Dynamic Evolution of Income Distribution and Credit-Constrained Human Capital Investment in Open Economies

Priya Ranjan[1]

Department of Economics

University of California, Irvine

Irvine, CA 92697

Abstract

This paper shows how the degree of credit-market imperfections affects the steady-state distributions of income and wealth, human capital investment, and the pattern of comparative advantage. The impact of trade liberalization on the accumulation of human capital depends on how it affects 1) the incentives to accumulate human capital; 2) the borrowing constraints facing human capital accumulation; and 3) the distribution of income and wealth. If the degree of credit market imperfections is low in the skill-abundant countries and high in the skill-scarce countries, then trade liberalization can increase investments in human capital in both types of countries.

Keywords: International Trade, Credit-Constraints, Human Capital, Income Distribution (JEL F11, O15)

[1] I would like to thank an anonymous referee and a co-editor for helpful comments. I would also like to thank J. Bhagwati, R. Findlay, A. Newman, and seminar participants at Rice, Toronto, UC-Berkeley, UC-Irvine, UCLA, and UC-Riverside for helpful suggestions and discussions on earlier versions of this paper. All remaining errors are mine.

Address for correspondence: Department of Economics, 3151 Social Science Plaza, Irvine, CA92697. Phone (949) 824-1926, Fax 949-824-2182, e-mail pranjan@uci.edu

1 Introduction

It is widely recognized that differences in human capital or skill across countries play an important role in determining differences in growth rates and per capita income levels across countries. As well, skill differentials form the basis of much of trade between the skill-abundant developed countries in the North and the skill-scarce developing countries in the South[1]. While trade based on skill differential always provides static gains from specialization, the dynamic gains depend on how the investment in human capital is affected. This makes the question of how trade affects skill accumulation an important one.

In a pioneering paper, Findlay & Kierzkowski (1983) extended the static Heckscher-Ohlin model of trade by endogenizing human capital accumulation to show that trade amplifies initial differences in factor endowments. The channel of influence is the Stolper-Samuelson effect of trade on factor prices which raises the reward of the abundant factor in each country. This provides further incentives to accumulate human capital in skill-abundant countries and does the opposite in skill-scarce countries. Similar results were also obtained by Grossman and Helpman (1991). Two recent papers -Cartiglia (1997) and Eicher (1999)- show that trade leads to convergence in human capital endowments. A key element of both these papers is that skill is used in the formation of skill (education sector uses skilled labor), and therefore, any rise in the price of skill has an adverse effect on skill accumulation. In Cartiglia (1997) the credit market is missing, therefore, investment in human capital has to be self-financed from the initial endowment. A trade induced rise in the skilled wage in a skill-abundant country increases the cost of education and hence exacerbates the borrowing constraint facing investment in human capital. In Eicher (1999) there is a domestic credit market where the savings of the unskilled workers are used to finance the investment in human capital. Now, a trade induced decrease in the unskilled wage in a skill-abundant country reduces the resources available for financing investment in human capital, while a rise in the cost of education increases the need for resources to finance investment in human capital[2]. Therefore, in both these papers trade liberalization reduces the investment in human capital in a skill-abundant country. The opposite happens in a skill-scarce country giving rise to the convergence in human capital endowments after opening up to trade. This literature, however, has ignored the impact of trade on investments in human capital coming through changes in the distribution of income and wealth. This is a serious omission because the distribution of wealth becomes an important determinant of investments in human capital in the presence of borrowing constraints. The main contribution of our paper lies in bringing the distributions of income and wealth to the centre of the discussion of the impact of trade on factor endowments. It endogenizes the wealth distribution in the presence of credit market imperfections and shows that changes in the distribution of wealth brought about by trade have additional and, in some cases offsetting effects on the accumulation of human capital.

It is first shown that there exists a unique invariant steady-state distribution of wealth for a given degree of credit-market imperfection. A decrease in the degree of credit-market imperfection implies an improvement in the steady state distribution of wealth in the first order stochastic dominance sense. The degree of credit-market imperfections affects investments in human capital both directly and indirectly by improving the steady-state distribution of wealth, and hence, it becomes a determinant of the pattern of comparative advantage. This new result shows how an institutional variable like the degree of credit-market imperfections can become a determinant of the pattern of comparative advantage.

Next it is discussed how trade liberalization affects investments in human capital by, among other things, altering the distribution of wealth. Trade liberalization increases the incentive to invest in human capital in a skill-abundant country. However, an increase in the skilled wage raises the cost of education, which worsens the borrowing constraint in a world with credit-market imperfections. In addition, the support of the steady-state distribution of wealth is widened. The widening of support at the bottom end has a negative impact on the investment in human capital. Intuitively, trade liberalization reduces the unskilled wage, which reduces the bequest of the unskilled, making their descendants more prone to borrowing constraint. The net effect on investments in human capital depends on the relative strengths of these three effects. The opposite happens in a skill-scarce country. There the negative incentive effect has to be balanced against the positive effect arising from the distributional changes and the relaxation of borrowing constraint due to reduced cost of education. The support of the steady state distribution of wealth shrinks to a smaller interval. The increase in the wealth of individuals at the bottom end caused by a rise in the unskilled wage has a positive impact on the investment in human capital of their descendants. The unskilled are able to leave a larger bequest, which relaxes the borrowing constraint for their descendants.

Finally, it is shown that if the degree of credit market imperfections is very low in the skill-abundant countries and very high in the skill-scarce countries, then trade liberalization is likely to increase investments in human capital in both types of countries. In the presence of a very low degree of credit market imperfections the positive rate of return effect will dominate in the skill-abundant countries, while in the presence of a very high-degree of credit market imperfections the positive effect arising from distributional change and the relaxation of borrowing constraint will dominate in the skill-scarce countries. Using data on the degree of contract enforcement as an indicator for the degree of credit market imperfections, we find that countries with low human capital endowment have a very high degree of credit market imperfections, while countries with high human capital endowment have a very low degree of credit market imperfections. A companion paper (Ranjan, 1997) using cross-country data finds a positive relationship between openness and investments in human capital in both rich and poor countries.

The present paper differs from the existing literature as follows. Unlike Eicher (1999) which is a representative agent type model, we allow for heterogeneity among agents in the inherited wealth and the ability to accumulate human capital. Cartiglia (1997) allows agents to have differing initial wealth but identical ability. Due to the assumption of identical ability in Cartiglia (1997), only the credit constraints are binding, while the incentive effect of trade on investments in human capital that gives rise to divergence in Findlay and Kierzkowski (1983) and Grossman and Helpman (1991), is non-binding both before and after trade liberalization. By allowing for heterogeneity in ability our paper allows both the incentive constraints and the borrowing constraint to be binding and is sufficient to generate the ambiguous effect of trade on factor endowments. Also, unlike Cartiglia (1997) where credit markets are completely missing, we model credit-market imperfections explicitly as arising from information problems. The most important difference is that the distribution of initial wealth, which is exogenous in Cartiglia (1997) and does not matter in other models due to perfect credit market assumption, evolves endogenously in our model through intergenerational bequests, and provides additional channels of influence of trade on human capital endowments. Like Cartiglia (1997) and Eicher (1999) we have also assumed that the cost of education depends on the skilled wage, however, the effect of trade working through changes in the distribution of wealth would arise even if the cost of education is independent of the skilled wage.

The plan of the rest of the paper is as follows. Section 2 sets up the basic model. Section 3 discusses the impact of trade liberalization on the steady state distribution of wealth and investments in human capital. Section 4 presents some concluding remarks.

2 The Model

The model spelled out below captures the elements of trade based on differences in skill endowments, making it suitable to analyze the impact of trade on skill accumulation.

2.1 Technology and Preferences

The economy can produce two final goods, H and L, with two factors of production: skilled labor, S, and unskilled labor, U. Both final goods employ both factors of production with a constant returns to scale technology. H is the high-tech good which is more skill intensive than the low-tech good L at all factor prices. L is also the numeraire good. The production functions are given by

$$H = A^h F^h(S_h, U_h) \tag{1}$$

$$L = A^l F^l(S_l, U_l) \tag{2}$$

In (1) and (2) above A^h and A^l are the productivity parameters, changes in which can capture biased technical changes.

The factor supplies are endogenously determined through the occupational choices of individuals. We assume a small open economy which takes the relative price of goods as given. The relative price of the high-tech good is denoted by p. Denote the skilled wage per efficiency unit of skill by w_s and unskilled wage by w_l. Given the above production structure, relative goods price fixes the relative factor price[3], $\omega = \dfrac{w_s}{w_l}$.

The education sector is modeled in a simple fashion to capture the fact that skill is used in the formation of skill. Denote the number of students by S and the amount of skill employed in the education sector to educate students by S_E, and assume the following functional form

$$S = QS_E \tag{3}$$

In (3) Q is a parameter capturing the productivity of the education sector. The above functional form for the education sector leads to a cost of education per student equal to qw_s, where $q = \frac{1}{Q}$[4]. The students in period t become skilled workers in period $t+1$. The total amount of skill in efficiency units is denoted by $\overline{S}$, which is allocated between production $(S_h + S_l)$ and education (S_E) sectors: $\overline{S} = S_h + S_l + S_E$.

The amount of human capital or skill that an individual acquires upon going to school depends on the talent that an individual is born with. We assume that the amount of skill acquired is independent of the educational inputs. We do so for two reasons: 1) to keep the analysis simple; and 2) the empirical evidence on the relationship between educational input and student performance is far from conclusive[5].

Each individual lives for two periods. The population born in each period is normalized to have measure 1. There is no population growth. Each individual is born with an endowment, $a_i \in [\underline{a}, \overline{a}]$, of the numeraire good and a talent or ability to acquire human capital, $\sigma_i \in [\underline{\sigma}, \overline{\sigma}]$. σ_i can be thought of as the efficiency units of human capital that an individual acquires upon going to school. The talent of each individual is public knowledge. The labor income of a skilled individual with talent σ_i is $\sigma_i w_s$. Denote the distribution function of talent by $F(\sigma)$, and the corresponding density function by $f(\sigma)$. The distribution of endowments evolves endogenously and is derived below.

In the first period an individual can either go to school or work as an unskilled worker. An individual who goes to school must pay an education cost of qw_s, and becomes a skilled worker in the second period. The unskilled work in both periods. For simplicity it is assumed that all consumption takes place in the second period only[6]. The unskilled save their first period wage and endowment. Each individual has a child in the second period of his life. All parents work in the second period of their lives, while some children work as unskilled and others go to school in the first period of their lives. Parents care about their children and leave them a bequest. We assume that parents have warm glow preferences over bequests. That is parents derive utility by giving to their children, independently of the extent to which their children actually benefit from the bequest. Apart from being analytically tractable, the warm glow preference for bequests formulation seems to have better microfoundations than the Ricardian formulation (see Andreoni, 1989).

In period t, young inherit a_t from their old parent. A simple form of utility function is assumed with C as an index of consumption and a being the bequest.

$$V = C^{\beta} a^{1-\beta} \tag{4}$$

$$C = C_h^{\theta} C_l^{1-\theta} \tag{5}$$

In (5) C_h is the consumption of the high-tech good, C_l is the consumption of the low-tech good. This specification of the utility function makes the indirect utility linear in income, and hence the expected utility is also linear in income, which makes the analysis of credit-market imperfections simple.

Denoting the market rate of interest by r, the incomes of unskilled and skilled individuals in the second period of their lives can be written as

$$y_u = (2+r)w_l + (1+r)a \tag{6}$$

$$y_s = \sigma w_s + (1+r)(a - q w_s) \tag{7}$$

If credit markets are perfect, then an individual decides to acquire skill or remains unskilled depending solely on his ability. Let us look at this decision for a given r, w_s and w_l. The lifetime utility from becoming skilled for an individual with endowment a_i and ability σ_i is given by

$$V_S^i = C^{\circ} * (\sigma_i w_s + (1+r)(a_i - q w_s)) \tag{8}$$

where C° is a constant which depends on the parameters of the utility function and the product price ratio p. Similarly, the lifetime utility of an individual deciding to remain unskilled is given by

$$V_U^i = C^\circ * (w_l + (1+r)(a_i + w_l)) \tag{9}$$

In equilibrium the marginal individual is indifferent between acquiring skill and remaining unskilled: $V_S^i = V_U^i$. This implies a threshold level of ability, σ^* given by

$$\sigma^* = \frac{(2+r)w_l + (1+r)qw_s}{w_s} \tag{10}$$

such that all individuals with $\sigma_i > \sigma^*$ invest in human capital by going to school, while the others remain unskilled.

