EXPECTATION SHOCKS AND LEARNING AS DRIVERS OF THE BUSINESS CYCLE

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Abstract. Psychological factors, market sentiments, and less-than-fully-rational shifts in beliefs are widely believed to play a role in the economy. Yet, they are rarely considered in macroeconomic models.

This paper evaluates the empirical role of expectational shocks on business cycle fluctuations. The paper relaxes the rational expectations assumption to exploit survey data on expectations in the estimation of a New Keynesian model, which allows for learning by economic agents. Expectation shocks affect the formation of expectations and capture waves of optimism and pessimism that lead agents to form forecasts that deviate from those implied by their learning model.

The empirical results uncover a crucial role for these novel expectations shocks as a major driving force of the U.S. business cycle. Expectation shocks regarding future real activity are the main source of economic fluctuations, accounting for roughly half of business cycle fluctuations.

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JEL classification: E31, E32, E52, E58.

Macroeconomists have been seeking for a long time to identify the causes of economic fluctuations. Empirical work has not reached definitive conclusions, but many researchers would agree that a variety of technology shocks, demand shock, monetary and fiscal policy shocks, in varying percentages, are responsible for the bulk of the business cycle.

In the past, however, economists also emphasised the importance of less conventional explanations for cyclical fluctuations. Psychological variables, in particular, were thought to play a crucial role in causing and amplifying business cycles. Keynes (1936), for example, attributed cycles to the action of "animal spirits", while Pigou (1927) discussed how business people's "errors of undue optimism or undue pessimism in their business forecasts" created fluctuations in industrial activity.

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Similar explanations, however, are rarely at the centre of the current generation of macroeconomic models. Their omission likely arises from the pervasive difficulty in measuring expectational or psychological shifts from observed realisations of macroeconomic variables.[1]

New Keynesian models, which are often used to characterise the interaction between macroeconomic variables and monetary policy, share this limitation, as they are similarly based on the idea that fluctuations are driven by exogenous structural shocks to technology, households' preferences, firms' mark-ups, and to policies. Yet, disturbances related to the formation of expectations, waves of optimism and pessimism, periods of generalised exuberance or gloom, which may be unrelated to fundamentals, may contribute in non-trivial part to economic fluctuations and, in such case, they should be taken into consideration in the formulation of monetary policy.

The main contribution of this paper is to propose a way to re-introduce these psychological elements in a monetary business cycle model, with the objective of investigating their contribution to economic activity. More specifically, the paper provides an attempt to evaluate the empirical importance of expectational shocks – which may be interpreted as exogenous changes in the private sector's degree of optimism or pessimism – as a source of aggregate economic fluctuations. These shocks affect the formation of expectations and cause changes in expectations that are unrelated to observed fundamentals, by making private economic agents more optimistic, or pessimistic, about the future state of the economy than it would be justified if they simply formed expectations from their perceived model of the economy and with beliefs derived from historical data.

The paper, therefore, relaxes the conventional assumption of rational expectations. To capture expectational swings, the paper exploits time series data on observed expectations, along with real-time data, to estimate a baseline New Keynesian model, which departs from the previous literature by including a potential role for psychological forces. The observed expectations are assumed to be formed from a near-rational expectations formation mechanism. Economic agents adopt a perceived model of the economy that has a similar structural form to the rational expectations solution of the system. Agents, however, do not know the reduced-form coefficients of the solution, but they can observe historical data on variables as output, inflation, and interest rates. Therefore, they exploit historical series to attempt to learn the reduced-form coefficients over time through constant-gain learning. They form expectations in each period from their perceived model, using the most recently updated parameter estimates and the data available in real-time.

The model with learning is found to be a good approximation of the expectations formation from the survey. The model, however, allows economic agents to depart from the numeric forecasts implied by the learning model. Private sector agents in some periods may be overly optimistic –

[1]Partial exceptions are the literatures on sunspots and news shocks, which will be discussed later in this section.

by forecasting a higher future output or lower inflation rate, for example, than implied by their learning model – or overly pessimistic. These waves of over-optimism and over-pessimism, which are exogenous to the state of the economy, are defined as the expectation shocks in the model. Different specific expectation shocks affect the formation of output, inflation, or policy expectations.

Survey data on expectations are exploited to extract both the best-fitting evolution of agents' beliefs and the expectation shocks over the sample. Both the agents' learning process and the properties of expectational shifts are thus not imposed a priori, but they are estimated from time series data on expectations, and from the dynamic interaction between expectations and realised variables within the structural model.

Preview of the Results. The empirical results reveal a large role for expectational shocks. These "optimism" and "pessimism" shocks, in particular related to future expectations about economic activity, are found to be a major source of business cycle fluctuations. Expectation shocks explain roughly half of business cycle movements, while the structural demand, supply, and policy shocks that have been typically considered in the literature explain the remaining half.

Fundamental demand shocks also have a large effect on output in the short run, but in a model that incorporates observed expectations and learning, they are far less persistent than found in previous literature. The adjustment of the economy after demand shocks is much faster than commonly implied by monetary DSGE models. The output gap response peaks only after few months after the shock and quickly vanishes to zero. Expectation shocks, on the other hand, cause a substantially more persistent adjustment. The effect on output is larger, delayed, and more long-lived than the corresponding effect provoked by structural demand shocks.

Fluctuations in inflation are also mostly driven by expectational shocks related to future real activity and future inflationary pressures.

Related Literature. The paper contributes to the literature on the role of learning and expectations in macroeconomics. While a large portion of the adaptive learning literature studies the convergence properties of systems with learning to the rational expectations equilibrium (e.g., Evans and Honkapohja, 1999, 2001), this paper is more directly related to the recent works that demonstrate the empirical relevance of learning in the economy during its transitional phase (e.g., Milani, 2007a, Eusepi and Preston, 2008). The learning specification used in this paper is particularly related to the one used in Bullard et al. (2008), who incorporate what that they define as "judgement" in the agents' forecasting model.

A methodological contribution of the paper consists in the use of data on expectations to estimate a general equilibrium model with learning by economic agents. Milani (2007a) estimates models with learning, but using only realised macroeconomic variables, and not data on forecasts.[2] This paper is, instead, more closely related to recent work by Ormeno (2009), who exploits the information in observed inflation forecasts to estimate models under learning, and by Del Negro and Eusepi (2009), who use it to judge whether state-of-the-art DSGE models under rational expectations or learning about the target fit inflation expectations from surveys.[3] The focus in these papers is different, as they don't introduce expectation shocks, which are a major focus here. In addition, this paper uses real-time data, as well as data on expectations about all the endogenous variables that enter the New Keynesian model.

More generally, within the extensive literature on the main sources of the economic fluctuations, the paper is particularly related to the studies that advance more "behavioural" theories for the business cycle and that assign a central role to expectations. Among those, a substantial literature in macroeconomics highlights the importance of self-fulfilling fluctuations driven by sunspots or "animal spirits".[4] The model presented in this paper, however, can generate self-fulfilling fluctuations without requiring the existence of sunspots and multiple equilibria (i.e., without requiring a failure of the Taylor principle in a New Keynesian model). The paper's focus on economic fluctuations driven by shifts in expectations has also points in common with the "news" shocks literature, although the transmission mechanisms and interpretations of the shocks clearly differ.[5]

1. Model

I assume a baseline New Keynesian model as a description of the behaviour of macroeconomic variables as output, inflation, and interest rates (e.g., Woodford, 2003, Galí, 2008):

$$
\begin{aligned}
y_t &= \widehat{E}_{t-1}\left[y_{t+1} - \sigma\left(i_t - \pi_{t+1} - r_t^n\right)\right] && (1.1)\\
\pi_t &= \widehat{E}_{t-1}\left[\beta\pi_{t+1} + \kappa y_t + u_t\right] && (1.2)\\
i_t &= \rho i_{t-1} + (1-\rho)[r_t^n + \chi_\pi \pi_{t-1} + \chi_y y_{t-1}] + \varepsilon_t. && (1.3)
\end{aligned}
$$

Equation (1.1) is the log-linearised Euler equation that arises from the households' optimal choice of consumption. Current output gap, denoted by y_t, depends on expectations about future output gap in $t+1$, and on the deviation of the real interest rate from the natural rate r_t^n. The coefficient σ denotes the elasticity of intertemporal substitution of private expenditures. The natural rate r_t^n

[2]Other papers (e.g., Milani, 2006, 2007b, 2008, 2009,a,b, Slobodyan and Wouters, 2009) estimated models with learning in a similar fashion.

[3]Carboni and Ellison (2009) also use available forecasts from the Greenbook to study and explain Federal Reserve policy during the Great Inflation.

[4]E.g., Azariadis (1981), Cass and Shell (1983), Benhabib and Farmer (1994, 1999), Farmer and Guo (1995), Farmer and Woodford (1997), and Woodford (1986, 1990, 1991), among others. Animal spirits in the New Keynesian model have been recently considered, although in a different fashion, also by De Grauwe (2009).

