Bargaining and Fighting in Hard Times*

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Abstract

Those inside and, especially, those controlling an organization – be it a firm, a committee, a cabinet ministry, or the state – often have better information about the organization’s resources and activities than outsiders do. This informational asymmetry may lead to costly, inefficient conflict between insiders and outsiders over the allocation of those resources. We formalize this conflict as a signaling game in which an incumbent government and an opposing faction – possibly a rebel group – vie for control of the state and the accompanying spoils. In order to avoid a challenge, the government must buy off or coopt the opposition by offering a share of the pie. But the size of the pie is private information. The government knows how large the spoils are but the opposition only has a rough idea about the spoils (e.g., oil prices are high or low, the economy is booming or in crisis). The unique perfect Bayesian equilibrium satisfying a common restriction on off-the-equilibrium-path beliefs (Cho and Kreps’ D1) is fully separating but inefficient. The opposition fights with positive probability even after learning the size of the pie (which the separating offers reveal). Empirical work indicates that poor economic conditions – hard times – make civil war and political conflict more likely. In the model, the equilibrium probability of fighting increases as times become harder. The probability of fighting is also increasing in the strength of the opposition and the uncertainty surrounding the size of the pie and decreasing in the cost of fighting.

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Bargaining and Fighting in Hard or Uncertain Times

Those inside and, especially, those controlling an organization – be it a firm, a committee, a cabinet ministry, or the state – often have better information about the organization’s resources and activities than outsiders do. This informational asymmetry may lead to costly, inefficient conflict between insiders and outsiders over the allocation of those resources. In resource-rich, developing countries, for example, the government frequently has private information about the revenues those resources bring. This lack of transparency is believed to facilitate corruption, make conflict more likely, and has led to international efforts to promote greater transparency, e.g., the Extractive Industries Transparency Initiative (Economist 2005, DFID a, nd.; DFID b, nd).

We study the effects of this asymmetry in a model in which an incumbent government and an opposition faction vie for control of the state and the spoils that come with it. The government knows how large the “pie” is but the opposition only has a rough idea of the size of the pie. The opposition knows, for example, whether times are “good” or “bad” (e.g., oil prices are high or low, the economy is booming or in recession) and therefore whether the pie is on average large or small. But the opposing faction is unsure precisely how large the spoils are in good times or how small they are in bad times.

This informational asymmetry casts a shadow over the government’s efforts to buy off or coopt the opposing faction and thereby prevent a costly conflict – a civil war – over the spoils. The government in the model makes a take-it-or-leave-offer which the opposition can accept or reject by fighting. Accepting ends the game in the agreed division of the spoils. Fighting destroys a fraction of pie with the winner getting what remains.

The opposition faces a simple but vexing strategic problem. If the opposing faction also knew the size of the pie, it would accept a low offer when the pie was small because the payoff to fighting and, if victorious, capturing the surviving spoils is also small. But the opposing faction cannot accept a low offer for sure when it is uncertain of the size of the pie. If it did, there would be nothing to deter the government from low-balling the opposition, i.e., offering a small amount when the pie is large. To prevent this,
the opposition rejects low offers and bargaining breaks down in inefficient fighting with positive probability.

Formally, the game is a standard signalling model with a continuum of types and actions. As is commonly the case with such games, the present model has a multiplicity of equilibria. However, only one of these equilibria is supported by “reasonable” off-the-equilibrium-path beliefs satisfying a common equilibrium refinement (Cho and Kreps’ (1987) condition D1).

This equilibrium is fully separating with the government’s offer strictly increasing in the spoils. The larger the pie, the more the government offers. Because pies of different sizes lead to different offers, the opposition knows how much there is to be divided as well as its payoff to fighting as soon as it receives an offer. Nevertheless, the opposing faction, although now certain of the size of the pie, fights with positive probability. The smaller the offer, the more likely the opposition is to fight.

This equilibrium has interesting theoretical properties in light of recent formal work on the causes of war. That work identifies two broad types of rationalist explanations of war: informational problems and commitment issues (Fearon 1995, Powell 2006).\(^1\) The former attributes ex-post inefficient fighting to asymmetric information; the later ascribes it to the actors’ inability to commit to any efficient outcome. Fighting in the present model clearly results from asymmetric information. Were there complete information, both the government and opposition would know the size of the pie at the outset, the government’s equilibrium offer would always be accepted, and there would be no fighting. But even though fighting in the model results from an informational problem, the decision to fight comes after the separating offers would seem to have solved the problem by revealing the size of the pie to the opposing faction. The analysis shows that the intuition that there should be no fighting once both the government and opposition know the size of the pie

\(^1\) Powell (2002) surveys much of this work.
The equilibrium also has an interesting empirical implication. Bargaining between the government and the opposing faction is more likely to break down during hard times. That is, the equilibrium probability of fighting if times are hard is larger than if times are good. This is in keeping with econometric work on civil war which generally finds that poor economic conditions—hard times—make conflict more likely (e.g., Collier and Hoefler 2004; Fearon and Laitin 2003; Miguel, Satyanath, and Sergenti 2004). A stronger opposition and more uncertainty about the spoils also make fighting more likely whereas higher costs reduce the probability of breakdown.

The next section presents the model and characterizes its equilibria. The two subsequent sections discuss the theoretical properties and empirical implications of the equilibrium analysis. The following section extends the model to allow for power sharing agreements with the opposition (e.g., bringing them into the government). These agreements create a trade off. Sharing power with the opposing faction provides a way of informing it about the size of the pie, but it also makes them more powerful. The analysis suggests that power sharing agreements are more likely to be tried in very bad times. A final section generalizes the analysis to a wide class of games satisfying a simple payoff regularity property. This class of games encompasses models of conflict such as ours and models of litigation such as that by Reinganum and Wilde (1986).