2.2 Imperfection in the Credit-Market

We make the following assumption about the rate of interest.

Assumption 1*:* Due to free international capital mobility, the individuals in this small open economy can lend any amount at the world rate of interest r.

They cannot borrow any amount at this rate of interest, however, due to the imperfections in the credit market described below.

A very simple form of imperfection in the credit market is assumed. The credit market is characterized by the possibility that a borrower may renege on a debt. To abstract from the bankruptcy issues assume that the parameters are such that the borrower can always afford repayment. The borrower can renege at the time of repayment. The borrower succeeds in fleeing with probability π, in which case she consumes her entire second period skilled wage σw_s. The borrower is caught with probability $1-\pi$, in which case her entire second period income is taken away by the lender. Reneging, therefore, yields an expected payoff of $\pi C^0 \sigma w_s$, while repaying yields a payoff of

$C^0(\sigma w_s - (1+r)(qw_s - a))$. Therefore, lenders will make loans that satisfy

$$C^0(\sigma w_s - (1+r)(qw_s - a)) \geq \pi C^0 \sigma w_s \tag{11}$$

Equation (11) implies the following threshold level of wealth for each ability

$$a^*(\sigma) = qw_s - \frac{1-\pi}{1+r}\sigma w_s = w_s(q - \frac{1-\pi}{1+r}\sigma) \tag{12}$$

Equation (12) can be written in an alternative form to show the threshold level of ability required for each level of wealth as follows.

$$\sigma(a) = \frac{(1+r)(qw_s - a)}{(1-\pi)w_s} = \frac{(1+r)q}{1-\pi} - \frac{(1+r)a}{(1-\pi)w_s} \tag{13}$$

(12) implies that for each level of σ, individuals having $a < a^*(\sigma)$ can not borrow adequately to invest in human capital. Individuals having $\sigma > \sigma^*$ and $a < a^*(\sigma)$ are rationed in the credit market in the sense that they would like to invest in human capital, but can not borrow enough to do so. The parameter π captures the degree of credit-market imperfections in the model. $\pi = 0$ corresponds to the first-best case, where the borrowing constraint does not bind because from (10) for $\sigma > \sigma^*$ we have $\sigma > (1+r)q$, therefore, if $\pi = 0$, then $a^*(\sigma) < 0$ in (12). The higher the π the lower the amount that individuals can borrow against their future earnings, and hence the more severe the borrowing constraint. In the extreme case if $\pi = 1$, individuals can not borrow at all against their future earnings, and therefore, education has to be completely self-financed.

It can be easily seen from (12) that the threshold level of collateral is decreasing in the ability of individuals. What this implies is that the higher the ability of an individual the less likely the individual is to be credit constrained. The intuition for this result is simple. The threshold level of wealth in (12) is the gap between the cost of education and the amount that individuals can borrow against their future earnings. The cost of education is unrelated to the ability of an individual, however, the amount they can borrow against their future earnings is positively related to their ability because their future earnings is positively related to their ability. Further, (13) makes it clear that wealthy individuals (those with high a) are less likely to be credit rationed because $\tilde{\sigma}$ is lower for them.

Equation (13) and (10) together imply that in the presence of credit market imperfections only individuals with $\sigma > \max\{\tilde{\sigma}(a), \sigma^*\}$ invest in human capital; others remain unskilled.

2.3 Distributional Dynamics

Now we look at the evolution of wealth distribution for this economy. We begin by considering the long run evolution of lineage wealth for a single lineage in this economy. We show that the probability distribution of lineage wealth converges to a unique stationary distribution. This stationary distribution can be interpreted as the steady state wealth distribution for the economy since all lineage wealth processes are identically and independently distributed, and since there is a continuum of lineages. To establish the convergence of the probability distribution of lineage wealth to a unique stationary distribution we appeal to results of convergence for monotonic Markov processes in Hopenhayn and Prescott (1992). Denoting the bequest function by $b(a, \sigma)$, the evolution of lineage wealth can be written as

$$a_{t+1} = b(a_t, \sigma_t) \tag{14}$$

Further, using the superscript ^{u} to denote the bequest of an unskilled parent and ^{s} to denote the bequest of a skilled parent, $b(a, \sigma)$ can be written as

$$\begin{aligned} b^u(a) &= (1-\beta)((1+r)a + (2+r)w_l) \ if \ \sigma < \max\{\tilde{\sigma}(a), \sigma^*\} && \text{(15a)} \\ b^s(a, \sigma) &= (1-\beta)(\sigma w_s + (1+r)(a - qw_s)) \ if \ \sigma \geq \max\{\tilde{\sigma}(a), \sigma^*\} && \text{(15b)} \end{aligned}$$

It should be noted from (15a) that the bequest of an unskilled parent does not depend on the level of ability of the parent. Let $\overline{a}$ and $\underline{a}$ be the highest and lowest sustainable wealth levels given as follows.

$$\overline{a} = \frac{(1-\beta)(\overline{\sigma}w_s - (1+r)qw_s)}{1-(1-\beta)(1+r)} \tag{16}$$

$$\underline{a} = \frac{(1-\beta)(2+r)w_l}{1-(1-\beta)(1+r)} \tag{17}$$

In order for $\underline{a}$ and $\overline{a}$ to be non-negative it is assumed that $(1-\beta)(1+r) < 1$. Since an individual with ability σ^* is indifferent between becoming skilled and remaining unskilled, the labor income of a skilled individual (net of the cost of education) with ability σ^* is equal to the labor income of an unskilled individual: $\sigma^* w_s - (1+r)qw_s = (2+r)w_l$. Therefore, (17) can be written as

$$\underline{a} = \frac{(1-\beta)(\sigma^* w_s - (1+r)qw_s)}{1-(1-\beta)(1+r)} \tag{18}$$

Define $\boldsymbol{b}$ as the level of ability such that $a^*(\boldsymbol{b}) = \underline{a}$ or alternatively, $\boldsymbol{e}(\underline{a}) = \boldsymbol{b}$. Assume the following.

Assumption 2 $\underline{\sigma} < \sigma^* < \boldsymbol{b} < \overline{\sigma}$

$\boldsymbol{b} < \overline{\sigma}$ ensures that individuals born with the highest possible ability can always find ways to invest in human capital no matter how poor they are born. This condition provides intergenerational mobility in the model. If this condition is not satisfied, then there will be a poverty trap in the model: Once an individual in any generation is unskilled, all his descendants are going to remain unskilled. Since, due to random ability shocks everyone has a positive probability of becoming unskilled, in the long run all individuals become unskilled, and the steady state distribution of wealth is concentrated at a single point $\underline{a}$. $\sigma^* < \boldsymbol{b}$ ensures that borrowing constraint binds for at least some individuals.

Now we are ready to show the existence of a unique invariant distribution when assumption 2 is satisfied. Let $A = [\underline{a}, \overline{a}]$ and let Ω denote the set of Borel subsets of A. Given that σ is i.i.d, the stochastic process of lineage wealth described by (14) is a stationary Markov process. The corresponding transition function: $P : A \times \Omega \rightarrow [0,1]$ is simply defined by

$$P(a,B) = prob\left\{b(a,\sigma) \in B\right\}, \text{ for all Borel subsets } B \in \Omega \tag{19}$$

The long run dynamic behavior implied by $P(.,.)$ is described by determining the existence of a unique invariant distribution G. For any wealth distribution $G(.)$, let $TG(.)$ be the Markov transformation of G defined by:

$$TG(B) = \int P(a,B)dG(a) \text{ for all Borel subsets } B \subset A$$

A wealth distribution G on A is invariant for P if for all Borel subsets $B \subset A$, one gets the following

$$TG(B) = G(B)$$

Figure 1 gives an idea of why an invariant distribution $G(.)$ exists for our Markov process of lineage wealth. We have plotted two bequest lines: $b^u(a)$ for the unskilled; and $b^s(a,\overline{\sigma})$ for the highest ability skilled. There is going to be a continuum of bequest lines in between for the skilled corresponding to each ability level ($> \sigma^*$) of the skilled. It can be seen from Figure 1 that if the lineage wealth at some date t is above $\overline{a}$ or below $\underline{a}$, then in finite time it will come back to the interval $[\underline{a}, \overline{a}]$. Once lineage wealth falls in this interval, it will remain in this interval forever. Thus, after a sufficient amount of time has elapsed, all lineages will find their wealth in the interval $[\underline{a}, \overline{a}]$. Figure 1 suggests that wealth lineages will move from any subset of $[\underline{a}, \overline{a}]$ to any other measurable subset of $[\underline{a}, \overline{a}]$. This is what gives a unique invariant distribution for lineage wealth. More formally the following can be proved.

Proposition 1 *Under assumption 2 there exists a unique invariant distribution G for the Markov process corresponding to $P(a,B)$. As well, for any initial wealth distribution G_0, the sequence $(T)^n(G_0)$ ($(T)^n$ is the nth iterate of T) converges to G.*

The proof is a straightforward application of Hopenhayn and Prescott (1992)'s analysis of existence, uniqueness and convergence properties of monotonic stochastic processes. The proof is contained in an appendix.

2.4 Pattern of Comparative Advantage

The model for the small open economy can be solved as follows. Given an exogenous product price ratio, p, factor prices are determined through the Stolper-Samuelson relationship between product prices and factor prices. Given the factor prices, the cost of education qw_s is determined. From Proposition 1 we know that starting from any initial distribution of wealth the economy will converge to a unique steady state distribution of wealth. Suppose that the steady state distribution of wealth is $G(a)$ defined over the support $[\underline{a}, \overline{a}]$. Recalling that $\hat{\sigma} = \tilde{\sigma}(\underline{a})$ is the level of ability above which an individual is unconstrained, the fraction of population investing in human capital in steady state is given below.

$$S = 1 - \int_{\underline{a}}^{\overline{a}} F(\max\{\sigma^*, \tilde{\sigma}(a)\})dG(a) = 1 - F(\sigma^*) - \int_{\sigma^*}^{\hat{\sigma}} G(a^*(\sigma))f(\sigma)d\sigma \tag{20}$$

The term under the integral on the right hand side in (20) captures the fraction of population that would like to invest in human capital, but is borrowing constrained. The amount of skill available in efficiency units is

$$\overline{S} = \int_{\sigma^*}^{\hat{\sigma}} [1 - G(a^*(\sigma))]\sigma f(\sigma)d\sigma + \int_{\hat{\sigma}}^{\overline{\sigma}} \sigma f(\sigma)d\sigma \tag{21}$$

Once, the endowment ratio is known, production of each good and the volume of trade can be easily calculated given the production functions and the utility function. A complete closed form solution with specific functional forms is given in the appendix.

Next we perform comparative statics with respect to π, the degree of credit market imperfection. It is easy to see what happens if $\pi = 0$. As discussed earlier, this corresponds to the first-best case, where borrowing constraint is not binding for any individual with ability greater than σ^*. Therefore, if $\pi = 0$, then $S = 1 - F(\sigma^*)$ and $\overline{S} = \int_{\sigma^*}^{\overline{\sigma}} \sigma f(\sigma) d\sigma$. The condition $\sigma^* < \hat{\sigma}$ mentioned in assumption 2 earlier ensures that the borrowing constraint binds for at least some individuals when $\pi > 0$.

In Figure 1 we draw a downward sloping curve to show the extent of borrowing constraint. The height of this curve is equal to $(1-\beta)(1+r)(a - qw_s) + (1-\beta)\tilde{\sigma}(a)w_s$, where $\tilde{\sigma}(a)$ (derived in equation (13)) is the threshold level of ability above which the borrowing constraint does not bind. The vertical gap between this curve and the bequest line corresponding to $b^u(a)(= b^s(a, \sigma^*))$, given by $(1-\beta)w_s(\tilde{\sigma}(a) - \sigma^*)$, can be understood as the extent of borrowing constraint for each level of wealth. If $\tilde{\sigma}(a) > \sigma^*$ for a particular a, it implies that the probability of this individual being credit constrained is $F(\tilde{\sigma}(a)) - F(\sigma^*)$. If $\tilde{\sigma}(a) < \sigma^*$, individuals with that level of wealth are unconstrained. The higher the wealth the lower the vertical gap between the two curves, and hence the lower the probability of being borrowing constrained. In Figure 1 individuals with $a > a'$ are unconstrained. The higher the π the higher the ability threshold, $\tilde{\sigma}(a)$, for each level of wealth, and thus higher the negatively sloped curve. Thus a higher π would imply a larger gap between the negatively sloped curve and the bequest line for the unskilled in Figure 1, implying a larger probability of being borrowing constrained. Therefore, each lineage has lower wealth for a longer time. The following Lemma is proved in the appendix.

