Strategic Differentiation in R&D Interaction between Firms

Kejun Song[1]

University of California, Irvine

Department of Economics

Email: kejuns@uci.edu

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Abstract

Most of the literature on innovation assumes that firms engaged in R&D competition that aims for exactly the same invention. However, both evidence and theory suggest that firms in high-tech industries are likely to target different innovations in their R&D activities. In this paper, we investigate the existence of Nash equilibrium with two otherwise identical firms choosing differentiated innovations in a R&D game. In addition, we study the impact of market conditions and intellectual property protection on such differentiated equilibrium.

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1. Introduction

In modern capitalist economies where imperfect competition is the norm, firms interact with each other in several forms. First there is interaction in setting price or quantity in an oligopoly market, which is the focus of much traditional industrial organization. Second, firms also interact with each other in product selection (Spence 1976; Dixit and Stiglitz, 1977) in monopolistic competitive market. In this article we are concerned with a third aspect in which interaction happens—in research and development. While the theory of product differentiation usually assumes that the set of available products is fixed and the production technology is given, we can't follow that in the theory of innovation and invention since the focus is on the discovery and development of new products or techniques. For firms in high-tech industries where R&D is a "matter of life or death", this third form interaction is essential to firms' decision making.

Since the influential article by Arrow (1962), most of the literature on innovation assumes that firms engaged in R&D competition that aims for exactly the same invention. Surveyed admirably in Reinganum (1989), R&D interaction under this assumption is more like an army race; timing of innovation becomes the variable of interest since the competition will end as soon as one firm succeeds. Relatively less work has been done to understand the issue of differentiated innovations, (Dasgupta & Stiglitz 1980, Judd 1985, Rosenkranz 1995, etc). In this paper, we investigate the existence of Nash equilibrium with two otherwise identical firms choosing differentiated innovations in a R&D game. In addition, we study the impact of market conditions and intellectual property protection on such differentiated equilibrium.

By nature, the various patent race models in the theory of innovations could be regarded as the counterpart of price competition models in oligopoly market since they both lead to ε preemption and sharpen the intense level of competition. The R&D game with differentiated innovation here would be the counterpart of differentiated product models since both dampen the intense level of competition and seek room for cooperation. The key insight here is: by differentiating from each other in R&D, firms have a larger joint surplus to share; therefore, timing of innovation is no longer of major interest.

Before we talk about how firms could differentiate their R&D, let's think for a second the possible sources of differentiation. Firstly, firm heterogeneity in risk aversion level, in age and size and in the industry conditions they face may contribute to their different choice in R&D, because such heterogeneity leads to relative advantages in R&D for different firms. For example, some recent literature (Vossen 1996, Shane and Katila 2003) founds small size firms are relatively strong in innovations where effects of scale are not important and where they can utilize their flexibility and proximity to market demand, such as new products or product-market combinations, modification to existing products for niche markets, and small-scale applications. Large firms are better at innovations that make use of economies of scale and scope, or require synergy effect in R&D and large scale application. Secondly, even if comparable firms are comparable in other aspects, they could have different information about the potential in innovations and choose to different priority in R&D. A case in point is IBM's misstep in personal computer. Back in 1980's, IBM's CEO thought there will be almost zero demand for personal computer in the 1990's and did not make the first move to develop PC. Another example is Apple missed the best opportunity to have Windows, the new operating system software when Bill Gates made the offer to it.

Lastly, even firms have the same information set; they will still choose to differentiate their R&D target in equilibrium if this is the best response to other players' choices. In this article we discuss differentiation in R&D interaction between firms in the pure strategic sense. It is inspired by Reinganum (1982) which found entry by a new firm is more likely with drastic innovation and persistence of the incumbent is more likely with incremental innovation. In a patent race model where both firms aim for the same innovation, the way to differentiate from a rival is either enter or quit the race; in a R&D game that allows for different innovation, there is more margin to be different from other players. Our current model aims to understand how intellectual property right protection and intrinsic relationship between different innovations affect firms' strategy in the R&D game. For the moment we ignore both heterogeneity in firm characteristics and variation in firms' belief about the value of innovations. In the future, we are going to incorporate firm heterogeneity and incomplete information to make the study more realistic and relevant.

