# Price Effects from Increased Competition on a Transportation Network

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**Abstract**

This paper models the price effects from increased competition in a transportation network, analyzing two routing set-ups. I model a game where a point-to-point (direct) network competes with a hub and spoke (indirect) network. I find that when competition is increased on a small transportation network, prices do not necessarily fall. It may be the case that a monopoly is the best strategy for obtaining higher quantities and lower prices. This occurs because of the economies of scale and economies of scope that result from the "cost-saving nature" of indirect routing.

# Introduction

Planners and policymakers tend to overlook the effect of how increases on a transportation network will have on the network, in particular profits, prices and quantities. Such effects are rarely modeled or discussed in cost-benefit analysis. This can lead to severe consequences, such as unexpected cost underestimation and inflated ridership forecasts. On average, new transportation projects tend to have lower projected costs than actual costs, and higher projected ridership than actual ridership. This result, in part, may be due to unexpected changes in prices, market structure, or consumer's preference for certain transportation modes. New transportation projects tend to encourage competition from other firms.

As an example, the cost-benefit analysis done for the EuroTunnel, which crosses the English Channel, showed significant returns for the project when it was first introduced. However, the EuroTunnel has had trouble repaying its debt and has had to refinance. The troubles the Eurotunnel has faced in comparison to the expectations laid out by the cost-benefit analysis may be due to not accounting for the effect of increased competition. Rail ridership was lower than expected and competition from the ferry industry increased. In addition to offering around the clock service, ferry companies merged to maintain market share in the freight and passenger sectors. The presence of the EuroTunnel has driven prices down and has provided social benefits that otherwise would not have been present.

My paper models transportation competition with two types of routing setups. One firm offers point-to-point service, while the other firm offers hub and spoke type, which I refer to as direct and indirect service, respectively. To keep the analysis simple, I assume that frequencies are the same among the two firms, that travel times are the same and that the two firms offer homogeneous transportation services. In my model, there are two competitors where one firm has the advantage of offering direct service whereas the second firm can only reach its destination by offering service through an intermediary point, labeled Y, in Figure 1. In this small and simplified transportation network, I show how increased competition on a network will not necessarily lower prices and increase consumer welfare. I also give an example where the two competing firms in a three point network may drive one firm out of the market on a particular route. Such a case will occur when one of the firms can take advantage of economies of scale that result from the "cost-saving" nature of indirect routing.

# Literature

There are a few transportation papers modeling competition in the airline industry. For example, "Competition and Mergers in Airline Networks," shows that increased competition on a network causes negative externalities, including reduced traffic and a loss in social surplus Brueckner and Spiller (1991). Other papers such as, "The Price Effects of International Airline Alliances," find that airline alliances reduce traffic on an international network and increase prices Bruckner and Whalen (2007). Others such as Givoni and Banister (Givoni and Banister) suggest integrating high-speed rail infrastructure into air infrastructure since competition can occur between air and rail along short-haul routes. Similar studies such as John Kay and Szymanski (John Kay and Szymanski) model competition between high speed rail and the ferry industry along the English Channel. My paper contributes to this literature by also including a model of increased transportation competition, but incorporating a perfect substitute. It is important to question the ridership predictions of the California High Speed Rail Authority cost benefit report, since it assumes a 55% ridership diversion from air to high speed rail, a particularly high diversion rate Brand and Keifer (Brand and Keifer). These papers set up the framework for modeling a small transportation network.

# Model

I model a transportation network model with two firms competing in a Cournot-Oligopoly fashion. The two firms compete to maximize profits, treating the other firm's output as given. I compare prices that result from increased competition under two routing setups. The first setup consists of two firms offering direct service and the second setup consists of one firm offering direct routing, while the other offers indirect type routing. This analysis can be applied to two airlines, two buses or even two railroads, competing on the same network. I try to capture the economies of scale that result from indirect routing. I argue that this type of economies of scale is similar to those realized on some real-world transportation networks.

### Demand and Cost Functions

I assume that the (inverse) demand function for round-trip transportation service between two points such as X to Y in Figure 1 is linear:

$$D_i(q_i) = p_i(q_i) = \alpha_i - q_i$$

Where $i$ is a subscript that can take on values 1, 2, or 3 corresponding to travel between X & Y, Y & Z, and X & Z, respectively, in Figure 1. I use $q_i$ to measure output on a particular route, such as, round trip passenger miles, and $p_i$ as the fare corresponding to the level of output for that particular route. In this paper, $\alpha_i$ measures the benefit minus the cost of travel. I assume that $\alpha_1 = \alpha_2 = \alpha_3 = \alpha > 1$, suggesting positive transportation demand. The slope of the demand curve is set to one for simplicity. The cost function I use for the firm offering direct service, as shown in Figure 1, has the following quadratic form:

$$C(q_1, q_2, q_3) = (1 - \gamma q_1)q_1 + (1 - \gamma q_2)q_2 + (1 - \gamma q_3)q_3$$