Lemma 1 If $\pi' > \pi$, then the steady state distribution of wealth under π dominates the steady state distribution of wealth under π' in the first order stochastic dominance sense: $G_{\pi'}(a) \geq G_{\pi}(a)$ for all a.

We can see the impact of a greater degree of credit market imperfections on the investments in human capital from equation (20). The level of talent, $\hat{\sigma}$, above which credit constraint is non binding is increasing in π ($\frac{\partial \hat{\sigma}}{\partial \pi} > 0$). Further, $\frac{\partial a^*(\sigma)}{\partial \pi} > 0$ from (12). These two combined with the result in Lemma 1 imply that $\frac{dS}{d\pi} < 0$ and $\frac{d\overline{S}}{d\pi} < 0$. This gives us the result summarized in Lemma 2 below.

Lemma 2 If $\pi' > \pi$, then the economy with lower degree of credit market imperfections has greater skill endowment in the steady state.

Lemma 1 and Lemma 2 imply the following proposition which is proved in the appendix.

Proposition 2 *Ceteris paribus, if the degree of credit-market imperfections in an economy is less than that in the average economy, then under the condition of balanced trade the former exports the skill intensive good.*

This result shows the link between an institutional variable like the degree of credit market imperfections and the pattern of trade based on skill endowment differences.

It is straightforward to show that if the distribution of talent in an economy dominates that in another economy in a first order stochastic sense, then the former has greater fraction of population investing in human capital and greater skill endowment in efficiency units. Therefore, differences in the distribution of ability can become a source of comparative advantage as well.

3 Impact of Trade Liberalization

The impact of trade liberalization for a small open economy will work through changes in the relative price of the high-tech good, p. For an economy exporting skill intensive good, trade liberalization means an increase in p, which raises the skilled wage w_s and reduces the unskilled wage w_l through the standard Stolper-Samuelson effect. We recall from (20) that the fraction of population investing in human capital is

$$S = 1 - F(\sigma^*) - \int_{\sigma^*}^{b} G(a^*(\sigma)) f(\sigma) d\sigma$$

where $\hat{\sigma}$ is the level of talent above which the borrowing constraint does not bind. Therefore, the impact of trade liberalization depends on how σ^*, $\hat{\sigma}$, $a^*(\sigma)$ and $G(a)$ are affected. Let us use the subscript A to denote the value of a variable before liberalization and T to denote its value after liberalization. From (16) and (17) we see that the support of the steady state distribution of wealth widens from $[\underline{a}_A, \overline{a}_A]$ to $[\underline{a}_T, \overline{a}_T]$. The impact of trade liberalization on the investment in human capital depends, on among other things, how the steady-state distribution of wealth changes from $G_A(a)$ to $G_T(a)$. The change in the investment in human capital upon trade-liberalization is given by

$$S_T - S_A = \underbrace{F(\sigma_A^*) - F(\sigma_T^*)}_{I} - \underbrace{[\int_{\sigma_T^*}^{\hat{\sigma}_T} G_T(a_T^*(\sigma)) f(\sigma) d\sigma - \int_{\sigma_A^*}^{\hat{\sigma}_A} G_A(a_A^*(\sigma)) f(\sigma) d\sigma]}_{II} \tag{22}$$

The first term on the $r.h.s.$ in (22) is the change in the fraction of population that would like to invest, while the second term is the change in the fraction of population that can afford to invest. The magnitude of the second term depends on how the borrowing constraint and the steady-state distribution of wealth change, which in turn depend crucially on the degree of credit market imperfections.

It can be easily seen from (10) that σ^* is decreasing in w_s and increasing in w_l. If the reward from acquiring skill $(w_s - (1+r)qw_s)$ rises, and the opportunity cost $((2+r)w_l)$ falls, people with lower ability will find it worthwhile to invest in education. Therefore, $\sigma_A^* - \sigma_T^* > 0$, and hence $F(\sigma_A^*) - F(\sigma_T^*) > 0$. This captures the positive rate of return effect on investment in human capital. If $\pi = 0$, then this is the only effect of trade liberalization on investment in human capital in a skill abundant country. When $\pi > 0$, the second term in (22) is non-zero and is discussed below.

From equation (12) we note that $a_T^*(\sigma) > a_A^*(\sigma)$. The intuition is as follows. An increase in the skilled wage increases the cost of education for all individuals identically. However, the increase in the future earnings potential of the skilled depends on the amount of talent of they possess. From (12) it is clear that at low levels of talent, the increase in the future earnings potential is less than the increase in the cost of education, giving rise to an increase in the threshold level of wealth. Therefore, the borrowing constraint becomes tighter for each level of talent. The impact of this on the investment in human capital is qualitatively similar to the impact of a higher degree of credit market imperfections discussed in Lemmas 1 and 2. This effect by itself will reduce the investment in human capital and worsen the distribution of income in the first-order stochastic sense. This effect is qualitatively similar to the effects in Cartiglia (1997) and Eicher (1999) arising because the cost of education depends on the skilled wage. It should be noted that the magnitude of this effect depends, among other things, on the degree of credit market imperfections: $\frac{\partial(a_T^*(\sigma)-a_A^*(\sigma))}{\partial\pi} > 0$. The greater the degree of credit market imperfections the greater the increase in borrowing constraints.

Finally, as was mentioned earlier, the support of the steady state distribution of wealth widens from $[\underline{a}_A, \overline{a}_A]$ to $[\underline{a}_T, \overline{a}_T]$. This change can be viewed as a shift in the probability mass from the center ($[\underline{a}_A, \overline{a}_A]$) to the tails ($[\underline{a}_T, \underline{a}_A]$ and $[\overline{a}_A, \overline{a}_T]$) of the wealth distribution. It is easy to see that all individuals in the interval $[\underline{a}_T, \underline{a}_A]$ are more credit constrained than anyone in the interval $[\underline{a}_A, \overline{a}_A]$ due to lower wealth as well as increased cost of education. Therefore, investment in human capital is likely to fall because of this distributional change. What happens is that a decline in the unskilled wage reduces the bequest of the unskilled, which makes their descendants more prone to borrowing constraint. The individuals in the interval $[\overline{a}, \overline{a}_T]$ have greater wealth, but face a higher cost of education. In general, the impact of distributional change depends on how the entire distribution of wealth changes.

The net impact on the investment in human capital depends on the relative strengths of these effects. If π is very small, then the rate of return effect is going to dominate the distributional effect (in the limit when $\pi = 0$ the distributional effect vanishes), and hence the investment in human capital is going to increase. As seen earlier, the worsening of borrowing constraint is positively related with π, therefore, the smaller the π the smaller the negative effect coming from the worsening of borrowing constraint and distributional change. Thus, for a small π the net impact of trade liberalization on the investment in human capital for a skill-abundant country is likely to be positive.

The opposite happens in a country having a comparative advantage in the unskilled intensive good. In this case the return on investment in human capital declines and, therefore, the first term in (22) is negative. However, there is a decrease in the fraction of population that is borrowing constrained because of two reasons: the borrowing constraint declines for each level of talent due a decrease in the skilled wage, and there is a positive distributional effect coming from an increase in the unskilled wage that enables the unskilled to leave a higher bequest, which allows their descendants to overcome the borrowing constraint. The net effect again depends on the relative strengths of these effects. The larger the degree of credit market imperfections, the greater the decrease in the fraction of population that is borrowing constrained, and hence the net effect is more likely to be positive.

If the incentive effect dominates in the skill-abundant countries, while the positive effect arising from distributional changes dominates in the skill-scarce countries, then a trade liberalization will increase skill endowments in all trading partners. This outcome is more likely if the degree of credit market imperfections is very small in the skill-abundant country and very large in the skill-scarce country. In the general case, it is difficult give precise parametric restrictions under which there is an increase in investment in human capital in both skill-abundant and skill-scarce countries because the investment in human capital depends on the entire shape of the distribution of wealth. However, below we give a numerical example to illustrate the possibility of trade liberalization increasing investment in human capital in both the skill-scarce and the skill-abundant countries, and then we provide some empirical evidence to support our contention that the degree of credit market imperfections is very high in the South and low in the North.

3.1 A numerical example

Below we construct a tractable example by taking a simple distribution of talent, which under some restrictions on parameters, yields multiple non-overlapping intervals as the support of the steady-state distribution of wealth. Most importantly for our purposes here, the transition probabilities between these intervals and the probability of investing in human capital do not depend on the exact wealth of the individual, but only on the interval in which the wealth of the individual belongs. The key parameters in the numerical example take the following values. The ability or talent shock takes 3 values: $\sigma_h = 2$, $\sigma_m = 1.5$, and $\sigma_l = 1$, each with probability 1/3. $(1-\beta) = 0.2$, $(1+r) = 1.1$, $q = 0.5$. The other details of the numerical example are gathered in an appendix[7].

Table 1 summarizes the results of the numerical exercise. In each case the fraction of population actually investing is the difference between the fraction of population that wants to invest and the fraction that is borrowing constrained.

Case of a Southern Country: The parameters are such that before liberalization everyone wants to invest in human capital. However, due to a high degree of credit market imperfections ($\pi = .78$) not every one with talent σ_l or σ_m can invest in human capital. There is a level of wealth $a(\sigma_m)$ such that only those with wealth $a > a(\sigma_m)$ can invest in human capital upon receiving a shock of σ_m. $a(\sigma_l)$ is similarly defined for ability σ_l. The steady-state distribution of wealth is defined over three intervals $I_l = [\underline{a}, a_{l1}]$, $I_m = [a_{m1}, a_{h1}]$, and $I_h = [a(\sigma_l), \overline{a}]$. The dynamics of wealth distribution are depicted in Figure 2 which captures the fact that those with wealth in I_l can invest only if they get a shock of σ_h, those in I_m can invest only after receiving a shock of σ_m or σ_h, and those in I_h always invest. Therefore, some individuals with wealth in the intervals I_l and I_m are borrowing constrained. As shown in Table 1 the fraction of population investing in human capital is 5/9 in this case.

Now when this economy opens up to trade, due to its comparative advantage in the unskilled intensive good, trade liberalization raises unskilled wage and reduces skilled wage. The dynamics of wealth distribution after liberalization are depicted in Figure 3. The steady-state-distribution of wealth is given over the following 3 intervals now: $I_l = [\underline{a}_T, a(\sigma_m)_T)$, $I_m = [a(\sigma_m)_T, a_{l2}]$, $I_h = [a_{m2}, \overline{a}_T]$. Now individuals with ability σ_l do not want to invest because the return on education decreases. This will reduce the fraction of population investing in human capital. However, a decrease in the skilled wage, combined with an increase in the unskilled wage makes it possible for some individuals, who were unable to do so earlier, to invest in human capital after receiving a shock of σ_m. This reduces the fraction of population that is borrowing constrained, and hence increases investment in human capital. In Figure 3 the only people who are borrowing constrained are those in the interval I_l receiving an ability realization of σ_m. The net effect of trade liberalization, which is a sum of these two effects, is to increase the fraction of population investing in human capital from 5/9 to 3/5. Numerical exercise also confirms that the larger the degree of credit market imperfections the larger the fraction of population that is borrowing constrained, and hence the larger the decrease in the fraction of population that is borrowing constrained upon trade liberalization. Thus, the net effect of trade liberalization on human capital investment is more likely to be positive in a Southern economy the larger its degree of credit market imperfections.

<u>Case of a Northern country:</u> Suppose in the North parameters are such that before liberalization only those receiving talent realizations of σ_m or σ_h want to invest . Also, the borrowing constraint is non-binding for all individuals before liberalization ($\pi = .67$ has been assumed for North in the numerical example). In this case the fraction of population investing in human capital before liberalization is 2/3.

