relationship between trade liberalization and collusive conduct appears to be that trade liberalization intensifies competitive pressures; therefore, the significant reductions in transport costs and trade barriers we have observed over the last several decades should have made markets more competitive. Yet, recent evidence on cartel activity suggests that cartels have continued to operate despite the existence of anti-trust enforcement.

A distinguishing characteristic of prosecuted cartels is that they have been international in nature, with firms from a number of countries participating in collusive arrangements covering market shares across national markets.[1] Evidence on the magnitude of such activities rests in the fact that the Department of Justice successfully prosecuted a number of international cartels in the 1990s, with fines from prosecutions accounting for more than 90% of the fines imposed in criminal antitrust cases annually. Interestingly, the prosecution of international cartels continues to the present day, with the Justice Department having successfully prosecuted five companies in an ongoing investigation of collusion in the Dynamic Random Access Memory cartel and having arrested executives from six countries for collusion in the marine hose industry.[2]

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[1] Of course, the existence of international market sharing arrangements is not altogether new. The first important case involving a cartel between US and foreign firms prosecuted by the Department of Justice was US vs. American Tobacco (1911). This case involved a cartel between a number of US firms and two British firms in which participants agreed to to stay out of each other's domestic markets and divide shares in third country markets. Evidence from recent international cartel cases indicates that market sharing arrangements are quite detailed, with market shares calculated to tenths of a decimal point and participating firms required to purchase from other firms when they exceed their allocation. See Bond (2004) for a discussion of recent examples and for a discussion of some of the legal issues involved in antitrust enforcement with international cartels.

[2] Examples of international cartels involving US and foreign firms that have been successfully prosecuted include a lysine and citric acid cartel (operating from 1991 to 1995), a vitamin cartel (1990 - 1999), a graphite electrodes cartel (1992-1997), and a synthetic rubber cartel (1999-2002). The lysine and citric acid cartel was estimated to have raised the price of citric acid (an additive to soft drinks, processed foods, and other household products) by 30% and the price of lysine (an additive to livestock feed) by 70%. The US Department of Justice (2000) reports that prosecution of participants in this cartel resulted in the imposition of a $100 million fine on the Archer-Daniel Midlands company, a U.S. firm, and a $50 million fine on a subsidiary of a German firm. In 1999, the government collected a record fine of $500 million from F. Hoffman-La Roche, a Swiss company involved in a conspiracy to fix the price of vitamins. The process of the DRAM cartel has resulted in $732 million in fines so far (US Department of Justice, 2007).

A report by the WTO (1997, Chapter 4) concluded that "...while the extent of cartel activities is intrinsically difficult to assess ... there are some indications that a growing proportion of cartel agreements are international in scope." Thus, it appears to be appropriate to re-consider the relationship between trade costs and collusive behavior in the context of a model that capture both the international nature of cartels and the fact that these cartels typically rely on coordinating market shares in multiple markets. That is precisely what this paper aspires to achieve. More specifically, it seeks to explore the relationship between trade costs, cartel profits, and national welfare when foreign and domestic firms participate in collusive multimarket arrangements.

Our benchmark model is similar to the one considered in Brander and Krugman (1983).[3] Like them, we consider a symmetric, two-country, homogeneous-goods, duopoly model in which firms interact in their (segmented by trade costs) domestic and export markets. To capture the possibility that firms may collude implicitly, in the spirit of Bernheim and Whinston (1990), we extend the Brander-Krugman analysis to allows these firms to pool their incentive constraints across national markets while coordinating their provision of supplies.

Our analysis reveals that the relationship between trade costs and collusive behavior is conditioned by the discount factor and is generally non-monotonic. We find that reciprocal trade liberalization (i.e., reciprocal tariff or transport cost reductions) is pro-competitive when both trade barriers and the discount factor are sufficiently low. In contrast, when trade barriers are low and the discount factor not too high, we show that reductions in trade barriers can be pro-collusive. We also show that if the level of trade costs exceeds a critical level (whose value depends on the discount factor), it is optimal for firms to share markets on a geographic basis with cartel members selling only in their own markets. In this region, trade costs protect the cartel by reducing deviation incentives in export markets. Thus, increases in trade

[3] These authors studied the implications of transport costs and imperfect competition for trade patterns and welfare under market segmentation.

barriers (or, conversely, trade liberalization) in this range raise (reduce) cartel profits and lower (raise) social welfare, in line with the conventional wisdom.

Our analysis also establishes that, for discount factors below a critical level, the optimal cartel agreement will call for the cross hauling of goods between markets when trade barriers are sufficiently low. This intriguing finding is due to the fact that deviation incentives are convex in output assignments to domestic and foreign markets; therefore, cartel agreements with the lowest deviation incentives are agreements that "average" outputs across markets by allocating firms similar market shares in each market. In short, to fulfill incentive compatibility, collusive conduct *will encourage* (not discourage) intra-industry trade.

An increase in trade costs may either enhance or reduce welfare, depending on the level of trade costs and their form (tariffs vs transport costs), and the value of the discount factor. In the case of tariffs, only the behavior of total output matters: welfare will rise or fall depending on whether total output rises or falls with the level of trade costs. In turn, this depends on whether trade costs facilitate or impede collusion. We establish the existence of ranges of discount factors and trade costs, for which trade cost increases result is cartel output increases. In these ranges, the welfare-maximizing level of tariffs is always positive, but may be either at or below the level at which trade is eliminated. The case of transport costs differs in that the increase in trade costs itself is welfare-reducing. As a consequence, the results turn out to be similar except that the welfare maximum is less likely to occur at a positive level of trade.

Although there is a substantial literature on the effects of trade barriers on collusion, our approach differs significantly. One branch of this literature focuses on the case in which home firms and foreign firms form a collusive arrangement that covers only the domestic market. An important paper in this area is Davidson (1984), which examined the impact of tariff policy on collusion with firms interacting in quantities in a single, homogeneous-goods market. In this setting, tariffs affect firms' incentives asymmetrically because they render deviations more attractive for the home country firm but not for the foreign firm. To maintain collusion, a unilateral tariff reduction requires cartel output to be reallocated from home firms to foreign firms.[4] Our analysis differs in that we focus on reciprocal changes in trade costs in the context of multimarket collusion, where profits of exporters are part of the efficient cartel agreement and changes in trade barriers affect all cartel members symmetrically.

Work in a second strand of the literature has examined the role of multimarket contact in sustaining collusion and is therefore related to ours. However, the tendency in this literature has been to focus on the idea that colluding firms will sell only in their own markets because the cross hauling of goods is costly. This point was first made by Pinto (1986), who extended the Brander-Krugman framework to investigate the effect of repeated firm interactions on collusive conduct and showed that trade would in fact be eliminated for some discount factor values if the firms select the monopoly output level. Our work differs in that it extends the analysis by characterizing the relationship between trade costs and collusive conduct and obtaining novel insights on the implications of trade liberalization for profits, the assignment of outputs to domestic and export markets, and welfare.[5]

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[4] In similar spirit, Rotemberg and Saloner (1989) compared the effects of tariffs and quotas with repeated price interactions between a domestic and a foreign firm. Placing cost asymmetries between firms at center stage, Fung (1992) studied the effects of economic integration (captured by a unilateral tariff reduction) on critical discount rates and through that on collusive conduct in a differentiated-goods model with quantity competition. He showed that integration promotes competition if the foreign firm is the low cost producer and retards competition if the foreign firm is relatively less efficient, provided the initial tariff is low. Focusing on the effects of trade policies on intra-group firm behavior, Syropoulos (1992) examined the problem of sustaining collusion among quantity-setting foreign oligopolists when products are differentiated and there are no domestic firms. In that case, higher tariffs make it easier to support a cartel with a given output level.

[5] In a related paper, Colonescu and Schmitt (2003) investigated how the move from a regime of "market segmentation" (where firms can price discriminate between domestic and export markets) to a regime of "economic integration" (where price discrimination is impossible) affects competitive conduct via its impact on minimum discount factors. Their analysis reveals that the transition to market integration is pro-competitive or anti-competitive depending on the degree of (dis)similarity of product markets. Our analysis differs in that we are concerned with the implications of reciprocal cuts in trade costs on incentive constraints and collusive outcomes, and pay special attention to the circumstances under which collusion does not necessarily lead to the elimination of trade flows. Lommerud and Sørgard (2001) also investigated the effects of reciprocal tariff cuts on multimarket collusion under repeated quantity- and price-setting games by symmetric homogeneous goods duopolists. However, these authors were mainly concerned with the effects of trade liberalization on minimum discount factors and did not study the exact circumstances under which incentive constraints are binding for firms and what that may mean for geographic collusion and welfare. Phillips and Mason (1996), considered a model of price competition similar to ours to explore the effects of a price ceiling in one market on multimarket collusion when incentive constraints are binding. They

Section II of the paper analyzes the benchmark model of a symmetric duopoly with homogeneous goods, with deviators from the collusive agreement being punished by reversion to the Nash equilibrium of the single-period game. In this section, we also extend the benchmark model to consider the effect of allowing $n > 1$ firms per country and more severe punishments. Section III extends the model further by considering the case in which goods are imperfect substitutes. The key features of this extension is that it implies intra-industry trade is desirable both to society and an unconstrained cartel — because of product differentiation — if trade costs are not very high. We first show the existence of a critical trade cost such that the minimum discount factor for sustaining the monopoly cartel will be increasing (decreasing) in trade costs when trade costs are less (greater) than the critical value. We then establish that the relationship between trade costs and welfare will be similar to those obtained in the benchmark model if the degree of substitutability is sufficiently high.

II. The Model with Homogeneous Goods and Quantity Competition

To examine the effects of reductions in trade costs in the presence of multimarket contact between firms, we consider a symmetric, two-country, quantity setting, duopoly model of trade. We first derive the effect of changes in trade costs on profits and welfare when firms engage in multimarket collusion under the assumption that firms use the threat of reversion to the Nash equilibrium as the punishment. We then extend the analysis to the case of oligopoly with $n > 1$ firms per country and demonstrate how the results generalize when more severe punishments are considered.

For simplicity, we refer to the two countries as "home" and "foreign." Our symmetry assumption is embodied in the following ideas. In each country i suppliers face the linear inverse demand function $p_i = A - Q_i$, where Q_i represents the total quantity sold. The marginal cost to each firm of delivering its product to its own market is constant. (Without loss of generality, we normalize this cost to 0.) However,

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showed that a mild price ceiling in one market causes firms to behave more collusively in the unregulated market.

a firm's involvement in trade is costly: for each unit of output shipped abroad, the exporting firm incurs a trade cost t. The important point is that trade costs segment national markets and introduce an asymmetry in the costs of delivering goods to domestic and export markets. Depending on whether it raises revenues or not, this trade cost can be identified with specific import tariffs or transportation costs.[6]

Keeping in mind that markets are segmented and quantity is the strategic variable, let q_i denote the quantity sold by the firm from country i in its own market and x_i the quantity sold in its export market. The global profit of the firm from market i, denoted Π_i, is the sum of the profit $[A - (q_i + x_j)]q_i$ obtained in its own market and the profit $[A - (q_j + x_i) - t]x_i$ obtained in its export market, where (q_j,x_j) denotes the quantities selected by the rival firm $j \neq i$.

We begin by characterizing the levels of outputs, profit and national welfare in the non-cooperative equilibrium of the one-shot game, where firm i chooses (q_i,x_i) to maximize global profits given (q_j,x_j). Under the aforementioned assumptions, it readily follows that $q_i = q_j$ and $x_i = x_j$ in the Nash equilibrium; we may, therefore, drop country and firm subscripts to focus on the output decisions of a representative firm. National welfare is measured as the sum of consumer surplus, global firm profits, and tariff revenues (if any). Letting $Q = q + x$ denote the level of output in each market, it can be verified that the consumer surplus in each market will be $Q^2/2$ and each firm's global profits will be $(A - Q)Q - tx$. If trade costs take the form of tariffs, a country's tariff revenues will be tx and thus its national welfare will be $W(Q) = AQ - \frac{1}{2}Q^2$. Since tariffs represent a private and not a social cost, tariff changes will improve consumer surplus and social welfare if and only if they cause total output to move in the direction of the competitive level, $Q = A$. In contrast, if trade barriers are identified with the real resource cost of transporting goods between markets, national welfare will be $V(Q,x,t) = W(Q) - tx$.

If firms were to interact non-cooperatively in a static environment, the model would coincide with

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[6] Here we abstract from the unilateral use of trade barriers for strategic policy purposes. We assume that changes in trade costs result either from technological change or from reciprocal multilateral tariff reductions that affect the countries symmetrically.

the "reciprocal dumping" model of Brander and Krugman (1983), and so the effects of trade costs on trade patterns and welfare would not differ in a substantive way. As a benchmark — and to also obtain a clear view of how the analysis and findings differ when firm collude under repeated multimarket contact — Lemma 1 summarizes the key features of the Nash equilibrium of the single-period game.

Lemma 1: *In the presence of symmetric trade costs, the Nash equilibrium output, profit and welfare levels can be summarized as follows:*

(a) *For trade barriers, t, no greater than the prohibitive level,* $\bar{t}^N \equiv A/2$,

(i) *output levels are:* $q^N(t) = (A + t)/3$ *and* $x^N(t) = (A - 2t)/3$;

(ii) *global profit,* $\Pi^N(t)$, *is strictly convex in* t *and minimized at* $t^{\Pi}_{\min} = A/5$;

(iii) *welfare under tariffs,* $W^N(t)$, *is strictly concave and decreasing in t*;

(iv) *welfare under transport costs,* $V^N(t)$, *is strictly convex in t and is minimized at* $t^V_{\min} = 4A/11$.

(b) *For trade costs* $t \geq \bar{t}^N$, *the values of output, profits and welfare are the same as at* $\bar{t}^N$.

(c) $\Pi^N(0) < \Pi^N(\bar{t}^N)$ *and* $W^N(0) = V^N(0) > V^N(\bar{t}^N) = W^N(\bar{t}^N)$.

As noted in Brander and Krugman (1983), identical goods are traded internationally in the non-cooperative Nash equilibrium. Reciprocal reductions in trade costs induce firms to compete more (less) aggressively in export (domestic) markets, resulting in higher export volumes, smaller domestic supplies and larger consumption levels. Furthermore, when trade barriers take the form of tariffs, the cross hauling of identical goods is not socially costly; therefore, welfare unambiguously rises as tariffs fall due to the just described pro-competitive effect. In contrast, when trade barriers take the form of transport costs, the cross hauling of goods is socially costly. For transport costs in the neighborhood of the prohibitive rate, $\bar{t}^N$, the resource cost of trade dominates the pro-competitive effect; as a result, a fall in *t* reduces welfare.

For transport costs in the neighborhood of unimpeded trade, the pro-competitive effect dominates; consequently, welfare rises with reductions in t. No matter whether trade barriers take the form of tariffs or transport costs, welfare is highest when $t = 0$.

