

Grade Inflation under the Threat of Students' Nuisance

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October 16, 2006

Abstract

Using a game theoretic model, this study suggests that professors commit grade inflation to avoid students' nuisance (lobbying) for more points. It concludes that the larger the size of a class, the more grade inflation for bad students. The study also seeks to explain the different lobbying behaviors of students, regardless of their grades. The empirical results show that, *ceteris paribus*, bad students receive better grades in larger classes than those in small classes.

1 Introduction and Literature Review

Grade inflation is generally attributed to student evaluations (McKenzie, 1975; Wallace and Wallace, 1998; Zangenehzadeh, 1988). Grades directly affect students' utility functions, so students rate professors according to the grades they receive. In order to get better evaluations from students, professors "bribe" students by giving easy As and Bs. Wallace and Wallace (1998) claim that the

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workload and expected grades directly reflect students' happiness. In order to receive good evaluations, a professor reduces the workload and inflates the grades of a class. McKenzie (1975) argues that by grade inflation, the professor shifts students' budget constraints out, so that students receive better bundles of leisure time and grade. Hence, "the professor's rating will rise because of "reduced standards"" (p. 101). Krautmann and Sander (1999) study the relationship between student evaluation and expected grade and other variables using two-stage least squares. They conclude that "faculty have the ability to "buy" higher evaluations by lowering their grading standards"(p. 61). Kanagaretnam, Mathieu, and Thevaranjan (2003) analyze grade inflation with joint maximizing behavior of students and professors. They suggest that over-emphasis on students' satisfaction may affect a professor's other activities including "academic research, service to the university, community at large and professional/academic organizations" (p. 9).

While these researchers provide an insightful explanation for grade inflation, previous work did not mention that teaching evaluations are of marginal importance in tenure and promotion considerations in research universities. Teaching evaluations may be important in universities and colleges that emphasize teaching over research. However, in research universities, research publications, rather than teaching evaluations, are essential in tenure/promotion considerations. Therefore, in research universities, faculty members would have little,

if any, incentive to improve their teaching evaluation by grade inflation. This study suggests that, in addition to the desire for a better evaluation and other established causes¹, grade inflation results from the potential *threat of students' nuisance*, especially in research universities.

This study defines students' nuisance as "students' repeatedly lobbying for better grades". Anecdotal evidence suggests that unsatisfied with their grades, college students annoy professors with "point lobbying". Thus, college professors are frequently besieged by students who request upward adjustments for their grades. True, professors are capable of grading mistakes. Yet, there are many cases where the lobbying student does not deserve the points he or she requests. There are also cases where the student lobbies at a margin—asking for two more points for a one-hundred-point quiz, which does not change his overall grade much. Of course, a rational student who maximizes his grade point average (GPA) only points out the professor's mistake where the student's work deserves more, rather than fewer, points. If so, the professor should reject the request of the student. Yet, the student may not give up. He may keep sending emails, dropping by the professor's office, making the same request, and asking for checking of his exam repeatedly. Such nuisance is costly to the professor in terms of his or her opportunity cost. To escape students' nuisance, the professor may choose to inflate students' grades (especially the grades of the bad students) in the first place, so there will be no or few students coming back to

¹See, for example, T. Bar (2006) and H. Rosovsky and M. Hartley (2006).

lobby for more points.

Additionally, it is not surprising that bad students, particularly failing students, often lobby for better grades. However, anecdotal evidence suggest that good students sometimes lobby as well. Some students just would not walk away with an “A-” — they “*have to*” receive an A.

This study suggests that students’ nuisance and the threat of students nuisance induce the professors to inflate grades. Moreover, it explains the lobbying behavior of all students, regardless of their grade point averages.

The rest of this paper proceeds as follows: section 2 introduces a dynamic game theoretic model that explains grade inflation under the threat of students’ nuisance, and the students’ lobbying behavior. Section 3 empirically tests the conclusion of the game theoretic model. Section 4 concludes.

2 The Nuisance Game

2.1 The Model

Players There are one professor p and J students in the game. Students are characterized by different degrees of intelligence, $i \in (0, 1)$, and different degrees of aggressiveness, $a \in (0, 1)$. Each student knows his or her degrees of intelligence and aggressiveness, but the professor is unable to observe them. However, the professor is able to determine who are the good and the bad

students based on the observation of students' grades.

Strategies The professor's strategy is a vector of zeros and ones:

$$S_p = \{w_j^t\}_{j=1,2,\dots,J,t=0,1,\dots,\infty}$$

where $w_j^t = 1$ if the professor inflates student j 's grade at time t ; $w_j^t = 0$ if the professor does not inflate student j 's grade at time t .

A student's strategy is how much effort to exert prior to the exam, and whether to lobby for a better grade after the exam.

$$S_j = \{e, l_t\}$$

where e is the effort level; $l_t = 1$ if the student lobbies at time t ; otherwise, $l_t = 0$

The Payoff Functions:

1. The professor's payoff function is expressed as

$$U_p = -N(x) - I(y)$$

The professor incurs a nuisance cost $N(x)$ if there are x occurrences of students' lobbying. For example, if there are two lobbying students, and each of them lobbies three times, $x = 6$. The nuisance cost function captures the professor's emotional and opportunity costs of students' lobbying.

The professor also incurs an integrity cost $I(y)$ when she inflates grades, where y is the number of students who receive inflated grades. For ex-

ample, if the professor inflates grades for all students, the integrity cost is $I(J)$. The grade inflation function captures the loss of integrity the professor suffers from grade inflation.

2. A Student's Payoff Function is expressed as

$$U_j = g^a - m \cdot e - C, \quad a \in (0, 1), \quad m < a \cdot i$$

$$g = e^i + I^*[\eta], \quad i \in (0, 1)$$

where g is the grade, e is the effort level, C is the lobby cost; a, i, m are the student's degrees of aggressiveness, intelligence, and laziness, respectively. The effort level enters the utility through two channels. A higher the effort level results in a better grade, which increases the utility. However, the student dislikes exerting effort, so a higher effort level also decreases the student's utility.