The dynamics of wealth distribution after liberalization in the North are depicted in Figure 4. The steady-state distribution of wealth in this case is defined over the intervals: $I_l = [\underline{a}_T, a(\sigma_m)_T)$, $I_m = [a(\sigma_m)_T, a(\sigma_l)_T)$, and $I_h = [a(\sigma_l)_T, \overline{a}_T]$. Figure 4 captures the fact that after trade liberalization the rate of return on human capital increases in such a way that everyone would like to invest in human capital. However, those with wealth in the interval I_l can not afford to invest if their ability realization is σ_m or σ_l, while those in the interval I_m can not afford to invest if their ability realization is σ_l. This happens because due to a decline in the unskilled wage some children of unskilled parents find their bequest inadequate to invest when their ability realization is low. This should be contrasted with the fact that before liberalization no one with ability σ_m was borrowing constrained (while those with ability σ_l did not want to invest before liberalization). Therefore, trade liberalization again produces two opposing effects. The positive rate of return effect has to be balanced against the negative effect coming from the increase in the fraction of population that is borrowing constrained. As shown in Table 1 the net effect for the chosen parametric configuration is positive: trade liberalization increases the fraction of population investing in human capital from 2/3 to 4/5. The numerical exercise also confirms that the negative effect arising from an increase in the fraction of population that is borrowing constrained is larger the larger the degree of credit market imperfections.

The proposition below summarizes the result on the impact of trade liberalization on credit-constrained investments in human capital when the distribution of wealth evolves endogenously.

Proposition 3 *The net effect of trade liberalization on investments in human capital depends on the relative strengths of the opposing effects coming from changes in the incentives to invest, and changes in the borrowing constraint and the distribution of wealth. If the degree of credit market imperfections is very low in the skill-abundant countries and very high in the skill-scarce countries, then trade liberalization is likely to increase the investment in human capital in both.*

The result above contrasts with the earlier results predicting divergence as in Findlay and Kierzkowski (1983) and Grossman and Helpman (1991) or a convergence as in Cartiglia (1997) and Eicher (1999). Also, in the last two papers it was shown that trade liberalization led to a decrease in the investment in human capital in the North. In contrast, our results imply that it is possible for the investment in human capital to increase in both the North and the South upon trade liberalization.

3.2 Some empirical evidence in support of the results

The degree of credit market imperfections in our theoretical model was captured by the probability of successful default. Empirically, this is going to depend on how well the contracts are enforced in a country. There are some data on the degree of contract enforcement collected by ICRG (see Knack and Keefer (1995) for details). We use one of their measures: Rule of Law (RLW). This indicator runs from 0 to 6. This variable reflects the degree to which the citizens of a country are willing to accept the established institutions to make and implement laws and adjudicate disputes. Therefore,it should be highly correlated with the degree of credit market imperfections. We find that this variable has a high positive correlation of .64 with the human capital endowments measured by the average years of secondary schooling of adult population. Also, for countries with human capital endowment in the bottom quartile (less than 0.44 years of average secondary schooling) the average value of RLW was 2.28, while for countries in the top quartile (greater than 1.72 years) the value was 4.7[8]. The difference between the two groups is almost one and half times the standard deviation for the entire sample. These numbers suggest that the degree of credit market imperfections is very high in countries with low skill endowment and very low in countries with high skill endowment, which is consistent with the conclusion in Proposition 3.

Our theoretical results depend on the link between wealth inequality and investment in human capital due to credit constraints, and the impact of trade on wealth inequality coming through the Stolper-Samuelson effect on wages. Williamson (1993) using data for 35 countries from 1960 to 1980 finds that secondary enrollment ratio is negatively correlated with inequality in the distribution of income (ratio of share of top 20% to bottom 40%) after controlling for other variables[9]. Our companion paper Ranjan (1997), using cross-country data on secondary and tertiary enrollment ratio and income inequality data from Deininger and Squire (1996) finds that greater inequality as measured by either Gini coefficient or fractile ratio (ratio of share of top 20% to bottom 20%) is significantly negatively related with investment in human capital after controlling for other variables like per capita income, regional dummies etc. Finally, Ranjan (1997) using several measures of openness finds positive correlation between openness and investments in human capital for both rich and poor countries after controlling for other determinants of investment in human capital, which is broadly consistent with the theoretical possibility shown in the numerical example that trade can increase investment in human capital in both trading partners.

4 Concluding Remarks

This paper constructs a dynamic general equilibrium model where the pattern of comparative advantage depends on the degree of credit market imperfections affecting human capital investments. Endogenizing wealth distribution by allowing for intergenerational transfers, the paper identifies a novel channel of influence of trade liberalization on investment in human capital. Changes in the distribution of wealth brought about by trade liberalization have additional and, in some cases offsetting effects on the accumulation of human capital. This makes it possible for trade liberalization to increase investments in human capital in both skill-abundant and skill-scarce countries. Therefore, trade liberalization could potentially yield dynamic gains to all trading partners.

Before ending the paper we discuss the reasons for introducing several special elements in the model and their implications. The reason for introducing heterogeneous ability is to allow for two way (both upward and downward) intergenerational mobility, which results in a unique invariant steady state distribution of income. In the absence of heterogeneous ability, there will be no intergenerational mobility, and the steady state distribution of income will be determined by the initial distribution of income as in Galor and Zeira (1993). However, heterogeneous ability alone is not sufficient to generate a unique steady state distribution. It is the partially open credit market along with heterogeneous ability that provides two way intergenerational mobility. Partially open credit market allows high ability individuals to invest in human capital even if they are born poor. If the credit market is completely missing, then heterogeneous ability will result only in downward mobility and no upward mobility. In this case in steady state everyone will become unskilled. Therefore, both heterogeneous ability and partially open credit market are essential features of the model. Also, the assumption that the ability of each individual is publicly known is a simplifying one, given our story of imperfection in the credit market. Without this assumption the contract in the credit market is much more complicated. However, modelling credit market imperfections in an alternative way a la Galor and Zeira (1993) will obviate the need for this assumption without changing any of the qualitative results in the paper. The reason for choosing our story of credit market imperfections is analytical tractability of the distributional dynamics. In our story, the bequest left by a skilled person does not depend directly on the degree of credit market imperfections, π, but using Galor and Zeira (1993) story will mean that for borrowers ($a < qw_s$) the bequest depends on the borrowing rate of interest i. For lenders the relevant rate of interest will continue to be r. Therefore, even among educated bequest functions differ depending on whether they are borrowers or lenders making the distributional dynamics more complicated.

The assumption that skill is used in the formation of skill is also not crucial for getting the new insights about trade affecting investment in human capital through changes in the distribution of wealth. Even if the direct cost of education is just a fixed amount of numeraire good, the changes in the steady state distribution of wealth will produce similar effects. Also, we can completely get rid of the direct cost of eduction, and have just the opportunity cost which is foregone wages. If we allow consumption in the first period, similar results will appear. Now borrowing constraint will affect consumption possibilities of those born poor who would like to invest in education. If they can not borrow for consumption in the first period, high marginal utility of first period consumption may not justify investments in human capital. This formulation of the model will make it applicable to the cases where there are no direct costs of education, such as primary education in most countries or public education more generally.

One shortcoming of the paper is that the model developed here is one of a small open economy rather than a two country case. The main reason for doing this analytical tractability. The small open economy assumption allows us to solve the model for an exogenous product price ratio and rate of interest. This makes the distributional dynamics a simple linear Markov process. If the rate of interest and product prices are endogenous, the wealth dynamics will become non-linear making it extremely difficult to study the steady-state wealth distribution. Future research will try to address this issue.

Appendix

Proof of Proposition 1: The proof proceeds in several steps. We first establish the monotonicity and monotone mixing property of the transition function and then use results derived in Hopenhayn and Prescott(1992) to prove the existence, uniqueness and convergence to an invariant distribution.

Lemma A.1: (Monotonicity of P): The transition function $P(a, B)$ is increasing in its first argument a in the following (first order stochastic dominance) sense: For all $(a, a') \in A^2, a \leq a' \Rightarrow \forall x \in A, P(a', [\underline{a}, x]) \leq P(a, [\underline{a}, x])$.

Proof: Denote the bequest of an unskilled parent by $b^u(a)$ and of a skilled parent by $b^s(a,\sigma)$.

Definition: For each $a \in A$ and $x \in A$ define $\sigma''(a)$ as follows: $b^s(a, \sigma''(a)) = x$.

Clearly, if $\sigma > \sigma''(a)$, then $b^s(a,\sigma) > x$. Also, $b^s(a,\sigma) > b^u(a)$ $\forall \sigma \geq \sigma^*$, from the incentive compatibility for investing in skill. Also, since $b^s(a,\sigma)$ is increasing in a , if $a' \geq a$, then $\sigma''(a') \leq \sigma''(a)$. Further define I as an indicator function as follows

$$\begin{aligned} I_{b^u(a)<x} &= 1 \; if \; b^u(a) < x \\ &= 0 \text{ otherwise} \\ I_{\sigma''(a)>\max\left\{\tilde{\sigma}(a),\sigma^*\right\}} &= 1 \text{ if } \sigma''(a) > \max\left\{\tilde{\sigma}(a), \sigma^*\right\} \\ &= 0 \text{ otherwise} \end{aligned}$$

For all $(x,a) \in A^2$,

$$P(a, [\underline{a}, x]) = F\left\{\max\left\{\tilde{\sigma}(a), \sigma^*\right\}\right\} I_{b^u(a)<x} + I_{\sigma''(a)>\max\left\{\tilde{\sigma}(a),\sigma^*\right\}} \int_{\max\left\{\tilde{\sigma}(a),\sigma^*\right\}}^{\sigma''(a)} f(\sigma)d\sigma \qquad (23)$$

(23) can also be written as

$$\begin{aligned} P(a, [\underline{a}, x]) &= F\left\{\max\left\{\tilde{\sigma}(a), \sigma^*\right\}\right\} I_{b^u(a)<x} + \\ & \left[F(\sigma''(a)) - F\left\{\max\left\{\tilde{\sigma}(a), \sigma^*\right\}\right\}\right] I_{\sigma''(a)>\max\left\{\tilde{\sigma}(a),\sigma^*\right\}} \qquad (24) \end{aligned}$$

We can write a similar expression for $P(a', [\underline{a}, x])$. Furthermore, if $I_{b^u(a')<x} = 0$, then $I_{\sigma''(a')>\max\left\{\tilde{\sigma}(a'),\sigma^*\right\}} = 0$ because $b^s(a', \sigma) > b^u(a')$ in the relevant range. Therefore, if $I_{b^u(a')<x} = 0$, then $P(a', [\underline{a}, x]) = 0$, hence $P(a', [\underline{a}, x]) \leq P(a, [\underline{a}, x])$ is trivially true. Thus, we only need to consider cases corresponding to $I_{b^u(a')<x} = 1$.

Case I: $I_{b^u(a')<x} = 1$ and $I_{\sigma''(a')>\max\left\{\tilde{\sigma}(a'),\sigma^*\right\}} = 1$. In this case $P(a', [\underline{a}, x]) = F(\sigma''(a'))$. Since $I_{b^u(a)<x} \geq I_{b^u(a')<x}$, therefore, $I_{b^u(a)<x} = 1$. Further, $\sigma''(a) \geq \sigma''(a') > \max\left\{\tilde{\sigma}(a'), \sigma^*\right\}$.

If $I_{\sigma''(a)>\max\{\tilde{a}(a),\sigma^*\}} = 1$, then $P(a,[\underline{a},x]) = F(\sigma''(a)) \geq F(\sigma''(a')) = P(a',[\underline{a},x]$. Otherwise, $P(a,[\underline{a},x]) = F\{\max\{\tilde{a}(a),\sigma^*\}\} \geq F(\sigma''(a)) \geq F(\sigma''(a')) = P(a',[\underline{a},x])$.

