Mind Changes and the Design of Reporting Protocols*

Wei Li†

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Abstract

In organizational, political, and financial settings, information is collected and reported by experts as it is received over time. This paper studies, in such dynamic situations, the incentives of an expert with reputational concerns to reveal his most recent information and the reporting protocol that induces the most truthful revelation. A principal receives sequential reports from an agent with privately known ability, who privately observes signals about the state of the world. The agent's signals are of different initial quality and, in contrast to previous work, also of different quality *improvement*. First, when a talented agent also improves faster, "mind changes" (inconsistent reports) may be a sign of high ability, whereas a mediocre agent still tends to repeat his early report. Second, requiring sequential reports creates an incentive to misreport the final, more accurate signal, but requiring a single report can only extract the agent's final, and not interim, opinion. As a result, sequential reports dominate when the principal's optimal decision is very sensitive to the reports' accuracy. A single report dominates when either the mediocre agent's signals improve faster, or when the agent is very unlikely to be talented.

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JEL classification: D82, C70, M50

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†University of California, Riverside. Email: wei.li@ucr.edu.

When the facts change, I change my mind. What do you do, sir? — John M. Keynes[1]

1 Introduction

In most economic models of communication, information is collected once and transmitted in a single piece from a sender to a receiver (Crawford and Sobel 1982, Aghion and Tirole 1997, Morris 2001). In many realistic settings, however, a sender receives *multiple* pieces of information over time and is asked to convey his opinion *multiple* times as more information comes in.[2] Formal sequential reports are frequently observed in congressional committees, accounting (Dye and Verrecchia 1995), capital budgeting (Arya and Sivaramakrishnan 1997), and the financial market (Penno 1985). Informally, consultants, doctors, and other professionals are often asked to convey their early opinions before giving a final report.

In many of these environments, information received later by the sender may contain less noise than that received earlier, and is thus of higher quality. For instance, a December survey of consumer demand for hybrid cars next year is likely to be more accurate than a similar one taken in August. Moreover, the sender's ability to observe the underlying true state of the world may improve as he becomes more familiar with the task at hand. In either case, the sender typically cares about how all his reports reflect on his ability. This paper investigates how an agent of privately known ability reacts strategically to improvement in the quality of his information under a sequential reports system. It also applies these insights to the optimal choice of reporting protocols. Namely, it identifies conditions under which the principal should require a report after the agent has received all the information, and conditions under which she should ask for sequential reports instead.

In the model, an agent delivers an interim report and a final report about the state of the world, based on his sequence of increasingly accurate private signals. After each signal, the agent sends a report to the principal, who makes a decision after the final report. Next, the true state becomes observable to all. The same game is repeated in the second stage. The agent can be of two privately known types: smart (type $\boldsymbol{H}$) or mediocre (type $\boldsymbol{L}$). A smart agent and a mediocre

[1] In reply to accusation of inconsistency: *The Economist*, 1999-12-18, p.47.

[2] Throughout the paper, the receiver of information who then makes decisions ("the principal") is female and the sender who receives and transmits information ("the agent") is male.

one differ not only in the *level* of signal quality, but also in the *slope* of signal quality improvement. A smart agent learns about the true state of the world with higher initial accuracy than a mediocre one, but his signal quality improvement may be higher or lower than that of a mediocre agent. In the second stage, the agent is paid the expected value of his information, which depends on how smart he is perceived to be. As a result, the agent has an incentive to boost his reputation.

The first main insight emerging from this model is that mind changes, or inconsistent reports, may signal high ability in equilibrium. This can happen when a smart agent's signals improve faster than those of a mediocre agent. Since the smart agent is more likely to receive and report an accurate first signal, a mediocre agent might want to "defend" his early report even when he receives conflicting signals. Thus, similar to Prendergast and Stole (1996), an agent may stick to a position that he gradually realizes is likely to be wrong because changing his mind makes him appear incapable of finding the true state of the world earlier. However, unlike in some existing models (Scharfstein and Stein 1990, Prendergast and Stole 1996), a mediocre agent is more likely to give consistent reports even though such consistency per se may indicate *low* ability in equilibrium.

The reason for this paradoxical result is that both consistency and accuracy matter in determining the agent's future wage, which is shown to be a convex function of his perceived ability. The wage is convex because the principal's second stage optimal decision may depend crucially on the reports' accuracy. For example, when a wrong decision leads to huge losses, an agent perceived to be very likely smart provides information of disproportionately higher value, and accordingly receives a disproportionally higher wage than a mediocre one. Specifically, being consistently right leads to the highest possible wage in the second stage, whereas being consistently wrong leads to the lowest. Therefore repeating one's early report is a risky "gamble" to receive the highest wage. Since a type L agent's improvement in signal quality is smaller, he is more willing to take on this gamble at the potential cost of appearing consistently wrong. Furthermore, the more convex is the wage function, the more likely a mediocre agent disregards his final signal and lies to appear consistent. Type H, however, has more confidence in his information *improvement*, and thus is willing to change his mind and follow his second, more informative signal when signals differ.[3]

[3] Some experimental and sociological evidence for increasing commitment to a wrong project is consistent with this model's prediction. See, for example, Staw (1976, 1981, 1992) and the references within. Wicklund and Braun

The second main insight of the model is therefore, when the principal's second stage decision depends strongly on the agent's report accuracy, mind changes are valued more as a sign of ability. That is, before the true state is observed, the agent who gives conflicting reports is more likely to be smart than one who gives consistent ones. This matters because the principal can improve her first stage decision using this sequencing information. When the principal's optimal decision is independent of the agent's type, i.e., her decision only depends on the reports, consistent reports are valued more in equilibrium. This result suggests that mind changes are valued more in professions and settings where optimal decisions are so sensitive to information accuracy that the wage function is highly convex. Examples of such professions and settings include major economic reforms, wars and high stake financial maneuvers.

The model so far demonstrates that truthful sequential reports may not be in the agent's interest due to reputational concerns. The principal, however, is only concerned with the accuracy of the reports and may want to choose a reporting protocol to encourage truthful reporting. One natural question is whether the principal should require sequential reports at all, given that the mediocre agent may repeat his initial signal and fail to convey his later, higher quality information. It would seem that requiring a single report (or reports) *after* the agent has received all signals is optimal because it eliminates the incentive to appear consistent.

An answer to the above question, and the third main insight of this paper, is that the optimal reporting protocol depends on how strongly the principal's decision depends on the reports' accuracy. The advantage of a final report system is that the agent will only be judged on its accuracy, thus he will report his best estimate of the state (his final signal) truthfully. The disadvantage is that the agent ignores his informative initial signal. Therefore if the principal's optimal decision depends primarily on which state of the world is more likely, a final report is optimal: the agent has no incentive to appear consistent by distorting his final report. Moreover, a final report system is also preferable when the mediocre agent's signals improve faster. On the other hand, the sequential reports system is optimal when the exact likelihood of each state is crucial to the principal's optimal decision. The two reports (even though the final report may be distorted) offer the principal

(1987) show that people who are more confident in their ability seem to be less committed to their early positions than the less confident ones.

finer information and may lead to a better decision than one truthful report under the final report system. Moreover, the sequencing of reports (whether reports are consistent or not) provides a better estimate of the agent's ability.

It is important to emphasize that this result hinges on the *timing*, not the *number*, of the reports. Despite the seeming similarity, it is shown that the sequential reports system cannot be replicated by requiring two reports at the end. Under the sequential reports system, an agent always reports his initial signal truthfully, even though he may distort his final report to appear consistent. If the principal requires both reports at the end, then the agent simply repeats his final signal, which is the agent's best estimate of the state, in order to appear both consistent and accurate. As a result, his first signal is lost in equilibrium, just like when one final report is required.

Previous research has shown that in a multi-agent setting, economic agents may want to be consistent with some early movers or existing consensus because they want to increase the market's perception of their ability (Scharfstein and Stein 1990).[4] In reputational herding models such as Scharfstein and Stein (1990), an agent wants to conform to the early mover because of a "smart people think alike" effect: smart agents receive signals that are correlated conditional on the state. Controlling for any information learned from the earlier mover, if smart agents' observations are independent conditional on the state, each agent will report according to his own signal and there will be no reputational herding or the incentive to appear consistent. Here, both reports are associated with the agent and thus reputational concerns distort his reports even when signals are conditionally independent.

Ottaviani and Sorensen (2005) analyze a static reputational cheap talk game with very general distributions of the state and the agent's (expert's) type. They find that full revelation, or truthtelling is generically impossible in this type of game. Either no informative equilibrium exists, or the expert can only communicate part of their information, for example, "high" or "low" despite a rich signal and message space. The current model adopts simple distributions of the agent's type and the state to zoom in on the dynamic aspect of the agent's incentive problem. That is, the focus

[4] Another reason is that they incorporate the information contained in the earlier actions before making their own decisions, as in the statistical herding models such as Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992).

is on the interaction between the agent's initial and final report given his reputational concerns.

More closely related to this paper, Prendergast and Stole (1996) consider a reputational concerns model in which a manager with privately known ability receives noisy signals about the true profitability of his investments over time, and the more capable manager receives signals with higher precision. In their model, the market infers each manager's precision from the period to period change in his investment choices. Initially, large changes indicate high quality information and therefore high precision relative to the prior, and each manager exaggerates out of reputational concerns. But eventually changes in investment indicate (many) past errors and everyone becomes too conservative. Therefore, exaggeration is beneficial only because the agent has no reputational stake in the prior, and "admitting" that a previous investment choice was bad always hurts reputation. One of the main insights of the present paper is that due to improvement in signal quality, admitting a previous mistake can indicate high ability in equilibrium. More generally, a goal of this paper is how such improvement in signal quality affects the agent's incentives to report truthfully and what type of reporting protocol can elicit the most truthful reports.

The paper proceeds as follows: Section 2 presents the model. Section 3 characterizes the equilibria and shows that, with improving signal quality, mind changes may signal high ability in equilibrium. Section 4 identifies the optimal reporting protocols in different environments. Several key assumptions and simple extensions are discussed in Section 5. Section 6 concludes. All proofs are collected in the appendix.

2 The Two Signal Model

A principal needs to make a decision based on two sequential reports from an agent. Although the model is clearly more general, this paper will couch it in a concrete story: the owner of a company needs to make an investment decision after reviewing a consultant's initial report (m_0) and final report (m_1) on a project's profitability. The profitability depends on the true state of world s, which is ex ante good or bad ($s \in \{g, b\}$) with equal probability. It is easiest to equate state with profitability: no investment yields zero, while investment brings profit g and b (net of investment cost) when the state is g and b respectively.[5] Moreover, it is assumed that the principal does not invest without further information, i.e., $g + b \leq 0$.

Before setting up the sequential reports game formally, it may be useful to provide some real world examples of the situations this model describes. First, in an application to stock markets, an analyst receives multiple pieces of information about a company over time and releases multiple stock recommendations. Eventually the company's true profitability becomes known and the investors can evaluate the analyst's ability. Second, in an application to the political arena, a politician announces a policy reform according to his private information. Later he receives new information and needs to decide whether to continue the reform or to change course. Later the truth becomes observable and the voters can discipline the politicians through elections.

