Parameter instabilities and monetary policy in a Small Open Economy: Does the Bank of England respond to the exchange rate?

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Abstract

This paper reconsiders whether monetary policy in small open economies responds to exchange rates by studying possible parameter instabilities in a Dynamic Stochastic General Equilibrium model. The main focus of the paper is to revisit preceding evidence on the response to exchange rate movements by the Bank of England and determine if its reaction function has remained constant throughout the sample. To this end, I estimate a small open economy general equilibrium model using Bayesian econometric techniques over rolling windows. I find overwhelming evidence of shifts in several parameters, including those related to the policy rule. Furthermore, posterior odds tests reveal a time-varying response to exchange-rate fluctuations by the monetary authorities. The results favor the model with the nominal exchange rate embedded in the policy rule for the initial subsamples. However, the evidence weakens and ultimately favors the model with no exchange rate for the latter ones. The paper also documents evident variations in the model dynamics derived by the instability of parameters via rolling-window impulse response functions and variance decomposition analysis.

Keywords: Small Open Economy; Monetary policy; Bayesian methods; DSGE models; Time-varying parameters.

JEL classification: E1, E52, E58, F41.

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1. Introduction

This paper aims to study the following problems: How stable are the structural parameters of a small open economy Dynamic Stochastic General Equilibrium model (DSGE)? If there are parameter instabilities, what are the macroeconomic implications that emerge from this time-varying behavior? Has the Bank of England (BoE) updated its response to exchange rate fluctuations?[1] To answer these questions, I study the evolving nature of the structural parameters of a small open economy DSGE model using data for the United Kingdom (UK). The model is estimated using Bayesian techniques over rolling samples. By comparing the estimates of each rolling sample, I show that there is strong evidence of parameter variations. Further, I find that these parameter instabilities modify the role of the nominal exchange rate in the BoE's reaction function across estimations. I accomplish this by performing rolling-window posterior odds tests against two specifications: one where exchange rate movements are part of the BoE's policy rule and an alternative specification where the response to this variable is restricted to be zero. I also document how possible parameter shifts may alter the model dynamics by conducting window-specific impulse response functions and study if the relative importance of shocks changes over time via forecast error variance decomposition.

In the paper, I adopt a rolling window approach to depict the possible variations in the model parameters over the sample period and show how the macroeconomy responds to these changes. In contrast to other methodologies that recognize the possibility of time variation in parameters, this procedure allows me to expose possible instabilities using conventional estimation techniques without forcing the data to fit between a finite set of states nor limiting the number of time-varying parameters.[2] Indeed, this strategy helps illustrate changes in all the parameters and facilitates a narrative on the evolving behavior of monetary policy, not to mention that it goes beyond the scope of this paper to test for different regimes in the UK economy. Nonetheless, a possible caveat to this approach is that it assumes economic agents are unaware of possible parameter drifts and believe the structure of the model to be invariant over time when they form expectations. While this could be partly correct, an alternative interpretation is that agents do recognize the instability of parameters, but the possible changes are of unknown form to them.

The paper contributes to several strands of literature, some of which inherently overlap

[1] As I will expand below, there is mixed evidence in the literature on whether monetary authorities in the UK respond to the nominal exchange rate when determining the policy rate.

[2] Namely, structural models with time-varying parameters or DSGEs with Markov-Switching regime changes in structural parameters or stochastic volatilities.

with each other. First, I use a structural model to document the conduct of monetary policy by monetary authorities. DSGE models have become the benchmark framework of recent macroeconomic research for monetary policy analysis. For instance, in the US context, these models have been widely used to investigate possible structural changes that the economy has experienced, such as differences in monetary policy during the high-inflation episode in the 1970s and subsequent periods,[3] or to estimate the Fed's inflation target.[4]

Second, I study the case of the UK and extend previous research on the response of central banks in small open economies to exchange rate fluctuations.[5] Having DSGE models become the standard tool in modern macroeconomics, it is not a surprise the development and uprising adoption of these models to study monetary policy in the open economy context[6]. A large body of literature, for example, has investigated the role of exchange rate movements in central banks' decisions when setting monetary policy. Arguably, Lubik and Schorfheide (2007) has been one the most influential papers in this arena. They use a small open economy structural model to investigate the premise that central banks consider information on exchange rates to determine the policy rate. One of the main findings of their paper is that the central banks of Australia and New Zealand do not react to this variable, though the Bank of Canada and the Bank of England do.

Consequently, numerous papers have built upon this result to explain monetary episodes in different economies or to test the robustness of this result when changing the properties of the model or the estimation technique. This paper falls in the latter alternative. In this regard, the present inquiry resembles recent work that re-evaluates whether the BoE reacts to exchange rate fluctuations using alternative approaches. Dong (2013) extends Lubik and Schorfheide (2007)'s framework and finds that monetary policy by the Bank of Canada, the Reserve Bank of New Zealand, and the BoE is not responsive to exchange rate movements when a limited exchange rate pass-through is introduced into the model. Caraiani and Gupta (2020) use a frequency-components approach and find evidence that not only does the BoE respond to exchange rate changes, but they focus on long-term depreciation movements.[7]

[3]See Lubik and Schorfheide (2004), Eo (2009), Milani (2008), Mavroeidis (2010), Coibion (2012), Traum and Yang (2011), Bhattarai, Lee, and Park (2012), Elias (2020), among other relevant contributions

[4]For example, Ireland (2007), Cogley and Sbordone (2008) Cogley, Primiceri, and Sargent (2010), Del Negro and Eusepi (2011), Milani (2020).

[5]In this setup, the UK has been an attractive case study to researchers due to the structural and economic changes that the economy has experienced over the last decades, not to mention its major participation in the world economy. See De Lipsis (2021) for a thorough list of these historical events.

[6]Influential theoretical contributions in this area include Smets and Wouters (2003), Gali and Monacelli (2005), Adolfson, Laséen, Lindé, and Villani (2007), Justiniano and Preston (2010b)

[7]An important remark to mention is that the sample is extended to include the zero lower bound period,

Third, although the paper focuses on the coefficients linked to the central banks' reaction function, I also document the time-varying nature of all the model parameters and shed light on the mechanisms that drive the model dynamics of open economy models. Indeed, numerous papers have considered parameters deviations for the study of monetary policy, although most are applications to the closed economy DSGE counterpart. Fernández-Villaverde, Rubio-Ramírez, Cogley, and Schorfheide (2007) find large variations in several parameters by estimating a medium-scale DSGE model where agents understand and are allowed to respond to policy changes. Galvao, Giraitis, Kapetanios, and Petrova (2016) build on Smets and Wouters (2007) and develop a time-varying DSGE with an added financial sector to evaluate how macroeconomic responses to financial friction shocks change over time.

In contrast, Bianchi (2012), Davig and Doh (2014), and Debortoli and Nunes (2014) use a regime-switching approach to analyze the Federal Reserve's behavior during the postwar period. In their findings, these papers support the common belief of a change in US monetary policy that started with the tenure of Paul Volcker as Chairmen of the Fed. However, Bianchi (2012) warns that the Federal Reserves' behavior is better described by a back and forth between passive and active regimes instead of a one-time-only regime change. On the open economy front, Liu and Mumtaz (2011) estimate a Markov switching open economy structural model to examine possible changes in the UK's macroeconomic dynamics and find substantial evidence of parameter variations.

This paper is more closely related to empirical work that uses a rolling window strategy. For example, Canova (2009), Canova and Ferroni (2012) Castelnuovo (2012), Hurtado (2014), and Ilabaca and Milani (2020) consider parameter instabilities in closed economy DSGE models by performing rolling window estimations. Furthermore, the closest paper that resembles the present analysis is Zamarripa (2021), who performs rolling window estimations on a small open economy DSGE model to document the conduct of monetary policy by the Bank of Mexico during the disinflation episode of 1995-2003. The paper shows that the policy feedback coefficients on output and the exchange rate systematically transitioned to lower values, while the response to inflation remained invariant. In this paper, however, I fully address the possibility that monetary authorities removed the nominal exchange rate from their reaction function. To my knowledge, this is the first application of rolling window estimations using UK data.

In general, the results derived from the present inquiry are of particular relevance for

which may likely affect the underlying comparison. I revisit this result in Section 4.3.

policy analysis as they provide new evidence on the importance of considering that 'structural' parameters may exhibit a time-varying component. For instance, monetary authorities could assign incorrect weights to the parameters that govern the policy rule when pursuing economic objectives and inadvertently create unintended macroeconomic effects. Similarly, failing to consider parameter instabilities would likely yield an inaccurate study of the propagation and relative importance of structural shocks or generate relatively poor forecasts.

The main empirical results of this paper are as follows. First, I find conclusive evidence of drifts in several model parameters. Most of these changes display clear transitions over the rolling windows. In particular, monetary policy is more assertive toward inflation in the initial samples and becomes more passive in the latter ones. The opposite is true for the authorities' behavior to the output gap: monetary policy steadily becomes more reactive. As for the response to exchange rate movements, posterior odds tests reveal a time-varying response to exchange-rate fluctuations by the monetary authorities. The results favor the model with the nominal exchange rate embedded in the policy rule for the initial samples. However, the evidence weakens and even suggests otherwise in the latter ones. This result is remarkably interesting, as it sides with both fronts of the debate on whether central banks in small open economies respond to the exchange rate by showing that monetary policy is not an invariant process. Mainly, the findings are of interest to the literature on optimal policy design in small open economies. Justiniano and Preston (2010b) show that it is not optimal for policymakers to respond to exchange rate variations within a class of generalized Taylor rules, suggesting an adapting behavior by the BoE. In terms of macroeconomic implications, the results show evident changes in the model dynamics. For instance, rolling-window impulse response functions expose periodic transitions and different degrees of persistence, while the relative contributions of forecast-error variance shares show transitory changes across the samples.

The rest of the paper is structured as follows. Section 2 outlines the structural Small Open Economy model. Section 3 describes the data, the rolling window procedure, and the estimation strategy. Section 4 presents the main estimation results by providing evidence of parameter instabilities, assessing the corresponding macroeconomic implications, and unveiling whether the BoE's has updated its response to exchange rate fluctuations. Section 5 concludes. Supplemental materials concerning the estimations are presented in the Appendix.

2. The model

The model specification is taken from Justiniano and Preston (2010b), which is a generalization of Monacelli (2005) and Gali and Monacelli (2005), but allowing for complete asset markets, habit formation, and indexation of prices to past inflation. Here, I summarize the reduced form equations, referring the reader to Justiniano and Preston (2010b) for a detailed derivation of the log-linearized model.[8]

$$(1+h)c_t = hc_{t-1} + E_t c_{t+1} - \sigma^{-1}(1-h)(i_t - \mathbb{E}_t\pi_{t+1}) + \sigma^{-1}(1-h)(\varepsilon_t^g - \mathbb{E}_t\varepsilon_{t+1}^g) \tag{1}$$

$$y_t = (1-\alpha)c_t + \alpha\eta(2-\alpha)s_t + \alpha\eta\psi_{F,t} + \alpha y_t^* \tag{2}$$

$$\Delta s_t = \pi_{F,t} - \pi_{H,t} \tag{3}$$

$$q_t = \psi_{F,t} + (1-\alpha)s_t \tag{4}$$

$$\Delta e_t = \Delta q_t + \pi_t - \pi_t^* \tag{5}$$

$$(1+\beta\delta_H)\pi_{H,t} = \delta_H\pi_{H,t-1} + \theta_H^{-1}(1-\theta_H)(1-\theta_H\beta)mc_t + \beta\mathbb{E}_t\pi_{H,t+1} \tag{6}$$

$$(1+\beta\delta_F)\pi_{F,t} = \delta_F\pi_{F,t-1} + \theta_F^{-1}(1-\theta_F)(1-\theta_F\beta)\psi_{F,t} + \beta\mathbb{E}_t\pi_{F,t+1} + \varepsilon_t^{cp} \tag{7}$$

$$\pi_t = \pi_{H,t} + \alpha\Delta s_t \tag{8}$$

$$\mathbb{E}_t\Delta q_{t+1} = (i_t - \mathbb{E}_t\pi_{t+1}) - (i_t^* - \mathbb{E}_t\pi_{t+1}^*) + \chi a_t + \varepsilon_t^\phi \tag{9}$$

$$c_t + a_t = \beta^{-1}a_{t-1} - \alpha(s_t + \psi_{F,t}) + y_t \tag{10}$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[\psi_\pi\pi_t + \psi_y y_t + \psi_{\Delta e}\Delta e_t] + \varepsilon_t^M \tag{11}$$

Eq. (1) denotes the log-linear approximation to the domestic household's Euler equation. Log of current consumption, c_t, is a function of expected future consumption, past consump-

[8] An overview of the microfoundations of the small open economy model is also available in the Appendix.

tion (via the assumption of habit formation in the household's preferences), the ex-ante real interest rate, $(i_t - \mathbb{E}_t\pi_{t+1})$, and a preference shock, ε_t^g. The parameters h and σ indicate the degree of habit persistence in consumption and the inverse of the intertemporal elasticity of substitution, respectively. Notice that disabling habit formation in the model, setting $h = 0$, returns the usual Euler equation.

Eq. (2) is derived by log-linearizing the goods market-clearing condition. Domestic output, y_t, is the sum of equilibrium domestic consumption and three elements of foreign variations (which, in turn, describe the foreign demand for the domestically produced good): the terms of trade, s_t, deviations from the law of one price, $\psi_{F,t} \equiv (e_t + p_t^*) - p_{F,t}$, and foreign output, y_t^*. In contrast to Monacelli (2005), import retailers are assumed to retain a small degree of pricing power when determining the domestic currency price of the imported good, thus leading to a violation of the law of one price. α is the import share (the share of foreign goods in the domestic consumption bundle), and η denotes the elasticity of substitution between domestic and foreign goods.

Eq. (3) is obtained by time differencing the bilateral terms of trade (i.e., the price of the foreign country's goods in terms of home goods). Differences in the terms of trade are a function of domestic price inflation, $\pi_{H,t}$, and domestic currency import price inflation, $\pi_{F,t}$. As usual, the mathematical operator Δ is used to denote first differences.

Eq. (4) portrays the relationship between the terms of trade and the real exchange rate. In particular, the real exchange, q_t, is explained by the deviations of the foreign price from the domestic currency price of imports and the heterogeneity of consumption bundles between the domestic and foreign economies. Time differencing this expression yields Eq. (5),[9] where Δe_t measures changes in the nominal exchange rate, π_t stands for CPI inflation, and π^* is foreign inflation.

Eq. (6) is obtained by log-linearizing the optimality conditions that arise from solving the domestic firms' price-setting problem. The resulting Phillips Curve implies that domestic price inflation is defined by the most recent observed inflation rate, the current marginal cost, $mc_t = \varphi y_t - (1+\varphi)\varepsilon_t^a + \alpha s_t + \sigma(1-h)^{-1}(c_t - hc_{t-1})$, where ε_t^a is an exogenous technology shock, and next-period inflation expectations, $\mathbb{E}_t\pi_{H,t+1}$. Compared to the closed-economy setup, domestic goods prices also respond to sources of foreign variation, namely the terms of trade, foreign output, and the deviations from the law of one price.[10] The structural parameter

[9] This is easier to notice from the original expression $q_t = e_t + p_t^* - p_t$. Eq. (4) is then derived using the fact that $p_t^* = p_{F,t}^*$ implied from the treatment of the rest of the world as a closed economy.