[5]E.g., Beaudry and Portier (2006), Jaimovich and Rebelo (2009), Christiano, et al. (2007), Lorenzoni (2009), Schmitt-Grohé and Uribe (2008), Milani and Treadwell (2009), Blanchard, et al. (2009).

acts as a disturbance in the IS equation, and it moves in response to aggregate taste, technology, and government spending shocks in the economy. Equation (1.2) represents a New Keynesian Phillips curve. The current inflation rate π_t depends on the expected inflation rate in $t+1$, on output gap in period t, and on the cost-push shock u_t. The coefficient β denotes the households' discount factor, while κ is a composite parameter, which denotes the slope of the Phillips curve, and is an inverse function of price rigidity. Equation (1.3) denotes a Taylor rule, which approximates monetary policy decisions in the economy. The monetary authority sets the policy instrument, a short-term nominal interest rate, in response to movements in inflation and the output gap. The reaction coefficients are denoted by χ_π and χ_y, while ρ accounts for the inertia of policy decisions.

The model departs in two ways from the benchmark New Keynesian framework. The assumption of rational expectations is relaxed. In the model, $\widehat{E}_t$ will, in fact, correspond to observed survey and market expectations, for which I will exploit actual data in the estimation, rather than rational, model-consistent, expectations. I will assume that the observed expectations are formed by agents from a near-rational expectations formation mechanism (detailed in the next section). Moreover, the model assumes that expectations are predetermined as in Giannoni and Woodford (2003): economic agents dispose only of information up to $t-1$ when forming expectations about variables in t and $t+1$ and when solving their maximisation problems ($\widehat{E}_{t-1}$ hence replaces $\widehat{E}_t$ in the model). This assumption is made here for empirical reasons, i.e. to match the timing in the Survey of Professional Forecasters and the information set (only up to $t-1$) that is available to the survey forecasters when forecasting period t and $t+1$ variables.[6,7]

1.1. *Expectations Formation in Real-Time*

The paper, therefore, abandons the conventional assumption of rational expectations and exploits, instead, survey and market data on expectations, which will be treated as observable variables in the estimation. I still assume that economic agents form expectations from a model of expectations formation, which aims to explain the observed expectations' data. The expectation formation mechanism consists of a rather small deviation from model-consistent rational expectations. Economic agents are assumed to form expectations according to a perceived law of motion (PLM), which has similar structural form to the minimum state variable solution of the model

[6]The assumption that agents dispose only of $t-1$ information when forming expectations is, however, common in the adaptive learning literature (e.g., Evans and Honkapohja, 2001), as it permits to avoid simultaneity issues. All results in the paper carry over to the case of period-t timing.

[7]One caveat needs to be noticed about the model's microfoundations under learning. I have assumed a model that is characterised by the same loglinearised equations that are obtained under rational expectations: only expectations of variables up to $t+1$ matter for the dynamics of current macroeconomic variables. Under subjective expectations and learning, however, Preston (2005, 2006, 2008) shows that long-horizon expectations may also enter the model. Honkapohja et al. (2003) discuss the conditions under which Preston's approach simplifies to yield the model in this paper.

under rational expectations. The PLM is, therefore, given by

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = a_t + b_t \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + \epsilon_t, \tag{1.4}$$

which resembles a VAR(1) in the model's endogenous variables. In contrast to the rational expectations case, however, agents are assumed to lack knowledge about the reduced-form parameters of the economy.[8] They are also assumed not to be able to observe the realisations of the shocks. This is seen as the most empirically realistic case, but, later in the paper, I will re-estimate the model under the alternative case in which agents are endowed with knowledge about the disturbances as well: the results remain unchanged. Economic agents try to infer the reduced-form parameters in (1.4) using the following constant-gain learning algorithm, through the updating rules

$$\begin{aligned} \widehat{\phi}_t &= \widehat{\phi}_{t-1} + \overline{\mathbf{g}} R_t^{-1} X_t \left(Y_t - X_t' \widehat{\phi}_{t-1} \right) && (1.5) \\ R_t &= R_{t-1} + \overline{\mathbf{g}} \left(X_t X_t' - R_{t-1} \right) && (1.6) \end{aligned}$$

where $Y_t \equiv [y_t, \pi_t, i_t]'$ is the vector of endogenous variables, $X_t \equiv \{1, Y_{t-1}\}$ is the matrix of regressors, and $\widehat{\phi}_t = (a_t', vec(b_t'))'$ collects the reduced-form coefficients. The first expression (1.5) describes the updating of agents' beliefs, while (1.6) illustrates the updating of the precision matrix R_t corresponding to the stacked regressors X_t. A crucial coefficient under learning is the constant gain coefficient $\overline{\mathbf{g}}$, which governs the rate at which agents discount past information when forming expectations about the future.

Expectations about future variables in t and $t+1$ are formed each period using the PLM (1.4) along with the most recently updated coefficients $\widehat{\phi}_t$ from (1.5), as:

$$\widehat{E}_{t-1} \begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix} = a_{t-1} + b_{t-1} \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{pmatrix} + \begin{pmatrix} e_{t-1}^{y_0} \\ 0 \\ e_{t-1}^{i} \end{pmatrix}, \tag{1.7}$$

and

$$\widehat{E}_{t-1} \begin{pmatrix} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \end{pmatrix} = a_{t-1} + b_{t-1} \widehat{E}_{t-1} \begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix} + \begin{pmatrix} e_{t-1}^{y_1} \\ e_{t-1}^{\pi} \\ 0 \end{pmatrix}. \tag{1.8}$$

In the empirical section, I will use time series data on one-period-ahead and two-period-ahead expectations $\widehat{E}_{t-1} y_t$, $\widehat{E}_{t-1} y_{t+1}$, $\widehat{E}_{t-1} \pi_{t+1}$, and $\widehat{E}_{t-1} i_t$, as observable variables.

The variables $e_t^{y_0}$, $e_t^{y_1}$, e_t^{π}, and e_t^{i} define the expectations shocks.[9] The shock $e_t^{y_0}$ indicates the expectational shock that refers to the output forecast between $t-1$ and t, while $e_t^{y_1}$ indicates the slightly longer-horizon shock related to the output forecast between t and $t+1$.

[8] Although the constant in the model solution under rational expectations will be equal to zero, economic agents are not endowed with this information and, therefore, they learn about the intercepts as well. In this way, the learning specification can permit to capture agents' misperceptions about the steady-state levels of inflation and interest rates and about the level of potential output or its trend.

[9] There are no expectations shocks related to $\widehat{E}_{t-1} \pi_t$ and $\widehat{E}_{t-1} i_{t+1}$, as these expectations do not directly enter the model.

The expectation shocks are allowed to be persistent: e_t^π and e_t^i follow the AR(1) processes

$$e_t^\pi = (1-\rho_e^\pi)\bar{\rho}_e^\pi + \rho_e^\pi e_{t-1}^\pi + \sigma_e^\pi \tilde{e}_t^\pi \quad (1.9)$$

$$e_t^i = (1-\rho_e^i)\bar{\rho}_e^i + \rho_e^i e_{t-1}^i + \sigma_e^i \tilde{e}_t^i, \quad (1.10)$$

where σ_e^π and σ_e^i denote the standard deviations of the expectational innovations, where $\tilde{e}_t^\pi$, $\tilde{e}_t^i$ $\sim N(0,1)$. The expectation shocks related to future output, instead, are allowed to be dynamically correlated. They evolve as a VAR(1):

$$\begin{bmatrix} e_t^{y_1} \\ e_t^{y_0} \end{bmatrix} = \begin{bmatrix} (1-\rho_e^{y_1})\bar{\rho}_e^{y_1} \\ (1-\rho_e^{y_0})\bar{\rho}_e^{y_0} \end{bmatrix} + \begin{bmatrix} \rho_e^{y_1} & \rho_{y_1,y_0} \\ \rho_{y_0,y_1} & \rho_e^{y_0} \end{bmatrix} \begin{bmatrix} e_{t-1}^{y_1} \\ e_{t-1}^{y_0} \end{bmatrix} + \begin{bmatrix} \sigma_e^{y_1} & 0 \\ 0 & \sigma_e^{y_0} \end{bmatrix} \begin{bmatrix} \tilde{e}_t^{y_1} \\ \tilde{e}_t^{y_0} \end{bmatrix}. \quad (1.11)$$

This assumption allows them to depend on each other, but it preserves the interpretation of each as an identifiable structural shock, as the variance-covariance matrix is still assumed to be diagonal.[10]

Expectations shocks are, therefore, identified as the exogenous component of expectations that is not related to observed fundamentals and not accounted for by the learning model. Data on expectations are exploited to provide information on the best-fitting learning process over the sample and to disentangle the part of expectations that is due to an endogenous response to the state of the economy and the exogenous expectation shock.