It is simply the sum of the costs for providing service on each route of the network. Note that this cost function allows for downward sloping marginal and average cost curves. This is because the cost per passenger decreases as the total number of passengers increases. In this setup, $\gamma$ measures the concavity of the cost function, and hence, the level of economies of scale. When $\gamma = 0$, the marginal cost curve is constant and equals the average cost curve, indicating neither economies nor diseconomies of scale. To capture the extra economies of scale that result from indirect type routing structure, as shown in Figure 3, I use the following cost structure.

$$C(q_1, q_2, q_3) = (1 - \gamma(q_1 + q_3))(q_1 + q_3) + (1 - \gamma(q_2 + q_3))(q_2 + q_3)$$

It is simply the sum of the costs for providing service on two of the routes, $q_1$ and $q_2$. Notice that the cost of providing $q_3$ service is incorporated in the cost of providing service along routes $q_1$ and $q_2$.

The next four sections describe the derivation for the optimal quantities, prices, and profits that result from four different routing scenarios. The first scenario derives the optimal quantities for a monopolist offering direct service on a three point network. The second scenario derives the optimal quantities for a duopolist, in which case the two firms offer direct service to the three points. The third scenario derives the optimal quantities for a monopolist offering indirect service to the three points. In other words, routes X & Y and Y & Z are serviced directly by the monopolist but if passengers desire to travel between X & Z, they need to ride route X & Y first and then Y & Z to get to their final destination. I assume passengers are willing to do so as long as they are appropriately compensated by a price discount. Scenario four, which is my unique contribution to the literature, is the scenario where two firms compete along the three destinations, one firm, which I will refer to as Firm A, offers direct service, whereas

Firm B offers indirect service as described in scenario three.

## Scenario 1: One Transportation Firm (Direct Service)

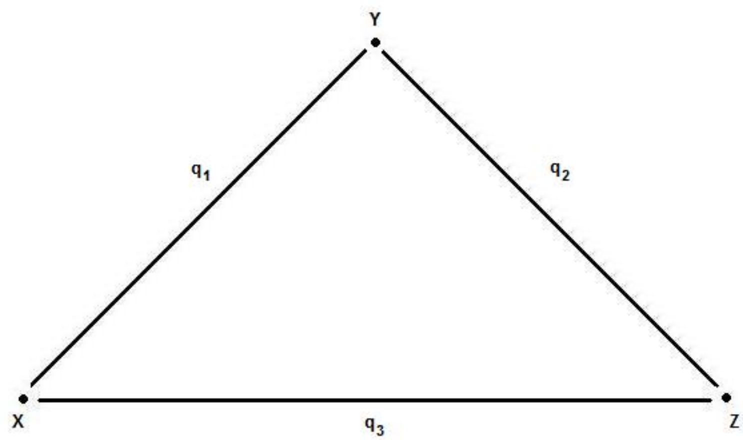


Figure 1: Monopoly - Direct Service

In this first scenario, Firm A offers direct point-to-point service and must maximize the following profit equation:

$$\text{Max}\Pi(q_1, q_2, q_3) = \sum_{i=1}^{3}[(\alpha - q_i)q_i] - \sum_{i=1}^{3}[(1 - \gamma q_i)q_i] \tag{1}$$

The first order conditions are:

$$\frac{\partial \Pi(q_1, q_2, q_3)}{\partial q_1} = \alpha - 2q_1 - 1 - 2\gamma q_1 = 0$$

$$\frac{\partial \Pi(q_1, q_2, q_3)}{\partial q_2} = \alpha - 2q_2 - 1 - 2\gamma q_2 = 0$$

$$\frac{\partial \Pi(q_1, q_2, q_3)}{\partial q_3} = \alpha - 2q_3 - 1 - 2\gamma q_3 = 0$$

The first order conditions tell the firm to set output on each route where marginal revenue equals marginal cost. The second-order condition requires that $\gamma < 1$, which requires that the marginal revenue curve to cross the marginal cost curve from above. Solving for $q$ yields the following profit-maximizing quantities:

$$q_1 = \frac{\alpha - 1}{2(1 - \gamma)} \tag{2}$$

$$q_2 = \frac{\alpha - 1}{2(1-\gamma)} \tag{3}$$

$$q_3 = \frac{\alpha - 1}{2(1-\gamma)} \tag{4}$$

I find prices charged for service on each of the following routes by substituting the optimized quantities into the demand functions for each route. I find profits by substituting the optimized values for quantity and corresponding prices into profit equation (1).