[10] This occurs directly through the marginal cost, and indirectly via the marke-clearing condition (see Eq. (2)).

δ_h depicts the degree of indexation to past inflation, θ_H is the fraction of firms that cannot optimally adjust their price each period, φ is the inverse elasticity of labor supply, and β is the traditional discount factor.

Similarly, a log-linear approximation of the retailers' optimality conditions renders Eq. (7). In this Phillips curve for import prices, domestic currency import price inflation is a function of its lag, deviations from the law of one price, expectations about the next period's inflation, $\mathbb{E}_t \pi_{F,t+1}$, and a cost-push shock, ε_t^{cp}, that captures inefficient variations in mark-ups. The parameter δ_F represents the indexation to previous import prices, and θ_F is the number of retail firms that cannot adjust prices.

Eq. (8) summarizes how domestic CPI and home goods prices are related. More precisely, and by substituting the terms of trade from Eq. (3), CPI inflation is defined as the weighted difference between domestic and imported goods price inflation (captured by the trade openness parameter).

Eq. (9) represents the log-linear version of an uncovered interest rate parity condition, which introduces the assumption of incomplete asset markets. The difference between the one-period-ahead expected and the current real exchange rate, $\mathbb{E}_t \Delta q_t$, depends on the gap between the domestic and foreign ex-ante real interest rates, the level of foreign assets position, a_t, and a risk premium shock, ε_t^{ϕ}. The debt elasticity with respect to the interest rate premium is governed by the structural parameter χ. Eq. (10) summarizes the foreign assets budget constraint.

Lastly, Eq. (11) embodies the BoE monetary policy reaction function. Monetary authorities set the nominal interest, i_t, rate following a Taylor-type rule that features persistence in nominal interest rates but also responds to current CPI inflation, domestic output, changes in the nominal exchange rate, and a monetary policy shock, ε_t^M. Parameters ψ_π, ψ_y, and $\psi_{\Delta e}$ represent the central bank's response to inflation, output, and the changes in the nominal interest rate, respectively. ρ_i is the interest-rate smoothing term.

Thus, Equations (1)-(11) characterize the domestic block of the model and describe the aggregate dynamics of the small open economy. In contrast, the foreign economy is assumed to be exogenous to the domestic economy[11] and is specified to follow an autoregressive process of order one:

[11] Strictly speaking, in Monacelli (2005) the underlying model assumes a world of two asymmetric economies, with one of them being small relative to the other (and which equilibrium is taken as exogenous).

$$\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + \varepsilon_t^{\pi^*} \tag{12}$$

$$y_t^* = \rho_{y^*}y_{t-1}^* + \varepsilon_t^{y^*} \tag{13}$$

$$i_t^* = \rho_{i^*}i_{t-1}^* + \varepsilon_t^{i^*} \tag{14}$$

Briefly, Equations (12)-(14) describe the paths for foreign inflation, foreign output, and foreign interest rates, with their associated shocks, $\varepsilon_t^{\pi^*}$, $\varepsilon_t^{y^*}$, $\varepsilon_t^{i^*}$, and corresponding autoregressive parameters, ρ_{π^*}, ρ_{y^*}, ρ_{i^*}, respectively. Together, the domestic and foreign blocks, along with the expectation terms and the exogenous disturbances (also assumed to evolve according to univariate autoregressive processes[12]), comprise a linear rational expectations model that can be rewritten in its state-space form as:

$$\Gamma_0 X_t = \Gamma_1 X_{t-1} + \Psi\epsilon_t + \Pi\eta_t \tag{15}$$

where X_t is a state vector that collects the domestic and foreign endogenous variables, the expectation terms, and the AR(1) disturbances, $\epsilon_t = [\varepsilon_t^M, \widehat{\varepsilon}_t^a, \widehat{\varepsilon}_t^g, \widehat{\varepsilon}_t^\phi, \widehat{\varepsilon}_t^{cp}, \varepsilon_t^{\pi^*}, \varepsilon_t^{y^*}, \varepsilon_t^{i^*}]'$ is a vector of i.i.d. exogenous innovations with mean zero and a corresponding standard deviation, σ, and η_t is a vector of expectation errors.

Given the state-space representation from Eq. (15), I use Sims (2002)'s procedure to solve the model under rational expectations. The algorithm then renders the solution in the form of:

$$X_t = F(\Theta)X_{t-1} + G(\Theta)\epsilon_t \tag{16}$$

where the matrices $F(\Theta)$ and $G(\Theta)$ are functions of the parameters of the model.

Eq. (16) represents the transition equation of the DSGE model. It expresses the state variables solely as functions of their lags and exogenous innovations. The transition equation, combined with a measurement equation, can then be used to evaluate the likelihood function and estimate the model parameters with the Kalman filter.

[12]Namely $\varepsilon_t^a, \varepsilon_t^g, \varepsilon_t^\phi$, and ε_t^{cp}, such that $\varepsilon_t^j = \rho^j\varepsilon_{t-1}^j + \widehat{\varepsilon}_t^j$ for $j = \{a, g, \phi, cp\}$.

3. Estimation approach

3.1 Data description

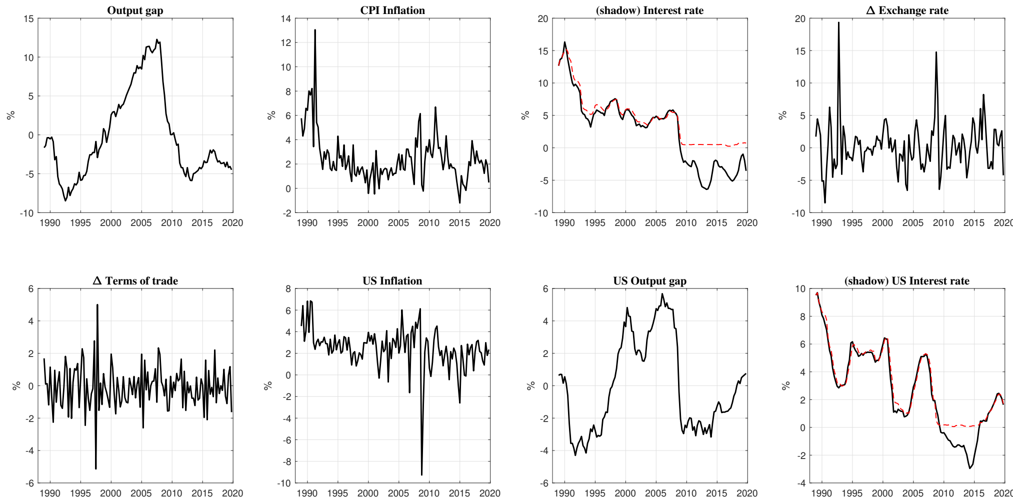
The empirical analysis uses quarterly observations on output, inflation, interest rates, real exchange rate changes, and terms of trade changes. I employ data that spans the years from 1989Q1 to 2019Q4 for the estimation exercise. Data series for the UK are obtained from the Office for National Statistics and the Bank of England databases. Output corresponds to real GDP per capita in log deviations from a linear trend. I calculate the inflation rate taking log difference of the Consumer Price Index (all goods) and scaled it by 400 to obtain annualized percentage rates. Real exchange rate changes are constructed using the UK/US bilateral nominal exchange rate and each country's CPI and then taking log differences to yield percentage changes. Terms of trade changes are computed using the ratio of import and export price indexes, also converted in log differences to obtain percentage changes. For the foreign block, I follow the standard approach of using US observables to approximate foreign variables. These data series are retrieved from the Federal Reserve of St. Louis (FRED) database. As with the domestic counterpart, foreign output is US real GDP per capita in log deviations from a linear trend, while foreign inflation corresponds to log differences of the US Consumer Price Index (scaled by 400). To deal with the zero lower bound situation imposed by the monetary policy instrument in both the UK and the US, I use Wu and Xia (2016)'s shadow interest rate equivalent (i.e., the nominal interest rate when the zero lower bound is not binding).[13] Lastly, all data series are seasonally adjusted and rescaled to have a zero mean. Figure 1 depicts a visual representation of the data.

3.2 Bayesian methodology and rolling-windows

As described before, the main objective of this paper is to document possible parameter instabilities, especially those embedded in the monetary policy rule. To this end, I illustrate the evolving behavior of the model parameters by conducting rolling-window Bayesian estimations. The rolling-window approach consists of repeated estimations of the model over different subsamples. Specifically, I use fifteen-year windows for the benchmark results, which I later compare to a twenty-year window exercise as a robustness check. The window size is maintained constant and considers increments of one year between each estimation.

[13]The UK and the US's shadow interest rates are available starting in 1990 throughout the end of the sample. I used the regular interest rate before that. The two series were converted from monthly to quarterly frequencies by taking the period's average.

Figure 1. Data series



Note: Red dashed line represents the effective interest rates for the UK and the US, respectively. Series are shown before being mean-zero rescaled.

This means that the first rolling-window uses data from 1989Q1 to 2003Q4; the second rolling-window uses data from 1990Q1 to 2004Q4; and so forth. Repeating this process over the full sample implies that the model is re-estimated seventeen times for the 15-year-window analysis, and twelve times when using twenty-year windows.

For the estimation of the DSGE model, I follow the Bayesian framework from Schorfheide (2000) and An and Schorfheide (2007). This approach is suitable to characterize the posterior distribution of the structural parameters that govern the small open economy model over each subsample. These parameters are jointly estimated using Bayesian methods and collected in the parameter vector Θ:

$$\begin{aligned}\Theta =&[\alpha, \sigma, \varphi, \theta_H, \theta_F, \eta, h, \delta_H, \delta_F, \rho_i, \psi_\pi, \psi_y, \psi_{\Delta e}, \rho_a, \rho_g, \rho_\phi, \rho_{cp}, \rho_{\pi^*}, \rho_{y^*}, \rho_{i^*},\\ &\sigma_M, \sigma_a, \sigma_g, \sigma_\phi, \sigma_{cp}, \sigma_{\pi^*}, \sigma_{y^*}, \sigma_{i^*}]'\end{aligned}$$

For each rolling-window, draws from the posterior distribution are generated using the random-walk Metropolis Hasting algorithm. I compute the posterior mode and the corresponding Hessian matrix employing standard optimization routines. Subsequently, I run 200,000 iterations and discard the initial 25% as burn-in. The Hessian is scaled accordingly to maintain a target acceptance rate between 25 and 30% on each subsample. For each draw, I obtain the likelihood of the model using the Kalman Filter and the state-space matrices derived from the rational expectations solution (see Eq. (16)).

3.3 Prior distributions

Table 1 summarizes the prior distributions for the model parameters. The choice of priors is based on Justiniano and Preston (2010b) and Lubik and Schorfheide (2007), although extended to consider Liu and Mumtaz (2011) as this paper offers a better and appropriate point of comparison in terms of the estimation approach and the country analyzed.

I follow Justiniano and Preston (2010b) and fix the discount factor and the debt elasticity to the interest rate premium coefficients at values of 0.99 and 0.01, respectively. However, I estimate the trade openness parameter jointly with the other model parameters using a Beta distribution centered at the average share for exports and imports to GDP in the UK over the sample period.[14] I assume the intertemporal elasticity of substitutions σ to follow a Gamma distribution centered at 1.2 with a standard deviation of 0.4. The priors for the inverse Frisch elasticity of labor supply φ and the elasticity of substitution between domestic and foreign goods parameters η are described using Gamma distributions with a mean of 1.5 and a standard deviation of 0.75. Calvo pricing parameters, θ_H and θ_F, are set to follow Beta distributions centered at 0.5 with standard deviations of 0.1. I specify the priors for the habit persistence parameter h and the indexation parameters, δ_H and δ_F, using Beta distributions with a mean of 0.5 and a standard deviation of 0.25. Regarding the parameters that govern the Taylor rule, I adopt the traditional assumption of using Gamma distributions to describe the policy response parameters. The authorities' response to inflation ψ_π is centered at 1.5, with a standard deviation of 0.30. Both the response to output ψ_y and exchange rate changes $\psi_{\Delta e}$ are set to have a mean of 0.25 and 0.13. For the interest rate smoothing parameter ρ_i, I use a Beta distribution mean centered around 0.5 with a standard deviation of 0.25. Given the nature and focus of this paper on studying the central bank's evolving behavior, I verify the robustness of the results by repeating the estimation exercise using an alternative set of priors on the policy rule. Lastly, the autoregressive coefficients of the structural shocks are all assumed to be persistent and follow Beta distributions, while the standard deviations of these disturbances are modeled to follow Inverse Gamma distributions.[15]

[14] Nonetheless, I had to impose a small standard deviation to avoid obtaining unreasonable low estimates. In preliminary estimations, I calibrated this parameter to the period's average to check for robustness and found similar overall results.

[15] As additional robustness checks, I also estimated all windows following the prior specifications from Liu and Mumtaz (2011) for both the autoregressive coefficients and the standard deviations of the exogenous

Table 1.
Prior distributions

Parameter	Domain	Density	P(1)	P(2)
β	0.99	-	-	-
χ	0.01	-	-	-
α	[0.1)	Beta	0.25	0.10
σ	$\mathbb{R}^+$	Gamma	1.20	0.40
φ	$\mathbb{R}^+$	Gamma	1.50	0.75
θ_H	[0.1)	Beta	0.50	0.10
θ_F	[0.1)	Beta	0.50	0.10
η	$\mathbb{R}^+$	Gamma	1.50	0.75
h	[0.1)	Beta	0.50	0.25
δ_H	[0.1)	Beta	0.50	0.25
δ_F	[0.1)	Beta	0.50	0.25
ρ_i	[0.1)	Beta	0.50	0.25
ψ_π	$\mathbb{R}^+$	Gamma	1.50	0.30
ψ_y	$\mathbb{R}^+$	Gamma	0.25	0.13
$\psi_{\Delta e}$	$\mathbb{R}^+$	Gamma	0.25	0.13
ρ_a	[0.1)	Beta	0.80	0.10
ρ_g	[0.1)	Beta	0.80	0.10
ρ_ϕ	[0.1)	Beta	0.80	0.10
ρ_{cp}	[0.1)	Beta	0.50	0.25
ρ_{π^*}	[0.1)	Beta	0.80	0.10
ρ_{y^*}	[0.1)	Beta	0.80	0.10
ρ_{i^*}	[0.1)	Beta	0.80	0.10
σ_M	$\mathbb{R}^+$	Inverse Gamma	0.50	4.00
σ_a	$\mathbb{R}^+$	Inverse Gamma	0.50	4.00
σ_g	$\mathbb{R}^+$	Inverse Gamma	1.50	4.00
σ_ϕ	$\mathbb{R}^+$	Inverse Gamma	0.50	4.00
σ_{cp}	$\mathbb{R}^+$	Inverse Gamma	0.50	4.00
σ_{π^*}	$\mathbb{R}^+$	Inverse Gamma	0.50	4.00
σ_{y^*}	$\mathbb{R}^+$	Inverse Gamma	1.50	4.00
σ_{i^*}	$\mathbb{R}^+$	Inverse Gamma	0.50	4.00

Note: P(1) and P(2) refers the mean and standard deviation for the Beta and Gamma distributions, and scale and shape for the inverse gamma distribution.

disturbances and found near-identical results.

4. Evidence of parameter drifts: empirical results

4.1 Posterior estimates and policy responses

I present the rolling window Bayesian estimates of all the model parameters in Figure 2. The figure shows the posterior median and 95% credible bands for each model parameter across subsamples. Roughly speaking, the parameters are displayed following this order: structural (with policy coefficients at the end) first, and then those that describe the exogenous shocks (autoregressive elements first, followed by the standard deviation of the innovations).

