An Empirical Study of Contest Success Functions — Evidence from the NBA

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1 Introduction

Sporting competitions are one of the most significant branches of the entertainment industry today. According to an official statistic in 1997, $14 billion in direct income was generated in U.S. by spectator sports in that year, which was about 0.17% of the U.S. GDP. The annual attendance at spectator sports in the same year was equal to 41% of the adult population, while annual household television viewing hours ran into the hundreds of millions. Analyzing the influential sports games and designing an optimal contest is both a matter of significant financial concern for the organizers, participating individuals and teams, and a matter of catering to the personal interest for millions of fans. Due to this reason, economic analysis and design of sport competition has gained more and more attention in recent years (see e.g. Lazear and Rosen, 1981; Szymanski, 2003; Blavatskyy, 2004).

This paper focuses on one of the most influential sporting contests, the National Basketball Association (NBA). From its inception as a league in 1949, the NBA had grown from a small league garnering little interest from the American public to a global juggernaut responsible for making basketball into the fastest growing sport in the world. Starting as a league with only eight teams, it has grown to 30 teams and continues to expand. As a fan, I hope to, through this study, gain more insight into the off-court behaviors of franchises in the NBA, and how that may affect on-court success. In addition, the method developed in this study can be extend to further research on other sports.

Many scholars classify sporting contests into two categories, individual sports and team sports, and emphasize the difference between them. The distinction between individual sports (such as chess, tennis, and golf) and team sports (such as basketball, football, and soccer) is that in the latter the outcome of a match depends on the interaction of efforts among the players on the same team whereas in the former there are no team production issues. Thus in team activities output depends not only on the marginal productivity of each player, but also on the change of player marginal productivity with respect to the effort of other team members, and this adds an extra dimension to the analysis of marginal productivity. However, the significance of this effect may vary from one sport to another. In many team sports, such as baseball and cricket, team members' marginal productivities are almost entirely independent. Even where interactions are more important, the economic implications are unclear and their effects uninvestigated. Many scholars (e.g. Szymanski, 2003) argue that the significance of sports team production function is neglectable because if interaction terms were truly of economic significance in some team sports, one might expect to see players offering themselves to the market as partnerships, as happen, for instance, with teams of bond traders or teams of consultants. Even in team sports where the labor market is open to such possibility (e.g. soccer, rugby, and cricket), players partnerships are barely known. There exists a substantial empirical literature concerning the estimation of sports team production functions (see e.g. Kahn, 1993). And I leave this issue to my future study.

A natural way to model the sporting competitions is using the tools developed in contests theory. A contest is a game in which contestants (agents) compete to capture a prize. Contestants exert some levels of effort presumably at the cost to enhance their chance to receive the prize. A key component of a contest is a probabilistic choice function that translates an individual's effort into her probability of winning. It is typically known as the Contest Success Function (CSF), which depends on both the effort contribution of the contestants (players) and their inherent abilities. The CSF differs fundamentally from previous studies which treat sporting contests as auctions, where the highest bidder wins with probability one. Here, the technology does not discriminate perfectly between effort levels and the highest bidder can only be certain of winning if all other contestants contribute no effort at all. This imperfect discriminating, yielding uncertainty of outcome, is one of the most attractive features of sporting competition itself.

Being a convenient tool to model contests, CSFs have been extensively used to study such activities as tournaments (e.g., Lazear and Rosen, 1981), political rent-seeking (e.g., Nitzan, 1991, 1994), patent races (e.g., Nti, 1997), etc. Two widely functional forms in rent-seeking contests were introduced in Tullock (1980) and McFadden (1984). Some of the properties of these functional forms as well others were analyzed by Hirshleifer (1989). Skaperdas (1996) introduced the axiomatic approach in deriving several classes of CSFs. An alternative approach, the stochastic approach, was developed by Jia (2005), which provides a helpful micro-foundation of the CSFs. A more flexible and complicated functional form of CSF, which is known as persuasion function, was recently discovered by Skaperdas and Vaidya (2005). It allows asymmetry among contestants and in some sense incorporates the team production processes into contest models.

The variety of functional forms of CSFs also leads some problems. Because of their different analytic properties, different functional forms of CSFs can easily give researchers whatever they want from their models. For instant, for some utility functions, the Tullock form (generalized ratio form) gives an interior equilibrium while the logit form (exponential functional form) yields a corner solution. This problem becomes especially severe in empirical study. How to choose the appropriate functional form, therefore, is a question of both theoretical and empirical importance.

The aim of my research is to use standard Bayesian model selection method to compare three extensively used functional forms of CSFs, the generalized ratio form, the logit form, and the probit form by using the NBA data. These three functional forms have been employed in literature without any particular reason other than analytical convenience. An empirical study, therefore, is necessary to develop a better understanding of any advantages or limitations of each functional form of the CSFs.

