A Dynamic Synthesis of Governance Theories*

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Abstract

Using evolutionary game theory, this paper models power dynamics of political systems. It synthesizes static political-economic analysis of power relations between the people, the ruler, and the elite. This dynamic synthesis suggests an empirically testable hypothesis: assuming that people's need for security empowers the elite and the ruler, and if the self interested elite and ruler will not allow a weak population to regain any power, in the universe of parsimonious models, there is no power sharing between all three players. A brief historical discussion examines both the assumptions and results of the analysis.

1 Introduction

This paper uses Saari's (1999) winding number methodology to explore the dynamics of competition for control decisions of collective interest among different groups in polities. The literature of political economy provides static two-player analysis. This paper integrates pair-wise static analysis into a dynamic framework. With minimal assumptions, this paper explores possible fixed points of power distribution between the people, the ruler, and the elite. The results suggest it is more likely that the people will be left powerless while the ruler and the elite share power. According to Finer (1997), historical data more frequently shows a power sharing equilibrium wherein people are left powerless. The contribution of this paper is two fold: adding the elite as a third player in the intrastate power game and providing a dynamic analysis.

Economists are aware that pairwise equilibria may no longer exist if a third player is added. A classical example of that is a game where two political candidates position themselves on the issue space to maximize the total number of voters. While there is an equilibrium with two players, there is none with three players. Reducing the number of players below that which is observed in the real world, the theoretical equilibrium will be an abstraction with no real world correspondence. Economist

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are well aware of games wherein an equilibrium exists, but is never observed in real experiments. An example of that is the limit cycle.

As illustrated by evolutionary analysis of the game rock, paper scissors, the existence of an equilibrium may not be useful in explaining observed data. Even though an equilibrium in that game exists, it is in the middle of a cycle and it is never reached. Through evolutionary analysis of a three player game, this paper characterizes the power dynamics implied by common assumptions in the literature. Some of these assumptions are the results of other papers, whenever possible, these papers are sited. More general sufficient conditions under which these assumptions are reasonable are included in the appendix. The paper also discusses the results of this dynamic synthesis in light of history.

The signers of the United States Constitution were concerned about government consolidating power. Today, some share these concerns. Others wonder if government serves the interest of the people or lobbying interests. Bernheim and Kartik (2004) study the impact of special interest on the quality of politicians. Corporations, lobbies, and the elite are increasingly demonstrating their influence on government decisions. There has been some public discussion of the behavior of elites and government. The Wall Street Journal (24 October 1990) notes:

> Republicans and Democrats have forged a political class to divvy up the spoils, fighting only over precisely how to pick pockets.

2 Literature Review

For tractability, to analyze power dynamics in polities, past work conducts static two-player game theoretic analysis . Early work by Roemer (1985) only considers government and the people. His analysis was followed by Buchanan and Faith (1987), then put into general equilibrium model by Grossman (1991). The aforementioned authors highlight the idea of power struggle within a state.

The idea of intra state power struggle is central to two literatures: positivists political economy and proprietary public finance. The two literatures recognize that all actors in a political system struggle for power. Grossman (2000) reviews positive theories in both political science and economics. Positive theories of economic policy typically assume that politicians and political organizations seek power either for its own sake or for instituting policies that to carry out their constituencies' will. Another approach taken by Bardhan (1990), Findlay (1990), and Grossman (1994) is the theory of proprietary public finance that treats the state's elite as an "autonomous actor". Findlay (1990) contrasts the usual view of the 'state as passive', with the theory of the 'autonomous state' that has strong incentives to consolidate power and extract political and economic rents.

The literatures of positivists political economy and proprietary public finance suggest a power struggle between the state as an autonomous self-interested agent and the people. Historically, the citizenry outside of government has not been treated equally. There are those who are considered the elite. The elite as a self-interested class is another player in this power game. Because of their wealth, influence, or real fighting power the elite are more mobile and effective so they command privileges. As long as the marginal benefit of appeasing the elite is more than the marginal cost of being unfair to the people, the ruler has no incentive to be just.

Historians such as North (1973) have pointed to the competition among rulers to attract or appease mobile, powerful subjects. Adding a the elite as a third player to the people-ruler game seems reasonable. However, classic analysis of this three player dynamic game is difficult. Even if it were technically possible, it would require the assuming the exact differential equations of the system; this would be a far fetched assumption. Yet, the existence of the elite class may be a significant factor in understanding power structure in political systems. Evolutionary game theory offers the possibility of a new dynamic synthesis of political economic theories about intrastate power struggle.

The Saari methodology provides general qualitative results without specifying the functional form of differential equations of the power flow. The methodology requires specifying the direction in which power tends to flow depends. Luckily, these tendencies are provided by pairwise strategic analysis. This allows this paper to use in some instances common assumptions from the literature and in other instances its pairwise results.

Whenever results from the literature are used, there is no loss of generality for two reasons. First, since only limiting results are used, the effect of assumptions about functional form is minimal. Second, appendix A contains a generalized game theoretic analysis version delineating sufficient conditions for all adopted assumptions. Lastly, along with the game theoretic analysis carried out by these authors, this paper supports its assumptions with intuition and historical evidence. Generally, it is sufficient to show that the marginal benefit at the end points of both players leads to an equilibrium directing the power flow inward or outward on any vertex on the simplex.

Each of these papers carries out game theoretic analysis that suggest the tendency of power near the vertices. This paper integrates the general assumptions and common results in the literature of political economy and provides qualitative descriptions of the flow of a single variable on power space common to all three players. Before exploring the details of the analysis and its results, this paper proceeds by reviewing the literature's contribution.

2.1 People, Power, and Protection

When people hold all the power, they empower the ruler to serve them. This assumption is the basis of the "social compact." In order for the ruler to perform its essential functions, the citizenry must subject itself to the ruler's power to act as the highest executive authority. People empower the ruler to enjoy benefits which include defense, the definition and enforcement of property rights and the provision of other public services. With power in hand, the ruler may choose to accumulate power to maximize rents.

Skaperdas and Konrad (2004) argue that the collective good, variously referred to as security, order, protection of property rights or simply protection, is a precondition for the provisioning of ordinary infrastructures, public goods, and generally for facilitating trade and economic development. Historically, it has also been the first type of good provided by states and is often considered the quintessential and defining attribute of a state. But, by its very nature, the protection apparatus is a double edged sword. People hired for protection may turn their guns against their client. Despite the risk of the ruler acting on its own behalf, people still choose to have a ruler. This paper's choice of adopting Skaperdas and Konrad (2004) is strengthened by the results of Grossman (2002).

Grossman (2002) shows that if the technology of predation is sufficiently effective, then having a

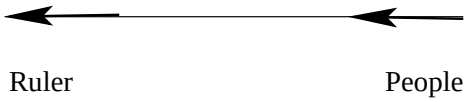

Figure 1: People Empower a Ruler, Rulers Maximizes Power

king, who can enforce a collective choice to allocate resources to secure producers product, is better for everyone, including both producers and potential predators, than not having a king, even though the ruler maximizes the consumption of a ruling class. This result suggests a great deal about the behavior of the people.