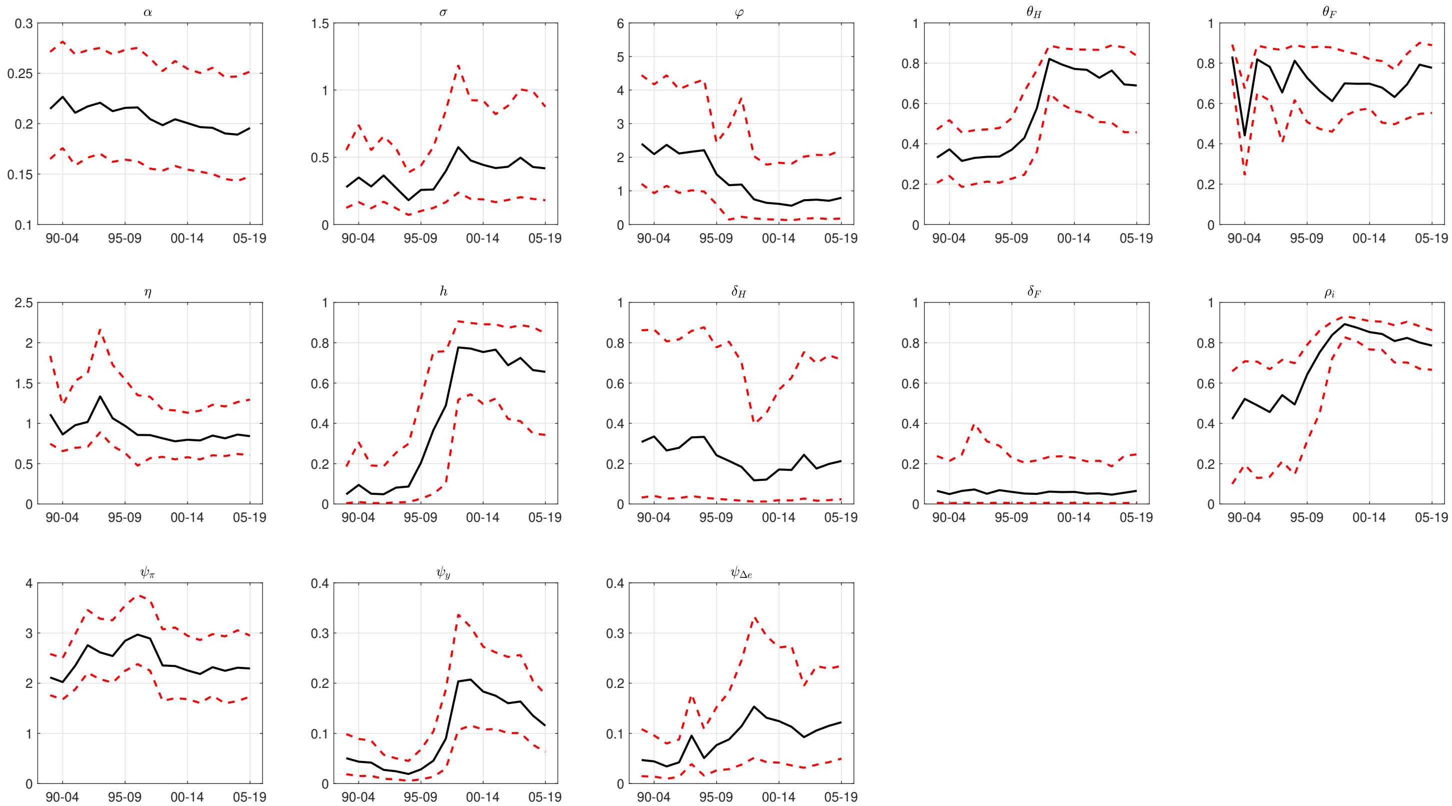
The trade openness parameter shows a steady decrease across the different windows, with posterior medians ranging between 0.18 and 0.22. In contrast, the UK's observed average share of exports and imports to GDP increased over the subsamples, moving from 0.23 in the first window to around 0.28 in the last one. Nevertheless, as previous research suggests (Lubik and Schorfheide (2005), Lubik (2006), and Justiniano and Preston (2010a)) this result is not surprising: an attempt to estimate the openness coefficient leads the estimation algorithm to choose parameter values that match the volatility of the data while complying with the cross-equation restrictions; thus, resulting in relatively lower estimates.

The inverse of the intertemporal elasticity of substitution shows an interesting behavior across the windows. In the first half of the estimations, the posterior medians range between 0.18-0.36 and then shift to higher values between 0.41-0.57 in the second half. One possible explanation of this transition to lower degrees of consumption growth responsiveness could be the relatively low interest rates that characterize the Zero Lower Bound period. These results seem to be consistent with similar research on small open economies.[16] Likewise, the inverse elasticity of labor supply also reveals an evident shift across the rolling windows, drifting from a posterior median of 2.4 in the initial windows to 0.79 in the last one.

Optimal price setting in home goods and imported goods render two contrasting results. The Calvo parameter of home good prices displays a progression from lower to higher posterior medians. For the lower estimates, the results suggest that firms reoptimize prices approximately every 1.5 quarters, while this happens every 3-5 quarters for the higher estimates. On the other hand, I find no evidence of a time-varying behavior by the imported-prices Calvo parameter. Instead, the posterior medians move back and forth between estimations, with rather similar credible bands, and suggest price re-optimization every 2-5 quarters.

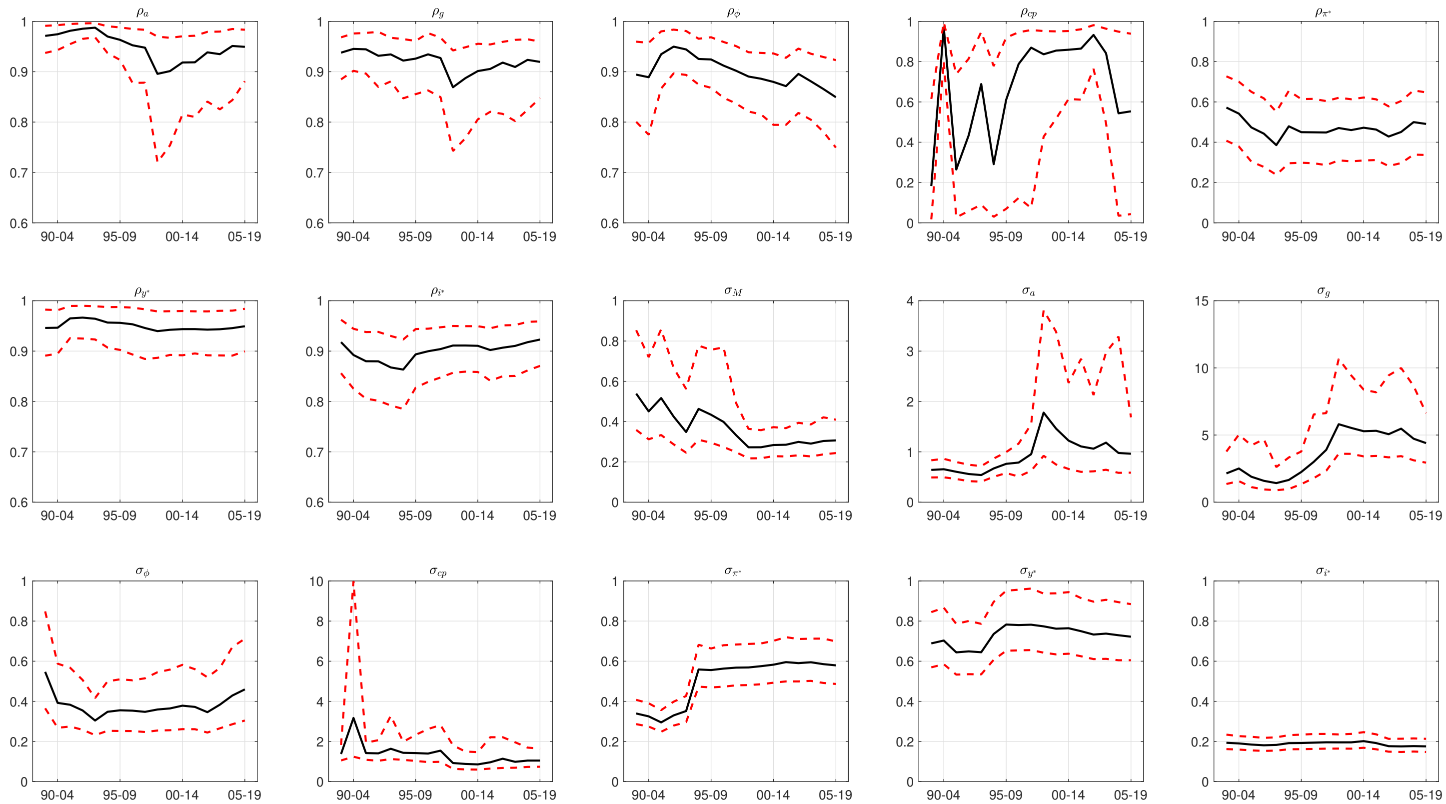
[16] In the UK context, for instance, Lubik and Schorfheide (2007) report a posterior mean of 0.36 (available in their working paper version), Caraiani and Gupta (2020) a posterior mean of 0.23. Liu and Mumtaz (2011) document values that range between 1.76-2.23 across alternative regimes. However, note that these estimates are derived using different sample periods and model specifications.

Figure 2. Rolling-window posterior estimates: structural parameters



Note: The figure shows the posterior median (solid line) of each sub-sample across the Metropolis-Hastings draws, along with 95% Bayesian credible interval bands (dashed lines).

Figure 2. (cont.) Rolling- window posterior estimates: autoregressive and standard deviations



Note: The figure shows the posterior median (solid line) of each sub-sample across the Metropolis-Hastings draws, along with 95% Bayesian credible interval bands (dashed lines).

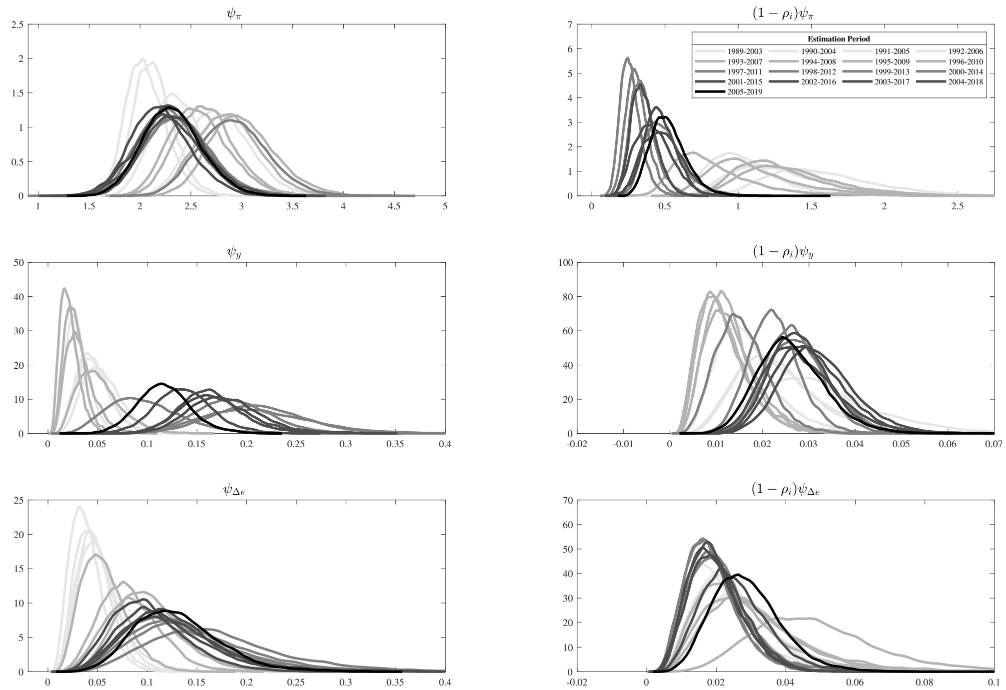
Posterior medians for the elasticity of substitution between home and foreign goods are close to unity in all windows, with slightly higher values in the initial subsamples that move below one in the latter ones. These findings consonate with related literature, although the estimates are still somewhat low compared to studies that use micro data.[17] The results for the habit formation parameter are interesting to note. The posterior medians seem to transition from estimates in the vicinity of 0.08 to ones in the neighborhood of 0.65-0.77. In comparison, Justiniano and Preston (2010b) and Liu and Mumtaz (2011) also find habits in consumption to play a smaller role than in other studies. They argue that these differences are likely due to the set of autoregressive shocks included in the model, particularly the fact that the persistence of home goods inflation is largely explained by the technology and preference shocks. Indeed, the results reveal similar patterns in the estimates for the standard deviations of these shocks and those encountered for the habit formation parameter. Moreover, it is remarking to note that the observable drift in all of these parameters starts when the windows include the data that concern the Zero Lower Bound period.

As for the price indexation parameters, I do not find substantial evidence of parameter variations. The densities for the price indexation of domestic goods do exhibit a transition to lower posterior medians, but the credible bands are too wide across all the estimations to sustain that inference. The imported goods indexation parameter displays a constant behavior over all windows.

For the parameters associated with the exogenous shocks, I find the following results. First, there is a high degree of persistence in almost all shocks. Except for the cost-push and the foreign inflation autoregressive parameters, most posterior medians stay above 0.8 for all subsamples. Second, only the persistence of the risk-premium shock exhibit an evident shift to slightly lower posterior medians in latter windows. Furthermore, the densities found for the autoregressive coefficient of the cost-push shock show visible instabilities across the estimations, with erratic behavior and wide credible bands. Third, the majority of the standard deviation parameters manifest a time-varying behavior. The standard deviation of the monetary policy innovation decreases in the first half of the windows and then stabilizes in the rest, with wider credible bands first and narrower later. The corresponding estimates for the technology and preference coefficients depict a similar narrative, moving from lower to higher values. Posterior distributions for the risk-premium standard deviation register a U-shape pattern across the estimations. Similar to the persistence coefficient, densities for

[17] See for example Obstfeld and Rogoff (2000). Refer also to Bajzik, Havranek, Irsova, and Schwarz (2020) for a survey of estimated values in individual studies.

Figure 3. Posterior distributions of policy coefficients and (effective) policy responses



the standard deviation of the cost-push shock are the most volatile but become relatively more stable in the latter windows while shifting to lower values.[18] Concerning the foreign block, only the standard deviation for foreign inflation drifts to larger posterior medians.[19]

Given the central role of monetary policy in this paper, and to facilitate a visual narrative of the evolution of these parameters, I display the BoE's policy coefficients (and the effective responses) separately in Figure 3. The figure portrays the same information as Figure 2 but overlaps the posterior distributions instead. I employ different shades of gray that periodically become darker to illustrate the transition of the rolling window densities over the different samples. The left-hand panels plot the masses for ψ_π, ψ_y, and $\psi_{\Delta e}$, while the right-hand panels show the distribution of the overall impact in the policy rule (i.e., accounting for the degree of interest rate smoothing. See Eq. (11)).

The policy parameters convey an interesting narrative in terms of drifts and uncertainty. The policy coefficient for inflation initially becomes more reactive but shifts back to more

[18]In contrast, Justiniano and Preston (2010b) also find the cost-push shock to be the most volatile across different small open economies.