The Model and Equilibria

In order to prevent a challenge, the government must buy off or coopt an opposing faction. To this end, the government begins the game knowing $\pi$, the size of the pie to be divided, and makes an offer $y \geq 0$ to the opposition that can accept the offer or fight. Accepting ends the game with the government and opposition receiving $\pi - y$ and

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In this context, we also discuss a closely related model in which two states know the size of the pie, e.g., the value of the territory over which they are bargaining, but the state making the take-it-or-leave-it offer has private information about the distribution of power. There is also complete separation and fighting in the unique equilibrium of this game satisfying D1.
Fighting destroys a fraction $\delta$ of the pie. If the opposing faction wins, which it does with probability $p$, it gets the surviving spoils; if the government wins it keeps the spoils. Thus, the payoffs to fighting for the government and opposition are $(1 - p)\sigma\pi$ and $p\sigma\pi$, respectively, where $\sigma = 1 - \delta$ is the fraction of the pie that survives the fight.

To formalize the informational asymmetry, let $\pi = c + r$ where $r$ has mean 0 and is distributed over $[r, \bar{r}]$ according to $H$ which has a continuous and strictly positive density $h$ over $(r, \bar{r})$. The government knows $c$ and $r$, but the rebels only observe $c$. The parameter $c$ measures the general climate of the times. The larger $c$, the larger $\pi$ is and the larger the rebels expect it to be (i.e., the larger $\int_{c}^{\bar{r}} \pi dH = c + \int_{c}^{\bar{r}} r dH$ is).

A pure-strategy for the government specifies the government’s offer as a function of its private information about the spoils: $y : [r, \bar{r}] \rightarrow [0, \bar{\pi}]$ where $\bar{\pi} = c + \bar{r}$. A strategy for the opposing faction defines the probability that the opposition accepts as a function of the government’s offer: $\alpha : [0, \bar{\pi}] \rightarrow [0, 1]$. As for what the opposition believes about the size of the spoils after receiving an offer, let $\Delta$ be the set of distributions over $[r, \bar{r}]$ and let $\mu(x) \in \Delta$ for all $x \in [0, \bar{\pi}]$ denote the opposition’s beliefs following an offer of $x$. Finally, a perfect Bayesian equilibrium (PBE) is a strategy profile $(y, \alpha)$ and beliefs $\mu$ such that the government can never profitably deviate from offering $y(\pi)$ given opposition’s strategy $\alpha(x)$, $\alpha(x)$ is a best reply to $x$ given $\mu(r|x)$, and $\mu$ is derived from $H$ and $y$ via Bayes’

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3 Strictly speaking, the government could offer more than there is to be divided ($y > \pi$) in which case the payoffs would be $\pi - \min\{y, \pi\}$ and $\min\{y, \pi\}$. However, these offers are strictly dominated and will never be made, so we simplify the notation by taking the payoffs to be $\pi - y$ and $y$ and $y \leq \pi$.

4 We only consider strategy profiles and equilibria in which the government plays a pure strategy.
The game has infinitely many PBEs. In some, the government pools on a specific offer, i.e., the government makes the same offer regardless of the size of the pie. In other semi-separating equilibria, the government’s offer varies with the spoils but does not fully reveal the exact size of the pie. In these equilibria, there are a set of cutpoints \( \pi_0 < k_1 < \cdots < k_N = \pi \) and a set of ever more favorable offers \( p\sigma\pi \leq y_1 < \cdots < y_N \leq p\sigma\pi \) such that the government proposes \( y_j \) if \( \pi \in (k_{j-1}, k_j) \). And, there is a fully separating equilibrium in which the government’s offer is strictly increasing in the size of the pie.

Incentive compatibility ensures that the equilibrium offers are weakly increasing in the spoils and that larger equilibrium offers are generally more likely to be accepted than smaller offers. More formally:

**Lemma 1:** Let \( (y, \alpha; \mu) \) be a PBE with \( y' = y(\pi'), y'' = y(\pi'') \), and \( \pi' < \pi'' \). Then:

(i) \( \alpha(y'') \geq \alpha(y') \);
(ii) if \( \alpha(y') > 0 \), then \( y'' \geq y' \);
(iii) if \( \alpha(y'') > 0 \) or \( \alpha(y') > 0 \) and if \( y'' > y' \), then \( \alpha(y'') > \alpha(y') \).

Proof: See the Appendix.

Although there is a surfeit of equilibria, only the separating equilibrium is predicated on reasonable out-of-equilibrium beliefs in the sense that they satisfy Cho and Kreps’ (1987) condition D1. This condition requires the opposition to disregard the possibility of facing \( \eta \) following an out-of-equilibrium offer \( z \) if there is another type \( \eta' \) such that the set of rationalizable actions that the opposing faction could take after \( z \) that would give \( \eta \) at least as much as its equilibrium payoff is a strict subset of the actions the opposition could take that would give \( \eta' \) more than its equilibrium payoff.