## Scenario 2: Two Transportation Firms (Direct & Direct Service)

In this scenario, competition is increased in the network from another firm that offers the same direct service over the entire network as in Figure 2. Each firm must maximize its profit function, taking its competitor's quantity as fixed, and find the necessary first order conditions on each route of the network. I compute reaction functions for the two firms and I set them equal to each other to find the Cournot equilibrium.

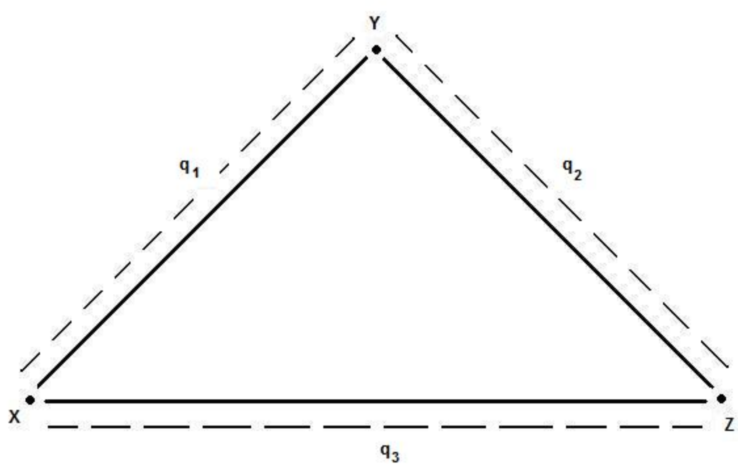


Figure 2: Duopoly - Direct & Direct Service

Firm A must maximize the following profit function:

$$\text{Max}\Pi(q_1, q_2, q_3) = \sum_{i=1}^{3}[(\alpha - (q_i^A + q_i^B))q_i^A] - \sum_{i=1}^{3}[(1 - \gamma q_i^A)q_i^A] \tag{5}$$

The first order conditions for Firm A are:

$$\frac{\partial \Pi(q_1, q_2, q_3)}{\partial q_1^A} = \alpha - 2q_1^A - q_1^B - 1 + 2\gamma q_1^A = 0$$

$$\frac{\partial \Pi(q_1, q_2, q_3)}{\partial q_2^A} = \alpha - 2q_2^A - q_2^B - 1 + 2\gamma q_2^A = 0$$

$$\frac{\partial \Pi(q_1, q_2, q_3)}{\partial q_3^A} = \alpha - 2q_3^A - q_3^B - 1 + 2\gamma q_3^A = 0$$

The optimized quantities for Firm A are the following:

$$q_1^A = \frac{(1-2\gamma)(\alpha-1)}{4(1-\gamma)^2 - 1} \tag{6}$$

$$q_2^A = \frac{(1-2\gamma)(\alpha-1)}{4(1-\gamma)^2 - 1} \tag{7}$$

$$q_3^A = \frac{(1-2\gamma)(\alpha-1)}{4(1-\gamma)^2 - 1} \tag{8}$$

Firm B must maximize the following profit function:

$$\text{Max}\Pi(q_1, q_2, q_3) = \sum_{i=1}^{3}[(\alpha - (q_i^B + q_i^A))q_i^B] - \sum_{i=1}^{3}[(1 - \gamma q_i^B)q_i^B] \tag{9}$$

The first order conditions for Firm B are:

$$\frac{\partial \Pi(q_1, q_2, q_3)}{\partial q_1^B} = \alpha - 2q_1^B - q_1^B - 1 + 2\gamma q_1^B = 0$$

$$\frac{\partial \Pi(q_1, q_2, q_3)}{\partial q_2^B} = \alpha - 2q_2^B - q_2^B - 1 + 2\gamma q_2^B = 0$$

$$\frac{\partial \Pi(q_1, q_2, q_3)}{\partial q_3^B} = \alpha - 2q_3^B - q_3^B - 1 + 2\gamma q_3^B = 0$$

The optimized quantities for Firm B are the following:

$$q_1^B = \frac{(1-2\gamma)(\alpha-1)}{4(1-\gamma)^2 - 1} \tag{10}$$

$$q_2^B = \frac{(1-2\gamma)(\alpha-1)}{4(1-\gamma)^2 - 1} \tag{11}$$

$$q_3^B = \frac{(1-2\gamma)(\alpha-1)}{4(1-\gamma)^2 - 1} \tag{12}$$

Notice that each firm's output is lower than the output produced if one firm operated as a monopolist. I find prices by substituting the sum of the quantities produced by each of the two firms on each of the three routes into each route's demand function. I find profits by substituting the optimized quantities and prices into each firm's profit equation.