[19]Arguably, this is also true for the standard deviation of foreign output.

passive estimates in latter samples. In contrast, monetary policy appears to become more assertive for output and exchange rate fluctuations, although associated with a higher degree of uncertainty. What is particularly remarking in the results is that, once the smoothing parameter is taken into account, a sharper description of the evolution of the BoE's monetary policy develops. In particular, the *effective* response to inflation steadily becomes more passive over the different windows. Moreover, there is a significant reduction in the degree of uncertainty associated with the overall impact on interest rates. For output, the effective response steadily becomes more reactive. As for the exchange rate changes, the results reveal a periodic overall reduction in the central bank's reaction when determining the policy rate. However, notice that the current methodology does not fully consider whether monetary authorities updated their response to this variable instead by completely removing it from their reaction function. Subsection 4.3 formally addresses this issue by performing posterior odds tests with an alternative model specification that features no exchange rate feedback in the policy rule.

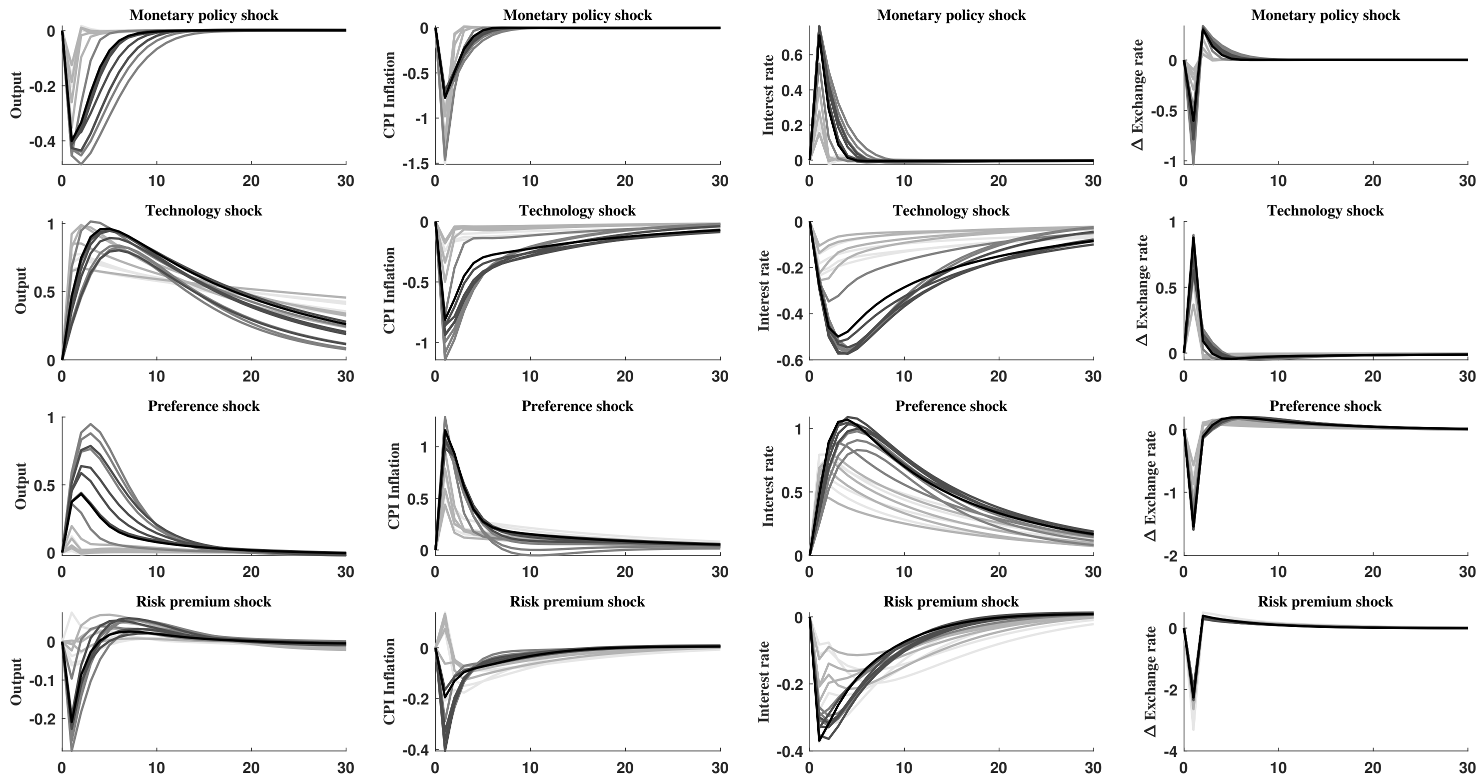
4.2 Evolving macroeconomic dynamics

In this section, I examine the associated implications to the UK macroeconomic dynamics. First, I start by computing rolling window impulse response functions, which are reported in Figure 4. The figure presents the response of the domestic variables (columns) to the various exogenous shocks (rows). For the sake of clarity, I focus the discussion on the time-varying differences depicted in the responses while abstracting to mention overlapping probability intervals, though these are available upon request. As before, I use different shades of gray to facilitate a visual narrative of the evolving dynamics across the subsamples.

The contractionary monetary policy shock has the expected sign-effect on domestic variables; it appreciates the domestic currency (initially) and lowers output and inflation. Across windows, the impulse responses shift outward (i.e., a larger impact effect in absolute value) and exhibit longer adjustments for output, interest rates, and exchange rates. For inflation, the results show lower and more persistent responses. Although it is difficult to isolate how individual parameters are associated with impulse response drifts, these results seem to match the evolution of monetary policy depicted in Figure 3, particularly the behavior of the effective policy responses (right panels).

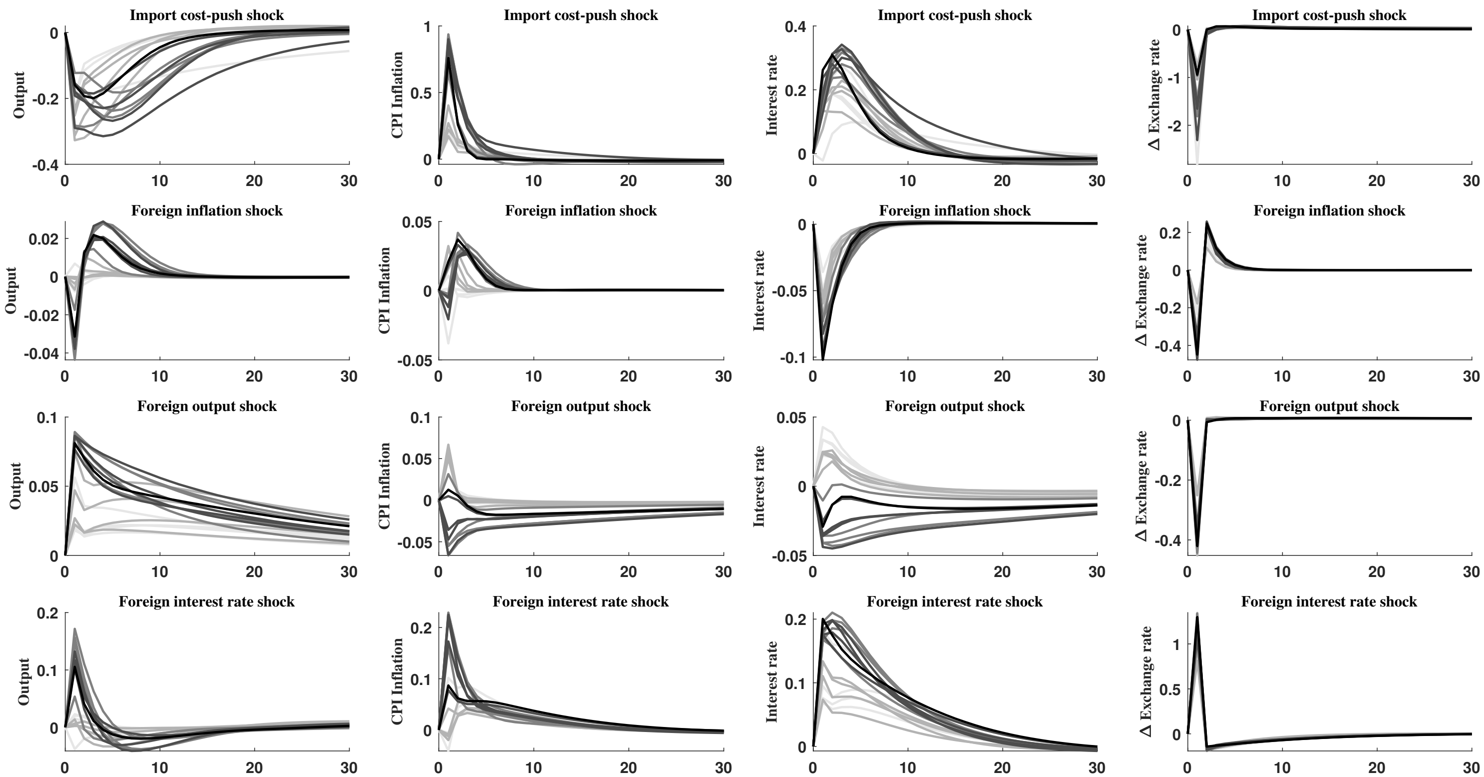
The responses to the technology and preference shocks become larger on impact and display longer adjustments, except for output, which narrative is not as evident through the rolling windows. In particular, for inflation, the interest rate, and exchange rates, the

Figure 4. Rolling-window Impulse Response Functions



Note: The impulse responses represent the median of each window across draws. Bayesian probability intervals omitted for clarity purposes. Legend is the same as in Figure 3 (lines get periodically darker for the more recent windows).

Figure 4. (cont.)



Note: The impulse responses represent the median of each window across draws. Bayesian probability intervals are omitted for clarity purposes. Legend is the same as in Figure 3 (lines get periodically darker for the more recent windows).

outward shift seems to occur in the first half of subsamples. For output, the responses reverse to zero at different paces for the technology shock and uniformly for the preference shock.

Impulse responses for the risk premium and cost-push shocks are among the most irregular across the estimation samples. Output and interest rate responses to the risk premium shock appear to become larger (in absolute value) and faster to adjust as the windows progress. Inflation initially responds positively to the risk premium shock, but the impact becomes negative for the rest of the samples. Still, the responses take similar periods to reverse back to zero. Concerning the cost-push shocks, inflation and interest rate impulse responses increase over time in their impact effects. However, for output, the lines instead oscillate around each other with no clear pattern that points toward a drifting effect. Exchange rate responses uniformly display a lower impact effect and virtually the same adjustment periods for both structural innovations.

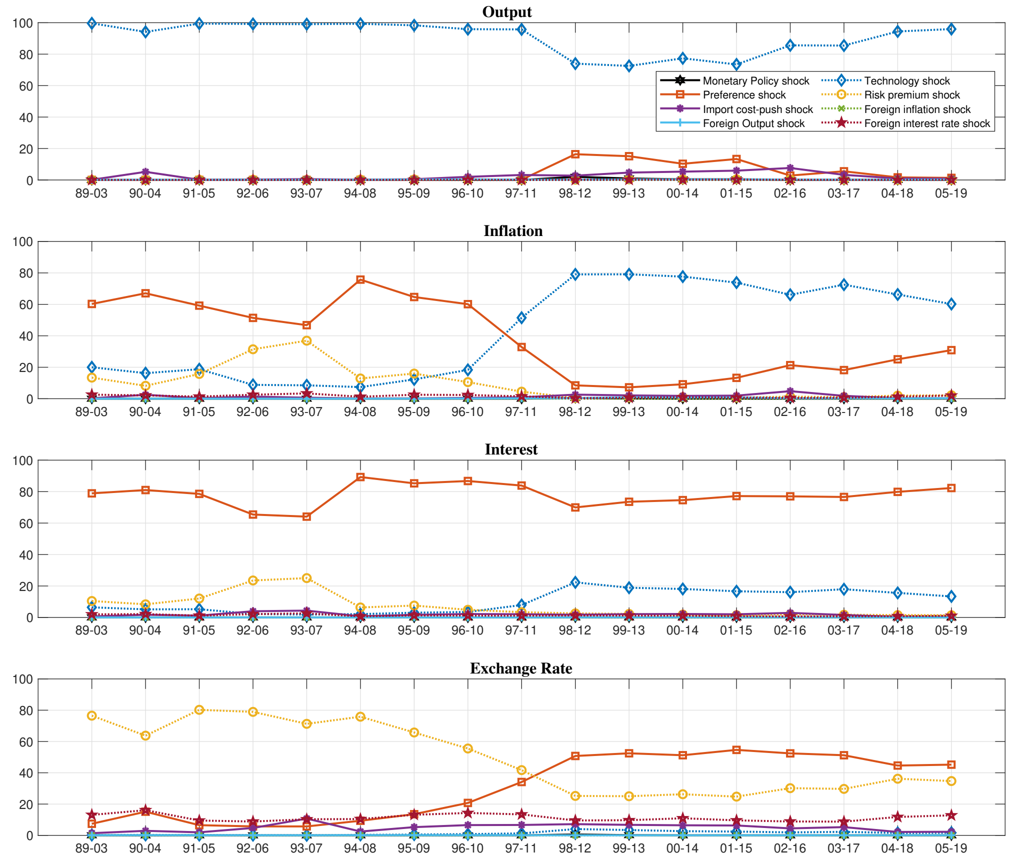
For the foreign block innovations, the results are as follows. The foreign inflation shock seems to escalate the impacts on output, interest rate, and the exchange rate. For output particularly, the adjustment also becomes slower. The time-varying behavior for the inflation responses, however, is unclear. The effects of the foreign output shock are somewhat similar for output and the exchange rate, increasing the impact effect over the samples. Nonetheless, for the output responses, it now renders different degrees of persistence. For the inflation and interest rate, the responses become smaller in magnitude while fluctuating between positive and negative impact values. Lastly, foreign interest rate shocks display greater impacts for all variables except inflation, which its response behavior is similar to the previous case.

I then investigate how the role of the structural shocks driving the UK macroeconomic performance has evolved by computing forecast error variance decomposition for each rolling window. Figure 5 illustrates the results. The figure represents the median forecast error variance share at the 8-period ahead horizon, but similar figures for the 4 and 24 horizons are also available in Appendix.

Output fluctuations are mainly explained by the technology shock, accounting for more than 75% of the variance across samples. The only evolving transition in the dynamics is presented by the contributions of the preference and the cost-push shocks, which seem to transition to larger shares after the 98-12 window and then return to their previous contributions in the last estimation period.

In contrast, inflation fluctuations exhibit various changes in the forecast error shares. The preference and risk premium shocks, which initially are among the main drivers of inflation's variability, decrease their role significantly in the second half of the samples. These shares,

Figure 5. Evolution of variance decomposition



Note: The figure shows the window-specific median forecast-error variance shares across MH draws at the 8-quarters ahead horizon.

on the other hand, are symmetrically gained by the technology shock, which accounts for 60-80% of inflation fluctuations in the latter samples.

The interest rate is also largely driven by the preference shock, which is consistently the major contributor to its variability, with forecast error variance shares of more than 60% in all windows. The visible transition here occurs between the risk premium and the technology shocks, the former being an important driver in the first half of windows and the latter in the rest.

