Vacancies, Unemployment, and the Phillips Curve

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Abstract

The canonical new Keynesian Phillips Curve has become a standard component of models designed for monetary policy analysis. However, in the basic new Keynesian model, there is no unemployment, all variation in labor input occurs along the intensive hours margin, and the driving variable for inflation depends on workers' marginal rates of substitution between leisure and consumption. In this paper, we incorporate a theory of unemployment into the new Keynesian theory of inflation. We show how a traditional Phillips curve linking inflation and unemployment can be derived and how the elasticity of inflation with respect to unemployment depends on structural characteristics of the labor market such as the matching technology that pairs vacancies with unemployed workers. We also derive a simple two-equation model for monetary policy analysis consistent with sticky prices and labor market frictions.

JEL: E52, E58, J64

1 Introduction

The canonical new Keynesian Phillips curve has become a standard component of models designed for monetary policy analysis. Based on the presence of monopolistic competition among individual firms, together with the imposition of stagged price setting, the new

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Keynesian Phillips curve provides a direct link between the underlying structural parameters characterizing the preferences of individual suppliers of labor and the parameters appearing in the Phillips curve.

However, in the basic new Keynesian model, all variation in labor input occurs along the intensive hours margin. In the standard sticky price, flexible wage model, the real wage and the marginal rate of substitution between leisure and consumption move together so that, at all points in time, households are supplying the amount of hours that maximize their utility, given the real wage. There are no unemployed workers; only hours worked per worker vary over the business cycle. As a consequence, the driving variable for inflation depends on workers' marginal rates of substitution between leisure and consumption. In its neglect of unemployment, the new Keynesian Phillips curve has a distinctly non-Keynesian flavor.

In contrast to this standard view of labor input, empirical evidence suggests that, at business cycle frequencies, most variation of labor input occurs at the extensive margin. In periods of below trend output, employed workers work fewer hours, but also fewer workers are employed. During periods of above trend output, employed workers work longer hours but also more workers are employed. These fluctuations in the fraction of workers actually employed reflect fluctuations in unemployment.

A growing number of papers have attempted to incorporate the extensive margin and unemployment into new Keynesian models. Example include Walsh (2003, 2005), Alexopoulos (2004), Trigari (2004), Christoffel, Kuester, and Linzert (2006), Blanchard and Galí (2005, 2006), Krause and Lubik (2005), Barnichon (2006), Thomas (2006), and Gertler and Trigari (2006). The focus of these earlier contributions has extended from exploring the implications for macro dynamics in calibrated models to the estimation of DSGE models with labor market frictions. In contrast to this earlier literature, we focus directly on the implications of the labor market specification for the Phillips curve and the connection between the structure of the labor market and the unemployment elasticity of inflation.

To draw a clear distinction with the previous literature, our model allows labor to adjust only along the extensive margin. Standard models allow adjustment only along the intensive margin. Trigari incorporates both margins, but marginal cost (and so inflation) is driven by the intensive margin. Consequently, the marginal rate of substitution between leisure hours and consumption is key, just as in standard new Keynesian models. Krause and Lubik depart from the Calvo model of price adjustment by assuming quadratic adjustment costs. In this case, all firms adjust each period, an implication that is not consistent with micro evidence on price adjustment. They also assume output adjustment occurs via fluctuations in the endogenous job destruction rate, which is not consistent with Hall's contention that this rate is roughly constant over the cycle. We retain the standard Calvo model of price adjustment and treat job destruction as exogenous.

Once we obtain an unemployment-based Phillips curve, we combine it with an unemployment version of the expectational IS equation derived from the households' Euler condition and the assumption of goods market clearing. This gives us a two equation representation of the economy, expressed in terms of an unemployment gap and inflation, that is comparable to the standard new Keynesian model. We use this model to derive the optimal commitment policy from a timeless perspective.

We see this paper as providing a link between the much older literature on Phillips curves which related unemployment and inflation (e.g., Gordon 197X) and the modern approach based on dynamic stochastic general equilibrium models. The older literature investigated the connection between unemployment and inflation from an empirical perspective with little formal theory to link the two.

The paper closest to ours in motivation is Blanchard and Galí (2006). They too are interested in developing a simple framework akin to the basic new Keynesian model but in which unemployment plans a central role. In contrast to the Mortensen-Pissarides search model we use, Blanchard and Galí assume firms face hiring costs that are increasing in the degree of labor market tightness (measured as new hires relative to unemployment). Their model does not incorporate vacancies and therefore does not provide for an endogenously generated Beveridge curve.

The rest of the paper is organizes as follows. The basic model is developed in section 2. The equilibrium of the model under flexible prices is discussed in section 3. Equilibrium in the presence of sticky prices is analyzed in section 4. A log-linearized version of the model is derived and the connections between labor market structure and the Phillip curve are discussed. Section 5 uses a simple two equation representation of the model to derive the optimal monetary policy targeting rule. Conclusions are summarized in section 6.