Grossman (2002) suggests that there are positive incentives for the people to empower a ruler. When people hold all but ε power, they empower a ruler to allow it to function. This is because ”even though the ruler maximizes the consumption of a ruling class,” having a king ”is better for everyone.” The tendency of power in this case would depend on the actions of the ruler. This tendency is also supported by the results of Skaperdas and Konrad (2004).

”Though self governance is best for a population, it faces problems of long term viability.”

This because of the free rider problem and the presence of ’competing predators.’ In the absence of a ruler anarchy rules.

The Skaperdas and Konrad (2004) model starts with anarchy and analyzes the strategic interaction between a protection monopoly and small competition. They also analyzes the market of competing protection racquets. Being small in the presence of larger predators requires large expenditures for security by individuals which leads to lower welfare levels than that of a subject of a warlord. This result suggests incentives on the part of the people to accept the reign of a ruler (a power monopolist,) or the elite(warlords) rather than totally decentralized power or(anarchy.) That is, if the ruler is weak, people may empower the elite to provide the same services the ruler is too weak to provide. Historically, when the ruler is weak (weakness or absence of centralized power), people turn to the nobles, tribal leadership, religious leadership. Having much to gain from this power the elite will not resist. Both the Skaperdas and Konrad’s (2004), Skaperdas (2005), and Benson’s (1996) work assume that warlords will take power away because of their comparative advantage in violence. This assumption is further strengthened by the results of Grossman (2002) and Axelrod (1996). The power flow is depicted on the people-elite power continuum.

Through evolutionary simulations, Axelrod (1996) also suggests the emergence of new political actors from a population of ordinary individuals. Some ordinary individuals accumulate power, until they become a part of the elite. The new political actors, Axelrod asserts, will behave much like autonomous states. This work is based on the assumption that specialists in violence will enforce a protection racquet and emerge as new players. Benson (1996) supports this assumption.

Benson (1996) suggests that in the state of anarchy, comparative advantage in violence will concentrate power. Whether it is true that the union of people will naturally form a government under the social compact, Rosseau’s social contract inspired many governments and constitutions around the world.

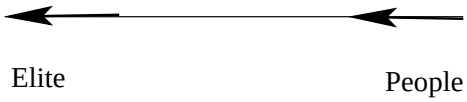

Figure 2: People Empower the Elite, The Elite Maximize Power

> Each of us places his person and all his power in common under the supreme direction of the general will; and as one we receive each members as an indivisible part of the whole.

The popularity of this assumption both academically and historically, supports its assumption in this model. The next step is to examine the power flow at two other continuums. Consider the elite strategic interaction with people, when the people are almost powerless.

As discussed earlier, proprietary public finance suggests that both the ruler and the elite maximize their power to gain rent. When the people are almost powerless, it is unlikely that they are able to reverse the flow of power in their direction. Grossman (2001) also suggests that the ruler has positive incentives to take power. This is intuitive. Given that the marginal benefit of the ruler due to the increase in power outweigh its costs, the ruler will not refuse this empowerment. This is shown on the continuum in Figure 1, by the flow of power (α) tending from the people to the ruler. The tendency represented by the arrow's direction. Grossman's (1994) central result of the analysis is that, if a relatively benevolent tax and spending policy is both necessary and sufficient for a high survival probability, then the rulers equilibrium policy is relatively benevolent. This implies that a ruler who can hold power despite unfair taxation will not be benevolent. By casual observation we can see that benevolent policies are not necessary for a high survival probability. On the continuum This power flow from the weak population towards the elite or the ruler is shown on the simplex upper end of the people-ruler edge and the left end of the people-elite edge. Having explained the power flow on edges connecting the people to both the ruler and the elite, now we turn our attention to the interaction between the ruler and the elite.

2.2 The Elite

The elite-ruler power flow is the resultant of the strategic interaction of both the ruler and the elite. Finer (1997) documents evidence that the strategy of nobles in the middle ages depended on the elite's power type. The behavior of the nobles who depend on the ruler to enforce property rights and bequeath wealth is different from those who inter-generationaly accumulate wealth and enforcement power.

2.2.1 The Elite with Real Power

From Asian tribal leadership to Mafia bosses in Italy, we observe many non governmental entities with real fighting power. Real power may allow the elite to break the ruler's monopoly on violence. It is clear by casual observation that these entities do not surrender power when government is weak. In a power oligopoly, a feud among warlords may ensue or the warlords may choose peace; Skaperdas (2002) calls it "peace in the shadow of war".

If the elite cooperate, once empowered to provide public goods the ruler is too weak to provide, the elite may need to empower a subset of them to be a central executive that oversees the enforcement

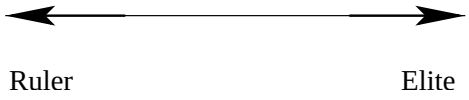

Figure 3: The Elite and the Ruler may compete for Power

Figure 4: Owing their Institutional Advantages to the Ruler, The Elite strengthen the Ruler

of their interests. In other words, to enjoy the rents of their power, the elite establishes an executive body to defend and exercises its privileges. This executive body, another self interested agent, may seek to maximize its power choosing to pursue their interest at the exclusion of other subsets of the elite. They may choose to consolidate power. Thus, evolving into into a ruler. If cooperation in the power oligopoly does not take place, the competition among warlords may turn into a feud.

In the feudal model, the competition of warlords over rents, may trigger a feudal state. The dominant force may succeed in consolidating power and securing a monopoly of protection as the ruler. Examples include Feudal Japan and China. Both of the cooperative scenario and the feudal one show that the flow near the elite-ruler vertex may point upward towards the ruler rather than rather than pointing at the elite. In sum, it seems more likely that even with real power the flow of power would head towards a central power. This also the case when the elite only have institutional power.

2.2.2 Elite with Institutional Power

The elite with institutional power and insufficient enforcement power may empower a ruler to strengthen law enforcement their claims on property. Finer (1997) points out that in the middle ages, kings would disarm the nobility and eventually endow them instead with land and money. This implies that the type of power affects the behavior of the ruler as well.

One paper that analyzes the strategy of the ruler against elites with real power is Skaperdas-Konrad (2004). The results of that paper suggest that the warlord with a near monopoly on protection chooses to consolidate power against its smaller competitors. This suggests that the ruler will consolidate power against the weak elite. However, this is not the only policy.

Whether or not the elite's power type changes the power flow on the simplex, since the flow of power is a state of the world contingent on both the actions of the players and uncertain outcomes. This paper will explore many other possible models in latter sections where the assumptions about the power flows are relaxed. This section will assume that the elite would empower a ruler. If one considers the elite's as simply wealthier more mobile people, this is consistent with our main assumption of the social compact, Grossman (2001), and the divide and rule strategy suggested by ADV (2003).