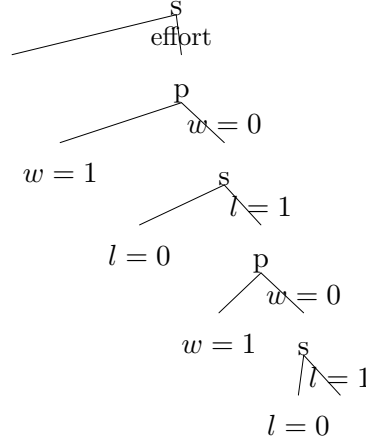
A student may boost his grade by working hard (a higher effort level) or by lobbying the professor for a better grade after the exam. If the professor inflates the student's grade, $I^* = 1$, and the student receives some positive adjustment of his grade, $\eta > 0$. If the professor does not inflate the student's grade, $I^* = 0$. The lobby cost C (such as the time the student spends on writing the lobbying email) is equal to zero if the student does not lobby; otherwise, $C = c > 0$.

The intelligence level, i , determines how effective a student studies. The higher the intelligence level, the more effective he studies. The aggressive-

ness level, a , determines how much a student cares about his academic performance. A more aggressive student has a higher marginal utility from a good grade than a less aggressive student does. The levels of intelligence and aggressiveness are determined by nature.

Game Timing :

1. Each student chooses his effort level prior to the exam.
2. The exam is taken and the professor determines whether to inflate student j 's grade ($w_j^0 = 1$) or not ($w_j^0 = 0$); $j = 1, 2, \dots, J$. Hence, the professor's first move is a vector of zeros and ones.
3. Each student j chooses to lobby ($l_j^0 = 1$) or not ($l_j^0 = 0$).
4. If student j chooses not to lobby ($l_j^0 = 0$), the game ends with him and his payoff is realized.
5. If the student j chooses to lobby ($l_j^0 = 1$), the game repeats from step 2.



Assumption 1

- a. The professor never makes any mistakes in grading. This assumption is to exclude the cases of incorrect grading.
- b. The professor nuisance cost function, $N(\cdot)$, is convex. This is intuitive: the marginal cost of nuisance is probably moderate if there were only two or three lobbying students. As the number of lobbying students rises to twenty or thirty, or if the same student lobbies repeatedly, lobbying becomes more costly to the professor. The convex nuisance cost function reflects the increasing marginal cost of nuisance.
- c. The professor's integrity cost function, $I(\cdot)$, is concave. In the beginning of grade inflation, the professor may feel guilty; but as she continues the grade inflation, she may not feel as uneasy as she felt in the beginning.
- d. For simplicity, assume the degree of laziness, m , and the cost of lobbying,

c , are the same for all students.

e. $m < a \cdot i$. The students are assumed not to be too disinclined in exerting effort. If the students were not into studying at all, they would not enter college in the first place.

h. No player discounts the future. If the game does not end, the payoffs for the professor and the never quitting students are $-\infty$.

i. All assumptions are common knowledge.

A Student's Choice of Effort Level A student determines his optimum effort level, e^* , by the first order condition, which is determined by his degrees of intelligence and aggressiveness.

$$\frac{\partial U}{\partial e} = 0$$

$$e^*(a, i) = \left(\frac{ai}{m}\right)^{\frac{1}{1-ai}} \quad (1)$$

Note that the optimum effort level e^* is always greater than one, since $ai > m$ by assumption 1e.

$$\frac{\partial e^*}{\partial i} = \frac{\left(\frac{ai}{m}\right)^{\frac{1}{1-ai}}}{i(1-ai)^2} [1 - ai + ai \cdot \ln\left(\frac{ai}{m}\right)] > 0 \quad (2)$$

PROPOSITION 1: Other things being equal, the higher the level of intelligence, the higher the optimum effort level. A student's optimal

effort level increases with his degree of intelligence. The smarter the student is, the more effective he studies, and the higher the marginal benefit of studying.

$$\frac{\partial e^*}{\partial a} = \frac{\left(\frac{ai}{m}\right)^{\frac{1}{1-ai}}}{a(1-ai)^2} [1 - ai + ai \cdot \ln\left(\frac{ai}{m}\right)] > 0 \quad (3)$$

PROPOSITION 2: Other things being equal, the higher the level of aggressiveness, the higher the optimum effort level. The more aggressive the student is, the more he cares about his grade; consequently, he is willing to exert more effort to study.

$$\frac{\partial g}{\partial i} \big|_{e=e^*} = \left(\frac{ai}{m}\right)^{\frac{i}{1-ai}} \left(\frac{1}{1-ai}\right)^2 (1 - ai + \ln\left(\frac{ai}{m}\right)) > 0 \quad (4)$$

PROPOSITION 3: Other things being equal, the higher the level of intelligence, the higher the grades. Grade is monotonically increasing in the level of intelligence, because grade is monotonically increasing in the effort level, and the optimum effort level is monotonically increasing in the level of intelligence.

$$\frac{\partial g}{\partial a} \big|_{e=e^*} = \left(\frac{ai}{m}\right)^{\frac{i}{1-ai}} \left(\frac{1}{1-ai}\right)^2 \frac{i}{a} (1 - ai + ai \cdot \ln\left(\frac{ai}{m}\right)) > 0 \quad (5)$$

PROPOSITION 4: Other things being equal, the higher the level of aggressiveness, the higher the grade. Grade is monotonically increasing in the level of aggressiveness, because grade is monotonically increasing in

the effort level, and the optimum effort level is increasing in the degree of aggressiveness.

A Student's Choice of Lobbying To determine whether to lobby for a better grade, a student considers the marginal benefit and the marginal cost of lobbying. He lobbies if the professor responds to lobbying with grade inflation, *and* if the marginal benefit is greater than the marginal cost of lobbying. The marginal cost of lobbying is c . The marginal benefit of lobbying can be calculated as follows: if the professor responds to lobbying with grade inflation, the utility is

$$U_j(e^*, l = 1) = [g(e^*) + \eta]^a - me^*$$

where η is the points the student receives from grade inflation.

If the student does not lobby, his utility is

$$U_j(e^*, l = 0) = [g(e^*)]^a - me^*$$

Hence, the marginal benefit of lobbying is

$$MB = [g(e^*) + \eta]^a - [g(e^*)]^a \quad (6)$$

Note that *the marginal benefit of lobbying is monotonically decreasing in the degree of intelligence.*

$$\frac{\partial MB}{\partial i} = \frac{\partial e^*}{\partial i} a(\ln(e^*)) (e^*)^i \left[\frac{1}{((e^*)^i + \eta)^{1-a}} - \frac{1}{((e^*)^i)^{1-a}} \right] < 0 \quad (7)$$

PROPOSITION 5: Other things being equal, the higher the degree of intelligence, the less likely the student is to lobby for a better grade.