For a context among n teams, team i invests some effort level x_i to enhance its probability of winning. Let $\{x_i\}_{i=1}^n$ be the effort vector, m be the noise parameter, and $\{c_i\}_{i=1}^n$ be the characteristic vector with $c_j \geq 0$, $j = 1, \cdots, n$. The generalized ratio form is given by

$$F_i(x_i, x_{-i}) = \frac{c_i x_i^m}{\sum\limits_{j=1}^{n} c_j x_j^m}. \tag{1}$$

With $c_i = c_j$ for any $i \neq j$, (1) is the famous Tullock form of CSF (see Tullock, 1980). Since the NBA games are played between two teams, the generalized ratio form (1) can be further simplified as

$$F_i(x_i, x_j) = \frac{c_i x_i^m}{c_i x_i^m + c_j x_j^m}. \tag{2}$$

The noise parameter is not well-defined in the logit CSFs, i.e., each team has distinct m_i's. The logit functional form is given by

$$F_i(x_i, x_{-i}) = \frac{c_i \exp(m_i x_i)}{\sum_{j=1}^{n} c_j \exp(m_j x_j)}. \tag{3}$$

Correspondingly, for the NBA data, the last expression can be simplified as

$$F_i(x_i, x_j) = \frac{c_i \exp(m_i x_i)}{c_i \exp(m_i x_i) + c_j \exp(m_j x_j)}. \tag{4}$$

It is easy to see that equation (2) can be rewritten as

$$F_i(x_i, x_j) = \frac{c_i \exp[m \ln(x_i)]}{c_i \exp[m \ln(x_i)] + c_j \exp[m \ln(x_j)]}, \tag{5}$$

which corresponds to a logit functional form with resource vector $\{\ln(x_i)\}_{i=1}^n$ and constraint $m_i = m$ for every i. Intuitively, the generalized ratio form will fit the data more poorly than the logit form due to the constraint, however, how does the nonlinear transformation $\{x_i\}_{i=1}^n \rightarrow \{\ln(x_i)\}_{i=1}^n$ affect the goodness-of-fit is ambiguous. Therefore, comparing these two forms against the data is still necessary before any conclusion being drawn.

Both the generalized ratio form and the logit form implicitly assume the contests satisfy the "Independence of Irrelevant Alternatives" property (see Skaperdas, 1996). This assumption may not be supported by the NBA data. The probit functional form accommodates contests which violate the "Independence of Irrelevant Alternatives"assumption, which has the form

$$F_i(x_i, x_j) = \Phi[s_{ij}(x_i, x_j)], \tag{6}$$

where Φ is the standard normal distribution function and s_{ij} is a linear function of x_i and x_j.

One potential difficulty is that efforts are not directly observable. In general, the only observable data is contestants' performances (output of the efforts) rather than efforts themselves. Performances are assumed to be functions of the contestants' efforts. The discrepancies between efforts and performances could be caused by some exogenous uncertainties, and these underlying uncertainties can bring about the randomness of the contests.

To overcome this difficulty, in this paper, I use the salary-on-court of each game as an index to capture team's effort level of that game. The amount of the ith team's salary-on-court of a certain game (x_i) is given by

$$x_i = \sum_{j=1}^{J(i)} w_{i,j} t_{i,j}, \tag{7}$$

where $J(i)$ is the total number of players present in that game, $w_{i,j}$ is the jth player's wage rate (dollar per minute), and $t_{i,j}$ is the time he players (minute). I suggest this index because of two reasons. First, intuitively, if a team wants to exert more effort in a certain match, it will send its best players into the court, which corresponds to a greater amount of salary-on-court in that game. Second, if a match is unimportant to a team, it is more likely that team use backup players to protect its best players from injury, which implies a lower salary-on-court in that game.

There are two legitimate concerns about the validity of this index. One is the possible distortion caused by the salary cap implemented by the NBA in 1984, the other is the long-term contracts between players and teams, which were often assigned before a season, may not capture the abilities of these players. However, the significance of these two effects may not be all that great. My argument goes as follows.

1. A "salary cap"is a payroll or cost-of-labor cap. The managers of the various franchises meet and agree to inhibit the labor market by refusing to spend more than a certain, pre-determined amount on payrolls, often less than some franchises already dole out. There are two broad types of caps: the "soft"cap and the "hard"cap. The NBA features a soft cap, which contains exceptions under which a team can be over. The current NBA cap was adopted in 1984-85. It limits payrolls of NBA teams to 57% of Basketball Related Income for the league. Basketball Related Income (BRI) requires some complicated math but it includes revenue from tickets, advertising, local concession and souvenir sales, local media and other league, team and arena income streams. In 2001-02 the cap was set at $42.5 million. Two teams, Detroit and the LA Clippers were under the cap; Detroit by $800,000 and the Clippers by $8.8 million. The 28 other teams were all over the cap, 16 teams by more than $10 million, and 2 teams by more than $40 million (Portland and the New York Knicks). The reason teams were able to exceed the cap by so much is because of the many exceptions (including the famous "Larry Bird Exemption") apply to players who do not exercise their free agent rights or who remain with the same team for more than a couple of years. The NBA explains that

   > The basic idea is to try to promote the ability for players to stay with their current team. Nobody likes it when a player plays with a team his entire career, the fans love him, he wants to stay and the team wants to keep him, but he has to leave because the team is unable to offer him a large enough contract.(`www.nba.com`)

   The NBA also has a "luxury tax"in effect. If average salaries league-wide for a season go over the agreed upon limit (55% of BRI) then there is an escrow fund (composed of 10% of player salaries) held back from players' paychecks that is reimbursed equally to franchises (thereby reducing the amount teams spend on player salaries) until the 55% limit is restored. If the 10% escrow fund is not sufficient, then the highest spending teams (the ones assumed responsible for the salaries being so far over the limit) are taxed the remaining amount to reimburse the other owners. Player salaries are also capped depending on tenure in the league, but there are so many exceptions to this rule that it rarely has a major impact on the salaries of star players, the ones the true cap on salaries is supposed to affect. Obviously the NBA salary cap and luxury tax structure are not overly constraining and therefore qualify as a soft cap.

