PRODUCT CYCLE AND WAGE INEQUALITY

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1. Introduction

The recent past has seen a considerable rise in wage inequality in developed as well as in developing countries. Katz and Autor (1999) report a 29 percent increase in the gap between the 90th percentile of earners and the 10 percentile of earners from the late 1970s to the mid 1990s in the United States. The gap increased by 27 percent over the same period in the United Kingdom and by 9 percent in the Canada. Feenstra and Hanson (1997) report a similarly high increase in wage inequality for Mexico, a developing country. Observations such as these have motivated a large body of literature that aims to understand the mechanisms leading to the rise in wage equality in an economy at a given point in time and in the long run. Broadly, these explanations can be categorized into two groups based on two different theories used to explain the problem at hand. The first emphasizes international trade with other countries (Wood, 1995; Dinopoulos and Segerstrom, 1999). The second centers on skill-biased-technical-change (STBC) hypothesis (Berman et al., 1998; Acemoglu, 2002).

International-trade based explanations are derived from the Stolper-Samuelson theorem, which describes a relation between the relative prices of output goods and relative factor rewards in real terms. When applied to the Heckscher-Ohlin model, the theorem predicts that the abundant factor in each country gains from trade and the scarce factor loses. This result works

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through the changes in product prices as each economy moves from autarky to an open economy. International trade between a developed (skilled labor) and a developing country (unskilled labor) results in a gain for skilled workers and a loss for unskilled workers in the developed country. Borjas and Ramey (1994), among many others, document a decrease in relative wage of unskilled workers with net imports of durable goods, giving empirical grounding to the hypothesis.

The trade-based explanations for rising wage inequality were later challenged on the basis of further evidence suggesting the absence of the Stolper-Samuelson mechanism. First, it was found that inequality is rising not only in developed economies but also in developing economies (Feenstra and Hanson, 1997), contradicting the finding of the theorem. Second, it was observed that even, when trade increased, domestic relative prices of imported goods in developed countries remained roughly constant (Lawrence and Slaughter, 1993). Since Stolper-Samuelson theorem's prediction about the change in relative wage works through changes in product prices, the evidence raises serious doubts about the trade explanation for increasing wage inequality. Krugman and Lawrence (1994) argued very forcefully that, even though the US trade with the rest of the world has increased manifold in the past, living standards in the economy is still determined by domestic factors. In their view, technological change and not increasing globalization plays a major role in explaining the current wage inequality.

The second type of explanation for the rise in wage inequality is based on the 'skill-biased-technological-change' (STBC) hypothesis. STBC is a change in the production technology that induces a bias in favor of skilled labor against unskilled labor. The bias occurs because the new technology exogenously increases the intrinsic productivity of skilled workers. The SBTC-based explanations are not free from criticism. The main criticism comes from the fact that a rise in the relative wage makes use of unskilled labor relatively cheap. Thus, an incentive exists to develop technologies which favors unskilled workers rather than skilled workers. This reasoning reveals the following puzzle: Why do technological advances tends to make skilled workers more efficient relative to unskilled wokers?

In my research, I propose the product-cycle based mechanism to explain the bias in favor of skilled workers. In contrast to SBTC based literature, the proposed model is based on the technological progress that is not biased towards any factor of production. Where in SBTC based model, wage inequality arises due to an increase in the relative productivity of skilled workers, in my proposed model wage inequality arises due to an an increase in the relative demand for skilled workers. The relative demand of skilled workers increases because of continuous standardization of production technology of a good.

The term 'product cycle' was first used by Vernon (1966) to describe a phenomena that most new goods are manufactured first in the countries where they were originally discovered and developed, and later in countries where production costs are lower, when products have been standardized. Vernon's implicit assumption was that, in the beginning the production function is not clearly specified such that production can only take place under the supervision of skilled engineers. As time progresses, the manufacturer gradually gains knowledge on how to produce the good without such assistance, and gradually production becomes less skill intensive.

Vernon's (1966) work pushed theorists to formalize the theory behind the cycle. The first notable formalization came from Krugman (1979). In his model, a developed country (industrialized north) innovates and produces new goods, and a developing country (south) produces old goods. Since agents in both economies have 'love of variety' type of preferences, there is trade between them. In Krugman's model, a new good becomes an old good over time with a lag specified exogenously. Since the south can imitate old goods and produce them more cheaply, southern manufacturers drive northern manufacturers of old goods out of the market.