Case II: $I_{b^u(a')<x} = 1$ and $I_{\sigma''(a')>\max\{\tilde{a}(a'),\sigma^*\}} = 0$. In this case $P(a',[\underline{a},x]) = F(\max\{\tilde{a}(a'),\sigma^*\})$. Again, $I_{b^u(a)<x} = 1$. Now, if $I_{\sigma''(a)>\max\{\tilde{a}(a),\sigma^*\}} = 1$, then $P(a,[\underline{a},x]) = F(\sigma''(a)) > F\{\max\{\tilde{a}(a),\sigma^*\}\}$. Since $\tilde{a}(a) > \tilde{a}(a')$, we get $F\{\max\{\tilde{a}(a),\sigma^*\}\} \geq F\{\max\{\tilde{a}(a'),\sigma^*\}\}$, hence $P(a,[\underline{a},x]) > P(a',[\underline{a},x])$. If $I_{\sigma''(a)>\max\{\tilde{a}(a),\sigma^*\}} = 0$, then $P(a,[\underline{a},x]) = F\{\max\{\tilde{a}(a),\sigma^*\}\} \geq F\{\max\{\tilde{a}(a'),\sigma^*\}\} = P(a',[\underline{a},x])$.*QED*

Lemma A.2:(Monotone Mixing Condition): The monotonic transition function P satisfies the following property:

For a $\tilde{a} \in (\underline{a},\overline{a})$ there exists an integer m such that:

$$P^m(\underline{a},[\tilde{a},\overline{a}]) > 0 \text{ and } P^m(\overline{a},[\underline{a},\tilde{a}]) > 0;$$

$P^m(a,B)$ above denotes the probability of reaching B from a after m generations. What the condition above implies is that even poorest individual will have his lineage wealth end up above $\tilde{a}$ after m consecutive high ability descendants; similarly, even the richest individual will have his lineage wealth end up below $\tilde{a}$ after m consecutive low ability descendants or borrowing constrained descendants. The fact that this condition is satisfied should be obvious from Figure 1. The formal proof is given below.

Proof: Take any $\tilde{a} \in (\underline{a},\overline{a})$. Then since $(b(a,\overline{\sigma}) - a)$ is always strictly positive and continuous on $[\underline{a},\tilde{a}]$, it remains uniformly bounded below by some positive number φ. Thus, there exists an integer n_1 such that: $n \geq n_1 \Leftrightarrow b^n(\underline{a},\overline{\sigma}) > \tilde{a}$, where $b^n(.,\overline{\sigma})$ denotes the n^{th} iterate of $b(.,\overline{\sigma})$. n_1 is less than or equal to the smallest integer n such that $n.\varphi > \tilde{a}$.

Similarly, since $[a - b(a,\underline{\sigma})]$ is strictly positive and continuous on $[\tilde{a},\overline{a}]$, it remains bounded below by some positive number ψ. Thus, there exists an integer n_0 such that: $n \geq n_0 \Leftrightarrow b^n(\overline{a},\underline{\sigma}) < \tilde{a}$, where $b^n(.,\underline{\sigma})$ denotes the n^{th} iterate of $b(.,\underline{\sigma})$. n_0 is less than or equal to the smallest integer n

such that $\overline{a} - n.\varphi < \hat{a}$.

Let $m = \max(n_0, n_1)$. Recalling that $f(\sigma)$ denotes the density function of σ, we have

$$
\begin{aligned}
P^m(\underline{a}, [\hat{a}, \overline{a}]) &\geq (f(\overline{\sigma}))^m > 0; \text{ and} \\
P^m(\overline{a}, [\underline{a}, \hat{a}]) &\geq (f(\underline{\sigma}))^m > 0.
\end{aligned}
$$

QED.

Existence of Invariant Distribution

The existence of an invariant distribution G for the Markov process defined by $P(a, B)$ follows from the monotonicity of P established in Lemma A.1 and from Corollary 4 in Hopenhayn and Prescott (1992).

Uniqueness and Convergence

This follows from the monotonicity of P established in Lemma A.1, its monotone mixing property established in Lemma A.2 and Theorem 2 in Hopenhayn and Prescott (1992).

Closed Form Solution for Specific Functional Forms

The Model

Production functions for the two goods are as follows:

$$H = A^h S_h^{\gamma} U_h^{1-\gamma} \tag{25}$$

$$L = A^l S_l^{\phi} U_l^{1-\phi} \tag{26}$$

Assume $\gamma > \phi$ to make H relatively more skill intensive at all factor prices. Perfect mobility of factors within sectors implies the following:

$$p\frac{\partial H}{\partial S} = \frac{\partial L}{\partial S} = w_s \tag{27}$$

$$p\frac{\partial H}{\partial U} = \frac{\partial L}{\partial U} = w_l \tag{28}$$

(27) and (28) can be solved for $s_h = \dfrac{S_h}{U_h}$ and $s_l = \dfrac{S_l}{U_l}$ to get the following:

$$
\begin{aligned}
s_h &= \left(\frac{A^l}{A^h}\right)^{\frac{1}{\gamma-\phi}} \left(\frac{\phi}{\gamma}\right)^{\frac{\phi}{\gamma-\phi}} \left(\frac{1-\gamma}{1-\phi}\right)^{\frac{\phi-1}{\gamma-\phi}} p^{-\frac{1}{\gamma-\phi}} &(29)\\
s_l &= \left(\frac{A^l}{A^h}\right)^{\frac{1}{\gamma-\phi}} \left(\frac{\phi}{\gamma}\right)^{\frac{\gamma}{\gamma-\phi}} \left(\frac{1-\gamma}{1-\phi}\right)^{\frac{\gamma-1}{\gamma-\phi}} p^{-\frac{1}{\gamma-\phi}} &(30)
\end{aligned}
$$

Thus, given the relative price ratio, p, factor intensities in the two sectors are determined by (29) and (30). (29) and (30) in turn determine the absolute returns to the two factors of production:

$$
\begin{aligned}
w_s &= (A^h)^{\frac{1-\phi}{\gamma-\phi}} (A^l)^{\frac{\gamma-1}{\gamma-\phi}} \phi \left(\frac{\phi}{\gamma}\right)^{\frac{\gamma(\phi-1)}{\gamma-\phi}} \left(\frac{1-\gamma}{1-\phi}\right)^{\frac{(\gamma-1)(\phi-1)}{\gamma-\phi}} p^{\frac{1-\phi}{\gamma-\phi}} &(31)\\
w_l &= (A^h)^{\frac{-\phi}{\gamma-\phi}} (A^l)^{\frac{\gamma}{\gamma-\phi}} (1-\phi) \left(\frac{\phi}{\gamma}\right)^{\frac{\gamma\phi}{\gamma-\phi}} \left(\frac{1-\gamma}{1-\phi}\right)^{\frac{(\gamma-1)\phi}{\gamma-\phi}} p^{\frac{-\phi}{\gamma-\phi}} &(32)
\end{aligned}
$$

Therefore, we see that w_s is positively related and w_l negatively related with the relative price of the high-tech good. As well, the relationship between A^h and the two wages is exactly same as between the relative price p and the two wages. Therefore, the impact of a skill-biased technical progress captured by an increase in A^h on wages is the same as the impact of an increase in the relative price of the high-tech good.

For a given p there is a unique $G(a)$ and the corresponding fraction of population investing in skill, S, and skill endowment in efficiency unit, $\overline{S}$ are given by (20) and (21), respectively. From the education sector production function the amount of skill used in the education sector is

$$S_E = \frac{1}{Q} S \tag{33}$$

Therefore, the amount of skill involved in direct production is $S_P = \overline{S} - S_E$. The factor endowment ratio relevant for goods production is given by $\dfrac{S_P}{2U}$, where $U = 1 - S$ is the fraction of population remaining unskilled in each generation. Therefore, given the relative product price, p, the relative factor supplies are determined.

Since the factor supplies are known, domestic production of H and L can be determined from the following relationship

$$\lambda s_h + (1-\lambda)s_l = \frac{S_P}{2U} = s \tag{34}$$

In (34) $\lambda = \dfrac{U_h}{2U}$ is the fraction of unskilled labor used in the high-tech sector. The condition required for obtaining a diversified equilibrium is $s \in (s_l, s_h)$. If the world price is such that this condition is violated then the economy will specialize in one of the two goods.

(34) implies that $\lambda = \dfrac{s - s_l}{s_h - s_l}$. This determines U_h and U_l. Since U_h, U_l, s_h, and s_l are known, production of H and L are determined from the production functions in (25) and (26).

What is left to determine is the consumption of H and L. The utility function of consumers in the second period of their lives is

$$C = C_h^{\theta} C_l^{1-\theta} \tag{35}$$

Denote the average level of steady state wealth by b. Denoting the aggregate income of each generation in the second period of their lives by Y', we get the following:

$$Y' = (1+r)b + (2+r)(1-S)w_l + \overline{S}w_s - (1+r)Sqw_s \tag{36}$$

If the aggregate income is Y', then $\beta Y'$ is spent on consumption and (1-β)Y' is spent on bequests. Let us define $Y = \beta Y'$. (35) implies the following for the levels of consumption of the two goods.

$$H_C = \sum C_h = \frac{\theta Y}{p}; L_C = \sum C_l = (1-\theta)Y \tag{37}$$

Therefore, we have determined the domestic availability and domestic consumption of both goods. The gap between domestic availability and domestic consumption is met through international trade. The material balance conditions for the two goods are as follows.

$$L_P + M = L_C \tag{38}$$

$$H_P - X = H_C \tag{39}$$

In (38) if $M > 0$, then it implies that the numeraire good is imported, while $M < 0$ implies that the numeraire good is exported. X in (39) has a similar interpretation for the high-tech good.

It can be easily checked that (38) implies (39) so that if the market for low-tech good clears, the market for high-tech good clears as well.

Substituting for L_C in (38) using (37)we get

$$L_P = Y - \theta Y - M \tag{40}$$

Denote the net borrowing or lending of each generation by I where

$$I = \mathbf{b} + (1 - S)w_l - Sqw_s \tag{41}$$

Using (41) above, (36) can be rewritten as

$$Y' = Uw_l + \overline{S}w_s + (1 + r)I \tag{42}$$

Further, from the constant returns to scale production function we get the following equality between the value of output and factor payments.

$$L_P + pH_P = 2Uw_l + w_s(\overline{S} - S_E) \tag{43}$$

Now substitute for L_p from (40) in (43) to get

$$pH_P = 2Uw_l + w_s(\overline{S} - S_E) - Y + \theta Y + M \tag{44}$$

Substitute for Y in (44) from (42) using the fact that $Y = \beta Y'$ and $\mathbf{b} = (1 - \beta)Y'$ and that the total cost of education is $Sqw_s = w_sS_E$, to get the following

$$pH_P = pH_C + M - rI \tag{45}$$

Using the balance of payment condition $pX + rI = M$ (45) can be written as (39). In (45) $I > 0$ implies that each generation is a lender in the capital market. However, in steady state in every period there will be a net inflow of capital because the repayment inclusive of interest payment is more than the lending. Thus, the economy will run a trade deficit ($M - pX > 0$). The opposite happens for an economy with $I < 0$.

Proof of Lemma 1: Let T_π and $T_{\pi'}$ be the Markov operators associated with the degrees of credit market imperfections π and π', respectively. Similarly, denote the unique invariant distributions associated with the degrees of credit market imperfections π and π' by $G_\pi(a)$ and $G_{\pi'}(a)$, respectively. Using the transition function defined earlier we can write the following.

$$T_{\pi'}G_{\pi'}(x) - T_\pi G_{\pi'}(x) = \int_{\underline{a}}^{\bar{a}} \left[P_{\pi'}(a, [\underline{a}, x]) - P_\pi(a, [\underline{a}, x])\right] dG_{\pi'}(a) \tag{46}$$

Next we show that $P_{\pi'}(a, [\underline{a}, x]) \geq P_\pi(a, [\underline{a}, x])\ \forall a$. Again, as in the proof of Lemma A.1 define $\sigma''(a)$ as the level of talent such that $b^s(a, \sigma''(a)) = x$. Using the transition function defined in (23) above and subscripting the variables which depend on the degree of credit-market imperfection, π, we can write the following.

$$\begin{aligned} P_\pi(a, [\underline{a}, x]) &= F\left\{\max\left\{\tilde{\sigma}_\pi(a), \sigma^*\right\}\right\} I_{b^u(a)<x} + \\ &\quad \left[F(\sigma''(a)) - F\left\{\max\left\{\tilde{\sigma}_\pi(a), \sigma^*\right\}\right\}\right] I_{\sigma''(a)>\max\left\{\tilde{\sigma}_\pi(a),\sigma^*\right\}} \end{aligned} \tag{47}$$

$P_{\pi'}(a, [\underline{a}, x])$ can be defined similarly. It should be noted that $I_{b^u(a)<x}$ and $\sigma''(a)$ do not depend on π because the amount of bequest does not depend on π directly. Further, it can be easily seen that $\tilde{\sigma}_{\pi'}(a) > \tilde{\sigma}_\pi(a)$. As well, as seen in the proof of Lemma A.1 if $I_{b^u(a)<x} = 0$ then $I_{\sigma''(a)>\max\left\{\tilde{\sigma}_{\pi'}(a),\sigma^*\right\}} = 0$ and $I_{\sigma''(a)>\max\left\{\tilde{\sigma}_\pi(a),\sigma^*\right\}} = 0$. Therefore, we only need to consider cases corresponding to $I_{b^u(a)<x} = 1$.