The empirical findings for the exchange rate variations depict several time-varying shifts. The most apparent ones are presented by the preference and risk premium shocks. Initially, the risk premium shock accounts for more than 60% of the variance, while the preference shock oscillates between 10-20%. After the 95-09 sample, risk premium and preference shares transition to values of 30-40% and 45-50%, respectively. At a smaller scale, the import cost-push shock becomes more important for the middle samples and the technology shock after the 98-12 window. Lastly, while being one of the main contributors explaining exchange rate fluctuations, the foreign interest shock did not display apparent shifts across the rolling samples.

The results are fairly similar at the 24-period ahead horizon, though a few differences are worth mentioning. The risk premium shock no longer displays a significant contribution explaining inflation and interest rate variability in the initial samples. In fact, in both scenarios, their shares seem to be absorbed by the technology shock. For the exchange rate fluctuations, the import cost-push shock plays a dominant role across all windows, while the contribution of the risk premium shock is more dormant in general.

4.3 Does the Bank of England respond to exchange rate fluctuations?

As discussed before, the estimation procedure does not consider whether monetary authorities include the exchange rate in their reaction function across the subsamples. In this section, I evaluate this possibility by estimating the model under the restriction $\psi_{\Delta e} = 0$ and comparing the marginal likelihoods of the two model specifications. The analysis is performed as in Lubik and Shorfheide (2004) but extended to contemplate the evolution of marginal likelihood across the rolling samples. The posterior estimates of this alternative specification are available in the Appendix.

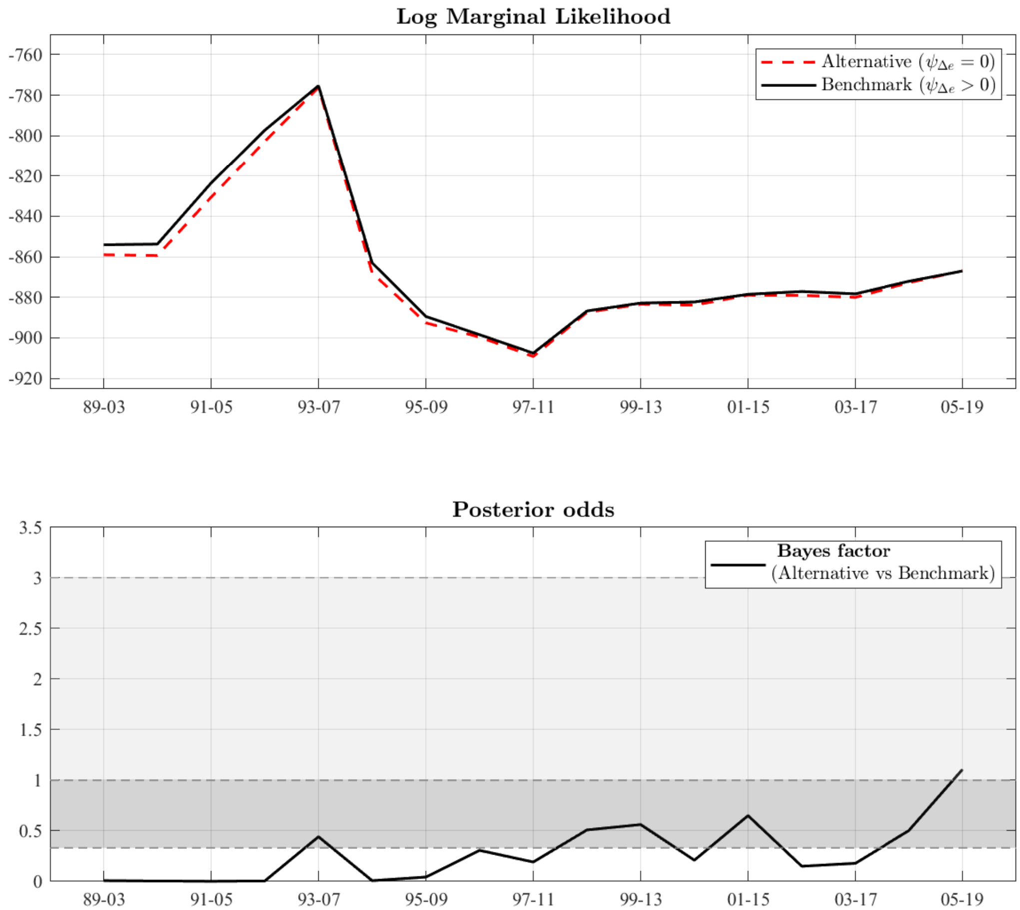
Figure 6 summarizes the results. The top panel shows the (log) marginal likelihood of the benchmark specification, where the nominal exchange rate is embedded in the policy rule ($\psi_{\Delta e} > 0$), and an alternative specification that assumes monetary policy is not responsive to this variable ($\psi_{\Delta e} = 0$).[20] The bottom panel presents the corresponding posterior odds (which assumes that the prior odds are one) for each rolling window. To facilitate the interpretation of the results, I use different shades of gray to illustrate anecdotal or weak evidence in favor of the specifications: light for the alternative and dark for the benchmark. White areas, in turn, suggest substantial or strong evidence: the upper area for the alternative and the lower area for the benchmark.[21]

The results render an interesting narrative. For the initial windows, the posterior odds

[20] For conciseness, I refer to them in the Appendix as $\mathcal{M}_1$ and $\mathcal{M}_2$, respectively.

[21] See Jeffreys (1998).

Figure 6. Evolution of Marginal Likelihood and window-specific Bayes factor



Note: The figure summarizes the posterior odds ratio of each rolling window of the hypothesis $\psi_e = 0$ versus $\psi_e > 0$. The light (dark) gray area denotes anecdotal evidence in favor of the alternative (benchmark). The upper (lower) white area suggests substantial or strong evidence for the alternative (benchmark). Prior odds are assumed to be equal to one.

tests favor the benchmark specification. This result is consistent with previous literature. For example, Lubik and Schorfheide (2007), whose sample roughly corresponds to the first window, find strong evidence that the Bank of England responds to exchange rate movements. However, the figure shows that this result is not robust across subsamples. Moreover, the gap between the marginal likelihoods shrinks as the windows progress. After the 93-07 window, the evidence in support of the benchmark becomes weak for some estimations and ultimately favors (slightly) the alternative in the last subsample. This result becomes more apparent when repeating the estimation exercise using 20-year windows, which I present in the robustness section.

As an additional exercise, I also re-estimate the model for both specifications using the full sample to compare the implications for the policy responses and the marginal likelihoods. The policy responses under this setup are available in the Appendix. In short, the posterior distributions for the effective policy responses are wider than most of the rolling window counterparts and tend to compute similar estimates to the mid subsample periods. In terms of the marginal likelihoods, I find a Bayes factor of 0.3679, slightly favoring the premise of a response to exchange rate movements by the BoE. This result contrasts the importance of considering parameter instabilities in the estimation of DSGE models.

4.4 Robustness analysis

4.4.1 20-year windows

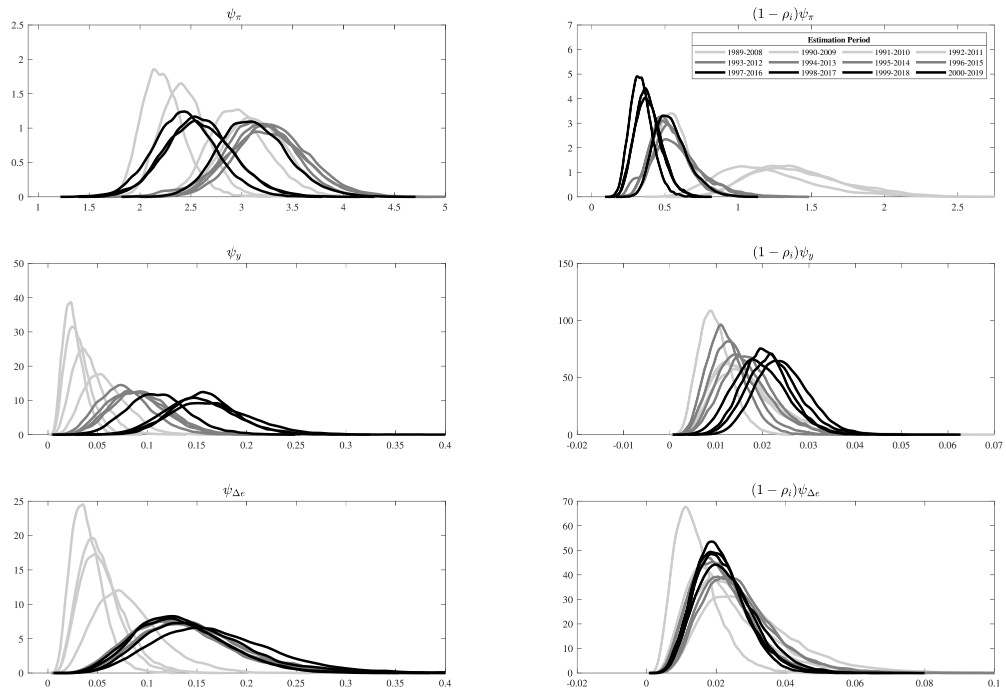
In this section, I check the robustness of the results by repeating the estimation procedure using 20-year rolling windows. The objective of this exercise is to verify that the width of the subsamples does not drive the main results. For brevity, here I report only the central findings concerning the evolving behavior of monetary authorities, though I provide the full ensemble of results in the Appendix.

Figure 7 presents the posterior distributions and effective policy responses for the 20-year windows. The results are overall alike and display a more linear transition across subsamples. Monetary policy becomes more passive for inflation and more active for output. As in Zamarripa (2021), this finding conforms with the notion that once central banks approach their inflation targets, they react relatively less to inflation and more to output.[22] For the exchange rate, the figure shows fairly steady effective responses.

Nonetheless, the striking difference is displayed by the posterior odds tests in Figure 8. The evolution of the marginal likelihoods and the window-specific Bayes factors becomes more apparent and signals an evident transition in the BoE reaction function. Previously, posterior odds ratios oscillated between strong and weak evidence supporting the benchmark specification. Now, after the 95-14 subsample, the preferred specification starts transitioning to the model with no exchange rates in the policy function. Although the evidence is weak,

[22] To provide some context, the UK adopted (retail) inflation targeting in 1992 with a band of 1-4%. In 2003, changed from RPIX inflation of 2.5% to the current CPI target of 2% (with a band of $\pm$ 1%).

Figure 7. Posterior distributions of policy coefficients and (effective) policy responses (20-year windows)



Note: The horizontal axis of the right panels is maintained consistent with Figure 3 for comparability reasons.

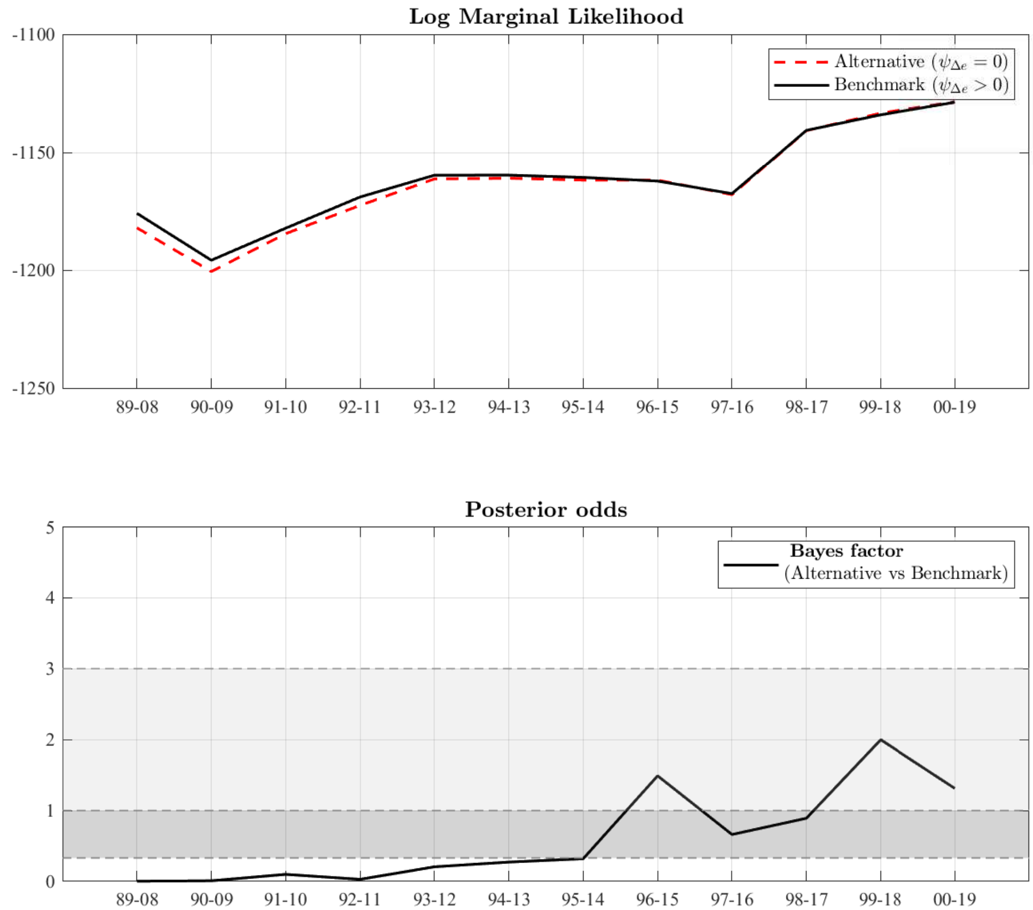
it is important to remark that the data no longer favors (substantially) the conception of exchange rate targetting by the BoE, as in Lubik and Schorfheide (2007).

Concerning the parameter instabilities and evolving macroeconomic dynamics, the overall results hold. In some instances, the rolling window estimates drift smoother across subsamples, but the transitions are comparable. The impulse responses also show more uniform shifts. There are slight differences in the median shares of the exogenous shocks for the forecast error variance decomposition but without disrupting the underlying results.

4.4.2 Alternative policy priors

Being the monetary policy parameters a focal element in the analysis, I also re-estimate the model relaxing the priors for the parameters of the BoE reaction function and assess the robustness of the parameter instabilities found earlier and the transition of the effective

Figure 8. Evolution of Marginal Likelihood and window-specific Bayes factor (20-year windows)



Note: The figure summarizes the posterior odds ratio of each rolling window of the hypothesis $\psi_e = 0$ versus $\psi_e > 0$. The light (dark) gray area denotes anecdotal evidence in favor of the alternative (benchmark). The upper (lower) white area suggests substantial or strong evidence for the alternative (benchmark). Prior odds are assumed to be equal to one.

policy responses. I increase the standard deviation for the three policy parameters and impose a uniform prior for the interest rate smoothing parameters.[23] Table 2 summarizes the new set of priors adopted.

[23] I also increase the prior mean of the response to output and the interest rate to account for the non-negative restrictions imposed by the distribution.

Table 2.
Alternative priors

Parameter	Density	Benchmark		Alternative priors	
		P(1)	P(2)	P(1)	P(2)
ρ_i	Beta/Uniform	0.50	0.25	0.00	1.00
ψ_π	Gamma	1.50	0.30	1.50	0.60
ψ_y	Gamma	0.25	0.13	0.75	0.30
ψ_e	Gamma	0.25	0.13	0.75	0.30

Note: P(1) and P(2) refers the mean and standard deviation for the Beta and Gamma distributions, and lower and upper bounds for the Uniform distribution.