Grossman and Helpman (1991) developed a model of product cycle based on endogenous growth theory. In endogenous growth models, whenever the discounted present value of the expected profits exceeds the current cost of development resources (skilled labor), entrepreneurs spend resources to bring new products to the market. The cost of developing a new product decreases in real terms as the number of already developed products increases in the economy. The reasoning is that available products represent disembodied knowledge in the economy. As disembodied knowledge in the economy increases, development costs decrease.

Krugman (1979), Grossman and Helpman (1991), and most if not all other existing analysis of product cycles focus on , the various implications of international trade. All these models have only one type of factor of production, hence they are not suitable to study wage inequality in a closed economy. These models also abstract from the Vernon's description of continuous standardization and use the same production function through the whole product life cycle. The line of research I propose is much closer, in spirit, to Vernon's description of continuous standardization of production process.[1] Also, the research incorporates two types of labor, skilled and unskilled, which is important to study wage inequality, particularly in a closed economy. Although, this part of research is similar to Ranjan (2005), my research differs from his in two respects. First, he uses a random and discrete

[1]Antras (2005) envisions a model, where he uses a continuous standardization process to describe the continuous change in factor requirements to produce the same good. Although, he uses two factors of production: skilled and unskilled workers, in his model both factor of production get the same wage, hence making his model unsuitable to study wage inequality. His main focus is to describe how an endogenous product cycle can arise due to incomplete contracts.

standardization process. Second, his model assumes that markets are perfectly competitive. My research, apart from studying a possible mechanism leading to wage inequality, will also contribute to the product cycle literature by formalizing a continuous standardization process, described first by Vernon. Further, the research aims to endogenize the standardization process. So far, I have written a base model to explain wage inequality in a closed economy which is presented in appendix. I will extend the study for an open economy and international-trade. The purpose for the extension is to show how the product cycle relates to the world inequality. The international-trade based model will provide a new price-differential based mechanism for the product cycle to take place.

2. The Base Model for Product Cycle with Continuous Standardization

Here, I present my model of endogenous product cycle. The model builds upon Grossman-Helpman (1991), yet differ in two important ways. First, my model incorporates two types of labor, skilled and unskilled, which is important to consider intra economy income distribution. Second, my formulation allows for continuous standardization of production function in the product cycle that represents continuous product development along the life cycle.

Consumers

Consider an economy populated by two types of infinitely lived workers, skilled (h) and unskilled (l), with population H and L, respectively. A worker of type $k \in \{h, l\}$, has a time-separable intertemporal lifetime utility function, U_k, with a common discount rate as ρ. The worker k's lifetime utility function depends on her instantaneous sub-utility function, $\mathrm{u}_k(\tau)$, which, in turn, depends on her instantaneous consumptions, $C_{kj}(\tau)$s where $j \in n(\tau)$, of $n(\tau)$ differentiated goods, available at time τ.

$$U_k(t) = \int_t^{\infty} e^{-\rho\tau} \log[u_k(\tau)] d\tau \tag{1}$$

$$u_k(\tau) = [\int_{j \in n(\tau)} {C_{kj}}^{\alpha} dj]^{\frac{1}{\alpha}}, \qquad \alpha \in (0, 1) \tag{2}$$

The assumption of CES (constant elasticity of substitution) implies that consumers have 'love of variety'. It also implies the elasticity of substitution between any two products is constant and equal to $\sigma = \frac{1}{1-\alpha} > 1$.