The out-of-equilibrium-beliefs satisfying D1 turn out to be very simple. Suppose as

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5 Because the parameter \( c \) is common knowledge, we abuse the notation slightly by taking \( y \) to be a function of \( \pi \) in order to simplify the exposition. Defining PBE’s with a continuum of types raises a number of technical issues. For example, no offer is made with positive probability in a separating equilibrium. Bayes’ rule therefore places no restriction on the opposition’s beliefs following any offer. It suffices for the present analysis to assume that if the set of types offering \( z \) has zero measure, then the support of the opposition’s beliefs following \( z \) is contained in the closure of the set : \( \{ \pi : y(\pi) = z \} \). See Ramey (1996) for a definition of a sequential or perfect Bayesian equilibrium with a continuum of types.
illustrated in Figure 1 that no type offers \( z \), i.e., there is no \( \pi \) for which \( z = y(\pi) \) where \( y(\pi) \) are the government’s equilibrium proposals. (Lemma 1(ii) ensures that \( y \) is nondecreasing.) Assume further that \( z \) is larger than some equilibrium offer that is accepted with positive probability: \( z > y(\pi) \) and \( \alpha(y(\pi)) > 0 \) for some \( \pi \). Then D1 implies that the opposition believes that it is facing the type \( \hat{\pi} \) at which the government’s equilibrium offers “jump” over \( z \).

More formally, let \( \pi^+ \) be the lowest type whose offer is accepted with positive probability in the PBE \((y(\pi), \alpha(x); \mu)\): \( \pi^+ \equiv \inf \{ \pi : \alpha(y(\pi)) > 0 \} \). Then,

**Lemma 2:** Take \((y(\pi), \alpha(x); \mu)\) to be a PBE satisfying D1. Assume further that \( z \) is an out-of-equilibrium offer between \( z \), i.e., \( z \notin \{ y(\pi) : \pi \in [\pi^+, \pi]\} \) such that \( p\sigma \pi > z > y(\pi) \) for some \( \pi > \pi^+ \). Then the opposition believes that it is facing \( \hat{\pi} \) with probability one where \( \hat{\pi} \equiv \sup \{ \pi : y(\pi) \leq z \} = \inf \{ \pi : y(\pi) \geq z \} \).

Proof: See the appendix.

It follows that all \( \pi > \pi^+ \) make distinct offers in any PBE satisfying D1. To sketch the intuition, assume the contrary. Then as depicted in Figure 1, there must be two types \( \pi' \) and \( \pi'' \) such that \( \pi^+ < \pi' < \pi'' \) and \( \pi' \) and \( \pi'' \) make the same offer \( \hat{y} \). Let \( \hat{\pi} = \inf \{ \pi : \hat{y} = y(\pi) \} \). Observe first that \( \hat{y} > p\sigma \hat{\pi} \). Because \( y(\pi) \) is nondecreasing, all \( \pi \in [\pi', \pi''] \) propose \( \hat{y} \). This interval has positive measure, and therefore the opposition’s payoff to fighting must be strictly larger than its payoff to fighting type \( \hat{\pi} \). That is, \( \int_{\{\pi:y(\pi)\}} p\sigma \pi d\hat{H}(\pi) > p\sigma \hat{\pi} \) where \( \hat{H} \) is the posterior of \( H \) given \( \hat{y} \). Moreover, Lemma 1 guarantees \( \alpha(y(\pi)) > 0 \) for all \( \pi > \pi^+ \). Hence the opposition accepts \( \hat{y} \) with positive probability and consequently must weakly prefer \( \hat{y} \) to fighting. This leaves \( \hat{y} \geq \int_{\{\pi:y(\pi)\}} p\sigma \pi d\hat{H}(\pi) > p\sigma \hat{\pi} \).

Now consider any offer \( z \) in the gap between \( p\sigma \hat{\pi} \) and \( \hat{y} \). If the challenger strictly prefers accepting \( z \) to fighting, then \( \alpha(z) = 1 \) and a contradiction results as those offering \( \hat{y} \) could profitably deviate to the lower offer \( z \). To see that the opposing faction does prefer accepting \( z \), suppose first that \( z \) is an equilibrium proposal, i.e., \( y(\pi) = z \) for some \( \pi \). Because \( y \) is nondecreasing and \( z < \hat{y} \), the opposition believes that \( \pi \) is bounded above by \( \hat{\pi} \) after being offered \( z \) as \( \sup \{ \pi : z = y(\pi) \} \leq \inf \{ \pi : y(\pi) \geq \hat{y} \} = \hat{\pi} \). Hence, the opposing faction’s payoff to fighting is bounded above by \( p\sigma \hat{\pi} \) which is strictly less than \( z \). If alternatively \( z \) is an out-of-equilibrium offer, then Lemma 2 ensures that the
Figure 1: The government’s offers.
opposition believes that it is facing sup\{\pi : y(\pi) \leq z\} \leq \hat{\pi} after z. The opposing faction’s expected payoff to fighting is therefore \( p\sigma\hat{\pi} \) and again strictly less than \( z \). Formally,

**Lemma 3:** Let \((y(\pi), \alpha(x); \mu)\) be a PBE satisfying condition D1 with \( \pi^+ \equiv \inf \{\pi : \alpha(y(\pi)) > 0\} \). Then all \( \pi > \pi^+ \) separate: \( y(\pi') < y(\pi'') \) whenever \( \pi^+ < \pi' < \pi'' \).

The remainder of this section characterizes the unique separating equilibrium that satisfy D1. Lemma 1 guarantees that \( y(\pi) \) is accepted with positive probability for any \( \pi > \pi^+ \). That lemma also implies that \( y(\pi) \) is weakly increasing in any PBE and therefore strictly increasing for \( \pi \geq \pi^+ \) because all \( \pi > \pi^+ \) separate. Again from Lemma 1, that \( y(\pi) < y(\pi') \) for \( \pi^+ \leq \pi < \pi' \) means that \( \alpha(y) \) is strictly increasing. Hence, \( 0 < \alpha(y(\pi)) < \alpha(y(\pi)) \leq 1 \) for \( \pi > \pi^+ \).