The results, which are available in the Appendix, show that the overall characteristics of the posterior densities hold. There is evidence of parameter instabilities across the model parameters and clear transitions in the policy coefficients. Not surprisingly, the response parameters are larger than before due to the influence of the prior means adopted for output and the exchange rate responses. Nonetheless, the higher posterior estimates for the interest smoothing term seem to balance this difference and ultimately render similar effective policy responses.

5. Conclusions

This paper revisited evidence on how central banks respond to the exchange rate by investigating the possibility of parameter instabilities estimating a small open economy DSGE model fitted to UK data over rolling windows.

The empirical results provide ample evidence of drifts on several model parameters. I find that monetary policy has become progressively more passive for inflation and more active for output. Concerning the exchange rate, I reconsidered earlier results on the response to this variable by the BoE by comparing posterior odds against an alternative with no exchange rate in the policy rule. The analysis reveals interesting results. For the initial windows, the marginal likelihood of the specification with exchange rate turned to be significantly superior. However, for latter samples, the evidence becomes weaker and, in some cases, even favors the model with no exchange rate policy feedback. These results become more apparent when increasing the window's width from 15 to 20 years.

By comparing the estimated impulse response functions, the paper shows evident differences in how the model responds to the exogenous shocks. This turns to be a significant finding that provides more context on the importance of accounting for parameter drifts in DSGE models and analysis of the macroeconomic dynamics. In the paper, I also document how various shocks become more important to explain the variance of the variables as the windows progress to latter samples.

Further, the paper aimed to provide a first look at how parameters drift in small open economies and the corresponding implications to the aggregate dynamics. In this sense, the results are to be interpreted concerning the UK's case. However, taking this inquiry together with similar findings in the literature, the evidence is consistent with the premise that central banks update the relative weights to these variables as the macroeconomy unfolds. In particular, these findings correlate with Dong (2013) and Zamarripa (2021), on that central banks in small open economies seem to have adjusted the weights in their policy rules around the periods when inflation targetting was adopted. Future research could reconcile these results and continue investigating recurrent patterns in the transition of structural parameters in the open economy context, such as during inflation episodes or when systematic changes in monetary policy occurred.

In the paper, I largely focused on showing the role of monetary policy and the evolution of the parameters that govern the BoE's reaction function. In light of the evidence derived from the posterior odds test, future studies could expand the analysis by considering optimal policy design within the rolling samples. It could also be worthwhile to extend the present study and explore how alternative assumptions about the expectation formation process may affect the path of parameter instabilities in small open economy DSGE models, such as with learning mechanisms or heterogeneous expectations.

From a policy standpoint, the results are of particular interest to policymakers as they show that 'structural' parameters in DSGE models may contain a time-varying component. As such, this paper shows that monetary policy is not an invariant process. Disregarding parameter instabilities could lead monetary authorities to assign incorrect weights to the variables in their policy rules when attempting to achieve economic objectives and unfold undesired macroeconomic effects. Likewise, ignoring the possibility of parameter drifts could lead to incorrect analysis of the propagation of shocks or produce relatively poor forecasts.

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A. Appendix

A.1 Small open economy model

This section sketches the microfoundations of the model employed in Section 2. The content is taken and summarized from Justiniano and Preston (2010b).

A.1.1 Households

Households are assume to maximize the following intertemporal problem:

$$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{\varepsilon}_{g,t} \left[\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

where N_t is the labor input; $H_t \equiv hC_{t-1}$ refers to an external habit taken as exogenous by the household; $\sigma, \varphi > 0$ are inverse elasticities of intertemporal substitution and labor suply, respectively; and $\tilde{\varepsilon}_{g,t}$ is a preference shock. C_t is a composite consumption index:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $C_{H,t}$ and $C_{F,t}$ are Dixit–Stiglitz aggregates of the domestic and foreign produced goods equal to

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad \text{and} \qquad C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where α corresponds to the share of foreign goods in the domestic consumption bundle; $\eta > 0$ is the elasticity of substitution between domestic and foreign goods; and $\varepsilon > 1$ refers to the elasticity of substitution between types differentiated domestic and foreign goods.

The only available assets are one-period domestic and foreign bonds. Hence, the flow budget constraint is given by

$$P_t C_t + D_t + \tilde{e}_t B_t = D_{t-1}(1 + \tilde{i}_{t-1}) + \tilde{e}_t B_{t-1}(1 + \tilde{i}^*_{t-1})\phi_t(A_t) + W_t N_t + \Pi_{H,t} + \Pi_{F,t} + T_t$$

for all $t > 0$, where D_t denotes the household's holding of one-period domestic bonds, and B_t holdings of one-period foreign bonds with corresponding interest rates $\tilde{i}_t$ and $\tilde{i}^*_t$. The nominal exchange rate is $\tilde{e}_t$. P_t, $P_{H,t}$, $P_{F,t}$ and P^*_t refer to the domestic CPI, domestic goods prices, the domestic currency price of imported goods and the foreign price, respectively, and are formally defined below. Wages W_t are earned on labor supplied and $\Pi_{H,t}$ and $\Pi_{F,t}$ denote profits from holding shares in domestic and imported goods firms. T_t denotes lump-sum taxes and transfers. Debt elastic interest rate premium is given by the function $\phi_t(\cdot)$, such that

$$\phi_t = \exp\big[-\chi(A_t + \tilde{\phi}_t)\big]$$

where

$$A_t \equiv \frac{\tilde{e}_{t-1}B_{t-1}}{\bar{Y}P_{t-1}}$$

is the real quantity of outstanding foreign debt expressed in terms of domestic currency as a fraction of steady-state output and $\tilde{\phi}_t$ a risk premium shock.

The budget constraint implicitly assumes that all households in the domestic economy receive an equal fraction of both domestic and retail firm. Thus, nominal income in each period is $W_tN_t + \Pi_{H,t} + \Pi_{F,t}$, which in equilibrium equals $P_{H,t}Y_{H,t} + (P_{F,t} - \tilde{e}_tP_t^*)C_{F,t}$ for all households.

The household's optimization problem requires allocation of expenditures across all types of domestic and foreign goods, both intratemporally and intertemporally. This yields the following set of optimality conditions.

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta} C_{H,t} \qquad \text{and} \qquad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\theta} C_{F,t}$$

for all i with associated aggregate price indexes for the domestic and foreign consumption bundles given by $P_{H,t}$ and $P_{F,t}$. Optimal allocation of expenditure across domestic and foreign goods imply the demand functions

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \qquad \text{and} \qquad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$

where

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

is the consumer price index.

The allocation of expenditures on the aggregate consumption bundle and optimal labor supply satisfy

$$\lambda_t = \tilde{\varepsilon}_{g,t}\big(C_t - H_t\big)^{-\frac{1}{\sigma}}$$
$$\lambda_t = \tilde{\varepsilon}_{g,t}\frac{P_t N_t^{\varphi}}{W_t}$$

and portfolio allocation is determined by the optimality conditions

$$\lambda_t \tilde{e}_t P_t = E_t\big[(1+\tilde{i}_t^*)\beta\phi_{t+1}\lambda_{t+1}\tilde{e}_{t+1}P_{t+1}\big]$$
$$\lambda_t P_t = E_t\big[(1+\tilde{i}_t^*)\beta\lambda_{t+1}P_{t+1}\big]$$

for the Lagrange multiplier λ_t.

A.1.2 Domestic producers

There is a continuum of monopolistically competitive domestic firms producing differentiated goods. Calvo-style price setting is assumed, allowing for indexation to past domestic goods price inflation. Hence, in any period t, a fraction $1-\theta_H$ of firms set prices optimally, while a fraction $0<\theta_H<1$ of goods prices are adjusted according to the indexation rule

$$\log P_{H,t}(i) = \log P_{H,t-1}(i) + \delta_H \pi_{H,t-1}$$

where $0 \leq \delta_H \leq 1$ measures the degree of indexation to the previous period's inflation rate and $\pi_{H,t} = \log(P_{H,t}/P_{H,t-1})$.

Since all firms having the opportunity to reset their price in period t face the same decision problem they set a common price $P'_{H,t}$. The Dixit–Stiglitz aggregate price index therefore evolves according to the relation

$$P_{H,t} = \left[(1-\theta_H)P_{H,t}^{\prime(1-\varepsilon)} + \theta_H\left(P_{H,t-1}\Big(\frac{P_{H,t-1}}{P_{H,t-2}}\Big)^{\delta_H}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Firms setting prices in period t face a demand curve

$$y_{H,T}(i) = \left(\frac{P_{H,t}(i)}{P_{H,T}}\cdot\Big(\frac{P_{H,T-1}}{P_{H,t-1}}\Big)^{\delta_H}\right)^{-\varepsilon}\big(C_{H,T}+C_{H,T}^*\big)$$

for all t and take aggregate prices and consumption bundles as parametric. Good i is produced using a single labor input $N_t(i)$ according to the relation $y_{H,t}(i) = \varepsilon_{a,t} N_t(i)$, where $\varepsilon_{a,t}$ is an exogenous technology shock.

The firm's price-setting problem in period t is to maximize the expected present discounted value of profits:

$$E_t \sum_{T=t}^{\infty} \theta_H^{T-t} Q_{t,T} y_{H,T}(i) \Big[P_{H,t}(i) \Big(\frac{P_{H,T-1}}{P_{H,t-1}} \Big)^{\delta_H} - P_{H,t} MC_t \Big]$$

where $MC_T = W_T / P_{H,T}\varepsilon_{a,T}$ is the real marginal cost function for each firm, assuming homogeneous factor markets. The factor θ_H^{T-t} in the firm's objective function is the probability that the firm will not be able to adjust its price in the next $(T - t)$ periods.

A.1.3 Retail firms

Retail firms import foreign differentiated goods for which the law of one price holds at the docks. In determining the domestic currency price of the imported good, firms are assumed to be monopolistically competitive. This small degree of pricing power leads to a violation of the law of one price in the short run.

In any period t, a fraction $1-\theta_F$ of firms set prices optimally, while a fraction $0 < \theta_F < 1$ of goods prices are adjusted given $\log P_{F,t}(i) = \log P_{F,t-1}(i) + \delta_F \pi_{F,t-1}$. The Dixit–Stiglitz aggregate price index consequently evolves according to the relation

$$P_{F,t} = \left[(1-\theta_F) P_{F,t}^{'(1-\varepsilon)} + \theta_F \left(P_{F,t-1} \Big(\frac{P_{F,t-1}}{P_{F,t-2}} \Big)^{\delta_F} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

and firms setting prices in period t face a demand curve

$$C_{F_T}(i) = \left(\frac{P_{F,t}(i)}{P_{F,T}} \cdot \Big(\frac{P_{F,T-1}}{P_{F,t-1}} \Big)^{\delta_F} C_{F,T} \right)^{-\varepsilon}$$

for all t and take aggregate prices and consumption bundles as parametric.

The firm's price-setting problem in period t is to maximize the expected present discounted value of profits:

$$E_t \sum_{T=t}^{\infty} \theta_H^{T-t} Q_{t,T} C_{F,T}(i) \Big[P_{F,t}(i) \Big(\frac{P_{F,T-1}}{P_{F,t-1}} \Big)^{\delta_F} - \tilde{e}_T P_{F,t}^{*}(i) \Big]$$

A.1.4 International risk sharing

From the asset-pricing conditions that determine domestic and foreign bond holdings, the uncovered interest rate parity condition

$$E_t \lambda_{t+1} P_{t+1} \left[(1+\tilde{i}_t) - (1+\tilde{i}_t^*) \left(\frac{\tilde{e}_{t+1}}{\tilde{e}_t} \right) \phi_{t+1} \right] = 0$$

The real exchange rate is defined as $\tilde{q} \equiv \tilde{e}_t P_t^* / P_{F,t}$. Since $P_t^* = P_{F,t}^*$, when the law of one price fails to hold, we have $\tilde{\Psi}_{F,t} \equiv \tilde{e}_t P_t^* / P_{F,t} \neq 1$ (the law of one price gap).

A.1.5 General equilibrium

Goods market clearing in the domestic economy requires

$$Y_{H,t} = C_{H,t} + C_{H,t}^*$$

The model is closed assuming foreign demand for the domestically produced good is specified as

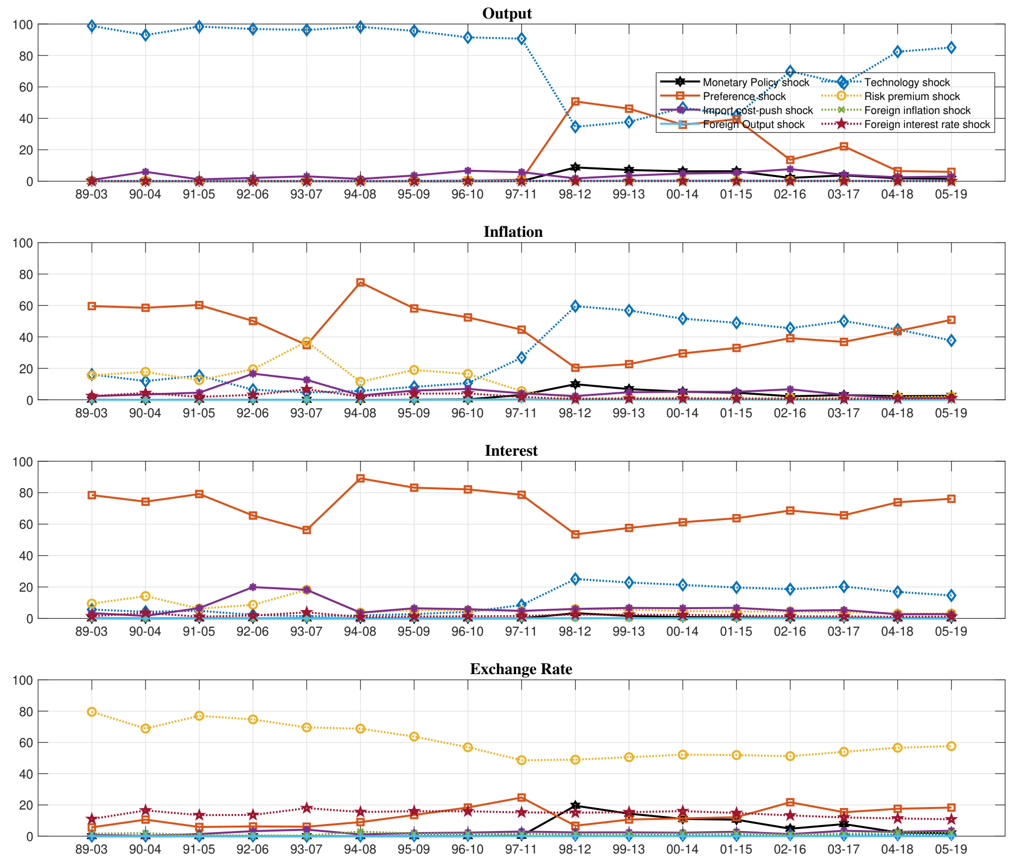
$$C_{H,t}^* = \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\lambda} Y_t^*$$

where $\lambda > 0$.

Domestic debt is assumed to be in zero net supply so that $D_t = 0$ for all t. The model considers a symmetric equilibrium in which all domestic producers and all retailers setting prices in period t set common prices $P_{H,t}$ and $P_{F,t}$, respectively. Households are assumed to have identical initial wealth, so that each faces the same period budget constraint and therefore makes identical consumption and portfolio decisions. Monetary policy is assumed to be conducted according to a Taylor-type rule. Fiscal policy is specified as a zero debt policy, with taxes equal to the subsidy required to eliminate the steady-state distortion induced by imperfect competition in the domestic and imported goods markets.