Case I: $I_{\sigma''(a)>\max\left\{\tilde{\sigma}_{\pi'}(a),\sigma^*\right\}} = 0$ and $I_{\sigma''(a)>\max\left\{\tilde{\sigma}_\pi(a),\sigma^*\right\}} = 0$. In this case $P_{\pi'}(a, [\underline{a}, x]) - P_\pi(a, [\underline{a}, x]) = F\left\{\max\left\{\tilde{\sigma}_{\pi'}(a), \sigma^*\right\}\right\} - F\left\{\max\left\{\tilde{\sigma}_\pi(a), \sigma^*\right\}\right\} \geq 0$.

Case II: $I_{\sigma''(a)>\max\left\{\tilde{\sigma}_{\pi'}(a),\sigma^*\right\}} = 1$ and $I_{\sigma''(a)>\max\left\{\tilde{\sigma}_\pi(a),\sigma^*\right\}} = 1$. In this case $P_{\pi'}(a, [\underline{a}, x]) - P_\pi(a, [\underline{a}, x]) = 0$

Case III: $I_{\sigma''(a)>\max\{\widehat{\sigma}_{\pi'}(a),\sigma^*\}} = 1$ and $I_{\sigma''(a)>\max\{\widehat{\sigma}_{\pi}(a),\sigma^*\}} = 0$. In this case $P_{\pi'}(a,[\underline{a},x]) - P_{\pi}(a,[\underline{a},x]) = F(\sigma''(a)) - F\{\max\{\widehat{\sigma}_{\pi}(a),\sigma^*\}\} \geq 0$ because $\sigma''(a) > \max\{\widehat{\sigma}_{\pi'}(a),\sigma^*\} \geq F\{\max\{\widehat{\sigma}_{\pi}(a),\sigma^*\}\}$.

Case IV: $I_{\sigma''(a)>\max\{\widehat{\sigma}_{\pi'}(a),\sigma^*\}} = 0$ and $I_{\sigma''(a)>\max\{\widehat{\sigma}_{\pi}(a),\sigma^*\}} = 1$. In this case $P_{\pi'}(a,[\underline{a},x]) - P_{\pi}(a,[\underline{a},x]) = F\{\max\{\widehat{\sigma}_{\pi'}(a),\sigma^*\}\} - F(\sigma''(a)) \geq 0$ because $\sigma''(a) < \max\{\widehat{\sigma}_{\pi'}(a),\sigma^*\}$ by definition.

Therefore, $P_{\pi'}(a,[\underline{a},x]) - P_{\pi}(a,[\underline{a},x]) \geq 0 \ \forall a$.

Thus, we get $T_{\pi'}G_{\pi'}(x) - T_{\pi}G_{\pi'}(x) \geq 0$ or $T_{\pi'}G_{\pi'}(x) = G_{\pi'}(x) \geq T_{\pi}G_{\pi'}(x)$. Since the Markov operator T_{π} is increasing (implied by the monotonicity of the transition function $P(a,B)$), we also get $T_{\pi}G_{\pi'}(x) \geq T_{\pi}(T_{\pi}G_{\pi'}(x))$. It follows that $G_{\pi'}(x) \geq (T_{\pi})^n G_{\pi'}(x)$, where $(T_{\pi})^n$ is the nth iterate of T_{π}. We know from Proposition 1 that for $n \to \infty$ $(T_{\pi})^n G_{\pi'}(x)$ converges to $G_{\pi}(x)$. Therefore, we have proved that $G_{\pi'}(x) \geq G_{\pi}(x)$.

<u>Proof of Proposition 2</u>: The proof uses closed form solution for the example given above as well as Lemma 1 proved above and Lemma 2 in the text.

Suppose the world is populated by a continuum of small open economies indexed by n. The only difference between these economies lies in the degree of credit market imperfections π, giving rise to a different steady state distribution of wealth, denoted by $G_n(a)$ for the nth economy. Now suppose the world relative price of the high-tech good settles at p, which is taken as given by all economies. Given this world price, all variables for a typical small open economy is determined as explained in the example above. Given the identical and homothetic preferences, all economies consume the two goods in the ratio $\frac{H_C}{L_C} = \frac{\theta}{(1-\theta)p}$. The production and availability ratios differ across economies depending on their endogenous skill endowment. Let us call an economy $\widehat{n}$ the average economy in the following sense. Its distribution of wealth is denoted by $G_{\widehat{n}}(a)$ and the corresponding skill endowment is $\overline{S}_{\widehat{n}}$. Given the above skill endowment, the production levels of the two goods H_p and L_P can be derived from the equation (34) in the example. The production levels for the average economy are such that the availability ratio for the two goods coincides with the consumption ratio. Since in steady state $\mathbf{b}_t = \mathbf{b}_{t+1} = \mathbf{b}$, the production ratio is same as the consumption ratio, i.e.

$$\frac{H_P^{\mathbf{b}}}{L_P^{\mathbf{b}}} = \frac{H_C}{L_C} = \frac{\theta}{(1-\theta)p} \tag{48}$$

Denote the export of high-tech good by X and the import of low-tech good by M. (48) above implies that the average economy does not export or import anything ($X^{\mathbf{b}} = M^{\mathbf{b}} = 0$). From the balance of payment condition this further implies that the average economy does not borrow or lend in the international capital market.

Now, suppose there is another economy $\overline{n}$ with smaller degree of credit market imperfections than $\mathbf{b}$. From Lemma 2 it is clear that $\overline{n}$ will have greater skill endowment than $\mathbf{b}$: $\overline{S}_{\overline{n}} > \overline{S}_{\mathbf{b}}$. Using the familiar Rybczynski theorem one can easily see that $H_P^{\overline{n}} > H_P^{\mathbf{b}}$ and $L_P^{\overline{n}} < L_P^{\mathbf{b}}$. Therefore,

$$\frac{H_P^{\overline{n}}}{L_P^{\overline{n}}} > \frac{H_P^{\mathbf{b}}}{L_P^{\mathbf{b}}} = \frac{\theta}{(1-\theta)p} \tag{49}$$

(49) implies that in the absence of trade the economy $\overline{n}$ will have an excess supply of the high-tech good, therefore, the autarchy price of the high-tech good in this economy will be lower than the world price. Thus, the economy $\overline{n}$ has a comparative advantage in the high-tech good. After opening up to trade, the following has to be true.

$$\frac{H_P^{\overline{n}} - X^{\overline{n}}}{L_P^{\overline{n}} + M^{\overline{n}}} = \frac{\theta}{(1-\theta)p} \tag{50}$$

(49) and (50) above imply that $X^{\overline{n}} > 0$ and $M^{\overline{n}} > 0$, if economy $\overline{n}$ has balanced trade, i.e. $pX^{\overline{n}} - M^{\overline{n}} = 0$. If this economy is a net importer of capital, then it has to run a trade surplus, in that case it is possible that $X^{\overline{n}} > 0$ and $M^{\overline{n}} < 0$. On the other hand, if this economy is a net exporter of capital, then it runs a trade deficit, and in this case it is possible that it imports both goods, i.e. $X^{\overline{n}} < 0$ and $M^{\overline{n}} > 0$. Thus in the case of balanced trade, the Heckscher-Ohlin pattern of trade is verified. In the case of unbalanced trade, it is possible for the economy to export or

import both the goods, however, it will never be the case that the pattern of trade is the opposite of that predicted by Heckscher-Ohlin theorem. QED.

References

[1] Andreoni, J., 1989. Giving with Impure Altruism: Applications to Charity and Ricardian Equivalence. Journal of Political Economy 97, 1447-1458.

[2] Barro, R., Lee, J., 1994. Data set for a panel of 138 countries. Available at http://www.nber.org.

[3] Cartiglia, F., 1997. Credit constraints and human Capital accumulation in the open economy. Journal of International Economics 43, 221-236.

[4] Chiu, W. H., 1998. Income inequality, human capital accumulation, and economic performance. Economic Journal 108, 44-59.

[5] Deininger, K., Squire, L., 1996. A new data set measuring income inequality. World Bank Economic Review 10(3), 565-91.

[6] Eicher, T., 1999. Trade, Development and converging growth rates: Dynamic gains from trade reconsidered. Journal of International Economics 48, 179-198.

[7] Findlay, R., Kierzkowski, H., 1983. International trade and human capital: a simple general equilibrium model. Journal of Political Economy 91, 957-78.

[8] Galor, O., Zeira, J., 1993. Income distribution and Macroeconomics. Review of Economic Studies 60, 35-52.

[9] Grossman, G., Helpman, E., 1991. Innovation and growth in the global economy. MIT Press, Cambridge.

[10] Hanushek, E., 1995. Interpreting recent research on schooling in developing countries. World Bank Research Observer 10, 227-246.

[11] Hopenhayn, H., Prescott, E., 1992. Stochastic Monotonicity and Stationary Distributions for Dynamic Economies. Econometrica 60, 1387-1406.

[12] Knack, S., Keefer, P., 1995. Institutions and Economic Performance: Cross-country tests using alternative institutional measures. Economics and Politics 7, 207-228.

[13] Lloyd-Ellis, H., 2000. Public Education, Occupational choice and the growth-inequality relationship. International Economic Review 41, 171-201.

[14] Psacharopoulos, G., 1985. Education for development, World Bank, Washington, D.C.

[15] Psacharopoulos, G., 1994. Returns to Investment in Education: A Global Update. World Development 22, 1325-1343.

[16] Ranjan, P., 1997. Income Inequality, Human Capital, and International Trade: An Empirical Analysis. ch.2 in Essays in Trade and Development, Ph.D dissertation, Columbia University, New York.

[17] Williamson, J., 1993. Human Capital deepening, inequality and demographic events along the asia-pacific rim. in Ogawa, N., et al (Eds), Human Resources in Development Along the Asia-Pacific Rim, Singapore, Oxford University Press.

[18] Wood, A., 1994. North-South trade, employment and inequality: changing fortunes in a skill driven world. Oxford, Clarendon Press.

Table 1

Results of Numerical Simulation

	Fraction of Population wanting to invest	Fraction of population borrowing constrained	Fraction of population actually investing
South before liberalization	1	$\frac{4}{9}$	$\frac{5}{9}$
South after liberalization	$\frac{2}{3}$	$\frac{1}{15}$	$\frac{3}{5}$
North before liberalization	$\frac{2}{3}$	0	$\frac{2}{3}$
North after liberalization	1	$\frac{1}{5}$	$\frac{4}{5}$

Note: Fraction of population actually investing = Fraction of population wanting to invest − Fraction of population borrowing constrained.

Footnotes

1.Looking at the exports of the South to the North, the share of manufactures as a proportion of non-fuel exports has gone up from 6% in 1955 to 71% in 1989. The North's export to the South continues to be dominated by manufactures; its share having risen from 73% to 79% over the same period. The difference lies in the fact that the Northern manufactures are more skill intensive than the Southern manufactures. (Source: Wood (1994))

2.Eicher (1999) has an endogenous growth model where the cost of human capital accumulation and the cost of technical change interact to give a convergence in growth rates as well. We have just noted his model's implications for convergence in human capital endowments.

3.This is true only in a diversified equilibrium where both goods are produced. Throughout the paper we assume a diversified equilibrium.

4.Eicher (1999) and Cartiglia (1997) model education sector in a similar fashion.

5.See Hanushek (1995) for a recent survey of empirical work on this issue.

6.Allowing for consumption in the first period will make the borrowing constraint more severe because individuals may want to borrow for consumption smoothing as well. The results of the paper remain qualitatively similar after allowing for first period consumption. See Chiu (1998) for a model that allows for first period consumption .