Reductions in trade costs have two conflicting effects on Nash equilibrium profits. The reduction in the cost of exporting raises a firm's profits in its export market. However, the corresponding increase in exports by the firm's rival reduces its profits in the domestic market. When trade barriers are low (high), the former (latter) effect dominates, thus giving rise to a negative (positive) relationship between trade barriers and firm profits. The highest level of profits is achieved for trade barriers at or above the prohibitive level, $\bar{t}^N$, since that effectively grants each firm a monopoly in its own market.

A. Collusion with Multimarket Contact

We now consider the possibility that the two firms may collude tacitly both in the home and foreign markets. In the spirit of Bernheim and Whinston (1990), we suppose multimarket collusion involves allocating to each firm a pair (q, x) that reflects the outputs q and x targeted for sale in the domestic and export markets, respectively.[7] Since the two firms are symmetric (in the sense that they face identical conditions in their respective domestic and export markets) we may focus, without loss of generality, on the global payoff of the representative firm. If a firm violates an implicit agreement in any one market it gets punished in all markets. (Of course, a firm that contemplates deviating from the collusive protocol in any one market will have an incentive in breaking the agreements in all markets.)

An important objective of our analysis in this section is to demonstrate that with multimarket contact the most profitable collusive arrangements involve *the cross hauling of goods* when trade barriers are sufficiently low and the no-deviation constraint is binding. This result stands in sharp contrast to the

[7] Spagnolo (2001) showed that in cases where a deviation in one market raises the payoff to deviating in another market, multimarket agreements might yield lower profits than single market collusive arrangements. Such a possibility does not arise in our model because deviation payoffs are independent across markets due to the assumption of constant marginal costs.

conventional wisdom on intra-industry trade in identical products, which suggests that colluding firms stay off their export markets presumably because this strategy maximizes joint profits. We argue that explicit consideration of incentive constraints alters this result in a fundamental way. Our second objective is to show that there exist circumstances under which reciprocal reductions in trade barriers (tariffs or transport costs) facilitate collusion which is detrimental to welfare. An important consequence of this finding is that welfare is not necessarily maximized under conditions of globally free trade.

The global profit of a firm under an agreement (q, x) is

$$\Pi^A(q, x, t) = [A - (q + x)](q + x) - tx, \tag{1}$$

where Π^A is concave in (q, x) and decreasing in t. In the special case of unimpeded trade (i.e., when t = 0), cartel profits are maximized for any combination of output levels that satisfy $q + x = A/2$. However, for $t > 0$, cartel profits are maximized when $q = A/2$ and $x = 0$. Since in this case every firm supplies the monopoly output to its home market and sells nothing abroad, we label it *maximal geographic collusion*. One of the key benefits of multimarket contact is that, in its presence, firms can reallocate shares across markets to the lowest cost producer so as to minimize total cartel costs.

If a firm deviates from a collusive agreement in its domestic (export) market that specifies quantity x (q) for its rival, the deviating firm's profit in its home market will be $\max_q (A - q - x)q = \frac{1}{4}(A - x)^2$ whereas its profit in the export market will be $\max_x [A - q - x - t]x = \frac{1}{4}(A - q - t)^2$. Thus, the total payoff to a firm that deviates from a cartel output pair (q, x) is the sum of these deviation payoffs; that is,

$$\Pi^D(q, x, t) = \frac{1}{4}[(A - x)^2 + (A - q - t)^2]. \tag{2}$$

Clearly, Π^D is decreasing in t and strictly convex in (q, x) for all non-negative output pairs that yield non-negative profits in each market. In what follows, the convexity of the deviation payoffs will play an important role because it will imply that the cross hauling of goods can be used to reduce firm incentives to deviate from collusive agreements when trade costs are low. To see this, note that at $t = 0$ the deviation profits obtained when firms share equally the monopoly output in each market (i.e., $\Pi^D(A/4, A/4, 0)$) is lower than the deviation profits obtained from a collusive agreement that entails maximal geographic collusion (i.e., $\Pi^D(A/2, 0, 0)$) due to the strict convexity of deviation profits.

We assume that a collusive agreement calls for a "credible" punishment in which firms switch to a subgame perfect equilibrium of the repeated game that forces the deviating firm to attain a lower payoff. Denote with $\delta > 0$ the representative firm's discount factor, and with $\Pi^P(t)$ its per period global payoff during the punishment phase. Then, for given t, an agreement (q, x) will be sustainable if

$$Z(q, x, t, \delta, \Pi^P(t)) \;=\; \Pi^A(q, x, t) \;-\; (1 - \delta)\Pi^D(q, x, t) \;-\; \delta\,\Pi^P(t) \;\geq\; 0. \tag{3}$$

The Nash equilibrium payoff characterized in Lemma 1 is a natural punishment, and so we will assume in the next section that $\Pi^P(t) = \Pi^N(t)$. However, for generality, we first derive some properties of the efficient cartel agreement that only depend on the requirement that $\Pi^P(t) \leq \Pi^N(t)$.

Let $F(t, \delta, \Pi^P(t)) = \{(q, x) \mid Z(q, x, t, \delta, \Pi^P(t)) \geq 0 \text{ and } q, x \geq 0\}$ be the set of incentive-compatible cartel agreements. The following Lemma shows that efficient agreements can be characterized using standard Lagrangian methods and provides some useful properties for these agreements.

Lemma 2: *For $t < \bar{t}^N$, the problem of maximizing the global profit of a cartel member subject to $(q, x) \in F(t, \delta, \Pi^P(t))$ can be represented by a saddle point problem for the Lagrangian function $\mathfrak{L}(q, x, \lambda, t, \delta, \Pi^P) = \Pi^A(q, x, t) + \lambda Z(q, x, t, \delta, \Pi^P)$. The maximum sustainable cartel profit, $\Pi^*(t)$, is the saddle point of this Lagrangian. Letting $X^*(t) = \{(q,x) \mid (q,x) \in \textbf{argmax}\ \mathfrak{L}(q, x, \lambda, t, \delta, \Pi^P)\}$ and $\Lambda^*(t) = \{\lambda \mid \lambda \in \textbf{argmax}\ \mathfrak{L}(q, x, \lambda, t, \delta, \Pi^P)\}$, the solution to this problem has the following* *properties:*

(a) *If* $t > 0$, *then* $X^*(t)$ *is a unique pair* (q^*, x^*) *satisfying* $q^* > x^* \geq 0$.

(b) *If* $t = 0$ *and*

(i) $\lambda^* > 0$, *then* $X^*(t)$ *is a unique pair* (q^*, x^*) *satisfying* $q^* = x^* > A/4$;

(ii) $\lambda^* = 0$, *then* $X^* = \{(q,x) \mid q + x = A/2;\ q, x \geq 0\}$ *and* $Z(q, x, t, \delta, \Pi^P(t)) \geq 0$.

(c) $Q^* = q^* + x^* \geq A/2$ *for* $t > 0$, *with strict equality if* $\lambda^* = 0$.

(d) *Assume that* $\Pi^P(t)$ *is differentiable. For* $t > 0$, *the profit function will be differentiable with*

$\Pi_t^*(t) = \mathfrak{L}_t(q, x, \lambda, t, \delta, \Pi^P)$. *At* $t = 0$,

$$\Pi_{t+}^*(0) = \max_{(q,x)\in X^*(t)} \min_{\lambda\in\Lambda^*(t)} \mathfrak{L}_t(q, x, \lambda, t, \delta, \Pi^P(t)), \quad \Pi_{t-}^*(0) = \min_{(q,x)\in X^*(t)} \max_{\lambda\in\Lambda^*(t)} \mathfrak{L}_t(q, x, \lambda, t, \delta, \Pi^P(t)) \quad (4)$$

At an interior solution with a binding incentive constraint, the FOCs for the efficient cartel output require $\Pi_q^A/\Pi_x^A = \Pi_q^D/\Pi_x^D$. (Subscripts q and x denote partial derivatives.) From (1) and (2), it follows that this condition requires $q^* = x^*$ when $t = 0$ and $\lambda > 0$. Intuitively, this is so because the strict convexity of the deviation payoff in (q, x) makes it attractive for the cartel to "average" outputs across markets when the incentive constraint is binding. In other words, cross hauling of identical products arises in a collusive agreement when $t = 0$ and the no-deviation constraint is binding ($\lambda > 0$).[8] If $t = 0$ and $\lambda = 0$, the optimal policy may be a correspondence because any $\{q, x\}$ combination that satisfies $q + x = A/2$ and the no-

[8] It is important to point out that Lemma 2 is robust to relaxation of the assumption that the demand curve is linear. For the case of a general inverse demand function, $f(Q)$, we can express a firm's deviation payoff as $\pi^D(Y, t) = \max_y [f(Y + y) - t)]y$, where Y is the aggregate output of the other firm, y and t are the firm's output and trade cost, respectively, in a given market. Let $\tilde{y}(Y, t)$ denote the firm's optimal deviation. Differentiation of π^D yields $\partial\pi^D/\partial Y = f'(Y + \tilde{y})\tilde{y} < 0$ since $f' < 0$. When demand is linear, the convexity of the deviation payoff follows from the fact that f' is constant and $\tilde{y}$ is decreasing in Y. In the general case, $\partial^2\pi^D/\partial Y^2 = -(f')^2/(2f' + \tilde{y}f'') > 0$ because the denominator is negative by the second-order condition on the choice of y. It follows that the deviation payoff will be convex in the general demand case as well. Thus, as long as the inverse demand function is such that $\Pi^A(q, x, t)$ is concave, the results of Lemma 2 will hold.

deviation constraint yields the monopoly profit level for the cartel. When the incentive constraint is slack, it may no longer be necessary to fully average outputs to deter deviations.

When $t > 0$, cross hauling of goods becomes costly and the cartel finds it more profitable to assign larger output shares to domestic firms in each market. However, for small trade costs, the no-deviation constraint may prevent the representative firm from sustaining complete geographic specialization. This suggests that in this case there exists a trade-off between incentive compatibility and profitability in the allocation of market shares. The result that total outputs in each market exceed the monopoly level when the no-deviation constraint binds will be useful below in deriving efficiency results.

Equation (4), which is the envelope theorem obtained by Milgrom and Segal (2002) for the case of a saddle point problem, shows that for $t > 0$ the profit function is differentiable because the output levels and the multiplier are uniquely determined. However, the right-hand and left-hand derivatives at $t = 0$ will not agree if $\lambda = 0$ due to the fact that the optimal cartel policy is a correspondence.[9]

B. The Effects of Changes in Trade Costs with Nash Punishments

Lemma 2 focused on the pattern of production for given trade costs t. We now turn to the analysis of the impact of changes in trade barriers on output, profits and welfare under a constrained efficient cartel agreement. In order to obtain several comparative statics results, we first need to know how changes in t affect the set of sustainable agreements. Using (1)-(3), we have that $Z_t = -x + (1-\delta)(A-q-t)/2 - \delta\Pi_t^P$. An increase in trade costs will reduce a firm's payoff to an agreement if $x > 0$, but it will also reduce its deviation payoff in the export market for $A - q - t > 0$. Clearly, calculation of the overall impact of a change in t on Z requires an assumption about the punishment payoff. In this section we derive results for the case in which punishment takes the form of permanent reversion to the Nash equilibrium of the one-shot game, so $\Pi^P(t) = \Pi^N(t)$. This assumption is relaxed in the following section.

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[9] If we considered import subsidies (i.e., $t < 0$), then the arguments of Lemma 2 would require $x > q$.

We begin by considering how changes in trade costs affect the ability to sustain the monopoly profit level. It is known from standard folk theorem arguments that the monopoly profit level $\Pi^M \equiv A^2/4$ can be supported as an outcome of the infinitely repeated game if the discount factor is sufficiently high. Denote with $\delta^c(t) \equiv [\Pi^D(A/2, 0, t) - \Pi^M]/[\Pi^D(A/2, 0, t) - \Pi^N(t)]$ the minimum discount factor that is required to sustain Π^M for $t \geq 0$. It can then be said that higher trade costs facilitate collusion — in the sense of expanding the range of discount factors for which the monopoly profit is self-enforcing — if $d\delta^c/dt < 0$. When $t > 0$, the cross hauling of goods is costly and the only way to capture the monopoly profit level is to choose maximal geographic collusion. Since $Z(q, x, t, \delta, \Pi^N(t))$ is increasing in δ, it follows from (1)-(3) and Lemma 1 that the monopoly output will be sustainable for all $\delta \geq \delta^c(t) = 9(A - 2t)/(13A + 22t)$. It can be easily verified that $\delta^c(t)$ is monotonically decreasing in t, with $\delta^c(t) \to 0$ as $t \to \bar{t}^N$ (=A/2). High trade costs facilitate the sustainability of the monopoly output level because they make it less appealing for exporters to enter the domestic market in the presence of maximal geographic collusion. This idea can also be captured by inverting $\delta^c(t)$ to obtain the result that for $\delta \in (0, 9/13]$ maximal geographic collusion can be sustained for $t \geq t^G(\delta) = \dfrac{13(\underline{\delta} - \delta)}{18 + 22\delta}$, where $\underline{\delta} \equiv \lim_{t \to 0^+} \delta^c(t) = 9/13$.

What may be more surprising is that there is also a sense in which trade liberalization facilitates collusion when trade barriers are low. To see this note that for $t = 0$ the monopoly profit can be attained with any allocation that satisfies $q + x = A/2$. From the strict convexity of the deviation payoffs, the lowest minimum discount factor for $t = 0$ will be attained by choosing $q = x = A/4$. This yields a minimum discount factor of $\delta^c(0) \equiv [\Pi^D(A/4, A/4, 0) - \Pi^M]/[\Pi^D(A/4, A/4, 0) - \Pi^N(0)] = 9/17 = 0.5294 < .6923 = 9/13 = \underline{\delta} \equiv \lim_{t \to 0^+} \delta^c(t)$. In other words, the cross hauling of goods facilitates collusion since allocations that are associated with more symmetric shares of the monopoly output in each market yield lower deviation payoffs. For the case of homogeneous products, this results in a discontinuity in the minimum discount factor that supports the monopoly profit level through multimarket contact at $t = 0$.

One might be tempted to view this discontinuity in the minimum discount factor at $t = 0$ as a theoretical curiosum because some transport cost frictions are likely to persist even when tariffs are completely eliminated. Not so. This result is a reflection of the fact that it is profitable for firms to use intra-industry trade as part of the efficient cartel agreement when $\delta < \underline{\delta}$, and has important practical implications. We will show that the cross hauling of goods will imply that collusive incentives will be decreasing in *t* (and thus social welfare will be increasing in *t*) in the neighborhood of $t = 0$ for $\delta \in [\delta^c(0), \underline{\delta})$. To substantiate this idea, we first provide another result which clarifies how output assignments in efficient cartel agreements vary with *t* and δ.