2. Literature Review

There seems to be two categories when we think about how firm would differentiate in R&D. Firstly, much similar to product differentiation, firm could choose R&D target that lead to innovations differ either vertically (like quality of a product) or horizontally (like brands). An example for horizontally differentiated innovations could be various game software that bear the same core code. An example of vertically differentiated is Dell computer that tries to meet largest fraction of potential demand with low- to mid-quality varieties, and IBM laptop that tries to meet the business customer's demand. Secondly, innovations can either substitute or complement each other and lead to different nature in R&D interaction between firms. This category incorporates the vertical/horizontal differentiation, since it takes into account complement innovations that are not considered in the typical product differentiation literature. But in the process of discovery and development of new product and techniques, we may well expect to see innovations to exhibit complementary relationship with each other.

Up to date, there is a moderate-size literature that explicitly deals with differentiated R&D activities. Both deterministic (Rosenkranz 1995, Motta 1992) and stochastic (Henry 2006) assumptions have been used to describe the discovery technology. Also, both horizontal (Henry 2006) and vertical (Rosenkranz 1995, Motta 1992) differentiation has been modeled as the form of differentiation. All of the literature we know seems to start from differentiated innovations and try to see what consequences it has on different issues given the question of interest. Beath et al (1987), among others investigate the consequences of a variety of product innovations on the outcome of a repeated patent auction. Motta (1992) and Rosenkranz (1995) investigate incentives toward Research Joint Venture. Henry (2006) used a horizontal differentiation model to show how sufficient differentiation in R&D could improve aggregate industry profits and therefore runner-up patents will increase incentives to innovate.

Our work is different in that we investigates when differentiated R&D choice will occur as a Nash equilibrium given some exogenous factors like the relationship between innovations, the strength of IP protection, etc. If differentiation is indeed the true mode in the world, hopefully our understanding on the conditions of its existence will provide more nuance insights when we address related issues mentioned above.

3. Basic Model

3.1 Preliminaries

Suppose two firms, A and B, engage in R&D activities that explore the same research idea. Let's assume that the research idea could lead to two distinct innovations with market value V_L and V_H respectively, $V_L<V_H$ via two different paths, L or H. Both A and B have the capability to conduct R&D for either version of innovation, and they simultaneously choose the version of innovation and an intensity level of R&D investment, x_j, $j = A, B$ at the beginning of the game.

The strategy set is therefore $\{(x_A : P_A), (x_B : P_B)\}$, $P_j = L, H$

$\{(x_A : L), (x_B : L)\}$	$\{(x_A : L), (x_B : H)\}$
$\{(x_A : H), (x_B : L)\}$	$\{(x_A : H), (x_B : H)\}$

For both methods, the R&D discovery process is random with probability p of success. Further, the probability is assumed to be linear in the intensity level of investment $x_j \in [0,1]$. That is, $p_L = \beta x_j ; p_H = \gamma x_j$ .For the same intensity level of investment, method that leads to the superior version innovation has a lower probability of success, so $\gamma < \beta$.

The payoff from R&D for each firm depends not only on the strategic choices of research path and investment intensity level, but also on the realization of the random discovery process. Table 1 gives the net value for firm A in different scenarios.

Outcome	Probabilities	Net Value for Firm A				
			L,L	L,H	H,L	H,H
Both Firm succeed	$p_A x_A p_B x_B$	V_{1A}	$0.5V_L$	$\alpha V_L + \tau$	$V_H - \tau$	$0.5V_H$
Neither firm succeeds	$[1 - p_A x_A][1 - p_B x_B]$		0	0	0	0
Only firm A succeeds	$p_A x_A [1 - p_B x_B]$	V_{2A}	V_L	V_L	V_H	V_H
Only firm B succeeds	$[1 - p_A x_A] p_B x_B$		0	0	0	0