Other things being equal, a higher level of intelligence not only increases the optimal effort level, but the studying is also more effective. As a result, the grade is high, and the marginal benefit of grade inflation is relatively low.

Marginal benefit of lobbying is monotonically increasing in the level of aggressiveness.

$$\frac{\partial MB}{\partial a} = \frac{\partial e^*}{\partial a} [(e^i + \eta)^a \ln(e^i + \eta) - e^{ia} \ln(e^i)] > 0 \quad (8)$$

It is intuitive that the marginal benefit of lobbying is monotonically increasing in the level of aggressiveness. The higher the degree of aggressiveness, the more important a good grade is to the student, and the higher the marginal benefit of lobbying.

PROPOSITION 6: Other things being equal, the higher the degree of aggressiveness, the more likely the student is to lobby for a better grade.

2.1.1 A Numeric Example

Consider the following numeric example of different levels of intelligence and aggressiveness. (Keep in mind that this example is different from a general

grade point example, as the highest grade is 5.8867.) Fix $m = 0.2$, $\eta = 0.02$.

Table 1 shows that the marginal benefit of lobbying is monotonically decreasing in intelligence, but monotonically increasing in aggressiveness. Table 2 shows that the optimal effort level is increasing in both levels of intelligence and aggressiveness.

Suppose the lobby cost is 0.01 for all students. If the professor responds to lobbying with grade inflation, students with a marginal benefit greater than 0.01 will lobby. By contrast, students whose marginal benefit is lower than 0.01 will not lobby. In tables 1, 2 and 3, the students who lobby are shown in shaded cells.

Table 1: Marginal Benefit

	$i = 0.55$	$i = 0.60$	$i = 0.65$	$i = 0.70$	$i = 0.75$
$a = 0.75$	0.0126	0.0120	0.0113	0.0105	0.0096
$a = 0.70$	0.0117	0.0111	0.0104	0.0097	0.0089
$a = 0.65$	0.0109	0.0103	0.0097	0.0090	0.0082
$a = 0.60$	0.0102	0.0096	0.0090	0.0084	0.0077
$a = 0.55$	0.0095	0.0090	0.0084	0.0078	0.0072

Table 2: Optimal Effort Level

	$i = 0.55$	$i = 0.60$	$i = 0.65$	$i = 0.70$	$i = 0.75$
$a = 0.75$	3.4288	4.3685	5.6887	7.6274	10.6292
$a = 0.70$	2.9006	3.5938	4.5187	5.7952	7.6274
$a = 0.65$	2.4695	2.9886	3.6511	4.5187	5.6887
$a = 0.60$	2.1116	2.5053	2.9886	3.5938	4.3685
$a = 0.55$	1.8098	2.1116	2.4695	2.9006	3.4288

Table 3: Grade

	$i = 0.55$	$i = 0.60$	$i = 0.65$	$i = 0.70$	$i = 0.75$
$a = 0.75$	1.9694	2.4221	3.0957	4.1463	5.8867
$a = 0.70$	1.7963	2.1544	2.6654	3.4210	4.5897
$a = 0.65$	1.6441	1.9288	2.3204	2.8741	3.6835
$a = 0.60$	1.5085	1.7351	2.0373	2.4484	3.0217
$a = 0.55$	1.3858	1.5659	1.7997	2.1074	2.5197

Same Effort Level, Different Grades Consider two students, student 1 ($i = 0.75, a = 0.70$) and student 2 ($i = 0.70, a = 0.75$). Student 1 is comparatively more intelligent but less aggressive, whereas student 2 is comparatively less intelligent but more aggressive. Both students have the same effort level (see table 2), but student 1 receives a better grade (see table 3). This example shows that students with the same effort level do not necessarily receive the same grades. In fact, intelligence is comparatively more important than aggressiveness in achieving a good grade.

Similar Grades, Different Lobbying Behaviors Student 2 ($i = 0.70, a = 0.75$) receives the third highest grade among all students (see table 3). Yet, he lobbies albeit his good grade. He is a typical aggressive good student. The following anecdote was recalled by a teaching assistant. A student (whose final grade was an A-) complained about her grade and lobbied for an “A”. The teaching assistant explained that the grade cannot be changed, and an “A-” is not a bad grade. The student replied: “But I *have to* get an A. I have never

received anything worse than an A.” Student 2 is an example of such a student. Nevertheless, a good student does not necessarily lobby. In fact, student 1 (who has a similar grade as student 2’s) does not lobby.

Information Asymmetry Table 3 shows that the professor cannot determine whether the students are to lobby by merely observing their grades. To predict whether a student is going to lobby, the professor needs the information about the student’s degrees of intelligence, i and aggressiveness, a . However, i and a are both private information unavailable to the professor.

Iso-grade Curves and Iso-Marginal Benefit Curves Figure 1 shows the iso-grade curves. The horizontal and vertical axes are the degrees of intelligence and aggressiveness, respectively. The slope of the iso-grade curves is the marginal rate of substitution between intelligence and aggressiveness.

$$MRS = \frac{\partial g / \partial i}{\partial g / \partial a} = \frac{1 - ai + \ln(\frac{ai}{m})}{\frac{i}{a} - i + i^2 \ln(\frac{ai}{m})}$$

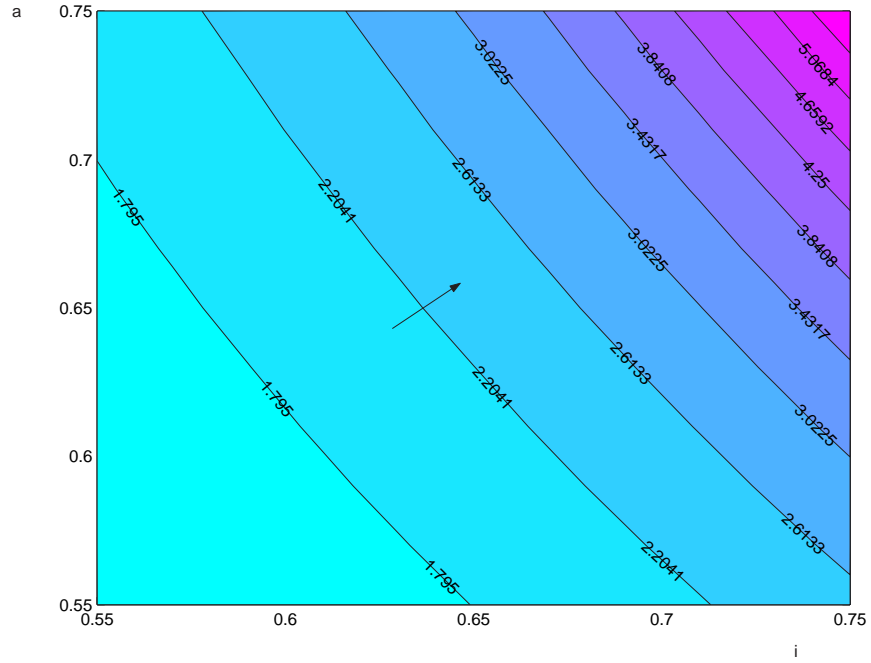
In this example, the slope is greater than one (in the absolute value), so the level of intelligence is more important than the level of aggressiveness in achieving a better grade. As propositions 3 and 4 suggest, the higher the degrees of intelligence and aggressiveness, the better the grade.