That \( \alpha(y) \in (0, 1) \) implies that the opposition is mixing between fighting and accepting and is, therefore, indifferent between these alternatives. Consequently, the government must be offering the opposing faction its certainty equivalent of fighting: \( y(\pi) = p\sigma\pi \) for \( \pi^+ < \pi < \hat{\pi} \).

As for the probability of acceptance, let \( y = p\sigma\pi \) and \( \hat{y} = p\sigma\hat{\pi} \) with \( \pi^+ < \pi < \hat{\pi} \). Because no type can profitably deviate,

\[
\begin{align*}
\alpha(y)(\pi - y) + (1 - \alpha(y))(1 - p)\sigma\pi & \geq \alpha(\hat{y})(\pi - \hat{y}) + (1 - \alpha(\hat{y}))(1 - p)\sigma\hat{\pi} \\
\alpha(\hat{y})(\hat{\pi} - \hat{y}) + (1 - \alpha(\hat{y}))(1 - p)\sigma\hat{\pi} & \geq \alpha(y)(\hat{\pi} - y) + (1 - \alpha(y))(1 - p)\sigma\hat{\pi}
\end{align*}
\]

Rewriting these inequalities and using the expressions for the government’s offers to eliminate \( \pi \) and \( \hat{\pi} \) gives

\[
\frac{\alpha(\hat{y})p\sigma}{y(1 - \sigma)} \geq \frac{\alpha(\hat{y}) - \alpha(y)}{\hat{y} - y} \geq \frac{\alpha(y)p\sigma}{\hat{y}(1 - \sigma)}. 
\]

Letting \( \hat{y} \) go to \( y \) then yields \( \alpha'(y) = \alpha(y)p\sigma/[y(1 - \sigma)] \). Solving this differential equation with the boundary condition \( \alpha(p\sigma\hat{\pi}) = 1 \) leads to \( \alpha(y) = [y/(p\sigma\hat{\pi})]^{\sigma p/(1 - \sigma)} \).

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\( ^6 \) It is straightforward to show \( y(\pi) = p\sigma\pi \) given \( y(\pi) = p\sigma\pi \) for \( \pi \in (\pi^+, \hat{\pi}) \).

\( ^7 \) If \( \alpha(p\sigma\hat{\pi}) < 1 \), then \( \hat{\pi} \) could profitably deviate by offering slightly more than \( p\sigma\hat{\pi} \) which would be accepted for sure.
Appendix shows that $\pi^+ = \pi$ and hence $\alpha(y) = [y/(p\sigma \pi)]^{\sigma/(1-\sigma)}$ for all $\pi \geq \pi$. This leaves:

**Proposition 1:** In the unique equilibrium satisfying D1, $y(\pi) = p\sigma \pi$, $\alpha(y) = 0$ if $y < p\sigma \pi$, $\alpha(y) = [y/(p\sigma \pi)]^{\sigma/(1-\sigma)}$ if $p \sigma \pi \leq y \leq p\sigma \pi$, and $\alpha(y) = 1$ if $y \geq p\sigma \pi$.

Proof: See the appendix.

**Fighting in Hard and Uncertain Times**

Empirical evidence indicates that hard times make civil war and political conflict in general more likely. There is a strong negative relation between income and the likelihood of civil war (e.g., Collier and Hoeffer 2004; Fearon and Laitin 2003; Miguel, Satyanath, and Sergenti 2004). Low income also makes coups more likely (Londregan and Poole 1990), and recessionary crises tend to undermine democratic regimes (Gasiorowski 1995).

In our model, hard times make conflict more likely, as does a stronger opposition and lower costs. Recalling that $\pi = c + r$ with $r$ distributed according to $H$ over $[\underline{r}, \overline{r}]$, the probability of fighting given the climate $c$ of the times is:

$$F = \int_{\underline{r}}^{\overline{r}} \left[ 1 - \left( \frac{c + r}{c + \overline{r}} \right)^{\frac{\sigma}{1-\sigma}} \right] dH(r)$$

We can then establish,

**Proposition 2:** Hard times (low $c$), a stronger opposition (higher $p$), and low costs to fighting (high $\sigma$), make fighting more likely.

Proof: The integrand in $F$ is decreasing in $c$: $\partial F/\partial c = -\frac{\sigma}{1-\sigma} \frac{1}{c+\overline{r}} \left( \frac{c+r}{c+\overline{r}} \right)^{\frac{\sigma}{1-\sigma} - 1} < 0$. Bad times (lower values of $c$) therefore make fighting more likely. Conversely, the integrand is easily shown to be increasing in both $p$ and $\sigma$. So stronger rebels make for more fighting ($\partial F/\partial p > 0$) while higher costs lead to less fighting ($\partial F/\partial \sigma < 0$).

The probability of fighting is also increasing in the uncertainty surrounding the size of the spoils. Let $G$ be a mean-preserving spread of $H$ defined by $\gamma = r + \mu$ where $\mu$ has mean zero and is distributed over $[\underline{\mu}, \overline{\mu}]$ according to $M$ with continuous density $m$ which is assumed to be positive over $(\underline{\mu}, \overline{\mu})$. Then the equilibrium probability of fighting given that the spoils are distributed according to $H$ is less than the probability of fighting if...
the spoils are distributed according to $G$.