2 The model economy

The model consists of households whose utility depends on the consumption of market and home produced goods. Households members are either employed (in a match) or searching for a new match. This means that we do not focus on labor force participation decisions. Households are employed by wholesale goods producing firms operating in a competitive market for the goods they produce. Wholesale goods are, in turn, purchased by retail firms who sell to households. The retail goods market is characterized by monopolistic competition. In addition, retail firms have sticky prices that adjust according to a standard Calvo specification. The modelling strategy of locating labor market frictions in the wholesale sector where prices are flexible and locating sticky prices in the retail sector among firms who do not employ labor provides a convenient separation of the two frictions in the model. A similar approach was adopted in Walsh (2003, 2005), Trigari (2005), and Thomas (2006).

2.1 Households

Workers can be either employed by wholesale firms in production activities, receiving a market real wage w_t, or unemployed, earning a fixed amount w^u of household production units. We assume that consumption risks are fully pooled; the consumption level of each worker would otherwise depend on its complete employment history. The optimality conditions for workers can be derived from the utility maximization problem of a large representative household with value function

$$\begin{aligned} W_t(N_t, B_t) &= \max\ U(C_t) + \beta E_t W_{t+1}(N_{t+1}, B_{t+1}) \\ st \quad P_t C_t + p_{bt} B_{t+1} &= P_t[w_t N_t + w^u(1 - N_t)] + B_t + P_t \Pi_t^r \end{aligned} \tag{1}$$

where C_t is consumption of each household's member, N_t is the fraction of the household's members currently employed, Π_t^r are profits from the retail sector, B_t is the amount of riskless nominal bonds held by the household, with price equal to p_{bt}. The price of a unit of teh consumption basket is P_t and is defined below. Consumption of market goods supplied by the retail sector is equal to $C_t^m = C_t - (1 - N_t)w^u$.

Consumption C_t^m is an aggregate of consumption purchased from the continuum of retail firms which produce differentiated final goods. The household preferences over the individual final goods from firm j, $C(j)$, are defined by the standard Dixit-Stiglitz aggregator, so that

$$
\begin{aligned}
E_t^m &= \int_0^1 P_t(j) C_t^m(j) dj = P_t C_t^m \\
C_t^m(j) &= \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon} C_t^m \\
P_t &= \left[\int_0^1 P_t(j)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}},
\end{aligned}
$$

where E_t^m is total expenditure by the household on consumption good purchases.

The intertemporal first order conditions yield the standard Euler equation:

$$\lambda_t = \beta E_t\{R_t \lambda_{t+1}\},$$

where R_t is the gross return on an asset paying one unit of consumption aggregate in any state of the world and λ_t is the marginal utility of consumption.

At the start of each period t, N_{t-1} workers are matched in existing jobs. We assume a fraction ρ $(0 \leq \rho < 1)$ of these matches exogenous terminate. To simplify the analysis, we ignore any endogenous separation.[1] The fraction of the household members who are employed evolves according to

$$N_t = (1-\rho)N_{t-1} + p_t s_t$$

where p_t is the probability of a worker finding a position and

$$s_t = 1 - (1-\rho)N_{t-1} \tag{3}$$

is the fraction of searching workers. Thus, we assume workers displaced at the start of period t have a probability p_t of finding a new job within the period (we think of a quarter as the time period). Consequently, the surplus value of a job to a worker expressed in terms of consumption (not utility) units is

$$V_t^S = w_t - w^u + \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right)(1-\rho)\left(1-p_{t+1}\right) V_{t+1}^S. \tag{4}$$

[1] Hall (xxxx) has argued that the separation rate varies little over the business cycle, although this position has been disputed by XXXX (xxxx). For a model with endogenous separation and sticky prices, see Walsh (2003).

Note that unemployment as measured after period t hiring is equal to $u_t \equiv 1 - N_t$.

2.2 Wholesale firms and wages

Production by wholesale firm i is

$$Y_{it}^w = Z_t N_{it}, \tag{5}$$

where Z_t is a common, aggregate productivity disturbance with a mean equal to 1 and bounded below by zero. Aggregating (5), $Y_t^w = Z_t N_t$.

Wholesale firms must post vacancies to obtain new employees. They lose existing employees at the rate ρ. To post a vacancy, a wholesale firms must pay a cost $P_t \kappa$ for each job posting. Since job postings are homogenous with final goods, effectively wholesale firms solve a static problem symmetric to the household's one: they buy individual final goods $v_t(j)$ from each j final goods producing retail firm so as to minimize the total expenditure, given that the production function of a unit of final good aggregate v_t is given by

$$\left[\int_0^1 v_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}} \geq v_t.$$

Therefore, total expenditures E^w on job postings and the demand for the final goods produced by retail firm j are given by

$$\begin{aligned} E_t^w &= \kappa \int_0^1 P_t(j) v_t(j) dj = \kappa P_t v_t \\ v_t(j) &= \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon} v_t, \end{aligned}$$

where $P_t = \left[\int_0^1 P_t(j)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$.