The intuition regarding the expectations formation works as follows. Agents usually form expectations in a near-rational way, by using past values of economic variables and their most recent beliefs about the structure of the economy to forecast future macroeconomic variables. But agents may deviate from these near-rational forecasts: they can be either more optimistic – by believing that future output will be higher than predicted by their learning model (or inflation lower) – or more pessimistic. One of the main goals of the paper will be to evaluate the role and empirical importance of these estimated exogenous waves of optimism and pessimism.

Economic agents base their optimising decisions on $t-1$ information. They observe the values of endogenous variables up to $t-1$ and they update their beliefs through (1.5) and (1.6) running regressions of the endogenous variables in $t-1$ on a vector of intercepts and on the variables in $t-2$; they can then form expectations about variables in t and $t+1$.

2. **Near-Rational Expectations Econometrics: Estimation Approach**

2.1. *Realised and Expectations Data*

I exploit available data on expectations, along with realised data on macroeconomic variables, to estimate the structural parameters of the model, to infer the economic agents' learning process over the sample, and to identify the expectations shocks. The expectations data are derived from

[10]This structure, however, is not crucial, as one could, instead, assume contemporaneous correlation in the innovations and then impose an identification condition, for example by assuming recursiveness, to compute the impulse responses and the variance decomposition.

the Survey of Professional Forecasters (SPF) when possible. I use the mean across forecasters regarding the one-period-ahead expected Nominal GDP (acronym NGDP) and the one-period-ahead expected Price level (PGDP). Expectations about real GDP are constructed by dividing the expected Nominal GDP data by the expected GDP Price Deflator from the survey. Expected inflation is calculated as the log of the expected two-quarter-ahead GDP Price Deflator minus the log of the expected one-quarter-ahead GDP Price Deflator.

Households' optimality conditions also require them to form expectations about one-period-ahead nominal interest rates. Such expectations are available from the SPF, but only starting from 1981:III. In the baseline estimation in the paper, however, I choose to exploit the longest possible sample and, hence, I derive expectations about future interest rates using the expectations theory of the term structure. This implies the relation $i_t^{6M} = \left(\frac{i_t^{3M} + \widehat{E}_t i_{t+1}^{3M}}{2}\right) + \bar{\varsigma}$, which states that the six-month yield is equal to the average between the current three-month yield and the expected three-month yield three months ahead, except for constant term premium $\bar{\varsigma}$, and which can be solved for $\widehat{E}_t i_{t+1}^{3M}$ at each t. Data on the three-month Treasury bill rate are used for i_{t-1}^{3M} and on the six-month Treasury bill rate for i_{t-1}^{6M}. The estimation will also be repeated using the expected interest rate from the SPF and the shorter post-1981 sample as a robustness check.[11]

To better explain the observed expectations and to more accurately identify the economic agents' learning process, it is desirable to exploit knowledge about their real-time information set. Such information is fortunately available from the SPF, since, in each quarter t, when forecasters receive the survey, they are asked about their perceptions about the values of the variables in $t-1$. When forming their expectations about variables in $t+1$, therefore, they use their best estimates for the variables in $t-1$, which are also available from the SPF. Moreover, one week before the survey is mailed to the forecasters, the BEA releases data about the values of the variables in $t-1$. Almost all forecasters in the survey simply report the BEA release as their perception of $t-1$ values.[12] Forecasters observe the values of the variables in $t-1$ when communicating forecasts for variables dated t and $t+1$. The forecasters' $t-1$ information set is available to the econometrician and, therefore, will be exploited in the estimation.

Real GDP is constructed using the real-time Nominal GDP series divided by the real time GDP Implicit Price Deflator, using the data from the SPF regarding the $t-1$ information set available to agents, i.e. using the BEA real-time data release about GDP. Inflation is constructed using real-time data on the price deflator (PQvQd), obtained from the 'Real Time Data Set for Macroeconomists', made available by Federal Reserve Bank of Philadelphia and described in Croushore and Stark (2001). As short-term nominal interest rate, I use the three-month Treasury bill (as forecasts data

[11] The correlation between forecasts derived from the expectations theory and forecasts from the SPF equals 0.978.
[12] *Survey of Professional Forecasters, Documentation, February 2010, update.*

from the SPF will also be available for this variable). The realised and expectations series regarding inflation, the growth rate of output, and interest rates, are shown in Figure 1.

2.2. *State-Space System*

The model, summarised by equations (1.1)-(1.3), and with expectations formed as in (1.7)-(1.8), can be written in state-space form as

$$\xi_t = A_t + F_t\xi_{t-1} + Gw_{t-1} \tag{2.1}$$

$$\Upsilon_t = H\xi_t + \Delta_0 + \Delta_1 t \tag{2.2}$$

where the state vector ξ_t includes the endogenous variables, expectations, as well as structural and expectational disturbances: $\xi_t = \left[y_t, \pi_t, i_t, r_t^n, u_t, \widehat{E}_{t-1}y_{t+1}, \widehat{E}_{t-1}\pi_{t+1}, ..., e_t^{y_1}, e_t^{y_0}, e_t^{\pi}, e_t^{i}\right]'$.[13] The model is estimated to match the following observable variables in Υ_t: output, inflation, short-term interest rate, output forecasts (one-period-ahead), output forecasts (two-period-ahead), inflation forecasts (two-period-ahead), interest rate forecasts (one-period-ahead). The baseline estimation assumes a linear trend for output. The estimation is, however, repeated under a variety of trend and potential output specifications, which are discussed later in the paper.[14]

2.3. *Priors*

Table 1 reports information about the prior distributions. There is large uncertainty on the value of the IES coefficient σ: I choose a Gamma prior with mean 1 and a rather large standard deviation equal to 0.75. For the slope of the Phillips curve coefficient κ, I assume a Gamma prior distribution with mean 0.25 and standard deviation equal to 0.177. Regarding the monetary policy rule, the feedback coefficients χ_π and χ_y follow Normal prior distributions with mean 1.5 and standard deviation 0.25 and mean 0.25 and standard deviation 0.125, while the interest-rate smoothing coefficient ρ is assumed to follow a Beta distribution with mean 0.8. The autoregressive coefficients in the AR processes for structural and expectational disturbances are assumed to follow weakly informative Beta prior distributions with mean 0.5 and standard deviation 0.26. Inverse Gamma distributions are chosen for the standard deviations of the shocks. An important parameter in the estimation is the constant gain coefficient. To minimise the influence of prior information and of assumptions about the learning process, I assume a non-informative Uniform prior distribution for the gain over the [0,1] interval.

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[13] In the estimation, I redefine the exogenous unobserved terms as follows: $\widehat{E}_{t-1}r_t^n = \tilde{r}_t^n$ and $\widehat{E}_{t-1}u_t = \tilde{u}_t$. This change of notation is not important for the results.

[14] The choice of using an output measure based on a linear trend as the benchmark case aims to capture the real-time forecasting process of actual economic agents over the whole sample: a deterministic trend is likely to be a better approximation of the detrending procedures that forecasters had in mind for a large portion of the sample than the theoretical New Keynesian definition of the gap.

2.4. *Bayesian Estimation*

The model is estimated using Bayesian methods over the 1968:IV-2009:I sample. The starting date coincides with the first quarter of availability of the survey forecasts. The likelihood of the state-space system (2.1)-(2.2) is derived using the Kalman filter. The Metropolis-Hastings algorithm is used to generate draws to approximate the posterior distribution. I run 500,000 draws, discarding an initial burn-in given by the first 25% draws.

Rather than imposing a learning process a priori and obtaining results that are conditional on a given learning process, I also estimate the learning parameters jointly along with the other structural parameters of the economy. In particular, both the constant gain coefficient and the uncertainty that characterises agents' initial beliefs, i.e. the variance-covariance matrix $R_{t=0}^{-1}$, are inferred from the estimation. The initial precision matrix $R_{t=0}$ is given by $R_{t=0} = \left[\overline{\mathbf{g}} \sum_{i=1}^{\tau} (1-\overline{\mathbf{g}})^{(i-1)} X_{\tau-i} X'_{\tau-i}\right]$, where τ indexes the pre-sample observations; therefore, by estimating a single coefficient, the constant gain $\overline{\mathbf{g}}$, the estimation provides evidence both on the learning speed and on the uncertainty surrounding initial beliefs. In this way, the best-fitting learning process can be extrapolated from time series data. This paper improves over previous work on the estimation of general equilibrium models with learning (e.g., Milani, 2007a, 2008) by exploiting actual data on expectations to best infer the evolution of the learning process over the sample. Pre-sample data for the 1947:I-1968:IV period are used to inform the choice of initial beliefs in the learning algorithm.