This is easy to see if $p\sigma/(1-\sigma) \leq 1$. The equilibrium probability of agreement is $A = 1 - F = \int_\gamma [(c+r)/(c+\bar{r})]^{\frac{\sigma}{1-\sigma}} dH$, and the integrand is concave because $p\sigma/(1-\sigma) \leq 1$. Jensen’s inequality then implies

$$\int_\gamma \left( \frac{c+r}{c+\bar{r}} \right)^{\frac{\sigma}{1-\sigma}} dH \geq \int_\gamma \left( \frac{c+\bar{r}}{c+\bar{r}} \right)^{\frac{\sigma}{1-\sigma}} dG$$

where $\bar{\gamma} = \bar{r} + \bar{\mu}$ and similarly for $\gamma$. Because $\bar{\gamma} > \bar{r}$,

$$\int_\gamma \left( \frac{c+r}{c+\bar{r}} \right)^{\frac{\sigma}{1-\sigma}} dH > \left( \frac{c+\bar{r}}{c+\bar{r}} \right)^{\frac{\sigma}{1-\sigma}} \int_\gamma \left( \frac{c+r}{c+\bar{r}} \right)^{\frac{\sigma}{1-\sigma}} dG$$

$$> \int_\gamma \left( \frac{c+\bar{r}}{c+\bar{r}} \right)^{\frac{\sigma}{1-\sigma}} dG$$

An increase in the uncertainty about the spoils therefore reduces the probability of acceptance and increases the probability of fighting when $p\sigma/(1-\sigma) \leq 1$. The appendix shows that the same holds even if $p\sigma/(1-\sigma) > 1$. Hence,

**Proposition 3:** Let $G$ be any mean preserving spread of $H$ defined by $\gamma = r + \mu$ where $\mu$ has mean zero and is distributed over $[\mu, \bar{\mu}]$ according to $M$ with continuous density $m$ which is positive over $(\mu, \bar{\mu})$. Then the probability of fighting if the spoils are distributed according to $G$ is strictly larger than if the spoils are distributed according to $H$.

**Proof:** See the appendix.

Although the probability of fighting is decreasing in $c$ and increasing in the uncertainty surrounding $r$, an example suggests that the fundamental cause of breakdown is not the climate of the times or uncertainty about the spoils per se. Rather, the probability of fighting is a function of the relative uncertainty. Suppose that $r$ is distributed uniformly over $[-u, u]$. Then the volatility of the spoils defined by $v = \sqrt{\text{var}(r)}/c$ measures the uncertainty in $\pi$ relative to the mean $c$. Solving for the probability of fighting and using
the fact that $v = u/(c\sqrt{3})$ gives:

$$F = 1 - \frac{(1 - \sigma)(c + u)}{2u[1 - \sigma(1 - p)]} \left[ 1 - \left( \frac{c - u}{c + u} \right)^{\frac{1-\sigma(1-p)}{1-\sigma}} \right]$$

$$= 1 - \frac{(1 - \sigma)(1 + v\sqrt{3})}{2v\sqrt{3}[1 - \sigma(1 - p)]} \left[ 1 - \left( \frac{1 - v\sqrt{3}}{1 + v\sqrt{3}} \right)^{\frac{1-\sigma(1-p)}{1-\sigma}} \right]$$

Thus, the probability of fighting is a function of the relative uncertainty surrounding the spoils.\(^8\)

Power Sharing: Why Not Reveal the Size of the Pie?

The opposition fights because it has to deter the government from bluffing by making small offers when the spoils are relatively large. Suppose, however, that there were some way the government could verifiably reveal the size of the pie to the opposing faction, i.e., a way that somehow made bluffing impossible. Then the government would reveal the spoils because this would increase its payoff. If, more specifically, the government verifiably reveals the size of the pie to the opposing faction before offering that faction its certainty equivalent $p\sigma\pi$, the opposition will accept this offer for sure rather than with probability $\alpha(p\sigma\pi) = (\pi/\bar{\pi})^{\sigma/>(1-\sigma)} < 1.\(^9\)$ As a result, verifiably revealing the spoils raises the government’s payoff from $\alpha(p\sigma\pi)(\pi - p\sigma\pi) + [1 - \alpha(p\sigma\pi)](1 - p)\sigma\pi =

\(^8\) There is a technical issue here. The notion of uncertainty or risk in Proposition 2 is second-order stochastic dominance whereas the measure of uncertainty in $v$ is the variance. Second-order stochastic dominance is not in general equivalent to an increase in variance. These two notions are, however, equivalent in the case of uniform distributions: the uniform distribution over $[-u, u]$ second-order stochastically dominates the uniform distribution $[-u', u']$ if and only if $u < u'$ which is equivalent to variance of the former, $u/\sqrt{3}$, being less than the variance of the latter, $u'/\sqrt{3}$.

\(^9\) Although the opposition are indifferent between accepting their certainty equivalent and fighting, they accept for sure for the same reason that they are sure to accept this offer in a complete-information, take-it-or-leave-it-offer game. If the government has verifiably revealed the spoils to be $\pi$, then the opposition accepts any $z > p\sigma\pi$ with probability one as it is sure to do strictly better by accepting than by fighting. But if in turn they do not accept $z = p\sigma\pi$ for sure, then the government has no best reply to the opposing faction’s strategy, and this strategy cannot be part of an equilibrium.
\[ \pi(1 - p\sigma) - [1 - \alpha(p\sigma\pi)]\pi(1 - \sigma) \text{ to } \pi(1 - p\sigma). \] Why, then, does the government not verifiably reveal its private information?