2.2.3 Divide and Rule

ADV (2003) suggest that if faced with two groups of people, the ruler gives some wealth to a group and not the other. This could make us consider that the ruler may surrender some power to the elite and enjoy their support or at least avoid their wrath. The divide and ruler strategy is as follows: the ruler will take revenue from all defector's in the people and give it to the elite. This result could be used within our institutional power model.

According to ADV (2003) , many kleptocrats are able to use a "divide-and-rule" strategy. Society's members need to cooperate in order to depose a kleptocrat. Such cooperation may be defused by imposing punitive rates of taxation on any citizen who proposes such a move, and redistributing the benefits to those who need to agree to it. This would intensify the collective action problem by threats which remain off the equilibrium path.

Kleptocratic policies are more likely when foreign aid and rents from natural resources provide rulers with substantial resources to buy off opponents. When opposition groups are shortsighted, when the average productivity in the economy is low, and when there is greater inequality between producer groups. Because more productive groups are more difficult to buy off. Faced with the choice of whom to appease, the ruler chooses to bestow benefits on the elite. This is because the marginal benefit of their approval is larger than that of ordinary people. This equilibrium is depicted by the arrow pointing towards government on the government people side and towards the elite on the ruler-elite side.

3 The Model

3.1 Pieces of a Political System

In his book, *The History of Government from the Beginning of Times*, Finer (1997) explains that polities consist of four main pure types of groups: The 'Palace', the 'Church', the 'Nobility' and the 'Forum.' These are euphemisms for the ruler, the religious establishment, the rich and powerful, and ordinary people. In other words, the dynamics of political systems is seen as the interaction of the administrating body, ordinary individuals, and a society of considerable status. This status is either economic, social, or religious. Building on Finer's classification, I generalize his model to include only three actors:

- The people
- The ruler
- The elite

Both the religious establishment and the nobility are two faces of the same coin. Both use their powers to influence the ruler. The Church lobbies through their base of support in the people and the 'Nobility' uses lineage, economic power, and sometimes armies. This makes individuals within that group more powerful than the ordinary citizen

Finer's distinction between the Church and the Nobility may be suitable for historical analysis, however, the distinction is not unnecessary for building this model. Whether it is the clergy, tribal leadership, or corporations, they are all elites. Whether this power is real or institutional, violent or economic,

it defines a group who are not in government, yet more powerful than the ordinary citizen yet not a direct part of the ruler's apparatus. The literature of political economy rarely makes the distinction between the ruler and other elites. Yet, it seems clear that there is one historically. One of this paper's main contributions is the making the distinction between the elites and the ruler and analyzing it in a formal model.

Historical examples of the ruler include a monarchy and its court, the bureaucrats and the military. Grossman (2000) contains further discussion of these these ideas. The word "ruler" is the group that comprises the political establishment.

To model the dynamics of power between the ruler, the people, and the elite, this paper focuses on the proportion of power held by each player. This proportional representation allows us to use the equilateral simplex the edges of which are the pairwise power continuum between each two of all three players. To synthesize the the above discussion into a dynamic frame work, the paper defines the variables of interest, the space on which the analysis is carried out, and the method of analysis.

3.2 Power Flow

Between the players, the general power flow depends on their strategies. The variables of the analysis: α_i ; $i = p,r,e$, is an abstract proportion of control over decisions of collective interest for each player. Since these variables are proportions, $\sum_i \alpha_i = 1$. Then S=$\{(\alpha_r, \alpha_e, \alpha_p) | \sum_i \alpha_i = 1, \alpha_i \geq 0 \; ; \forall i\}$ is the simplex wherein the interaction between the pairs can be represented. Local dynamics near the endpoints provides a picture of the global resultant tendency of power distribution amongst the three players.

Since institutional power is closely tied to the proportion of resources held and the potential of force, a distinction is made only when it affects the outcome of the analysis. As long as the continuity of α is not violated, one can assume it a function of real fighting power. Since power is divided among the players, $\alpha = (\alpha_p, \alpha_e, \alpha_r) \in S$. On the equilateral simplex, $\alpha = (1,0,0)$ denotes power lying solely with people. The interpretation of is anarchy and the continuum through self-governance to autocracy where $\alpha = (0,0,1)$ means the ruler has all the power. The analysis in this case does not take a space of strategies but builds on the static equilibria of the strategy space near the vertices of the power simplex.

Alternatively, the state of α could also be thought of as the level of power centralization. Then the people vertex would mean complete decentralization. The ruler vertex would mean that the ruler has all the power and so on. This vertex represents a monopoly on power. The elite vertex represents an oligopoly on power and protection.

The edges of the simplex indicate a power split between the two players on its end points. Since α is a state of the world and not a strategy, this means that alpha uses a mapping from the strategy space to the power flow space.

This mapping is the outcome of the pairwise strategic interaction between players. This section of the paper integrates the assumptions and results from the literature discussed in section 2. For necessary and sufficient conditions on the players' utility functions resulting in a Nash Equilibriums that suggest the power tendencies depicted see Appendix.

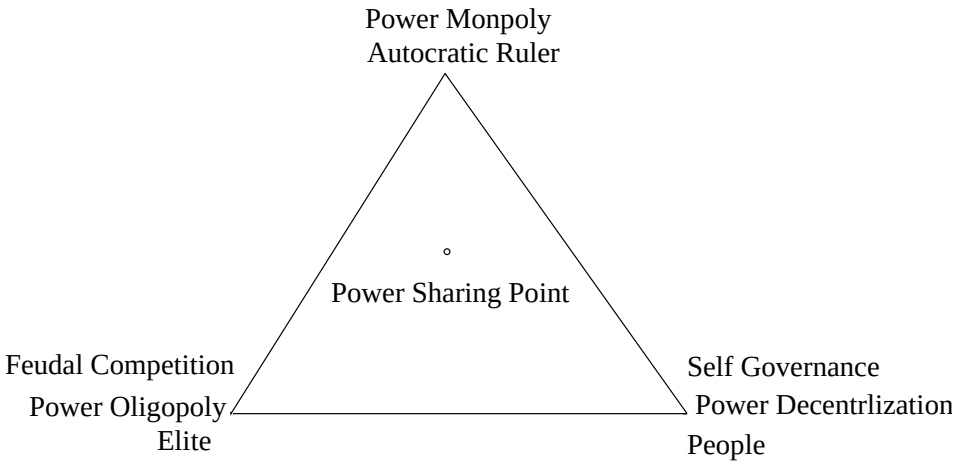

Figure 5: The Continuum of Power Distribution

4 Simplex Analysis

Following Saari's (1999) seminal work on dynamic analysis in the social sciences, we analyze the depicted simplex synthesizing the results from political economy that provides us with local information around the vertices. Once local information is provided, the next step is to characterize the net spin on the global level. This is similar to the sum of the changes equaling the overall change. To compare the global and local spin on the simplex, this paper uses winding numbers.