3. **Near-Rational Expectations Econometrics: Empirical Results**

3.1. *Posterior Estimates*

Table 1 shows the posterior mean and 95% credible intervals for each estimated parameter. The estimates indicate a posterior mean for the sensitivity of inflation to output κ equal to 0.035 and for the elasticity of intertemporal substitution σ equal to 0.236. The estimates of the monetary policy rule coefficients are consistent with the vast majority of previous studies: they indicate a large degree of policy inertia ($\rho = 0.95$) and reaction coefficients to inflation and output equal to 1.417 and 0.221. The data are informative about the best-fitting learning process in the sample. The posterior estimate for the constant gain parameter is equal to 0.0196, with a 95% credible interval between 0.015 and 0.025. This estimate, which is obtained by fitting the learning process to expectations data, is remarkably similar to the estimate of $\overline{\mathbf{g}} = 0.0187$ found in Milani (2007a) in an estimation that used information on realised variables only.

The main interest of the paper lies in identifying the effects of structural and expectational shocks. The posterior estimates suggest a relatively low persistence of the structural disturbances. The autoregressive coefficients have mean equal to 0.351 for the demand shock r_t^n and to 0.171 for the supply shock u_t. Since the model lacks "mechanical" sources of persistence as habit formation and inflation indexation and, yet, the structural shocks are characterised by a substantially lower persistence than usually obtained in these models, the estimation suggests that the inclusion of observed expectations and learning can successfully induce realistic levels of persistence in the system.[15] This finding also hints that the properties of estimated shocks may critically change when survey expectations are included as observables (Del Negro and Eusepi, 2009, similarly find that inflation expectations data alter their target shock estimates). The main novelty in the estimation lies in identifying the expectational shocks. These are found to be generally quite persistent. The autoregressive coefficients have posterior means equal to 0.854 for $e_t^{y_1}$, 0.422 for e_t^{π}, and 0.627 for e_t^{i}, while the coefficient is smaller for $e_t^{y_0}$ ($\rho_e^{y_0} = 0.231$). I have allowed the expectational disturbances related to output expectations to be dynamically correlated: I find that $e_t^{y_0}$ is strongly connected to the previous period $e_{t-1}^{y_1}$ ($\rho_{y_0,y_1} = 0.722$).

The evolution of all the estimated beliefs over the sample and a detailed discussion, here omitted to save space, can be found in the expanded working paper version (Milani, 2010, Figure 2).

3.2. *Expectation Shocks as Drivers of Economic Fluctuations*

I derive impulse response functions for the macroeconomic variables in the model to both structural and expectation shocks. The impulse responses in the figures denote averages over the sample and across draws (using the last 10,000 MCMC draws) and are shown along with their respective 2.5 and 97.5% percentiles.

Figure 2 overlaps the impulse responses of detrended output to the aggregate demand natural rate shock r_t^n and to the output expectation shock $e_t^{y_1}$, which can be interpreted as an "optimism" shock. The effect of the structural demand shock is rather large on impact, but its transmission is relatively quick: the peak of the output response already occurs in the second quarter after the shock. The expectation shock leads to a much more persistent output response. The peak occurs after a year and a half and the effect is larger and more long-lived than that of the structural shock.

Figure 3 shows the response of inflation to cost-push and natural rate shocks, along with the response to expectational shocks regarding future inflationary pressures and real activity. The supply shock and the expectational shock regarding future inflation die off rather quickly, in slightly more than a year. Demand shocks induce a more inertial adjustment. In particular, the response of inflation to the output expectation shock is more pronounced and sluggish. The expectational

[15]This result is consistent with Milani (2007a) and Ormeno (2009), who also document the role of learning on macroeconomic and inflation persistence. The success in matching the data without additional frictions is also likely to be related to the pervasive result that survey forecasts are known to have a superior performance in forecasting macro variables (e.g., Ang et al., 2007).

shock $e_t^{y_1}$ (and $e_t^{y_0}$ as well), therefore, resembles a demand shock, as an increase in optimism about the future state of the economy moves output and inflation in the same direction.

Table 2 reports the outcome of the forecast error variance decomposition: the table shows the mean shares across the last 10,000 MCMC draws, along with the 2.5 and 97.5 percentiles, for different horizons (4, 12, 20 quarters), to capture the importance of shocks at business cycle frequencies.

Expectation shocks regarding future output are the main source of economic fluctuations. These expectational shocks can account, in fact, for 53-54% of fluctuations (at horizons equal to 12 and 20 quarters). Natural rate shocks explain around 20%, monetary policy shocks 22%, and cost-push shocks 3% of fluctuations. Structural shocks are, however, more important in the very short run: natural rate shocks account for 41% of fluctuations at the one year horizon and for a larger share at even shorter horizons.

Expectational shocks are also the main contributor to the variability of inflation. Cost-push shocks explain 27% of its variance (at the 20 quarters horizon), while expectation shocks regarding future inflation explain 17% and expectation shocks regarding future real activity explain 33%. The role of structural cost-push shocks is again larger for fluctuations within the one-year horizon (for which, they arrive at a share of 50% or more).

The expectation shock regarding future output is shown in Figure 4, along with vertical bands denoting NBER recession dates. The figure shows that expectation shocks quickly fall during recessions and become negative, indicating an increasing aggregate pessimism (that is, economic agents form expectations about future economic conditions that fall below what their near-rational model and their updated beliefs would suggest). The expectation shock begins to fall right before the economy enters a recession and increases before the recession ends. The degree of optimism is usually at the highest in the middle of an expansion.[16,17]

The model specification also permits to study the determinants of observed private agents' expectations. Expectations are affected by developments in the economy. Fundamental shocks can explain roughly one third of expectation data regarding output, inflation, and interest rates. But expectations are mostly driven by expectational innovations. Purely expectational shocks, in fact, account for roughly 60% of the variance in output, inflation, and interest rate expectations.

[16]From the figure it seems that agents are on average more optimistic in the first part of the sample than in the second. This feature, however, is not robust to the use of the alternative output gap measures that will be discussed in section 3.5. The dynamics over recessions and expansions is, instead, consistent across different specifications.

[17]This relation with the cycle seems to distinguish the identified expectation shocks from "news" shocks, which are gaining popularity as sources of fluctuations. Sims (2009), for example, shows that news shocks are negatively correlated with current and future detrended output and display a low positive correlation with output around nine quarters ahead; news shocks often do not worsen in correspondence of recessions, and, in several episodes, they fall to negative levels in periods that do not constitute recessions. Canova et al. (2010) also show graphs that indicate that news shocks are mostly acyclical and large and positive in some recessions.

It is important to point out that the results on the importance of expectation shocks do not arise from a serious misspecification of the learning model or from its failure to match expectations data. Figure 5 plots the agents' near-rational expectations derived from the learning model's PLM versus the observed survey forecasts. The series track each other remarkably well. The worst fit is observed in the case of inflation, but it is still very satisfactory over the sample, with the possible exception of the 1976-1977 observations in which the learning model would imply a downward revision in inflation expectations, which, instead, does not materialise in actual expectations data.[18] The importance of expectation shocks stems in part from the result that the exogenous waves of optimism and pessimism appear quite persistent over time. While structural shocks may have large effects on the economy in the short run, they are here not strongly serially correlated and their transmission is rapid. Shifts in market sentiments, instead, take a long time to reverse direction.

The empirical results may be taken to suggest that macroeconomic model building should be revisited. A substantial modelling effort is directed toward incorporating frictions in current DSGE models to match the sluggish response of macro variables to structural shocks. But this paper suggests that the response to aggregate demand and supply shocks may be faster than commonly thought. Under the paper's framework, sluggishness in the economy is, instead, induced by learning and by slow-moving expectational shifts.

3.3. *Expectational Shocks: Sentiment versus Alternative Interpretations.*

The paper has emphasised the interpretation of the e_t disturbances as expectational shocks, which reflect exogenous shifts in economic agents' optimism and pessimism. Since e_t shocks have been found to play a major role over the business cycle, it is necessary to check that these shocks do not spuriously capture other distinct elements, such as superior information set by agents, misspecification in the model, measurement error, and so forth.

First, to test whether expectation shocks are simply capturing the action of variables and shocks that are omitted from the model, but which may influence the business cycle, I can regress $e_t^{y_1}$ (since this is the specific expectation shock that has been found to be the most important for fluctuations) on factors as real oil prices, credit conditions (proxied by the BAA-AAA credit spread), and productivity growth (nonfarm business sector output per hour series). Moreover, when forming expectations, economic agents may have additional information, which is not included in their PLM: for example, they may have information about future monetary policies from credible policy announcements. To test whether $e_t^{y_1}$ captures forward-looking information about Fed's policy,

[18]The evidence that the learning model provides a good fit of survey expectations is in line with Ormeno (2009), and suggest learning as a mechanism that may reduce the role of measurement error in Del Negro and Eusepi (2009).

therefore, I add as a regressor the anticipated "news" shocks about future monetary policies identified in Milani and Treadwell (2009). To verify, instead, that $e_t^{y_1}$ actually reflects market sentiment, I relate it to the University of Michigan Consumer Sentiment indicator. In particular, the relevant indicator is the part of consumer sentiment that cannot be explained as a direct response to changes in macroeconomic conditions. Therefore, I add to the regression the identified innovation in consumer sentiment from a VAR that also includes detrended GDP, inflation, and the federal funds rate (identified using a Cholesky decomposition).