## Scenario 3: One Transportation Firm (Indirect Service)

In this scenario, Firm B offers the same transportation service as in Scenario 1 but can only service the network by going through an intermediary stop at Y in Figure 3. This firm must maximize the following profit equation:

$$\text{Max}\Pi(q_1, q_2, q_3) = \sum_{i=1}^{3}[(\alpha - q_i)q_i] - \sum_{i=1}^{2}[(1 - \gamma(q_i + q_3))(q_i + q_3)] \qquad (13)$$

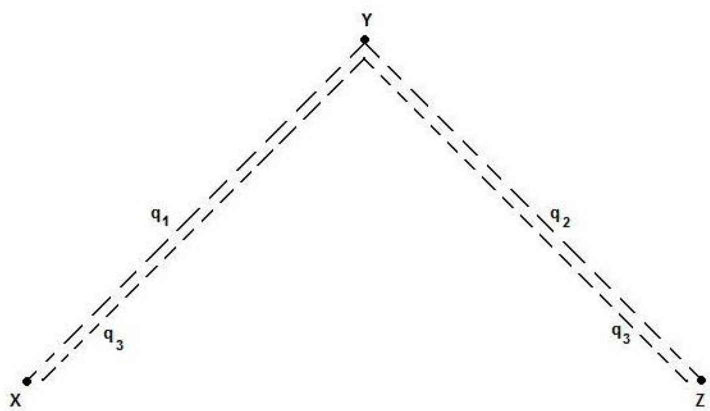


Figure 3: Monopoly - Indirect Service

The first order conditions are:

$$\frac{\partial\Pi(q_1, q_2, q_3)}{\partial q_1} = \alpha - 2q_1 - 1 + 2\gamma(q_1 + q_3) = 0$$

$$\frac{\partial\Pi(q_1, q_2, q_3)}{\partial q_2} = \alpha - 2q_2 - 1 + 2\gamma(q_1 + q_3) = 0$$

$$\frac{\partial\Pi(q_1, q_2, q_3)}{\partial q_3} = \alpha - 2q_3 - 1 + 2\gamma(q_1 + q_3) - 1 + 2\gamma(q_2 + q_3) = 0$$

Solving for $q_1, q_2$ and $q_3$ yields:

$$q_1 = \frac{\alpha - 1 + 2\gamma q_3}{2(1 - \gamma)} \qquad (14)$$

$$q_2 = \frac{\alpha - 1 + 2\gamma q_3}{2(1 - \gamma)} \qquad (15)$$

$$q_3 = \frac{\alpha - 2 + 2\gamma(q_1 + q_2)}{2(1 - 2\gamma)} \qquad (16)$$

Notice that the third equation has a "$2\gamma$" term as opposed to having only a "$\gamma$" term as is the case for direct service. This term represents the increase in economies of scale that result from servicing the network by going through the intermediary point Y. The second order condition requires that $\gamma < 1/3$. Also, note that the first order conditions for $q_1$ and $q_2$ are both functions of $q_3$ and that the first order condition for $q_3$ is a function of $q_1$ and $q_2$. These must be solved simultaneously to find a solution that is not a function of the choice variables. Doing so yields:

$$q_1 = \frac{\delta(\alpha - 1) + \gamma[(1-\gamma)(\alpha - 2) + 2\gamma(\alpha - 1)]}{2(1-\gamma)\delta} \tag{17}$$

$$q_2 = \frac{\delta(\alpha - 1) + \gamma[(1-\gamma)(\alpha - 2) + 2\gamma(\alpha - 1)]}{2(1-\gamma)\delta} \tag{18}$$

$$q_3 = \frac{(1-\gamma)(\alpha - 2) + 2\gamma(\alpha - 1)}{2\delta} \tag{19}$$

Where $\delta = [(1-2\gamma)(1-\gamma) - 2\gamma^2]$. I find prices by substituting $q_1, q_2$ and $q_3$ into each route's demand function. I find profits by substituting prices and quantities into the monopolist's profit function. Notice that there must exist some level of economies of scale. If economies of scale are too high, however, then the marginal cost curves will be too steep and will not intersect the marginal revenue curves at all.