2.1 Environment and Information

The agent works in two stages $N = 0, 1$.[6] In each stage, the true state of the world s is, independently, either good (g) or bad (b). Events *within stage* 0 proceed as follows:

- At $t = 0$: the agent gets a fixed wage w_0;
- At $t = 0.5$: the agent receives his first signal $i_0 \in \{g, b\}$ and then sends an initial report $m_0 \in \{g, b\}$ as to which state his initial signal indicates;
- At $t = 1$: the agent receives his second signal $i_1 \in \{g, b\}$ and then sends a final report $m_1 \in \{g, b\}$ as to which state his second signal indicates;
- At $t = 2$: the principal makes the investment decision $a \in \{0, 1\}$ based on the reports;
- At $t = 2.5$: the true state of world becomes observable to all but not verifiable.

Stage 1 repeats the above process: the agent receives a fixed wage w_1 and then delivers two sequential reports. The principal makes her investment decision and the game ends. Timing of this

[5] The nontrivial case is when $g > 0$, $b < 0$.

[6] Some career concern models such as Scharfstein and Stein (1990) employ a reduced form second stage in which the agent's wage is his posterior probability of being talented. Modeling two full stages, however, makes it possible to study explicitly the *shape* of the agent's wage function in the second stage, which influences the agent's truthtelling incentives in the first period. See Lemma 1 for a characterization of the wage function.

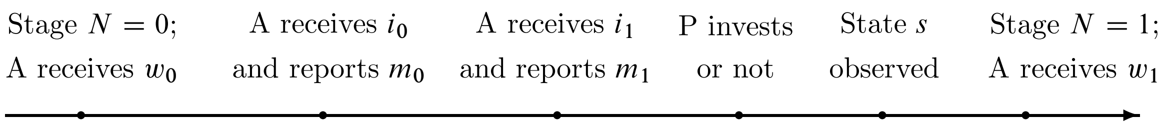


Figure 1: Timeline

game is illustrated in Figure 1. Notice that even though the principal only decides after the agent's reports, it is not crucial to the results of the model as long as she can observe the second report. In the examples above, the investors or voters may take some action based on the early report. As long as they can observe the second report, incentives similar to the current model's would arise because the principal can still use both reports and the (later) realized state to evaluate the agent.

The agent is one of two types: $\theta \in \{H, L\}$. An agent is smart (type H) with probability η and mediocre (type L) with probability $1 - \eta$. While the distribution of the state and that of the agent's type are common knowledge, only the agent knows his type. The agent receives two private signals i_0, i_1, which are independent conditional on the state. Let $p_0 \equiv Pr(i_0 = s|H, s)$ and $p_1 \equiv Pr(i_1 = s|H, s)$ denote the qualities of type H's signals. Assume that type L's initial signal is uninformative, and let $r \equiv Pr(i_1 = s|L, s)$ be the quality of type L's second signal.[7] The second signal is assumed to be more accurate than the first for both types of agent:

$$p_1 > p_0; \quad r > \frac{1}{2}.$$

The above specification combined with the symmetric distribution of the state means that the initial signal i_0 itself is not informative about ability, i.e., $Pr(i_0 = g|H) = Pr(i_0 = g|L) = \frac{1}{2}$.[8] This allows the analysis to focus on the dynamic incentive problems due to the improvement in signal quality because in equilibrium the agent is not tempted to lie in his first report. Moreover, type H's signals are assumed to be more accurate than the corresponding ones of type L, that is, $p_0 > \frac{1}{2}$; and $p_1 \geq r$.[9]

[7] The assumption that type L's first signal is uninformative simplifies the analysis. As long as type H's initial signal is more accurate, the main results hold with slight modifications.

[8] Allowing asymmetric state distribution introduces potential lying in the agent's first report in addition to the dynamic incentive problems. For example, when state g is much more likely than state b, the smart agent is more likely to observe state g because his initial signal is more accurate. This gives type L an incentive to report $s = g$ with some probability even when his first signal is b. This effect surfaces in Prendergast (1993) and Prat (2005).

[9] Which type is smarter is not well defined if $r > p_1$.

2.2 Payoffs

The principal and the agent are risk neutral, but the principal cannot transfer the ownership of the project to the agent (e.g. due to credit constraints). Let m^N be the history of reports in stage N and Π^N be the stage N profit. Also let $\hat{\eta} \equiv Pr(H|m^0, s)$ be the principal's posterior estimate of the agent being type H, given his first stage reports as well as the observed state, then:

$$\Pi^0 = \sum_{m^0} [Pr(g, m^0)g + Pr(b, m^0)b]a(m^0, \eta); \quad \Pi^1 = \sum_{m^1} [Pr(g, m^1)g + Pr(b, m^1)b]a(m^1, \hat{\eta}).$$

The principal chooses action $a \in \{0, 1\}$ after each report sequence m^N to maximize her net expected profit: $E\Pi = \Pi^0 - w_0 + E[\Pi^1 - w_1|m^0, s]$.

The agent cannot be paid conditional on the accuracy of reports because the true state s is assumed to be unverifiable (though observable), and no contract can be written on the reports as is standard in the cheap talk literature. Thus the agent is only motivated by his second stage wage. Assuming the principal operates in a perfectly competitive environment, the agent's second stage wage is simply the expected value of his information conditional on the principal's updated belief of him being type H. Thus all rents accrue to the agent if he is perceived as talented.[10]

Assume the agent reports truthfully in the second stage (later shown to be part of an equilibrium). Let $a^*(m^1, \hat{\eta})$ denote the principal's optimal action given reports m^1 and the posterior estimate of the agent's talent $\hat{\eta}$. The agent's wage is $w(\hat{\eta}) = \Pi^1(m^1, \hat{\eta})|_{a=a^*} \equiv V(a^*(\hat{\eta}))$.[11]

Lemma 1 *(1) $w(\hat{\eta})$ is a convex, non-decreasing and piecewise-linear function of $\hat{\eta}$, the posterior probability that the agent is smart; (2) $w(\hat{\eta})$ is affine and strictly increasing if the principal's optimal action $a^*(m^1, \hat{\eta})$ is independent of $\hat{\eta}$.*

As shown by Blackwell (1953), the value function of the agent's information is convex in the principal's beliefs about the agent's type. This is because the principal can make better (and potentially different) decisions given two different posterior distributions of the agent's type than

[10] As a result, the principal is only concerned about her first stage profit in choosing optimal reporting protocol. If the expert market is not perfectly competitive, then the principal may gain (partially) from her updated knowledge of the agent's talent in the second stage. This is discussed further in part B of Section 5.

[11] The agent's wage is the value of his information over what the principal would obtain by default, which is zero because her optimal decision without further information is assumed to be no investment.

when she is constrained to make the best decision given a convex combination of these two type distributions. Intuitively, imagine that the agent's type is known in the second stage. Then, for any given report sequence, the principal can choose the most profitable action given the agent's type. Thus she can do no worse than if she has to choose an action knowing only the agent's type distribution.

Example 1: A simple convex payoff function. Suppose that state $s = b$ is sufficiently bad that the principal is only willing to invest if she strongly believes that $s = g$. Then the implicit incentives here are straightforward:

$$w(\hat{\eta}) = \begin{cases} 0, & \text{if } \hat{\eta} < \eta_1 \\ \tau_1(\hat{\eta} - \eta_1), & \text{if } \hat{\eta} \in [\eta_1, \eta_2) \\ \tau_2(\hat{\eta} - \eta_2) + \tau_1(\eta_2 - \eta_1), & \text{if } \hat{\eta} \in [\eta_2, 1] \end{cases}$$

where $\tau_1 < \tau_2$.[12] The agent is "fired" after the first stage if his perceived ability is below cutoff value η_1. Otherwise, he is retained and his wage depends on where $\hat{\eta}$ falls in: he gets either a good wage or a star wage. Intuitively, even the best news from a (likely) mediocre agent is not enough to change the principal's decision from no investment (the default) to investment. But good news from an agent quite likely to be smart may induce her to invest and get higher expected profit. □

The exact shape of the payoff function depends on the difference of signal quality between types as well as the project-specific values g, b. One special case is when the principal's optimal action depends only on the reports she receives regardless of the agent's type. This occurs when $rg + (1-r)b \geq 0$, that is, report sequences (g, g) or (b, g) from even a mediocre agent yield a non-negative expected profit. Thus the principal will invest if there is *any* probability that the agent is smart and the reports are positive. And the wage function $w(\hat{\eta}) = \frac{rg+(1-r)b}{2} + \frac{g-b}{2}(p_1 - r)\hat{\eta}$.

Albeit simple, this lemma shows that the reduced form approach used in many reputational concerns models is a special case. In those models, the agent maximizes the posterior probability that he is smart because his future wage is linear in such probability (Scharfstein and Stein 1990,

[12] This requires $\frac{r}{2}g + \frac{1-r}{2}b \leq 0$, and $(1-p_0)p_1 g + p_0(1-p_1)b > 0$. The first inequality means that report sequence (b, g) from type L is not good enough news about the state to warrant investment, while the second one means that the same sequence from type H is. More formally, $\tau_1 = \frac{1}{2}[g(p_0 p_1 - \frac{r}{2}) + b((1-p_0)(1-p_1) - \frac{1-r}{2})]$, $\tau_2 = \frac{1}{2}(g-b)(p_1 - r)$.

Prendergast and Stole 1996). Such an approach implicitly assumes that the principal's future decision problem is not very sensitive to the agent's forecasting accuracy. One economic implication, to be explored partially below, is that in professions where key information is provided by experts most concerned about their reputation, the implicit incentive itself may be convex. Therefore even if the agents themselves are risk neutral, the implicit incentive structure encourages risk-taking behavior. Moreover, the higher are the premiums on the accuracy of expert's advice, the more convex the implicit incentives become and the experts may take on bigger risks.

2.3 Equilibrium

The analysis adopts the concept of Perfect Bayesian Equilibrium (PBE), in which the agent's strategy is a function that maps his type, his signals, as well as the history of reports (if any) to new report(s). In a PBE, the principal infers the agent's true signals and updates her belief about his type from the reports before making a decision. The agent's reports depend on the principal's inference. In the first stage, the strategy of the agent is $m_0 : \Theta \times I_0 \rightarrow \Delta(g, b)$ and $m_1 : \Theta \times I_0 \times M_0 \times I_1 \rightarrow \Delta(g, b)$. His strategy in the second stage is similarly defined. The equilibrium consists of a triple $(m^*, a^*, \hat{\eta})$ such that: $m^*(\theta, I) = \text{argmax}_m \; Ew(\hat{\eta})$; and $a^* = \text{argmax}_{a \in \{0,1\}} \; \Pi(a, m)$, where $\hat{\eta}$ is the principal's posterior belief that the agent is smart given his strategy. This belief is updated by Bayes' rule whenever possible.

Two well known equilibrium multiplicity problems exist in cheap talk games. First, there always exist "babbling" equilibria in which all messages are taken to be meaningless and ignored by the receiver.[13] The following analysis restricts attention to characterizing informative equilibria and to identifying when they exist. Because the principal requires multiple reports in this model, there may exist a particular type of informative equilibrium in which one report is useful but the other is completely uninformative, i.e. no information can be transmitted in that report in any equilibrium due to the agent's incentive problems.[14]

[13] Papers such as Farrell (1993) argue that the babbling equilibria are frequently implausible in games with some common interest. Blume, Yong-Gwan, and Sobel (1993) show that they are unstable in the long run from an evolutionary viewpoint.