Second, to more thouroughly assess the hypothesis that expectation shocks may capture omitted information in the model – or superior information set by private agents than allowed for by their PLM – I also test, in a second regression, whether $e_t^{y_1}$ can be explained by a group of dynamic factors $(F_{1,t}, ..., F_{7,t})$, which are extracted from a large set of macroeconomic variables and taken from Belviso and Milani (2006).[19] The two regressions are shown below (s.e. in parenthesis):

$$\begin{aligned} e_t^{y_1} &= \widehat{c} + \underset{(0.05)}{0.83} e_{t-1}^{y_1} - \underset{(0.05)}{0.02} Oil\ Price_t + \underset{(0.06)}{0.03} Credit\ Spread_t + \underset{(0.03)}{0.016} \Delta Prod_t + \qquad (3.1) \\ &\quad - \underset{(0.09)}{0.13} MP\ News_t + \underset{(0.36)}{1.57} Sentiment_t + \widehat{\varepsilon}_t, \qquad R^2 = 0.73,\ \textit{Sample: } 1969-2008; \\ &= \widehat{c} + \underset{(0.07)}{0.80} e_{t-1}^{y_1} + \underset{(0.04)}{0.04} F_{1,t} - \underset{(0.07)}{0.05} F_{2,t} + \underset{(0.04)}{0.02} F_{3,t} + \underset{(0.04)}{0.002} F_{4,t} + \underset{(0.05)}{0.05} F_{5,t} + \qquad (3.2) \\ &\quad + \underset{(0.02)}{0.001} F_{6,t} + \underset{(0.05)}{0.01} F_{7,t} + \underset{(0.45)}{2.09}\ Sentiment + \widehat{\varepsilon}_t, \qquad R^2 = 0.74,\ \textit{Sample: } 1969-1998. \end{aligned}$$

From regression (3.1), it can be seen that the expectational innovations identified from the DSGE estimation are not capturing the effect of omitted variables, whereas they are correlated with exogenous innovations in aggregate sentiment. In the factor-augmented regression (3.2), the factors do not have explanatory power. The sentiment indicator is again the only significant regressor.

Therefore, it is unlikely that $e_t^{y_1}$ represents superior information by agents or omitted factors in either the PLM or the baseline model. The shock $e_t^{y_1}$ appears indeed correlated with measures of sentiment, in line with the interpretation given in the paper. This remains true if alternative sentiment indicators are used (again identifying the innovation component). Focusing on regression (3.1), for example, the expectation shock is significantly related to the Conference Board (CB) Consumer Confidence - Expectations index (t-stat= 4.44, sample 1969-2008), the difference between the CB percentage of business respondents indicating that they expect better business conditions six months ahead and the percentage of those expecting worse business conditions (t-stat= 4.89, sample 1969-2008), the CB Business Executive Confidence Index (t-stat= 2.03, sample 1988-2008), and, from the Duke Fuqua School of Business - CFO Magazine Business Outlook Survey, the

[19]Seven factors were obtained from 145 macroeconomic indicators and were interpreted in that paper as a real activity factor $(F_{1,t})$, an inflation factor $(F_{2,t})$, an interest rate factor $(F_{3,t})$, a financial market factor $(F_{4,t})$, a money factor $(F_{5,t})$, a credit factor $(F_{6,t})$, and an expectation/survey factor $(F_{7,t})$.

difference between the percentages of respondents that indicate they are more optimistic about the economy and those that are less optimistic (t-stat= 2.63, sample 2001-2008).[20]

Finally, rather than omitted information, the expectation shock may simply reflect the existence of large measurement error in the data. To rule out this possibility, I re-estimate the general equilibrium model, but now allowing for measurement error (either i.i.d. or AR(1)) in the observable expectations data. The estimation should be able to disentangle expectation shocks from measurement error, by exploiting the correlation between expectations data and realised variables. The posterior estimates (not shown) remain similar and the standard deviations of measurement error terms fall close to zero (below 0.02). The variance decomposition, therefore, yields the same conclusions: expectation shocks still explain half of fluctuations even when measurement error is included. This finding somewhat differs from the one in Del Negro and Eusepi (2009), who find that inflation expectations are mostly explained by measurement error in a model with rational expectations, but learning only about the target. This paper's results (as well as those in Ormeno, 2009), may suggest parametric learning as an important component in fitting survey data.[21]

3.4. *The Role of Learning*

To quantify the role of endogenous changes in expectations through learning, rather than exogenous changes, in contributing to fluctuations, I can re-estimate the model, but now endowing agents with the same PLM they would have under rational expectations:

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \overline{a} + \overline{b} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + \overline{c} r^n_{t-1} + \overline{d} u_{t-1} + \epsilon_t. \tag{3.3}$$

Agents now know the values of the structural disturbances, the correct coefficient values $\overline{a}$, $\overline{b}$, $\overline{c}$, and $\overline{d}$ (constant, since agents have already learned the truth), and recognise that the intercept vector contains only zeros. The expectation shocks are identified as the part of the observed survey and market expectations that deviates from the forecasts implied by the rational expectations PLM. The variance decomposition is recomputed under this new scenario.

Under the rational expectations PLM, expectation shocks now account for a larger share of fluctuations: the expectational shock about output accounts for 89% of output fluctuations, while the expectational shock about future inflation accounts for 62% of inflation fluctuations. The results may be taken to indicate that out of 89% of output gap fluctuations that may be attributed to expectational shocks if the economy were assumed to have already converged to the rational expectations equilibrium, 40% may be rationalised as the endogenous response due to learning,

[20] All data series have been obtained from IHS Global Insight.

[21] A more serious analysis of the relation between measurement error, as in Del Negro and Eusepi (2009), and expectation shocks, however, is beyond the scope of this paper.

while the remaining 50%, found before, is due to exogenous expectation shocks.[22] In the case of inflation, out of 62% of fluctuations attributed to expectation shocks under rational expectations, the most part ($\approx 45\%$) is due to a near-rational response to changes in the economy and learning.

3.5. *Robustness*

I have performed a number of robustness checks and the results are not sensitive. Some of the major checks are described here, while others, along with a detailed table showing the variance decomposition shares, can be found in the working paper version (Milani, 2010, Table 4).

The results do not hinge on the choice of a peculiar output gap measure: all conclusions are robust to a wide variety of detrending options and to different ways to characterise potential output. I can also relax the assumption that agents know the coefficients in the trend equation (i.e., Δ_0 and Δ_1), by allowing them to learn about the trend over time (so far, misperceptions about the trend were captured only by intercepts in the learning rule). The output gap can be alternatively constructed as the log deviation of real GDP from the CBO's estimate of potential GDP. I repeat the analysis under the assumptions that agents either know the growth rate of potential GDP, or that they have to learn about it. The model is also re-estimated using the theoretical definition of the output gap, i.e. the deviation of output from its flexible-price level, and using growth rates of real GDP, and expectations about the growth rates from the SPF as observables, with the implied output gap in the model obtained from the filtering procedure. In all these cases, expectational shocks referring to future output gaps are always the dominant source of fluctuations: their shares go from 48% to almost 70%. The share of the natural rate shock is smaller, with posterior means below 20%.

It is likely that economic agents learn about the dynamics of different macroeconomic variables at various speeds; therefore, I allow the gain coefficients to differ across variables. The posterior means for the constant gain related to inflation and output are equal to 0.0179 and 0.0296. Learning about Federal Reserve's policy rule has been slower: the posterior mean for $\bar{\mathbf{g}}_i$ is 0.005. The implied impulse responses and variance decomposition, however, remain similar to those in the baseline case.

In the baseline estimation, I have assumed that economic agents learn using a PLM that corresponds to a VAR(1) in the model's endogenous variables. I verify the robustness of the results to assuming that agents can observe the structural shocks as well. Expectation shocks regarding future real activity are confirmed to be the main driver of economic fluctuations as they explain 57% of output gap variability.

[22]There is an important caveat here. The results are obtained assuming that the economy is characterised by the same model that was used under learning. In this scenario, expectation shocks become even more important as expectations derived from the new PLM fall very far from the observed expectations. But if the model was re-estimated under rational expectations (and shutting down the expectation shocks), the structural coefficients would likely change in the effort of matching the endogenous variables and the results might significantly differ.

While data on expectations about output and inflation were obtained from the SPF, in the baseline estimation I have extracted interest rate forecasts from the term structure of interest rates. Data on expectations about future interest rates, however, are also available from the SPF, but starting from 1981:III. I can repeat the estimation for the post-1981 sample and using SPF forecasts for all series. The role of expectation shocks regarding future monetary policy choices on the business cycle is confirmed to be small. Moreover, the results indicate that expectation shocks were not only important in the 1970s, but they also represent the main source of output fluctuations in the more stable post-1981 period. Unexpected monetary policy shocks, instead, were considerably more important in the first subperiod.