Assuming only the continuity of the differential equation $\frac{d\alpha}{dt}$, winding numbers allows us to characterize the dynamics of of power. The method of winding numbers requires the calculation of the directional spin along the edges of the simplex. Before discussing the the details of the analysis, consider figure 6 that puts together the aforementioned results from political economy. The flows and the name of the publications that support them are displayed.

4.1 Political Economy, a Dynamic Synthesis

4.2 Calculating Winding Numbers

To calculate the global winding number, trace the directional flow along the edge of the triangular simplex. When travelling along the simplex in a counterclockwise direction, compute the number of times the "arrows" make a complete revolution. If the rotations are in a clockwise direction, the answer will be negative. One can proceed in this manner netting out the complete revolutions turned. For more on this please see appendix B.

Since we start and end at the same place, the number of revolutions must be an integer. An easy way to compute this number is to put a pencil on any vertex of the simplex pointing in the direction of the arrowhead and rotating away from the simplex. The pencil will rotate to match the direction of the different arrow heads representing the direction flow on the edge of the simplex. The goal is to count the number of times the pencil rotates during this journey. The other part of this analysis is computing

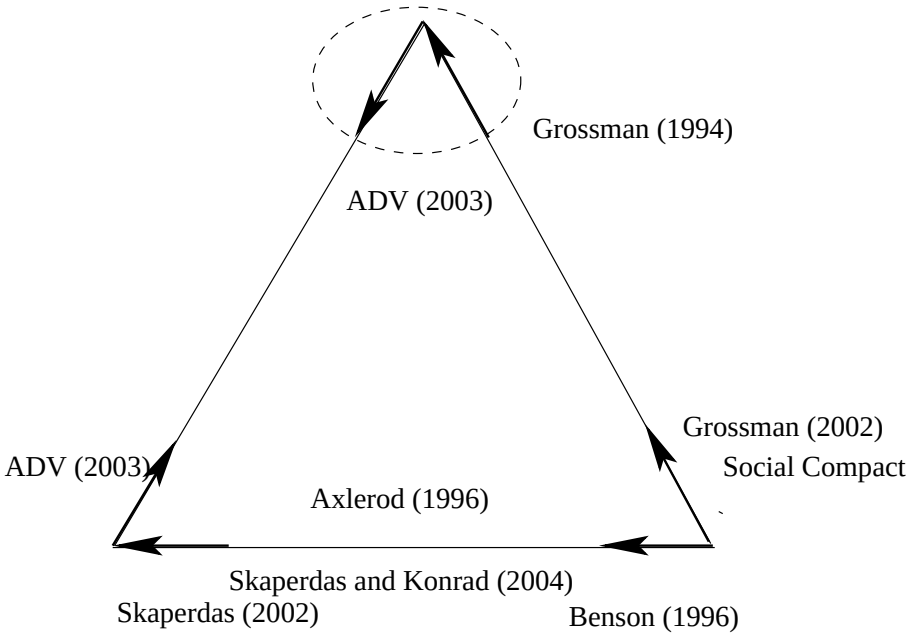

Figure 6: The Literature of Political Economy

the sum of local indices.

To compute the sum of local indices, identify the index for each critical point. Those are points where arrows push or pull on each other. This local index will be the product of the signs of the two arrows at each fixed point. If the arrows are in different directions, the local index would be -1. If the arrows point in the same direction, the local index would be +1.

Keep in mind that the opposite vector of and Eigen-vector is also an Eigen-vector. This is why the arrow outside the simplex is in the opposite direction of the ones inside it. Hence, there are only three possibilities for the local equilibria. All arrows going in is a sink (a point attracting all around it)and has a local index of +1; all arrows going out is a source(a point repelling all around it) and has a local index of +1. Otherwise the local index is a -1 indicating an unstable threshold. The focal point of all these details is the following relation:

$$\text{Global Winding Number} = \sum_{local equilibria} \quad i(e)$$

If one finds that the assumed flows in the model create a global directional spin that is not equal to the sum of local indices, then at least one interior fixed point must exist and the their sum with the other local indices is equal to the global winding number. Consider Figure 7.

Notice that the global winding number in Figure 7 is zero. If the sum of local indices is equal to the global winding number, it means that there is no interior fixed point of any kind. Then, the distribution of power is on the edges or vertices leaving at least one player powerless. Violating the above relation suggests the existence of a fixed point inside the simplex where power is shared by all three players. The analysis also allows us to investigate whether the interior fixed point is a stable or an unstable point.

DEFINITION 1 Let $\Omega = \bigcup M_S$ and M_S," and M_s="The model associated with the scenario S." Scenario

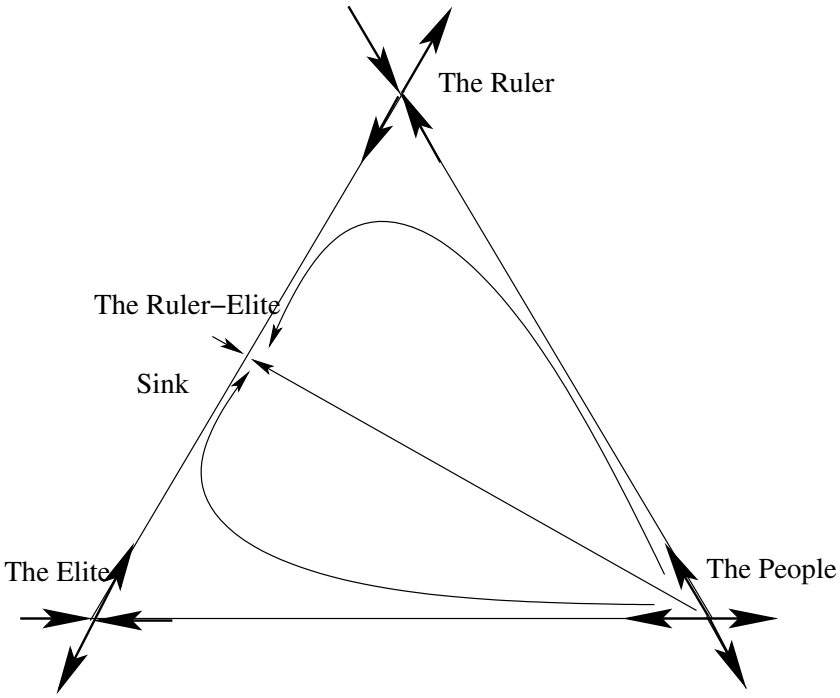

Figure 7: People Empower a Ruler, Rulers Maximizes Power

S="The ordered set of flows on all three edges of the Simplex ξ." e_{M_S}="The set of equilibrium points possible for M_S"Then, $\Omega^\star = \bigcup M^* : M_S^\star = M(S) : min|e|$

THEOREM 1 Given that $\alpha' = f(\alpha)$ such that f is a continuous function and given that $M \in \Omega^\star$ if:

1. People lose or surrender power to either the ruler or the elite to gain security.
2. Ruler evolves out of an elite power struggle, need for adminstration or to support property rights when the elite are nearly all powerful,
3. Once devoid of power, the people are unable to regain it.
4. The self-interested ruler practices the divide and rule strategy

$$e_{M_S^*} \bigcap \{\alpha | \quad \alpha_i > 0 \quad \forall i\} = \emptyset$$

In words: given the continuity of the differential equation governing the power flow, and within the universe of parsimonious models, it is impossible to have an interior equilibrium wherein the ruler, the elite, and the people share power.