[14] This differs from standard babbling equilibrium in which the agent babbles because the principal thinks it is meaningless. In the informative equilibrium here the agent babbles in one report because the signal cannot be revealed

Second, there exists an unimportant type of multiple equilibria: because the meaning of messages in cheap talk games is endogenously determined in equilibrium, any permutation of messages across meanings yields another equilibrium. This paper deals with this problem by assuming that both the principal and agent use and understand the *literal* meaning of the reports; whether they think the reports are credible depends on the equilibrium strategies.[15]

3 Equilibrium Information Revelation

This section categorizes the equilibrium strategies of the principal and the agent, focusing on how information revelation depends on both the initial difference and the improvement in the agent's signal quality.

First, there always exists a truthtelling equilibrium in the second stage such that the agent always reports both signals truthfully. The reason is that the agent's wage $w(\hat{\eta})$ does not depend on his second stage performance, and he has no further reputational concerns because the second stage is the end of his career. Since there is no conflict of interest between the principal and the agent, it is assumed that this truthtelling equilibrium is always played in the second stage. What is interesting is the agent's equilibrium behavior in the first stage.

In the first stage, assume that both types of agents report the first signal i_0 truthfully (shown later to be part of the equilibrium). Without loss of generality, the agent's continuation pure strategy after receiving i_1 is either: always report true i_1; or always repeat $m_0 = i_0$.[16] Observe that it cannot be an equilibrium for type H to always report i_1 truthfully and for type L to always repeat his first report regardless of i_1, or vice versa. Suppose so, then type H and type L can be distinguished perfectly on the equilibrium path when $i_0 \neq i_1$, in which case L has a

in any equilibrium.

[15] See also Footnote 16 for an example. Myerson (1989) and Farrell (1993) show that this type of multiple equilibria disappears in a rich language such as English, because both the sender and the receiver may use the literal meaning of a message but may not believe its content.

[16] By restricting attention to the literal meaning of messages, a lot of uninteresting equilibria are eliminated. For example, here the agent can use other strategies such as always reporting the opposite of i_0 or i_1. But it does not change the essence of the equilibrium if each type uses an opposite strategy because one can simply redefine i_0 or i_1. That is, suppose there exists a full revelation equilibrium in which everyone reports the opposite of their true signals and the principal knows that the reports are the opposite of the signals, then this equilibrium is equivalent to one in which everyone just reports the true signals.

strong incentive to deviate and pretend to be H. Therefore there can be at most three possible continuation equilibria: a "full revelation equilibrium" in which both types of agent report their second signal truthfully; a "full pooling equilibrium" in which both types simply repeat their initial report, and finally, a "partial revelation equilibrium" in which the agent plays a mix strategy by repeating his initial report with some probability.

3.1 Signal Quality Improvement and the Agent's Equilibrium Incentives

This subsection focuses on how the agent's incentive to report his second, more informative signal truthfully depends on his type and the signal quality improvement. Since the agent of both types receives signals of increasing quality, it is necessary to define a measure of signal quality improvement. A smart agent is considered to improve faster than a mediocre one if the following condition holds:

$$\frac{1-r}{r} \geq \frac{p_0(1-p_1)}{p_1(1-p_0)}, \tag{1}$$

while a mediocre agent is considered to improve faster if it does not. The left hand side of inequality (1) measures the probability ratio that a type L agent's second signal is wrong versus his second signal is right; the right hand side is the same ratio for type H. This inequality compares the confidence of an agent in his second signal *relative* to the first when the signals disagree. When it holds, type H trusts his second signal more than type L when he receives conflicting signals, and the opposite is true when it fails to hold.

Consider a benchmark case when both types of agent report truthfully. A comparison of posterior probabilities that the agent is smart given his reports and the observed true state suggests that both the accuracy and consistency of reports indicate high ability:

$$Pr(H|i_0 = i_1 = s) > Pr(H|i_0 \neq s, i_1 = s) > Pr(H|i_0 = s, i_1 \neq s) > Pr(H|i_0 \neq s, i_1 \neq s).$$ [17]

Denote the above four posterior probabilities respectively as (CR), (R), (W), and (CW) such that CR stands for consistently right; R for a right change of mind; W for a wrong change of mind, and lastly, CW stands for consistently wrong. The above inequalities suggest that, for example, given

[17] Exact expressions are in Appendix B, the analysis of Proposition 1.

the correct final report, a change of mind is bad for agent's reputation because it means that he is wrong at the beginning. But being consistently wrong is even worse.

In the current model, however, the agent may not report truthfully due to reputational concerns. The following proposition characterizes the agent's equilibrium behavior when sequential reports are required.

Proposition 1 *There exist cutoff values* p_0^L, p_0^H, *and* $\overline{\eta} \in (\frac{1}{3}, 1]$ *such that if* $\eta < \overline{\eta}$,[18]
(1.1) when the smart agent improves faster: if $p_0 \leq p_0^L$, *a unique full revelation equilibrium exists in which the agent reports both signals truthfully. If* $p_0 \geq p_0^L$, *there exists a unique partial revelation equilibrium in which the agent reports his first signal truthfully* ($m_0 = i_0$). *In the second report, type* H *always reports truthfully. Type* L *reports truthfully if* $i_0 = i_1$, *but repeats* m_0 *with probability* $\pi^* \in (0, 1)$ *if* $i_1 \neq i_0$. *Moreover, the mediocre agent's lying probability* π^* *increases with* p_0.
(1.2) when the mediocre agent improves faster: if $p_0 \leq p_0^H$, *a unique full revelation equilibrium exists. If* $p_0 \geq p_0^H$, *a full pooling equilibrium exists in which the agent reports* $m_0 = m_1 = i_0$. *Moreover, the second report is uninformative in any equilibrium.*[19]
(1.3) when the agent's signal quality does not improve ($p_0 = p_1, r = \frac{1}{2}$), *a full pooling equilibrium exists in which the agent reports* $m_0 = m_1 = i_0$. *Moreover, the second report is uninformative in any equilibrium.*

Proposition 1 shows first that when p_0 is relatively low, the agent tries to deliver an accurate final report, which is quite important to the principal's updated belief of the agent's type. When the agent's signals disagree, the more he believes in his later (and better) signal, the less attractive repeating his first report becomes. Intuitively, lying and repeating the first report is likely to lead to a consistently wrong sequence of reports, yielding the lowest reputational payoff. Therefore, when p_0 is sufficiently close to $\frac{1}{2}$, both H and L have (almost) uninformative first signals and the final report is the key indicator of ability. Thus the agent reports their second signal truthfully in equilibrium. *Ceteris paribus*, the faster an agent's signals improve, the more value he attaches to

[18] All these cutoff values are defined in the proof contained in Appendix B. The condition on η guarantees the monotonicity of the mixing probability with respect to p_0, the key parameter. In many cases, for example, when $p_0 \approx 1$ or when p_1 is sufficiently larger than r, $\overline{\eta} \approx 1$ and the restriction is trivial.

[19] One off-equilibrium belief that supports this equilibrium is that when the principal observes $m_0 \neq m_1$, she believes that the probability the agent is smart is 0.

the second signal because it is "better late than never": he can look like H who is unlucky in the first signal but finds out about the true state later.

Second, the better the smart agent's first signal is, the more likely the mediocre agent repeats his first report after receiving conflicting signals. Intuitively, as p_0 becomes higher, a correct first report is increasingly more likely to reflect high ability. Consider an extreme example where type H's first signal is perfect ($p_0 = 1$), then regardless of a type L agent's second signal, he repeats his first report with probability one. Any mind change shows that he is type L for sure, while repeating his first report makes him appear smart with some probability.

More subtly, however, it is not sufficient that type H receives better signals than L in *absolute* terms for him to report truthfully. Rather, type H's higher relative improvement in signal quality is crucial. For example, suppose $p_0 \approx p_1$ and the signals differ. Type L believes that his second signal is correct with probability r, which is higher than $\frac{1}{2}$, the likelihood H thinks that his second signal is correct. In this case, type H has less relative confidence in his second signal and is more tempted to repeat his first report than type L, even though *both* his signals are more accurate. Then type L must imitate and both types repeat their initial report in a pooling equilibrium.

Finally, the last part of Proposition 1 shows that even though the agent is still better informed if he receives multiple signals of the same quality, the principal will not gain additional information from requiring multiple reports. To see this, note that without quality improvement both types of agent have the same estimate of the true state when they receive conflicting signals ($Pr(g|g, b; \theta) = \frac{1}{2}$). Therefore reporting the true second signal does not increase his probability of giving a correct final report nor the principal's posterior that he is smart. Hence both types of agent repeat the first report in equilibrium. This implies that strict improvement in signal quality, not just better information, is crucial for the principal to benefit from the agent's multiple signals.

Even with strict improvement in signal quality, the agent may still inefficiently repeat his initial report. Such inefficiency may be quite high: the principal's information may deteriorate significantly even if the probability that the agent is smart is negligible. This occurs because a mediocre agent may repeat his first uninformative report with a high probability to appear smart despite a high quality second signal. Consider the following example:

Example 2: One good apple may ruin the barrel. Suppose that $\eta = 0.001$, $p_1 = 1, r = 0.9$ and $w(\hat{\eta}) = \hat{\eta}$. That is, the agent is extremely likely to be mediocre and type L's second signal is very accurate. In equilibrium, however, type L agent repeats his first report with probability $\pi^*(p_0) = 9.982p_0 + 0.078p_0^2 - 9$. Clearly, π^* increases in p_0. When $p_0 = 0.95, \pi^* = 0.5$. Hence a type L agent lies against his highly informative signal i_1 and uses his totally uninformative signal i_0 half of the time, even though the prior probability he is smart is only one out of a thousand. □

3.2 Value of Consistent Reports

One interesting question is whether the sequencing of reports alone carries any information about the agent's ability. If that is the case, the principal may use such information to improve her first stage decision before she can observe the true state. In particular, in areas such as political reforms, major joint ventures overseas, or risky medical procedures, the principal (voters, investors or patients) may be only able to observe the true state after a relatively long time lag. In the interim period, though, she may benefit from better information of the agent's ability derived from his messages. Moreover, a major insight of the herding models is that consistency (with early movers or existing consensus) is valued by the market as a sign of talent, which gives rise to a lot of empirical work examining whether consultants and forecasters are biased toward consistency.

Formally, the market is considered to value consistency more if $Pr(H|m_0 = m_1) > Pr(H|m_0 \neq m_1)$ and to value mind changes more otherwise. Proposition 1 shows that the principal may receive consistent reports despite conflicting signals. The following proposition describes, in the first stage, when consistent reports signal higher ability and when mind changes do.

Proposition 2 *When there exists a unique partial revelation equilibrium under the sequential reports system, before observing the true state,*
(2.1) the principal values consistency more than mind changes when type L's equilibrium mixing probability $\pi^* \leq (2p_0 - 1)(2p_1 - 1)$, which occurs when $w(\hat{\eta})$ is affine and strictly increasing.
(2.2) the principal values mind changes more than consistency when type L's equilibrium mixing probability $\pi^* > (2p_0 - 1)(2p_1 - 1)$, which occurs when $w(\hat{\eta})$ is sufficiently convex.

Proposition 2 may appear counterintuitive: if the principal does not value consistency in equilibrium, why should a mediocre agent lie against his second, more informative signal to appear consistent? Instead, a mediocre agent should simply tell the truth when he receives conflicting signals. Observe, however, the wage function is very convex when the principal values highly accurate reports disproportionally more than somewhat accurate ones.