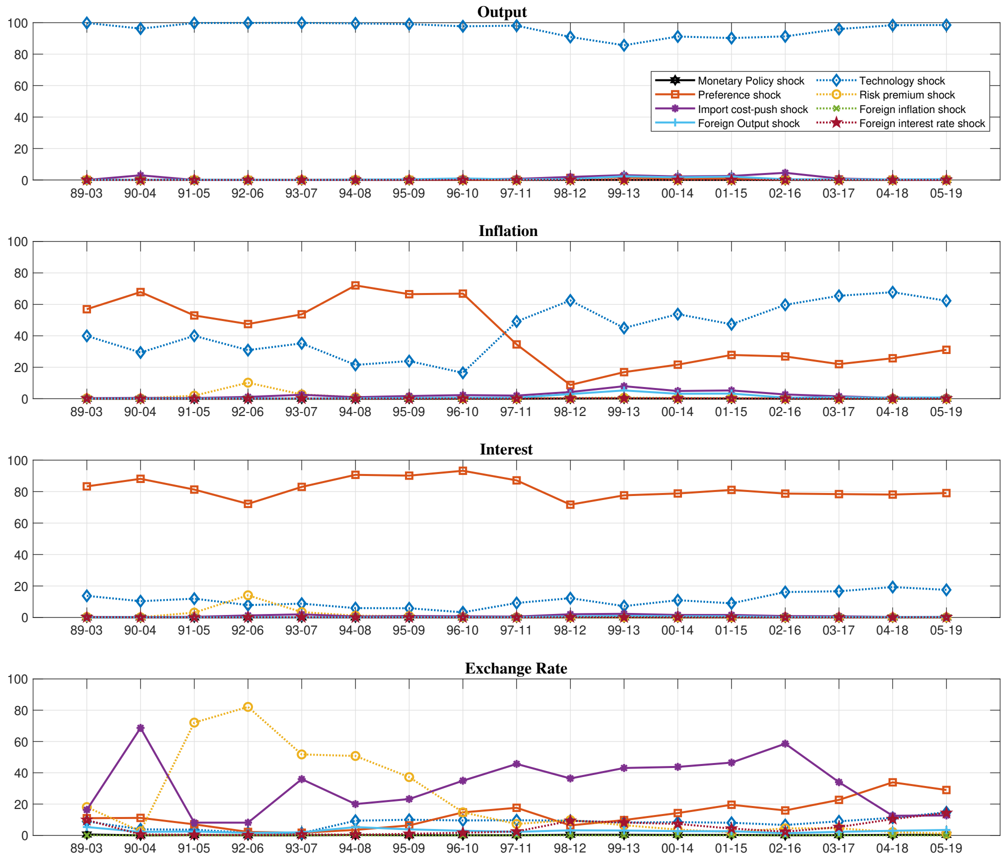
A.2 15-year windows

Figure 9. Evolution of variance decomposition (4-quarters ahead horizon)



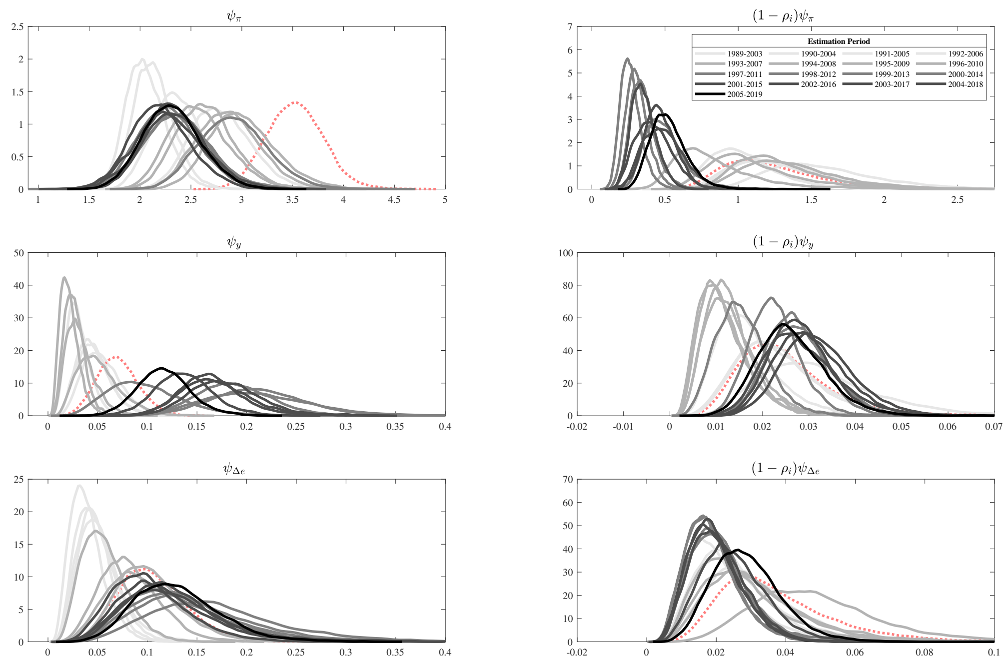
Note: The figure shows the window-specific median forecast-error variance shares across MH draws at the 4-quarters ahead horizon.

Figure 10. Evolution of variance decomposition (24-quarters ahead horizon)



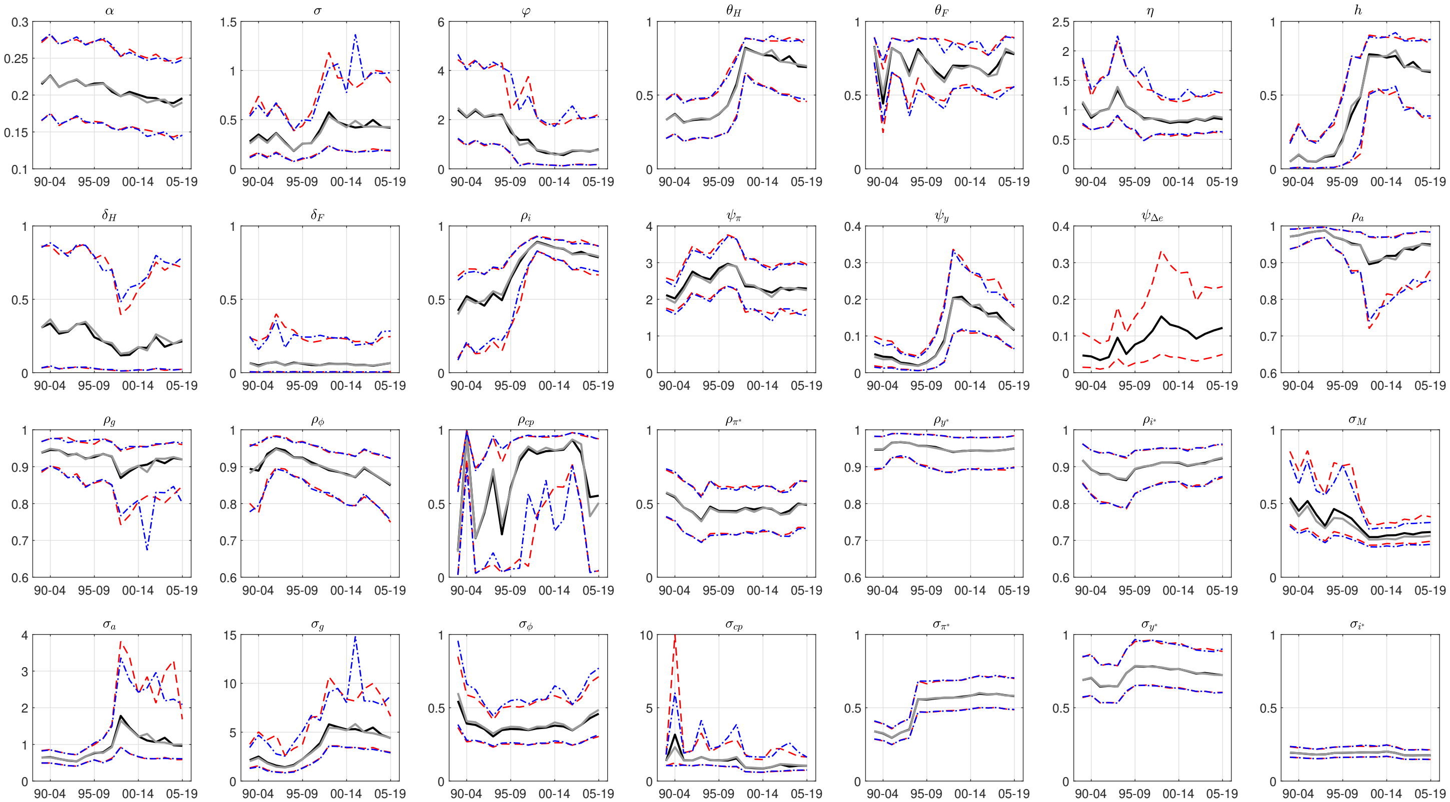
Note: The figure shows the window-specific median forecast-error variance shares across MH draws at the 24-quarters ahead horizon.

Figure 11. Posterior distributions of policy coefficients and policy responses (with full-sample estimates)



Note: The red dashed densities refer to full-sample estimates.

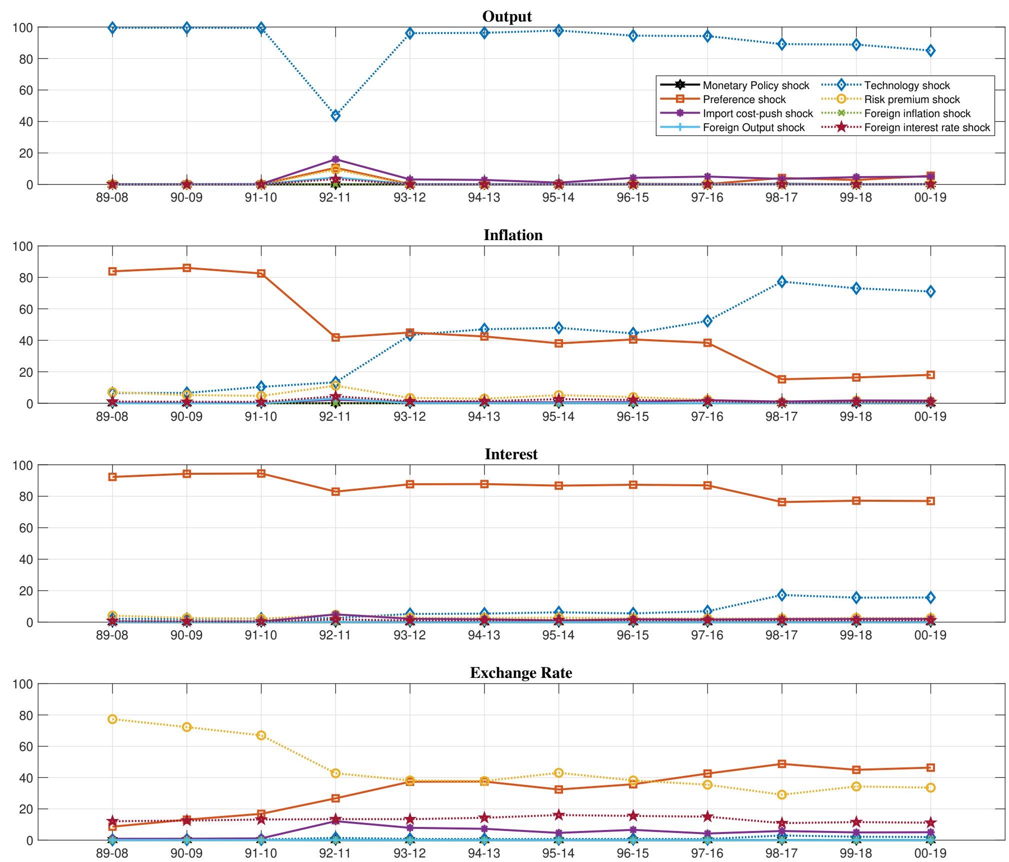
Figure 12. Rolling-window posterior estimates ($\mathcal{M}_1$ vs $\mathcal{M}_2$ - 15-year windows)



Note: The figure shows the posterior median (solid lines) of each sub-sample across the Metropolis-Hastings draws, along with 95% Bayesian credible interval bands (dashed lines). Black solid lines and red interval bands describe $\mathcal{M}_1$ estimates, gray solid lines and blue interval bands describe $\mathcal{M}_2$ estimates.

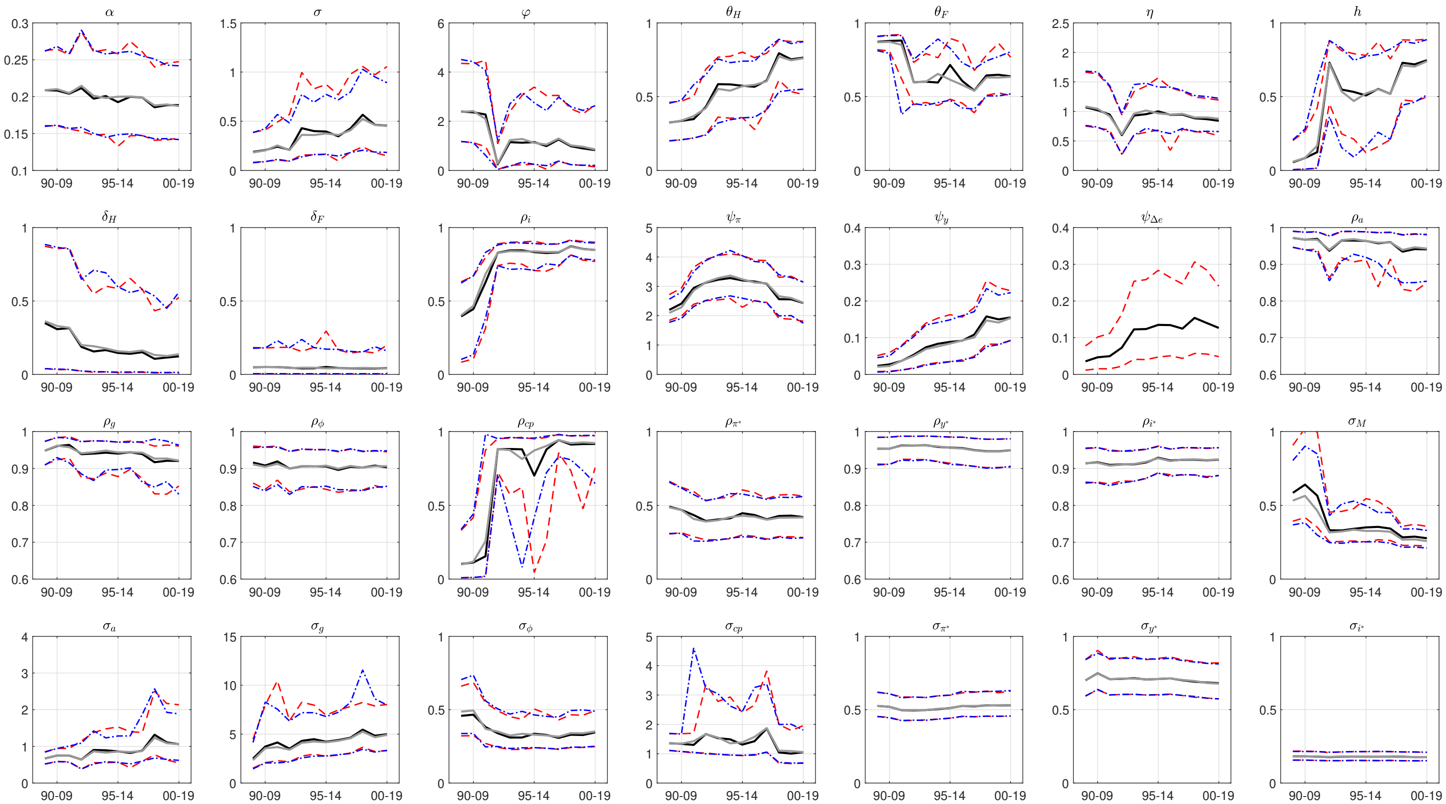
A.3 20-year windows

Figure 13. Evolution of variance decomposition (8-quarters ahead horizon)