2. An easy regression analysis shows the player's actual game statistics contributed to their salaries. This is because a player's statistics are a reflection on what he is accomplishing on the job, just as in any other business field, a worker's salary is dependent on her performance. I used basic statistics such as minutes per game and points per game to determine if their wages could be explained by their actual talents.

   In order to check if a player's statistics have a positive affect on his salary, I used minutes per game, assists per game, rebounds per game, and points per game as explanatory variables. I believe that minutes per game is important in the regression because the more the athlete earns, the more important is his position and hence the more minutes the athlete should play. Points per game, rebounds, and assists are included because they are valid measures of the productivity of that player to the team.

   A simple regression gives the following result:

   $$Y = -0.716 - 0.425A - 0.0756M + 0.742P + 0.0536R, \quad (8)$$

   where A, M, P, and R are assists, minutes, points, and rebounds per game, respectively. And salary Y is measured in millions. By performing individual tests, I found that the only variable that has a significant positive affect on salary is points per game, which directly relates to the outcome of a game. Further hypothesis testing shows that the model is significant at 95% confident level and the adjusted R-square equals to 46%. This result shows that individual player's salary is consistent with his measurable statistics, or his efforts. Therefore, salary-on-court does capture the team's efforts in a certain match.

2 Empirical Analysis

My empirical analysis proceed in stages. First, I estimated equations (2), (3), and (6) using the NBA data. Next, I compare these three models by compute their marginal likelihoods and the corresponding Bayes factors. Finally, I average these three models with respect to the Bayes factors and "predict"the NBA game results of 2004-2005 season, which provides another verification of the goodness-of-fit of my estimation.

Equations were estimated in the form

$$s_{ij} = (\beta_0^i + \beta_1^i soc_i) - (\beta_0^j + \beta_1^j soc_j) + \beta_2^i HA + \epsilon_{ij}, \tag{9}$$

where s_{ij} is the relative cost team i spends versus team j, HA is a dummy variable that indicates the game was played home or away for team i, soc_i and soc_j are salary-on-court for team i and team j respectively, and ϵ_{ij} is a random error term. The β_0^i's capture the fixed effect of each team. For identification purpose, I set $\beta_0^{Atlanta}$ equal to zero. If the theory of contests is correct, higher salary-on-court should lead to higher probability of winning for team i and lower team j's winning percentage; hence estimates of β_1^i and β_1^j should be both positive. If any home advantage exists, β_2^i should also be positive.

2.1 Data Description

Data are available at `http://www.basketball-reference.com`, `http://www.dfw.net/\%7epatricia/index.html`, and the NBA' s official website `http://www.nba.com` for each game from 1991-1992 season to 2003-2004 season on the scores, minute played, and salaries for all players. The reason the data of 2004-2005 and 2005-2006 seasons been exclude from my estimation is that I want to further check the accuracy of my predictions against these data.

From 1991-1992 season to 2003-2004 season, 29,066 NBA games had been played, including pre-season, regular season, playoffs and off-season. With 292,651 pieces of game statistics information and 5,827 pieces of salary data been collected from the internet, I am able to compute the salary-on-court index and game result for every game. For the game statistics, the accuracy is guaranteed by the NBA; while cautions must be taken about the salary data, although they have been claimed to be "as accurate as possible"by their providers, because they assign minimum salaries to all players without salary data in 1991 through 2004. The average salary-on-court indexes of all the NBA teams from 1991-92 season to 2003-04 season are listed in Table 1, which helps the readers to gain some basic idea of the magnitude of soc's.

2.2 Estimation

Estimates are reported in table 2, 3, and 4 for the generalized ratio form, logit form, and probit form respectively.

As seen in the tables, more salary-on-court invested, as measured by *soc*, is seen to lead to higher probability of winning. Furthermore, most elements of the coefficient $\{\beta_1^i\}_{i=1}^{29}$ corresponds to t-values greater than 2, which indicates the coefficient β_2 is statistically significant. As expected, the home-advantage effect, as measured by $\{\beta_2^i\}_{i=1}^{29}$'s, does exist, although some of them are not statistically significant at 95% confidence level. All teams