When both firms succeed and they are targeting the same version of innovation, each gets half of the market value; alternatively, when both succeed but they are targeting different versions of innovation, the successful low firm gets a depreciated market return $\alpha V_L, \alpha \in (0,1)$ plus a licensing fee $\tau \geq 0$ collected from the successful high firm. The more specific market niche an innovation is defined for, the closer α is to 1; and stronger IP protection on the blocking patent contained in the low version innovation, the higher licensing fee τ to be paid by the successful high firm. Therefore, net payoff for the successful high firm is $V_H - \tau$ .Finally, total cost of R&D is assumed to be concave in the intensity level of investment.

$$C = \frac{1}{k} x_j^{\ 2}.$$

Given the setting up of the model, each firm solves the following problem:

$$(x_j, P_j) \in \arg\max \pi[(x_j, P_j), (x_{-j}, P_{-j})], P_j = L, H; \forall j = A, B$$

$$E(\pi_A) = p_A x_A p_B x_B V_{1A} + p_A x_A (1 - p_B x_B) V_{2A} - \frac{1}{k} x_A{}^2$$

$$E(\pi_B) = p_A x_A p_B x_B V_{1B} + p_B x_B (1 - p_A x_A) V_{2B} - \frac{1}{k} x_B{}^2$$

$$\frac{dE(\pi_A)}{dx_A} = p_A p_B x_B V_{1A} + p_A (1 - p_B x_B) V_{2A} - \frac{2}{k} x_A = 0$$

$$\frac{dE(\pi_B)}{dx_B} = p_A p_B x_B V_{1B} + p_B (1 - p_A x_A) V_{2B} - \frac{2}{k} x_B = 0$$

$$\Rightarrow x_A{}^* = \frac{k}{2}[p_A V_{2A} + p_A p_B x_B (V_{1A} - V_{2A})]$$

$$x_B{}^* = \frac{k}{2}[p_B V_{2B} + p_A p_B x_A (V_{1B} - V_{2B})]$$

$\therefore$

$$x_A{}^* = \frac{p_A V_{2A} + \frac{k}{2} p_A p_B^2 (V_{1A} - V_{2A}) V_{2B}}{\frac{2}{k} - \frac{k}{2} p_A^2 p_B^2 (V_{1A} - V_{2A})(V_{1B} - V_{2B})}$$

$$x_B{}^* = \frac{p_B V_{2B} + \frac{k}{2} p_A^2 p_B (V_{1B} - V_{2B}) V_{2A}}{\frac{2}{k} - \frac{k}{2} p_A^2 p_B^2 (V_{1A} - V_{2A})(V_{1B} - V_{2B})}$$

where V_{1j} is the payoff for firm j when both firms succeed; V_{2j} is the payoff for firm j when it is the only successful innovator. p_A, p_B is the parameter in the probability of success for firm A and firm B's choice on path, respectively.

Table 2 gives the choice of investment intensity level of firm A as best response in all four cases.

x_A^* : calculate from the general form of x_A^* and payoffs in Table 1.			
L,L	L,H	H,L	H,H
$\frac{\beta V_L (1 - \frac{k}{4}\beta^2 V_L)}{\frac{2}{k} - \frac{k}{8}\beta^4 V_L{}^2}$	$\frac{\beta V_L + \frac{k}{2}\beta\gamma^2[(\alpha-1)V_L + \tau]V_H}{\frac{2}{k} + \frac{k}{2}\beta^2\gamma^2[(\alpha-1)V_L + \tau]\tau}$	$\frac{\gamma V_H - \frac{k}{2}\beta^2\gamma V_L \tau}{\frac{2}{k} + \frac{k}{2}\beta^2\gamma^2[(\alpha-1)V_L + \tau]\tau}$	$\frac{\gamma V_H (1 - \frac{k}{4}\gamma^2 V_H)}{\frac{2}{k} - \frac{k}{8}\gamma^2 V_H{}^2}$

Lemma 1: With the above assumptions, firm's expected payoff in equilibrium is equal (proportional) to its total cost of R&D, $C = \frac{1}{k} x_j^{*2}$ where x_j^* is the best response intensity level of investment to the other player's strategy (x_j, P_j) .