Figure 2: Partial Revelation Equilibrium with Convex Payoff Function

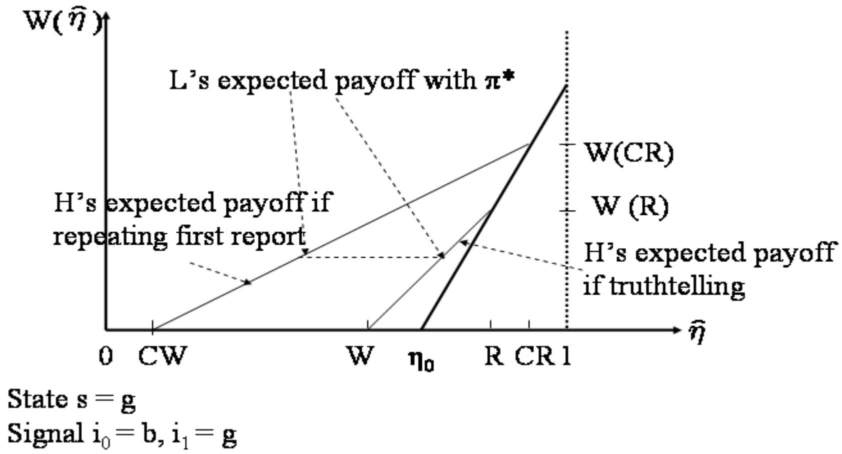


Figure 2 illustrates the agent's tradeoff between appearing consistent and reporting truthfully with a convex wage function. When a type L agent receives inconsistent signals, repeating his first report leads to the best future wage $w(CR)$ with probability $1-r$ or the worst future wage $w(CW)$ with probability r. If he follows his true second signal and gives inconsistent reports, he receives $w(R)$ or $w(W)$ with probability r and $1-r$ instead. Although consistent reports are more likely to be wrong, the cost is relatively small given his lack of confidence in the second signal comparing to type H. The benefit of the riskier consistent reports is that they can give him a higher expected payoff than mind changes. In contrast, the smart agent reports his second signal truthfully because he has a different probability distribution over the outcomes. His second signal is so much better than his first that repeating his first signal is very likely to lead to the worst payoff of all: being consistently wrong.

This result suggests that both consistency and mind changes may signal high ability, depending on the environment and the project involved. On one hand, in forecasting for financial crises, speculative currency attacks, preparation for major natural disasters, initiation of political reforms or wars, extremely high quality information may be necessary because the potential cost of wrong decisions is catastrophic. In this case, experts who change their minds may be more valued because they demonstrate confidence in their later and better information. On the other hand, when the project is more routine and relies less on highly precise information, consistency may be more valued. Examples may include short term forecast of consumer demand for existing products, fact-finding procedures in law, and cost benefit analysis of many regular government policies.

Proposition 1 and 2 together yield some interesting testable predictions. On one hand, when the smart expert's initial signal is not too superior, both types of agent are likely to report signals truthfully in a full revelation equilibrium. In this case, if the expert is more likely to be mediocre, we may observe a lot of mind changes in the wrong direction. That is, mind changes are positively correlated with predicting errors even though there is relatively little bias involved. Moreover, the market values experts giving consistent reports. This prediction is consistent with some empirical result such as Ehrbeck and Waldmann (1996), who used experts' forecasts of the discount rate on new issues of U.S. Treasury bills over time to test forecasting bias. Their data show that experts changed forecasts too much and forecasters who make large changes in forecasts have larger errors. On the other hand, when the payoff is sufficiently convex, a mediocre agent reports too consistently and mind changes become negatively correlated with forecasting errors.[20]

4 Optimal Reporting Protocol

Section 3 shows that requiring sequential reports may induce a mediocre agent to lie and report too consistently, and Example 2 shows that the resulting inefficiency can be quite significant. A frequently observed alternative is for the principal to require report(s) *after* the agent has received both signals. This section compares these two reporting systems by first investigating incentives

[20] To see who are more likely to make forecasting errors in the present model is to compare the ex ante probability of observing W and CW (R and CR make no error). Suppose that the true state is 0, then the probability of observing $(0,1)$ and $(1,1)$ are respectively: $p_0(1-p_1)\eta + \frac{1}{2}(1-r)(1-\pi^*)(1-\eta)$; $(1-p_0)(1-p_1)\eta + \frac{1}{2}(1-r+r\pi^*)(1-\eta)$. The former is bigger when $\pi^* \approx 0$.

generated when the principal requires a final report and then characterizing which system enables the principal to make better decisions.

Recall that in the second stage, the agent receives the full expected value of his information. Therefore the optimal reporting protocol here is one that elicits the most truthful reports and leads to the highest first stage profit for the principal. The principal can require one final report (m^f) or a final report sequence after the agent has learned both signals ($\vec{m}^f = (m_0, m_1)$). When one final report is required, the agent's future wage depends solely on its accuracy. Clearly, the agent should report his best estimate of the state based on the signals. When a sequence of final reports is required, however, the results are more subtle:

Proposition 3 *(3.1). If the principal requires one report m^f, then in equilibrium both types of agent report $m^f = i_1$, regardless of their first signal.*

(3.2). If the principal requires report $\vec{m}^f = (m_0, m_1)$ after the agent receives both signals, and if type H's signal improves faster (inequality (1) holds), both types of agent report $m_0 = m_1 = i_1$. Moreover, there does not exist an equilibrium in which m_0 is informative.

The first part of Proposition 3 shows that the principal receives the true final signal when she requires m^f. The reason is clear: the agent should report his higher quality second signal, which leads to a higher posterior estimate of his ability in expectation than reporting $m^f = i_0$. The final report m^f, however, is not a sufficient statistic of the agent's signals because both signal sequences (b, g) and (g, g) lead to the same report, whereas the principal forms very different opinions of the true state given these sequences. A natural alternative is to require a vector of report at the end, $\vec{m}^f = (m_0, m_1)$, about the signals the agent received.[21]

Despite some seeming similarity, however, Proposition 3 shows that requiring $\vec{m}^f$ at the end differs markedly from the sequential reports model in Section 3. The key is to observe that the agent has little incentive to lie in his initial report when sequential reports are required, which commits him to that report to some extent. When $\vec{m}^f$ is required, however, inducing truthful initial report becomes more difficult because the agent, knowing both signals, can and will modify

[21] Requiring a vector of final reports is equivalent to requiring a probabilistic assessment of the state. See Section 5 for more details.

his whole report sequence in any way to get the highest reputational payoff in the next stage.

In particular, the second part of Proposition 3 shows that when the smart agent improves faster, full revelation cannot be an equilibrium even if a unique full revelation equilibrium exists in the sequential reports case. Part of the reasoning is familiar: if there were a full revelation equilibrium, consistent reports would signal higher ability, and the agent would deviate and report more consistently. In the sequential reports system, type H agent reports inconsistent signals truthfully because his second report is more likely to be accurate and the benefit of a correct final report outweighs any cost of inconsistency. In the final report(s) system, there is no need for type H to send inconsistent reports: by reporting $m_0 = m_1 = i_1$, he is likely to be accurate as well as consistent. Since type H agent has more relative confidence in his second signal than type L, he is more confident that his consistent reports are likely to be correct and he will receive the highest posterior $Pr(H|m_0 = m_1 = s)$. This, however, drives type L agent to give more consistent reports too. Thus in equilibrium, the first signal is lost.

Having shown the similarity of requiring m^f and $\vec{m}^f$, the following proposition compares the final report system with the sequential reports system:

Proposition 4 *(4.1) The principal should require sequential reports when both types of agent report truthfully in a full revelation equilibrium. Formally, this occurs if $\eta < \overline{\eta}$, and 1) inequality (1) holds and $p_0 \leq p_0^L$; or 2) inequality (1) does not hold and $p_0 \leq p_0^H$.*
(4.2) The principal should require one final report when both types of agent report $m_0 = m_1 = i_0$ in a sequential report system. Formally, this occurs if $\eta < \overline{\eta}$, inequality (1) does not hold and $p_0 \geq p_0^H$.
(4.3) When there exists a partial revelation equilibrium in a sequential reports system, the principal should require a final report when the expert market is extremely poor ($\eta \approx 0$). However, when the expert market is not too poor, and/or when the principal needs highly precise reports ($Pr(s = g|m^0)$ sufficiently large), the principal should require sequential reports.

The first part of Proposition 4 shows that if the expert reports both signals truthfully when sequential reports are required, the principal is better off knowing two true signals than knowing only the second signal: better information leads to better decision and (weakly) more profit in the first stage. The agent, however, may not report both signals truthfully. In particular, as shown in Proposition 1, when the mediocre agent's signals improve faster, the second report is completely uninformative and only i_0 is truthfully revealed. The second part of Proposition 4 shows that in this case, requiring a final report is better because it elicits the true second signal. Given the improvement in signal quality, a true second signal is more useful to the principal.[22]

The previous two cases are relatively straightforward because one reporting system clearly elicits more truthful reports than the other. Sometimes, however, as the third part of Proposition 4 shows, the optimal reporting protocol depends on the particular projects and professional specific parameters g, b. The principal should choose a final report when the inefficiency of the sequential reports system is sufficiently high. As illustrated in Example 2, the prior probability of the agent being smart is so low that even if a talented agent reports truthfully, the principal is much more likely to receive two uninformative reports from a mediocre one. Thus on average, she makes better decision with the final report system.

However, sequential reports are more valuable when the decision problem is very sensitive to accuracy of the reports. One final report, as a coarser estimate of the true state, does not contain all the information revealed in the sequential reports. This matters when the principal's decision requires high precision. For example, suppose that the principal needs to be convinced that $s = g$ with such a high probability that she would not invest under the final report system even if $m^f = g$, then one final report is useless for her.[23] But if the equilibrium lying probability of type L is not too high, then the principal may invest instead after hearing two good reports and earns higher expected profit $(Pr(g|g,g) > Pr(g|g))$.

Proposition 4 shows that if the optimal decision does not depend on highly precise information, final report system is optimal. Routine operations such as cost benefit analysis of everyday policies in the government fall into this category. When the optimal decision relies crucially on precise information, however, the optimal protocol is more subtle and interesting. For example, in international

[22] This result is related to the single signal model in Prat (2005), which shows that the principal may prefer not to observing an agent's action (similar to a report here) because then the agent is likely to conform to the ex ante more likely action of a smart agent. Instead, she is better off observing the outcome of such action (whether the report correctly predicted the state) because the agent's reputation increases with the good outcome and he will choose the most efficient action.

[23] Formally, when $[p_1\eta + r(1-\eta)]g + [(1-p_1)\eta + (1-r)(1-\eta)]b \leq 0$.

currency speculations, implementing large scale development projects, or launching a major war, the precision of information is of paramount concern due to the catastrophic financial and human costs of wrong decisions. The very importance of accuracy in these professions or environment leads to great premium on talent and in turn very convex implicit incentives. Thus if the smart expert is exceedingly rare, the principal may be better off requiring only a final recommendation–requiring early reports is likely to lead mediocre agents to repeat themselves to protect their reputation. Such escalating commitment to initial decisions was observed in the Vietnam war (Staw 1976). But when the talent is not extremely rare, the principal may prefer sequential reports because the path of reports reveals finer information.[24] Under a final report(s) system, the agent can fabricate early reports and potentially lead the principal to believe too strongly in a certain direction.

5 Discussion and Extensions

First, this section discusses the role of several assumptions on communication and the expert market. Then two plausible extensions of this model are discussed: i) allowing the agent's ability to be symmetric information and ii) allowing the agent to receive additional informative signals. These extensions further illustrate how subtlely the agent's incentive to report truthfully depends on the expert's signal structure.