One answer may be that there is simply no way to do so in a way that is not vulnerable to bluffing. Suppose, alternatively, that the government can reveal the spoils to the opposition – possibly by bringing it into the government through a power-sharing agreement. But revealing information is costly. In particular, verifiably revealing the size of the pie to the opposing faction also shifts the distribution of power in its favor by increasing the probability that the opposition prevails from \( p \) to \( p + \Delta \).

This shift introduces a commitment problem alongside the original informational problem. If the opposition could commit to accepting \( p\sigma\pi \) and to not exploiting its enhanced bargaining power, then the government would reveal the spoils, the opposing faction would accept the government’s offer, and there would be no fighting. But the opposition cannot commit to this and will fight if offered anything less than \( (p + \Delta)\sigma\pi \). Thus, verifiably revealing the spoils to the opposition also raises the cost of buying it off from \( p\sigma\pi \) to \( (p + \Delta)\sigma\pi \). If this cost is too large, the commitment problem swamps the informational problem, and the government foregoes the opportunity to reveal the spoils.

More formally, assume that the government can reveal the spoils at the outset of the game or make an offer to the opposing faction. If the government reveals the spoils, the game ends with payoffs \( \pi[1 - (p + \Delta)\sigma] \) and \( (p + \Delta)\sigma\pi \) for the government and opposition respectively. If the government makes an offer, the game proceeds as before.

In equilibrium, the government either shares power or offers \( p\sigma\pi \). It shares power and thereby avoids any risk of fighting if \( \pi[1 - (p + \Delta)\sigma] \geq \alpha(p\sigma\pi)\pi(1 - p\sigma) + [1 - \alpha(p\sigma\pi)](1 - p)\sigma\pi \) where \( \alpha(p\sigma\pi) = (\pi/\pi)^{p\sigma/(1-\sigma)}. \) Hence, the government shares power if times are bad enough and the shift in power is small enough, i.e., if \( \pi < \pi\left[(1 - \sigma - \Delta\sigma)/(1 - \sigma)\right]^{(1-\sigma)/(p\sigma)}. \)

A More General Model

The preceding discussion has focused on an incumbent government and an opposition group vying for control of the state. But the model analyzed above is just one member of a simple class of signaling models that exhibit the same general equilibrium behavior: The
unique equilibrium satisfying D1 is separating and simple to characterize. This section describes this class of games, characterizes their equilibria, and briefly discusses two other members of this class, one a model of war and the other of litigation (the last one due to Reinganum and Wilde, 1986).

To be continued...

Conclusion

An incumbent government is likely to be better informed about the spoils that come with controlling the state than is an opposing, out-of-power faction. This informational asymmetry creates a vexing strategic problem for the government and opposing faction vying for control of the state. Because fighting is costly, the government prefers to buy off or coopt a potential challenger by offering to share some of the spoils with it. But even if the opposing faction would be willing to accept a lower offer if the spoils were known to be small and therefore the value of winning control of the state was less, the opposition cannot simply accept low offers when it is uncertain of the spoils. If it did, there would be nothing to deter the government from making low offers when the spoils were large. To discourage these low-ball offers, the opposing faction rejects low offers with positive probability and the bargaining breaks down in costly fighting.

When modeled as a signaling game, this dilemma leads to very simple equilibrium behavior. In the unique PBE satisfying Cho and Kreps’ (1987) D1 restriction on beliefs, the government offers the opposing faction the latter’s certainty equivalent of fighting. The larger the spoils, the more the government offers and the higher the probability the opposition accepts the proposal. Harder times, a stronger opposition, lower costs, and greater uncertainty about the spoils all make it more likely that the bargaining will break down in costly fighting.

The government bears the inefficiency cost of fighting and therefore would like to verifiably reveal the spoils to the opposition if that were possible and costless. If, however, revealing the spoils to the opposing faction also makes it more powerful, then the government may face a commitment problem. The government would prefer to reveal the spoils
to the opposition if the latter could commit itself to not exploiting its enhanced bargaining power. When the opposition cannot commit to this, the government will reveal the spoils only if the shift in power it induces is sufficiently small and times are bad enough.
Appendix

Proof of Lemma 1: Let \((y, \alpha; \mu)\) be a PBE with \(y' = y(\pi')\), \(y'' = y(\pi'')\) and \(\pi' < \pi''\). Incentive compatibility then implies:

\[
\alpha(y')(\pi' - y') + (1 - \alpha(y'))(1 - p)\sigma\pi' \\
\geq \alpha(y'')(\pi'' - y'') + (1 - \alpha(y''))(1 - p)\sigma\pi''
\]

(A1)

\[
\alpha(y'')(\pi'' - y'') + (1 - \alpha(y''))(1 - p)\sigma\pi'' \\
\geq \alpha(y')(\pi' - y') + (1 - \alpha(y'))(1 - p)\sigma\pi'
\]

(A2)

To establish (i) subtract (A1) from (A2) to obtain \((\pi'' - \pi')[\alpha(y'') - \alpha(y')][1 - \sigma(1 - p)] \geq 0\) which then leaves \(\alpha(y'') \geq \alpha(y')\).