Total expenditure on final goods by households and wholesale firms is

$$
\begin{aligned}
E_t &= E_t^w + E_t^m \\
&= \kappa \int_0^1 P_t(j) v_t(j) dj + \int_0^1 P_t(j) C_t^m(j) dj \\
&= \int_0^1 P_t(j) Y_t^d(j) dj \\
&= P_t(C_t^m + \kappa v_t)
\end{aligned}
$$

where $Y_t^d(j) = \kappa v_t(j) + C_t^m(j)$ is total demand for final good j.

The number of workers available for production at firm i is given by

$$N_{it} = (1-\rho)N_{it-1} + v_{it} q(\theta_t),$$

where v_{it} is the number of vacancies the firm posts and $q(\theta_t)$ is the probably of filling a vacancy. This probability is a function of aggregate labor market tightness θ_t, equal to the ratio of aggregate vacancies v_t and the aggregate number of workers searching for a job s_t ($\theta_t \equiv v_t/s_t$). At the aggregate level, workers available for production in period t equal

$$N_t = (1-\rho)N_{t-1} + v_t q(\theta_t) \tag{7}$$

Wholesale firms sell their output in a competitive market at the price P_t^w. The real value of the firm's output, expressed in terms of time t consumption goods, is $P_t^w Y_{it}/P_t = Y_{it}/\mu_t$, where $\mu_t = P_t/P_t^w$ is the markup of retail over wholesale prices.

Let Π_{it} denote firm i's period t profit. The wholesale firm's problem is to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\lambda_{t+i}}{\lambda_t} \right) \Pi_{it+j},$$

where

$$\Pi_{it+j} = \mu_{t+i}^{-1} Y_{it+j}^w - \kappa v_{it+j} - w_{t+j} N_{it+j}$$

and the maximization is subject to (7) and (5) and is with respect to Y_{it}^w, N_{it}, and v_{it}. Vacancy costs, κv_{it}, and the wage are expressed in terms of consumption goods. Let φ and ψ be the Lagrangian multipliers on (7) and (5). Then the first order conditions for the firm's problem are

$$\text{For } Y_{it}^{w}\text{: } \mu_t^{-1} - \psi_{it} = 0$$

$$\text{For } v_{it}\text{: } -\kappa - \varphi_{it} q(\theta_t) = 0$$

$$\text{For } N_{it}\text{: } \mu_t^{-1} Z_t - w_t + \varphi_{it} - \beta(1-\rho) E_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \varphi_{it+1} = 0$$

The first two of these conditions imply

$$\psi_{it} = \psi_t = \left(\frac{1}{\mu_t}\right) \text{ for all } t$$

and

$$\varphi_{it} = -\frac{\kappa}{q(\theta_t)} \text{ for all } t.$$

Thus, reflecting the competitive market for the output of wholesale firms, each such firm charges the same price and the shadow prices of a filled job is equal across firms.

Using these results in the last first order condition yields

$$\frac{\kappa}{q(\theta_t)} = \frac{Z_t}{\mu_t} - w_t + \beta(1-\rho) E_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \frac{\kappa}{q(\theta_{t+1})}. \tag{8}$$

We can rewrite this equation as

$$w_t = \frac{Z_t}{\mu_t} - \frac{\kappa}{q(\theta_t)} + \beta(1-\rho) E_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \frac{\kappa}{q(\theta_{t+1})}$$

The real wage is equal to the marginal product of labor Z_t/μ_t, minus the expected cost of hiring the matched worker $\kappa/q(\theta_t)$ (a vacancy is matched with probability $q(\theta_t)$, so the number of vacancies to be posted such that expected hires equals one is $1/q(\theta_t)$ each of which costs κ), plus the expected saving the following period of not having to generate a new match, all expressed in units of the final good. Note that if $\kappa = 0$, this yields the standard result that $w_t = Z_t/\mu_t$.

The value of a filled job is equal to $\kappa/q(\theta_t)$. To see this, let V_t^V and V_t^J be the value to the firm of an unfilled vacancy and a filled job respectively. Then

$$V_t^V = -\kappa + q(\theta_t) V_t^J + [1 - q(\theta_t)]\, E_t \beta \left(\frac{\lambda_{t+1}}{\lambda_t}\right) V_{t+1}^V.$$

Free entry implies that $V_t^V = 0$, so

$$V_t^J = \frac{\kappa}{q(\theta_t)}. \tag{9}$$

2.2.1 Wages

Assume the wage is set in Nash bargaining with the worker's share equal to b. Let V_t^S be the surplus to the worker of a match relative to not being in a match. Then the outcome of the wage bargain ensures

$$(1-b)V_t^S = bV_t^J = \frac{b\kappa}{q(\theta_t)}, \tag{10}$$

where the job posting condition (9) has been used. Since the probability of a searching worker being employed is $p_t = M_t/s_t = \theta_t q(\theta_t)$ where M_t is the number of new employer-worker matches formed in t, the value of the match to the worker (4) can be rewritten as

$$V_t^S = w_t - w^u + \beta(1-\rho)E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left[1-\theta_{t+1}q(\theta_{t+1})\right]V_{t+1}^S. \tag{11}$$