A. Assumption on the Message Space. To begin with, the agent in this model reports simply that the state is good or bad. One question is that whether more information can be communicated in a richer message space, e.g. if the agent reports his belief of the state distribution instead ($m = \beta g + (1-\beta)b, \beta \in [0,1]$). However, the binary signals with known signal qualities for both types of agent mean that there are only six possible state distributions, and that the agent can always report a distribution that is type H's. For example, a mediocre agent who receives $i_0 = g$ would not report truthfully that he believes that both states are equally likely because doing so identifies him as a mediocre type.[25] Thus the reports are no more informative than when

[24] Sequential reports are also better for sorting talent in the second stage. This effect is discussed further in Section 5, part B.

[25] Similarly, after the second signal, he would choose a report that maximize the expected wage, using the distribution of type H. That is, the agent reports that he believes the state is g with $Pr(s = g|i_0 = i_1 = g, H)$ is equivalent to reporting he received two good signals.

the messages were limited to g or b. If the state distribution is more general, however, interval equilibria with finite many number of messages/reports may arise as shown by Crawford and Sobel (1982) and Ottaviani and Sorensen (2005).

Even in the present setting, a state distribution that should not arise can mean that the agent does not want to reveal his signal. Looking in this light, this question is similar to voluntary disclosure of information: the agent can reveal his signals, or refuse to do so by reporting an impossible state distribution. There exist additional pooling equilibria in which the agent chooses not to reveal anything in his report, but this type of equilibria depend strongly on the principal's beliefs.[26] One practical implication is that even when final reporting is optimal, if the principal cannot forbid voluntary, informal reports, smart agent may have a strong incentive to report his initial signal and drive everyone to send an early report.

B. Assumption of Perfect Competition. In this model, the market is perfectly competitive and thus the expert obtains all the value of information he provides. With imperfect competition, the principal may profit from her updated knowledge of the agent's talent in the second stage. This affects the principal's incentives in two ways: she may arrange the projects in the two stages such that the second stage project is far more important, and/or choose a reporting system that does not lead to the highest first stage profit, but gives a more precise estimate of the agent's type to increase her second stage expected profit. This may increase the agent's incentive to appear smart in the first stage because even though he may obtain only part of his future value of information, the project choice implies that the overall value of his information is far higher in the second stage.

C. When Ability is Symmetric Information. The incentives of an agent to report truthfully depend crucially on his signal quality improvement, which in turn depends on whether he is smart or mediocre. In some professions (or stages of one's career), the agent may not know how smart he is. Appendix C considers such a model, where all other modeling assumptions remain the

[26] One such equilibrium is for the principal to believe that anyone who reports a possible state distribution is mediocre, then the agent will always report some impossible distribution instead. This type of pooling equilibria may not be reasonable under some natural beliefs, though. Suppose that the principal believes that each type is equally likely to use a state distribution that should not occur in the initial report, then for p_0 sufficiently close to $\frac{1}{2}$ or p_0 sufficiently high, type H can be shown to prefer reporting his true signal i_0, thus type L must do so as well. Intuitively, in these two cases, type H has a strong incentive to report his higher quality first signal to distinguish himself, which breaks the putative pooling equilibrium. However, this depends crucially on the principal's belief upon receiving this type of report and is not the focus of this paper.

same except that only the agent's type distribution's known. One important insight emerges from the symmetric information case: consistent reports always signal high ability in equilibrium (Proposition 5, Appendix C). Intuitively, both the principal and the agent himself believe that a smart agent is more likely to be consistent. Thus both types of agent face very similar incentives to lie when he receives conflicting signals. This implies that whether consistency or mind changes is more valuable as a sign of talent also depends on how well the agent knows his own ability. In professions where one's talent is unknown to all parties, consistency is more valued. High quality information and fast improvement are not enough to ensure that a smart agent changes his mind and acknowledge a (likely) early mistake: the agent needs to know how good he is.

D. Additional Signals. This model shows that sequencing of reports provides information about the agent's ability. If, however, the agent receives additional signals, sequencing of reports is important in another way: the agent's incentive to report truthfully is path-dependent. An extension of this model to a three signal setting illustrates a new tradeoff the agent faces: to appear consistent early to show his confidence in the early signals or to appear consistent late so that his reports are more likely to be correct (Li 2004).

On one hand, the principal may want to require the third report when the improvement in the smart agent's signal quality levels off. In this case, a mediocre agent may lie against his true third signal in equilibrium if he has lied against his second to appear consistent. This "escalation effect", however, improves type L's incentive to report the true second signal because he knows that once he lies in the second report, he is likely to lie again in the next report and suffer from a big loss in accuracy after three wrong reports. On the other hand, the principal may not want to require the third report when the smart agent's signal quality improves a lot in the final signal but the mediocre one's does not. In equilibrium, a type L agent may lie more in his early reports because he can still change his mind later without appearing too mediocre. This type of counterintuitive effect on the agent's overall incentives should be considered in designing reporting systems with many signals.

6 Conclusion

When experts are asked to give sequential reports based on private signals of increasing quality, both the sequencing and the accuracy of the reports become a signal of ability. A mediocre agent then tends to repeat his initial report to appear consistent even when the market values mind changes as a prized sign of the fast learners and the talented.

This paper shows that the implicit incentive structure itself is generally convex and may encourage risk taking behavior even with risk-neutral agents. One natural question is how these implicit incentives evolve, e.g., whether the wage becomes less convex over time as an agent's ability becomes better known. Another question concerns the optimal reporting protocol if the agent receives many signals over time. The model here suggests that requiring very early report may cause the mediocre agent to commit to a position too early just to appear smart, but requiring late reports alone makes it difficult for the principal to obtain finer information. How the principal optimizes over both the number and the timing of reports is a question of further research.

APPENDIX A: PROOFS (EXCEPT PROPOSITION 1)

Proof of Lemma 1:

(1) First, recall from the text that the principal's expected profit in the second stage is: $\Pi^1(m^1,\hat{\eta}) = \sum_{m^1}[Pr(g,m^1)g + Pr(b,m^1)b]a(m^1,\hat{\eta})$. Let $\pi^1(m^1,\hat{\eta}) \equiv Pr(g,m^1)g + Pr(b,m^1)b$, the expected profit of seeing m^1, then we have:

$$\pi^1(g,g,\hat{\eta}) = \frac{r}{4}g + \frac{1-r}{4}b + \frac{\hat{\eta}}{2}[g(p_0p_1 - \frac{r}{2}) + b((1-p_0)(1-p_1) - \frac{1-r}{2})];$$
$$\pi^1(b,g,\hat{\eta}) = \frac{r}{4}g + \frac{1-r}{4}b + \frac{\hat{\eta}}{2}[g((1-p_0)p_1 - \frac{r}{2}) + b(p_0(1-p_1) - \frac{1-r}{2})];$$
$$\pi^1(g,b,\hat{\eta}) = \frac{1-r}{4}g + \frac{r}{4}b + \frac{\hat{\eta}}{2}[g(p_0(1-p_1) - \frac{1-r}{2}) + b((1-p_0)p_1 - \frac{r}{2})];$$
$$\pi^1(b,b,\hat{\eta}) = \frac{1-r}{4}g + \frac{r}{4}b + \frac{\hat{\eta}}{2}[g((1-p_0)(1-p_1) - \frac{1-r}{2}) + b(p_0p_1 - \frac{r}{2})].$$

Clearly, each $\pi^1(m^1,\hat{\eta})$ is affine in $\hat{\eta}$, and they can be ranked as $\pi^1(g,g,\hat{\eta}) > \pi^1(b,g,\hat{\eta}) > \pi^1(g,b,\hat{\eta}) > \pi^1(b,b,\hat{\eta})$ for any given $\hat{\eta}$. If $\pi^1(m^1,\hat{\eta}) \geq 0$, the principal should choose $a^* = 1$ after m^1, $a^* = 0$ otherwise. Thus $\Pi^1(m^1,\hat{\eta})|_{a=a^*}$ is simply the sum of $\pi^1(m^1,\hat{\eta})$ where $a^*(m^1,\hat{\eta}) = 1$ and is (piece-wise) linear in $\hat{\eta}$.

Second, by assumption, the default action is $a = 0$ ($g + b \leq 0$). Thus we can show that $\pi^1(g,b,\hat{\eta}) < 0$. As a result, $a^* = 0$ if the reports are (g,b) or (b,b). What happens if the reports are

(b,g) or (g,g)? Note that both $\pi^1(g,g,\hat{\eta})$ and $\pi^1(b,g,\hat{\eta})$ are increasing in $\hat{\eta}$. Therefore the slopes of $\Pi^1(m^1,\hat{\eta})|_{a=a^*}$ can be at most one of the following three: zero (when b is sufficiently negative that the principal never invests), or that of $\pi^1(g,g,\hat{\eta})$ (when only (g,g) leads to investment) or the sum of the slopes (both (b,g), (g,g) lead to investment). In all three cases, the expected profit, and thus the wage is nondecreasing in $\hat{\eta}$.

Next, consider two posterior estimates of the agent's type η_1 and η_2, and let $\eta = \gamma\eta_1 + (1-\gamma)\eta_2, \gamma \in [0,1]$ denote a convex combination of η_1, η_2. Let $V(a_1^*(\eta_1)), V(a_2^*(\eta_2)), V(a^*(\eta))$ denote the respective wages of the agent in the second stage given these posterior distributions and when the principal takes optimal action. Then,

$$\begin{aligned}
\gamma V(a_1^*(\eta_1)) + (1-\gamma)V(a_2^*(\eta_2)) &\geq \gamma V(a^*(\eta_1)) + (1-\gamma)V(a^*(\eta_2)) \\
&= \gamma \sum_{m^1} \pi^1(m^1,\eta_1)a^*(\eta) + (1-\gamma)\sum_{m^1} \pi^1(m^1,\eta_2)a^*(\eta) \\
&= \sum_{m^1} \big[\gamma\pi^1(m^1,\eta_1) + (1-\gamma)\pi^1(m^1,\eta_2)\big]a^*(\eta) \\
&= V(a^*(\eta)).
\end{aligned}$$

The second equality is true because the action is constrained to be the optimal one give η. The third equality is true because each $\pi^1(m^1,\hat{\eta})$ is affine in $\hat{\eta}$. Thus the wage function $w(\hat{\eta})$ is convex.