Proposition 1: *For a duopoly with homogeneous goods, the most profitable sustainable cartel agreements,* $\square^*(t) = (q^*, x^*)$, *have the following features:*

(a) *If* $\delta \in [\underline{\delta}, 1)$ *and*

(i) $t > 0$, *then* $X^*(t) = (A/2, 0)$

(ii) $t = 0$, *then* $X^*(t) = \{(q^*, x^*) \mid q^* + x^* = A/2;\ x \in [0, A/2]\}$.

(b) *For* $\delta \in (0, \underline{\delta})$, *there exists a trade barrier level* $t_x(\delta) < t^G(\delta)$ *such that the optimal cartel policy falls into one of the following three types, depending on the level of the trade barrier:*

(i) *If* $t \geq t^G(\delta)$, *then* $q^* = A/2$ *and* $x^* = 0$.

(ii) *If* $t \in [t_x(\delta), t^G(\delta))$, *then* $q^* > A/2$ *and* $x^* = 0$ *with* $\partial q^*/\partial t < 0$.

(iii) *If* $t \in (0, t_x(\delta))$, *then* $q^* > x^* > 0$ *with* $q^* + x^* > A/2$.

(c) *If* $t = 0$ *and*

(i) $\delta \in [\delta^c(0), \underline{\delta})$, *then the optimal cartel policies are correspondences satisfying* $q^* + x^* = A/2$ *and* $x^* \in [\frac{A}{4}(1 - g(\delta)), \frac{A}{4}(1 + g(\delta))]$, *where* $g(\delta) \equiv \left[\dfrac{\delta - \delta^c(0)}{\delta^c(0)(1-\delta)}\right]^{1/2} < 1$

(ii) $\delta \in [0, \delta^c(0))$, *then* $q^* = x^* > A/4$.

For $\delta \geq \underline{\delta}$, the firms are sufficiently patient so they can sustain the monopoly profit for all trade cost levels using maximal geographic collusion. For $\delta < \underline{\delta}$, maximal geographic collusion is sustainable only when trade barriers are sufficiently high (i.e., $t \geq t^G(\delta)$). When $t \in [t_x(\delta), t^G(\delta))$ trade barriers are sufficiently high that they eliminate the possibility of cross hauling of goods and, more interestingly, induce firms to supply larger output levels domestically — and thus sustain lower profits — as compared to full monopoly. Further, trade cost reductions in this region are pro-competitive because they raise the payoff to deviating in export markets and thus require the cartel to expand its domestic output level to satisfy the no-deviation constraint. For $t < t_x(\delta)$, trade costs are sufficiently low that cross hauling becomes desirable because the no-deviation constraint is relaxed. In summary, incentive-compatible collusion narrows the range of trade costs under which intra-industry trade takes place (as compared to the Nash equilibrium), but it does not bring about the elimination of intra-industry trade for $t \in [0, t_x(\delta))$ when $\delta < \underline{\delta}$.

It should be noted that, in the special case of unimpeded trade, $X^*(0)$ is a correspondence for $\delta > \delta^c(0)$ because there is a continuum of output pairs that can attain the monopoly output level. The minimum volume of trade that is consistent with sustaining the monopoly output, denoted $x^{\min}(0)$, ranges from $A/4$ at $\delta = \delta^c(0)$ to 0 when $\delta \geq \underline{\delta}$. As the discount factor increases in this interval, less cross hauling of goods is required to maintain the monopoly profit level because a larger weight is placed on the losses from future punishments that ensue following a deviation.

Since the level of welfare for both the tariff and transport cost cases is determined only by total output in each market when there is either free trade or no trade at all (i.e., when either $t = 0$ or $x^* = 0$), Proposition 1 yields several immediate results on the relationship between welfare and trade barriers. Part (a) reveals that maximal geographic collusion is sustainable (i.e., there is no trade) for all t when $\delta \geq \underline{\delta}$; therefore, welfare and profits will be independent of trade costs in this region. Part (b) points out that for $\delta < \underline{\delta}$ profits (welfare) will be increasing (decreasing) in t on the interval $[t_x(\delta), t^G(\delta))$ for both the tariff and transport cost cases. Since $x^* = 0$ and $\partial q^*/\partial t < 0$ in this interval, welfare is decreasing in t for either welfare measure because higher trade barriers result in lower domestic output levels. Finally, parts (b) and (c) show that $W^*(0) = V^*(0) < V^*(t_x(\delta)) = W^*(t_x(\delta))$ for $\delta \in [\delta^c(0), \underline{\delta})$. Since for this interval of discount factors the monopoly output level can be sustained under free trade but not when $t = t_x(\delta)$, welfare must be increasing on average as t is raised from 0 to $t_x(\delta)$.

For the case of tariffs we obtain an even stronger result. Since by Lemma 2(c) the total output supplied to each market exceeds the monopoly output, we will have $W^*(0) < W^*(t)$ for all $\delta \in [\delta^c(0), \underline{\delta})$ and $t \in (0, t^G(\delta))$. This provides an important sense in which trade liberalization is pro-collusive for values of the discount parameter in the range $[\delta^c(0), \underline{\delta})$ because it establishes that welfare must be maximized at a positive value of t.

On the other hand, we also know that payoffs under a collusive agreement will approach those under the static non-cooperative equilibrium as $\delta \to 0$. Since welfare $W^*(t)$ is monotonically decreasing in t on $[0, \bar{t}^N]$ in the non-cooperative case, we know that the pro-collusive effect of trade liberalization will disappear for sufficiently low δ. Using our comparison of welfare at free trade with that at the tariff level at which there is no trade under the efficient collusive agreement, we show in the Appendix that there will be a critical value of the discount factor $\delta' \in (0, \delta^c(0))$ such that $W^*(0) > W^*(t_x(\delta))$ for $\delta < \delta'$.

The following proposition summarizes these results:

Proposition 2: *Welfare and profit levels at $t = 0$ and $t \geq t_x(\delta)$.*

(a) *For $\delta \in [\underline{\delta}, 1)$, profits and welfare are at the global monopoly level for all t (i.e., $\Pi^*(t) = \Pi^M$ and $w^*(t) = W^M$).*

(b) *For $\delta \in (0, \underline{\delta})$, profits are increasing in t and welfare is decreasing in t for $t \in [t_x(\delta), t^G(\delta))$. Profits and welfare are at the global monopoly level for $t \geq t^G(\delta)$.*

(c) *There exists a value $\delta' \approx 0.4387 \in (0, \delta^c(0))$ such that*

(i) $W^*(0) < W^*(t_x(\delta))$ *for* $\delta \in (\delta', \underline{\delta})$.

(ii) $W^*(0) > W^*(t_x(\delta))$ *for* $\delta \in [0, \delta')$.

Part (c) of Proposition 3 points out that for $\delta \in (\delta', \underline{\delta})$, world welfare under free trade is less than world welfare at the smallest trade cost level that is consistent with the elimination of trade; therefore, in this case, world welfare is not maximized under free trade. This raises the question of whether welfare is maximized at an interior point with positive intra-industry trade or whether it is maximized at $t = t_x(\delta)$ where there is no intra-industry trade. A similar question also arises for $\delta \in [0, \delta')$, where welfare at $t = 0$ exceeds welfare at $t = t_x(\delta)$. An analytic characterization of welfare for $t \in (0, t_x(\delta))$ is intractable because changes in trade costs affect both domestic and export supply levels. Nonetheless, it is possible to answer the question of whether or not the endpoints are local optima by characterizing the impact of trade costs on welfare at the boundaries of these intervals.

Proposition 3: *The welfare and profit effects of changes in trade barriers at* $t = 0$ *and* $t = t_x(\delta)$.

For $\delta \in [0, \underline{\delta})$, *the following results hold for the behavior of welfare and profits at the boundaries of the region at which trade is part of the efficient cartel agreement:*

(a) *For* $t = 0$ *and*

(i) $\delta \in (\delta^c(0), \underline{\delta})$

- $\lim_{t \to 0^+} \frac{\partial W^*(t)}{\partial t} > \lim_{t \to 0^+} \frac{\partial V^*(t)}{\partial t} > 0$ *and* $\lim_{t \to 0^+} \frac{\partial \Pi^*(t)}{\partial t} = -x^{\min}(0, \delta) < 0$

(ii) $\delta \in [0, \delta^c(0))$

- $\lim_{t \to 0^+} \frac{\partial V^*(t)}{\partial t} < \lim_{t \to 0^+} \frac{\partial W^*(t)}{\partial t} < 0$ *and* $\lim_{t \to 0^+} \frac{\partial \Pi^*(t)}{\partial t} < 0$.

(b) *For* $t = t_x(\delta)$,

(i) $\lim_{t \to t_x(\delta)^-} \frac{\partial W^*(t)}{\partial t} \gtreqless 0$ *as* $\delta \gtreqless \delta''$ *for some* $\delta'' \approx 0.5465 \in (\delta^c(0), \underline{\delta})$

(ii) $\lim_{t \to t_x(\delta)^-} \frac{\partial V^*(t)}{\partial t} > 0$ *for* $\delta \in [0, \underline{\delta})$

(iii) $\quad \lim_{t \to t_x(\delta)^-} \dfrac{\partial \Pi^*(t)}{\partial t} > 0 \;\; \mathit{for} \;\; \delta \in [0, \underline{\delta})$.

For $\delta \in (\delta^c(0), \underline{\delta})$, Proposition 1 showed that the monopoly output level is sustainable for $t = 0$ but not for t in the neighborhood of $t = 0$. It then follows from Lemma 2(c) and (4) that profits must be decreasing in t and output must be increasing in t in the neighborhood of $t = 0$. Since intra-industry trade is required to sustain the monopoly output at $t = 0$, profits will necessarily fall when this trade becomes costly. Since in the case of tariffs the direction of change in welfare is determined solely by the change in total output, the resulting increase in output at $t = 0$ implies that welfare will be increasing in t in the neighborhood of free trade. Somewhat more surprisingly, welfare will be increasing in t in the neighborhood of $t = 0$ for the transport cost case as well because the pro-competitive effect of an increase in trade costs is sufficiently large to offset the resource cost associated with the use of cross hauling of goods to support the cartel in this region. For $\delta < \delta^c(0)$, on the other hand, the resource cost effect dominates the pro-competitive effect and thus both $W^*(t)$ and $V^*(t)$ are decreasing in t at $t = 0$, as noted in part (a) of Proposition 3. In other words, unimpeded trade is a local (but not necessarily a global) optimum for both types of trade barriers when $\delta \in (0, \delta^c(0))$.

Part (b) of Proposition 3 reveals that, in the case of transport costs, $V^*(t)$ is always increasing in t as $t \to t_x(\delta)^-$. However, in the case of tariffs, $W^*(t)$ is increasing or decreasing in t depending on whether the actual discount factor is high or low, respectively. This suggests that while $t_x(\delta)$ is always a local maximum in the case of transport costs, $t_x(\delta)$ is a local maximum for tariffs only when the discount factor is sufficiently high. As $t \to t_x(\delta)^-$, local output q^* rises but the volume of imports x^* falls. As a consequence, the direction of change in overall output $Q^* = q^* + x^*$ appears to be ambiguous. In the Appendix we show that total output rises or falls as $t \to t_x(\delta)^-$ when $\delta \in (\delta'', \underline{\delta})$ or $\delta \in (0, \delta'')$, respectively, thus explaining the behavior of $W^*(t)$ at the prohibitive tariff level. The reason $V^*(t)$ always rises as $t \to t_x(\delta)^-$ in the case of transport costs is because $\square^*(t)$ includes the resource cost of trade, tx^*, and this cost is reduced as the volume of trade falls in the neighborhood of this endpoint.

In contrast to the static non-cooperative case where free trade is not only welfare superior to autarky but also a global optimum (Lemma 1), collusion with multimarket contact generates a host of new possibilities. For example, in the case of transport costs, autarky may dominate free trade and $t_x(\delta)$ may be a global maximum for $\delta \in [\delta', \underline{\delta})$. Perhaps more interestingly, this possibility also arises in the case of tariffs when $\delta \in (\delta'', \underline{\delta})$. Moreover, since $W^*(0) < W^*(t_x(\delta))$ for $\delta \in [\delta', \underline{\delta})$ (Proposition 2(c)), $\partial W^*/\partial t < 0$ as $t \to t_x(\delta)^-$ for $\delta < \delta''$ (Proposition 3(b)), and $\delta' < \delta''$, it will necessarily be the case that $W^*(t)$ attains its global maximum at an interior $t \in (0, t_x(\delta))$ for $\delta \in (\delta', \delta'')$ (i.e., in the presence of restricted intra-industry trade). Upon further reflection, Propositions 2 and 3 suggest that the relationship between welfare and tariffs may be quite complex indeed — $W^*(t)$ may in fact have several local optima — for intermediate values of the discount factor.

For additional insight, we elaborate on the implications of Propositions 2 and 3 for the shape of the welfare functions with the help of Fig. 1 which plots $W^*(t)$ (solid- and dotted-line curves) and $V^*(t)$ (dashed- and dotted-line curves) under the optimal cartel agreements. Since $\delta' < \delta^c(0) < \delta'' < \underline{\delta}$, it was natural to consider discount factor values in intervals $(0, \delta')$, $(\delta', \delta^c(0))$, $(\delta^c(0), \delta'')$ and $(\delta'', \underline{\delta})$. Thus, the simulations in Fig. 1 supplement Propositions 2 and 3 by illustrating where global optima occur and by highlighting the behavior of welfare on the interior of the regions with trade (i.e., for $t \in [0, t_x(\delta))$, where precise analytic results are impossible.[10]

For $\delta \geq \underline{\delta}$, the monopoly profit level is sustainable for all t; thus welfare under tariffs and transport costs equals $W^M = V^M = 3A^2/8$.

For $\delta = 0.62 \in (\delta'', \underline{\delta})$, welfare at $t_x(0.62)$ exceeds welfare at $t = 0$ by Proposition 2. Moreover, as

[10] It should be noted that the critical values $t_x(\delta)$ and $t^G(\delta)$ are homogeneous of degree one in A, while the critical discount factors δ', $\delta^c(0)$, δ'' and $\underline{\delta}$ are homogeneous of degree zero in A. As a result, the shape of the welfare curves for the respective δ values illustrated in Fig. 1 are representative of those obtained for any choice of A, although the scaling of t will vary with the particular A value chosen.

shown in Proposition (3), welfare is increasing in t in the neighborhood of free trade as well as in the neighborhood of $t_x(0.62)$ for both types of trade costs; and, as predicted in Proposition 2, welfare is unambiguously decreasing in t for $t \in [t_x(\delta), t^G(\delta)]$. The simulations further show that for this value of δ both $W^*(t)$ and $V^*(t)$ are increasing on the entire interval $[0, t_x(\delta)]$; thus, as already anticipated, $t_x(\delta)$ is indeed the global maximum under both welfare measures.