Finally, the baseline model assumed constant gain learning, but it didn't incorporate any actual source of parameter variation in the model. This may be seen as unrealistic. The model is, therefore, re-estimated under the assumption that there is now a structural break in the monetary policy rule coefficients in correspondence of the start of Volcker's tenure as Chairman of the Federal Reserve, which is not known by private agents. The estimates indicate that the coefficient of response to inflation increases from around 1 to 1.93 and the response to the output gap declines from 0.33 to close to 0. All the paper's results, however, are robust to the assumption of a time-varying monetary policy rule, and expectation shocks still explain roughly half of output fluctuations.

4. **Conclusions**

While economists have recognised for a long time that psychological forces, changes in market sentiments, shifts in confidence, and so forth, may exert a large influence on economic fluctuations, the current generation of macroeconomic models typically excludes them from the analysis.

This paper argues that these forces, in the form of exogenous expectational shifts, such as waves of optimism and pessimism, should be brought back to the centre of macroeconomics.

The paper has estimated a baseline New Keynesian model and exploited observed survey data on expectations or expectations extrapolated from the market. In this way, the paper has allowed a departure from the conventional rational expectations hypothesis, which is dominant in macroeconomics. The observed expectations were assumed to be formed, instead, from a near-rational mechanism. Economic agents, however, were allowed to deviate each period from the forecasts that were implied by their learning model and that were hence justified as an endogenous response to the state of the economy. The deviations are captured by expectation shocks in the model.

The empirical evidence has shown that expectational shocks, particularly those related to future real activity, may play a large role in driving the business cycle. These shocks can explain half of economic fluctuations over the sample.

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		Prior Distributions			Posterior Distributions	
Descr.	Param.	Distr.	Mean	95% Prior Int.	Mean	95% Credible Int.
Slope PC	κ	Γ	0.25	[0.03-0.7]	0.035	[0.019-0.053]
IES	σ	Γ	1	[0.1-2.92]	0.236	[0.03-0.55]
IRS	ρ	B	0.8	[0.46-0.98]	0.95	[0.91-0.98]
Feedback Infl.	χ_π	N	1.5	[1.01-1.99]	1.417	[0.97-1.86]
Feedback Gap	χ_y	N	0.25	[0.01-0.49]	0.221	[0.06-0.43]
Autoregr. Demand shock	ρ_r	B	0.5	[0.05-0.95]	0.351	[0.19-0.50]
Autoregr. Cost-push shock	ρ_u	B	0.5	[0.05-0.95]	0.171	[0.04-0.31]
Std. Demand shock	σ_r	Γ^{-1}	0.5	[0.1-1.92]	0.77	[0.69-0.86]
Std. Cost-push shock	σ_u	Γ^{-1}	0.5	[0.1-1.92]	0.297	[0.27-0.33]
Std. MP shock	σ_ε	Γ^{-1}	0.5	[0.1-1.92]	0.207	[0.19-0.23]
Autoregr. Exp. shock y_{t+1}	$\rho_{e^{y_1}}$	B	0.5	[0.05-0.95]	0.854	[0.68-0.98]
Autoregr. Exp. shock y_t	$\rho_{e^{y_0}}$	B	0.5	[0.05-0.95]	0.231	[0.08-0.41]
Autoregr. Exp. shock π_{t+1}	ρ_{e^π}	B	0.5	[0.05-0.95]	0.422	[0.28-0.56]
Autoregr. Exp. shock i_t	ρ_{e^i}	B	0.5	[0.05-0.95]	0.627	[0.51-0.74]
Depend. $e_t^{y_1}$ on $e_{t-1}^{y_0}$	ρ_{y_1,y_0}	N	0	[-0.98-0.98]	-0.009	[-0.13-0.15]
Depend. $e_t^{y_0}$ on $e_{t-1}^{y_1}$	ρ_{y_0,y_1}	N	0	[-0.98-0.98]	0.722	[0.50-0.91]
Std. Exp. shock y_{t+1}	$\sigma_{e^{y_1}}$	Γ^{-1}	0.5	[0.1-1.92]	0.286	[0.26-0.32]
Std. Exp. shock y_t	$\sigma_{e^{y_0}}$	Γ^{-1}	0.5	[0.1-1.92]	0.342	[0.30-0.38]
Std. Exp. shock π_{t+1}	σ_{e^π}	Γ^{-1}	0.5	[0.1-1.92]	0.203	[0.18-0.23]
Std. Exp. shock i_t	σ_{e^i}	Γ^{-1}	0.5	[0.1-1.92]	0.087	[0.08-0.10]
Constant gain	$\bar{\mathbf{g}}$	U	0.5	[0.025-0.975]	0.0196	[0.015-0.025]

Table 1 - Prior distributions and Posterior estimates, baseline model.

Note: Γ denotes Gamma distribution, B denotes Beta distribution, N denotes Normal distribution, Γ^{-1} denotes Inverse Gamma distribution, and U denotes Uniform distribution. Posterior means and 95% credible intervals have been calculated over 500,000 Metropolis-Hastings draws, discarding an initial burn-in of 25% draws. The sample is 1968:III-2009:I.

	π_t	y_t	i_t	$\widehat{E}_{t-1}\pi_{t+1}$	$\widehat{E}_{t-1}y_{t+1}$	$\widehat{E}_{t-1}y_t$	$\widehat{E}_{t-1}i_t$
$h = 4$							
Cost-Push Shock u_t	0.507 [0.42,0.61]	0.022 [0.01,0.03]	0.02 [0,0.06]	0.233 [0.18,0.32]	0.031 [0.02,0.05]	0.025 [0.01,0.04]	0.019 [0.01,0.05]
Natural Rate Shock r_t^n	0.05 [0.03,0.09]	0.413 [0.32,0.51]	0.011 [0,0.03]	0.044 [0.03,0.07]	0.262 [0.20,0.34]	0.366 [0.28,0.45]	0.011 [0,0.02]
MP Shock ε_t	0.05 [0.04,0.07]	0.083 [0.06,0.12]	0.943 [0.87,0.99]	0.08 [0.06,0.10]	0.092 [0.07,0.12]	0.077 [0.06,0.11]	0.824 [0.76,0.88]
Expect. Shock e_t^π	0.317 [0.23,0.39]	0.01 [0,0.02]	0.013 [0,0.04]	0.588 [0.50,0.66]	0.014 [0.01,0.03]	0.01 [0,0.02]	0.012 [0.01,0.03]
Expect. Shock e_t^{y1}	0.05 [0.02,0.09]	0.410 [0.31,0.50]	0.01 [0,0.013]	0.033 [0.02,0.05]	0.526 [0.43,0.61]	0.420 [0.33,0.50]	< 0.01 [0,0.009]
Expect. Shock e_t^{y0}	0.013 [0.01,0.02]	0.050 [0.03,0.08]	< 0.01 [0,0.01]	0.011 [0.01,0.02]	0.063 [0.03,0.10]	0.091 [0.05,0.13]	< 0.01 [0,0.004]
Expect. Shock e_t^i	< 0.01 [0,0.004]	< 0.01 [0,0.01]	< 0.01 [0,0.01]	< 0.01 [0,0.006]	< 0.01 [0,0.009]	< 0.01 [0,0.007]	0.120 [0.08,0.17]
$h = 12$							
Cost-Push Shock u_t	0.297 [0.20,0.44]	0.03 [0.01,0.06]	0.021 [0,0.06]	0.133 [0.09,0.20]	0.033 [0.02,0.07]	0.032 [0.02,0.06]	0.022 [0.01,0.06]
Natural Rate Shock r_t^n	0.078 [0.04,0.13]	0.198 [0.12,0.29]	0.072 [0.01,0.14]	0.081 [0.04,0.13]	0.125 [0.07,0.19]	0.158 [0.09,0.24]	0.075 [0.01,0.14]
MP Shock ε_t	0.127 [0.09,0.17]	0.201 [0.12,0.32]	0.666 [0.40,0.96]	0.152 [0.11,0.20]	0.206 [0.11,0.32]	0.202 [0.12,0.31]	0.610 [0.37,0.87]
Expect. Shock e_t^π	0.187 [0.13,0.26]	0.02 [0.01,0.04]	0.02 [0,0.06]	0.323 [0.23,0.43]	0.022 [0.01,0.04]	0.021 [0.01,0.04]	0.018 [0.01,0.06]
Expect. Shock e_t^{y1}	0.288 [0.17,0.40]	0.515 [0.38,0.66]	0.206 [0.02,0.40]	0.288 [0.18,0.39]	0.574 [0.43,0.70]	0.542 [0.41,0.68]	0.210 [0.05,0.39]
Expect. Shock e_t^{y0}	0.014 [0.01,0.03]	0.03 [0.01,0.05]	0.01 [0,0.04]	0.014 [0.01,0.03]	0.03 [0.01,0.05]	0.037 [0.02,0.06]	0.01 [0,0.04]
Expect. Shock e_t^i	< 0.01 [0,0.004]	< 0.01 [0,0.006]	< 0.01 [0,0.003]	< 0.01 [0,0.005]	< 0.01 [0,0.006]	< 0.01 [0,0.005]	0.05 [0.03,0.07]
$h = 20$							
Cost-Push Shock u_t	0.271 [0.18,0.41]	0.03 [0.01,0.06]	0.02 [0,0.05]	0.12 [0.08,0.19]	0.032 [0.01,0.06]	0.031 [0.01,0.06]	0.017 [0,0.04]
Natural Rate Shock r_t^n	0.075 [0.04,0.13]	0.193 [0.11,0.29]	0.07 [0.01,0.13]	0.076 [0.04,0.13]	0.126 [0.07,0.20]	0.155 [0.09,0.24]	0.073 [0.02,0.13]
MP Shock ε_t	0.14 [0.08,0.23]	0.223 [0.12,0.39]	0.53 [0.26,0.90]	0.16 [0.1,0.25]	0.227 [0.12,0.39]	0.226 [0.12,0.39]	0.481 [0.24,0.83]
Expect. Shock e_t^π	0.171 [0.11,0.24]	0.02 [0.01,0.04]	0.015 [0,0.05]	0.286 [0.20,0.40]	0.022 [0.01,0.04]	0.021 [0.01,0.04]	0.015 [0,0.05]
Expect. Shock e_t^{y1}	0.32 [0.20,0.45]	0.499 [0.35,0.64]	0.35 [0.07,0.57]	0.336 [0.22,0.49]	0.554 [0.40,0.68]	0.522 [0.37,0.66]	0.362 [0.11,0.58]
Expect. Shock e_t^{y0}	0.014 [0.01,0.03]	0.03 [0.01,0.05]	0.01 [0,0.04]	0.014 [0.01,0.03]	0.03 [0.01,0.05]	0.036 [0.02,0.06]	0.01 [0,0.04]
Expect. Shock e_t^i	< 0.01 [0,0.003]	< 0.01 [0,0.006]	< 0.01 [0,0.003]	< 0.01 [0,0.005]	< 0.01 [0,0.005]	< 0.01 [0,0.005]	0.037 [0.02,0.06]