PROOF 1 The proof is carried out by comparing the Global Winding Number to the sum of the local indices. Global Winding Number =0 $\sum_{localequilibria} \quad i(e) = 1 - 1 + 1 - 1 = 0$
QED

LEMMA 1 There is a stable power sharing between the ruler and the elite that leaves people powerless.

PROOF 2 The set of possible Equilibria in the simplex ξ=$\{(1,0,0),(0,1,0),(0,0,1)\}\bigcup\{\alpha\in(0,\alpha_e,\alpha_r)|\quad\alpha_i<1\quad\forall i\}\bigcup\{\alpha\in(\alpha_p,0,\alpha_r)|\quad\alpha_i<1\quad\forall i\}\bigcup\{\alpha\in(\alpha_p,\alpha_e,0)|\quad\alpha_i<1\quad\forall i\}\bigcup\{\alpha|\quad\alpha_i>0\quad\forall i\}$

All cases $\begin{cases} i(e(1,0,0))=+1, & \text{People vertex is a source;it can not be stable equilibrium;}\\ i(e(0,0,1))=-1, & \text{The Ruler vertex has local index of -1; it is an unstable point ;}\\ i(e(0,1,0))=-1, & \text{The negative inde implies instability at the elite vertex;}\\ i(e(Ruler-Elite))=+1, & \text{The Ruler-Elite Edge is a sink ;}\\ \text{By inspection,} & \text{No equilibria on any other edge ;}\\ \text{By theorem 1,} & \text{No interior equilibrium .QED} \end{cases}$

When the founding fathers of the American Revolution were drafting the Constitution they were concerned about the potential for tyranny. This is because of its historical prevalence. This model shows one reason why it would be the case. The dynamic flow of power inside the simplex is depicted in Figure 7.

From the point of origin, once the elite gains sufficient power, the flow goes to the ruler elite sink. If not, power will flow all the way to the ruler and then back down to the ruler elite sink. Otherwise, power would flow from the people to the elite and then up to the ruler elite sink. It is not clear if power sharing fixed point is to the advantage of the ruler or the elite. It would depend on the specifics of the situation modelled. Finer points to the historical variety of power sharing between the ruler and the elite.

The model discussed above is the simplest possible model. Without changing the global or local indices, a number of fixed point along the edges could be added. This helps to add sophistication to the model. However, this added sophistication will not add an interior power sharing equilibrium. If there is an equilibrium wherein people hold some power it would require fixed points to exist on the triangle edge between the people and the ruler.

Historically, examples that fall outside of autocracy are infrequent. Until the French revolution, if any, there were few examples where people held any power. Skaperdas (2004) points to possible exceptions in early Mesopotamia, ancient Greece, and late medieval Italy. Adding the elite into the picture allows us to rule out these examples because they did not include slaves, women, or people without property. A group enjoying privileges at the exclusion of others because of their power and wealth is this model's definition of the elite. Until the past three hundred years, few counter examples to this paper's result exist. The more frequent observation is power sharing between the elite and the ruler.

It is not an equal distribution of power. More power would either be held by the ruler or the elite. While Lemma 1 result: The elite and the ruler sharing power at the exclusion of all others characterizes many observed outcomes, it relies on assumptions about behavior and policy. The policy of the ruler, the behavior of the elite, and the reaction of the people. One era rife with such examples is the middle ages.

Jones (1981) describes the revolutions of the middle ages as "a struggle between taxes and rents." The struggle between the ruler and the elite ensued over the market for protection. In many instances, the monarch crushed the nobles to maintain a monopoly on security. In the feudal model, the elite enjoy enforcement power and they are unwilling to surrender it. The uncertain outcome of this struggle

prompts us to explore alternative flows on the ruler-elite edge. To analyze this alternative setting, the next section relaxes the assumptions used in synthesis model.

5 Relaxing the Assumptions of the Synthesis

This section explores alternative models. The first section selected and synthesized relevant papers from the literature. This section relaxes all assumptions about the flow of power on the ruler-elite edge. There are Four possible models:

1. The Divide and Rule model(Synthesis Model): the ruler and the elite cooperate to exclusively share power.
2. The Feudal model: both the ruler and the elite consolidate power at the vertex.
3. The Centralized Power model: all power on the ruler-elite edge flows to the ruler.
4. Power Oligopoly: all power on the ruler-elite edge flows to the elite creating a power oligopoly.

5.1 The Feudal Model

Consider the feudal model where there is a power struggle between the elite and the ruler. Instead of cooperating, both the ruler and the elite seek to consolidate power when the other is weak. Naturally, the flow is most likely in the direction of the strongest. As mentioned above, Skaperdas and Konrad (2004) suggest that a powerful ruler will consolidate power against the elite.

Like any profit seeker, the theory of proprietary public finance assumes that the proprietor of ruler enterprize maximizes 'political rents.' Assuming this objective as a generic property of rulers and the model assumes that rulers will consolidate power in pursuit of maximal taxation. The arrows pointing towards the top of the triangle signify the assumption that if the ruler was dominant, it will seek to consolidate power. For more on this see Grossman (2000).

THEOREM 2 Given that $\alpha' = f(\alpha)$ such that f is a contentious function and given that $M \in \Omega^{\star}$ if:

1. People will lose or surrender power to either the ruler or the elite to gain security.
2. Both the elite and the ruler take power away from a weak population.
3. Once almost powerless, people are unable to reagin power.

$$e_{M_S^*} \bigcap \{\alpha | \quad \alpha_i > 0 \quad \forall i\} = \emptyset$$

In words, given the continuity of the differential equation governing the power flow, and within the space of parsimonious models, it is impossible to have an interior equilibrium wherein the ruler, the elite, and the people share power.