For $\delta = 0.53 \in (\delta^c(0), \delta'')$ welfare under transport costs $V^*(t)$ continues to exhibit the same behavior as in the previous case. However, while the dependence of welfare under tariffs $W^*(t)$ is also similar to the previous case, now there is an important difference: $W^*(t)$ is non-monotonic and attains a global maximum at some $t \in (0, t_x(\delta))$, where intra-industry trade is present.

When $\delta = 0.45 \in (\delta', \delta^c(0))$, welfare under free trade is a local optimum for both measures by Proposition 3. Still, the free trade welfare level is less than the welfare level at $t_x(0.45)$ for both types of trade costs by Proposition 2, and $W^*(t)$ is decreasing (but $V^*(t)$ is increasing) in the neighborhood of $t_x(0.45)$ by Proposition 3. This unveils the non-monotonicity of both welfare measures on the interval $[0, t_x(\delta)]$ noted above. It also reveals how the nature of trade barriers matters with regards to the global optimum. In this case, the global optimum with tariffs requires some intra-industry trade but the global optimum with transport costs requires no trade at all.

For $\delta = 0.41 \in (0, \delta')$, welfare under both measures remains non-monotonic and behaves similarly to the previous case. There are several important differences though. The welfare level at $t = 0$ now exceeds the level at $t = t_x(0.41)$ and thus, in the transport cost case, the global optimum is at free trade. Interestingly, in the case of tariffs, there are still two local optima, but the global maximum continues to occur at some $t \in (0, t_x(\delta))$ where intra-industry is present.

Eventually, when the discount factor declines to sufficiently low levels, free trade emerges as the global optimum for both welfare measures and the associated level of welfare rises (not shown). In contrast, the welfare levels at $t = t_x(\delta)$ fall, converging to W^M as $\delta \to 0$.

Fig. 1 also highlights an interesting difference between the tariff and transport cases with regard to the effect of a change in δ. In the case of tariffs, welfare rises with decreases in the discount factor for a given tariff. The lower the discount factor, the greater the difficulty of sustaining collusion and thus the higher output levels and welfare. In contrast, no such general relationship seems to exist between welfare and time preferences in the case of transport costs. In fact, welfare falls below the level associated with maximal geographic specialization (i.e., below W^M) when transport costs are sufficiently high and the discount parameter is sufficiently low. This is so because $w^*(t) \to W^N(t)$ as $\delta \to 0$, which implies that along with the welfare-promoting increases in output come increases in the overall cost of trading.

It should be emphasized that the discount parameter here may reflect both the market discount on future profits, the time that elapses between violations of collusive agreements and their detection by cartel members, as well as the probability that the cartel relationship may be terminated due to other reasons — such as, for example, detection of collusive behavior by antitrust authorities. Although a complete model of antitrust enforcement is beyond the scope of this paper, the above logic suggests that stricter enforcement of cartels by antitrust authorities could be captured here with low levels of δ. This suggests that the welfare-maximizing level of trade costs will vary with the amount of resources that are devoted to antitrust enforcement.

C. Collusion with n (>1) *Firms per Country*

The model can be extended in a straightforward manner to the oligopoly case in which there are more than one firms in each country (i.e., $n > 1$). In this section we undertake this extension focusing on the pro-collusive effects of trade liberalization when trade costs are low, and on the protective effects of trade costs when these costs are high.

When there are more than one firms per country, maximal geographical collusion entails splitting equally the monopoly output (i.e., $A/2$) between the domestic firms, yielding $(q, x) = (A/(2n), 0)$. At $t = 0$, the monopoly profit can also be sustained through cross hauling that gives each firm the same output in every market so that $(q, x) = (A/(4n), A/(4n))$. However, because the deviation payoffs are strictly convex in outputs for $n > 1$, the latter agreement is easier to sustain. As a result, there is a discontinuity in the minimum discount factor at $t = 0$, just as in the case of $n = 1$.

Let $\delta^c(t,n)$ denote the minimum discount factor that is capable of sustaining the monopoly output level with n firms. As in the case of duopoly, there is a discontinuity in the minimum discount factors at $t = 0$. We show in the Appendix that the magnitude of the jump in the minimum discount factor at $t = 0$ is $\delta^c(0,n) - \lim_{t\to 0}\delta^c(t,n) = 4n(1+n)^2/[(1+12n+4n^2)(1+6n-18n^2+16n^3+8n^4)]$. It can be verified that this expression is decreasing in n and converges to 0 as $n\to\infty$; therefore, the effect of increasing the number of firms is to reduce the magnitude of this jump at $t = 0$. Since $\delta^c(t,n)$ is continuous in t for $t > 0$, it follows that for $\delta\in(\delta^c(0,n), \lim_{t\to 0}\delta^c(t,n))$, there will always exist an interval of trade costs $T(\delta,n)$ such that $\delta < \delta^c(t,n)$ for $t\in T(\delta,n)$; therefore, the monopoly profit will be sustainable at $t = 0$ but not for $t\in T(\delta,n)$. Furthermore, social welfare will fall if trade barriers take the form of tariffs. This establishes one sense in which trade liberalization is pro-collusive in the neighborhood of $t = 0$.

But there is another way to prove that trade liberalization will cause social welfare to fall and, as a consequence, trade liberalization to be pro-collusive for $\delta\in(\delta^c(0,n), \lim_{t\to 0}\delta^c(t,n))$ in the neighborhood of $t = 0$ for tariffs as well as transport costs. Utilizing the methodology employed in proving part (a.i) of Proposition 3 we can demonstrate that $\lim_{t\to 0^+} dQ^*/dt > 0$ and $\lim_{t\to 0^+} x^* = A[1-(2n-1)g(\delta)]/(4n)$ for $\delta\in(\delta^c(0,n), \lim_{t\to 0}\delta^c(t,n))$. Using these ideas in the definitions of welfare implies $\lim_{t\to 0^+} dW^*/dt > 0$ for tariffs and $\lim_{t\to 0^+} dV^*/dt > 0$ for transport costs.[11]

With regards to high trade costs, it was shown that for $n = 1$ and $t \geq A/2$ it is unprofitable for a firm to deviate in the export market when the local firm has been assigned the monopoly output. As a result, the monopoly output is sustainable for all $\delta \geq 0$ if $t \geq A/2$ and $n = 1$. With $n > 1$, deviations by exporting

[11] Additional details and the formal proof on these findings can be provided from the authors upon request.

firms will also be unprofitable when $t \geq A/2$ under maximal geographic collusion. However, since there is more than one local firm, maximal geographic collusion requires the discount factor to be high enough to deter deviations by local firms in the domestic market. Since in this case the minimum discount factor is $\delta^c(A/2,n) = (n+1)^2/(1+6n+n^2)$ we would need $\delta > \delta^c(A/2,n)$.[12] Moreover, since $\Pi^N(t) = A^2/(n+1)^2$ and $\Pi^A(A/(2n),0,t) = A^2/(4n)$ are constant, and $\Pi^D(A/(2n),0,t) = A^2(n+1)^2/(16n^2) + (A/2-t)^2/4$ is decreasing in t for $t \in [A/(n+1), A/2)$, it follows that $\delta^c(t,n)$ is decreasing in t. Thus, in this region, trade liberalization must be pro-competitive because its raises the appeal of deviations by exporting firms in the local market.

The precise dependence of collusive conduct, profits and welfare on trade costs when $t \in (0, \bar{t}^N)$ is considerably more complex. It can be shown that for given $n > 1$, $\delta^c(t,n)$, $\Pi^*(t)$, $W^*(t)$ and $V^*(t)$ are non-monotonic functions of trade costs. Still, as we've just seen, the qualitative features of the analysis in the duopoly case remain intact.

D. Collusion with More Severe Punishments

As has been emphasized by Abreu (1986) and others, a cartel could support more profitable agreements by choosing punishments that are more severe than reversion to the Nash equilibrium. In this section we illustrate how the results with Nash punishments can be generalized when we consider punishments that satisfy $\Pi^P(t) \leq \Pi^N(t)$ for the case of $n = 1$. As in the previous section, here we discuss the behavior of minimum discount factors in the neighborhood of $t = 0$ and $t = A/2$.

If $\Pi^P(t)$ is continuous in t, there will be a discontinuity in the minimum discount factor at $t = 0$ as in the case with Nash punishments. Since the results of Lemma 2 apply for $\Pi^P(t) \leq \Pi^N(t)$, efficient cartel agreements that sustain the monopoly profit with $t = 0$ will use $q = x = A/4$. For $t > 0$, the minimum discount factor for monopoly profit utilizes maximal geographical specialization with $(q, x) = (A/2, 0)$. It

[12] Note that the critical value of trade costs that eliminates deviation profits by exporters in export markets is $\bar{t}^D = A/2$ and it exceeds the value that prohibits trade in the Nash equilibrium, which is $\bar{t}^N = A/(n+1)$, for $n > 1$.

follows from the strict convexity of Π^D and its symmetry at $t = 0$ that $\Pi^D(A/4, A/4, 0) < \Pi^D(A/2, 0, 0)$, so $\delta^c(0) < \underline{\delta} \equiv \lim_{t \to 0} \delta^c(t)$. Thus, for $\delta \in (\delta^c(0), \underline{\delta})$, we may use an argument similar to that in the previous section to establish an interval $(0, \hat{t}(\delta))$ such that $\delta < \delta^c(t)$ for $t \in (0, \hat{t}(\delta))$.[13] By Lemma 2, we obtain $\Pi^*(t) < \Pi^M$ and $Q^*(t) > A/2$ for $t \in (0, \hat{t}(\delta))$. Starting from any t in this interval, the elimination of trade barriers will raise cartel profits. This establishes the pro-competitive effects of raising tariffs in the neighborhood of $t = 0$ with more severe punishments when the associated payoffs are continuous in t.

It was established in the case of Nash punishments that $\delta^c(t)$ is decreasing in t with $\lim_{t \to \bar{t}^N} \delta^c(t) = 0$. However, with more general punishments, $\delta^c(t)$ need not be monotonic in t for $t > 0$ without further restrictions on $\Pi^P(t)$. Still, the fact that $\Pi^D(A/2, 0, A/2) = \Pi^M$ is sufficient to ensure $\lim_{t \to \bar{t}^N} \delta^c(t) = 0$ when $\Pi^P(t) \leq \Pi^N(t)$.[14] Since $\delta^c(t)$ is continuous in t, this shows that increases in t must enhance collusion for values of t that are sufficiently high.

III. Multimarket Collusion in Duopoly with Imperfect Substitutes

We now extend the model to consider the case in which products are imperfect substitutes on the demand side. As before, there are two countries with one firm in each country and each firm having a zero marginal production cost. The inverse demand function of good i sold in market k is $p_{ik} = A - q_{ik} - \gamma q_{jk}$, where $j \neq i$ and $\gamma \in (0, 1)$ captures the degree of substitutability between any two goods. This demand function maintains the equal size of the two markets and imposes a symmetry between home and foreign firms by requiring an equal degree of substitutability between all products.

The following Lemma summarizes the properties of the Nash equilibrium of the one-shot game:

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[13] The continuity of $\delta^c(t)$ and the result that $\delta^c(t) \to 0$ as $t \to \bar{t}^N$ ensures the existence of a t such that $\delta^c(t) = \delta$. Choosing $\hat{t}(\delta)$ to be the smallest value of t satisfying this condition, it follows from the definition of the minimum discount factor that monopoly profits will be unsustainable for $t \in (0, \hat{t}(\delta))$.

[14] For $t > 0$, the monopoly profit can be sustained through maximal geographic specialization for $\delta \geq \delta^c(t) \equiv [\Pi^D(A/2, 0, t) - \Pi^M]/[\Pi^D(A/2, 0, t) - \Pi^P(t)]$. If $\Pi^P(\bar{t}^N) < \Pi^N(\bar{t}^N)$ $(= \Pi^M)$, then $\delta^c(\bar{t}^N) = 0$ follows from the fact that $\Pi^D(A/2, 0, \bar{t}^N) = \Pi^M$. If $\Pi^P(\bar{t}^N) = \Pi^N(\bar{t}^N)$, then $\lim_{t \to \bar{t}^N} \delta^c(t) = 0$ follows from the fact that $\Pi_t^D(A/2, 0, \bar{t}^N) = 0$ and $\Pi_t^P(\bar{t}^N) \geq \Pi_t^N(\bar{t}^N) > 0$.

Lemma 3: *In the Cournot duopoly model with product differentiation,* $\gamma \in (0, 1)$*, and symmetric trade barriers, the Nash equilibrium output, profit, and welfare levels can be summarized as follows:*

(a) *For* $t \leq \bar{t}^N \equiv A(1-\gamma/2)$

(i) *output levels are*: $q^N(t,\gamma) = \dfrac{A(2-\gamma) + \gamma t}{(4-\gamma^2)}, \quad x^N(t,\gamma) = \dfrac{A(2-\gamma) - t}{(4-\gamma^2)}$; (5)

(ii) *global profit,* $\Pi^N(t,\gamma)$*, is strictly convex in t and minimized at* $t^{\Pi}_{\min} = \dfrac{A(2-\gamma)^2}{4+\gamma^2} < \bar{t}^N$;

(iii) *welfare under tariffs,* $W^N(t,\gamma)$, *is strictly concave and decreasing in t*;

(iv) *welfare under transport costs,* $V^N(t,\gamma)$, *is strictly convex in t and is minimized at*

$$t^V_{\min} = \frac{A(2-\gamma)^2(3+\gamma)}{12 - \gamma^2} < \bar{t}^N.$$

(b) *For* $t \geq \bar{t}^N$*, the values of output, profits and welfare are the same as at* $\bar{t}^N$.

(c) $\Pi^N(0,\gamma) \lesseqgtr \Pi^N(\bar{t}^N,\gamma)$ as $\gamma \gtreqless 2/(\sqrt{2}+1)$ and

$W^N(0,\gamma) = V^N(0,\gamma) > V^N(\bar{t}^N,\gamma) = W^N(\bar{t}^N,\gamma)$.

The properties of the Nash equilibrium with imperfect substitutability are quite similar to those of the homogeneous goods case. In particular, the volume of trade is decreasing in *t* and welfare with tariffs (transport costs) is strictly concave (convex) in *t*. Moreover, welfare is maximized at free trade for either the tariff or transport cost case. The important difference is that trade itself is beneficial when goods are differentiated (i.e., when $\gamma < 1$). This is reflected in the fact that the profit level at free trade may exceed the level under a prohibitive tariff when goods are sufficiently dissimilar. When goods are distant substitutes, the gain in profits on domestic sales due to a prohibitive tariff is simply not as large as the loss of profits in the foreign market.

A. Efficient Cartel Agreements

In the absence of enforcement problems, the most profitable cartel with $t > 0$ does not necessarily require the elimination of trade for $\gamma \in [0,1)$. Due to product differentiation the cartel can earn higher profits by allowing some trade to take place as long as trade costs are not too high. Letting $\Pi^M(t) = \max_{q,x} \Pi^A(q,x,t)$, the following lemma summarizes the impact of trade barriers on output, profit, and welfare levels when the cartel is unconstrained in its maximization of global profits.