Teams	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
Atlanta Hawks	91879	113500	121800	125810	148220	223560	276850	281880	299290	217260	246750	421030	262840
Boston Celtics	117800	131670	130250	114380	132720	91154	119840	211630	261090	288260	383770	347660	229150
Charlotte Hornets(NO)	73025	103540	101840	140640	138250	171170	177860	177770	245320	272080	231000	216030	270880
Cleveland Cavaliers	118610	148510	151260	135350	130680	117610	178630	206430	252060	171910	237180	235860	232540
Chicago Bulls	124440	117220	104280	127810	444100	454870	483680	134280	103810	161020	203730	253600	220180
Dallas Mavericks	78901	51195	90734	124910	120730	132840	198900	244430	249230	337620	435050	402920	503680
Denver Nuggets	74092	91256	116850	113470	138460	122410	92703	262900	261050	292450	245620	228270	211370
Detroit Pistons	97117	87371	75457	139460	179360	195820	177220	152690	230210	204480	274890	256350	235040
Golden State Warriors	80301	90465	81563	98093	180030	200240	139890	164970	166070	198750	295540	257380	245890
Houston Rockets	98348	100530	118450	150260	166960	191910	155370	254030	222580	281650	209000	336530	328770
Indiana Pacers	99757	97789	128520	152290	260930	229610	259910	256460	396420	377000	344360	314300	321000
Los Angeles Clippers	82831	128450	109580	99536	128680	113500	107870	105680	116540	138090	175440	251780	219670
Los Angeles Lakers	86292	122010	142650	129360	160740	199510	239290	222900	421440	421810	437130	473430	417370
Miami Heat	80276	101030	120470	119910	151060	217260	248900	281230	366650	387190	403580	234480	339050
Milwaukee Bucks	72929	69809	74164	124310	154680	179660	163740	140790	342530	371120	364400	337790	203100
Memphis Grizzlies(VAN)	0	0	0	0	81124	96985	175660	119790	234610	250090	188110	208070	262630
Minnesota Timberwolves	58803	93702	104830	111180	134600	142420	239520	196530	343510	388070	406110	435570	517390
New Jersey Nets	91508	107560	138800	131470	114920	123090	216030	179870	323430	245550	332110	310030	287590
New York Knickerbockers	81150	114830	121780	288760	138450	394490	296790	384840	484850	489500	454660	467100	323160
Orlando Magic	52200	110890	166620	203930	295140	272180	202660	257920	97910	229570	246030	252240	212610
Philadelphia 76ers	112060	96549	103760	118200	83536	164380	184540	187560	290760	345330	370800	390860	287520
Portland Trail Blazers	102050	139090	141690	150180	145810	182810	301580	332950	512160	567320	611100	396860	368930
Phoenix Suns	67733	111960	168340	206620	204930	202310	235350	245250	307560	242620	347880	407910	283160
Seattle Supersonics	88111	85318	150110	172070	221170	233880	230480	206060	344560	383700	327270	307320	280000
Sacramento Kings	72255	79841	108960	131180	133600	156510	99384	198280	286840	354930	407260	438570	415180
San Antonio Spurs	108540	113460	154920	187340	211660	112930	241990	258380	342190	309940	359800	307820	251390
Toronto Raptors	0	0	0	0	73670	125930	96724	178790	191740	287820	348560	182880	302820
Utah Jazz	92662	110390	115210	117440	170960	184930	192570	201920	365120	371230	340380	348580	116120
Washington Wizards	60948	75510	81320	130030	175290	305460	265220	204920	382500	258160	147270	199110	207890

Table 1: Average soc's($)

Table 2: Estimation Results (Generalized Ratio Form of CSF)

	Teams	β_0(t-value)	m(t-value)
1	Atlanta Hawks	1(N/A)	1.2701(41.036)
2	Boston Celtics	0.9244(-1.2279)	1.2701(41.036)
3	Chicago Bulls	1.1418(2.0389)	1.2701(41.036)
4	Cleveland Cavaliers	1.1447(2.1193)	1.2701(41.036)
5	Dallas Mavericks	0.8075(-3.2477)	1.2701(41.036)
6	Denver Nuggets	0.7933(-3.4872)	1.2701(41.036)
7	Detroit Pistons	1.3191(4.3495)	1.2701(41.036)
8	Golden State Warriors	0.9607(-0.6093)	1.2701(41.036)
9	Houston Rockets	1.4856(6.0500)	1.2701(41.036)
10	Indiana Pacers	1.1759(2.5445)	1.2701(41.036)
11	Los Angeles Clippers	0.9765(-0.3538)	1.2701(41.036)
12	Los Angeles Lakers	1.5810(6.9250)	1.2701(41.036)
13	Memphis Grizzlies	0.6247(-6.0437)	1.2701(41.036)
14	Miami Heat	1.1840(2.6448)	1.2701(41.036)
15	Milwaukee Bucks	0.9965(-0.0550)	1.2701(41.036)
16	Minnesota Timberwolves	0.8787(-1.9779)	1.2701(41.036)
17	New Jersey Nets	0.9892(-0.1699)	1.2701(41.036)
18	Oklahoma City Hornets	1.4918(6.1546)	1.2701(41.036)
19	New York Knickerbockers	1.0796(1.1808)	1.2701(41.036)
20	Orlando Magic	1.2300(3.2105)	1.2701(41.036)
21	Philadelphia 76ers	0.8216(-3.0647)	1.2701(41.036)
22	Phoenix Suns	1.4343(5.5073)	1.2701(41.036)
23	Portland Trail Blazers	1.0297(0.4454)	1.2701(41.036)
24	Sacramento Kings	1.2192(3.0369)	1.2701(41.036)
25	San Antonio Spurs	1.8875(9.5397)	1.2701(41.036)
26	Seattle Supersonics	1.6228(7.3549)	1.2701(41.036)
27	Toronto Raptors	0.9750(-0.3453)	1.2701(41.036)
28	Utah Jazz	2.1818(11.690)	1.2701(41.036)
29	Washington Wizards	0.7570(-4.2783)	1.2701(41.036)

Table 3: Estimation Results (Logit Functional Form of CSF)