Table 2 - Forecast Error Variance Decomposition.

Note: The table reports shares of the variance of inflation, the output gap, the nominal interest rate, expected inflation, expected output gap (one and two-period ahead), and expected nominal interest rate, that are explained by each structural and expectational shock. The entries in the table denote posterior means calculated over the last 10,000 MCMC draws; the numbers below each entry in square brackets denote 95% posterior density intervals. The variance decomposition is calculated for business cycle horizons equal to 4, 12, and 20 quarters.

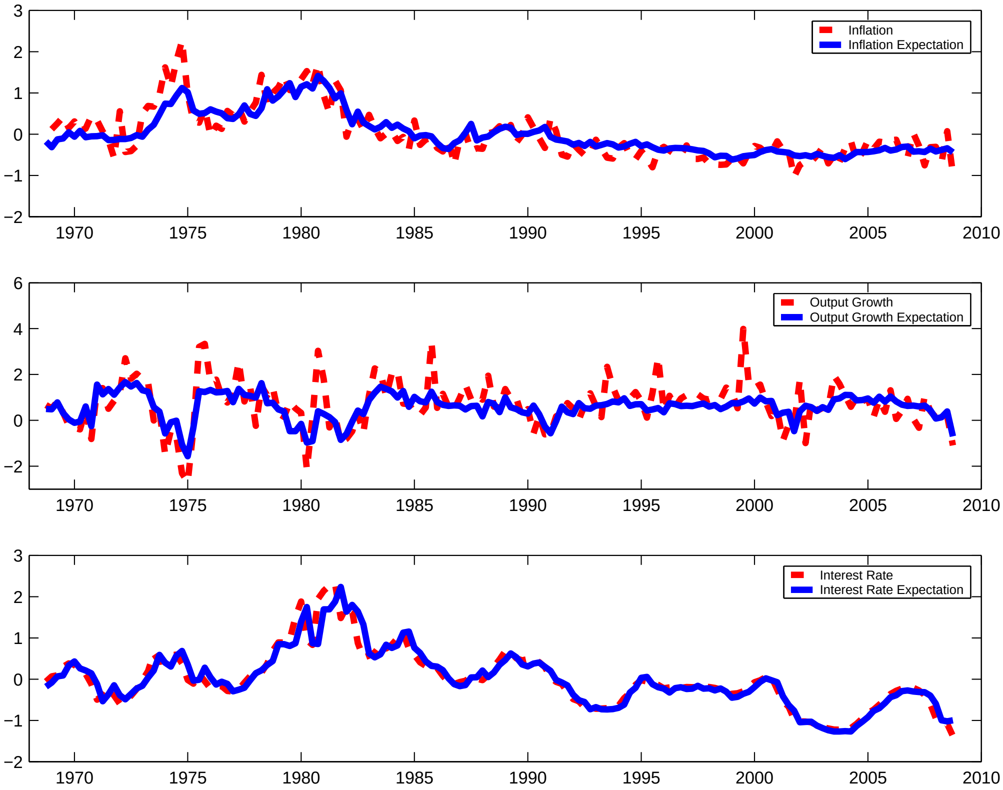

FIGURE 1. Realised Variables and Expectations.

Note: The first panel shows realised inflation (π_{t+1}) and inflation expectations ($\widehat{E}_{t-1}\pi_{t+1}$) from the Survey of Professional Forecasters. The second panel shows output growth (y_t) along with output growth expectations ($\widehat{E}_{t-1}y_t$) from the Survey of Professional Forecasters. The third panel shows the three-month nominal interest rate (i_t) along with interest rate expectations ($\widehat{E}_{t-1}i_t$) extracted from the term structure of interest rates. Realised values and expectations regarding inflation and interest rates are shown in deviation from their sample averages and expressed as quarterly rates.

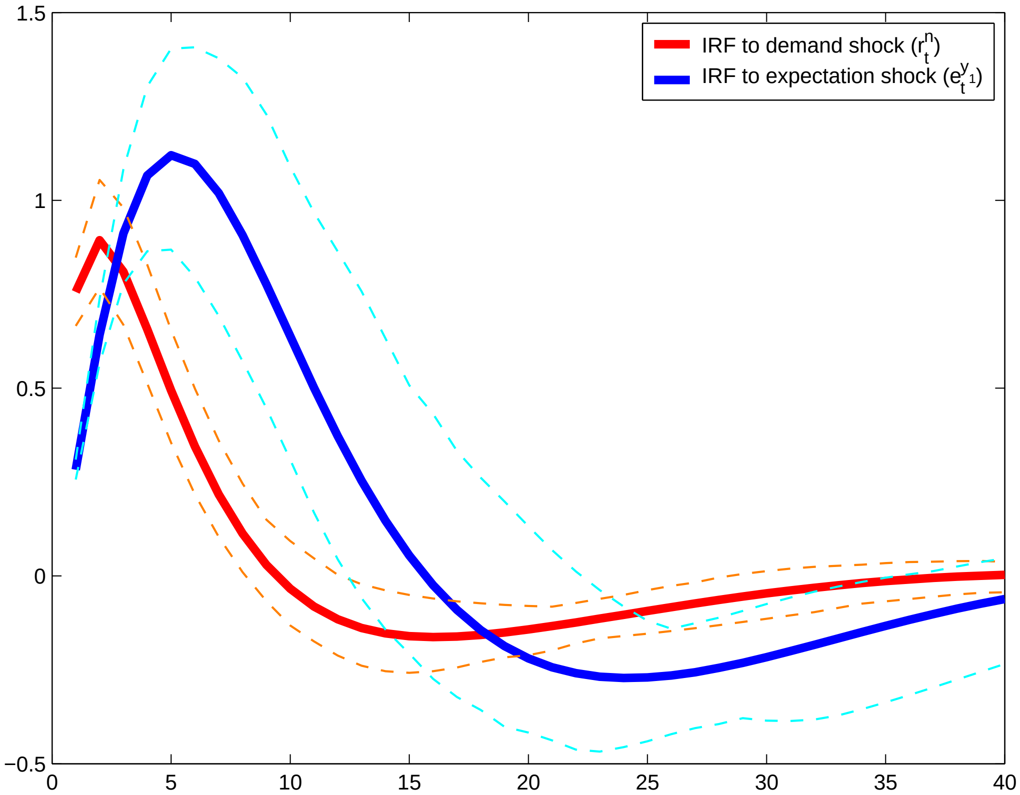

FIGURE 2. Impulse response function of the output gap to the natural rate shock and the expectation shock about future output.

Note: Solids lines in the figure denote mean impulse responses over the sample, calculated over the last 10,000 MCMC draws. Dashed lines denote 95% error bands.