Proof (See Appendix A)

Proposition 1: Existence of a differentiated Nash equilibrium for the R&D game

By Lemma 1, finding the conditions where we have the differentiated case as Nash equilibrium of the game

$$E[\pi_A^*(x_A^*, H \mid x_B^*, H)] < E[\pi_A^*(x_A^*, L \mid x_B^*, H)]$$
$$E[\pi_B^*(x_B^*, L \mid x_A^*, L)] < E[\pi_B^*(x_B^*, H \mid x_A^*, L)]$$

is equivalent to finding conditions when

$$(x_A^*, H) \mid (x_B^*, H) < (x_A^*, L) \mid x_B^*, H)$$
$$(x_B^*, L) \mid (x_A^*, L) < (x_B^*, H) \mid (x_A^*, L)$$

From table 2, when the following two inequality hold,

$$\frac{\gamma V_H (1 - \frac{k}{4}\gamma^2 V_H)}{\frac{2}{k} - \frac{k}{8}\gamma^4 {V_H}^2} < \frac{\beta V_L + \frac{k}{2}\beta\gamma^2[(\alpha - 1)V_L + \tau]V_H}{\frac{2}{k} + \frac{k}{2}\beta^2\gamma^2[(\alpha - 1)V_L + \tau]\tau} \quad (1)$$

$$\frac{\beta V_L (1 - \frac{k}{4}\beta^2 V_L)}{\frac{2}{k} - \frac{k}{8}\beta^4 {V_L}^2} < \frac{\gamma V_H - \frac{k}{2}\beta^2\gamma V_L \tau}{\frac{2}{k} + \frac{k}{2}\beta^2\gamma^2[(\alpha - 1)V_L + \tau]\tau} \quad (2)$$

a differentiated Nash equilibrium exists.

Collary Stability of the differentiated Nash equilibrium

The reaction curves in a differentiated Nash equilibrium are

$$x_A^* = \beta V_L + \beta\gamma x_B[(\alpha-1)V_L+\tau]$$
$$x_B^* = \gamma V_H - \beta\gamma x_A \tau$$
$$\frac{\partial x_A^*}{\partial x_B} = \beta\gamma[(\alpha-1)V_L+\tau]$$
$$\frac{\partial x_B^*}{\partial x_A} = -\beta\gamma\tau < 0$$

The firm targeting the high version innovation will treat investment from the other player as complement; however, the firm with the low version innovation will not necessarily treat its opponent's investment as complement, depending on the sign of $\frac{\partial x_A^*}{\partial x_B}$.

If $(\alpha-1)V_L+\tau<0$, both firms use investment as strategic complement, the Nash equilibrium is stable. If not, the Nash equilibrium is not stable even it exists for some relevant parameter range.

3.2 Discussion

Inequality (1) and (2) are complicated to give any analytic results, so we made a couple of simplifying steps. By normalizing β, the parameter in success probability for the low innovation to 1, let $V_H = aV_L, a>1$, divide both sides by V_L, multiply by $2/k$, We have

$$\frac{\gamma a}{1+\frac{k}{4}\gamma^2 a V_L} < \frac{1+\frac{k}{2}\gamma^2 a[(\alpha-1)V_L+\tau]}{1+(\frac{k}{2})\gamma^2[(\alpha-1)V_L+\tau]\tau}$$

$$\frac{1}{1+\frac{k}{4}V_L} < \frac{\gamma a-\frac{k}{2}\gamma\tau}{1+(\frac{k}{2})\gamma^2[(\alpha-1)V_L+\tau]\tau}$$

Reparameterize

$$t \equiv \frac{k\gamma\tau}{2}, \qquad A \equiv \gamma a, \quad B \equiv kV_L$$

We have

$$4At^2 + t[2\gamma(\alpha-1)AB - 4A - A^2B\gamma] + [4A - 4 - AB\gamma - \frac{\gamma}{2}(\alpha-1)AB(4+AB\gamma)] < 0$$
$$4t^2 + t[2\gamma(\alpha-1)B + 4 + B] + [4 - 4A - AB] < 0$$

In order to find parameter ranges for both inequalities, we start with special values for α and τ, the two parameters of most interest. If we set $\alpha = 1, \tau = 0$, then the above inequalities become

$$4A - 4 - AB\gamma < 0$$

$$4 - 4A - BA < 0$$

Both of which will hold when $\frac{4}{4+B} < A < \frac{4}{4-\gamma B}, \; B < \frac{4}{\gamma}$.