	Teams	β_0(t-value)	$\beta_1 \times 10^6$(t-value)	β_2(t-value)
1	Atlanta Hawks	0(N/A)	2.1220(4.4597)	0.43654(4.0333)
2	Boston Celtics	-0.6624(-4.3482)	4.8345(9.9847)	0.30827(2.8837)
3	Chicago Bulls	-0.3446(-2.3393)	4.2523(11.784)	0.27854(2.4938)
4	Cleveland Cavaliers	0.1639(0.8407)	1.3862(1.5489)	0.53642(4.9080)
5	Dallas Mavericks	-1.6875(-11.177)	8.2934(22.603)	0.28295(2.4631)
6	Denver Nuggets	-1.0124(-6.2730)	5.0949(8.3888)	0.66987(6.1139)
7	Detroit Pistons	-1.2071(-6.8932)	9.8971(13.386)	0.38962(3.5747)
8	Golden State Warriors	-0.3879(-2.3480)	3.1093(4.6370)	0.35499(3.2610)
9	Houston Rockets	0.0537(0.3280)	3.6872(6.6351)	0.39558(3.5300)
10	Indiana Pacers	-0.2750(-1.6813)	4.2841(9.1881)	0.52780(4.7090)
11	Los Angeles Clippers	-1.4001(-7.8922)	9.8323(10.662)	0.37652(3.4134)
12	Los Angeles Lakers	-0.3097(-2.0545)	5.5364(15.921)	0.37316(3.1925)
13	Memphis Grizzlies	-2.6055(-10.943)	12.620(12.633)	0.33759(2.3497)
14	Miami Heat	-0.4667(-3.0778)	4.9218(12.060)	0.47157(4.2830)
15	Milwaukee Bucks	-1.1080(-7.4585)	6.9318(16.214)	0.43125(3.9422)
16	Minnesota Timberwolves	-1.5259(-10.141)	7.7803(22.430)	0.34615(3.0549)
17	New Jersey Nets	-0.9046(-5.7803)	6.2630(12.318)	0.46442(4.2914)
18	Oklahoma City Hornets	-0.6456(-3.7976)	7.2871(10.345)	0.50355(4.3863)
19	New York Knickerbockers	0.5018(3.2920)	1.3091(4.2607)	0.46383(4.0738)
20	Orlando Magic	-0.7044(-4.1691)	6.3884(10.693)	0.46279(4.1802)
21	Philadelphia 76ers	-1.4818(-9.7782)	8.2148(19.078)	0.18385(1.6684)
22	Phoenix Suns	0.3565(2.1694)	2.4137(5.0567)	0.49930(4.3612)
23	Portland Trail Blazers	0.1652(1.1198)	2.3227(8.4826)	0.35492(3.1787)
24	Sacramento Kings	-1.2569(-8.4253)	8.2173(21.183)	0.64776(5.6364)
25	San Antonio Spurs	-0.5167(-2.9716)	7.6201(13.509)	0.48393(4.0155)
26	Seattle Supersonics	0.6831(3.9810)	1.6181(3.1372)	0.51457(4.3969)
27	Toronto Raptors	-1.2083(-6.7474)	7.4578(12.549)	0.20627(1.5382)
28	Utah Jazz	0.5158(3.2824)	3.1100(6.6605)	0.62141(5.0896)
29	Washington Wizards	-0.8263(-5.4147)	4.2476(9.0332)	0.37165(3.4204)

Table 4: Estimation Results (Probit Functional Form of CSF)

	Teams	β_0(t-value)	$\beta_1 \times 10^6$(t-value)	β_2(t-value)
1	Atlanta Hawks	0(N/A)	1.2512(4.3061)	0.26821(4.0475)
2	Boston Celtics	-0.4105(-4.3851)	2.9102(9.8698)	0.18906(2.8750)
3	Chicago Bulls	-0.2255(-2.4939)	2.6112(12.039)	0.17662(2.5775)
4	Cleveland Cavaliers	0.1038(0.8649)	0.7630(1.398)	0.33187(4.9560)
5	Dallas Mavericks	-1.0160(-11.166)	4.9195(23.351)	0.17533(2.5437)
6	Denver Nuggets	-0.6209(-6.3145)	3.0873(8.4436)	0.40514(6.0881)
7	Detroit Pistons	-0.7357(-6.8569)	5.9670(13.241)	0.23729(3.5602)
8	Golden State Warriors	-0.2431(-2.4004)	1.8816(4.6350)	0.21752(3.2731)
9	Houston Rockets	0.0284(0.2832)	2.2112(6.5655)	0.24060(3.5454)
10	Indiana Pacers	-0.1651(-1.6397)	2.5630(9.0032)	0.32541(4.7885)
11	Los Angeles Clippers	-0.8591(-7.9644)	5.9680(10.717)	0.22680(3.3917)
12	Los Angeles Lakers	-0.1749(-1.9015)	3.2708(16.088)	0.22320(3.2013)
13	Memphis Grizzlies	-1.5339(-11.227)	7.4283(12.965)	0.19550(2.2963)
14	Miami Heat	-0.2843(-3.0454)	2.9445(11.911)	0.29116(4.3481)
15	Milwaukee Bucks	-0.6819(-7.5065)	4.2041(16.258)	0.25983(3.8987)
16	Minnesota Timberwolves	-0.9242(-10.133)	4.6447(22.921)	0.21003(3.0985)
17	New Jersey Nets	-0.5546(-5.7766)	3.7732(12.202)	0.28526(4.3032)
18	Oklahoma City Hornets	-0.3852(-3.6959)	4.3364(10.159)	0.31176(4.4517)
19	New York Knickerbockers	0.3104(3.3219)	0.7526(4.0327)	0.28227(4.1258)
20	Orlando Magic	-0.4278(-4.1515)	3.8372(10.658)	0.28427(4.2167)
21	Philadelphia 76ers	-0.9073(-9.8450)	4.9608(19.258)	0.11469(1.7141)
22	Phoenix Suns	0.2196(2.1832)	1.4216(4.9039)	0.30260(4.4050)
23	Portland Trail Blazers	0.1061(1.1733)	1.3607(8.2820)	0.21456(3.1826)
24	Sacramento Kings	-0.7584(-8.3663)	4.8833(21.650)	0.39378(5.6927)
25	San Antonio Spurs	-0.2928(-2.7742)	4.4953(13.517)	0.28593(4.0299)
26	Seattle Supersonics	0.4220(4.0549)	0.9206(2.9810)	0.31150(4.4739)
27	Toronto Raptors	-0.7384(-6.8108)	4.4920(12.666)	0.11879(1.4657)
28	Utah Jazz	0.3175(3.3278)	1.8271(6.5427)	0.37673(5.2441)
29	Washington Wizards	-0.5042(-5.3944)	2.5702(8.9471)	0.22257(3.3730)

are favored in their home fields. For the β_0's, most estimates are statistically significant. It indicates that teams are heterogeneous even after we controlled the salary-on-court and Home/Away(HA) variables.

