

# Technical Appendix to "Position-specific Information in Social Networks: Are You Connected?"

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## Abstract

This appendix provides supplemental results to those presented in "Position-specific Information in Social Networks: Are You Connected?" *Mathematical Social Sciences* 56: 283-295, 2008

## A Efficiency

**Proposition 1** *Fix  $v \in V$ . If  $\sum_{i \in N} v_i > c$  then the set of efficient networks is the set of minimally connected networks. If  $\sum_{i \in N} v_i < c$  then the unique efficient network is the empty network.*

**Proof.** Suppose a non-empty efficient network. Minimality follows directly since the distance between links does not matter so long as a path exists. Connectedness also follows: if an efficient network had a component  $N_i$  with  $n_i < n$ , then it must generate value  $\sum_{i \in N_i} v_i$  greater than cost  $c$ ; but adding a link between  $i$  and  $j \notin N_i$  will generate value  $\sum_{i \in N_i} v_i$  to  $j$ , which means net social utility must go up since the link costs only  $c$ . Thus, any non-empty efficient network must be minimally connected.

Given  $v$ , whether an efficient network is empty or minimally connected will depend on which generates the highest sum of utilities. Any minimally connected network generates

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$n \sum_{i \in N} v_i - (n - 1)c$ , and the empty network generates  $\sum_{i \in N} v_i$ . Comparing these gives the proposed condition. ■

## B Complete Characterization of Full Information Nash Equilibrium Networks

In the full information case, each player fully observes  $s$  and  $v$  in equilibrium. Thus, no  $i$  will change her link decisions if, given  $v$ , her choice  $s_i$  is a best response to what the others actually do  $s_{-i}$ . This is exactly a (pure) Nash equilibrium:

**Definition 1** *Fix  $v$ . A (pure) Nash Equilibrium (NE) of the network game is a strategy profile  $(s_i^*)_{i \in N}$  such that for each  $i \in N$ ,  $u_i(s_i^*, s_{-i}^* | v) \geq u_i(s'_i, s_{-i}^* | v) \forall s'_i \in S_i$ .*

It follows that the set of network equilibria under full information is the set of NE. An additional definition will help to characterize this set.

**Definition 2** *Fix  $(v, s)$ . Say that component  $N_i$  in  $s$  has a low-valued, link-receiving sub-component (LLS) if there exists an  $i, j \in N$  with  $s_{ij} = 1$ , such that (i) setting  $s_{ij} = 0$  partitions  $N_i$  into two separate components  $N'_i$  and  $N'_j$ , and (ii)  $\sum_{k \in N'_j} v_k < c$ .*

Intuitively, any  $i$  would remove a link to  $j$  if  $i$  knows  $j$  is in a LLS since the link provides marginal benefits less than the marginal cost  $c$ . We can now describe  $E_{full}(v)$ , the set of network structures that can be sustained as equilibria under full information given type profile  $v$ .

**Proposition 2** *Fix  $v$ . If  $v_i < c$  for all  $i$ , then  $E_{full}(v)$  contains the empty network and all LLS-free minimally connected networks. If  $v_i > c$  for at least one  $i$ , then  $E_{full}(v)$  contains all LLS-free minimally connected networks.*

The main logic is straightforward and follows from that used by Bala and Goyal (2000) in their examination of the symmetric types case. If the network is not minimal, then

there must be a redundant link that could be removed to make someone better off. Thus, any network must be minimal. If the network is not empty but is disconnected, and if all individuals in their disconnected components prefer remaining in their component to removing all their links, then connecting separate components makes all parties strictly better off. The reason is that the value of any component must exceed  $c$  for an individual to want to remain in it, and this implies that connecting separate components will be in all parties' interests. Finally, if all types are low valued then the empty network is an equilibrium because initiating a link to an isolated player yields few benefits.

We can say more about  $E_{full}(v)$  and obtain intuition about the model by making assumptions about  $v$ . In general, as the  $v_i$ 's increase, the set of Nash networks increases, i.e.,  $E_{full}(v) \supseteq E_{full}(v')$  if  $v_i \geq v'_i$  for all  $i$ . The intuition follows from the proposition: if you pick an  $s$  with LLSs and then increase the values, then those LLSs might no longer be LLSs, which would mean they are sustainable as an equilibrium. In fact, the number of equilibria becomes extremely large as more players become high-valued ( $v_i > c$ ) because a link-receiving subcomponent with high-valued player is high-valued and not a LLS. The following corollary, though not comprehensive, illustrates this basic logic.