## Scenario 4: Two Transportation Firms (Direct & Indirect Service)

In this scenario, there are the two firms, with Firm A offering direct service whereas Firm B offers indirect service as illustrated in Figure 4. The two firms must compete by servicing all points on the network but taking into account its competitor's quantity as fixed. The two firms compete in a Cournot-Oligopoly fashion. I compute reaction functions for the two firms and I find the Cournot equilibrium where the two reaction functions intersect. To maintain consumer equilibrium, I assume that passengers will not mind the intermediate stop at Y as long as they are compensated with a lower price than if they had traveled directly from X to Z. That is, consumers with higher values of time are willing to pay a higher portion of their income to travel directly to their destination and arrive earlier by choosing Firm A. Consumers with lower values of time are indifferent to the intermediate stop as long as they are compensated for the added travel time and will choose Firm B. Consumers with higher values of time will have to be compensated by a greater amount to offset the the cost for stopping at point Y. In this scenario $q_1, q_2$ and $q_3$ are now, $q_1 = q_1^A + q_1^B$, $q_2 = q_2^A + q_2^B$ and $q_3 = q_3^A + q_3^B$.

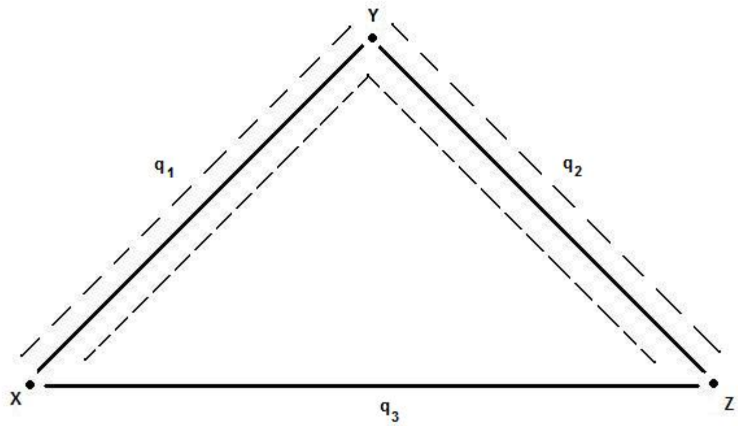


Figure 4: Duopoly - Direct & Indirect Service

Firm A maximizes the following profit equation:

$$\text{Max}\Pi(q_1, q_2, q_3) = \sum_{i=1}^{3}[(\alpha - (q_i^A + q_i^B))q_i^A] - \sum_{i=1}^{3}[(1 - \gamma q_i^A)q_i^A] \tag{20}$$

Notice that this is the same profit function as equation 1, however, now the firm must take into account Firm B's quantity as fixed.

The first order conditions for Firm A are:

$$\frac{\partial\Pi(q_1, q_2, q_3)}{\partial q_1^A} = \alpha - 2q_1^A - q_1^B - 1 + 2\gamma q_1^A = 0$$

$$\frac{\partial\Pi(q_1, q_2, q_3)}{\partial q_2^A} = \alpha - 2q_2^A - q_2^B - 1 + 2\gamma q_2^A = 0$$

$$\frac{\partial\Pi(q_1, q_2, q_3)}{\partial q_3^A} = \alpha - 2q_3^A - q_3^B - 1 + 2\gamma q_3^A = 0$$

Solving for $q_1^A, q_2^A$ and $q_3^B$ yields Firm A's reaction functions:

$$q_1^A(q_1^B) = \frac{\alpha - 1 - q_1^B}{2(1-\gamma)} \tag{21}$$

$$q_2^A(q_2^B) = \frac{\alpha - 1 - q_2^B}{2(1-\gamma)} \tag{22}$$

$$q_3^A(q_3^B) = \frac{\alpha - 1 - q_3^B}{2(1-\gamma)} \tag{23}$$

I compute the reaction functions for Firm B. This is done by maximizing the following profit equation:

$$\text{Max}\Pi(q_1, q_2, q_3) = \sum_{i=1}^{3}[\alpha - (q_i^A + q_i^B)q_i^B] - \sum_{i=1}^{2}[(1 - \gamma(q_i^B + q_3^B))(q_i^B + q_3^B)] \quad (24)$$

Notice that this is the same profit function as equation 5, however, now the firm must take into account Firm A's quantity as fixed.

The first order conditions are:

$$\frac{\partial \Pi(q_1, q_2, q_3)}{\partial q_1^B} = \alpha - 2q_1^B - q_1^A - 1 + 2\gamma(q_1^B + q_3^B) = 0$$

$$\frac{\partial \Pi(q_1, q_2, q_3)}{\partial q_2^B} = \alpha - 2q_2^B - q_2^A - 1 + 2\gamma(q_2^B + q_3^B) = 0$$

$$\frac{\partial \Pi(q_1, q_2, q_3)}{\partial q_3^B} = \alpha - 2q_3^B - q_3^A - 1 + 2\gamma(q_1^B + q_3^B) - 1 + 2\gamma(q_2^B + q_3^B) = 0$$