The most interesting implication is the marginal effects of the salary-on-court indexes and the HA variables, which are reported in Table 5. As seen in the table, $10,000$ dollars extra expenditure increases teams winning chance by 0.3%-2.97%, which suggests that some teams, e.g., Memphis Grizzlies, Los Angeles Clippers, and Detroit Pistons, may be remarkably benefited from hiring more expensive players; while the same strategy may not improve the performances of teams like New York Knickerbockers, Cleveland Cavaliers, and Seattle Supersonics. Meanwhile, playing games home increases teams' winning chance by 0.05%-0.15%. This number seems to be small. However, it may cause huge differences when two very closed teams play against each other.

Let

$$\alpha_{i,t} = \hat{\beta}_0^i + \overline{soc_{i,t}}\hat{\beta}_1^i + 0.5\hat{\beta}_3^i, \tag{10}$$

where $\hat{\beta}_j^i$, $j = 1, 2, 3$, are the estimates of the coefficients of team i and $\overline{soc_{i,t}}$'s are the average soc indexes of team i in season t. The 0.5 comes from the average of the HA_i dummies. Looking at $\alpha_{i,t}$'s should be able to tell us the ranking of team i in season t, i.e., for a given t_0, teams' standings will be revealed by comparing $\alpha_{i,t0}$ across all i's. Although a total ranking of the NBA teams is unavailable, I can pick the 16 teams with the 16 largest $\alpha_{i,t}$'s and compare them with the teams that enter the playoffs. Ideally, the estimated results will match the real data perfectly. However, my matching suggests that the $\alpha_{i,t}$'s are more or less distorted by the tournament structure of the NBA games. The NBA divides its franchise into two conferences. And both Eastern and Western conferences consist of two divisions. Only the four best teams of each division enter the playoffs. Hence a team may not enter the playoffs if its division standing is poor in spite of having a better performance than

Table 5: Marginal Effects (%)

	Teams	soc($\times 10^4$)	HA
1	Atlanta Hawks	0.50	0.10
2	Boston Celtics	1.15	0.07
3	Chicago Bulls	1.04	0.07
4	Cleveland Cavaliers	0.30	0.13
5	Dallas Mavericks	1.95	0.07
6	Denver Nuggets	1.23	0.15
7	Detroit Pistons	2.36	0.09
8	Golden State Warriors	0.75	0.08
9	Houston Rockets	0.88	0.09
10	Indiana Pacers	1.02	0.12
11	Los Angeles Clippers	2.37	0.09
12	Los Angeles Lakers	1.30	0.09
13	Memphis Grizzlies	2.94	0.08
14	Miami Heat	1.17	0.11
15	Milwaukee Bucks	1.67	0.10
16	Minnesota Timberwolves	1.84	0.08
17	New Jersey Nets	1.49	0.11
18	Oklahoma City Hornets	1.72	0.12
19	New York Knickerbockers	0.30	0.11
20	Orlando Magic	1.51	0.11
21	Philadelphia 76ers	1.97	0.04
22	Phoenix Suns	0.57	0.12
23	Portland Trail Blazers	0.54	0.08
24	Sacramento Kings	1.93	0.15
25	San Antonio Spurs	1.79	0.11
26	Seattle Supersonics	0.36	0.12
27	Toronto Raptors	1.78	0.05
28	Utah Jazz	0.73	0.14
29	Washington Wizards	1.02	0.09

another team which enters. Due to this structural distortion, the further a team goes, the less consistent the estimates get. Nonetheless, my estimates of $\alpha_{i,t}$'s still work fairly well. For example, in 13 consecutive seasons, eight times my estimated playoffs teams differ from the real results by less than three teams, three times they differ by three teams, and twice by four teams. The discrepancy gets greater when it comes to the champion team. From the 1991-92 to 2003-04 season, only five times does my estimate give the champion team correctly.

2.3 Bayesian Model Comparison

Logit and probit models have been extensively analyzed by many scholars, especially psychologists. Many researches suggested that they are undistinguishable empirically. For instance, Burke and Zinnes (1965) compared a Case-V Thurstone model ($\mathcal{T}$) and a Luce model ($\mathcal{L}$), which are the psychologists' way to call the probit and logit models respectively, and they found:

> Unfortunately, the nature of the solutions makes it very difficult to design an experiment for deciding between the theories... For the Gulliksen-Tukey (1958), Guilford (1954), and Thurstone (1959) data, the $\mathcal{T}$ predictions are considerably better than the $\mathcal{L}$ predictions.

Meanwhile, in another paper by Hohle (1966), the author found that

> (a) neither model provided uniformly satisfactory representations for the data, and (b) (for) all six sets of data were more accurately represented by Model II ($\mathcal{L}$) than by Model I ($\mathcal{T}$).