Lemma 4: *For $\gamma \in [0,1)$, the output, profits and welfare levels of an unconstrained cartel maximizing global profits are as follows:*

(a) *For* $t \leq \bar{t}^M \equiv A(1-\gamma)$

(i) *output levels are*: $q^M(t,\gamma) = \dfrac{A(1-\gamma) + \gamma t}{2(1-\gamma^2)}, \quad x^M(t,\gamma) = \dfrac{A(1-\gamma) - t}{2(1-\gamma^2)}$; (6)

(ii) *global profit,* $\Pi^M(t,\gamma)$, *is decreasing in t and* γ, *with* $\Pi^M_{tt} > 0$ *and* $\Pi^M_{\gamma\gamma} > 0$*;*

(iii) *welfare under tariffs,* $W^M(t,\gamma)$, *is decreasing in t and* γ, *and strictly concave in t;*

(iv) *welfare under transport costs,* $V^M(t,\gamma)$, *is decreasing in t and* γ, *and strictly convex in t;*

(b) *For* $t \geq \bar{t}^M$, *the values of output, profits and welfare are the same as at* $\bar{t}^M$.

It is also easy to show that the lower the degree of substitutability between goods, the more costly the elimination of trade to the cartel. Furthermore, the level of trade barriers at which trade is eliminated in the unconstrained efficient cartel, $\bar{t}^M$, is lower than the level at which trade is eliminated in the Nash equilibrium, $\bar{t}^N$; that is, $\bar{t}^M < \bar{t}^N$ for $\gamma \in (0,1)$.

As in the previous section, we can define $F(t, \delta, \gamma, \Pi^N(t))$ to be the set of $(q,x) \in \mathbb{R}^2_+$ pairs satisfying the no-deviation constraint (3) when punishment involves permanent reversion to the Nash equilibrium. The agreement payoff will be strictly convex in (q,x) for $t \geq 0$ when $\gamma \in [0,1)$, and the deviation payoff from an agreement, $\Pi^D(q,x,t)$, will be strictly convex in (q,x); thus, the set of sustainable agreements will be convex, and arguments similar to the ones used in Lemma 2 can establish that there is a unique agreement that maximizes cartel profits subject to the no-deviation constraint. The uniqueness of the efficient cartel agreement is strengthened for the case of $\gamma < 1$, since it also holds when the no-deviation constraint does not bind at $t = 0$ due to the uniqueness of $q^M(0,\gamma) = x^M(0,\gamma)$. The efficient agreements have the properties that $q^* > x^*$ if $t > 0$ and $q^* = x^* \geq q^M$ if $t = 0$. With imperfect substitutability and $t = 0$ there are two reasons for firms to equalize their market shares: deviation incentives are lower and payoffs are maximized when market shares are equal. For $t > 0$, the cartel will choose to allocate larger output shares to domestic firms in each market, as in the case with perfect substitutability.

B. Minimum Discount Factors and Trade Barriers

One way to illustrate the impact of trade costs on collusion is to evaluate their impact on the minimum discount factor associated with sustaining the most profitable cartel agreement. Let $\delta^c(t,\gamma)$ denote the minimum discount factor that is capable of supporting the efficient cartel agreement $\{q^M(t,\gamma), x^M(t,\gamma)\}$ defined in (6). The following result, which summarizes the impact of changes in t on the minimum discount factor for $\gamma \in (0,1)$, is proven in the Appendix.

Proposition 4: *When goods are imperfect substitutes (i.e., $\gamma \in [0,1)$), the minimum discount factor that supports the most collusive outcome, $\delta^c(t,\gamma)$, has the following properties:*

(a) $\delta^c(t,\gamma)$ *is increasing in t for* $t \in (0, \bar{t}^M)$.

(b) $\delta^c(t,\gamma)$ *is decreasing in t for* $t \in (\bar{t}^M, \bar{t}^N)$, *with* $\lim_{t \to \bar{t}^{N-}} \delta^c(t,\gamma) = 0$.

(c) $\delta^c(\bar{t}^M,\gamma) - \delta^c(0,\gamma)$ *is increasing in* γ *and* $\lim_{\gamma \to 1} \bar{t}^M = 0$ *for* $\gamma \in [0,1)$.

For $t > t^M$, the payoff under the most collusive outcome is independent of trade costs because $x^M = 0$. It then follows that the minimum discount factor will be decreasing in t iff the average payoff from a deviation, $(1-\delta)\Pi^D(q^M, x^M, t) + \delta\,\Pi^N(t)$, is decreasing in t when evaluated at $\delta^c(t,\gamma)$. Increases in t will reduce Π^D, since they render deviations into the export market less attractive, and will have an ambiguous effect on Π^N. In the Appendix we show that the former effect dominates; therefore, increases in t facilitate the sustainability of collusion in this region. For $t \in (0, \bar{t}^M)$, the payoff under the most collusive agreement is decreasing in t because $x^M > 0$. It is shown in the Appendix that this effect must dominate any potential negative effects on the average deviation payoffs, so that the minimum discount factor is increasing in t on this interval.

Fig. 2a illustrates the properties of $\delta^c(t,\gamma)$ noted in parts (a) and (b) of Proposition 4 for $\gamma = 0.98$. Part (c) of Proposition 4 reveals that as γ increases, the range of discount factors (trade barrier values) over which the minimum discount function is increasing in t becomes larger (smaller); thus, the pro-competitive effect of increasing t is strongest when goods are very close substitutes. The discontinuity in the minimum discount factor obtained when $\gamma = 1$ results from $\lim_{\gamma \to 1} [\delta^c(\bar{t}^M,\gamma) - \delta^c(0,\gamma)]/\bar{t}^M = \infty$. On the other hand, $\lim_{\gamma \to 0} [\delta^c(\bar{t}^M,\gamma) - \delta^c(0,\gamma)]/\bar{t}^M = 0$ and $\lim_{\gamma \to 0} (\bar{t}^N - \bar{t}^M) = 0$; therefore, the minimum discount factor function approaches a horizontal line when the degree of substitutability between goods vanishes.

Fig. 2a also reveals that the interval of discount parameters can be divided into three regions — as in Proposition 1 where $\gamma = 1$. For high values of the discount factor (i.e., $\delta > \delta^c(\bar{t}^M,\gamma)$), $\Pi^M(t)$ can be sustained for all t. For intermediate values of the discount factor (i.e., $\delta \in (\delta^c(0,\gamma), \delta^c(\bar{t}^M,\gamma))$), $\Pi^M(t)$ can be sustained only for $t \in [0, t^E(\delta,\gamma)]$ and for $t \geq t^G(\delta,\gamma)$, where $t^E(\delta,\gamma)$ ($t^G(\delta,\gamma)$) denotes the value of the trade barrier associated with the positively (negatively) sloped section of the $\delta^c(t,\gamma)$ locus at δ. For sufficiently low values of the discount factor (i.e., $\delta < \delta^c(0,\gamma)$), $\Pi^M(t)$ is sustainable only if trade costs that are sufficiently high (i.e., $t > t^G(\delta,\gamma)$ so that maximal geographic collusion is sustainable).

For (δ, t) values where the no-deviation constraint binds and $\Pi^*(t) < \Pi^M(t)$, it can be shown that there will exist a critical trade cost $t_x(\delta,\gamma) \in [\bar{t}^M(\gamma), t^G(\delta,\gamma)]$ (with equality if δ equals $\delta^c(\bar{t}^M,\gamma)$ or 0) such that $x^*(\delta,\gamma) = 0$ and $q^*(\delta,\gamma) > A/2$ for $t \in [t_x(\delta,\gamma), t^G(\delta,\gamma)]$, with $\partial q^*/\partial t < 0$ on this interval. The optimal cartel policy will involve (constrained efficient) intra-industry trade (i) if $\delta < \delta^c(0,\gamma)$ and $t \in [0, t_x(\delta,\gamma))$, and (ii) if $\delta \in (\delta^c(0,\gamma), \delta^c(\bar{t}^M,\gamma))$ and $t \in (t^E(\delta,\gamma), t^G(\delta,\gamma))$. These results on trade patterns parallel those of Proposition 1 for the case of homogeneous goods.

C. Welfare and Trade Costs

We now turn to the impact of trade costs on welfare. In the case of perfect substitutability, it was shown in Proposition 2 that there is a range of discount factors for which the level of welfare under free trade $(t = 0)$ is lower than the level associated with the smallest value of the trade cost that eliminates trade $(t = t_x(\delta))$. This finding provides an average sense in which trade liberalization has a pro-collusive effect over the interval $[0, t_x(\delta)]$, and is due to the fact that the only value of intra-industry trade with identical (homogeneous) goods is that it relaxes the no-deviation constraint. When goods are imperfect substitutes, however, the elimination of trade has a social cost as well.

The following result, which is proven in the Appendix, extends the analysis of Proposition 2 to the case of γ < 1.

Proposition 5: *Some welfare comparisons for imperfect substitutes.*

(a) *If* $\delta \in \lim_{\gamma \to 1} (\delta^c(0,\gamma), \delta^c(\bar{t}^M,\gamma))$ *there will exist a* $\gamma^0(\delta) \in (0,1)$ *such that* $W^*(0,\gamma) < W^*(t_x(\delta,\gamma),\gamma)$ *for all* $\gamma \in (\gamma^0(\delta), 1)$.

(b) *There exists a* $\gamma^1(\delta) \in (0.5, 1)$ *such that* $W^*(0,\gamma) > W^*(t_x(\delta,\gamma),\gamma)$ *for all* $\delta \in (0, \delta^c(\bar{t}^M,\gamma))$ *if* $\gamma < \gamma^1(\delta)$.

Part (a) shows that the result of Proposition 2, which identified a range of discount factors for which the free trade welfare level was below the value at which trade was eliminated, will continue to hold if γ is sufficiently close to 1. This is established by showing that national welfare under the cartel agreement is continuous in γ. This ensures that, if $W^*(t_x(\delta,\gamma),\gamma) - W^*(0,\gamma) > 0$ at $\gamma = 1$, there will exist an $\varepsilon > 0$

such that the inequality holds at $\gamma = 1 - \varepsilon$. Part (b) is obtained by showing that if γ is sufficiently different from 1, the cost of eliminating trade is sufficiently high that the welfare level in any sustainable solution with no trade must be less than the unconstrained monopoly level at $t = 0$.

Proposition 3 established a local result that trade liberalization is welfare-reducing at $t = 0$ when $\gamma = 1$ and $\delta \in \lim_{\gamma \to 1} (\delta^c(0,\gamma), \delta^c(\bar{t}^M,\gamma))$. In this interval, the monopoly output was sustainable at $t = 0$ but not for $t > 0$. A similar result at $t = 0$ for $\gamma \in (0,1)$ and $\delta \in (\delta^c(0,\gamma), \delta^c(\bar{t}^M,\gamma))$ cannot be obtained because the efficient cartel agreements are sustainable in the neighborhood of $t = 0$ as illustrated in Proposition 4 and Fig. 2a. Since the efficient cartel is sustainable in a neighborhood of $t = 0$, profits and welfare must both be decreasing in t at $t = 0$ by Lemma 4. However, increases in t can be welfare-increasing when t rises sufficiently that the no-deviation constraint binds. This is illustrated in Fig. 2b, which shows the relationship between trade costs and welfare with $\delta = 0.55$ and $\delta = 0.61$. For $t \in (0, t^E(0.55, 0.98))$ and $\gamma = 0.98$, the no-deviation constraint is slack in both cases so $W^*(t,0.55) = W^*(t,0.61) > V^*(t,0.55) = \square^*(t,0.61)$. Welfare is lower in the case with transport costs because the efficient cartel involves positive levels of trade. Similarly, welfare coincides in the two cases for $t \geq t^G(0.55,0.98)$ where maximal geographic collusion is sustainable.

When trade costs exceed the critical value $t^E(0.55,0.98)$, welfare is increasing in t because increases in t tighten the no-deviation constraint and force the cartel to produce higher output levels. This increase in welfare is monotonic to the point at which trade is eliminated if trade barriers takes the form of transport costs. When trade costs take the form of tariffs, the welfare optimum occurs at some $t < t_x(0.55,0.98)$ — with intra-industry being present — at the low discount factor $(\delta = 0.55)$. In contrast, at the higher discount factor $(\delta = 0.61)$ welfare is maximized at $t_x(0.61,0.98)$, in the absence of intra-industry trade.

The analysis of the case of imperfect substitutability highlights the robustness of the result that trade liberalization will be pro-collusive when trade costs are low. If the pro-collusive effect is measured in terms of a positive relationship between t and the minimum discount factor, the pro-collusive effect holds for $\delta \in (\delta^c(0,\gamma), \delta^c(\bar{t}^M,\gamma))$ for all γ. If the pro-collusive effect is measured as a positive relationship between t and the level of social welfare, then this result will hold for γ sufficiently close to 1.

IV. Conclusion

This paper has generated a collection of new results on the relationship between trade costs, collusive behavior and welfare when firms are engaged in multimarket collusion. In summarizing our findings, we will emphasize several conclusions that we feel are robust to assumptions regarding the number of firms, degree of product substitutability, and type of punishment. The first result has to do with trade patterns. Contrary to conventional wisdom, we demonstrate that multimarket collusion does not necessarily result in the elimination of cross hauling of goods. Because the deviation payoffs of cartel members are convex in output assignments, cartel agreements that assign similar shares in domestic and export markets are easier to sustain when trade costs are not very high. In other words, multimarket collusion can be the driving force behind trade in identical products.

The second result is that trade liberalization can be pro-collusive when trade costs are already low. This is so essentially because deviation incentives are lower in agreements that involve cross hauling. One implication of the preference for cross hauling, which held for all of the cases that we examined, is that the minimum discount factor for sustaining the monopoly profit level at $t = 0$ is less than the minimum discount factor associated with a range of positive trade costs. For the case of a duopoly with homogeneous goods, we identified a range of discount factors for which this implied that welfare at the free trade welfare level would be lower than at the minimum level of trade costs at which the cartel prefers to eliminate trade. (This result remained valid in the case of imperfect substitutes when the degree of substitutability between goods is sufficiently high.) This result is consistent with the observation that international cartels continue to persist despite recent decreases in trade and transport costs. It is also relevant to the analysis of cartels within a country, where transport costs may result in the creation of regional market segmentation.

A third robust conclusion is that trade liberalization will be pro-competitive in regions where trade barriers are high so that the cartel chooses to eliminate trade. In the neighborhood of a (sufficiently high) threshold level of trade barriers that eliminate trade, there exists a region of values within which trade liberalization raises deviation incentives in export markets and thus increase the difficulty of sustaining collusion. Interestingly, in this case, reductions in trade barriers lead to higher output levels and lower cartel profits, even though the reduction in trade barriers may not result in positive trade flows. This result also holds in the case of a duopoly with imperfect substitutability, as well as in the case with perfect substitutability with more than one firm per country.