Solving for $q_1^B, q_2^B$ and $q_3^B$ yields the following optimized quantities:

$$q_1^B(q_1^A) = \frac{\alpha - 1 + 2\gamma q_3^B - q_1^A}{2(1-\gamma)}$$

$$q_1^B(q_2^A) = \frac{\alpha - 1 + 2\gamma q_3^B - q_2^A}{2(1-\gamma)}$$

$$q_3^B(q_3^A) = \frac{\alpha - 2 + 2\gamma(q_1^B + q_2^B) - q_3^A}{2(1-2\gamma)}$$

Just like in Scenario 3, the three functions above are functions of $q_1^B, q_2^B$ and $q_3^B$ and must be solved to find a solution that is not a function of the choice variables. Doing so yields the following reaction functions:

$$q_1^B(q_1^A, q_2^A, q_3^A) = \frac{\delta(\alpha - 1 - q_1^A) + \gamma[(1-\gamma)(\alpha - 2 - q_3^A) + 2\gamma(\alpha - 1 - (q_1^A/2 + q_2^A/2))]}{2(1-\gamma)\delta} \quad (25)$$

$$q_2^B(q_1^A, q_2^A, q_3^A) = \frac{\delta(\alpha - 1 - q_2^A) + \gamma[(1-\gamma)(\alpha - 2 - q_3^A) + 2\gamma(\alpha - 1 - (q_1^A/2 + q_2^A/2))]}{2(1-\gamma)\delta} \quad (26)$$

$$q_3^B(q_1^A, q_2^A, q_3^A) = \frac{(1-\gamma)(\alpha - 2 - q_3^A) + 2\gamma(\alpha - 1 - (q_1^A/2 + q_2^A/2))}{2\delta} \quad (27)$$

Where, once again, $\delta = [(1-2\gamma)(1-\gamma) - 2\gamma^2]$. Note that these reaction functions are identical to equations 17, 18 & 19, except that the reaction functions depend negatively on the competitor's quantity. I set the reaction functions equal to each other to find the Cournot equilibrium. Because there are six equations and six unknowns, it is much easier to use linear algebra to solve for the six quantities. Equations 21-23 and 25-27 can be written in vector form as $\mathbf{q_A} = \mathbf{D_A^{-1}}[\mathbf{C_A} - \mathbf{q_B}]$ and $\mathbf{q_B} = \mathbf{D_B^{-1}}[\mathbf{C_B} - \mathbf{B_B q_A}]$, respectively. I find the Cournot equilibrium by setting the reaction functions equal to each other, in other words, $\mathbf{q_A^*} = \mathbf{R_A}(\mathbf{q_B^*})$ and $\mathbf{q_B^*} = \mathbf{R_B}(\mathbf{q_A^*})$ Mas-Colell (1995). Doing so yields, the following optimized quantities in vector form:

$$\mathbf{q_A^*} = [\mathbf{I} - \mathbf{D_A^{-1}}\mathbf{D_B^{-1}}\mathbf{B_B}]^{-1}[\mathbf{D_A^{-1}}\mathbf{C_A} - \mathbf{D_A^{-1}}\mathbf{D_B^{-1}}\mathbf{C_B}]$$

$$\mathbf{q_B^*} = [\mathbf{I} - \mathbf{D_B^{-1}}\mathbf{B_B}\mathbf{D_A^{-1}}]^{-1}[\mathbf{D_B^{-1}}\mathbf{C_B} - \mathbf{D_B^{-1}}\mathbf{B_B}\mathbf{D_A^{-1}}\mathbf{C_A}]$$

The explicit optimized quantities for Firm A are shown by the following:

$$q_1^A = \frac{\alpha(-20\gamma^2 + 20\gamma - 3) + 24\gamma^2 - 24\gamma - 3}{(48\gamma^3 - 100\gamma^2 + 60\gamma - 9)} \tag{28}$$

$$q_2^A = \frac{\alpha(-20\gamma^2 + 20\gamma - 3) + 24\gamma^2 - 24\gamma - 3}{(48\gamma^3 - 100\gamma^2 + 60\gamma - 9)} \tag{29}$$

$$q_3^A = \frac{\alpha(-28\gamma^2 + 24\gamma - 3) + 8\gamma(3\gamma - 2)}{(48\gamma^3 - 100\gamma^2 + 60\gamma - 9)} \tag{30}$$

The explicit optimized quantities for Firm B are shown by the following:

$$q_1^B = \frac{\alpha(1 - 2\gamma)^2(2\gamma - 3) + 4\gamma^2 - 6\gamma + 3}{(48\gamma^3 - 100\gamma^2 + 60\gamma - 9)} \tag{31}$$

$$q_2^B = \frac{\alpha(1 - 2\gamma)^2(2\gamma - 3) + 4\gamma^2 - 6\gamma + 3}{(48\gamma^3 - 100\gamma^2 + 60\gamma - 9)} \tag{32}$$

$$q_3^B = \frac{(1 - 2\gamma)[9 - 10\gamma + \alpha(4\gamma^2 - 3)]}{(48\gamma^3 - 100\gamma^2 + 60\gamma - 9)} \tag{33}$$