A consumer k can solve her maximization problem in two stages. In first stage, she finds the consumption of good i at time t, $C_{ki}(t)$, as to maximize her instantaneous utility, $u_k(t)$, given prices of all available goods at that time and assuming instantaneous expenditure, $E_k(t)$. In Second stage, she solves for the time pattern of expenditures, $E_k(t)$, that maximizes her life-time utility, U_k.

$$C_{ki} = \frac{p_i^{-\sigma}}{\int_{k \in n(t)} p_k^{1-\sigma} dk} E_k(t) \tag{3}$$

As there are H identical skilled workers and L identical unskilled worker, the instantaneous demand function of good i in the economy can be given as

$$Y_i(t) = L \cdot C_{li} + H \cdot C_{hi} = \frac{p_i^{-\sigma}}{\int_{k \in n(t)} p_k^{1-\sigma} dk} E(t) \tag{4}$$

where $E(t) = L \cdot E_l(t) + H \cdot E_h(t)$, is economy's total expenditure at time t.

$$= \lambda p_i^{-\sigma} \quad \text{where, } \lambda = \frac{E(t)}{\int_{k \in n(t)} p_k^{1-\sigma} dk}, \text{ and } \sigma = \frac{1}{1-\alpha}$$

While solving for the time pattern of expenditures, $E_k(t)$, that maximizes U_k, k needs to satisfy her inter-temporal budget constraint. The budget constraint depends on her wage, $w_k(t)$, asset holding at time t, $\mathcal{A}_k(t)$, and instantaneous interest rest, $\dot{R}(t)$, prevailing in the capital market. Assuming a consumer can lend and borrow freely in the capital marker, k's budget constraint is

$$\int_t^\infty e^{-[R(\tau)-R(t)]} E_k(\tau)d\tau \quad = \quad \int_t^\infty e^{-[R(\tau)-R(t)]} w_k(\tau)d\tau + \mathcal{A}(t) \qquad (5)$$

where $R(t)$ is a cumulative interest factor from time 0 to time t that a worker faces in the capital market. It is given by $R(t) = \int_0^t \dot{R}(t)$.

Consumption of goods, i's, $(i \in n(\tau))$ for the worker k, obtained in first stage and given by C_{ki}'s lead to an indirect utility function $u_k(\tau)$ which is weakly separable in the level of k's spending, $E_k(\tau)$, and in a function of prices of differentiated goods. This implies that the life time utility function of worker k, U_k, is separable in the time path of her spending and instantaneous function of prices.[2] Thus the maximization of k's life time utility, subject to her intertemporal constrains, provides the optimal time pattern of spending for her, which is given as

$$\frac{\dot{E}_k}{E_k} = \dot{R} - \rho,$$

It is important to note that the worker k can be either a skilled worker or an unskilled worker. It implies that above expenditure pattern is true for any worker, skilled or unskilled, in the economy, thus it is also true for the expenditure pattern, $E(t)$, for the whole economy. $E(t)$ is equal to $L \cdot E_l(t) + H \cdot E_h(t)$.[3]

$$\frac{\dot{E}}{E} \quad = \quad \dot{R} - \rho \qquad (6)$$

Above condition implies that the value of spending for the economy's grow at the instantaneous rate equal to the instantaneous interest rate corrected by given future discount rate.

Producers

The number of potential products is infinite. To begin production of one of

[2]Mathematical formulation is shown in Appendix A

[3]$\frac{\dot{E}_h}{E_h} = \frac{\dot{E}_l}{E_l} = \frac{L \cdot \dot{E}_l + H \cdot \dot{E}_h}{H \cdot E_h + L \cdot E_l}$

potential differentiated goods, the producer needs to learn how to produce that good. All new producers incur a development (research) cost to start production. A new producer does not want to develop an already existing type, as this leads to a Bertrand competition between two same type of products. In a Bertrand competition, competitors have to set price of the good equal to the marginal cost. Since the learning process is costly, a new producer would never able to recover the development cost of the good, had she developed an already existing product.

After a new producer learns how to produce a good, production takes place under the constant return to scale technology. Assuming Cobb-Douglas function as production technology:

$$Y = \zeta h^z l^{1-z} \qquad 0 \leq z \leq 1 \tag{7}$$

where $\zeta = z^{-z}(1-z)^{1-z}$.

The labor market is perfectly competitive. If w_h and w_l denote the wage for a skilled worker and an unskilled worker, respectively, the associated unit cost function can be written as

$$c(w_h, w_l) = w_h^z w_l^{1-z}$$

Further, Shephard's lemma gives the optimal units of skilled worrker, (a_h), and unskilled worker, (a_l), to produce a unit good as a function of wages of skilled and unskilled workers.

$$\begin{aligned} a_h &= z \cdot w^{z-1} \qquad (8) \\ a_l &= (1-z) \cdot w^z \end{aligned}$$

where $w = \frac{w_h}{w_l}$.