In the case where there is more than one firm per country, collusive agreements have intra-national and international elements (i.e., collusion may involve firms within and across countries). Our analysis of this case focused on two extreme cases: the neighborhood of unimpeded trade (where $t = 0$ and thus domestic and foreign firms face identical cost functions), and when trade barriers are sufficiently high so that foreign firms are excluded from the domestic market. The detailed analysis of intermediate cases of trade costs (where trade costs create asymmetries between foreign and domestic firms but are not large enough to eliminate trade) remains an area for future research.

Our analysis has taken the location of production facilities as exogenously given, so firms must necessarily export to serve foreign markets. An interesting and important extension is to allow for the possibility that firms may choose to build a plant in a foreign country. The observation that cartel deviation incentives are smallest when firms have symmetric costs in a market suggest that firms might have an incentive to locate plants in each market in order to facilitate collusion.

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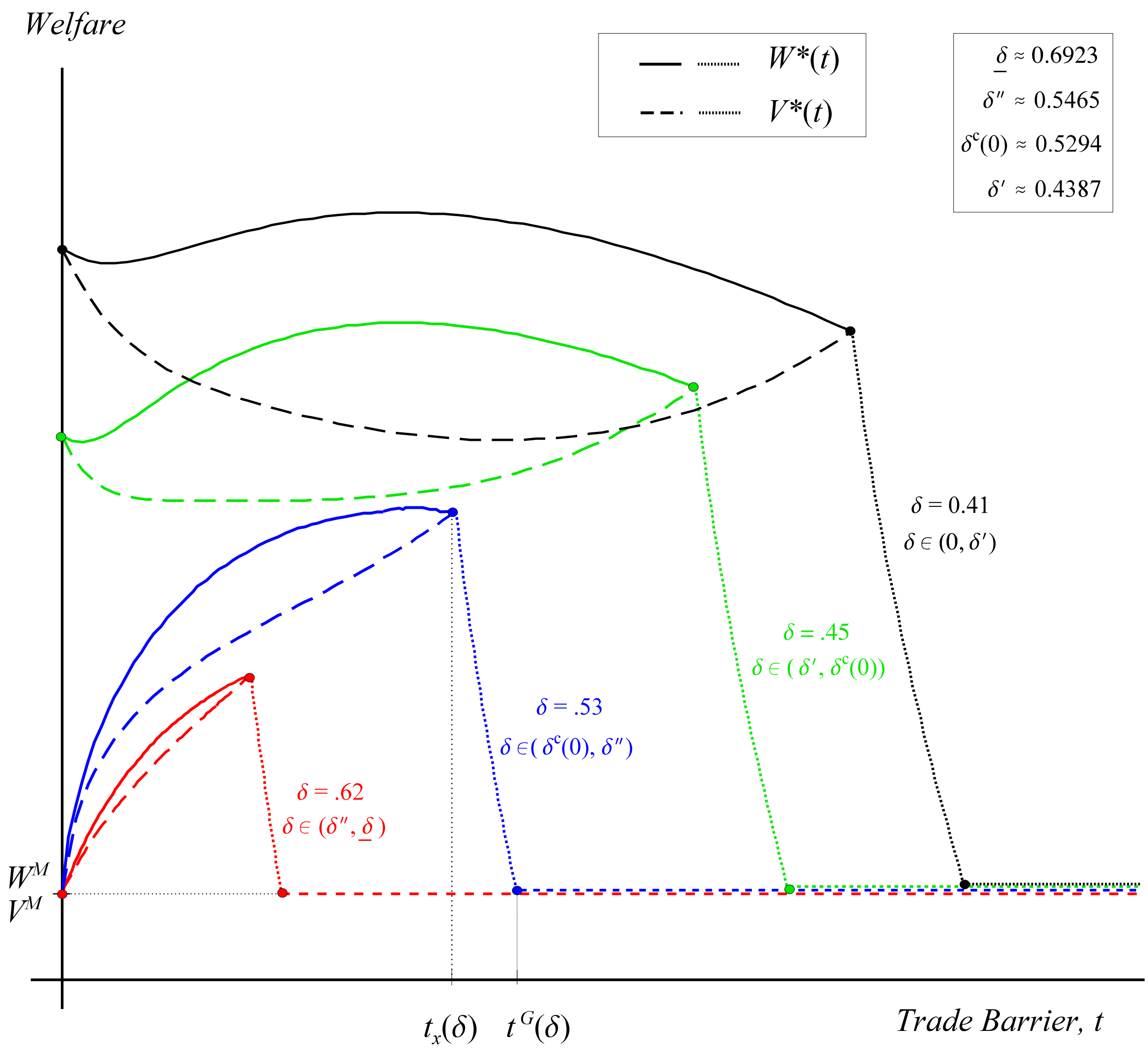

Figure 1: The Effect of Trade Costs on Welfare for Various Discount Factors

(The case of tariffs is shown with the solid-line curves and the case of transport costs with dashed-line curves when the two differ)

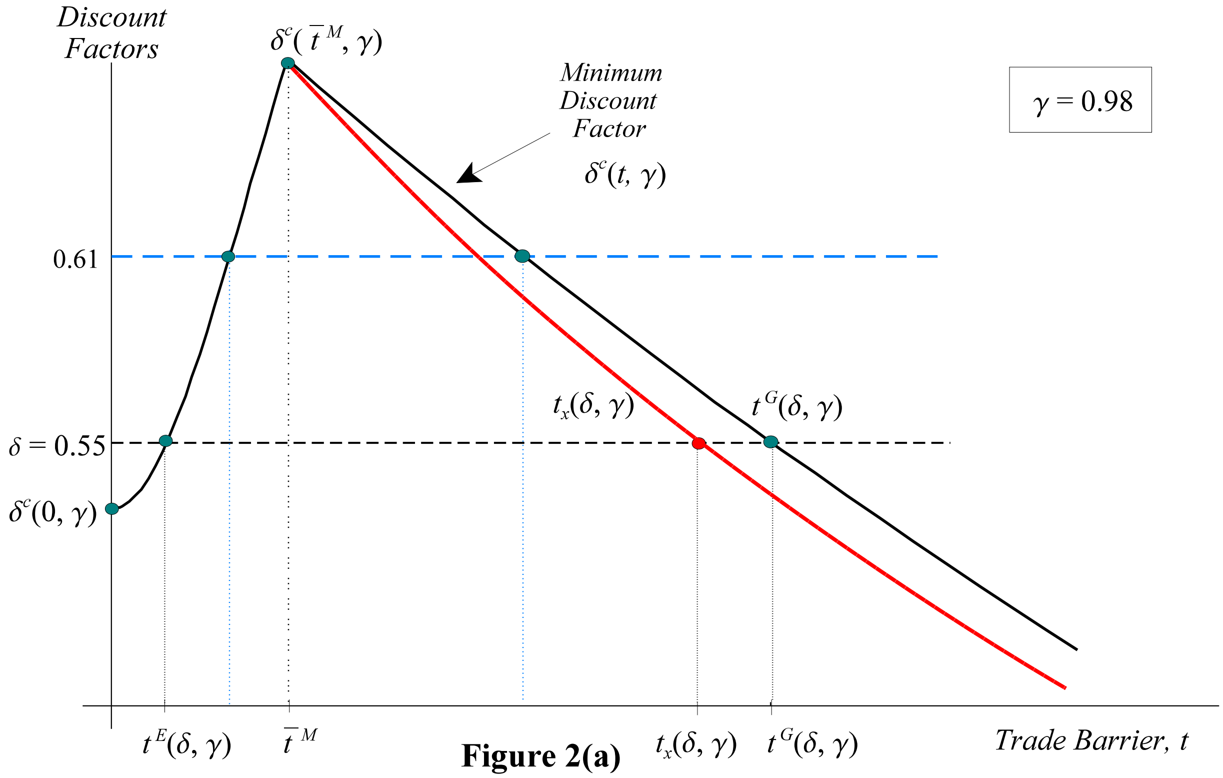

Figure 2(a)

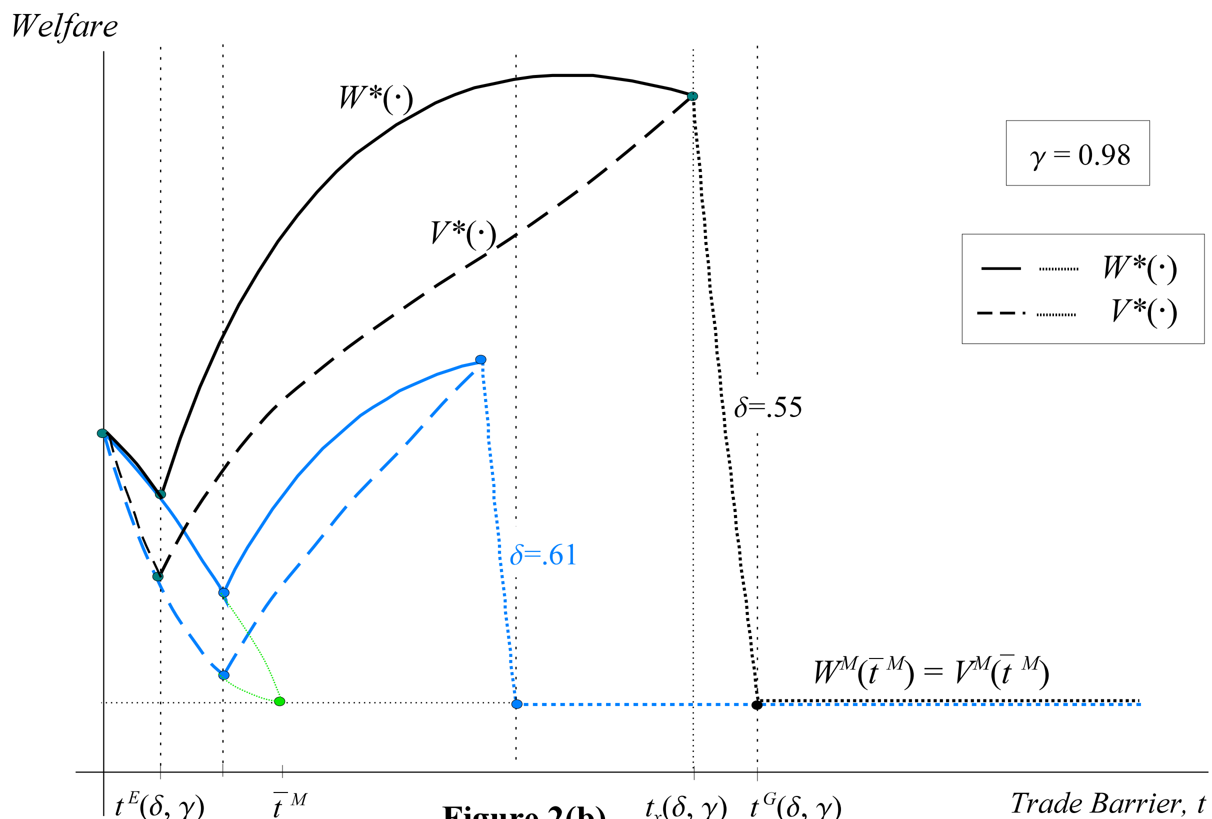

Figure 2(b)

The Dependence of Welfare and Minimum Discount Factors on Trade Barriers (Differentiated Goods)

Appendix

Proof of Lemmas 1 and 3: We derive the Nash equilibrium for the most general case, which utilizes the inverse demand function $p_{ik} = A - q_{ik} - \gamma \sum_{j \neq i} q_{jk}$ for firm *i* in market *k*, and where there are $n \geq 1$ firms in each country with $\gamma \in [0, 1]$. Let $\pi(y, Y, c) = (A - y - \gamma Y - c)y$ be the profit of a representative firm that has a marginal cost *c* and supplies output *y* when all other firms supply *Y*. The best response function of this firm is $\tilde{y}(Y, c) = (A - \gamma Y - c)/2$ and its maximum profit is $\tilde{\pi}(Y, c) = \pi(\tilde{y}, Y, c) = \tilde{y}(Y, c)^2$. The Nash equilibrium is then obtained by solving $\tilde{y}((n-1)q + nx, 0) - q = 0$ and $\tilde{y}(nq + (n-1)x, t) - x = 0$ together with the requirement that $x \geq 0$. Lemma 1 is derived using $n = 1$, while Lemma 3 is derived using $n = 1$ and $\gamma \in [0, 1)$. The discussion in Section II.C utilizes the solution with $n > 1$ and $\gamma = 1$. ||

Proof of Lemma 2: We establish this result for the general case of a duopoly with $\gamma \in [0, 1]$. It is straightforward to show from the definitions of Π^A and Π^D that $Z(q, x, t, \delta, \Pi^P(t))$ is concave in (q,x) so the feasible set will be convex in (q,x). Since Π^A is a concave function being maximized over a convex constraint set, the first-order conditions of the Lagrangian will be necessary and sufficient for an agreement to be optimal if the feasible set contains an interior point satisfying $Z(q, x, t, \delta, \Pi^P(t)) > 0$ (Takayama (1993), Theorems 2.1 - 2.2).

To show existence of an interior point, note that $Z(q^N(t), x^N(t), t, \delta, \Pi^P(t)) \geq 0$, with strict equality if $\Pi^P = \Pi^N$. Differentiation of *Z* yields $z_q(q^N(t), x^N(t), t, \delta, \Pi^P(t)) = -\delta\gamma x^N(t)$ and $Z_x(q^N(t), x^N(t), t, \delta, \Pi^P(t)) = -\delta\gamma q^N(t)$. It then follows by the continuity of *Z* that for $\delta > 0$ and $t < \bar{t}^N$ there exists an $\varepsilon > 0$ such that $Z(q^N - \varepsilon, x^N - \varepsilon, t, \delta, \Pi^P) > 0$.

The necessary conditions for a maximum of the Lagrangian at an interior solution can be written as

$$\Pi_j^A = \theta \Pi_j^D \quad \text{for} \quad j \in \{q, x\}, \quad \theta \equiv \frac{\lambda(1 - \delta)}{1 + \lambda} \in [0, 1) \tag{A.1}$$

where $\theta > 0$ is just a transformation of the Lagrange multiplier, λ. For $\theta > 0$ these expressions can be solved to obtain

$$q^*(\theta) = \frac{1}{2}\left[z^* + \frac{t(2-\theta\gamma)}{4(1-\gamma)+\theta\gamma^2}\right], \quad x^*(\theta) = \frac{1}{2}\left[z^* - \frac{t(2-\theta\gamma)}{4(1-\gamma)+\theta\gamma^2}\right], \quad with \quad z^* \equiv \frac{(2+\theta\gamma)(2A-t)}{4+\gamma(4+\theta\gamma)} \quad \text{(A.2)}$$

The condition for $\mathbf{x}^* \geq 0$ is satisfied as long as

$$t \leq t_x(\theta) = \frac{A[2+\theta\gamma][4(1-\gamma)+\theta\gamma^2]}{4(2-\gamma^2\theta)}. \quad \text{(A.3)}$$

Equations (A.2) - (A.3) describe an interior solution at which (A.1) holds with equality for $i = q, x$. If $\mathbf{t} > t_x(\theta)$, (A.2) cannot be satisfied with $\mathbf{x}^* \geq 0$. The optimal cartel will then involve a corner solution, which is characterized by

$$q^*(\theta,t) = \frac{A(2+\theta\gamma) - t\theta\gamma}{4+\gamma^2\theta}, \qquad x^*(\theta,t) = 0 \qquad for \quad t > t_x(\theta). \quad \text{(A.4)}$$

At a corner solution, (A.1) holds with strict equality for $j = q$ and $\Pi_x^A \leq \theta\Pi_x^D$.