To demonstrate (ii), assume \(\alpha(y') > 0\) and rewrite (A1) to get \(\alpha(y'')(y'' - y') \geq [\alpha(y'') - \alpha(y')][\pi' - y' - \sigma(1 - p)\pi']\). Because \(y'\) is accepted with positive probability, it must bring \(\pi'\) as least as much as it would get by fighting. So, \(\pi' - y' \geq \sigma(1 - p)\pi'\). This along with part (i) implies \([\alpha(y'') - \alpha(y')][\pi' - y' - \sigma(1 - p)\pi'] \geq 0\). Part (i) also ensures that \(\alpha(y'') \geq \alpha(y') > 0\) which leaves \(y'' \geq y'\). As for the case in which \(\alpha(y') = 0\) but \(\alpha(y'') > 0\), the previous argument immediately yields \(y'' \geq y'\).

As for (iii), again take \(\alpha(y') > 0\) and \(y'' > y'\). Rewriting (A2) gives \(\alpha(y'')(y'' - y') \leq [\alpha(y'') - \alpha(y')][\pi'' - y'' - \sigma(1 - p)\pi'']\). The left side of this inequality is positive. And, \(\alpha(y') > 0\) implies \(\alpha(y'') > 0\). Because \(y''\) is accepted with positive probability, agreeing to \(y''\) must bring \(\pi''\) as least as much as it would get by fighting. So, \(\pi'' - y'' \geq (1 - \sigma(1 - p))\). Hence, \(\alpha(y'') > \alpha(y')\).

Now suppose \(\alpha(y'') > 0\). If \(\alpha(y') = 0\), there is nothing to show. If \(\alpha(y') > 0\), the previous argument ensures \(\alpha(y'') > \alpha(y')\). □

Proof of Lemma 2: Let \((y(\pi), \alpha(x); \mu)\) be a PBE, and take \(z\) be any out-of-equilibrium offer such that \(z > y(\pi^+)\) and \(\alpha(y(\pi^+)) > 0\) for some \(\pi^+\). The set of strategies that are mixed best responses to \(z\) given some set of beliefs is simply \(\alpha \in [0, 1]\) as any \(\alpha\) is a best reply to \(z\) if the opposition believes \(\pi = z/(p\sigma)\). Moreover, deviating to \(z\) from \(y(\pi)\) given
\(\alpha\) is weakly profitable if:

\[(\pi - z)\alpha + (1 - \alpha)(1 - p)\sigma \pi \geq \alpha(y(\pi)) [\pi - y(\pi)] + [1 - \alpha(y(\pi))] (1 - p)\sigma \pi\]

\[\alpha \geq \alpha^*(\pi) \equiv \alpha(y(\pi)) \left(\frac{\pi[1 - \sigma(1 - p)] - y(\pi)}{\pi[1 - \sigma(1 - p)] - z}\right)\]

Hence, the set of strategies \(\alpha\) for which deviating from \(y(\pi)\) are strictly and weakly profitable are, respectively, \(D(z, \pi) \equiv (\alpha^*(\pi), 1]\) and \(D^0(z, \pi) \equiv [\alpha^*(\pi), 1]\).

There are now two cases to be considered. In the first, \(\pi^+ \leq \pi < \pi'\) and \(y' = y(\pi') < z\). Then \(\alpha^*(\pi) > \alpha^*(\pi')\) and consequently \(D^0(z, \pi) \subset D(z, \pi')\). To see that \(\alpha^*(\pi) > \alpha^*(\pi')\), note that \(z > y'\) implies:

\[1 + \frac{z - y'}{\pi[1 - \sigma(1 - p)] - z} > 1 + \frac{z - y'}{\pi'[1 - \sigma(1 - p)] - z}\]

\[\alpha(y') \left(\frac{\pi[1 - \sigma(1 - p)] - y'}{\pi[1 - \sigma(1 - p)] - z}\right) > \alpha(y') \left(\frac{\pi'[1 - \sigma(1 - p)] - y'}{\pi'[1 - \sigma(1 - p)] - z}\right)\]

But incentive compatibility implies \(\alpha(y)[\pi[1 - \sigma(1 - p)] - y] \geq \alpha(y')[\pi[1 - \sigma(1 - p)] - y']\), so

\[\alpha(y) \left(\frac{\pi[1 - \sigma(1 - p)] - y}{\pi[1 - \sigma(1 - p)] - z}\right) > \alpha(y') \left(\frac{\pi'[1 - \sigma(1 - p)] - y'}{\pi'[1 - \sigma(1 - p)] - z}\right)\]

\[\alpha^*(\pi) > \alpha^*(\pi')\]

Because \(D^0(z, \pi)\) is a strict subset of \(D(z, \pi')\), condition D1 requires the opposition to conclude that it is not facing \(\pi\). But for any \(\pi\) such that \(\pi^+ \leq \pi < \sup\{\pi : y(\pi) < z\}\), there exists a \(\pi'\) such that \(\pi^+ \leq \pi < \pi' < \sup\{\pi : y(\pi) < z\}\). D1 thus has the opposition’s putting probability zero on facing any \(\pi\) such that \(\pi^+ \leq \pi < \sup\{\pi : y(\pi) < z\}\).

If \(\pi < \pi^+\), then \(\alpha(y(\pi)) = 0\). Hence, \(\pi\) weakly prefers fighting to offering an arbitrarily small amount more than \(y(\pi^+)\) which is sure to be accepted with positive probability. This implies \((1 - p)\sigma \pi \geq \pi - y(\pi^+)\) which means \((1 - p)\sigma \pi \geq \pi - z\). Consequently, \(\pi\)
strictly prefers fighting to offering \( z \) for any \( \alpha \), so \( D^0(z, \pi) = \emptyset \). D1 therefore excludes all \( \pi < \pi^+ \) as well.