Cobb-Douglas production technology with constant factor intensity is ubiquitous in Economics literature. To capture the standardization of production technology, as envisioned by Vernon (1966), I consider a cobb douglas production function where factor intensity is changing continuosly with time. When a new product is innovated, the good is produced solley by skilled labors. As time progresses, skilled workers are able to train unskilled workers about the new production technology.

[4]

z as a function of time span (τ) captures the standardization process. The standardization implies that output elasticity of unskilled workers increase as product becomes older, and of skilled workers decreases. If maturity period, from beginning of production to the time when production is solely function of unskilled workers, is θ, then basic characteristics of standardization can be given as

$$z(0) = 1, \quad z'(\tau) < 0, \quad \text{and} \lim_{\tau \to \theta} z(\tau) = 0.$$

In this model, the standardization process is linear and exogenous in nature. The exogenous nature of standardization process indicates that the only cost of standardization is the time spent on producing new goods. The process, I have in mind, is "Learning by Doing". It is given as[5]

$$z(\tau) = \begin{cases} 1 - \frac{\tau}{\theta} & \text{for } z \leq \theta \\ 0 & \text{for } z > \theta \end{cases} \tag{9}$$

Consumers' CES type of preference lead to an iso-elastic demand curve for a unique differentiated good.

$$Y_i(t) = \lambda \left[p_i(t)\right]^{-\sigma}, \quad \sigma > 1 \tag{10}$$

[4]Since $\lim_{z\to 1} z^{-z}(1-z)^{1-z} = 1$ & $\lim_{z\to 0} z^{-z}(1-z)^{1-z} = 1$, the production function is continuous in z.

[5]Antras (2005) envisions a slightly different standardization process in which product-development intensity of the good is inversely related to product maturity. For this, he proposes a exponential standardization process. My model can accommodate his standardization process.

λ is a parameter given in equation(4) is a parameter that the producer takes as given. Such producers maximizes profit by setting a price p, that is a fixed mark up over marginal cost of production.

Since product function is of constant return of scale (CRS) type, marginal cost of production is equal to unit cost of production. For a good i at time t, this can be given in terms of unit factor prices, wages of skilled (w_h), unskilled workers (w_l) and time (τ) since the product is in the market..

$$c_i(\tau, t) = [w_h(t)]^{z(\tau)}[w_l(t)]^{1-z(\tau)} \tag{11}$$

Thus corresponding price of the good i is:

$$p_i(\tau, t) = \frac{[w_h(t)]^{z(\tau)}[w_l(t)]^{1-z(\tau)}}{\alpha} \tag{12}$$

And the producer of good i, which developed τ time ago, earns instantaneous profit at time t given by

$$\pi_i(\tau, t) = (1-\alpha) \cdot p_i(\tau, t) \cdot Y_i(\tau, t) \tag{13}$$

If relative factor price, in this case, relative wage, is denoted as $\omega = \frac{w_h}{w_l}$, the marginal skilled and unskilled labor requirement to produce one more unit of good i developed at time t and available in the market for period τ can be given by $a_h(\tau, t)$ and $a_l(\tau, t)$ respectively.

$$a_h(\tau, t) = z \cdot [\omega(t)]^{(z(\tau)-1)}; \qquad a_l(\tau, t) = (1 - z(\tau)) \cdot [\omega(t)]^{z(\tau)} \tag{14}$$

The derived demand for labor for each differentiated good is simply the unit labor requirement multiplied by the demand of that good. Total skilled and unskilled labor engaged in productive activities can be given and H_p and L_p respectively.

$$
\begin{aligned}
L_p &= \int_{j\in n(t)} a_l(j,t)Y(j,t)dj. \quad (15)\\
H_p &= \int_{j\in n(t)} a_h(j,t)Y(j,t)dj.
\end{aligned}
$$

Product Development

Following endogenous growth literature, particularly Romer (1990); Grossman and Helpman (1991), I assume that the resources dedicated to research leads to two types of outputs. First, a direct output that is ability to produce a new differentiated product from the pool of infinitely feasible products. It gives the developer a monopoly over the production of the new good and earns her a stream of monopolistic profit. Second is the side and unintended output. The development of each new good leads to the addition of general knowledge available in the economy. The underlying assumption is that such knowledge has widespread scientific applicability and it increases the productivity of any such development efforts in the future.