The term $[1-\theta_{t+1}q(\theta_{t+1})]$ arises since workers who are in a match at time t but who do not survive the exogenous separation hazard at $t+1$ may find a new match during $t+1$.[2]

Using (11) in (10),

$$\frac{b\kappa}{q(\theta_t)} = (1-b)\left(w_t - w^u\right) + \beta(1-\rho)E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left[1-\theta_{t+1}q(\theta_{t+1})\right]\frac{b\kappa}{q(\theta_{t+1})}.$$

Solving this for the wage and substituting the result into (8), one obtains an expression for the real wage:

$$w_t = (1-b)w^u + b\left[\frac{Z_t}{\mu_t} + \beta(1-\rho)E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\kappa\theta_{t+1}\right]. \tag{12}$$

Substituting (12) into (8), one finds that the relative price of wholesale goods in terms of retail goods is equal to

$$\frac{P_t^w}{P_t} = \frac{1}{\mu_t} = \frac{\tau_t}{Z_t}, \tag{13}$$

[2] See the appendix for details.

where

$$\tau_t \equiv w^u + \left(\frac{1}{1-b}\right)\left\{\frac{\kappa}{q(\theta_t)} - \beta(1-\rho)E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left[1-b\theta_{t+1}q(\theta_{t+1})\right]\frac{\kappa}{q(\theta_{t+1})}\right\} \tag{14}$$

summarizes the impact of labor market conditions on the relative price variable.

It is useful to contrast expression (13) with the corresponding expression arising in a new Keynesian model with a Walrasian labor market. The marginal cost faced by a retail firm is P_t^w/P_t. In a standard new Keynesian model with sticky prices, marginal cost is proportional to the ratio of the marginal rate of substitution between leisure and consumption (equal to the real wage) and the marginal product of labor. Since the marginal product of labor is equal to Z_t, (13) shows how, in a search model of the labor market, the marginal rate of substitution is replaced by a labor-cost expression that depends on the worker's outside productivity, w^u, and current and expected future labor market conditions via θ_t and θ_{t+1}. If vacancies could be posted costlessly ($\kappa = 0$), then $\tau_t = w^u$ as firms only need to pay workers a wage equal to worker's outside alternative. When $\kappa > 0$, matches have value and the wage will exceed w^u. The wage, and therefore marginal cost, varies with labor market tightness.

2.3 Retail firms

Each retail firm purchases wholesale output which it converts into a differentiated final good that is sold to households and wholesale firms. The retail firms cost minimization problem implies

$$MC_t^n = P_t MC_t = P_t^w$$

where MC^n is nominal marginal cost and MC is real marginal cost.

Retail firms face a Calvo process for adjusting prices. Each period, there is a probability $1-\omega$ that a firm can adjust its price. Since all firms that adjust their price are identical, they all set the same price. Given MC_t^n, the retail firm chooses $P_t(j)$ to max

$$\sum_{i=0}^{\infty}(\omega\beta)^i E_t\left[\left(\frac{\lambda_{t+i}}{\lambda_t}\right)\frac{P_t(j)-MC_{t+i}^n}{P_{t+i}}Y_{t+i}(j)\right]$$

subject to

$$Y_{t+i}(j) = Y_{t+i}^d(j) = \left[\frac{P_t(j)}{P_{t+i}}\right]^{-\varepsilon} Y_{t+i}^d \tag{15}$$

where $Y_t^d = \frac{E_t}{P_t}$ is aggregate demand for the final goods basket. The standard pricing equation obtains. These can be written as

$$[(1+\pi_t)]^{1-\geq} = \omega + (1-\omega)\left[\frac{\tilde{G}_t}{\tilde{H}_t}(1+\pi_t)\right]^{1-\varepsilon}, \tag{16}$$

where

$$\tilde{G}_t = \mu\lambda_t\mu_t^{-1}Y_t + \omega\beta\tilde{G}_{t+1}(1+\pi_{t+1})^{\varepsilon}$$

$$\tilde{H}_t = \lambda_t Y_t + \omega\beta\tilde{H}_{t+1}(1+\pi_{t+1})^{\varepsilon-1}$$

and λ_t is the marginal utility of consumption.