(2) If the principal's action $a(m^1;\hat{\eta})$ is independent of the agent's posterior ability $\hat{\eta}$, then her decision depends on the report sequence only. This implies that for a given m^1, $\pi^1(m^1,\hat{\eta})$ is strictly positive or negative for all $\hat{\eta}$. From part (1), we know that $a^* = 0$ if the reports are (g,b) or (b,b). Two possibilities remain: either $p_0p_1g + (1-p_0)(1-p_1)b \leq 0$, in which case $\pi^1(g,g,\hat{\eta}) \leq 0$ for all $\hat{\eta}$, then $w(\hat{\eta}) = 0$. Or when $rg + (1-r)b \geq 0$, in which case $\pi^1(g,g,\hat{\eta}) > \pi^1(b,g,\hat{\eta}) \geq 0, \ \ \forall\hat{\eta}$. Thus $w(\hat{\eta}) = \sum_{m^1} \pi(m^1,\hat{\eta})a(m^1,\hat{\eta}) = \frac{rg+(1-r)b}{2} + \frac{g-b}{2}(p_1-r)\hat{\eta}$, which is affine (linear if $rg+(1-r)b = 0$) and strictly increasing in $\hat{\eta}$. ‖

Proof of Proposition 2:

(2.1) First, in a partial information revelation equilibrium, the principal's beliefs about the agent's type before the state realizes but after receiving consistent and inconsistent reports are respectively:

$$\begin{aligned}
Pr(H|m_0 = m_1) &= \frac{[p_0p_1 + (1-p_0)(1-p_1)]\eta}{[p_0p_1 + (1-p_0)(1-p_1)]\eta + \frac{1}{2}(1+\pi^*)(1-\eta)} \\
Pr(H|m_0 \neq m_1) &= \frac{[p_0(1-p_1) + (1-p_0)p_1]\eta}{[p_0(1-p_1) + (1-p_0)p_1]\eta + \frac{1}{2}(1-\pi^*)(1-\eta)}
\end{aligned}$$

Simple calculation shows that $Pr(H|m_0 = m_1) \geq Pr(H|m_0 \neq m_1)$ if $\pi^* \leq (2p_0-1)(2p_1-1)$ and $Pr(H|m_0 = m_1) < Pr(H|m_0 \neq m_1)$ otherwise.

Second, in a partial revelation equilibrium, IC_1^L binds, as shown in the proof of Proposition 1. Let $f(r,\pi^*) \equiv (w(CR) - w(W))(1-r) - (w(R) - w(CW))r = 0$. Since $\frac{\partial f}{\partial r} < 0, \frac{\partial f}{\partial \pi^*} < 0$, by the implicit function theorem, $\frac{d\pi^*}{dr} < 0$. That is, the mixing probability π^* decreases with r because the expected value of giving consistent reports decreases with r and that of giving inconsistent reports increases with r. Thus in a partial information revelation equilibrium, the highest mixing probability is obtained for any given p_0, p_1 at $r = \frac{1}{2}$. When w is affine and strictly increasing, the maximum π^* is obtained at $CR - W = R - CW$. However, at the cutoff value $\pi^* = (2p_0 - 1)(2p_1 - 1)$, straightforward calculations show that $CR - W < R - CW$, therefore the equilibrium mixing probability in this case is smaller than $(2p_0 - 1)(2p_1 - 1)$. This shows that consistency is always valued more as a signal of talent when w is affine and strictly increasing.

(2.2) When the principal's optimal decision depends on $\hat{\eta}$, w is convex. Then holding everything else equal, for IC_L^1 to bind, the mediocre type needs to repeat his first report with higher probability than he would when w is affine and strictly increasing as in part (2.1). For example, as illustrated in Figure 2 of the text, where $w(W) = w(CW) = 0$, the incentive constraint IC_1^L simplifies into $w(CR)(1 - r) < w(R)r$. At $r = \frac{1}{2}$, $\pi^* = 2p_0 - 1$, which is larger than the cutoff value $(2p_0 - 1)(2p_1 - 1)$ and the principal values mind changes more. ‖

Proof of Proposition 3:

(3.1) When only one final report is required, the agent chooses m^f to maximize his expected wage: $w(Pr(H|m^f = s))Pr(m^f = s|i_0, i_1, \theta) + w(Pr(H|m^f \neq s))Pr(m^f \neq s|i_0, i_1, \theta)$. When $i_0 = i_1$, it is clear that the agent reports $m^f = i_0 = i_1$. If $i_0 \neq i_1$, $Pr(m^f = i_0 = s|i_0, i_1, \theta) \leq Pr(m^f = i_1 = s|i_0, i_i, \theta)$. Moreover, since $w(\hat{\eta})$ is non-decreasing and $Pr(H|m^f = s) > Pr(H|m^f \neq s)$, reporting $m^f = i_1$ gives a higher expected wage.

(3.2) When the principal requires $\vec{m}^f = (m_0, m_1)$ after the agent receives both signals, the agent will report truthfully if he receives the highest expected wage doing so. First, suppose that he receives inconsistent signals, e.g., $i_0 = b, i_1 = g$, then the following three ICs must hold for truthful revelation:

$$
\begin{aligned}
w(R)Pr(g|b,g,\theta) + w(W)Pr(b|b,g,\theta) &\geq w(CR)Pr(g|b,g,\theta) + w(CW)Pr(b|b,g,\theta) \quad (2)\\
w(R)Pr(g|b,g,\theta) + w(W)Pr(b|b,g,\theta) &\geq w(W)Pr(g|b,g,\theta) + w(R)Pr(b|b,g,\theta)\\
w(R)Pr(g|b,g,\theta) + w(W)Pr(b|b,g,\theta) &\geq w(CW)Pr(g|b,g,\theta) + w(CR)Pr(b|b,g,\theta)
\end{aligned}
$$

In particular, IC (2) simplifies into the following:

$$
\begin{aligned}
&[w(CR) - w(R)](1 - p_0)p_1 \leq [w(W) - w(CW)]p_0(1 - p_1), \text{ for type } H;\\
&[w(CR) - w(R)]r \leq [w(W) - w(CW)](1 - r), \quad \text{for type } L.
\end{aligned}
$$

Recall from the proof of Proposition 1 that when the agent reports truthfully, $w(CR) - w(W) \geq w(R) - w(CW)$ at $p_0 = \frac{1}{2}$, and the gap increases with p_0 when $\eta < \overline{\eta}$. Moreover, $(1 - p_0)p_1 > p_0(1 - p_1)$. Thus for all p_0, IC (2) fails to hold if the agent reports truthfully because $[w(CR) - w(R)](1 - p_0)p_1 > [w(W) - w(CW)]p_0(1 - p_1)$. Hence no full revelation equilibrium exists because the agent receives higher expected wage reporting $m_0 = m_1 = g$ instead.

Next, if inequality (1) holds, compare IC (2) for type H and L, we can see that there are two possible cases: first, if the IC for type H binds, the one for type L hold strictly. That is, if the smart agent reports $m_0 = i_0$ with positive probability, then type L always reports truthfully. Simple algebra can show, however, that the relative reputational payoff of consistent reports increases with type H's mixing probability, thus type H will report consistently and type L needs to report consistently as well. Second, if IC (2) for L binds, the one for H fails to hold, which implies that type H strictly prefers to report consistently when type L is indifferent. Therefore no partial revelation equilibrium exists.

The only possible type of equilibrium is a pooling equilibrium in which both type H and L report $m_0 = m_1$. Similar to part (3.1), the agent receives higher expected wage by reporting

$m_0 = m_1 = i_1$ than i_0. One reasonable belief that supports this equilibrium is for the principal to believe that $Pr(m_0 \neq m_1) = \epsilon$ for both types. That is, both types are equally likely to make mistakes and report $m_0 = i_0 \neq m_1$ instead. Simple calculations show that neither type wants to deviate under this belief.

Second, consider the case when the signals are consistent, e.g., $i_0 = i_1 = g$, there are also three incentive constraints:

$$\begin{aligned}
w(CR)Pr(g|g,g,\theta) + w(CW)Pr(b|g,g,\theta) &\geq w(CW)Pr(g|g,g,\theta) + w(CR)Pr(b|g,g,\theta)\\
w(CR)Pr(g|g,g,\theta) + w(CW)Pr(b|g,g,\theta) &\geq w(W)Pr(g|g,g,\theta) + w(R)Pr(b|g,g,\theta)\\
w(CR)Pr(g|g,g,\theta) + w(CW)Pr(b|g,g,\theta) &\geq w(R)Pr(g|g,g,\theta) + w(W)Pr(b|g,g,\theta)
\end{aligned}$$

From the part above, we can see that $[w(CR)-w(R)]Pr(g|b,g,\theta) > [w(W)-w(CW)]Pr(b|b,g,\theta)$ for all $p_0 > \frac{1}{2}$. Moreover, since $Pr(g|g,g,\theta) \geq Pr(g|b,g,\theta) > \frac{1}{2}$, all the above incentive constraints hold strictly when the agent reports truthfully. Thus in equilibrium, the agent reports truthfully when the signals agree. ‖

Proof of Proposition 4:

To begin with, given that $g + b \leq 0$, the principal will not invest at all if the final report is b or if the sequential reports are (b, b) or (g, b). Let $\Pi_f^0(g), \Pi_s^0(g), \Pi^0(g,g), \Pi^0(b,g)$ denote respectively the expected first stage profit when the one final report is g; the profit under the full pooling equilibrium of the sequential reports system when both reports are g, and the expected profits after report sequence (g, g) and (b, g) in a sequential report system. As in Proposition 1, $\pi^* \in [0, 1)$ is the equilibrium mixing probability in the sequential reports system. We have:

$$\Pi_f^0(g) = \frac{r}{2}g + \frac{1-r}{2}b + \frac{(p_1 - r)\eta}{2}(g-b); \qquad \Pi_s^0(g) = \frac{1}{4}g + \frac{1}{4}b + \frac{(p_0 - 0.5)\eta}{2}(g-b);$$

$$\Pi^0(g,g) = \frac{r + (1-r)\pi^*}{4}g + \frac{1-r+r\pi^*}{4}b + \frac{\eta}{2}\left[g(p_0 p_1 - \frac{r+(1-r)\pi^*}{2}) + b((1-p_0)(1-p_1) - \frac{1-r+r\pi^*}{2})\right];$$

$$\Pi^0(b,g) = \frac{(1-\pi^*)r}{4}g + \frac{(1-\pi^*)(1-r)}{4}b + \frac{\eta}{2}\left[g((1-p_0)p_1 - \frac{(1-\pi^*)r}{2}) + b(p_0(1-p_1) - \frac{(1-\pi^*)(1-r)}{2})\right].$$

(4.1) In this case, Proposition 1 shows that both types of agent report truthfully, which provides the principal with the best information possible.

(4.2) In this case, Proposition 1 shows that both type H and L report $m_0 = m_1 = i_0$. When one final report is required, both types report $m_0 = m_1 = i_1$. Recall that $g + b < 0$, thus the expected profit is 0 if the report in either case is b. When $m = g$, simple calculation shows that $\Pi_f^0(g) - \pi_s^0(g) = \frac{1}{2}[(p_1 - p_0)\eta + (r - \frac{1}{2})(1-\eta)](g-b) > 0$: thus the expected profit is higher under the final report system.

(4.3) When type H improves faster (inequality (1) holds) and $p_0 \geq p_0^L$, a unique partial revelation equilibrium exists with sequential reports with a positive mixing probability π^*. There are two possible cases. First, a comparison of the expected profits shows that $\Pi_f^0(g) \geq \Pi^0(g,g) + \Pi^0(b,g)$ for $\eta \approx 0$. In this case, the expected profit under sequential reports depends (almost) entirely on the reports provided by the mediocre agent, whose initial signal is useless. Thus the principal should require one final report because she gets the truthful second signal.

Second, consider the case when η is not too close to 0 and the principal needs highly accurate information. Suppose b is so negative that $\Pi_f^0(g) = \frac{1}{2}[p_1\eta + r(1-\eta)]g + \frac{1}{2}[(1-p_1)\eta + (1-r)(1-\eta)b] \leq 0$, then the final report has no value to the principal because she will not invest even if $m^f = g$. However, compare the expected profit, we can show that $\Pi^0(g,g) > \Pi_f^0(g)$ when the mixing probability π^* is not too high. The reason is that here type L lies relatively little and thus g, g is a better signal of state g. Second, $\Pi^0(b,g) > \Pi_f^0(g)$ when the mixing probability π^* is very high and p_1 is sufficiently high. The reason is that here type L repeats his initial report so much that b, g is almost surely a sign of type H, whose second signal is very accurate. Thus sequential reports may offer more precise information than the final report and changes the optimal decision of the principal. ‖

APPENDIX B: ANALYSIS AND PROOF OF PROPOSITION 1

This section first describes the general problem the agent faces after receiving his second signal. Then it provides some basic characterizations before proving Proposition 1, the main result of this paper.