Following Grossman and Helpman (1991), if k denotes the level of disembodied knowledge capital in the economy and a_d, a fixed productivity parameter in the product development sector, the resources required to come up with a new product could be given as a_d/k units of skilled labor.[6] Total available number of products in the economy can be used as proxy for the disembodied knowledge capital. If H_d is the number of high skilled laborers involved in the development work, the rate of development $\dot{n}$ can be given as

$$
\dot{n} = \frac{n \cdot H_d}{a_d} \quad (16)
$$

The model allows for free entry. So, the discounted value of the cumulative profit for an individual producer should equal to her development cost at

[6] Grossman and Helpman (1991) describes the requirement to come up with a new product in very similar fashion. Since, their model has only one type of factor of production, the same factor is used for the development. Here, I shy away from using both factors of production as this will unnecessarily complicate the model without adding any extra insight.

time t, $V(t)$.

$$V(t) = \frac{a_d w_h(t)}{n(t)} = \int_0^\infty e^{[R(t+\tau)-R(t)]}\pi_i(\tau, t+\tau)d\tau \tag{17}$$

Where τ denotes how long the product is in the market.

Above derived equation can completely determine the evolution of the economy from any initial conditions. Provided $E(0)$ is consistent with long term convergence, the economy attains a steady state.

3. Steady-State Analysis

I am interested in determining long term rate of product development and the distribution of income in the economy. In the steady state, I denote the growth rate of number of products ($\frac{\dot{n}}{n}$) by g. Since, total expenditure in the economy, $E(t)$, is equal to the number of products in the economy, in the steady state, the economy's expenditure grows at constant rate g. The growth rate for both wages (skill and unskilled) would also be the same as g. The equation (33) implies:

$$\dot{R} = g + \rho \tag{18}$$

The equation gives the instantaneous interest rate as summation of future discount factor and the growth rate of number of varieties. In the previous section, value function described in equation (17) also gives instantaneous interest rate in the economy.

Differentiating equation (17)[7]

$$\dot{V}(t) = \frac{dV}{dt} = \left[\frac{\dot{w}_h}{w_h} - \frac{\dot{n}}{n}\right]V(t) = -\pi_i(0,t) + \dot{R}\cdot V(t). \tag{19}$$

$$\dot{R} = \frac{\pi_i(0,t)}{a_d w_h(t)/n(t)} + (\frac{\dot{w}_h}{w_h} - \frac{\dot{n}}{n}) \tag{20}$$

The equation (20) provides a a no-arbitrage condition for the development market. On the right hand side the first term is instantaneous profit rate

[7] $\pi_i(\tau, t)$ is independent of t in the steady state.

and the second term is capital gain. Summation of these two rates gives instantaneous interest rate.

In the steady state, capital gain is equal to zero, as growth rate in wage is equal to the growth in the number of products.

$$\dot{R} \quad = \quad \frac{\pi_i(0,t)}{a_d w_h(t)/n(t)} \tag{21}$$

Using equations (4) and (13), I find

$$\pi_i(0,t) = (1-\alpha)\cdot\frac{[p_i(t)]^{1-\sigma}}{\int_{j\in n(t)}[p_j(t)]^{1-\sigma}dj}E(t) \tag{22}$$

In the very beginning of product cycle, the product costs $\frac{w_h(t)}{\alpha}$, as it is solely produced by the skilled labor. In the steady state at any particular time, the price of any good depends on how long (maturity period) the product has been available and the prevailing wage rate at that time, which is again the function of number of products available in the market. In the steady state, number of products in the economy is evolving at constant growth rate, g. As product development pattern is known, the denominator in the equation (22) can be easily calculated.