These quantities seem to be positive with $\alpha$ and $\gamma$ but this is not necessarily the case. There are regions where $\alpha$ cannot be "too" high because the marginal revenue and marginal cost curves may intersect at negative regions making the corresponding quantities infeasible. The same applies to $\gamma$ since it measures the steepness of the marginal cost curve. A marginal cost curve that is steeper than the marginal revenue curve will yield an intersection in a negative region, or may even yield a situation where the two curves do not intersect at all. To find appropriate solutions for quantity, prices, marginal revenue, marginal cost, and profits, I perform a grid search on all the possible values of $\alpha$ and $\gamma$ combinations, which requires computer simulation.

# Numerical Example & Discussion

To illustrate these concepts I take one point centrally located within the grid and calculate quantities, prices, marginal costs, marginal revenues and profits. When $\alpha = 6.00$ and $\gamma = 0.08$, then the following table is calculated.

| | **Monopoly** | | **Duopoly** | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Direct | Indirect | Direct & Direct | | | Direct & Indirect | | |
| | | | Firm A | Firm B | Total | Firm A | Firm B | Total |
| $q_1$ | 2.72 | 2.97 | 1.76 | 1.76 | 3.52 | 2.89 | 1.97 | 4.86 |
| $q_2$ | 2.72 | 2.97 | 1.76 | 1.76 | 3.52 | 2.89 | 1.97 | 4.86 |
| $q_3$ | 2.72 | 2.95 | 1.76 | 1.76 | 3.52 | 1.80 | 1.68 | 3.49 |
| $p_1$ | 3.28 | 3.03 | - | - | 2.48 | - | - | 1.13 |
| $p_2$ | 3.28 | 3.03 | - | - | 2.48 | - | - | 1.13 |
| $p_3$ | 3.28 | 3.05 | - | - | 2.48 | - | - | 2.51 |
| $MR_1$ | 0.57 | 0.05 | 2.48 | 2.48 | - | -1.76 | -0.83 | - |
| $MR_2$ | 0.57 | 0.05 | 2.48 | 2.48 | - | -1.76 | -0.83 | - |
| $MR_3$ | 0.57 | 0.11 | 2.48 | 2.48 | - | 0.71 | 0.83 | - |
| $MC_1$ | 0.57 | 0.05 | 0.72 | 0.72 | - | 0.54 | 0.42 | - |
| $MC_2$ | 0.57 | 0.05 | 0.72 | 0.72 | - | 0.54 | 0.42 | - |
| $MC_3$ | 0.57 | 0.11 | 0.72 | 0.72 | - | 0.71 | 0.83 | - |
| $\Pi$ | 20.38 | 20.76 | 17.03 | 17.03 | - | 5.12 | -12.17 | - |

The table indicates that both monopolists do well under this combination of parameter values. The monopolist offering indirect service, does slightly better by offering more quantity and lower prices, yet still achieving higher profits than the monopolist offering direct service. The duopolists offering direct service also seem to perform well by offering more total traffic at lower prices, however neither of the two firms perform as well when it comes to profits. Each has slightly lower profits than the two monopolists. When analyzing the duopoly direct and indirect scenario, the two firms seem to perform poorly. Both firms experience negative marginal revenues on $q_1$ and $q_2$ and Firm B operates at a loss. In addition, negative marginal revenues indicate that the firms may be offering too much service on the network. I perform a grid search to find if there are any other parameter values that yield healthier numbers, but the grid suggests that no other $\alpha$, $\gamma$ combination does. Either quantities, marginal revenue, marginal costs, prices, profits, or a combination of the above are consistently negative.

Does this suggest that the model is incorrect? Not necessarily. Intuition indicates that there may be too much traffic offered on the network, and thus possibly corner solutions exist, especially on $q_1$ and $q_2$. The two firms may have to re-route and re-optimize their profit function. Doing so may yield higher profits for each. If the two firms were to re-route, the most obvious routing for two firms would be for each firm to monopolize and offer service on the routes that have the lowest marginal costs. From the table above, Firm A will most likely earn higher profits by offering service on $q_3$, since it has the lowest marginal cost on this route, and no service on $q_1$ and $q_2$. Analogously, Firm B will earn higher profits by offering service on $q_1$ and $q_2$, since it has the lowest marginal costs on these routes, with no service on $q_3$. Offering no service on $q_3$ may sound enigmatic, but this occurs when passengers are required to purchase two tickets to a particular destination without any form of transfer or discount. Passengers must purchase two tickets as if he or she were purchasing two goods. Offering $q_3$ service, on the other hand, implies that passengers can purchase a "transfer" ticket at a slightly discounted rate, for example. The scenario just described may look like the one illustrated in Figure 5. Another possible scenario would be for both firms to even out traffic on the network and perform as a duopolist, each splitting the market offering direct service on the network. This would lead both firms to Scenario 2 described earlier. Such a scenario, however, will depend on each firm's level of aggressiveness.