Assume that the agent reports $m_0 = i_0$ in his initial report. After receiving the second signal i_1, the agent needs to choose m_1 to maximize: $\sum_s w(Pr(H|m_0, m_1; s))Pr(s|i_0, i_1, \theta)$, where $Pr(s|i_0, i_1, \theta)$ is the probability that the true state is s based on the agent's information and his type, and $w(Pr(H|m_0, m_1, s))$ is the agent's future wage given his reports and the later realized true state. For both types of agent to report $m_1 = i_1$ truthfully, four incentive constraints given the history (consistent signals or not) as well as the agent's type must hold:

$$
\begin{aligned}
&(IC_1^L) \qquad (w(CR) - w(W))Pr(b|b,g,L) \leq (w(R) - w(CW))Pr(g|b,g,L);\\
&(IC_2^L) \qquad (w(CR) - w(W))Pr(g|g,g,L) \geq (w(R) - w(CW))Pr(g|g,b,L);\\
&(IC_1^H) \qquad (w(CR) - w(W))Pr(b|b,g,H) \leq (w(R) - w(CW))Pr(g|b,g,H);\\
&(IC_2^H) \qquad (w(CR) - w(W))Pr(g|g,g,H) \geq (w(R) - w(CW))Pr(b|g,g,H).
\end{aligned}
$$

First, compare probabilities $Pr(s|i_0, i_1, \theta)$ in the above incentive constraints:

$$
\begin{aligned}
&\frac{Pr(g|g,g,L)}{Pr(b|g,g,L)} = \frac{r}{1-r} > 1 > \frac{Pr(b|b,g,L)}{Pr(g|b,g,L)} = \frac{1-r}{r};\\
&\frac{Pr(g|g,g,H)}{Pr(b|g,g,H)} = \frac{p_0 p_1}{(1-p_0)(1-p_1)} > 1 > \frac{Pr(b|b,g,H)}{Pr(g|b,g,H)} = \frac{p_0(1-p_1)}{p_1(1-p_0)}.
\end{aligned}
$$

Assuming that the wage differences on both sides of the IC are non-negative (shown later to be true in equilibrium), then observe that (1): if either IC_1^H or IC_1^L binds, IC_2^H and IC_2^L hold strictly. That is, if either type is (weakly) willing to report $m_1 = i_1$ after receiving opposite signals, the agent strictly prefers to report truthfully after consistent signals. (2) If inequality (1) holds, then $\frac{Pr(b|b,g,H)}{Pr(g|b,g,H)} < \frac{Pr(b|b,g,L)}{Pr(g|b,g,L)}$ by definition. This means that if IC_1^L holds or binds, IC_1^H holds strictly. That is, when a smart agent improves faster, type H strictly prefers to report $m_1 = i_1$ if type L weakly prefers to do so. (3) If inequality (1) does not hold, then if IC_1^H holds or binds, IC_1^L holds strictly.

Second, consider the agent's wage $w(Pr(H|m_0, m_1, s))$ in the above incentive constraints. As described in the text, suppose that type L repeats his initial report with probability π, the principal's posterior beliefs of the agent's ability are:

$$
\begin{aligned}
(CR) \qquad Pr(H|g,g,g) &= \frac{p_0 p_1 \eta}{p_0 p_1 \eta + \frac{1}{2}[r + (1-r)\pi](1-\eta)}; \\
(CW) \qquad Pr(H|g,g,b) &= \frac{(1-p_0)(1-p_1)\eta}{(1-p_0)(1-p_1)\eta + \frac{1}{2}[(1-r) + r\pi](1-\eta)}; \\
(W) \qquad Pr(H|g,b,g) &= \frac{p_0(1-p_1)\eta}{p_0(1-p_1)\eta + \frac{1}{2}(1-r)(1-\pi)(1-\eta)}; \\
(R) \qquad Pr(H|g,b,g) &= \frac{(1-p_0)p_1\eta}{(1-p_0)p_1\eta + \frac{1}{2}(1-\pi)r(1-\eta)}.
\end{aligned}
$$

Consider the case when the agent reports i_1 truthfully ($\pi = 0$). Lemma 1 shows that $w' \geq 0$, thus:

$$
\frac{\partial}{\partial p_0}(w(CR) - w(W)) \geq w'(W)\frac{\partial}{\partial p_0}(CR - W), \text{ and } \frac{\partial}{\partial p_0}(w(R) - w(CW)) \leq w'(R)\frac{\partial}{\partial p_0}(R - CW). \quad (3)
$$

Taking derivatives with respect to p_0, we have:

$$
\begin{aligned}
sign\left(\frac{\partial(CR-W)}{\partial p_0}\right) &= sign\left(-(p_1(1-p_1)\eta^2 - \frac{r(1-r)(1-\eta)^2}{4p_0^2})\right); \\
sign\left(\frac{\partial(R-CW)}{\partial p_0}\right) &= sign\left(p_1(1-p_1)\eta^2 - \frac{r(1-r)(1-\eta)^2}{4(1-p_0)^2}\right). \quad (4)
\end{aligned}
$$

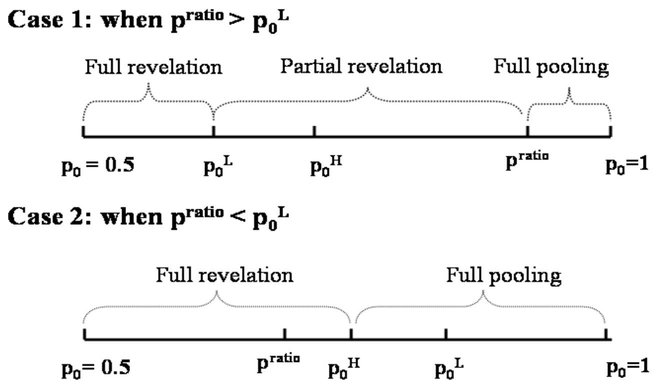
Simple calculations show that $p_1(1-p_1)\eta^2 - \frac{r(1-r)(1-\eta)^2}{4p_0^2} < 0$ for all p_0, p_1, r if $\eta \leq \frac{1}{3}$. Let $\overline{\eta}$ solve $p_1(1-p_1)\eta^2 - \frac{r(1-r)(1-\eta)^2}{4p_0^2} = 0$, then $\overline{\eta} \in (\frac{1}{3}, 1]$ (see also the remark on η after the proof). When $\eta \leq \overline{\eta}$, the left hand side (LHS from now on) of IC_1^L increases in p_0 and the right hand side (RHS from now on) decreases with it. We are now ready to prove the claims in the proposition.

(1.1) The case when the smart agent also improves faster. Then inequality (1) holds, which is equivalent to $p_0 \leq p^{ratio} \equiv \frac{1-r}{r+p_1-2rp_1}$. To start with, at $p_0 = \frac{1}{2}$, it is easy to see that $w(CR) - w(W)) = w(R) - w(CW)$ and all the four ICs hold strictly. At $p_0 = 1$, the agent always needs to be consistent ($w(R) = w(W) = w(CW) = 0$) and IC_1^L is clearly violated if the agent reports truthfully. Since the LHS of IC_1^L increases in p_0 and the RHS decreases in p_0, there exists a probability p_0^L such that IC_1^L binds when the agent reports truthfully ($\pi = 0$). Similarly, let p_0^H be the probability that IC_1^H binds when the agent reports truthfully.

Observe, however, that the three values p_0^L, p_0^H, p^{ratio} cannot be ranked in general. Because $p^{ratio} \in (\frac{1}{2}, 1]$ is a function of p_1, r, but p_0^L, p_0^H are functions of p_1, r, η, g, b. But we can show that there are only two possible cases, illustrated in the following Figure 3: first, if $p^{ratio} \geq p_0^L$, then $p_0^L < p_0^H < p^{ratio}$. Second, if $p^{ratio} < p_0^L$, then $p^{ratio} < p_0^H < p_0^L$.

To see the first case, note that IC_L^1 binds at p_0^L by definition. If $p_0^L < p^{ratio}$, IC_H^2 still holds at p_0^L, thus $p_0^L < p_0^H$. Moreover, it cannot be $p_0^H > p^{ratio}$ because in that case, IC_H^2 binds first at p_0^H while IC_L^1 holds strictly, which implies that $p_0^L > p_0^H > p^{ratio}$, a contradiction. Thus it must be $p_0^L < p_0^H < p^{ratio}$. Similar reasoning shows the second case.

Figure 3: Equilibrium behavior when p_0 varies



For all $p_0 \leq p_0^L \leq p^{ratio}$, IC_1^L holds and from the discussion above, IC_1^H holds. Moreover, since at $p_0 = \frac{1}{2}$, $w(CR) - w(W)) = w(R) - w(CW)$ and the LHS increases while the RHS decreases with p_0, IC_2^H and IC_2^L hold strictly. A unique full revelation equilibrium exists.

But if $p_0^L \leq p_0 < p^{ratio}$, then IC_1^L is violated and there is no full revelation equilibrium. Consider the mixing strategy that when $i_1 \neq m_0$, type L repeats m_0 with probability π. Then $w(CR) - w(W)$ decreases with π while $w(R) - w(CW)$ increases with π, because the more likely a type L agent repeats his initial report, the less likely that the principal thinks that the agent is smart after hearing consistent reports. Thus the LHS of IC_1^L decreases with π while the RHS increases with π. At $\pi = 1$, i.e., when L always repeats m_0, $LHS < RHS = w(1)$. When $LHS \geq RHS$ at $\pi = 0$, there exists a mixing probability $\pi^* \in (0, 1)$ such that $LHS = RHS$ and thus IC_1^L binds. When IC_1^L binds, all the other three IC hold strictly and we have a partial revelation equilibrium.

In order to see that π^* increases in p_0, let $f(p_0, \pi^*) \equiv (w(CR) - w(W))(1 - r) - (w(R) - w(CW))r = 0$. Since $\frac{\partial f}{\partial p_0} > 0$, $\frac{\partial f}{\partial \pi^*} < 0$, by the implicit function theorem, $\frac{d\pi^*}{dp_0} > 0$. Case 1 of Figure 3 illustrates how the continuation equilibria change as p_0 changes when H improves faster. If $p_0 > p^{ratio}$, however, the equilibrium behavior falls into the case when type L improves faster, which we turn to presently.

(1.2) When type L improves faster (inequality (1) does not hold), then $p_0 > p^{ratio}$. Similar to part (1.1), for p_0 sufficiently close to $\frac{1}{2}$, there still exists a full revelation continuation equilibrium. However, in this case IC_H^1 binds first as p_0 increases. Thus for all $p_0 < p_0^H$, IC_H^1 holds and the equilibrium is full revelation.

For $p_0 > p_0^H$, however, IC_H^1 is violated even though IC_L^1 still holds strictly. Suppose that type H repeats his first report with probability y when $i_1 \neq m_0$. Simple calculation shows that the LHS increases with y, thus type H will repeat his first report with probability one: the more type H repeats his first report, the more consistent signals signal high type. As a result, type L has to repeat his initial report because mind change leads to zero posterior probability of being smart. This continuation equilibrium can be supported by the principal's belief that $Pr(H|m_0 \neq m_1) = 0$.