Figure 2 shows the iso-marginal benefit curves. Marginal benefit of lobbying is monotonically decreasing in the degree of intelligence, but monotonically

increasing in the degree of aggressiveness, as suggested by propositions 5 and 6.

Combine figures 1 and 2, one observes that if the lobby cost is sufficiently high, the proportion of bad and “high marginal benefit” student is greater than the proportion of good and “high marginal benefit” student.

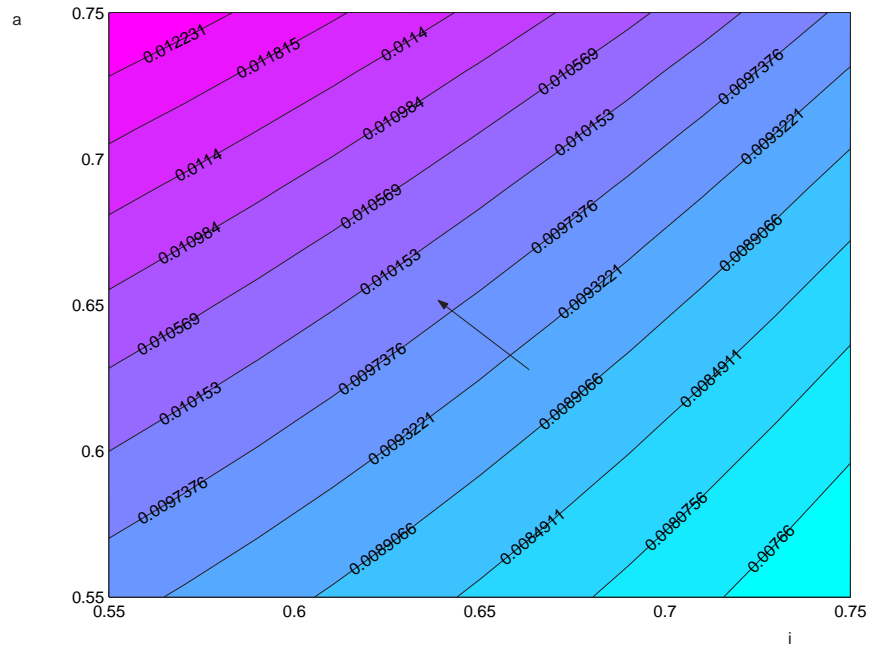
Figure 1: Iso-grade Curves



Assumption 2 (to be proved as a lemma) Let the set of bad students be B , and the set of high marginal benefit students be MB . Suppose the cost of lobbying is sufficiently high. Then,

$$Pr(B \cap MB) > Pr(B^c \cap MB) \quad (9)$$

Figure 2: Iso-Marginal Benefit Curves



2.1.2 Equilibria

One can guess and verify the following three equilibria: the “no grade inflation equilibrium”, the pooling equilibrium, and the separating equilibrium. Recall that the professor’s strategy is whether to inflate each student’s grade, and each student’s strategy is a pair of effort level and whether to lobby for a better grade.

The “No Grade Inflation” equilibrium To be completed

The Pooling Equilibrium Consider the following strategies of the professor and the students:

$$s_p(pooling) : \left(\{w_j^t\}_{j \in B, t=0,1,\dots,\infty} = \{1\}; \{w_j^0\}_{j \in B^c} = \{0\}; \right. \\ \left. \{w_j^t\}_{t=1,\dots,\infty} = 1 \text{ if } l_{t-1} = 1 \right)$$

$$s_j = \left(e_j = \left(\frac{ai}{m} \right)^{\frac{1}{1-ai}}; l_j^t = 1 \text{ if } MB > c \text{ and } w_j^t = 0; \text{ otherwise, } l_j^t = 0 \right)$$

The professor's strategy is to inflate grades for all bad students, but not to inflate grades for any good student, at $t = 0$. At $t = 1, \dots, \infty$, the professor inflates grades for whoever lobbies. Each student chooses his effort levels according to his degrees of intelligence and aggressiveness. Moreover, if the marginal benefit is greater than the cost of lobbying, *and* if the professor does not inflate the student's grade at time 0, the student will lobby. If the professor inflates the student's grade, *or* if the marginal benefit is lower than the cost of lobbying, the student will not lobby. As a result, bad students do not lobby (since their grades are inflated at time 0 already), and the better students who have high marginal benefit of lobbying lobby once, and the professor inflates the grades for bad students and the good students who lobby. In equilibrium, bad students do not lobby, but the threat of their lobbying induces the professor to inflate their grades. The payoffs for the professor and the students are expressed as follows:

$$U_p(s_p(pooling), s_j)_{j=1,2,\dots,J} = -I([Pr(B) + Pr(B^c \cap MB)] \times J) - N(Pr(B^c \cap MB) \times J)$$

$$U_{j \in B}(s_p, s_j = (e^*, l = 1)) = [g(e^*) + \eta]^a - me^*$$

$$U_{j \in B^c} \left(s_p, s_j = (e^*, l = 1) \right) = [g(e^*) + \eta]^a - me^* - c$$

$$U_{j \in B^c} \left(s_p, s_j = (e^*, l = 0) \right) = [g(e^*)]^a - me^*$$

where $e^*(a, i) = \left(\frac{ai}{m} \right)^{\frac{1}{1-ai}}$, and $Pr(\cdot)$ is the proportion of the set of students.