2.4 Market Clearing

Aggregating the budget constraint (1) over all households yields

$$P_t C_t^m = P_t w_t N_t + P_t \Pi_t^r.$$

Since the wholesale sector is in perfect competition, profits Π_{it} are zero for each i firm and

$$\frac{P_t^w}{P_t} Y_t^w = w_t N_t + \kappa v_t.$$

In turn, this implies

$$C_t^m = \frac{P_t^w}{P_t} Y_t^w - \kappa v_t + \Pi_t^r. \tag{17}$$

Profits in the retail sector are equal to

$$\begin{aligned}
\Pi_t^r &= \int \left[\frac{P_t(j)}{P_t} - \frac{P_t^w}{P_t}\right] Y_t^d(j)dj \\
&= \frac{1}{P_t}\int P_t(j)Y_t^d(j)dj - \frac{P_t^w}{P_t}\int Y_t^d(j)dj
\end{aligned}$$

Since for each good j market clearing implies $Y_t^d(j) = Y_t(j)$, and since the production function of final goods is given by $Y_t(j) = Y_t^w(j)$, we can write profits of the retail sector as

$$\Pi_t^r = Y_t^d - \frac{P_t^w}{P_t} Y_t^w,$$

where $Y_t^w = \int Y_t^w(j)dj$. Then (17) gives aggregate real spending:

$$Y_t^d = C_t^m + \kappa v_t. \tag{18}$$

Finally, using the demand for final good j in (15), the aggregate resource constraint is

$$\begin{aligned}\int Y_t(j)dj &= \int Y_t^w(j)dj = Z_t \int N_t(j)dj = Z_t N_t \\ &= \int \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon} Y_t^d dj = \int \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon} [C_t^m + \kappa v_t] dj,\end{aligned}$$

or

$$Y_t^w = Z_t N_t = [C_t^m + \kappa v_t] \int \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon} dj. \tag{19}$$

Aggregate consumption is given by

$$C_t = C_t^m + w^u(1 - N_t).$$

A more compact way of rewriting the resource constraint can be obtaining be writing (18) and (19) as:

$$\begin{aligned}Y_t^d &= C_t^m + \kappa v_t \\ Y_t^w &= Y_t^d f_t,\end{aligned}$$

where f_t is defined as

$$f_t \equiv \int_0^1 \left[\frac{P_t(z)}{P_t}\right]^{-\varepsilon} dz$$

and measures relative price dispersion across retail firms.

3 Equilibrium with flexible prices

With flexible prices in both the wholesale and retail sectors, the markup is a constant and equal to $\mu \equiv \epsilon/(\epsilon - 1) > 1$. In addition, $P_{jt} = P_t$ for all retail firms. Letting the matching function be constant returns to scale Cobb-Douglas, given by $\eta v_t^{\xi} u_t^{1-\xi}$ with $0 < \xi < 1$ and $\eta > 0$, the probability of filling a vacancy is given by $q_t = \eta \theta_t^{\xi - 1}$ and the probability of finding a job is $p_t = \theta q_t = \eta \theta_t^{\xi}$. Assume utility is $U(C_t) = C_t^{1-\sigma}/(1-\sigma)$. Then, consumption, employment, and labor market tightness satisfy the job posting condition, the goods market equilibrium condition, and the employment transition equation.

Consider the steady-state equilibrium for this economy. Using (13), (14), the definitions of market consumption together with the goods market clearing condition and (7), steady-state employment, consumption, and labor market tightness satisfy the following three equations:

$$f(\bar{\theta}) \equiv [1-\beta(1-\rho)]\,\eta^{-1}\bar{\theta}^{1-\xi} + b\beta(1-\rho)\bar{\theta} = \left(\frac{1-b}{\kappa}\right)\left(\frac{1}{\mu} - w^u\right). \tag{20}$$

$$\bar{N} = \frac{\eta\bar{\theta}^{\xi}}{\rho + (1-\rho)\eta\bar{\theta}^{\xi}} \tag{21}$$

$$\bar{C} = \left\{\frac{1}{\mu} - (1-\rho)\left[w^u(1-\eta\bar{\theta}^{\xi}) - \kappa\bar{\theta}\right]\right\}\bar{N} - \kappa\bar{\theta} \tag{22}$$

The economy displays a recursive structure; (20) determines $\bar{\theta}$, (21) then determines $\bar{N}$ as a function of $\bar{\theta}$, and finally (22) determines $\bar{C}$. Note also that (21) and (22) are independent of b which determines how the job surplus is split between workers and firms.

The right side of (20) is independent of θ and positive for $\mu w^u < 1$; this condition ensures it is efficient for individuals to engage in market production. The left hand side is strictly increasing in θ, $f(0) = 0$ and $\lim_{\theta\to\infty} f(\theta) = \infty$, so there exists a unique solution $f(\bar{\theta}) = \left(\frac{1-b}{\kappa}\right)\left(\frac{1}{\mu} - w^u\right)$. Labor market tightness is decreasing in the cost of posting vacancies, labor's share of the job match surplus, and the outside opportunity wage. An increase in the retail markup (a rise in μ) also reduces labor market tightness by reducing the returns to posting vacancies; it does so by raising the cost in terms of wholesale goods to posting vacancies (recall, κ is fixed in consumption units).

3.1 The efficient allocation

The market equilibrium in this economy is subject to three types of distortions: monopoly power in the retail goods market, sticky prices, and externalities in the labor market matching process. The efficient allocation, subject to the constraints implied by the matching process, is given by the maximization of

$$E_t \sum_{i=0}^{\infty} \beta^i \left(\frac{C_{t+i}^{1-\sigma}}{1-\sigma}\right)$$

subject to

$$C_t = Z_t N_t - \kappa v_t + w_t^u (1 - \theta_t q_t) s_t,$$

and

$$N_t = (1-\rho) N_{t-1} + v_t q(\theta_t),$$

where $v_t = s_t \theta_t$ and $s_t = 1 - (1-\rho) N_{t-1}$.