In a comprehensive survey, Batchelder (1986) claimed that

> The effort to decide between Luce's and Thurstone's theories was frustrated by the fact that despite the case that the two theories entail quite different theoretical constraints on choice probabilities, they are practically indistinguishable

with data. More precisely, ... for every member of $\mathcal{L}$ there is a member of $\mathcal{T}$ (and vice versa) that would require an unrealistic amount of choice data to achieve any reasonable power in testing between them statistically.

Differs from their frequentists' methods, in this paper, I compare the probit and logit models using the Bayesian model comparison method. Bayesian treat the two candidate models as hypothesis. Rather than artificially designing some goodness-of-fit statistic, Bayesian choose a natural criterion, the Bayes factor, to compare alternative models. The Bayes factor for model 1 versus model 2 is defined as

$$B_{12} = \frac{f(y|M_1)}{f(y|M_2)}, \tag{11}$$

where

$$f(y|M_i) = \int_{\Theta_i} f(\theta_i|M_i)\mathfrak{L}(y|\theta_i, M_i)d\theta_i \tag{12}$$

is the marginal likelihood of model i, $i = 1, 2$ (see Klepper and Poirier 1981).

The interpretation of the Bayes factor, after it has been calculated, is given by Jeffrey (1961) and Kass and Raferty (1993). He suggested the following criterion as the "order of magnitude"interpretation of B_{12}:

$$\begin{array}{lllll} 1 & < B_{12} & < & \infty, & \text{evidence supports } M_1, \\ 10^{-1/2} & < B_{12} & < & 1, & \text{every slight evidence against } M_1, \\ 10^{-1} & < B_{12} & < & 10^{-1/2}, & \text{slight evidence against } M_1, \\ 10^{-2} & < B_{12} & < & 10^{-1}, & \text{strong evidence against } M_1, \\ 0 & < B_{12} & < & 10{-}2, & \text{decisive evidence against } M_1. \end{array}$$

The key step in the Bayesian model comparison is computing a good approximation to the marginal likelihood. For our nonlinear regression models (logit, probit), the main difficulty is that the marginal likelihood cannot be expressed directly as a posterior moment, and consequently the problem cannot be treated directly as a special case of the simulation-consistent approximation of posterior moments. Fortunately, there are computational methods specifically tailored to this kind of problem. Especially, I use two different methods to compute the marginal likelihoods of probit and logit models respectively.

2.3.1 Evaluating the Marginal Likelihood in the Probit Model

In a probit model A, the observables are a $T \times K$ matrix of covariates $X = [x_1, \ldots, x_T]'$ and a corresponding set of T binary outcomes y, with

$$P(y_t = 0|x_t, A) = \Phi(\beta' x_t), \tag{13}$$

$$P(y_t = 1|x_t, A) = 1 - \Phi(\beta' x_t). \tag{14}$$

If we introduce the latent variables $\tilde{y}_t = \beta' x_t + \epsilon_t$, $\epsilon_t \sim N(0, 1)$, then the first outcome, $y_t = 0$, corresponds to $\tilde{y}_t \leq 0$ and the second to $\tilde{y}_t > 0$. Let I_t be an index function of $(0, \infty)$. And the conditionally conjugate prior distribution is $\beta|X, A \sim N(\underline{\beta}, \underline{H}^{-1})$.

The joint distribution of both observables and unobservables in the probit model is

$$\begin{aligned} & f(\beta, \tilde{y}, I|X, A) = f(\beta|X, A) f(\tilde{y}|\beta, X, A) f(I|\tilde{y}, X, A) \\ \propto \quad & \exp[-\frac{1}{2}(\beta - \underline{\beta})' \underline{H}(\beta - \underline{\beta}) + (\tilde{y} - X\beta)'(\tilde{y} - X\beta)] \prod_{t=1}^{T} I(\tilde{y}_t). \end{aligned}$$

It is straightforward to show that

$$\beta|\tilde{y}, I, A \sim N(\overline{\beta}, \overline{H}^{-1}), \tag{15}$$

where

$$\overline{H} = \underline{H} + X'X, \tag{16}$$

$$\overline{\beta} = \overline{H}^{-1}(\underline{H}\underline{\beta} + X'\tilde{y}). \tag{17}$$

Moreover, in the distribution of $\tilde{y}|I, \beta, X, A$, the elements $\tilde{y}_t$, known as probits, are independent. These conditional posterior distributions are the basis of a very simple Gibbs sampling algorithm, which is first proposed in Albert and Chib (1993).

After the posterior simulator been constructed, I can proceed to evaluate the marginal likelihood of the probit model A. I follow the method proposed by Gelfand and Dey (1994), which is known as the density ratio approximation method.

Notice

$$f(y|A) = \int_{\Theta_A} f(\beta|A) f(y|\beta, A) d\beta. \tag{18}$$

. Given the output of a posterior simulator, $\beta^{(m)} \sim f(\beta|y, A)$, and evaluations of the prior density $f(\beta|A)$ and data density $f(y|\beta^{(m)}, A)$, we can approximate (18) by

$$M^{-1} \sum_{m=1}^{M} \frac{f(\beta^{(m)})}{f(y|\beta^{(m)}, A) f(\beta^{(m)}|A)} \xrightarrow{a.s.} [f(y|A)]^{-1}, \tag{19}$$

where M is the number of iteration. This result is given by Geweke (1999). Obviously, the term $f(\beta^{(m)}|A)$ can be evaluated in the prior, and