Equations (A.2) and (A.4) can be used to establish (a) and (b) of Lemma 2 for the case of $\gamma = 1$. For $t > 0$, Π^A is strictly quasi-concave and F is a convex set, so the solution must be unique. The result that $t > 0$ (= 0) implies $q^* > (=)\ x^*$ at an interior solution follows immediately from (A.2). A corner solution with $q^* = 0$ cannot arise for $t > 0$ because it would lead to a violation of $\mathbf{x}^* \geq 0$ from (A.2).

To establish part (c), note that for $\theta > 0$ the necessary condition for the choice of $Q = q+x$ with $\gamma = 1$ requires $A - 2Q^* < 0$. If the punishment payoff is absolutely continuous, then equation (4) in part (d) follows immediately from Corollary 5 of Theorem 5 in Milgrom and Segal (2002).

If $\gamma \in [0,1)$, the cartel quantity is unique for all $t \geq 0$ because Π^A is strictly quasi-concave even at $t = 0$. The result that $t > 0$ (= 0) implies $q^* > (=)\ x^*$ continues to hold, as does the result that a corner solution with $q^* = 0$ cannot arise. ||

Proof of Proposition 1: The discussion in the text showed that the monopoly profit is sustainable for all t if $\delta \in (\underline{\delta}, 1)$ and for $t \geq t^G(\delta)$ if $\delta \in (0, \underline{\delta})$, which established (a), (b.i) and (c.i). Here we derive the optimal production pattern for $\left\{(\delta, t) \mid \delta < \underline{\delta} \textit{ and } t < t^G(\delta)\right\}$, where the no deviation constraint will bind and cartel profits will be below the monopoly level.

We first establish that for $\delta \in (0, \underline{\delta})$, there exists a $t_x(\delta) \in (0, t^G(\delta))$ such that $x^* = 0$ for $t \geq t_x(\delta)$ and $x^* > 0$ for $t \in [0, t_x(\delta))$. Letting $\Delta(q,x,t) \equiv \Pi_q^A(q,x,t)\Pi_x^D(q,x,t) - \Pi_q^D(q,x,t)\Pi_x^A(q,x,t)$, a corner solution with output $(q, 0)$ will be optimal if $Z(q, 0, t, \delta, \Pi^N(t)) = 0$ and $\Delta(q, 0, t) \leq 0$. Let $\{q_x(\delta), t_x(\delta)\}$ be the values at which these two conditions hold with equality. It is convenient to solve for these values by using (A.3) to define $t_x(\theta)$, and then to derive $q_x(\theta)$ and $\delta_x(\theta)$ as solutions to $\Delta(q, 0, t_x(\theta)) = 0$ and $Z(q, 0, t_x(\theta), \delta, \Pi^N(t_x(\theta))) = 0$. This yields

$$q_x(\theta) = \frac{A(4 - \theta^2)}{2(4 - \theta)} \qquad \textit{and} \qquad \delta_x(\theta) = \frac{9(1-\theta)(4+\theta^2)}{52 + 4\theta + 29\theta^2 + 5\theta^3}. \tag{A.5}$$

The function $\delta_x(\theta)$ is decreasing in θ and maps [0, 1] onto $[0, \underline{\delta}]$. Letting $h \equiv \delta_x^{-1}$, we obtain the result that for each $\delta \in (0, \underline{\delta})$, there will exist unique values $t_x(h(\delta))$ and $q_x(h(\delta))$ at which there is a corner optimum with $\Delta(q_x(\delta), 0, t_x(\delta)) = 0$. It is clear from (A.5) that $q_x(\delta) > A/2$ for $\theta \in (0, 1)$, and it follows from (A.3), (A.5) and the definition of $t^G(\delta)$ that $t^G(\delta) - t_x(\delta) = \dfrac{A\theta(1-\theta)(16 + 12\theta + 20\theta^2 - 3\theta^3)}{2(4-\theta)(6-\theta)(8 - 2\theta + 3\theta^3)} > 0$ for $\theta \in (0, 1)$ (or, equivalently, for $\delta \in (0, \underline{\delta})$).

We now show that a corner solution will be optimal for $t \in (t_x(\delta), t^G(\delta)]$ and it cannot be optimal for $t \in [0, t_x(\delta))$. This is shown by, first, establishing the existence of a q satisfying $Z(q, 0, t, \delta, \Pi^N(t)) = 0$ and $\Delta(q, 0, t) \leq 0$ for all t on the former interval and, second, by showing that no q can satisfy these conditions on the latter interval. First, we note that $Z(q, 0, t, \delta, \Pi^N(t))$ is strictly concave in (q, t), with $Z(A/2, 0, t^G(\delta), \delta, \Pi^N(t^G(\delta))) = 0$ and $Z(A/2, 0, A/2, \delta, \Pi^N(A/2)) = 0$. The $Z = 0$ locus is illustrated in Fig.

A.1, which makes use of the fact that the slope at $t^G(\delta)$ is $-(9+11\delta)/(18(1-\delta))<0$, and the slope approaches $-\infty$ as $t \to A/2$ for $\delta < 1$. Now, the $\Delta(q,0,t) = 0$ locus can be described by the function $q^0(t) = \left[A - 3t + (A^2 + 10At + t^2)^{1/2}\right]/4$, which is a strictly concave function for $t \in [0, A/2]$, with $q(0) = q(A/2) = A/2$. Since $\Delta_q(q,0,t) > 0$, we will have $\Delta(q,0,t) < 0$ below the $q^0(t)$ locus. The $Z = 0$ and $\Delta = 0$ loci will have two intersections, one at the pair $\{q_x(\delta), t_x(\delta)\}$ defined by (A.3) and (A.5) and one at $\{A/2, A/2\}$. The former intersection will occur in the negatively sloped portion of the Z = 0 locus since its slope at $t_x(\delta)$ is $-\dfrac{(4-\theta^2)(12-2\theta+5\theta^2)}{(1-\theta)(4+\theta)(4+5\theta^2)} < 0$ for $\theta \in (0,1)$. It then follows from the above properties that $\Delta(q,0,t) < 0$ along the $Z = 0$ locus for $t \in (t_x(\delta), t^G(\delta)]$; therefore, $x^* = 0$ in this region. This also ensures that there can exist no q satisfying $\Delta(q,0,t) \leq 0$ and $Z = 0$ for $t \in [0, t_x(\delta))$, so $x^* > 0$ for $t \in [0, t_x(\delta))$. Finally, $\partial q^*/\partial t = -Z_t(q,0,t,\delta,\Pi^N(t))/Z_q(q,0,t,\delta,\Pi^N(t)) < 0$ for $t \in (t_x(\delta), t^G(\delta)]$ follows from the above and the fact that $Z(q, 0, t, \delta, \Pi^N(t))$ is strictly concave in (q, t). ||

Extension of Proposition 1 to $\gamma \in (0,1)$: The above line of argument can be used to establish the following result, which extends Proposition 1(b) to the case of $\gamma \in (0,1)$.

Suppose $\delta \in (0, \delta^c(\bar{t}^M, \gamma))$, *where* $\delta^c(t,\gamma)$ *is derived in Proposition 5. Then,*

(a) *there exists a* $t^G(\delta,\gamma)$ *such that* $q^* = A/2$ *and* $x^* = 0$ *for* $t \geq t^G(\delta,\gamma)$.

(b) *there exists a* $t_x(\delta,\gamma) < t^G(\delta,\gamma)$ *such that* $q^* > 0$ *and* $x^* = 0$ *for* $t \in [t_x(\delta,\gamma), t^G(\delta,\gamma))$ *and* $\partial q^*/\partial t < 0$. *For* $t < t_x(\delta,\gamma)$, $q^*, x^* > 0$.

Proof of Proposition 2: Parts (a) and (b) follow immediately from parts (a) and (b) of Proposition 1 and the discussion in the text. The comparison of welfare and profit between $t = 0$ and $t = t^G(\delta)$ follows from the discussion in the text. Similarly, $W^*(t)$ and $V^*(t)$ coincide for the equilibria with specialization for $t \in [t_x(\delta), t^G(\delta)]$, so $W_t^* = (A - q^*)(\partial q^*/\partial t) < 0$ and $\Pi_t^*(t) = (A - 2q^*)(\partial q^*/\partial t) > 0$ on this interval by Proposition 1(b).

For $\delta < \delta^c(0)$, the no-deviation constraint will bind at $t = 0$. By Lemma 2(b), the efficient cartel agreement will satisfy $Z(q, q, 0, \delta, \Pi^N(0)) = 0$ which yields

$$q^*(0,\delta) = x^*(0,\delta) = \frac{A}{3}\left(\frac{9-5\delta}{9-\delta}\right) \quad for \quad \delta \le \delta^c(0). \tag{A.6}$$

Now $W^*(0) - W^*(t_x(\delta))$ > (<) 0 if $\phi(\delta) \equiv Q^*(0,\delta) - Q^*(t_x(\delta),\delta)$ > (<) 0 or, utilizing (A.5) and recalling the properties of $\delta_x(\theta)$, if $\phi(\theta) \equiv 2q^*(0,\delta_x(\theta)) - q_x(\theta) \equiv \frac{A(2+\theta)^2(-8+28\theta-14\theta^2+9\theta^3)}{6(4-\theta)(24+4\theta+14\theta^2+3\theta^3)}$ > (<) 0. After some straightforward (but tedious) algebra, $\phi(\theta)$ can be shown to be increasing in θ. In addition, it can be verified that $\phi(h(\delta^c(0))) = -\frac{A\theta(1-\theta)}{2(4-\theta)} < 0$ and $\phi(1) = A/6 > 0$. It follows that there will exist a $\theta' \in (h(\delta^c(0)), 1)$ such that $\phi(\theta') = 0$. Defining $\delta' \equiv h(\theta')$, it then follows from the properties of $\phi(\theta)$ and $\delta_x(\theta)$ that $\delta' < \delta^c(0)$; therefore, $W^*(0) - W^*(t_x(\delta))$ > (<) 0 if δ < (>) δ'. Solving $\phi(\theta') = 0$ gives $\theta' \approx 0.3282$ which implies $\delta' \approx 0.4387$. ||

Proof of Proposition 3: An optimum satisfying (A.1), $q^* > 0$, $x^* \ge 0$, and a binding incentive constraint requires $\Delta(q^*, x^*, t) = 0$ and $Z(q^*, x^*, t, \delta, \Pi^N(t)) = 0$. Totally differentiating these conditions gives

$$[-A + 4(Q^* - x^*) + 3t]dQ^* + [2A - 4Q^* - t]dx^* = [2A - 3Q^* + x^* - 2t]dt \tag{A.7a}$$

$$[A - 2Q^* + \frac{1}{2}(1-\delta)(A - Q^* + x^* - t)]dQ^* - [t - \frac{1}{2}(1-\delta)(Q^* - 2x^* + t)]dx^* =$$

$$[x^* - \frac{2\delta}{9}(A - 5t) - \frac{1}{2}(1-\delta)(A - Q^* + x^* - t)]dt \tag{A.7b}$$

Proposition 3 can be established by solving (A.7) and evaluating at the optimal cartel values to obtain

$$\frac{\partial W^*}{\partial t} = (A - Q^*)\frac{\partial Q^*}{\partial t} \qquad and \qquad \frac{\partial V^*}{\partial t} = \frac{\partial W^*}{\partial t} - t\frac{\partial x^*}{\partial t} - x^*. \tag{A.8}$$

Part (a.i): For $t = 0$ and $\delta \in (\delta^c(0), \underline{\delta})$, Proposition 1(c) established that the optimal cartel policy is any pair

$\{q^*, x^*\}$ satisfying $Q^* = q^* + x^* = A/2$ and $x^* \in [0, A(1-g(\delta))/4]$. Following Lemma 2 and Proposition 1, we evaluate (A.7) using $\lim_{t \to 0^+} x^* = A(1-g(\delta))/4$, where $g(\delta) \equiv \left[\frac{\delta - \delta^c(0)}{\delta^c(0)(1-\delta)}\right]^{1/2} \in (0,1)$ from Proposition 1(c). Substituting this output level in (A.7) and solving for dQ^* and dx^* gives

$$\lim_{t \to 0^+} \frac{\partial Q^*}{\partial t} = \frac{3 - g(\delta)}{4g(\delta)} > 0 \quad \textit{for} \quad \delta \in (\delta^c(0), \underline{\delta}).$$

Clearly, in the case of tariffs, $\lim_{t \to 0^+} \frac{\partial W^*}{\partial t} = \frac{A}{2} (\lim_{t \to 0^+} \frac{\partial Q^*}{\partial t}) > 0$. For transport costs, we have

$$\lim_{t \to 0^+} \frac{\partial V^*}{\partial t} = \lim_{t \to 0^+} \frac{\partial W^*}{\partial t} - \lim_{t \to 0^+} x^* = \frac{A}{8} \left[\frac{3(1-g(\delta)) + 2g(\delta)^2}{g(\delta)}\right] > 0.$$

Part (a.ii): For $t = 0$ and $\delta \in (0, \delta^c(0))$, the output levels are unique and given by (A.6). Utilizing these values in (A.7)-(A.8) gives $\frac{\partial Q^*}{\partial t} = -\frac{9 - 5\delta}{3(9 - \delta)} < 0$ and hence $\frac{\partial W^*(0)}{\partial t} < 0$. Since, in the case of transport costs, $\frac{\partial V^*(0)}{\partial t} = \frac{\partial W^*(0)}{\partial t} - x^*(0,\delta)$ will also have $\frac{\partial V^*(0)}{\partial t} < 0$.