If $n(t)$ is total number of product in the economy at time t, τ time ago in infinitesimal time interval $d\tau$, the number of new products that enter the economy would be $n(t)g \cdot e^{-g\tau}$. Since these product entered the economy τ time ago their price can be given as function of wage rates of skilled and unskilled workers and maturity period τ. Products with maturity period less than θ are still produced by using both factors of production while products with maturity period greater than θ are produced solely by unskilled workers.

$$\begin{aligned}\int_{j\in n(t)}[p_j(t)]^{1-\sigma}dj &= \int_0^\infty [p(\tau)]^{1-\sigma}n(t)ge^{-g\tau}d\tau \\ &= \int_0^\theta \left[\frac{w_h^{1-\frac{\tau}{\theta}}w_l^{\frac{\tau}{\theta}}}{\alpha}\right]^{1-\sigma} n(t)ge^{-g\tau}d\tau + \int_\theta^\infty \left[\frac{w_l}{\alpha}\right]^{1-\sigma} n(t)ge^{-g\tau}d\tau \\ &= \left[(\frac{w_h}{\alpha})^{1-\sigma}n(t)g\frac{1-e^{-\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}}{(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}\right] + (\frac{w_l}{\alpha})^{1-\sigma}n(t)e^{-g\theta}\end{aligned} \tag{23}$$

Using above equation and $E(t) = n(t)$, instantaneous profit given in the equation (22) can be written as

$$\pi_i(t) = \frac{(1-\alpha)}{g\left[\frac{1-e^{-\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}}{g+\frac{(1-\sigma)\ln(\omega)}{\theta}}\right]+\frac{e^{-g\theta}}{\omega^{1-\sigma}}} \tag{24}$$

substituting equations (18) and (24) in the equation (21), I find **no-arbitrage condition for products development** in terms of variables relative wage (w), growth rate in number of products, g, and parameters maturity period, θ, elasticity of substitution σ and a product development parameter, a_d.

$$H + \frac{L}{\omega} = a_d \cdot \frac{\rho+g}{1-\alpha} \cdot \left[g\frac{1-e^{-\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}}{g+\frac{(1-\sigma)\ln(\omega)}{\theta}}+\frac{e^{-g\theta}}{\omega^{1-\sigma}}\right] \tag{25}$$

Labor Market Clearing Condition

No one in the economy is unemployed. It means that both skilled and unskilled labor market clear. Equation (15) gives the expression for labor involved in productive activities. I can obtain the expression for labor involved in development activities from equation (16). The labor market claering condition can be written as

$$\begin{aligned} H &= H_p + H_d \\ L &= L_p \end{aligned} \tag{26}$$

To calculate H_p given in the equation (15), First, I obtain $Y_i(\tau, t)$ using equations (4) and (23).

$$Y_i(\tau,t) = \begin{cases} \dfrac{\alpha\cdot\omega^{-\sigma}\cdot\omega^{\frac{\sigma\tau}{\theta}}(H\omega+L)}{n(t)\left[g\cdot\omega^{1-\sigma}\left(\dfrac{1-e^{-\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}}{(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}\right)+e^{-g\theta}\right]} = \dfrac{k\omega^{\sigma(\frac{\tau}{\theta}-1)}}{n(t)} & \text{if } 0 \le \tau \le \theta \\ \dfrac{\alpha\cdot(H\omega+L)}{n(t)\left[g\cdot\omega^{1-\sigma}\left(\dfrac{1-e^{-\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}}{(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}\right)+e^{-g\theta}\right]} = \dfrac{k}{n(t)} & \text{if } \theta \le \tau \le \infty \end{cases} \tag{27}$$

Where k is a term independent of τ and t.

Substituting equation (27) and equation (14) in equation (15), and the fact that after a product matures, it's production needs only unskilled workers, I find expression for skilled workers and unskilled workers engaged in the productive activiy.

$$\begin{aligned} H_p &= \int_0^\theta (1-\frac{\tau}{\theta})\omega^{-\frac{\tau}{\theta}}\left[\frac{k\cdot\omega^{\frac{\sigma\tau}{\theta}}\omega^{-\sigma}}{n(t)}\right]n(t)ge^{-\tau g}d\tau \\ &= kg\omega^{-\sigma}\left[\left(\frac{1-e^{-\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}}{(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}\right)-\frac{1}{\theta}\left(\frac{1-e^{-\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}\left(1+\theta\left(g+\frac{(1-\sigma)\ln(\omega)}{\theta}\right)\right)}{(g+\frac{(1-\sigma)\ln(\omega)}{\theta})^2}\right)\right] \end{aligned} \tag{28}$$