In equilibrium, the professor inflates grades for $Pr(B) \times J$ bad students at $t = 0$ and $Pr(B^c \cap MB) \times J$ better students at $t = 1$. Therefore, the professor's integrity cost is $I \left([Pr(B) + Pr(B^c \cap MB)] \times J \right)$. Her nuisance cost is $N \left(Pr(B^c \cap MB) \times J \right)$, because $Pr(B^c \cap MB) \times J$ students (the good and high marginal benefit students) lobby. The proof and the condition of the pooling equilibrium comes after the discussion of the separating equilibrium.

The Separating Equilibrium Consider the following strategies of the professor and the students:

$$s_p(separating) : (\{w_j^0\}_{j=1,2 \dots J} = 0; \{w_j^t\}_{t=1, \dots \infty} = 1 \text{ if } l_{t-1} = 1)$$

$$s_j = (e_j = \left(\frac{ai}{m} \right)^{\frac{1}{1-ai}}; l_j^t = 1 \text{ if } MB > c \text{ and } w_j^t = 0; \text{ otherwise, } l_j^t = 0)$$

The professor's strategy is not to inflate any student's grade at $t = 0$. At $t = 1$, she inflates the grade for whoever lobbies. Note that the student's strategy is the same as that in the pooling equilibrium. He lobbies if the marginal benefit is greater than the cost of lobbying *and* if his grade is not inflated in the first place. Otherwise, he does not lobby.

The payoffs for the professor and the students are expressed as follows:

$$U_p(s_p(\textit{separating}), s_j)_{j=1,2,\dots,J} = -I(Pr(MB) \times J) - N(Pr(MB) \times J)$$

$$U_j(e^*, l = 1) = [g(e^*) + \eta]^a - me^* - c$$

$$U_j(e^*, l = 0) = [g(e^*)]^a - me^*$$

Again, $e^*(a, i) = (\frac{ai}{m})^{\frac{1}{1-ai}}$. In the separating equilibrium, the professor does not inflate anyone's grade at $t = 0$. At $t = 1$, $Pr(MB) \times J$ students lobby. Hence, the professor's nuisance cost is $-N(Pr(MB) \times J)$. The professor then inflates these students' grades to send them away, so her integrity cost is $-I(Pr(MB) \times J)$.

A teaching assistant recalls the following anecdote. After a quiz, a student lobbied for half a point out of ten. The teaching assistant referred to the text book to prove that the student's wrong answer does not deserve half more point. Yet, the student kept arguing for half an hour until this teaching assistant finally gave up, saying: "OK, I just give you half a point to get rid of you." This is an example of the separating equilibrium— giving students the points they want to send them away.

The condition and proof of the separating equilibrium follows.

Pooling or Separating Equilibrium? Recall that the integrity cost function $I(\cdot)$ is concave, and the nuisance cost function $N(\cdot)$ is convex. It can be shown² that there exists only one $J^* > 0$ such that

$$\begin{aligned} & -I\left([Pr(B) + Pr(B^c \cap MB)] \times J^*\right) - N\left(Pr(B^c \cap MB) \times J^*\right) \\ & = -I\left(Pr(MB) \times J^*\right) - N\left(Pr(MB) \times J^*\right) \quad (10) \end{aligned}$$

That is to say, there exists only one J^* such that the professor's payoff is the same for the pooling and the separating strategies. Moreover,

$$\begin{aligned} & -I\left([Pr(B) + Pr(B^c \cap MB)] \times J\right) - N\left(Pr(B^c \cap MB) \times J\right) < \\ & \quad -I\left(Pr(MB) \times J\right) - N\left(Pr(MB) \times J\right) \quad \forall J < J^* \\ & -I\left([Pr(B) + Pr(B^c \cap MB)] \times J\right) - N\left(Pr(B^c \cap MB) \times J\right) > \\ & \quad -I\left(Pr(MB) \times J\right) - N\left(Pr(MB) \times J\right) \quad \forall J > J^* \end{aligned}$$

Hence, if $J > J^*$, the professor plays the pooling strategy; if $J < J^*$, the professor plays the separating strategy. The theory predicts that, other things being equal, *the larger the class size, the more likely the professor is to play the pooling strategy, which results in more grade inflation for the bad students.*

PROPOSITION 7. (The Pooling Equilibrium): Suppose $J > J^*$. Then, there exists a pooling equilibrium such that the professor inflates grades for all *bad* students at time 0, the good and high mar-

²See appendix A for the mathematical proof.

ginal benefit students lobby, and no bad students lobby.

PROOF: Given students' strategies, the professor may choose between pooling and separating strategies. Since $J > J^*$, the professor is better off by choosing the pooling strategy. Given that the professor inflates the grades for all bad students, no bad students lobby. The good students who have high marginal benefit lobby for better grades. In equilibrium, bad students never lobby. However, bad and high marginal benefit students pose a credible off equilibrium threat that induces the professor to inflate grades. \square

PROPOSITION 8. (The Separating Equilibrium): Suppose $J < J^*$. Then, there exists a separating equilibrium such that the professor does not inflate grades for any student at $t = 0$; the high marginal benefit students lobby and receive inflated grades at $t = 1$.

PROOF: If $J < J^*$, the professor is better off by choosing the separating strategy. According to the professor's separating strategy, any student who would like to receive an inflated grade must lobby at least once. The high marginal benefit students would then lobby once, while the low marginal benefit students do not lobby. In equilibrium, there are $Pr(MB) \times J$ lobbying occurrences, and the professor inflates the grades for $Pr(MB) \times J$ students. \square

2.2 Policy Implication

Changing Students' Cost of Lobbying Raising the lobbying cost discourages students from lobbying. For instance, a professor may re-grade the whole exam if a student asks for grade adjustments. The student is told that his grade “may go up as well as go down” after the re-grading. The uncertainty increases the student's lobbying cost. A professor may also require the student to submit a report of justification if he wishes to lobby for more points for an exam or a problem set. Such a requirement also raises the cost of lobbying.

The emergence of emails drastically decreases students' lobbying cost. To avoid students' lobbying, the professor may refuse to read students' emails. If students have questions, they are required to visit the professor during his office hours. Visiting the professor is more costly than merely sending an email, especially for shy students. If the lobbying channel of emails is cut off, lobbying may be reduced.