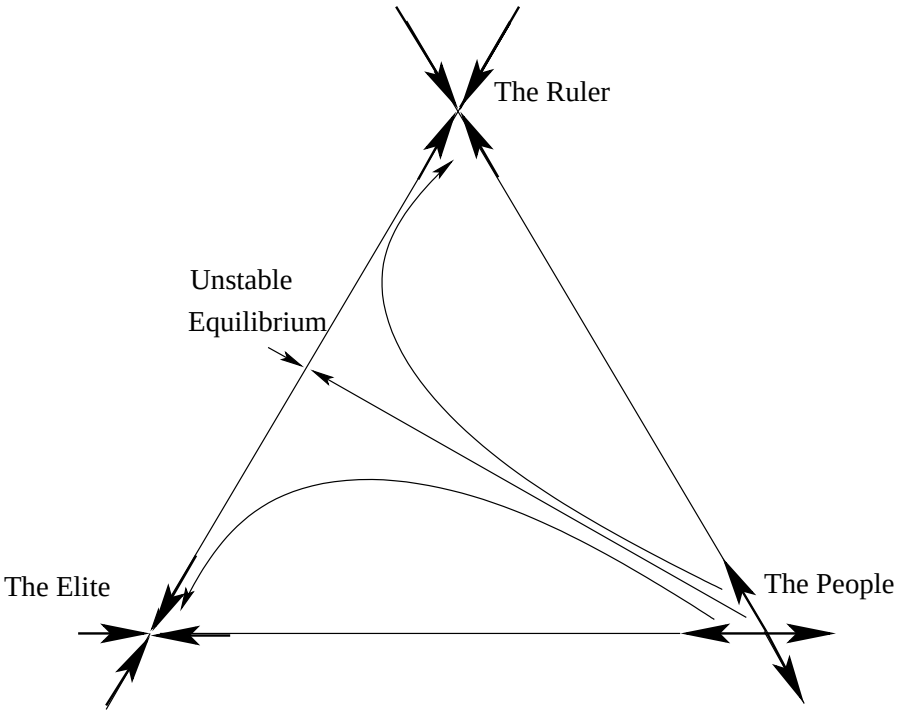

Figure 8: The Feudal Model

PROOF 3
$$\begin{cases} \text{Global Winding Number} = \sum_{localequilibria} & i(e) = 0 & \text{The Divide and Rule model;} \\ \text{Global Winding Number} = \sum_{localequilibria} & i(e) = 1 & \text{The Feudal model ;} \\ \text{Global Winding Number} = \sum_{localequilibria} & i(e) = 1 & \text{The Tyrannical Model;} \\ \text{Global Winding Number} = \sum_{localequilibria} & i(e) = 1 & \text{The Warlords model: .QED} \end{cases}$$

LEMMA 2 In the feudal model, stable equilibrium either lies with elite alone or with the ruler alone.

PROOF 4 ξ=$\{(1,0,0),(0,1,0),(0,0,1)\}\bigcup\{\alpha \in (0,\alpha_e,\alpha_r)| \quad \alpha_i < 1 \quad \forall i\}\bigcup$
$\{\alpha \in (\alpha_p,0,\alpha_r)| \quad \alpha_i < 1 \quad \forall i\}\bigcup\{\alpha \in (\alpha_p,\alpha_e,0)| \quad \alpha_i < 1 \quad \forall i\}\bigcup\{\alpha| \quad \sum_i \alpha_i < 1\}$

$$\begin{cases} i(e(1,0,0)) = +1, & \text{People vertex is a source;it can not be stable equilibrium;} \\ i(e(0,1,0)) = +1, & \text{The Elite vertex is a sink;} \\ i(e(0,0,1)) = +1, & \text{The Elite vertex is a sink;} \\ i(e(Ruler - Elite)) = -1, & \text{The Ruler-Elite is unstable threshold ;} \\ \text{By inspection,} & \text{No equilibria on any other edge ;} \\ \text{By theorem 1,} & \text{No interior equilibrium .QED} \end{cases}$$

5.2 Tyranny of Sorts

From old monarchs to modern day dictatorships, history is ripe with examples of tyrannical rulers consolidating all power. The 'Stalins' of the world are known to trust no one else with power. The result of this model is intuitive since the ruler's power seems to go unchecked. Relaxing the synthesis assumptions allows the possibility where all power flows to the ruler as depicted in figure 7.

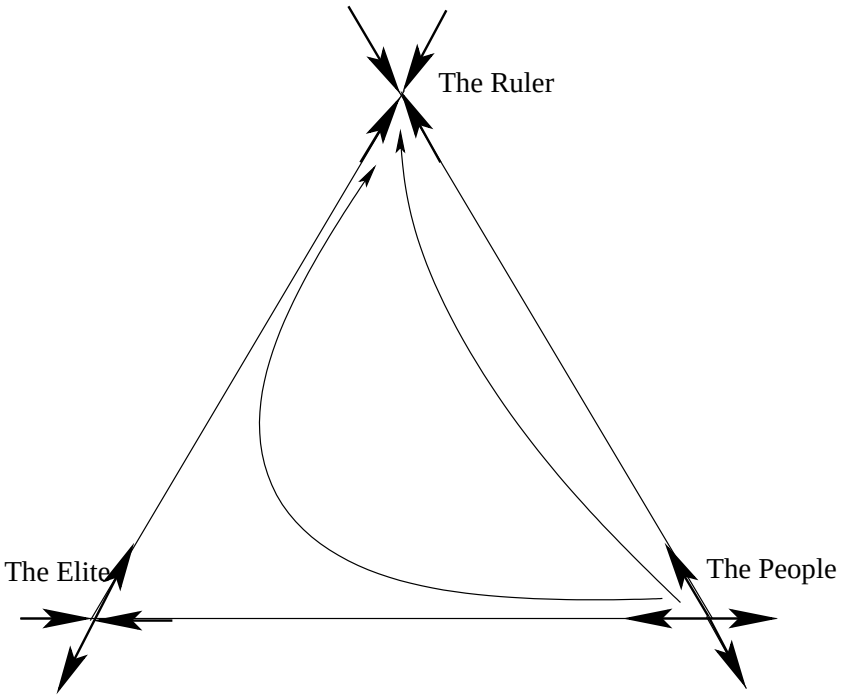

Figure 9: The Tyrannical Model: Power Monopoly

5.3 The Warlords

5.3.1 Possible Sophistication

While added sophistication on any of the edges could create fixed points, it will not alter the impossibility of a power sharing fixed point without assuming an alternative flow path from the proposed edges.

Examples of complete control by the elite are less common. Finer (1997) , documents only one example in 18th century Poland where the nobility was in complete control. The king was one of them and the veto of any member of the nobility can stop any proposition. In that sense the nobility was the ruler, which means that the elite became the executive and the flow of power points from the all powerful elite towards the ruler. This as an example of the ruler fixed equilibrium rather than an elite equilibrium. More appropriate examples are those found in power oligopolies, where there are warlords and they are constantly trying to dominate each other; feudal Japan and the period of warring states in China. This is the history of Arabia from the 400 AD to 700 AD. Often an emerging power is success in creating a near monopoly on in a central area, in areas away from that center there are pockets that remain outside the reach of central power.

Skaperdas (1999) analyzes the condition of this warlord oligopoly. ”Because war is destructive, human beings are typically risk averse, and there exist numerous complementarities in production and consumption, we can expect peace in the shadow of war to be most often preferable by all parties. Actual war can take place because of incomplete information about the preferences and capabilities of the adversaries but also, somewhat surprisingly, when the shadow of the future is sufficiently long.”