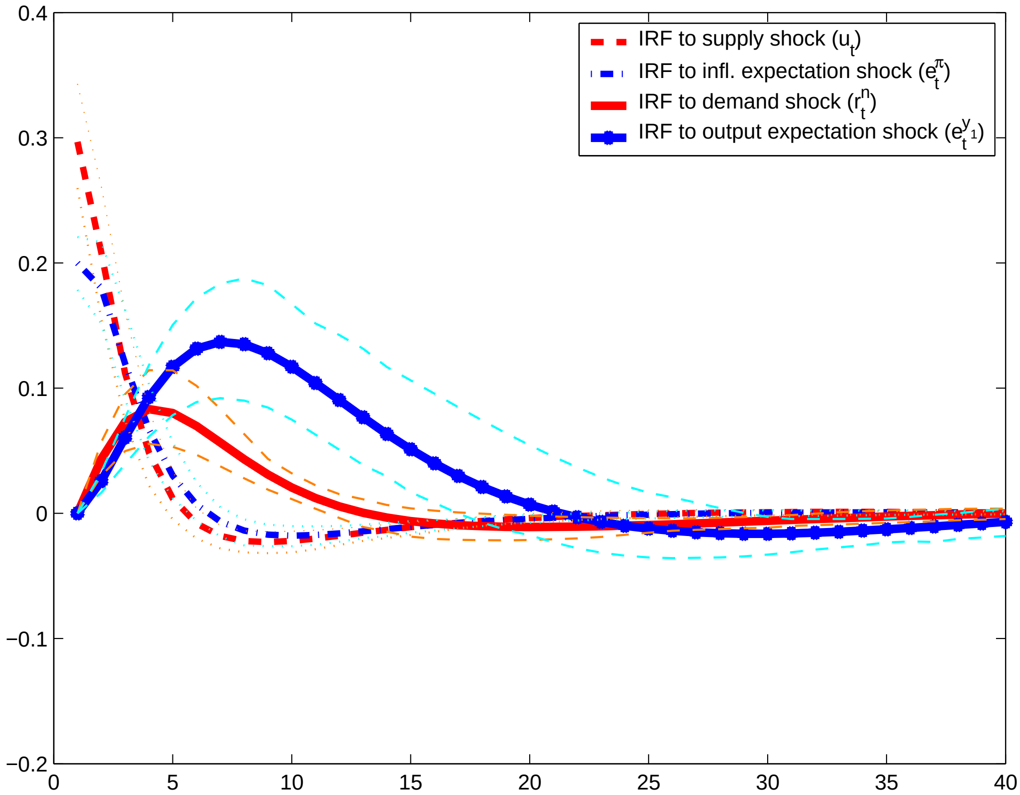

FIGURE 3. Impulse response function of inflation to the cost push-shock, to the natural rate shock, and to the expectation shocks about future output and about future inflation.

Note: Solid lines in the figure denote mean impulse responses over the sample, calculated over the last 10,000 MCMC draws. Dashed lines denote 95% error bands.

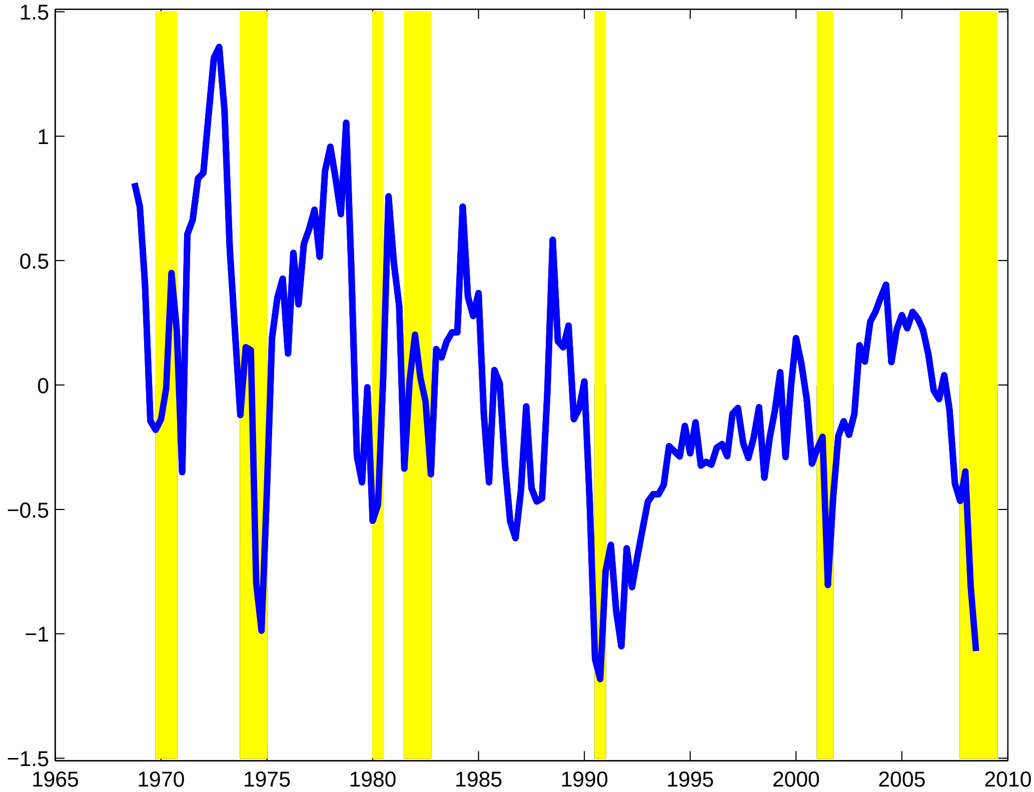

FIGURE 4. Expectation shock about future real activity ($e_t^{y_1}$) and NBER recession dates (yellow vertical bands).

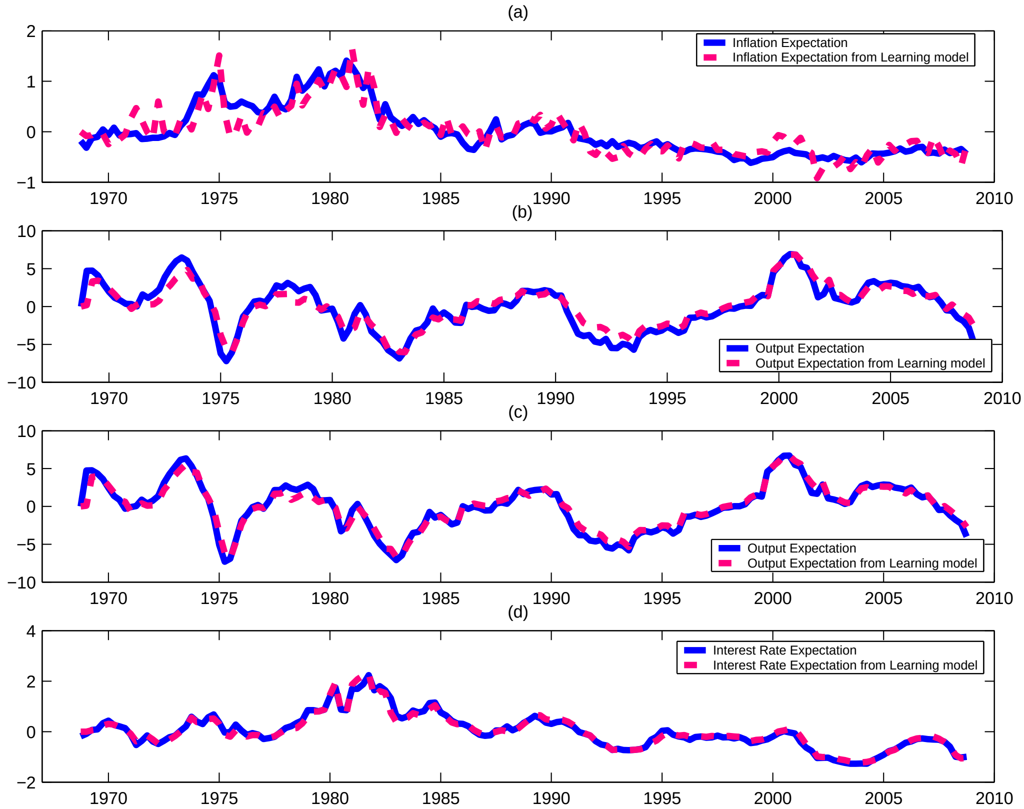

FIGURE 5. Observed Survey Expectations and Expectations from economic agents' PLM.

Note: The first panel compares inflation expectations ($\widehat{E}_{t-1}\pi_{t+1}$) from the Survey of Professional Forecasters (solid line) and the near-rational expectations from the agents' learning model (dashed line), obtained as averages across MCMC draws. The second and third panels show output gap expectations ($\widehat{E}_{t-1}y_{t+1}$ and $\widehat{E}_{t-1}y_t$) from the Survey of Professional Forecasters and near-rational expectations from the learning model. The third panel shows interest rate expectations ($\widehat{E}_{t-1}i_t$) extracted from the term structure of interest rates along with expectations from the learning model.