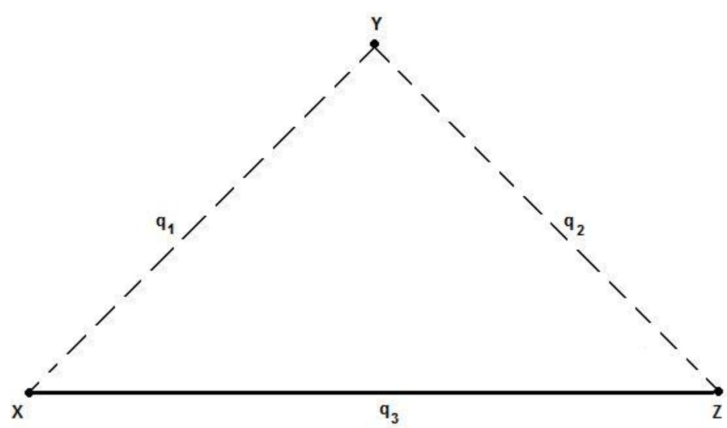


Figure 5: Two Monopolies - Direct & Indirect Service

If, however, Firm B begins to experience strong economies of scale, it may threat to start offering service on $q_3$, forcing Firm A out of the market. The grid suggests that such a scenario is possible at high levels of $\gamma$. This would lead back to Scenario 2, where the monopolists with the higher economies of scale, is able to take advantage of its routing setup and offer the most service at lower prices, without threat of entry from other firms. Such a scenario is the quintessential example of a natural monopoly. The natural monopolist can take advantage of both its economies of scale and economies of scope to offer service on the entire network.

The scenarios described above are too simplistic to put forth against data. The scenarios ignore differences in speed, travel times, preferences and operating frequencies. However, it helps illustrate how the "cost-saving" nature of an indirect network is more sensitive to the level of economies of scale. These types of routing setups are commonly seen in buses and airlines. For example, no two bus companies offer the exact same routes as each other. If they did, buses may run empty through these areas. Buses need to operate in areas where they can have a slight monopoly on service to be profitable. Of course, in high density areas, such as in a central business district, this may not be the case since there will always be a sufficient demand for service.

# Conclusion

In the first stage of the game, prices do fall below Duopoly Direct-Direct Service. However, if the two firms were to play a second stage, and I allow the firms to choose which routes to provide service on based on positive profits, it is highly possible that each firm will choose to offer service on the route that yields each monopoly power and thus positive profits. As economies of scale become stronger and stronger, this may lead to strong economies of scope and force one firm out of the market. Having only one firm, instead of two firms, will likely yield lower prices, higher quantities and higher profits Bailey and Friedlander (1982).

A possible extension to this paper may be to calculate a fifth scenario and find out how the two firms will compete in a duopoly fashion with both firms offering Indirect-Indirect service. I predict that each firm will provide this type of service as long as profits are positive, demand is high enough and when there are sufficient economies of scale present. I would need to calculate the best response functions and perform the appropriate grid search to find out if this scenario is actually feasible. It may turn out that this case may or may not yield profits for the two firms and the network may or may not be better off if only one firm provides service. There may be sufficient economies of scale and scope to justify a monopolist Braeutigam (1999).

# References


Bailey, E. E. and A. F. Friedlander (1982). Market structure and multiproduct industries. *Journal of Economic Literature 10*.

Braeutigam, R. R. (1999). *Essays In Transportation Economics and Policy*. Brookings.

Brand, D. and M. Keifer. Independent ridership and passenger revenue projects for high speed rail alternatives in california. *California High Speed Rail Authority*.

Bruckner, J. and W. Whalen (2007). The price effects of international airline alliances. *Journal of Law and Economics* (43).

Brueckner, J. and P. Spiller (1991). Competition and mergers in airline networks. *International Journal of Industrial Organization*.

Givoni, M. and D. Banister. Airline and railway integration. *Transport Policy 13*.

John Kay, A. M. and S. Szymanski. The economic benefits of the channel tunnel. *Economic Policy 4*.

Mas-Colell, A. e. a. (1995). *Microeconomic Theory*.