Changing Students' Gross Payoff of lobbying If the utility function of the grade were a step function, a student is more likely to lobby if his grade is close to the next higher rung. For example, if his grade falls into a B+ range but his grade is close to an A-, he is more likely to lobby than if his grade barely falls into the B+ range. A student whose grade is close to the next higher rung has a higher payoff in lobbying. The professor could avoid students' lobbying by curving in a way that no students' grades are close to the next higher rung.

3 Empirical Testing

The conclusion of section 2.1 suggests that, other things being equal, the larger the class size, the more grade inflation for bad students. In other words, *ceteris paribus*, *bad students receive better grades in large classes than those in small classes*. In this section, this conclusion is taken as a testable hypothesis.

Class size influences grades through the channel of students' lobbying as well as other channels. For instance, class size determines the interactions between the professor and the students. The larger the class size, the fewer interactions between the professor and the students, and the worse the learning, which reflects on students' grades. For bad students, the "size effect" (learning effect from class size) and the "nuisance effect" (grade inflation from class size) coexist but work in different directions in the class size variable.

Class Size Effect (Learning Effect) The empirical literature about size effect on education production have provided mixed results. Lazear (2001) offers a review about how some researchers find no, or even reversed, class size effects. He explains this "class size effect puzzle" with the theory of optimal class size. The optimal class size is positively associated with students' attention span and discipline in the classroom. In fact, several schools gather problematic students together in a small class, and the better-behaved students in a bigger class. Lazear suggests that reverse causation problem results in smaller or

reversed class size effect in the empirical literature. He also emphasizes that class size matters: “[p]reschool classes are smaller than large lecture classes for college students. How is this to be explained if class size is irrelevant?” (p. 778) Besides Lazear’s argument, Bedard and Kuhn (2005) also find class size has a large and negative and nonlinear effect on student’s evaluation on professors’ effectiveness, which may explain why so many parents are willing to pay extra tuition to send their children to private colleges with small class sizes.

Nuisance Effect The nuisance effect in this study suggests that the larger the class, the higher the grades for bad students due to grade inflation. If the magnitude of the nuisance effect outweighs that of the class size effect (learning effect), one would observe a positive association between grades and the interaction between bad students and class size.

3.1 Data

Office of Institutional Research Data for this study come mainly from the Office of Institutional Research (OIR), the University of California, Irvine.

The targeted data include all students who enter UC Irvine as freshmen and have taken econ 100A, B, C from fall 2001 to spring 2004: the grades students earned for econ 100 series, class size, students’ UCI GPA, high school GPA, SAT math and verbal scores, students’ major (economics or not), dummy variables that indicate whether students have passed several prerequisites, the

quarter in which the class was offered, and which professor taught the class.

Unfortunately, due to privacy and legal issues, the Office of Institutional Research (OIR) at UC Irvine is unable to provide the desired data. The OIR worries that a complete record of students' course grades, SAT scores, UCI GPA and high school GPA would make students identifiable even if the records were previously de-identified. After several discussions with the University of California attorneys, the OIR agreed to release the de-identified and additionally restricted data.

The first restriction in the data are students' grades. Instead of reporting raw grades, OIR labels A+, A and A- as A; B+, B, and B- as B; C+, C and C- as C; D+, D and D- as D. F and incomplete grades are labeled as "O". One cannot apply linear regression to the model, because the cardinal information is lost by the restriction. Instead, an *ordered probit model* is appropriate, since various grade levels are ordered by different rungs (the As, the Bs, and so on). If a student takes any class for more than one time, only the first grade is included in the sample. For instance, if a student failed econ 100A and then passed it when he retook it, only his first failing grade is recorded. This restriction is to maintain the consistency of students' grades and abilities.

The second restriction on the data are students' GPAs at UCI. GPAs are aggregated into several categories. For example, if the student's GPA is 2.47, it is labeled as "2.41-2.5". GPA below 1.9 are aggregated altogether.

Students' SAT math score, SAT verbal score, and high school GPA are combined to make an "admission index". The formula for this index is 400 times high school GPA, plus SAT math score, plus SAT verbal score.

Rather than providing the dummy variables that indicate whether students have passed several prerequisites such as econ 20A-C (basic economics courses), math 2A-B (calculus), math 3C (linear algebra), math 4 (math for economists), and so on, OIR provides "the number of prerequisite passed". For instance, if a student have passed econ 20A and math 2A before taking econ 100, the "number of prerequisite passed" is labeled as "2". The drawback of this restriction is the identical treatment of all prerequisites. For example, passing math 2A is assumed to have the same effect on the students' human capital as passing econ 20B does.

OIR also provides a major dummy variable that indicates whether students are econ majors or not. Yet, a double major in econ are mistakenly labeled as "non-major". This mistake can be potentially problematic.

WebSOC This study gathers professors' information from WebSOC, a web search tool provided by the UCI registrar. WebSOC keeps records, including professors' names and class enrollment of all UCI classes offered from winter 2001 up to date. The professors' names and class size provided by WebSOC are matched to the class size provided by OIR to determine the professor of each class.

However, the match might not be perfect. A small number of students registered or dropped classes at the end of the quarter. Therefore, there can be a small gap (such as two students) between the record of WebSOC and the data provided by OIR. Fortunately, since different classes have large variations in their sizes, the author is able to match most classes and the professors.

3.2 Model

The model seeks to explain grade inflation by students' lobbying and the interaction between bad students and the class size. The student's ordered grade for econ 100 series is the dependent variable. The interaction between bad students and the class size is the main explanatory variable. Students' grades reflect not only grade inflation but also the students' abilities and disciplines, which must be controlled for. The ordered probit model can be expressed as:

$$G = f(badsize, size, x\beta)$$

$$y^* = \beta_1 badsize + \beta_2 size + x\beta + \epsilon \quad (11)$$

where G is the ordered grade, y^* is the latent variable for the ordered grades, "size" is the class size, "badsize" is the interaction between bad students and class size, and $x\beta$ are the control variables. "badsize" is created by the following steps: the dummy variable "bad" is equal to one if the student's GPA is less

or equal to 2.85; otherwise, “bad” is equal to zero. (Note that the average UVI GPA of students are 3.055, 3.076, 3.099 for econ 100A, B, C, respectively.) “bad” is then interacted with class size, called “badsize”. The expected sign for “size” is negative according to the class size literature. The expected sign for “badsize” is positive, according to the nuisance theory discussed earlier.

Students’ abilities and discipline are controlled for by the admission index (the combination of SAT scores and high school GPA), UCI GPA, the number of prerequisite passed, and a major dummy variable. The expected signs of all these control variables are positive.