The appendix shows that the condition for an efficient allocation is

$$\begin{aligned} Z_t &= w^u + \frac{1}{\xi} \frac{\kappa}{q(\theta_t)} - \frac{1}{\xi} \beta (1-\rho) E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \frac{\kappa}{q(\theta_{t+1})} \\ &\quad + \beta (1-\rho) \left(\frac{1-\xi}{\xi} \right) E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \kappa \theta_{t+1}, \end{aligned} \tag{23}$$

where ξ is the elasticity of matches with respect to labor market tightness.

From (13) and (14), the private market equilibrium implies

$$\begin{aligned} \frac{Z_t}{\mu_t} &= w^u + \frac{1}{1-b} \frac{\kappa}{q(\theta_t)} - \beta \left(\frac{1-\rho}{1-b} \right) E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \frac{\kappa}{q(\theta_{t+1})} \\ &\quad + \left(\frac{b}{1-b} \right) \beta (1-\rho) E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \kappa \theta_{t+1}. \end{aligned} \tag{24}$$

Inspection reveals that (24) is equivalent to (23) when $\mu_t = 1$ (perfect competition in the retail goods market and flexible prices) and $\xi = 1 - b$. This last condition is the standard Hosios (1990) condition for search market efficiency.

4 Equilibrium with sticky prices

When prices are sticky ($\omega > 0$), the retail price market up (equivalently, the marginal cost of retail firms) can vary. The complete set of equilibrium conditions

$$C_t^{-\sigma} = \beta E_t \left\{ R_t C_{t+1}^{-\sigma} \right\}. \tag{25}$$

$$\frac{Z_t}{\mu_t} = w^u + \frac{1}{1-b} \kappa \frac{1}{\eta} \theta_t^{1-\xi} - \kappa \beta \left(\frac{1-\rho}{1-b} \right) E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{1}{\eta} \theta_{t+1}^{-\xi} - b \right) \theta_{t+1}. \tag{26}$$

$$C_t = Z_t N_t + \left[w^u (1 - \eta \theta_t^{\xi}) - \kappa \theta_t \right] s_t \tag{27}$$

$$N_t = (1-\rho) N_{t-1} + \eta \theta_t^{\xi} \left[1 - (1-\rho) N_{t-1} \right], \tag{28}$$

$$s_t = 1 - (1-\rho) N_{t-1} \tag{29}$$

$$Z_t N_t = Y_t \int_0^1 \left[\frac{P_t(z)}{P_t} \right]^{-\varepsilon} dz \tag{30}$$

$$Y_t = C_t - w^u (1 - N_t) + \kappa s_t \theta_t \tag{31}$$

$$[(1+\pi_t)]^{1-\varepsilon} = \omega + (1-\omega) \left[\frac{\tilde{G}_t}{\tilde{H}_t} (1+\pi_t) \right]^{1-\varepsilon} \tag{32}$$

$$\tilde{G}_t = \mu \lambda_t \mu_t^{-1} Y_t + \omega \beta \tilde{G}_{t+1} (1+\pi_{t+1})^{\varepsilon} \tag{33}$$

$$\tilde{H}_t = \lambda_t Y_t + \omega \beta \tilde{H}_{t+1} (1+\pi_{t+1})^{\varepsilon - 1} \tag{34}$$

and a specification for monetary policy.

4.1 Log linearization of the sticky price equilibrium

The standard new Keynesian model is typically log-linearized to obtain three equations: the Phillips curve, an expectational IS curve, and a specification of monetary policy. These three relationships jointly determine inflation, the output gap (output relative to the flex-price equilibrium output), and the nominal rate of interest. The model developed in section 2 can also be log-linearized and reduced to a three equation system, one involving inflation, unemployment, and the nominal interest rate.

Let $\hat{x}_t$ denote the log deviation of a variable x around its steady-state value, and let $\tilde{x}_t$ denote the deviation of $\hat{x}_t$ around its flexible-price equilibrium value. A variable without a time subscript denotes the steady-state value. Using (30) and (31) to eliminate Y_t (yielding $Z_t N_t = [C_t - w^u (1-N_t) + \kappa s_t \theta_t] f_t$, where f is the measure of relative price dispersion), and then linearizing this equation, together with (25) - (29) and (32) - (34) results in the following system for consumption, employment, the markup, labor market tightness, the number of searching workers, 'post-hiring' unemployment, and inflation:

$$\hat{c}_t = E_t \hat{c}_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1})$$

$$\hat{c}_t = \left(\frac{N}{C} \right) z_t + \left(\frac{N}{C} \right) (1 - w^u) \hat{n}_t - \left(\frac{\kappa v}{C} \right) \left(\hat{s}_t + \hat{\theta}_t \right)$$

