Section 3 of [1] deals with the case $s > i$. The claimed results are correct. But Proposition 4 is inaccurately stated and its proof contains an error. The proof supposed that because the second derivative of π is positive wherever it exists, the function has no interior maximum. But the interior maximum might be at a point where the derivative does not exist. (For example, consider the minimum of two linear functions, one increasing and the other one decreasing).

In this supplement we prove the claimed result: the profit-maximizing policy either has continuous sales with $p = w$, or else has sales only at the ordering points $0, T, 2T, \ldots$.

In the following observations we consider a policy of discrete sales, with sales at τ_0 and $\tau > \tau_0$, and exactly one other sales point in this interval. We investigate the effect of choosing the time for the internal sales point. Without loss of generality, we assume that $\tau_0 = 0$, since the difference is a constant independent of the decision which we consider.

Recall that in any no-sales interval (t_1, t_2), a proportion $\beta = \frac{s}{i+s}$ of the arrivals in this interval prefer to buy the good at t_1, and a proportion $1-\beta$ prefer to buy at t_2. By assumption, $s > i$ and therefore $\beta > 1/2$.

Observation *If $\tau < T$ then the costs associated with selling at $x \in (0, \tau)$ or at $\tau - x$ are identical.*
Proof: Consider selling at x. The quantity sold at time x is $\lambda[(1-\beta)x + \beta(\tau - x)]$; the quantity sold at time τ is $\lambda(\tau - x)(1-\beta)$. The inventory holding cost is

$$\begin{aligned} C(x) &= \lambda h\{x[(1-\beta)x + \beta(\tau - x)] + [\tau(\tau - x)(1-\beta)]\} \\ &= \lambda h[(x^2 + (\tau - x)^2)(1-\beta) + x(\tau - x)]. \end{aligned}$$

This expression is symmetric in x and $\tau - x$. ■

Observation *Suppose that $T = \tau$. Selling at $x < \tau - x$ is preferred to selling at $\tau - x$.*
Proof: The cost associated with selling at x is $\lambda h\{x[(1-\beta)x + \beta(\tau - x)]$. The cost associated with selling at $\tau - x$ is higher, namely $\lambda h\{(\tau - x)[(1-\beta)(\tau - x) + \beta x]$. ■

Observation *Suppose that the length of a no-sales intervals cannot exceed Δ, and that $\Delta < \tau < 2\Delta$. Then the firm minimizes its costs by selling at time $x = \tau - \Delta$.*
Proof: Since the no-sales interval is bounded by Δ, the unique sales point $x \in (0, \tau)$ satisfies $x \geq \tau - \Delta$. The cost associated with selling at $x = \tau - \Delta$

is $C(x) = \lambda h\{(x^2 + \Delta^2)(1-\beta) + x\Delta\}$. Suppose instead the firms sells at time $x + \epsilon$, where $0 < \epsilon < \Delta - x$. Then

$$\begin{aligned} C(x+\epsilon) &= \lambda h\{[(x+\epsilon)^2 + (\Delta-\epsilon)^2](1-\beta) + (x+\epsilon)(\Delta-\epsilon)\} \\ &= \lambda h\{[\Delta^2 + x^2 - 2\epsilon(\Delta - x) + 2\epsilon^2](1-\beta) + x\Delta - \epsilon^2 + \epsilon(\Delta-\epsilon)\} \\ &= C(x) + \lambda h\{\epsilon(\Delta - x)[1 - 2(1-\beta)] + \epsilon^2[2(1-\beta) - 1]\} \\ &= C(x) + \lambda h\{[1-2(1-\beta)]\epsilon[\Delta - x - \epsilon]\} \ > C(x). \end{aligned}$$

The inequality applies because $0 < \epsilon < \Delta - x$ and $\beta > 1/2$. ■

Recall that the price p is determined by the longest no-sales interval.