$$f(y|\beta^{(m)}, A) = \prod_{t=1}^{T} \Phi(-x_t' \beta^{(m)})^{(1-y_t)} \Phi(x_t' \beta^{(m)})^{y_t} \tag{20}$$

can be computed to machine accuracy rapidly. The only thing left is $f(\beta^{(m)})$, a p.d.f. constructed from the posterior simulator output. By theorem 8.1.2 of Geweke (2005), it is constructed as follows:

$$f_\alpha^{(M)}(x) = \frac{(2\pi)^{-K/2} |\Sigma^{(M)}|^{-1/2}}{1-\alpha} \exp[-\frac{1}{2}(x-\mu^{(M)})'(\Sigma^{(M)})^{-1}(x-\mu^{(M)})] I_{X_\alpha^{(M)}}(x), \tag{21}$$

where $\mu^{(M)}$ and $\Sigma^{(M)}$ are sample mean and variance respectively, α is a predetermined number between 0 and 1, and $X_\alpha^{(M)}$ is the truncated highest density region of size $1-\alpha$.

Putting all three pieces, $f(\beta^{(m)}|A)$, (20), and (21), together, I can compute the approximated marginal likelihood of the probit model.

2.3.2 Evaluating the Marginal Likelihood in the Logit Model

The only difference between probit and logit is that the likelihood function. Because the likelihood of logit does not have a conjugate prior, I cannot compute the posterior $\beta|y, X, A$ analytically. A Metropolis-Hasting procedure is necessary within the Gibbs sampling, i.e., for every iteration of $\beta|A$, I have to make draws from the posterior distribution $\beta|y, X, A$ and compute their mean as one posterior draw.

To be more specific, I assign $\beta|A \sim N(\underline{\beta}, \underline{H}^{-1})$ as the prior. The likelihood is then given by

$$f(y|\beta, X, A) = \prod_{t=1}^{T} f(y_t|\beta, X, A) = \prod_{t=1}^{T} \frac{\exp(\beta' x_t)}{1 + \exp(\beta' x_t)}. \tag{22}$$

By Bayes theorem, I have

$$f(\beta|y, X, A) \propto f(\beta|X, A) f(y|\beta, X, A). \tag{23}$$

There is no simple way to draw from this posterior, and so the Metropolis-Hasting algorithm is used. After many times iterations within the Metropolis-Hasting procedure, I compute the mean of all the simulations and call it a single draw from the posterior[1]. Based upon this draw, the Gibbs sampling loop keeps rolling. And I need to record all the posterior simulates.

Following the same logic discussed in the last section, the density ratio method gives me the approximation of the marginal likelihood of the logit model.

Following the procedures described above, I compute the marginal likelihoods of the generalized ratio, logit, and probit models. The corresponding Bayes factors are calculated according to (11). The results are listed in Table 6(not yet).

2.4 Prediction

Bayesian Point Prediction:

$$f(y^\star|y) = \int_{\Theta, M} f(y^\star|y, \theta) f(\theta|M_i) d\theta dM \tag{24}$$

is the Bayesian Point Predictor, given the possible model from 1 to n, where θ_i is the hierarchical parameter of model i....

[1]For a more detailed discussion, see Train (2003).

3 Concluding Remarks

This paper has provided nonexperimental evidence that the CSFs do characterize some sporting competitions. My analysis of data from the 1991-2004 NBA game results suggests several important features of the game. First, as characterized by the generalized ratio form of CSF, the "noise parameter" m is considerably large, which implies that the NBA games are pretty "discriminating" in the sense that the contest outcome significantly depends on the contributed efforts of the teams. This result provides us further understanding of the NBA games. Second, as shown by both logit and probit regressions, efforts (salary-on-court) influence teams' performance. Higher salary-on-court index leads, other things equal, to higher probability of winning. Third, home-advantage effect exists in NBA games. Other things being equal, teams generally have a better chance to win in their home fields.

Another finding of this paper is that it compares three different CSF models. By using Bayesian model comparison method, I compute the marginal likelihoods of the generalized ratio, logit, and probit form of CSFs. Based upon the corresponding Bayes factors, I show that:...

My work is only an initial empirical study of the CSFs, and there are a number of directions in which future research might proceed. First, replications and extensions using data from other sports in which absolute measures of effort are available, the structure of tournaments differs would obviously be desirable. Professional boxing is particularly attractive in this regard because boxing games are played between two individuals rather than two teams. Its result therefore does not involve any interdependence among team members. Analysis using data from a much longer period would also be desirable; my analysis focuses on how teams allocate effort/concentration within a game but show very little evidence on how different timings influence how many resources the NBA teams devote to in games.

Second, all my analysis are derived from simple two-party models that yield implications for the input/output of a team. Generalization to n-person tournaments would yield implications about the entire distribution of game results one might expect to observe, and empirical analysis of the distribution of final results could then be undertaken.

Third, my analysis assumed that the effort/concentration levels influence game results equally across all teams. It is, obviously, an unrealistic assumption. Teams can choose different strategies against different teams, and depending on a team's ability relative to the rest of its rivals or its standing after each round, different strategies may be pursued. Models that also included the choice of strategies undoubtedly would yield additional empirical implications.

Finally, while analysis of sports contests are of interest in themselves, there is the broader question of the extent to which contest theory can help to provide an explanation for the various behaviors we observe among other competitions. As is well known, situations in which opportunities exist for one contestant to sabotage a rival's performance are rarely conducive to contest models (see Lazear 1989). Nevertheless, devising ways to theoretically and empirically address this question should rank high on the research agenda of economists interested in contest issues.

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