$$\begin{aligned} L_p &= \int_0^\theta \frac{\tau}{\theta}\omega^{1-\frac{\tau}{\theta}}\left[\frac{k\omega^{-\sigma}\omega^{\frac{\sigma\tau}{\theta}}}{n(t)}\right]n(t)ge^{-\tau g}d\tau+\int_\theta^\infty\left[\frac{k}{n(t)}\right]n(t)ge^{-\tau g}d\tau \\ &= k\left[\frac{g\omega^{1-\sigma}}{\theta}\left(\frac{1-e^{-\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}\left(1+\theta\left(g+\frac{(1-\sigma)\ln(\omega)}{\theta}\right)\right)}{(g+\frac{(1-\sigma)\ln(\omega)}{\theta})^2}\right)+e^{-\theta g}\right] \end{aligned} \tag{29}$$

Using equation (16), I obtain number of skilled workers engaged in development activity.

$$H_d = g\cdot a_d \tag{30}$$

Now, similar to no arbitrage condition for products development, I can write market clearing conditions and call it **product market no arbitrage condition** in terms of variables relative wage (ω), growth rate in number of products, g, and parameters, maturity period, θ, elasticity of substitution σ and a product development parameter, a_d.

$$\frac{H-a_dg}{L_p}=\frac{\frac{1-e^{-\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}}{(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}-\frac{1}{\theta}\left[\frac{1-(1+\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta}))e^{-\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}}{(g+\frac{(1-\sigma)\ln(\omega)}{\theta})^2}\right]}{\frac{\omega}{\theta}\left[\frac{1-(1+\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta}))e^{-\theta(g+\frac{(1-\sigma)\ln(\omega)}{\theta})}}{(g+\frac{(1-\sigma)\ln(\omega)}{\theta})^2}\right]+\frac{e^{-\theta g}}{g\omega^{-\sigma}}} \tag{31}$$

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Appendix A

It implies that $u_k(\tau)$ can be written as $u_k[p(\tau), E(\tau)] = E_k(\tau) f(p(\tau))$. I can rewrite the life time utility function, equation (1), as

$$U_k(t) = \int_t^\infty e^{-\rho\tau} [\log E(.) + \log f(p(.))] d\tau$$

Lagrangian using equation (32) and life time budget constraint (5):

$$\begin{aligned} \mathcal{L} &= \int_t^\infty e^{-\rho\tau} [\log E(\tau) + \log f(p(\tau))] d\tau \\ &\quad -\mu_t [\int_t^\infty e^{-[R(\tau)-R(t)]} (E(\tau) - W(\tau)) d\tau - \mathcal{A}_j(t)] \end{aligned}$$

μ_t denotes the lagrangian multiplier on the budget constraint. The first order condition for maximizing $U_k(t)$ can be written as

$$\int_t^\infty [e^{-\rho\tau} \frac{1}{E(\tau)} - \mu_t e^{-[R(\tau)-R(t)]}] d\tau = 0$$

For having above expression equal to zero, the expression inside integration $[e^{-\rho\tau}\frac{1}{E(\tau)} - \mu_t e^{[R(\tau)-R(t)]}]$ should equal to zero at every point of time.

$$e^{-\rho\tau} \frac{1}{E(\tau)} = \mu_t e^{-[R(\tau)-R(t)]}$$

Taking log on both of the sides and differentiating with respect to t gives

$$\frac{\dot{E}_k}{E_k} = \dot{R} - \rho, \qquad k \in \{L, H\}$$

The economy's total expenditure in period t is $E(t) = L.E_l(t) + H.E_k(t)$. This gives:

$$\frac{\dot{E}}{E} = \frac{\dot{E}_L}{E_L} = \frac{\dot{E}_H}{E_H} = \dot{R} - \rho \tag{32}$$

In the model, there is no monetary authority. So, I am free to give an arbitrary value to one of variables in the model. Following Grossman and Helpman (1991), I fix the economy's expenditure at every time equal to the number of products available at that time, $E(t) = n(t)$. This is a kind of normalization which describes that if prices of all goods in the economy would have been same the expenditure on each of these good would have been 1. Using this normalization and equation (32) the instantaneous interest in the economy can be given as

$$\dot{R} = \rho + \frac{\dot{n}}{n} \tag{33}$$

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