Corollary *Suppose that T and the maximum length of a no-sales interval Δ are fixed, and $T = (k+\alpha)\Delta$ where $k \in \{1, 2, \ldots\}$ and $0 \le \alpha < 1$. Then, the firm maximizes profits by selling at $0, \Delta, 2\Delta, \ldots, (k-1)\Delta$, and $(k-1+\alpha)\Delta$.*
Proof: By the first observation, the order of no-sales intervals, except for the last one, has no effect on the firm's costs. Combined with the second observation, this implies that the last interval is the longest, or of length Δ. By the third observation, at most one interval is strictly shorter than Δ. Using again the first observation, we assume that this is the interval before the last. ■

From this point we analyze policies that satisfy the properties given in the corollary.

At the end of each of the first $k-2$ no-sale intervals the firm sells a quantity $\lambda\Delta$. Of this quantity, $(1-\beta)\Delta$ is sold to customers who desired the good at an earlier time, and $\beta\Delta$ to customers who buy it earlier than their most desired time of purchase. Similarly, at $t = (k-1)\Delta$ the firm sells $\Delta(1-\beta+\alpha\beta)$, and at $t = (k-1+\alpha)\Delta$ the firm sells $\Delta(\alpha(1-\beta)+\beta)$. The total inventory holding cost is therefore

$$\begin{aligned} C_I &= \Delta^2[(1+2+\cdots+k-2) + (k-1)(1-\beta+\alpha\beta) + (k-1+\alpha)(\alpha - \alpha\beta + \beta)] \\ &= \Delta^2[(1+2+\cdots+k-1) + (k-1)\alpha + \alpha^2 + \alpha\beta(1-\alpha)] \\ &\ge \Delta^2\left[\frac{k(k-1)}{2} + (k-1)\alpha + \alpha^2 + \frac{\alpha(1-\alpha)}{2}\right] \\ &= 0.5\Delta^2(k+\alpha)(k+\alpha-1). \end{aligned}$$

The inequality holds because $s > 1/2$.

Since $\alpha < 1$, by Corollary 1 of [1] $p = w - \Delta \frac{is}{i+s}$, and the profit rate is

$$\pi = \lambda \left(w - \Delta \frac{is}{i+s} \right) - \frac{0.5\lambda h \Delta^2 (k+\alpha)(k+\alpha-1) + A}{(k+\alpha)\Delta} \equiv \lambda w - C.$$

Keeping α and k at constant levels while optimizing Δ we obtain that

$$C \geq 2\sqrt{\left(\frac{is}{i+s} + \frac{h(k+\alpha-1)}{2} \right) \frac{\lambda A}{k+\alpha}}.$$

Lemma *If* $h\left(\frac{1}{i} + \frac{1}{s}\right) \leq 2$ *then* $C \geq C_{EOQ}$.

Proof: Recall from (12) in [1] that $C_{EOQ} = \sqrt{2\lambda h A}$. Hence, it suffices to prove that $2\left(\frac{is}{i+s} + \frac{h(k+\alpha-1)}{2}\right)\frac{1}{k+\alpha} \geq h$, or equivalently, that $\frac{2}{k+\alpha}\frac{is}{i+s} \geq h\left(1 - \frac{k+\alpha-1}{k+\alpha}\right)$. The last inequality follows from the assumption of the lemma. ∎

Lemma *If* $h\left(\frac{1}{i} + \frac{1}{s}\right) \geq 2$ *then* $C \geq C_{0,T,\ldots}$, *where* $C_{0,T,\ldots}$ *is the minimum cost of a policy that sells only at the beginning of the cycle.*

Proof: Recall from (10) that $C_{0,T,\ldots} = 2\sqrt{\frac{\lambda A i s}{i+s}}$. Hence, it suffices to prove that $\left(\frac{is}{i+s} + \frac{h(k+\alpha-1)}{2}\right)\frac{1}{k+\alpha} \geq \frac{is}{i+s}$, or equivalently, $\frac{h(k+\alpha-1)}{2(k+\alpha)} \geq \frac{is}{i+s}\left(1 - \frac{1}{k+\alpha}\right)$. The last inequality follows from the assumption of the lemma. ∎

References

[1] A. Glazer and R. Hassin, "A deterministic single-item inventory model with seller holding cost and buyer holding and shortage costs," *Operations Research* **34** (1986) 613-618.