3.2.1 Comparative Static

Since both inequalities always hold given the above ranges for A and B, we can study the marginal effects of α and τ in the neighborhood of $\alpha = 1, \tau = 0$ on the existence of differentiated Nash equilibrium.

$$\frac{\partial LHS}{\partial \alpha}|_{\tau=0} = 2\gamma ABt - \frac{\gamma}{2}AB(4+AB\gamma)|_{\tau=0} = -\frac{\gamma}{2}AB(4+AB\gamma) < 0$$
$$\frac{\partial LHS}{\partial \alpha}|_{\tau=0} = 2\gamma Bt|_{\tau=0} = 0$$

Therefore, all else equal, when α drops from 1, the first inequality is less likely to hold, while the second inequality is not affected. On the whole, a differentiated N.E. is less likely to occur. This corresponds to our initial observation for the stability of differentiated NE. In the neighborhood of $\alpha = 1, \tau = 0$, a stable NE requires changes in certain direction for α or τ. Here, as α drops from 1, the low firm is apparently less willing to stay with the differentiation strategy; instead it prefers to direct compete with the high firm by joining the race, especially when the difference in success rates of two innovations is small, a large γ.

$$\frac{\partial LHS}{\partial \tau}|_{\alpha=1} = \{8At + [2\gamma(\alpha-1)AB - 4A - A^2B\gamma]\} \cdot \frac{\partial t}{\partial \tau}|_{\alpha=1} = (8At - 4A - A^2B\gamma) \cdot \frac{\gamma k}{2}$$
$$\frac{\partial LHS}{\partial \tau}|_{\alpha=1} = \{8t + [2\gamma(\alpha-1)B + 4 + B]\} \cdot \frac{\partial t}{\partial \tau}|_{\alpha=1} = [8t + 4 + B] \cdot \frac{\gamma k}{2}$$

Holding $\alpha = 1$, $\tau = 0$ belongs to range $[\frac{2}{k\gamma}(-\frac{1}{2} - \frac{B}{8}), \frac{2}{k\gamma}(\frac{1}{2} + \frac{AB}{8})]$, where the first derivative is negative and the second is positive. Again, this means the differentiated equilibrium, although exists for certain parameter range, is not a stable one. Any increase in license fee would drive the high firm further away from the differentiation strategy, although the firm choosing low is not worse off within the range. The width of this acceptable range depends also on relative difficulty level of the high innovation. The more difficult it is to achieve the high innovation; the less obvious is the transition window from stable to unstable stage. Another interesting observation is that if the low firm charges an unreasonable high license fee, (greater than the upper bound), then the first inequalities also become positive and it is impossible to maintain any differentiated equilibrium. Intuitively, this could mean that a firm may foresee that a unreasonable high licensee fee is not acceptable by its competitor so it would rather to do the high innovation if his competitor is doing high. Further, the existence of an upper bound here implies that there should be a cap in blocking patent's license fee if the differentiated NE is socially desirable.