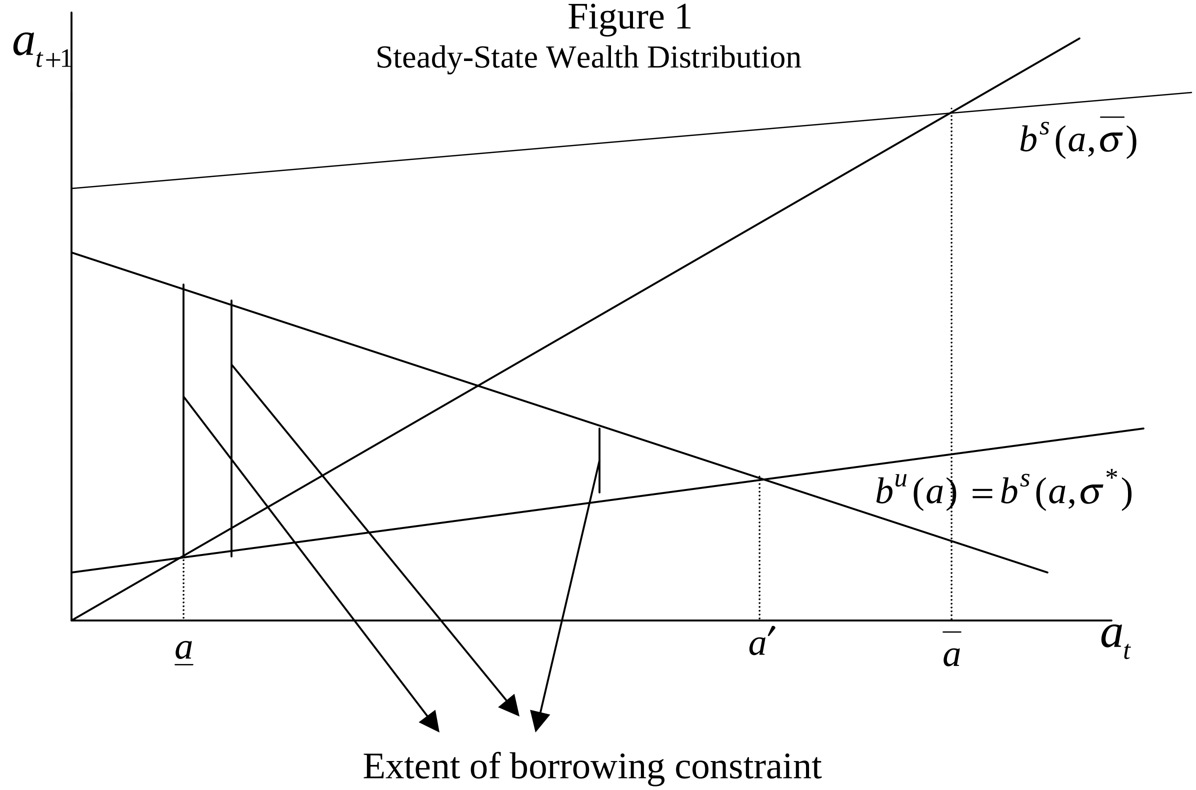
7.This appendix is not being published to conserve space. It is available from the author's website at http://orion.oac.uci.edu/~pranjan/research.html

8. The data on average years of secondary schooling are from Barro-Lee (1994). The data on RLW are averages for the period 1982-85. Similar results were obtained using other indicators collected by ICRG like Repudiation of Contracts by Governments, Risk of Expropriation etc. Also, similar results were obtained using data on average years of tertiary education rather than secondary education.

9.Lloyd-Ellis (2000) finds a statistically significant negative impact of income inequality measured by Gini coefficient on secondary enrollment ratio in a sample of 35 countries.

Figure 1

Steady-State Wealth Distribution



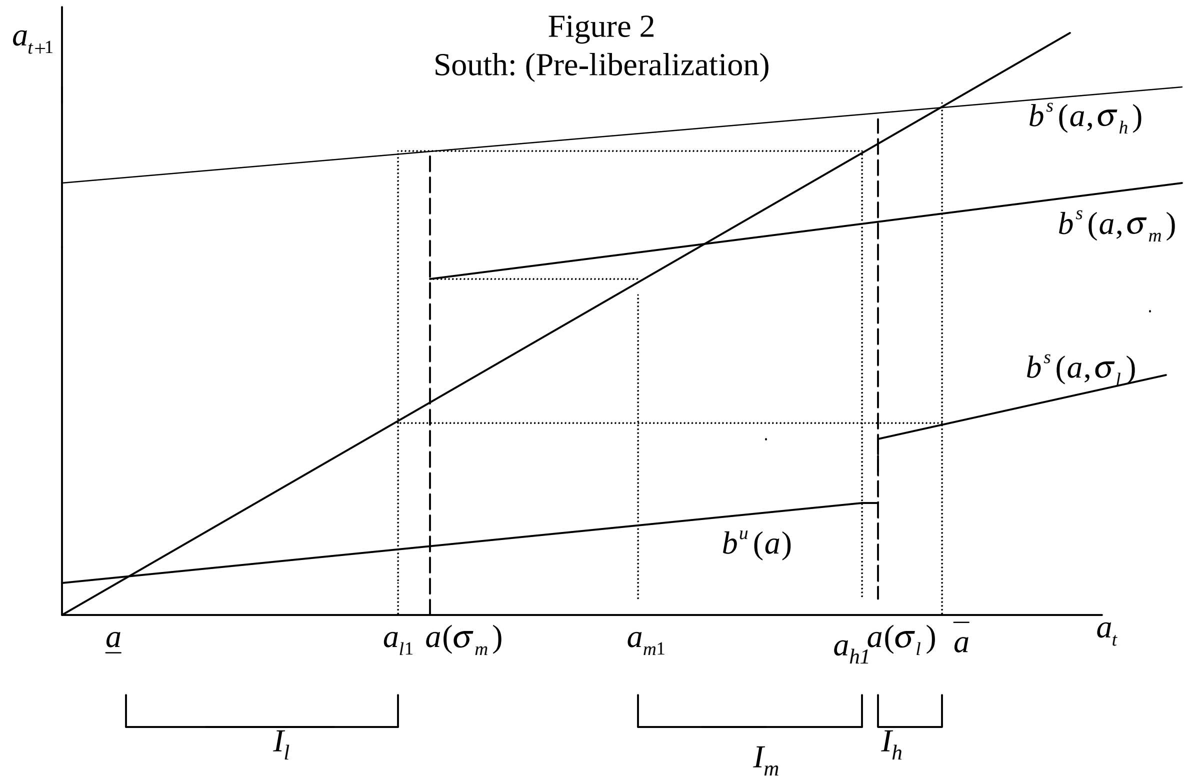


Figure 2
South: (Pre-liberalization)
a_{t+1}
$b^s(a,\sigma_h)$
$b^s(a,\sigma_m)$
$b^s(a,\sigma_l)$
$b^u(a)$
$\underline{a}$
a_{l1}
$a(\sigma_m)$
a_{m1}
a_{h1}
$a(\sigma_l)$
$\bar{a}$
a_t
I_l
I_m
I_h

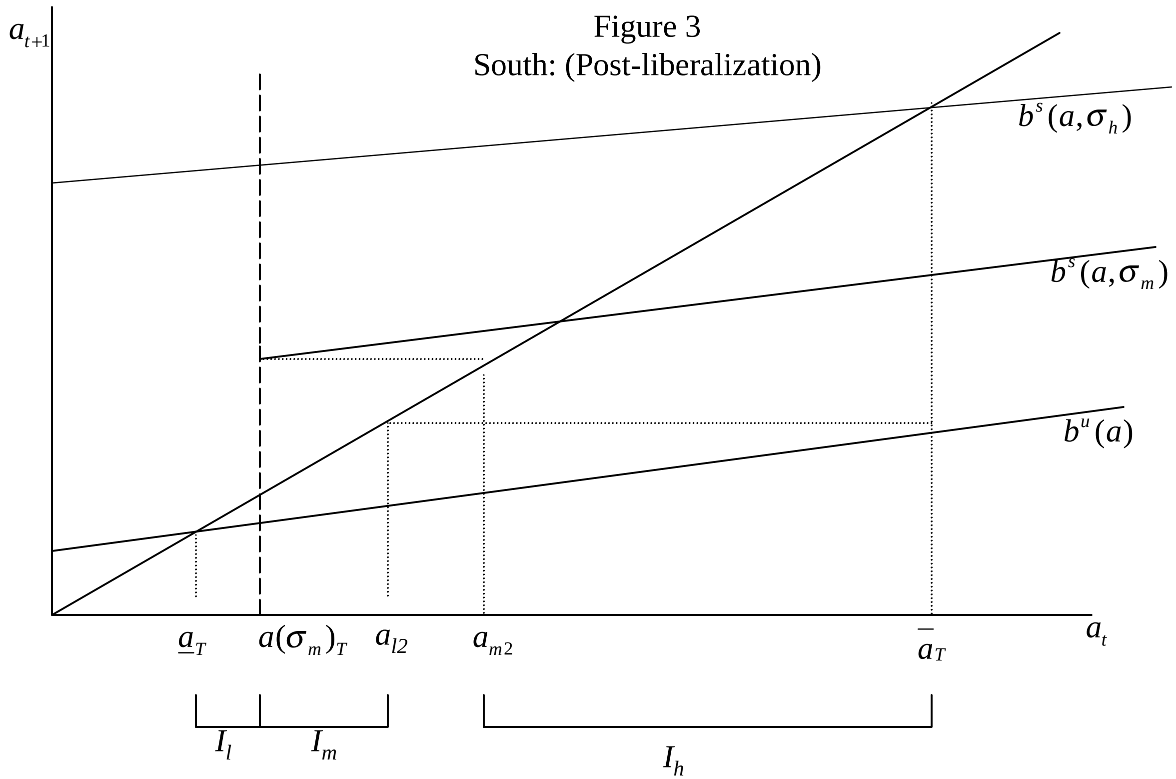


Figure 3
South: (Post-liberalization)

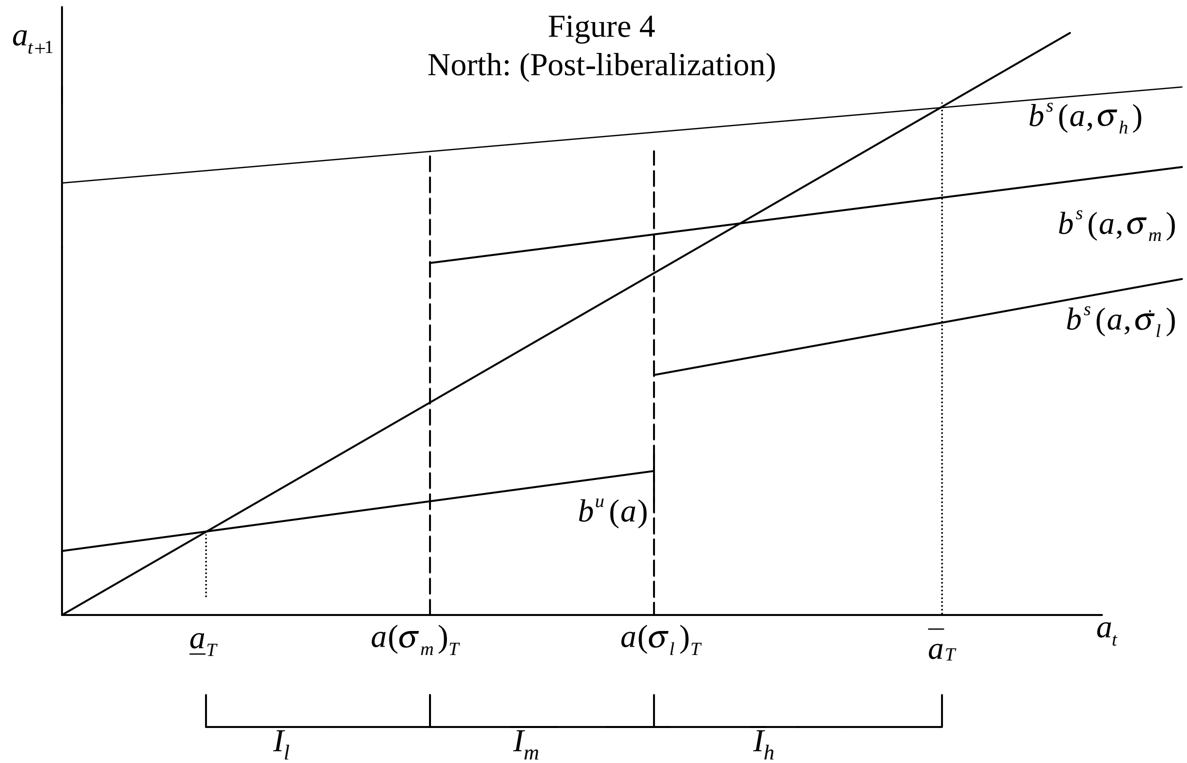


Figure 4
North: (Post-liberalization)

Appendix D
Parametric Restrictions for the numerical example

Denote $(1-\beta)$ by β' and $(1+r)$ by r'. The numerical example is based on the following parameter values: $\beta' = .2, r' = 1.1, q = .5, \sigma_l = 1, \sigma_m = 1.5, \sigma_h = 2.$

South before liberalization

Since we assumed that everyone wants to invest in the South before liberalization, the return on education must satisfy the following condition.

$$\frac{w_l}{w_s} < \frac{\sigma_l - r'q}{1+r'} \tag{1}$$

For the parameters chosen above, this implies $\frac{w_l}{w_s} < .214$.