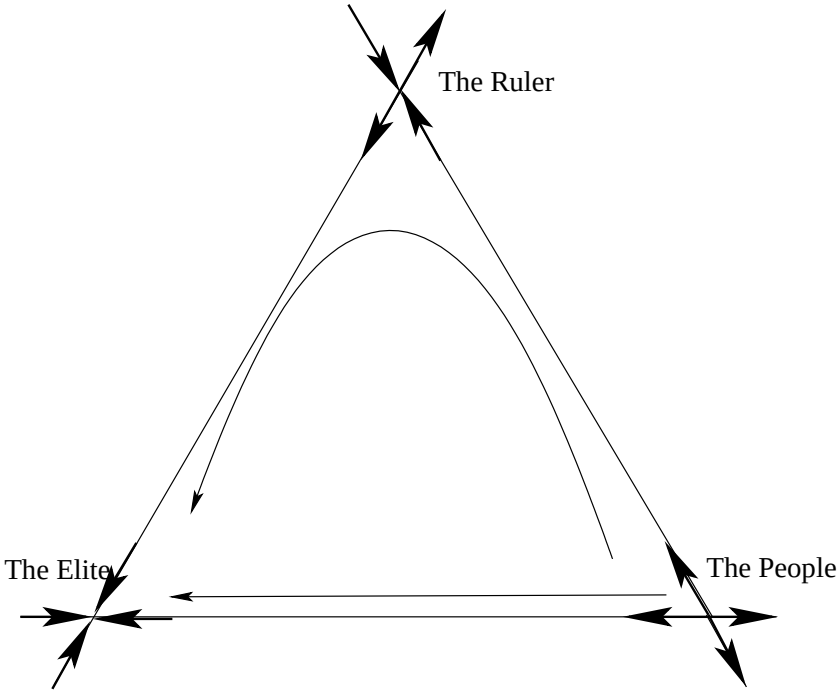

Figure 10: Warlords Competition: Power Oligopoly

The winding number analysis of power oligopoly is straight forward and no different than the analysis of power monopoly.

The infrequent observation of the elite exclusively holding power for extended periods focuses the attention on the Synthesis Model and the Feudal Model.

6 Summary and Conclusion

The synthesis model analyzes one likely scenario. It brings together works in political economy in a dynamic setting. Relaxing some of the assumptions made in the synthesis model, offers one several possible scenarios. These scenarios are different power tendencies on the edges of the simplex. These scenarios provide different models. The different models would seem to be alternatives, but they should not be viewed as final. Again because the power flows are tendencies based on strategic interactions and uncertain outcomes, the tendencies may change if any player in the pairwise game changes their strategy.

For example, like the nobility in the Middle Ages the elite may resist the ruler. This generated Feudal Dynamics a stage where there is no interior equilibrium. If the ruler was unable to establish a centralized power, an oligopoly of power may result in warlord competition. On the other hand, if the ruler is successful in subduing the elite then this change in the flow will be best modelled by the Tyrannical model. However, risking a struggle against the elite, an outcome of which may be complete loss of power, one often used policy is to share some power with the elite to gain support, then the

outcome of that policy is best described by the Synthesis Model. While these models can be viewed as possibilities in a probability space the measure of which would depend on both the strategies and their associated probability of success, one must not think of the results as static. In other words, as discussed earlier, a political system might go through all stages as the actors better adjust their strategies in response to others and in response to events. For example, in the feudal model, the power struggle between the ruler and the elite may end with the success of the ruler in consolidating control. Once the elite have been de-clawed, the flow near the elite vertex may switch from drawing power into supporting the new ruler in hopes of gaining favor.

Under very general conditions, this paper was able to characterize many frequently observed power distributions in world history. Relaxing the assumption made in this paper will expand the set of possible models. With no assumptions made, there are 110 possible parsimonious models. Relying only on the most robust assumption, that of social compact, leaves 40 possible dynamic flows to consider. This is an avenue for future work.

Appendices

A Game Theoretic Analysis

Normal Form of the Game on the People Ruler Power Continuum

People/Ruler	Consolidate Power	Not Consolidate
Revolt	$u_r(C,R)$,$u_p(C,R)$	$u_r(NC,R)$,$u_p(NC,R)$
Not Revolt	$u_r(C,NR)$, $u_p(C,NR)$	$u_r(NC,NR)$,$u_p(NC,NR)$

Since revolt has a probability of success call it θ then the Utility of both players is not the expectation over the probability of successful revolt. Notice that the probability of a successful revolt is a function people's level of power. The Ruler has the option to penalize a revolting population by taking away all power in the event of failure.

$$
\begin{gathered}
u_r(C,R) = \theta(0) * u_r(0) + (1-\theta(0)) * u_r(1) \\
u_p(C,R) = \theta(0) * u_p(1) + (1-\theta(0)) * u_p(0) \\
u_r(NC,R) = \theta(\varepsilon) * u_r(0) + (1-\theta(\varepsilon)) * u_r(1-\varepsilon) \\
u_p(NC,R) = \theta(\varepsilon) * u_p(1) + (1-\theta(\varepsilon)) * u_r(0) \\
u_r(C,NR) = u_r(1) \\
u_p(C,NR) = u_r(0) \\
u_r(NC,NR) = u_r(1-\varepsilon) \\
u_p(NC,NR) = u_p(\varepsilon)
\end{gathered}
$$

for the ruler to consolidate then $u(C) \geq u(NC)$: $\begin{cases} u_r(C,R) \geq u_r(NC,R), \\ u_r(C,NR) \geq u_r(NC,NR), \end{cases}$

For a simultaneous move game these expressions are equivalent to:
$\theta(0) * u_r(0) + (1-\theta(0)) * u_r(1) - \theta(\varepsilon) * u_r(0) - (1-\theta(\varepsilon) * u_r(1) \geq 0$

and $u_r(1) - u_r(1-\varepsilon) \geq 0$ This is equivalent to :

$$(u_r(0) - u_r(1-\varepsilon)) * (\theta(0) - \theta(\varepsilon)) \geq u_r(1-\varepsilon) - u_r(1)$$

for values near the edge of the continuum:

$$\begin{array}{c} \lim_{\varepsilon \to 0} \quad \frac{[u_r(0)-u_r(1-\varepsilon)]*[\theta(0)-\theta(\varepsilon)]}{\varepsilon} \geq \frac{u_r(1-\varepsilon)-u_r(1)}{\varepsilon} \text{ and:} \\ \lim_{\varepsilon \to 0} \quad \frac{u_r(1)-u_r(1-\varepsilon)}{\varepsilon} \geq 0 \end{array}$$

Notice that for α_{people} to end up at zero they have to revolt at any level and fail. The probability of failure $1-\theta(\alpha)$ becomes the object of interest. Consider the CSF being a popular class of functions for the success in contests. Roemer (1985) suggests that the the decision to revolt is linked to that the probability of success. Even without the total loss of power as deterrent, values near the edge of the continuum yields the following conditions:

$$\begin{array}{c} \lim_{\varepsilon \to 0} \quad \frac{u_r(0)-u_r(1)*[\theta(0)-\theta(\varepsilon)]}{\varepsilon} \geq \frac{u_r(1-\varepsilon-u_r(1)}{\varepsilon} \text{ and} \\ \lim_{\varepsilon \to 0} \quad \frac{u_r(1)-u_r(1-\varepsilon)}{\varepsilon} \geq 0 \qquad \text{or equivalently:} \\ \theta'(0) * [u(1) - u(0)] \leq u_r'(1-\varepsilon) \text{ and} \\ u_r'(1-\varepsilon) \geq 0 \\ [u_r(0) - u_r(1)] * [\theta(0) - \theta(\varepsilon)] \geq u_r(1-\varepsilon) - u_r(1) \end{array}$$