3.2.2 Special Cases

Recall in re-parameterization, A is the product of γ, which measures the relative risk of succeeding in the high innovation compared to that of the low innovation, and a, which measures the relative lucre of the high innovation compared to the low innovation. So A is a combined measure of the relative attractiveness of the high innovation compared to the low one. The range for A we derived previously seems to suggest that one is a reasonable value for the relative attractiveness for small B. Since $B \equiv kV_L$ and it is natural to assume that total cost $C = \frac{1}{k} x_j^{*2}$ is bounded by the market return of innovations, V_L, V_H. It is naturally see that B shall not be very large. Because k and V_L can not be large at the same time. Because when k is large, R&D is cheap, so every one will choose to do R&D and aim for the high innovation. When k is small, R&D is expensive, firms will be conservative doing R&D. Therefore, the product k and V_L shall be bounded by a reasonable value. For convenience in following analysis, we pick one to be the value for both A and B and discuss some special cases.

$$4t^2 + t[2\gamma(\alpha-1) - 4 - \gamma] + [-\gamma - \frac{\gamma}{2}(\alpha-1)(4+\gamma)] < 0$$

$$4t^2 + t[2\gamma(\alpha-1) + 5] - 1 < 0$$

Scenario 1: $\tau = 0$

When there is no IP protection but there is some overlapping in market returns of the two innovations, for both inequalities to hold we need $\alpha > 1 - \frac{2}{4+\gamma} > 0$

Intuitively, when chances are that the firm targeting the high version innovation will succeed relatively easily, a higher γ , the low successful firm will tend to target a more specific market, a higher α to protect depreciation in market return in case the high innovation occurs in the differentiated equilibrium. Or, in such equilibrium, there exists a tradeoff between the difficulty in successful of high innovation and the ability to control the market of low innovation.

Scenario 2: $\alpha = 1$

When market returns to the low innovation is immune to the high innovation, the range for license fee is $\tau \in [0, \frac{V_L}{4\gamma}(\sqrt{41} - 5)]$. It is bounded by zero to the left, means the low firm could stay with differentiation with no licensee fee. But the upper bound of license fee is determined by the second inequality for the high firm. This is quite intuitive as the high firm in this case will tend to deviate from differentiation. The more likely the high innovation is going to be successful, the less room to bargain for license fee since two innovations may not differ by much.

Scenario 3: $\alpha = 0$

When the high innovation completely replaces the low innovation in differentiated equilibrium, there exists a range for the license fee to be collected from the high successful firm.

$$\tau \in [\frac{V_L}{2}, \frac{V_L}{4}(2 - \frac{5}{\gamma} + \sqrt{\frac{41 + 4\gamma^2 - 20\gamma}{\gamma^2}})]$$

The upper bound for τ , is increasing as γ drops from 1 to zero. Intuitively, this means when there is greater difficulty to achieve the high innovation, there is more room to collect the license fee for the blocking patent owned by the low successful firm.

3.3 Robustness

To be finished.

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Appendix A

Lemma 1

Given the assumptions and general form of x_j^*, we have

$$E(\pi_A{}^*) = p_A x_A{}^* V_{2A} + p_A p_B (V_{1A} - V_{2A}) x_A{}^* x_B{}^* - \frac{1}{k} x_A{}^{*2}$$
$$= p_A x_A{}^* V_{2A} + p_A p_B (V_{1A} - V_{2A}) x_A{}^* [\frac{k}{2} [p_B V_{2B} + p_A p_B x_A{}^* (V_{1B} - V_{2B})] - \frac{1}{k} x_A{}^{*2}$$
$$= x_A{}^* [p_A V_{2A} + \frac{k}{2} p_A p_B{}^2 (V_{1A} - V_{2A}) V_{2B}] + x_A{}^{*2} [\frac{k}{2} p_A{}^2 p_B{}^2 (V_{1A} - V_{2A})(V_{1B} - V_{2B}) - \frac{1}{k}]$$
$$= x_A{}^* \cdot x_A{}^* [\frac{2}{k} - \frac{k}{2} p_A^2 p_B^2 (V_{1A} - V_{2A})(V_{1B} - V_{2B})] + x_A{}^{*2} [\frac{k}{2} p_A{}^2 p_B{}^2 (V_{1A} - V_{2A})(V_{1B} - V_{2B}) - \frac{1}{k}]$$
$$= \frac{1}{k} x_A{}^{*2}$$

Similarly,

$$E(\pi_B{}^*) = \frac{1}{k} x_B{}^{*2}$$

Appendix B

(To be typed)