Part (b): As shown in the proof of Proposition 1, (A.3) and (A.5) define the values $t_x(\theta)$, $q_x(\theta)$ and $\delta(\theta)$ satisfying the no deviation constraint and the tangency condition at a corner solution with $x^* = 0$ for any $\theta \in (0,1)$. Substituting these values in (A.7)-(A.8) and solving yields the effect of reductions in t on Q^* and x^* at $t_x(\theta)$.

$$\frac{\partial Q^*(t_x(\theta))}{\partial t^-} = \frac{(2+\theta)k(\theta)}{(4+\theta)(4+5\theta^2)(16+36\theta-12\theta^2+5\theta^3)} \tag{A.9a}$$

$$\frac{\partial x^*(t_x(\theta))}{\partial t^-} = -\frac{2(4-\theta)(112 + 64\theta + 48\theta^2 - 16\theta^3 + 17\theta^4)}{(4+\theta)(4+5\theta^2)(16+36\theta-12\theta^2+5\theta^3)} \tag{A.9b}$$

where $k(\theta) \equiv 64 - 400\theta + 400\theta^2 - 376\theta^3 + 112\theta^4 - 25\theta^5$. Clearly, $\textit{sign}\,(\partial Q^*/\partial t^-) = \textit{sign}\, k(\theta)$ whereas $\partial x^*/\partial t^- < 0$.

(i) Since $\mathbf{sign}\,(\partial W^*/\partial t^-) = sign\,(\partial Q^*/\partial t^-) = sign\,k(\theta)$ for $t = t_x(\theta)$, we can establish the result by determining the sign of $k(\theta)$ for $\theta \in (0, 1)$. Since $k(\theta)$ is continuous with $k(0) = 64$ and $k(1) = -225$, $\mathbf{k}(\theta)$ must have at least one zero on [0, 1]. The equation $k(\theta) = 0$ can be solved to show that it has a single real root $\theta'' = 0.190003$, so $k(\theta) > (<)\ 0$ if $\theta < (>)\ \theta''$. Defining $\delta'' \equiv \delta_x(\theta'')$ from (A.5), and noting that $\delta'' \approx 0.5465 > \delta^c(0) \approx 0.5294$, it then follows that $\partial W^*/\partial t^- > (<)\ 0$ if $\delta > (<)\ \delta''$ for $t = t_x(\theta)$.

(ii) Utilizing (A.9) in the welfare decompositions in (A.8) it can be shown that

$$\frac{\partial V^*(t_x(\theta))}{\partial t^-} = \frac{(4-\theta^2)J(\theta)}{2(4-\theta)(4+\theta)(4+5\theta^2)(16+36\theta-12\theta^2+5\theta^3)}$$

where $J(\theta) = 128 - 352\theta + 1200\theta^2 - 624\theta^3 + 376\theta^4 - 78\theta^5 + 25\theta^5$. It is now straightforward to verify that $J(\theta)$ is minimized at some $\theta_{\min} \in (0, 1)$ and $J(\theta_{\min}) > 0$; therefore, $\partial V^*/\partial t^- > 0$ for $t = t_x(\theta)$.

(iii) Noting that $\Pi^*_{t^-} = \mathfrak{L}^*_{t^-}$ and utilizing (A.1), (A.3) and (A.4) yields

$$\frac{\partial \Pi^*(t_x(\theta))}{\partial t^-} = -\lambda\left[\delta\frac{\partial \Pi^N}{\partial t^-} + (1-\delta)\frac{\partial \Pi^D}{\partial t^-}\right] = \frac{A(4-\theta^2)(1-\theta)(12-2\theta+5\theta^2)}{(4-\theta)(52+4\theta+29\theta^2+5\theta^3)} > 0. \qquad \|$$

Analysis of the *n*-firm case (Section IIC): With n firms per country, the relevant payoff functions for the multimarket cartel problem are

$$\Pi^A(q, x, t) = [A - n(q+x)](q+x) - tx\,;$$

$$\Pi^D(q, x, t) = \frac{\{[A - ((n-1)q + nx)]^2 + [A - (nq + (n-1)x) - t]^2\}}{4} \quad for\ t \le \frac{A}{2}$$

$$\Pi^N(t) = \frac{2A(A-t) + (1+2n+2n^2)t^2}{(1+2n)^2} \quad for\ t \le \frac{A}{n+1};\quad \Pi^N(t) = \left(\frac{A}{1+n}\right)^2 \quad for\ t \ge \frac{A}{n+1}.$$

Since Π^A is concave in (q,x) and Π^D is strictly convex in (q,x), the feasible set will be convex and the solution to the cartel problem will have features similar to those obtained in Lemma 2. For $t > 0$, the minimum discount factor for sustaining monopoly profits is obtained using maximal geographic collusion, which yields $\delta^c(t,n) = [\Pi^D(A/(2n),0,t) - \Pi^A(A/(2n),0,t)]/\Pi^D(A/(2n),0,t) - \Pi^N(t)]$. Evaluating at $t = 0$ yields $\delta^c(t,n) = (1+2n-2n^2+8n^4)/(1+6n-18n^2+16n^3+8n^4)$. The minimum discount factor for sustaining cross hauling in the absence of trade barriers is $\delta^c(0,n) = [\Pi^D(A/(2n),0,0) - \Pi^A(A/(2n),0,0)]/\Pi^D(A/(2n),0,0) - \Pi^N(0)] = (1+2n)^2/(1+12n+4n^2)$. Taking the difference between these and evaluating at $t = 0$ yields the result in the text.

For $t \in [A/(n+1), A/2)$, the Nash payoff is independent of t. Since Π^D is decreasing in t on this interval, $\delta^c(t,n)$ will be decreasing in t as well.

Proof of Proposition 4: The minimum discount factor to sustain the most collusive output can be expressed as $\delta^c(t,\gamma) = [\Omega(t) - 1]/[\Omega(t) - \Psi(t)]$, where $\Omega(t) \equiv \Pi^D(q^M, x^M, t)/\Pi^M(t)$ and $\Psi(t) \equiv \Pi^N(t)/\Pi^M(t)$ (<1). This yields $sign\, \delta^c_t(t,\gamma) = sign\, [(1-\Psi)\Omega_t + (\Omega-1)\Psi_t]$. There are two possibilities: $t \in [0,\bar{t}^M)$ and $t \in (\bar{t}^M, \bar{t}^N)$. For $t \in [0,\bar{t}^M)$, $\Omega_t = \dfrac{A(2A-t)\gamma^3 t}{32(1-\gamma^2)^2\Pi^M(t)^2} \geq 0$ and $\Psi_t = -\dfrac{A(2A-t)\gamma^2 t}{(1-\gamma^2)(4-\gamma^2)^2\Pi^M(t)^2} \leq 0$; therefore, $(1-\Psi)\Omega_t + (\Omega-1)\Psi_t = \dfrac{A(2A-t)\gamma^7 t}{32(1-\gamma^2)^2(4-\gamma^2)^2\Pi^M(t)^2} \geq 0$ which implies $\delta^c_t(t,\gamma) \geq 0$. For $t \in (\bar{t}^M, \bar{t}^N)$, $\Pi^M(t) = \Pi^M(\bar{t}^M) = A^2/2$; therefore, $\Omega_t = -[A(2-\gamma) - 2t]/A^2 \leq 0$ and $\Psi_t = -8[A(2-\gamma)^2 - (4+\gamma^2)t]/[A(4-\gamma^2)]^2$. But Ψ_t is positive iff $t > A(2-\gamma)^2/(4+\gamma^2) = t^{\Pi}_{\min}$, the value at which profit in the Nash equilibrium is minimized (Lemma 3). Utilizing these expressions we obtain $(1-\Psi)\Omega_t + (\Omega-1)\Psi_t = -4\gamma(\bar{t}^N - t)^2/[A^3(4-\gamma^2)] < 0$. These results establish part (a) and (b).

To establish (c), we first obtain $\delta^c(\bar{t}^M,\gamma) = \dfrac{(4-\gamma^2)^2}{(4-\gamma^2)^2 + 4(4-3\gamma^2)}$ and $\delta^c(0,\gamma) = \dfrac{(2+\gamma)^2}{(2+\gamma)^2 + 4(1+\gamma)}$. It can be verified that $\delta^c_\gamma(0,\gamma) > 0$ and $\delta^c_\gamma(\bar{t}^M,\gamma) > 0$. Now note that

$$\rho(\gamma) \equiv \frac{\delta^c(\bar{t}^M,\gamma)}{\delta^c(0,\gamma)} = \frac{(2-\gamma)^2[(2+\gamma)^2 + 4(1+\gamma)]}{(4-\gamma^2)^2 + 4(4-3\gamma^2)} \quad \text{where} \quad \frac{d\rho(\gamma)}{d\gamma} = \frac{4\gamma^2(4-\gamma^2)(24+\gamma^2)}{[(4-\gamma^2)^2 + 4(4-3\gamma^2)]^2} > 0$$

for $\gamma \in [0,1]$. Moreover, $\rho(0) = 1$ and $\rho(\bar{t}^M) = 17/13 > 1$; therefore, $\rho(\gamma) \geq 1$ for $\gamma \in (0,1]$. From the above, it follows that $\delta^c_\gamma(\bar{t}^M, \gamma) - \delta^c_\gamma(0, \gamma) > \delta^c_\gamma(\bar{t}^M, \gamma) - \rho(\gamma)\delta^c_\gamma(0, \gamma)$. But the expression in the right-hand side is positive because it is the condition that ensures $\rho'(\gamma) > 0$. This proves part (c). ||

Proof of Proposition 5: Part (a): The constraint set is a continuous correspondence in γ. It then follows from Berge's maximum theorem that $\Pi^*(t, \delta, \gamma)$ is continuous in γ and the optimal cartel outputs $q^*(t, \delta, \gamma)$ and $x^*(t, \delta, \gamma)$ are upper hemi-continuous. We know that there will be unique solutions to this problem if $t > 0$ and $\gamma \in [0,1]$ or if $t = 0$ and $\gamma \in [0,1)$, so we have the stronger result that the output functions will be continuous in γ on these intervals. Since welfare is continuous in γ, q, and x, welfare will also be a continuous function on these intervals. For $t = 0$ and $\gamma = 1$, X^* will be a correspondence for $\delta \in (\delta^c(0), \underline{\delta})$ by Lemma 2(bii). However, these output levels will be welfare equivalent so welfare is continuous in γ for $\gamma \in [0,1]$ for all $t \geq 0$.

Part (b): We first solve for the output level with geographic specialization at which social welfare is equal to that at the maximum profit cartel outputs at free trade, which is the value q at which $W(q^M(0), x^M(0)) = W(q, 0)$. The solution is denoted $q^a(\gamma) = A[1 - (2\gamma^2 + \gamma - 1)^{1/2}/(2^{1/2}(1+\gamma))]$, which is decreasing for $\gamma \in [0.5, 1]$ with $q^a(.5) = A$ and $q^a(1) = A/2$. For $\gamma < 0.5$, the goods are sufficiently poor substitutes that the free trade welfare level cannot be sustained with specialization even when outputs are at the competitive (i.e., welfare maximizing) level. Using (A.2) and (A.3), we can solve for the maximum sustainable level of output at a corner solution for $\theta \in (0,1)$ to be $q^b(\gamma) = 2A[(2 - (4 - \gamma^2)^{1/2}]/\gamma^2$, which is increasing on (0,1) with $\lim_{\gamma \to 0} q^b(\gamma) = 1/2$ and $q^b(1) = 2(2 - 3^{1/2})$.

At $t = 0$, we have $W^*(0) > W^M$, with strict equality for $\delta > \delta(\bar{t}^M, \gamma)$. It then follows that a sufficient condition for $W^*(0) > W^*(t_x(\delta, \gamma))$ is that $q^a(\gamma) > q^b(\gamma)$. Using the properties of $q^a(\gamma)$ and $q^b(\gamma)$ derived above, it follows that there will exist a $\gamma^0 \in (.5, 1)$ such that $W^*(0) > W^*(t_x(\delta, \gamma))$ for $\gamma < \gamma^0$. ||

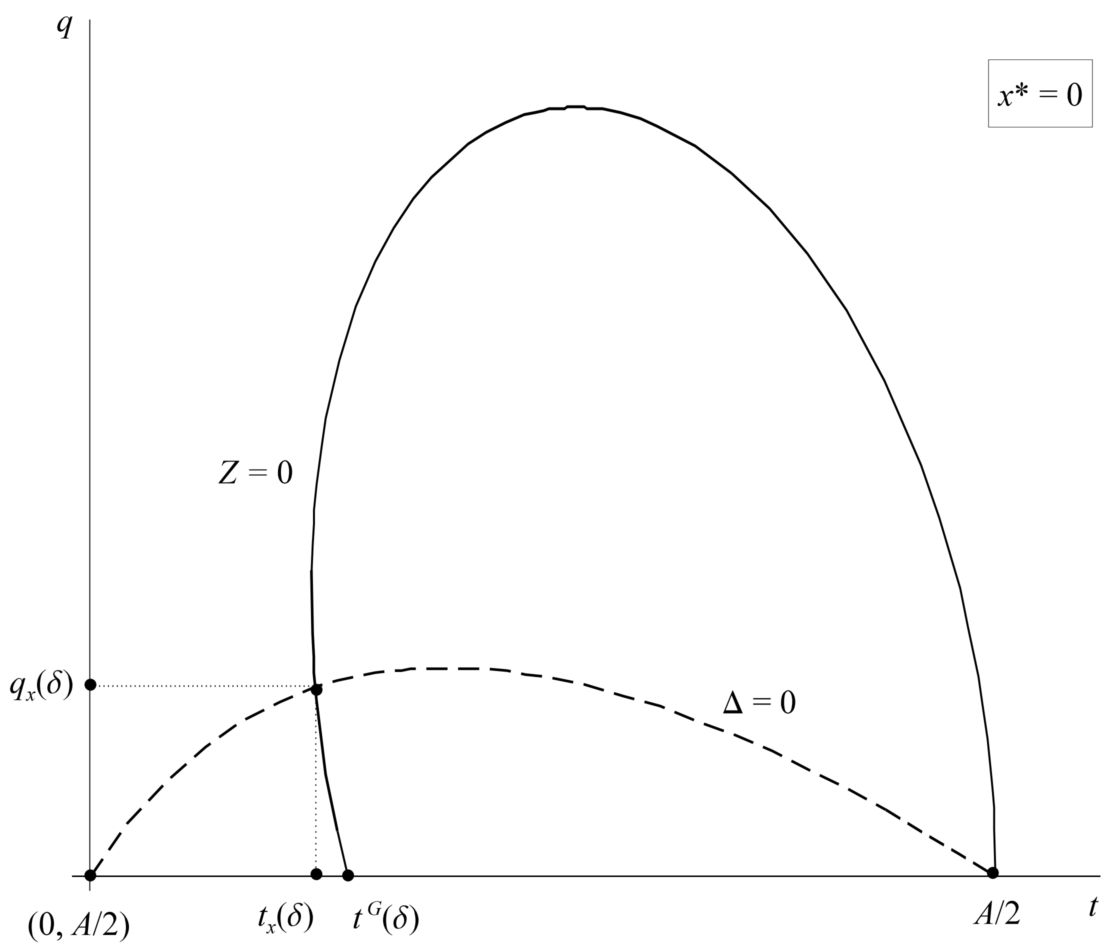

Figure A.1: Locus of (q, t) pairs consistent with a corner optimum ($Z(q, 0, t) = 0$ and $\Delta(q, 0, t) \leq 0$)