The continuation equilibrium behavior when type L learns faster as p_0 increases is illustrated as Case 2 in Figure 3.

(1.3) When neither type's signal improves ($p_0 = p_1 = p, r = \frac{1}{2}$),[27] then the agent's belief after receiving conflicting signals becomes $Pr(g|b, g, \theta) = \frac{1}{2}$. Thus the key truthtelling constraints IC_L^1 and IC_H^1 become the same: $w(CR) - w(W) \leq w(R) - w(CW)$. However, this is violated for all $p > \frac{1}{2}$. Suppose that in equilibrium, type H and L repeat the first report with probability y and π respectively. Then clearly, truthtelling ($y = \pi = 0$) cannot be part of the equilibrium. Moreover, for any π, the LHS of the incentive constraint increases with y, while the RHS decreases with y as in part (1.2). Thus type H will repeat his first report with probability one and type L must do so as well. Thus the equilibrium is one of full pooling in which $m_0 = m_1 = i_0$.

Finally, given the continuation equilibrium above, we need to check whether the agent wants to report $m_0 = i_0$. Type L's first signal is completely uninformative and thus he is indifferent. Type H prefers to report $m_0 = i_0$ if the following incentive constraint is true:

$$\begin{aligned} & w(CR)p_0p_1 + w(R)(1-p_0)p_1 + w(W)p_0(1-p_1) + w(CW)(1-p_0)(1-p_1) \\ \geq \; & w(CR)(1-p_0)p_1 + w(R)p_0p_1 + w(W)(1-p_0)(1-p_1) + w(CW)p_0(1-p_1); \\ \Rightarrow \; & (w(CR) - w(R))p_1 \geq (w(CW) - w(W))(1-p_1). \end{aligned} \tag{5}$$

The LHS of the above is the expected reputational payoff if type H agent reports truthfully, while the RHS are the expected payoff if he reports $m_0 \neq i_0$. IC (5) holds if $CR \geq R$ and $W \geq CW$. For a full revelation continuation equilibrium, the discussion in part (1) shows that $CR > R > W > CW$ and IC (5) holds. For a partial revelation continuation equilibrium, recall that $W \geq CW$ at $\pi = 0$, and W increases in π while CW decreases in π. Thus in a partial information revelation equilibrium, $W \geq CW$. Also, in equilibrium, $CR \geq R$, otherwise type L should deviate by reducing his mixing probability and receive higher payoff. Therefore IC (5) holds in a partial revelation equilibrium. Last, in a full pooling equilibrium, IC (5) simplifies to $w(CR)p_1 \geq w(CW)(1-p_1)$, which is always true. Thus both types of agent report $m_0 = i_0$, regardless of the type of continuation equilibrium. ‖

Remark: strong expert market. One remaining question is that what happens if the expert market is very good, i.e., the fraction of smart agent η exceeds the cutoff value $\overline{\eta} \in [\frac{1}{3}, 1]$ in Proposition 1. The limit $\overline{\eta}$ is necessary to guarantee the monotonicity of the agent's incentives with respect to p_0, the key parameter of interest. Because the wage function depends on parameter values g, b, the cutoff $\overline{\eta}$ is a lower bound derived using the limit case of an affine wage function. From inequality (3) and (4), we can see that Proposition 1 holds for even higher $\overline{\eta}$ in general, and thus it is less restrictive. When η is very high, however, we can see from Equation 4 that the sign of the LHS and the RHS depends on the specific w and a general characterization cannot be given. However, in a special case, the following limiting result holds.

Corollary 1 *When $w(\hat{\eta})$ is affine and strictly increasing in $\hat{\eta}$, a full revelation equilibrium exists for p_0 sufficiently close to $\frac{1}{2}$, A pooling equilibrium such that $m_0 = i_0 = m_1$ exists when p_0 is sufficiently close to 1, and the second report is uninformative in any equilibrium.*

Proof: similar to that of Proposition 1. ‖

APPENDIX C: WHEN ABILITY IS SYMMETRIC INFORMATION

All assumptions remain the same as the two signal model in the text, except that now both the principal and the agent only know the agent's type distribution. Then the following result holds:

[27] This can be generalized to type L receives informative signals as long as $r < p$.

Proposition 5 *When $\eta \leq \overline{\eta} \in [\frac{1}{3}, 1)$, there exists a cutoff $\hat{p}_0$ such that:*
(5.1) If $p_0 \leq \hat{p}_0$, there exists a unique full revelation equilibrium in which the agent always reports $m_0 = i_0, m_1 = i_1$. Moreover, $\hat{p}_0 > p_0^L$ if type H improves faster, and $\hat{p}_0 > p_0^H$ if type L does.
(5.2) If $p_0 > \hat{p}_0$, then there exists a full pooling equilibrium in which the agent always repeats his first report, and the principal believes that $Pr(H|m_0 \neq m_1) = \eta$.
(5.3) The market always values consistent reports more than mind changes in equilibrium, i.e., $Pr(H|m_0 = m_1) > Pr(H|m_0 \neq m_1)$.

Proof: first, find the continuation equilibrium assuming that the agent reports i_0 truthfully. Suppose that $m_0 = i_0 = g$, then the following two truth-telling ICs need to hold, depending on whether $i_0 = i_1$ (instead of four in the asymmetric information model):

$$[w(CR) - w(W)]Pr(g|g,g) \geq [w(R) - w(CW)]Pr(b|g,g); \quad (IC_1)$$
$$[w(CR) - w(W)]Pr(g|g,b) \leq [w(R) - w(CW)]Pr(b|g,b). \quad (IC_2)$$

The agent's own estimate of the state given his signals are:

$$Pr(g|g,g) = \frac{p_0 p_1 \eta + \frac{r}{2}(1-\eta)}{[p_0 p_1 + (1-p_0)(1-p_1)]\eta + \frac{1}{2}(1-\eta)}; \quad Pr(g|g,b) = \frac{p_0(1-p_1)\eta + \frac{1-r}{2}(1-\eta)}{[p_0(1-p_1) + (1-p_0)p_1]\eta + \frac{1}{2}(1-\eta)}.$$

Since $Pr(g|g,g) > Pr(g|g,b)$, if (IC_2) binds, (IC_1) holds strictly. Moreover, $Pr(g|g,b)$ strictly increases in p_0. Assume that the agent repeats his initial report with probability τ, his posterior reputations are respectively:

$$(CR) \quad Pr(H|g,g;g) = \frac{p_0[p_1 + (1-p_1)\tau]\eta}{p_0[p_1 + (1-p_1)\tau]\eta + \frac{1}{2}[r + (1-r)\tau](1-\eta)};$$

$$(CW) \quad Pr(H|g,g;b) = \frac{(1-p_0)[(1-p_1) + p_1\tau]\eta}{(1-p_0)[(1-p_1) + p_1\tau]\eta + \frac{1}{2}[(1-r) + r\tau](1-\eta)};$$

$$(W) \quad Pr(H|g,b;g) = \frac{p_0(1-p_1)\eta}{p_0(1-p_1)\eta + \frac{1}{2}(1-r)(1-\eta)}; \quad (R) \quad Pr(H|g,b;g) = \frac{(1-p_0)p_1\eta}{(1-p_0)p_1\eta + \frac{1}{2}r(1-\eta)}.$$

Note that if the agent reports truthfully ($\tau = 0$), the posterior reputations CR and CW are identical to those with asymmetric information at $\pi = 0$. Moreover, since the agent does not know his type and lie with the same probability τ, W and R don't vary with τ.

(5.1) Similar to Proposition 1, at p_0 sufficiently close to $\frac{1}{2}$, both incentive constraints hold strictly and a unique full revelation equilibrium exists. The agent reveals both signals truthfully because neither type's initial signal is very accurate and thus a correct final report is much more important in influencing the principal's beliefs. When $\eta \leq \overline{\eta}$, the LHS increases with p_0 and the RHS decreases with it. Thus at $p_0 = \hat{p}_0$, the agent is indifferent between repeating his first report or reporting his second signal truthfully. Moreover, because $Pr(g|g,b,H) < Pr(g|g,b) < Pr(g|g,b,L)$, the cutoff value $\hat{p}_0 > p_0^L$ if type H improves faster, and $\hat{p}_0 > p_0^H$ if type L improves faster. Intuitively, if the agent receives inconsistent signals, he believes that with some probability he is mediocre and his second signal is not too accurate, and with some probability that he is smart and the second signal is very accurate. Therefore he has more incentive to report the second signal truthfully than in the asymmetric information case.

(5.2) When $p_0 > \hat{p}_0$, IC_2 does not hold at $\tau = 0$. Moreover, the wage gap between consistent and inconsistent reports $\Delta_w \equiv [w(CR) - w(W)]Pr(g|g,b) - [w(R) - w(CW)]Pr(b|g,b)$

increases with p_0. Thus the agent is tempted to appear more consistent by repeating his first report with positive probability. The expected wage after inconsistent report does not vary with the mixing probability τ, as described above. The expected wage of giving consistent reports $(w(CR)Pr(g|g,b) + w(CW)Pr(b|g,g))$ increases with τ for $p_0 \leq \overline{p}_0$ and decreases with τ for $p_0 > \overline{p}_0$. Thus for $p_0 \leq \overline{p}_0$, the agent will increase τ until $\tau = 1$, thus we have a full pooling equilibrium. When $p_0 \in (\overline{p}_0, 1]$, simple calculation can show that even though Δ_w decreases in τ, but it is still positive at $\tau = 1$, thus the agent will repeat his initial report as well.

This equilibrium can be supported by the principal's belief that $Pr(H|m_1 \neq m_0) = \eta$ (but it works whenever $Pr(H|m_1 \neq m_0) \leq \eta$). That is, the principal thinks that a smart agent is no more likely to make mistakes and give inconsistent reports than an mediocre one. Finally, recall that the agent's first signal is uninformative about his type, then reasoning similar to that of Proposition 1 shows that in equilibrium the agent always reports $m_0 = i_0$.

(5.3) As the above two parts show, in equilibrium, either the agent reports i_1 truthfully ($\tau = 0$) or he repeats his initial report probability $\tau = 1$, let us compare the probability that the agent is considered smart conditional on his reports.

$$Pr(H|m_1 = m_0) = \frac{\big[p_0 p_1 + (1-p_0)(1-p_1) + [p_0(1-p_1) + p_1(1-p_0)]\tau\big]\eta}{\big[p_0 p_1 + (1-p_0)(1-p_1) + [p_0(1-p_1) + p_1(1-p_0)]\tau\big]\eta + \frac{1+\tau}{2}(1-\eta)};$$

$$Pr(H|m_1 \neq m_0) = \frac{[p_0(1-p_1) + p_1(1-p_0)]\eta}{[p_0(1-p_1) + p_1(1-p_0)]\eta + \frac{1}{2}(1-\eta)}.$$

Simple calculations show that $Pr(H|m_1 = m_0) \geq Pr(H|m_1 \neq m_0)$ at $\tau = 1$, and $Pr(H|m_0, m_1 = m_0)$ decreases in τ. Thus consistent reports always signal higher ability in equilibrium. Intuitively, both the principal and the agent himself believe that H is more likely to receive consistent signals. When $i_0 \neq i_1$, the agent has relatively little confidence of his talent and prefers giving more consistent report to appear smart. ‖

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