Econ 100A, B, and C are taught by different professors in different seasons (winter, spring, summer and fall). To control for professor and season specific effects, the author includes dichotomous variables for each. Econ 100A is offered in summer, fall and winter; 100B, winter, spring and summer ; 100C, spring, summer and fall. The academic year starts with the fall quarter. Econ 100A, B, and C are supposed to be taken in sequence and each is the prerequisite of the subsequent course. A well-motivated student takes econ 100 series in an academic year; that is to say, he takes econ 100A in the fall quarter, 100B in the winter quarter, and 100C in the spring quarter. Hence, other things being equal, for econ 100A, students are expected to do better in fall quarters than in the winter quarters. Likewise, for econ 100B, students are to do better in winter quarters than in the spring quarters, and for econ 100C, students are to

do better in spring quarters than in fall quarters. Moreover, students who took econ 100 C in a fall quarter may perform especially poorly, since there is a long summer (when few students study) between the fall quarter and the winter or spring quarter, when econ 100B is offered.

Unfortunately, the problem of collinearity makes it impossible to control for professor's tenure level, since there is only one tenured professor in the data.

3.3 Empirical Results

Tables 4, 5, 6 and 7 show the results of the ordered probit regression for econ 100A, B, C and all econ 100 students, respectively.

Class Size In general, class size is negatively associated with students' grades, 100B being an exception. The coefficient on class size is statistically significant at the one percent level and negatively associated with the grade of a student for the regression for 100C and the aggregated classes of the econ 100 series. The mixed result agrees with the literature that researchers have found both positive and negative class size effects (Lazear, 2001; Bedard and Kuhn, 2006).

“Badsize” The interaction between bad students and class size is the most interesting variable. The result shows that, other things being equal, bad students do better in a large class than those in a small class. “Badsize” is positive

and statistically significant at the one percent level for econ 100A, B and all econ 100 series, and statistically significant at the five percent level for econ 100C.

The nuisance theory of this paper suggests that bad students do better in larger classes because of the threat of students' lobbying. The empirical results agree with this prediction.

There could be other reasons for the students to do better in a large class. Other things being equal, proctoring an exam is harder in a larger class. Therefore, it is easier to cheat in a larger class, all else equal. Easy cheating could be the reason that bad students do better in large class. However, this argument is based on one assumption: cheating does help bad students to achieve better grades. According to anecdotal evidence, when bad students cheat, they tend to sit together in an exam to copy one another's *wrong* answers, because none of them studied. Not only does this kind of cheating not help bad students to achieve better grades, but cheating among the bad students makes it easier for the professor to catch them. Therefore, cheating is probably not why bad students do better in larger classes.

Control Variables Admission index, UCI GPA, number of prerequisites passed, and the major dummy variable control for students' discipline and aptitude. All these variables are expected to be positively associated with grades.

The sign of admission index are in general the same as we expected, 100B being an exception; but it is statistically insignificant. UCI GPA is positively associated with grades and statistically significant at the one percent level for all regressions.

The number of prerequisite passed are not statistically significant, and one of them (econ 100B) has the opposite expected sign. This is most likely due to the nature of the data. Recall that the Office of Institutional Research (OIR) is unable to provide data on each individual prerequisite, but only supplies the total number of prerequisite of each student. In reality, passing some prerequisites may be more important than passing other prerequisites. For example, Butler et al. show that the grade earned for a calculus class (math 170) has a greater positive impact on the performance of advanced microeconomics than the grade earned on basic macroeconomics class does (Butler, Finegan and Siegfried, 1998, p195). If so, other thing being equal, a student who passes two prerequisites could surpass a student who passes three prerequisites. The identical treatment of the prerequisites could cause the variable to be statistically insignificant, or to have the opposite expected sign.

The result of the variable “major” is perplexing. Economics majors are expected to outperform non-majors in these upper level econ classes. Nevertheless, the regression results of econ 100A and 100C show that majoring in econ is negatively associated with grades, although the results are both statis-

tically insignificant. One possible explanation for the wrong sign is that OIR have mistakenly labeled “double major in econ” as “non-major”.

Professor and Season Controls From fall 2001 to spring 2004, econ 100A was taught by four teachers; econ 100B by four professors, and econ 100C by five teachers. Professor and season dummy variables control for professor and season effects. Yet, there are several restrictions. For instance, for econ 100A, there is perfect collinearity between professor 2 and summer (econ 100A was offered in summer only once, taught by professor 2, and professor 2 did not teach 100A in any other seasons.). Therefore, professor 2 is dropped from econ 100A. Likewise, professor 10 is dropped from econ 100C because of perfect collinearity between professor 10 and summer. For all econ 100 series students, teachers’ genders are controlled for. Since there are only two female professors, one female professor is dropped. Interestingly, other things being equal, students of a female professor receive higher grades than their peers whose professors’ are male.

Table 4: Ordered probit estimates of econ 100A

Variable	Expected sign	Coefficient	(Std. Err.)	z value
size	—	-0.001	(0.001)	-1.19
badsize	+	0.001	(0.000)	3.35***
admin_index	+	0.001	(0.000)	3.06***
ucigpa	+	1.874	(0.142)	13.24***
major	+	-0.033	(0.083)	-0.40
num_preq_passed	+	0.010	(0.032)	0.31
professor_1	?	-0.220	(0.113)	-1.95*
professor_3	?	0.333	(0.100)	3.33***
winter	?	0.319	(0.167)	1.91*
fall	?	-0.042	(0.212)	-0.20
_cut1		4.516	(0.585)	
_cut2		4.984	(0.585)	
_cut3		6.577	(0.594)	
_cut4		7.964	(0.605)	

Teacher 2 is dropped due to collinearity.