$$\hat{n}_t = \rho \xi \hat{\theta}_t + (1-\rho) [1 - \theta q(\theta)] \hat{n}_{t-1}$$

$$\hat{s}_t = \left(\frac{s-1}{s}\right)\hat{n}_{t-1}$$

$$\hat{u}_t = -\left(\frac{N}{1-N}\right)\hat{n}_t$$

$$\pi_t = \beta E_t \pi_{t+1} - \delta \hat{\mu}_t$$

$$\begin{aligned} \mu_t \;\; = \;\; & z_t - A\,(1-\xi)\,\hat{\theta}_t \\ & -A\beta\,(1-\rho)\,[1-b\theta q(\theta)]\,E_t\,(i_t - E_t\pi_{t+1}) \\ & +A\beta\,(1-\rho)\,[1-\xi-b\theta q(\theta)]\,E_t\hat{\theta}_{t+1}, \end{aligned}$$

where

$$\delta \equiv \frac{(1-\omega)(1-\omega\beta)}{\omega},$$

and

$$A \equiv \mu\left(\frac{1}{1-b}\right)\frac{\kappa}{q(\theta)}.$$

To close the system, it is necessary to specify the behavior of the monetary authority, and this is done below.

The expressions for inflation and the markup illustrate how labor market tightness affects inflation. A rise in labor market tightness reduces the retail price markup, increasing the marginal cost of the retail firms. This leads to a rise in inflation. Expected future labor market tightness also affects current inflation. For a given $\hat{\theta}_t$, a rise in $E_t\hat{\theta}_{t+1}$ increases the markup and reduces current inflation.[3] It does so through its effects on current wages. Expectations of labor market tightness increase the incentive of firms to post vacancies. This would normally lead to a rise in current tightness. However, since the coefficient on $E_t\hat{\theta}_{t+1}$ measures the impact on μ_t when $\hat{\theta}_t$ remains constant, wages must fall to offset the rise in vacancies that would otherwise occur and keep $\hat{\theta}_t$ constant. Finally, there is a cost channel effect in that the real interest rate has a direct impact on μ_t and therefore on inflation (see Ravenna and Walsh 2006). This arises since it is the present discounted value of expected future labor market conditions that matter.

We can further simplify the system of equations to obtain a form more easily comparable to the standard new Keynesian model. Noting that $\hat{n}_t = -\left(\frac{1-N}{N}\right)\hat{u}_t$ and

[3] In our baseline calibration, $1-\xi-b\theta q(\theta) > 0$.

$s_t = \left(\frac{1-s}{s}\right)\left(\frac{1-N}{N}\right)\hat{u}_{t-1}$, the equation for the evolution of employment can be expressed as

$$\hat{\theta}_t = -\left(\frac{1-N}{N}\right)\left(\frac{1}{\rho\xi}\right)\left[\hat{u}_t - (1-\rho)\left[1-\theta q(\theta)\right]\hat{u}_{t-1}\right]. \tag{35}$$

The goods market clearing condition can be written in terms of consumption and unemployment:

$$\begin{aligned}
\hat{c}_t &= \left(\frac{N}{C}\right) z_t - \left(\frac{1-N}{C}\right)(1-w^u)\hat{u}_t - \left(\frac{\kappa v}{C}\right)\left(\frac{1-s}{s}\right)\left(\frac{1-N}{N}\right)\hat{u}_{t-1} \\
&\quad + \left(\frac{\kappa v}{C}\right)\left(\frac{1-N}{N}\right)\left(\frac{1}{\rho\xi}\right)\left[\hat{u}_t - (1-\rho)\left[1-\theta q(\theta)\right]\hat{u}_{t-1}\right] \\
&= a_1\hat{u}_t - a_2\hat{u}_{t-1} + \left(\frac{N}{C}\right) z_t,
\end{aligned}$$

where

$$a_1 = \left(\frac{1-N}{N}\right)\left[\left(\frac{\kappa v}{C}\right)\left(\frac{1}{\rho\xi}\right) - \left(\frac{N}{C}\right)(1-w^u)\right]$$

$$a_2 = \left(\frac{\kappa v}{C}\right)\left(\frac{1-N}{N}\right)\left[\left(\frac{1}{\rho\xi}\right)(1-\rho)\left[1-\theta q(\theta)\right] + \left(\frac{1-s}{s}\right)\right].$$

Letting $\alpha \equiv a_1/(a_1+a_2)$ and $\tilde{\sigma} = \sigma(a_1+a_2)$, this result allows us to rewrite the Euler condition as

$$\hat{u}_t = \alpha E_t\hat{u}_{t+1} + (1-\alpha)\hat{u}_{t-1} + \left(\frac{N/C}{a_1+a_2}\right)(E_t z_{t+1} - z_t) - \tilde{\sigma}^{-1}(i_t - E_t\pi_{t+1}). \tag{36}$$