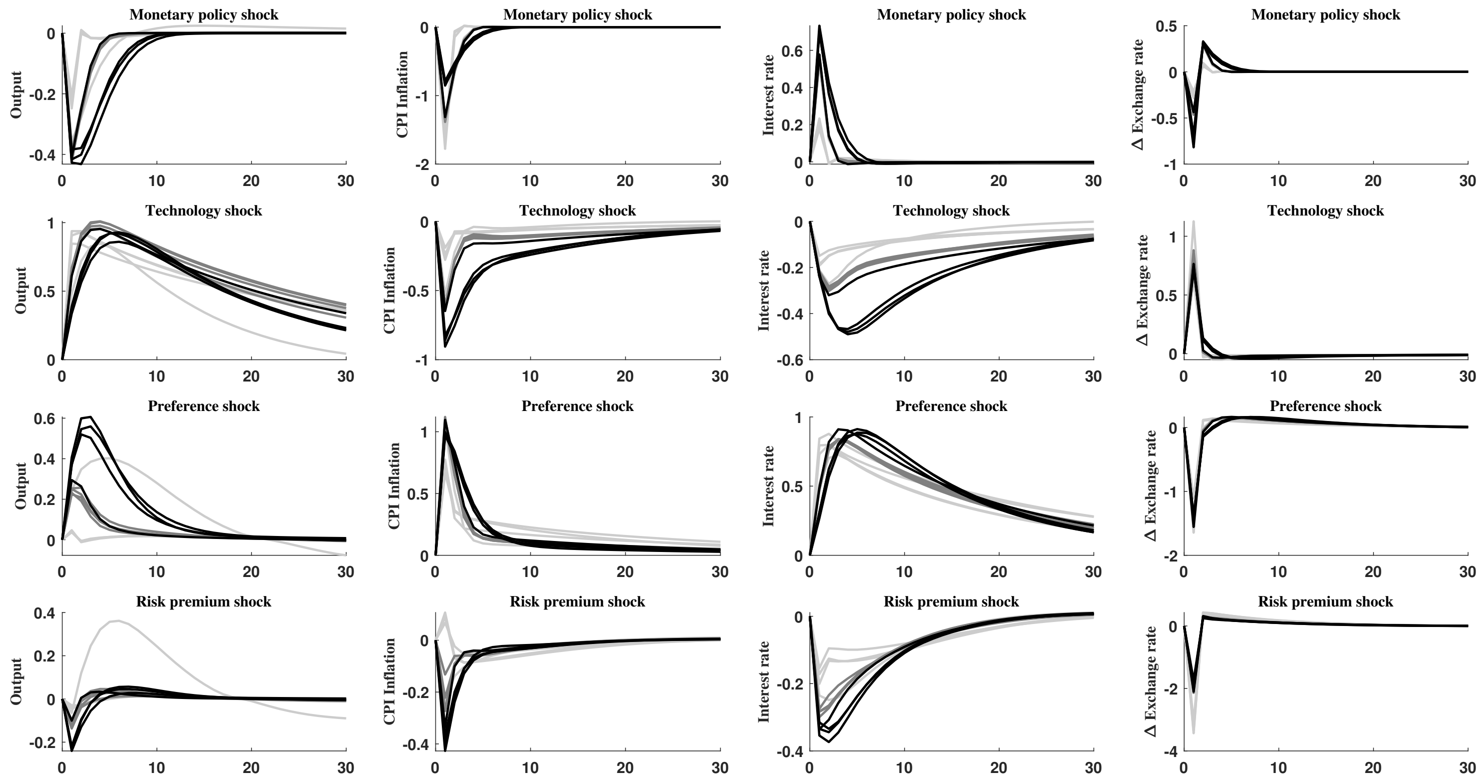
Note: The figure shows the window-specific median forecast-error variance shares across MH draws at the 8-quarters ahead horizon.

Figure 14. Rolling-window posterior estimates ($\mathcal{M}_1$ vs $\mathcal{M}_2$ - 20-year windows)



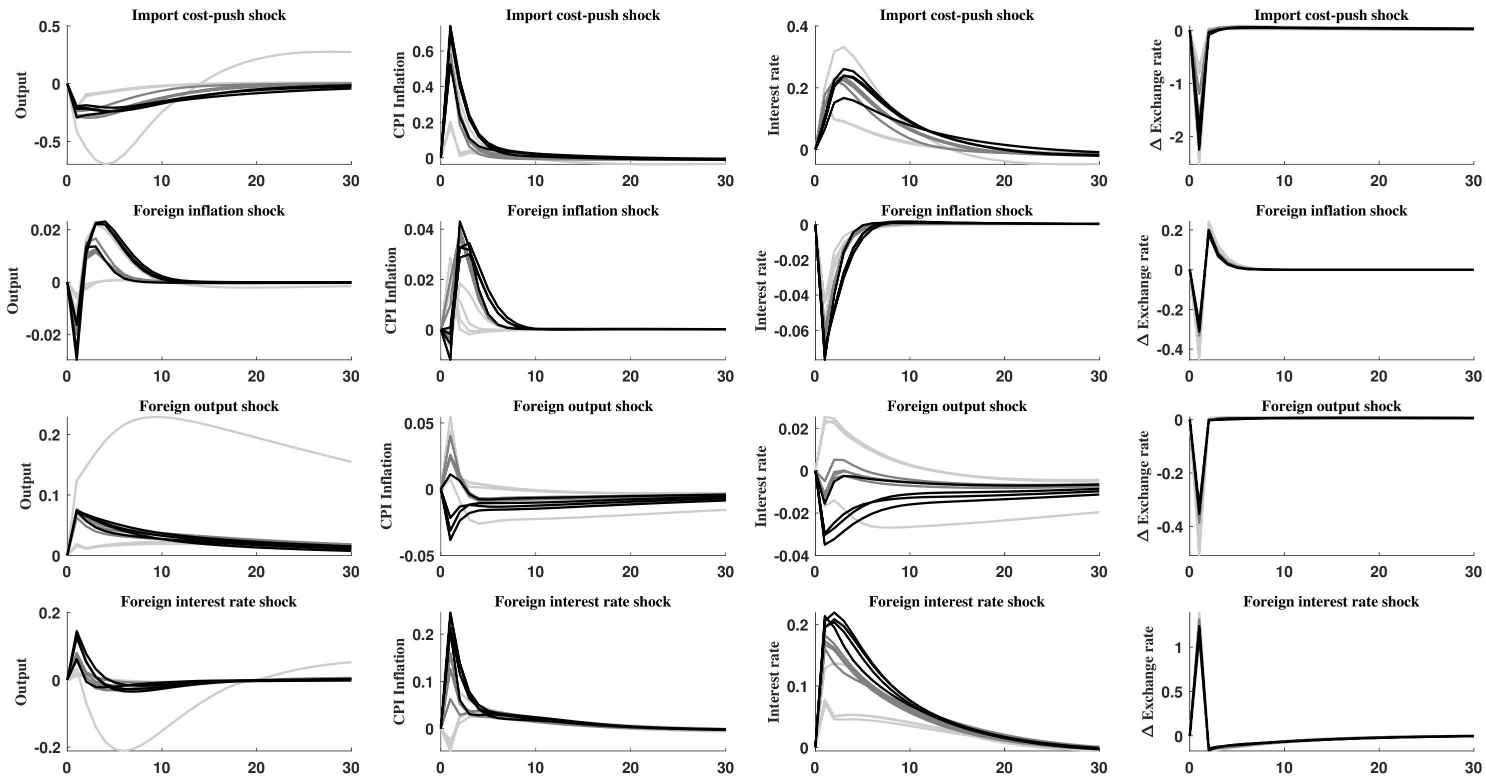
Note: The figure shows the posterior median (solid lines) of each 20-year rolling window across the Metropolis-Hastings draws, along with 95% Bayesian credible interval bands (dashed lines). Black solid lines and red interval bands describe $\mathcal{M}_1$ estimates, gray solid lines and blue interval bands describe $\mathcal{M}_2$ estimates.

Figure 15. Rolling-window Impulse Response Functions (20-year windows)



Note: The impulse responses represent the median of each window across draws. Bayesian probability intervals omitted for clarity purposes. Legend is the same as in Figure 7 (lines get periodically darker for the more recent windows).

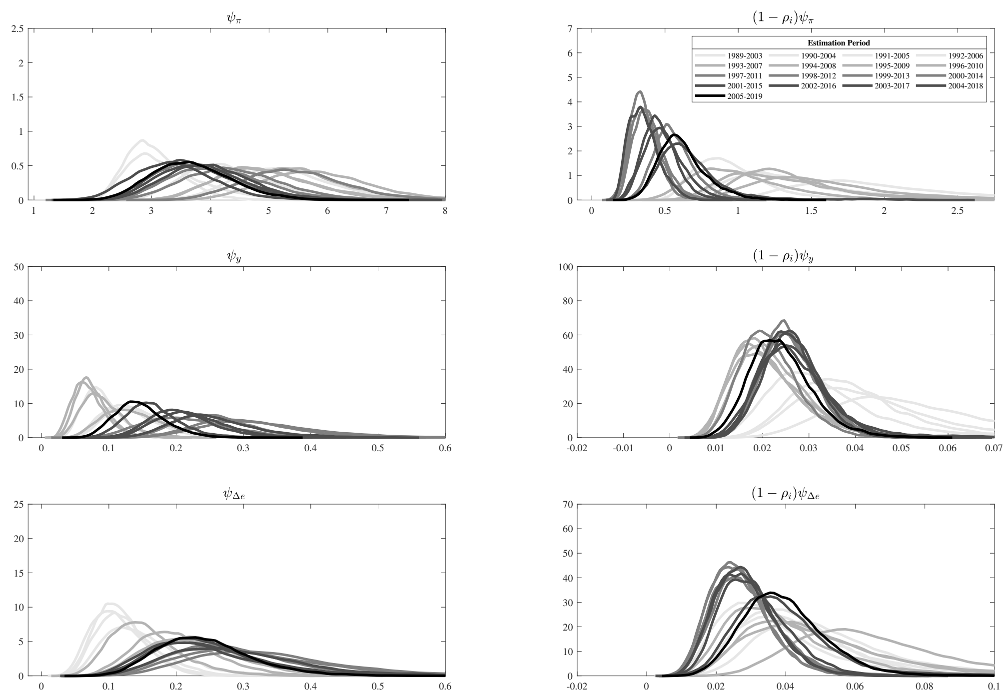
Figure 15. (cont.)



Note: The impulse responses represent the median of each window across draws. Bayesian probability intervals are omitted for clarity purposes. Legend is the same as in Figure 7 (lines get periodically darker for the more recent windows).

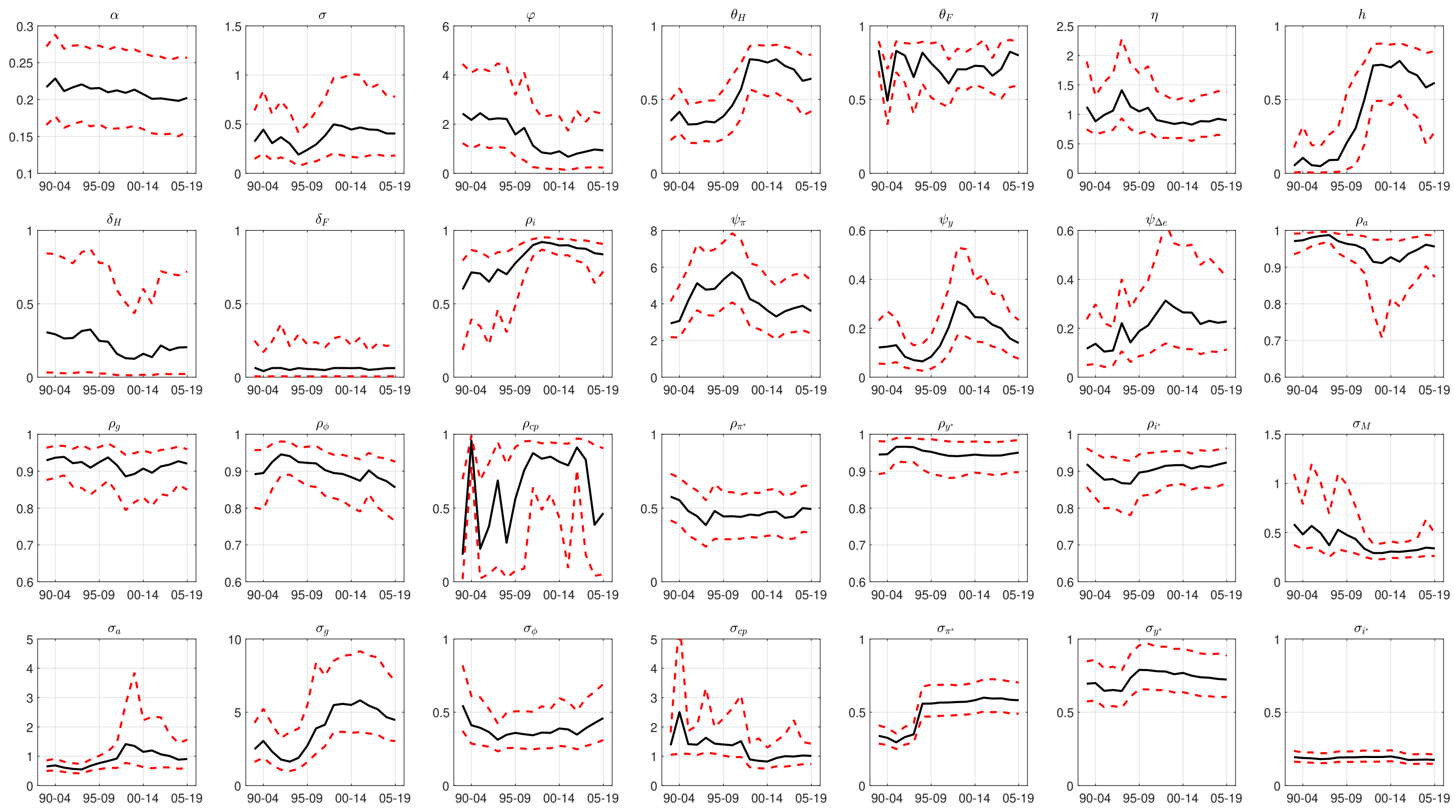
A.4 Alternative priors

Figure 16. Posterior distributions of policy coefficients and policy responses (Alternative priors)



Note: The horizontal axis of the right panels is maintained consistent with Figure 3 for comparability reasons.

Figure 17. Posterior distributions of policy coefficients (Alternative priors)



Note: The figure shows the posterior median (solid lines) of each sub-sample across the Metropolis-Hastings draws, along with 95% Bayesian credible interval bands (dashed lines).