**Corollary 1:** *Fix  $v$ .*

(a) *If  $\sum_{j \neq i} v_j < c$  for at least two  $i$  and  $\sum_{j \neq i} v_j \geq c$  for all other  $i$ , then the empty network is the unique Nash network.*

(b) *If  $\sum_{j \neq i} v_j < c$  for exactly one  $i$  and  $\sum_{j \neq i} v_j \geq c$  for all other  $i$ , then the connected, periphery-sponsored star with  $i$  in the center is the only Nash network.*

(c) *If  $v_i \geq c$  for all  $i$ , then  $E_{full}(v)$  contains the set of all minimally connected networks.*

Proving this corollary is straightforward using logic similar to that used by Bala and Goyal (2000) when examining the symmetric case. The key difference is that  $\sum_{j \neq i} v_j$  is the largest possible benefit that  $i$  can receive in a network instead of  $(n - 1)v_0$ . Yet, as in the

symmetric case, if this sum is less than the cost of forming one link for all  $i$ , then there is no scenario in which any  $i$  will form any links, and the empty network is the only equilibrium network. If, on the other hand,  $v_i \geq c$  for all  $i$ , then  $\sum_{j \neq i} v_j$  is also greater than  $c$ , and any subcomponent must not be low-valued. Any minimally connected network would now be an equilibrium.

Also note that since any minimally connected network is efficient, any non-empty equilibrium is efficient, and these efficient equilibria always exist except when the  $v_i$ 's are extremely low. Thus, the full information NE comprise a useful benchmark. If we have inefficient non-empty equilibria under incomplete information or imperfect monitoring, then it will be due to the change in information available to the players. Of course, for some  $v$ , the empty network might be both the only equilibrium and inefficient. This occurs when both  $\sum_{i \in N} v_i > c$  but and  $v_i < c$  for all  $i$ . Standard reasoning applies: an individual considers only her own marginal benefits of a link and not the social benefits, which due to the positive externalities will exceed her marginal benefits.

## C Decrease in Observation

An immediate implication of the GCE and  $x/y$ -link definitions is that any equilibrium structure under a certain level of information is also an equilibrium under less information.

**Lemma 1** *Fix  $v$ . Then  $E_{x/y}(v) \subseteq E_{x'/y'}(v)$  for all  $x'$  and  $y'$  such that  $1 \leq x' \leq x$  and  $0 \leq y' \leq y$ .*

The proof is trivial. Larger observation places greater restrictions on the equilibrium beliefs. If a network meets the stricter restrictions of higher observation it will meet the looser restrictions of lower observation, and the additional restrictions under  $x$  and  $y$  will further refine the set of equilibria, so that there may be equilibria in  $E_{x'/y'}(v)$  not in  $E_{x/y}(v)$ . According to this lemma, limited observation does not necessarily prevent a network from achieving what it could achieve in equilibrium under full information. However, when

observation is limited, individuals may sustain incorrect beliefs in the form of assigning non-zero probability to a state  $s'$  not equal to the true state  $s^*$  in equilibrium.

## D Decay with 1/0-link Observation

**Proposition 3** *Fix  $v$ , and suppose there is flow decay  $\delta$ .  $E_{1/0}(v)$  consists of all  $s^*$  without direct redundant links such that  $u_i(s^*|v) \geq v_i$  for all  $i$ .*

**Proof.** *Necessity.* Follows directly since any redundant link would be within a player's observational range, and a player who does not meet the participation constraint is better off removing all link initiations.

*Sufficiency.* Consider a network that does not have redundant links and where the participation constraint is met. Let  $\pi_i^*$  assign probability 1 to  $s'$  where  $N_i = N_i^{1/0}$  and all others are isolated, and probability 1 to  $v'$  where  $v'_i = v_i$ ,  $v_j = \frac{u_i^* - v_i}{|I_i|} + c$  with  $\varepsilon > 0$  small for all  $j \in I_i$ , and  $v_k = 0$  for all  $k \notin N_i^{1/0}$ . With such beliefs, equilibrium conditions (i)-(iii) are met for each  $i$ . ■

Note that the wording used to describe the set of equilibrium networks in Corollary 2 and Proposition 8 is identical. The only difference is that decay reduces the overall value of most components, thus making fewer networks meet the participation constraint. Thus, the set of equilibria with flow decay may be smaller than that without flow decay even though I describe them in similar manners. The idea is akin to that of link switching used to find strict equilibria except here it is link addition instead of switching. A player must observe the presence of another link and be able to tell if the marginal benefit of having a direct link to someone already indirectly connected is better than the marginal cost of the link. With  $x = 1$  and  $y = 0$ , she is not aware of this opportunity. Also notice that increasing  $x$  and  $y$  leads to more efficient networks even if it means more cycles. This contrasts with the elimination of smaller cycles as  $x$  increases when there is no flow decay.

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