In order for the borrowing constraint to be binding for at least some individuals a sufficient condition is that it is binding for some individuals with talent σ_m. This would be true if $a(\sigma_m) > \underline{a}$, which upon simplification yields

$$\frac{w_l}{w_s} < \frac{1-\beta' r'}{\beta'(1+r')}[q - (\frac{1-\pi}{r'})\sigma_m] \tag{2}$$

For $\pi = .78$ the r.h.s of (2) is .37. Therefore, $\frac{w_l}{w_s} < .37$ is needed to satisfy (2).

Further, it was assumed that everyone with talent σ_h can afford to invest. This implies $\underline{a} > a(\sigma_h)$ which upon simplification yields

$$\frac{w_l}{w_s} > \frac{1-\beta' r'}{\beta'(1+r')}[q - (\frac{1-\pi}{r'})\sigma_h] \tag{3}$$

For $\pi = .78$ the r.h.s. of (3) is .189. Therefore, $\frac{w_l}{w_s} > .189$ satisfies (3).

Figure 2 assumes that $a_{m1} > a(\sigma_m)$, which upon simplification yields

$$\pi < \frac{\frac{\sigma_m}{r'} - q}{\frac{\sigma_m}{r'} - \beta' \sigma_m} \tag{4}$$

This condition is satisfied for $\pi < .81$.

To ensure that individuals in I_l transit to I_m upon receiving σ_h, the condition is $\beta'(\sigma_h w_s + r'(a_{l1} - q w_s)) < a(\sigma_l)$, which upon simplification yields

$$\pi > 1 - \frac{r'q}{(1-\beta' r')\sigma_l} + \beta'^2 r'^2 + \beta' \frac{\sigma_h}{\sigma_l} + \frac{\beta'^3 r'^3 \sigma_h}{(1-\beta' r')\sigma_l} \tag{5}$$

This condition is satisfied for $\pi \geq .78$.

To ensure that individuals in the interval I_h transit to I_l where they are borrowing constrained when their talent is σ_l or σ_m we need $a(\sigma_m) > a_{l1}$, which upon simplification yields.

$$\pi > 1 - \frac{r'q}{(1-\beta' r')\sigma_m} + \beta' r' \frac{\sigma_l}{\sigma_m} + \frac{\beta'^2 r'^2 \sigma_h}{(1-\beta' r')\sigma_m} \tag{6}$$

This condition is satisfied for $\pi > .759$.

A sufficient condition for those in I_m to transit to I_h upon receiving a shock of σ_h is $\beta'[\sigma_m w_s + r'a(\sigma_m) - r'qw_s] > a(\sigma_l)$. This upon simplification yields

$$\pi < 1 - \frac{r'q - \beta' r' \sigma_h}{\sigma_l - \beta' r' \sigma_m} \tag{7}$$

This condition is satisfied if $\pi < .83$.

Finally, the condition required for people in I_h to transit to I_m after receiving σ_m is $a(\sigma_l) > \beta'[\sigma_m w_s + r'\overline{a} - r'qw_s]$. This upon simplification becomes

$$\pi > 1 - \frac{r'q}{(1-\beta' r')\sigma_l} + \beta' r' \frac{\sigma_m}{\sigma_l} + \frac{\beta'^2 r'^2 \sigma_h}{(1-\beta' r')\sigma_l} \tag{8}$$

This condition is satisfied for $\pi > .75$.

The analysis above implies that for our chosen parameter values (4)-(8) are satisfied for $.81 > \pi \geq .78$ and for $\pi = .78$ the conditions on the relative wage are satisfied for $.189 < \frac{w_l}{w_s} < .214$. So, $\pi = .78$ for the South in our numerical example.

<u>South after liberalization</u>

After liberalization also we need the conditions $a(\sigma_m)_T > \underline{a}_T$ and $\underline{a}_T > a(\sigma_h)_T$ to remain valid. Therefore, (2) and (3) must remain satisfied.

We also need $a(\sigma_m)_T < a_{l2}$ implying $w_{sT}(q - \frac{1-\pi}{r'}\sigma_m) < \beta'(r'\overline{a}_T + (1 + r')w_{lT})$, which upon simplification yields

$$\frac{w_{lT}}{w_{sT}} > \frac{1 - \beta' r' + \beta'^2 r'^2}{\beta'(1+r')(1-\beta' r')} q - \frac{1-\pi}{\beta' r'(1+r')}\sigma_m - \frac{\beta' r' \sigma_h}{(1-\beta' r')(1+r')} \tag{9}$$

For $\pi = .78$ the r.h.s. of (9) is .277. Therefore, $\frac{w_l}{w_s} > .277$ satisfies (9).

To ensure that individuals in I_l transit to I_h after liberalization a sufficient condition is $\beta' \sigma_h w_s + \beta' r' \underline{a} - \beta' r' q w_s > a_{m2}$, which upon simplification yields

$$\frac{w_{lT}}{w_{sT}} > \frac{1 - \beta' r'}{\beta'(1+r')} q + \frac{1 - \beta' r'}{\beta' r'(1+r')}(\pi \sigma_m - \sigma_h) \tag{10}$$

(10) is easily satisfied for any non-negative wage ratio for $\pi \leq .93$.

To ensure that individuals in I_h transit to I_m upon receiving a shock of σ_l and not to I_l we need $\beta' r' a_{m2} + \beta'(1 + r')w_{lT} > a(\sigma_m)_T$. Upon simplification this yields

$$\frac{w_{lT}}{w_{sT}} > \frac{q}{\beta'(1+r')} - \frac{(1-\pi)(1-\beta'^2 r'^2)}{\beta' r'(1+r')}\sigma_m - \frac{\beta' r' \sigma_m}{(1+r')} \tag{11}$$

For $\pi = .78$ the r.h.s. of (11) is .353. Therefore, $\frac{w_l}{w_s} > .353$ satisfies (11).

To ensure that individuals in I_m transit to I_l upon receiving a shock of σ_l the condition required is $\beta' r' a_{l2} + \beta'(1 + r')w_{lT} < a(\sigma_m)_T$. Upon simplification this yields

$$\frac{w_{lT}}{w_{sT}} < \frac{1 - \beta' r' + \beta'^3 r'^3}{\beta'(1+r')(1-\beta'^2 r'^2)} q - \frac{(1-\pi)}{\beta' r'(1+r')(1+\beta' r')}\sigma_m - \frac{\beta'^2 r'^2 \sigma_h}{(1+r')(1-\beta'^2 r'^2)} \tag{12}$$

For $\pi = .78$ the r.h.s. of (12) is .355. Therefore, $\frac{w_l}{w_s} < .355$ satisfies (12).

So, all the conditions for Figure 3 are satisfied for $\pi = .78$ if $\frac{w_l}{w_s} = .354$. So, a rise in the relative unskilled wage from .21 to .354 will produce the dynamics represented in Figure 3.

North before liberalization

We assumed that the borrowing constraints are not binding before liberalization in the North. A sufficient condition for this is $a(\sigma_m) < \underline{a}$, which upon simplification yields

$$\frac{w_l}{w_s} > \frac{1-\beta' r'}{\beta'(1+r')}[q - (\frac{1-\pi}{r'})\sigma_m] \tag{13}$$

For $\pi = .67$ this condition is satisfied for $\frac{w_l}{w_s} > .09$.

North after liberalization

We need conditions (2) and (3) to be satisfied after liberalization. Note that (2) is violated before liberalization as a result borrowing constraint is not binding for anyone. For $\pi = .67$ (2) is satisfied for $\frac{w_l}{w_s} < .09$ and (3) is satisfied for any $\frac{w_l}{w_s} > 0$.

In order for an individual with wealth in I_l to transit to I_h after receiving σ_h the condition required is $\beta'(\sigma_h w_{sT} + r'(\underline{a}_T - q w_{sT})) > a(\sigma_l) = w_{sT}(q - \frac{1-\pi}{r'}\sigma_l)$. Upon simplification this yields

$$\frac{w_{lT}}{w_{sT}} > \frac{1-\beta'^2 r'^2}{\beta'^2 r'(1+r')}q - \frac{(1-\beta' r')(1-\pi)}{\beta'^2 r'^2(1+r')}\sigma_l - \frac{(1-\beta' r')}{\beta' r'(1+r')}\sigma_h \tag{14}$$

For $\pi = .67$ (14) is satisfied for any $\frac{w_l}{w_s} > 0$.

In order for an individual with wealth in I_m to transit to I_l after receiving σ_l the condition required is $\beta'(r' a(\sigma_l) + (1+r')w_l) < a(\sigma_m)$. Upon simplification this yields

$$\frac{w_{lT}}{w_{sT}} < \frac{1-\beta' r'}{\beta'(1+r')}q + \frac{(1-\pi)}{(1+r')}\sigma_l - \frac{(1-\pi)}{\beta' r'(1+r')}\sigma_m \tag{15}$$

For $\pi = .67$ (15) is satisfied for any $\frac{w_l}{w_s} < .0146$.

Next the condition required to ensure that an individual with wealth in I_m transits to I_h after receiving σ_m or σ_h is $\beta'(\sigma_m w_{sT} + r'(a(\sigma_m) - q w_{sT})) > a(\sigma_l)$. This upon simplification yields

$$\pi < \frac{\sigma_l - r' q}{\sigma_l - \beta' r' \sigma_m} \tag{16}$$

The r.h.s. of (16) equals .6716. Therefore, (16) is satisfied for $\pi < .6716$.

In order for an individual with wealth in I_h to transit to I_m after receiving σ_l we need the following conditions: $\beta'(\sigma_l w_{sT} + r'(a(\sigma_l) - q w_{sT})) > a(\sigma_m)$ and $\beta'(\sigma_l w_{sT} + r'(\overline{a} - q w_{sT})) < a(\sigma_l)$. Upon simplification these two conditions yield

$$\pi < 1 - \frac{r' q - \beta' r' \sigma_l}{\sigma_m - \beta' r' \sigma_l} \tag{17}$$

$$\pi > 1 - \frac{r' q}{(1-\beta' r')\sigma_l} + \beta' r' + \frac{\beta'^2 r'^2 \sigma_h}{(1-\beta' r')\sigma_l} \tag{18}$$

The r.h.s. of (17) is .742. Therefore, (17) is satisfied for $\pi < .742$. The r.h.s of (18) is .64. Therefore, (18) is satisfied for $\pi > .64$.

From the above it is clear that the $.64 < \pi < .6716$ satisfies conditions (16)-(18). We chose $\pi = .67$ so that the condition (15) is satisfied for positive wage ratio. Therefore, for $\pi = .67$ and for a decrease in $\frac{w_l}{w_s}$ from .22 before liberalization to .0146 after liberalization all the conditions required for the dynamics described in Figure 4 are satisfied.

Suppose instead $\pi = .72$ in the North. The only condition that is violated is (16). So, now individuals with wealth in I_m remain in I_m after receiving a shock of σ_m. In this case the post-liberalization fraction of population investing in human capital is 3/4.