Turning to the second case, assume \( \pi^+ \leq \pi < \pi' \) and \( y = y(\pi) > z \). Then an analogous argument starting with \( y - z > 0 \) yields \( \alpha^*(\pi) < \alpha^*(\pi') \) which leads directly to \( D^0(z, \pi') \subset D(z, \pi) \). D1 now requires the opposition to put zero probability on facing \( \pi > \inf \{ \pi : y(\pi) > z \} \). Because \( y(\pi) \) is nondecreasing, \( \sup \{ \pi : y(\pi) < z \} = \inf \{ \pi : y(\pi) > z \} \), and the only belief satisfying D1 has the opposing faction believing that they are facing the \( \pi \) for which this equality holds. \( \square \)

**Proof of Lemma 3:** Let \( (y(\pi), \alpha(x); \mu) \) be a PBE satisfying condition D1. Arguing by contradiction, there must be two types \( \pi' \) and \( \pi'' \) such that \( \pi^+ < \pi' < \pi'' \) and \( \pi' \) and \( \pi'' \) make the same offer \( \hat{y} \). Let \( \hat{\pi} = \inf \{ \pi : \hat{y} = y(\pi) \} \).

It follows that \( \hat{y} > p\sigma\hat{\pi} \). Because \( y(\pi) \) is nondecreasing, all \( \pi \in [\pi', \pi''] \) propose \( \hat{y} \). This interval has positive measure, and therefore the rebel’s payoff to fighting must be strictly larger than the payoff to fighting the lowest type. Formally, \( \int_{\{x: \hat{y} = y(\pi)\}} p\sigma\pi d\hat{H}(\pi) > \int_{\{x: \hat{y} = y(\pi)\}} p\sigma\hat{\pi} d\hat{H}(\pi) = p\sigma\hat{\pi} \) where \( \hat{H} \) is the posterior of \( H \) given \( \hat{y} \). Lemma 1 guarantees \( \alpha(y(\pi)) > 0 \) for all \( \pi > \pi^+ \). Thus, the opposition accepts \( \hat{y} \) with positive probability which leaves \( \hat{y} \geq \int_{\{x: \hat{y} = y(\pi)\}} p\sigma\pi d\hat{H}(\pi) > p\sigma\hat{\pi} \).

Now consider any offer of slightly less than \( \hat{y} \), i.e., some \( z \in (\hat{y} - \varepsilon, \hat{y}) \) for an \( \varepsilon \) small enough to ensure \( z > p\sigma\hat{\pi} \). If the opposition strictly prefers accepting \( z \) to fighting, then \( \alpha(z) = 1 \) and a contradiction results as those offering \( \hat{y} \) could profitably deviate to the lower offer \( z \). To see that the opposition does prefer accepting \( z \), suppose that \( z \) is an equilibrium proposal, i.e., \( y(\pi) = z \) for some \( \pi \). Because \( y \) is nondecreasing and \( z < \hat{y} \). The opposing faction therefore believes that \( \pi \) is bounded above by \( \hat{\pi} \) after being offered \( z \) since \( \sup \{ \pi : z = y(\pi) \} \leq \inf \{ \pi : y(\pi) \geq \hat{y} \} = \hat{\pi} \). Hence, the opposition’s payoff to fighting is bounded above by \( p\sigma\hat{\pi} \) which is strictly less than \( z \). If \( z \) is an out-of-equilibrium offer, then the argument in the second case in the proof of Lemma 2 implies that the opposition believes it is facing \( \sup \{ \pi : y(\pi) \leq z \} \leq \hat{\pi} \) after \( z \). The opposing faction’s expected payoff to fighting is therefore \( p\sigma\hat{\pi} \) and again strictly less than \( z \). \( \square \)

**Proof of Proposition 1:** It remains to be shown that \( \pi^+ = \underline{\pi} \) and \( \alpha(y) = [y/(p\sigma\underline{\pi})]^{\sigma p/(1-\sigma)} \)
for all $\pi \geq \overline{\pi}$ if $\alpha(y) = [y/(p\sigma\overline{\pi})]^{\sigma p/(1-\sigma)}$ for all $\pi \geq \pi^+$. Suppose the contrary. If $\pi^+ > \overline{\pi}$, then all $\pi < \pi^+$ receive $(1 - p)\sigma\pi$ because $\alpha(y(\pi)) = 0$ for $\pi < \pi^+$. But $(\pi^+/\overline{\pi})^{\sigma p/(1-\sigma)} > 0$. So some $\pi$ less than but arbitrarily close to $\pi^+$ with equilibrium payoffs arbitrarily close to $(1 - p)\sigma\pi^+$ can offer slightly more than $p\sigma\pi^+$ and thereby obtain a payoff arbitrarily close to $(\pi - p\sigma\pi^+)(\pi^+/\pi)^{\sigma p/(1-\sigma)} + [1 - (\pi^+/\pi)^{\sigma p/(1-\sigma)}](1 - p)\sigma\pi = (1 - p\sigma\pi) + \pi(1 - \sigma)(\pi^+/\pi)^{\sigma p/(1-\sigma)}$. This yields a profitable deviation and a contradiction if $\pi$ is close enough to $\pi^+$. This contradiction means $\pi^+ = \overline{\pi}$, the equilibrium is fully separating, and $\alpha(y) = [y/(p\sigma\overline{\pi})]^{\sigma p/(1-\sigma)}$ for all $y \geq p\sigma\overline{\pi}$. \(\square\)

Proof of Proposition 3: to be added. \(\square\)
References