Log likelyhood: -1080.1767; Num. of Observations: 1008

***: significant at 0.01 level; *: significant at 0.1 level.

Table 5: Ordered probit estimates of econ 100B

Variable	Expected sign	Coefficient	(Std. Err.)	z value
size	—	0.004	(0.001)	3.60***
badsize	+	0.002	(0.001)	2.81***
admin_index	+	0.000	(0.000)	-0.25
ucigpa	+	2.514	(0.135)	18.59***
major	+	0.117	(0.089)	1.32
num_preq_passed	+	-0.023	(0.034)	-0.67
professor_5	?	-0.050	(0.193)	-0.26
professor_2	?	-0.498	(0.200)	-2.49**
professor_7	?	0.308	(0.197)	1.56
winter	?	-1.466	(0.318)	-4.61***
spring	?	-0.545	(0.245)	-2.23**
_cut1		4.991	(0.616)	
_cut2		5.763	(0.617)	
_cut3		7.135	(0.627)	
_cut4		8.922	(0.646)	

Log likelyhood: -982.1631; Num. of Observations: 934

***: significant at 0.01 level; **: significant at 0.05 level.

Table 6: Ordered probit estimates of econ 100C

Variable	Expected sign	Coefficient	(Std. Err.)	z value
size	—	-0.184	(0.056)	-3.27***
badsize	+	0.001	(0.000)	2.21**
admin_index	+	0.000	(0.000)	0.96
ucigpa	+	2.215	(0.169)	13.13***
major	+	-0.005	(0.096)	-0.05
num_preq_passed	+	0.029	(0.036)	0.79
professor_8	?	-5.628	(1.996)	-2.82***
professor_9	?	-4.173	(1.564)	-2.67***
professor_2	?	0.710	(0.363)	1.96**
fall		7.404	(2.560)	2.89***
spring		41.730	(13.207)	3.16***
_cut1		-11.085	(4.903)	
_cut2		-10.549	(4.901)	
_cut3		-9.151	(4.900)	
_cut4		-7.546	(4.898)	

Professor 11 is the baseline professor. Teacher 10 is dropped due to collinearity.

Log likelyhood: -886.91601; Num. of Observations: 815

***: significant at 0.01 level; *: significant at 0.1 level.

4 Conclusion

This study discussed the possibility of grade inflation under the threat of students' nuisance with a game theoretic model. The model suggest that, *ceteris paribus*, the larger the class size, the more grade inflation for bad students. The empirical results show that bad students receive higher grades in a large class than those in a small class. This evidence supports the results of the game theoretic model.

There are more to be done, though. First, the data requested from the OIR are too restrictive. Useful information was lost in the banding of the data. Moreover, the study lacks data that that directly link grades to students' nui-

Table 7: Ordered probit estimates for all econ 100 students

Variable	Expected sign	Coefficient	(Std. Err.)	z value
size	—	-0.002	(0.000)	-4.25***
badsize	+	0.001	(0.000)	3.38***
admin_index	+	0.000	(0.000)	2.63**
gpa	+	2.076	(0.087)	23.73***
num_prereq_passed	+	0.001	(0.019)	0.04
major	+	0.001	(0.051)	0.01
100B	?	-0.331	(0.204)	-1.62
100C	?	-0.054	(0.245)	-0.22
fall	?	-0.248	(0.218)	-1.14
winter	?	-0.059	(0.184)	-0.32
spring	?	-0.245	(0.229)	-1.07
female	+	1.121	(0.433)	2.59**
professor1	?	0.564	(0.432)	1.30
professor2	?	0.462	(0.377)	1.22
professor4	?	0.716	(0.433)	1.65
professor5	?	-0.525	(0.232)	-2.26**
professor6	?	1.190	(0.427)	2.79***
professor7	?	1.106	(0.436)	2.54**
professor8	?	0.586	(0.384)	1.53
professor9	?	0.815	(0.384)	2.12**
professor10	?	0.608	(0.451)	1.35
_cut1		4.753	(0.512)	
_cut2		5.345	(0.512)	
_cut3		6.767	(0.515)	
_cut4		8.316	(0.521)	

Professor 3 is dropped due to collinearity. Teacher 11 is the baseline teacher.

Log likelihood: -2999.2948; Num. of Observations: 2757

***: significant at 0.01 level; **: significant at 0.05 level.

sance behavior. Future work for this project includes acquiring less restrictive data and conducting a survey that studies students' nuisance behavior and grades directly.

A Mathematical Proof

Let $Pr(B) + Pr(B^c \cap MB) = \alpha$, $Pr(MB) = \beta$, $Pr(B^c \cap MB) = \gamma$. Obviously, $\alpha > \beta > \gamma$. It can be shown that with the professor's convex nuisance cost function, $N(\cdot)$ and the concave integrity cost function, $I(\cdot)$, \exists only one $J^* > 0$ such that

$$I(\alpha J^*) + N(\gamma J^*) = I(\beta J^*) + N(\beta J^*)$$

PROOF: Suppose there exists $J_0 > 0$ and $J_1 > 0$ ($J_0 \neq J_1$) such that the above equality holds. Without loss of generality, assume that $J_1 > J_0$. Recall that $N'(\cdot) > 0$, $N''(\cdot) > 0$, $I'(\cdot) > 0$, and $I''(\cdot) < 0$. Thus,

$$I(\alpha J_0) + N(\gamma J_0) = I(\beta J_0) + N(\beta J_0)$$

$$I(\alpha J_1) + N(\gamma J_1) = I(\beta J_1) + N(\beta J_1)$$

which implies

$$I(\alpha J_0) - I(\beta J_0) = N(\beta J_0) - N(\gamma J_0)$$

$$I(\alpha J_1) - I(\beta J_1) = N(\beta J_1) - N(\gamma J_1)$$

Take derivative with respect to N at both sides,

$$\alpha I'(\alpha J_0) - \beta I'(\beta J_0) = \beta N'(\beta J_0) - \gamma N'(\gamma J_0)$$

$$\alpha I'(\alpha J_1) - \beta I'(\beta J_1) = \beta N'(\beta J_1) - \gamma N'(\gamma J_1)$$

Since $N(\cdot)$ is convex, $\beta N'(\beta J_1) - \gamma N'(\gamma J_1) > \beta N'(\beta J_0) - \gamma N'(\gamma J_0)$. Therefore, $\alpha I'(\alpha J_1) - \beta I'(\beta J_1) > \alpha I'(\alpha J_0) - \beta I'(\beta J_0)$, which implies that

$$\alpha[I'(\alpha J^0)] - \alpha I'(\alpha J_1) < \beta[I'(\beta J^0)] - \beta I'(\beta J_1) \quad (12)$$

Define $f(\phi) = I'(\phi J_0) - I'(\phi J_1)$, then

$$f'(\phi) = J_0 I''(\phi J_0) - J_1 I''(\phi J_1) > 0$$

since $I'' < 0$ and $J_1 > J_0$. Therefore, $f(\phi)$ is increasing in ϕ , which contradicts equation (12). \square

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