Similar analysis from the point of view of people gives the following conditions:

$$\begin{cases} u_p(0) \geq \theta(0)u_p(1) + [1-\theta(0)]u_P(FailedRevolution), & \text{Not revolting in response to power consolodation} \\ u_p(\varepsilon) \geq \theta(\varepsilon)u_p(1) + [1-\theta(\varepsilon)]u_P(FailedRevolution), & \text{Not revolting even with some power left.} \end{cases}$$

These conditions yield this familiar expression: $[u_p(1) - u_p(failedrevolution)] \geq u'(0)$

For any θ such that $\theta'(0) = 0$ the conditions are rather simple. Notice that for many versions of the CSF, the popular theoretical measure of success in contests, this conditions is satisfied. Furthermore, the game between the elite and the people is identical to the one above. The conditions above are those necessary for the assumed tendency of the ruler to consolidate power. The conditions required for tendency of people to empower the elite or the ruler are:

$$\begin{array}{ll} u_p'(1-\varepsilon) \geq 0 & \text{and } u_r'(\varepsilon) \geq 0 \\ u_p'(1-\varepsilon) \geq 0 & \text{and } u_e'(\varepsilon) \geq 0 \end{array}$$

Lastly, consider the conditions under which the once all powerful, the elite and the ruler will not allow the people to gain any power. Let $g_i(\alpha)$="the share of power level player i gets on the edge." for $i \in$ the elite, the ruler$\}$ Then, player i will choose to prevent the people from regaining ε power if and only if: $u_i(g_i(1)) \geq u_i(g_i(1-\varepsilon))$It is clear this condition is enough to prevent people from regaining power. Notice that at edge of the continuum we obtain similar conditions:

$$\begin{array}{c} u_i'(1) \geq u_i'(1-\varepsilon) \\ \frac{du_i}{dx}(1) \geq \frac{du_i}{dx}(1-\varepsilon) \\ \frac{du_i}{dg}(1)\frac{dg}{dx}(1) \geq \frac{du_i}{dg}(1-\varepsilon)\frac{dg}{dx}(1-\varepsilon) \end{array}$$

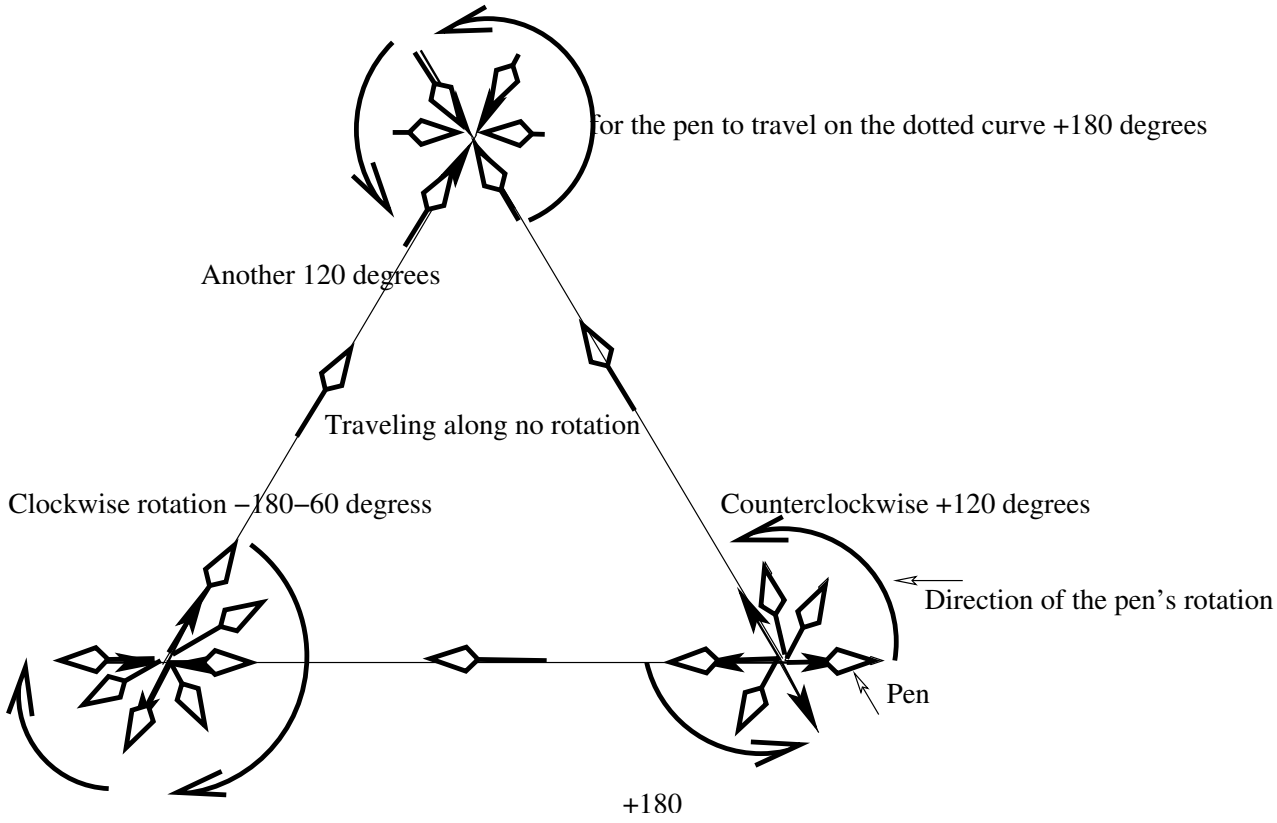

Figure 11: How to Calculate the Global Winding Number

B Winding Numbers

B.1 Definition

As the name suggests, the winding number w(L,0) of a closed loop L about the origins 0 is simply the net number of revolutions of the direction of z as it traces out L once in its given sense. A good example of net rotation is a nut on a bolt. If one spins the nut back and forth in opposite direction, the final distance from its starting point measure the nut's net rotation.

The shapes we deal with in this paper belong to the set of simple shapes. The global winding number is calculated with respect to a point inside the simplex. The local indices are calculated with respect to the critical point considered.

Notice that the net of positive and negative rotations is 360 degrees or +1 rotations. Now consider the local equilibriums. As mentioned earlier, if all the arrows are in the same direction then the local index is +1 otherwise it is a -1. Starting from the lower right hand vertex to the top vertex and down to the lower vertex the indices are: +1+1-1=1. Notice that the global winding number and the local are equal, implying no interior equilibrium in the space of parsimonious models.

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