Using (35), the expression for the price markup can be expressed as

$$\mu_t = z_t + h_1\hat{u}_t - h_2\hat{u}_{t-1} - h_3 E_t\hat{u}_{t+1} - h_4(i_t - E_t\pi_{t+1}), \tag{37}$$

where

$$h_1 = A\left(\frac{1-N}{N}\right)\left(\frac{1}{\rho\xi}\right)\left\{(1-\xi) + \beta(1-\rho)\left[1-\xi-b\theta q(\theta)\right](1-\rho)\left[1-\theta q(\theta)\right]\right\}$$

$$h_2 = A(1-\xi)\left(\frac{1-N}{N}\right)\left(\frac{1}{\rho\xi}\right)(1-\rho)\left[1-\theta q(\theta)\right] > 0$$

$$h_3 = A\beta\left(1-\rho\right)\left(\frac{1-N}{N}\right)\left(\frac{1}{\rho\xi}\right)\left[1-\xi-b\theta q(\theta)\right]$$

and

$$h_4 = A\beta\left(1-\rho\right)\left[1-b\theta q(\theta)\right] > 0.$$

Using this expression for the markup in the inflation adjustment equation yields a new Keynesian Phillips curve expressed in terms of expected future inflation, unemployment, lagged unemployment, expected future unemployment, and the real rate of interest:

$$\pi_t = \beta E_t\pi_{t+1} - \delta h_1\hat{u}_t + \delta h_2\hat{u}_{t-1} + \delta h_3 E_t\hat{u}_{t+1} + \delta h_4\left(i_t - E_t\pi_{t+1}\right) - \delta z_t. \tag{38}$$

Let r_t^{flex} be the real interest rate in the flexible-price equilibrium. Then with flexible prices, (36) becomes

$$\hat{u}_t^{flex} = \alpha E_t\hat{u}_{t+1}^{flex} + (1-\alpha)\,\hat{u}_{t-1}^{flex} + \left(\frac{N/C}{a_1+a_2}\right)\left(E_t z_{t+1} - z_t\right) - \tilde{\sigma}^{-1} r_t^{flex}. \tag{39}$$

Subtracting this from (36), and letting $\tilde{u}_t$ denote the deviation of inflation from the flexible price equilibrium,

$$\tilde{u}_t = \alpha E_t\tilde{u}_{t+1} + (1-\alpha)\,\tilde{u}_{t-1} - \tilde{\sigma}^{-1}\left(i_t - E_t\pi_{t+1} - r_t^{flex}\right). \tag{40}$$

Similarly, in the flex-price equilibrium, $\hat{\mu}_t = 0$, so (37) becomes

$$0 = z_t + h_1\hat{u}_t^{flex} - h_2\hat{u}_{t-1}^{flex} - h_3 E_t\hat{u}_{t+1}^{flex} - h_4 r_t^{flex}. \tag{41}$$

Subtracting this from (38),

$$\pi_t = \beta E_t\pi_{t+1} - \tilde{h}_1\tilde{u}_t + \tilde{h}_2\tilde{u}_{t-1} + \tilde{h}_3 E_t\tilde{u}_{t+1} + \tilde{h}_4\left(i_t - E_t\pi_{t+1} - r_t^{flex}\right) \tag{42}$$

where $\tilde{h}_i = \delta h_i$.

Equations (40) and (42) provide the two equation parallel in the presence of labor market search frictions to the two equations of a standard new Keynesian model. Three important differences are apparent. First, the expectational IS curve expressed in terms of unemployment contains both expected future unemployment *and* lagged unemployment. To match empirical evidence, it is common in new Keynesian models to assume habit persistence in consumption as this leads to the presence of lagged consumption in the

IS equation. In the present framework, lagged unemployment appears due to the search frictions in the labor market.

Second, all the coefficients in the IS equation depend on the structural parameters that characterize the labor market. In the standard new Keynesian model, they depend only on preference parameters from the representative agent's utility function.

Third, current, lagged, and expected future unemployment affect the inflation rate.

Finally, there is a cost channel in that the real interest rate has a direct impact on inflation. This will affect the impact of monetary policy by generating a supply-side channel through which monetary policy affects inflation.

4.2 Unemployment and the Phillips Curve

In this section, we investigate the dependence of the unemployment-inflation relationship on labor market frictions. We can derive two different versions of the Phillips curve linking unemployment and inflation. The first, given by (42), is repeated here:

$$\pi_t = \beta E_t \pi_{t+1} - \tilde{h}_1 \tilde{u}_t + \tilde{h}_2 \tilde{u}_{t-1} + \tilde{h}_3 E_t \tilde{u}_{t+1} + \tilde{h}_4 E_t \left(i_t - E_t \pi_{t+1} - r_t^{flex} \right). \tag{43}$$

The coefficients on current, lagged, and future unemployment in this equation reflect the impact of the unemployment gap on inflation, holding the real interest rate constant. However, the real interest and the unemployment gap are linked by the expectational IS equation (40). This relationship can be used to eliminate the real interest rate from (42), yielding

$$\pi_t = \beta E_t \pi_{t+1} - \left(\tilde{h}_1 + \tilde{h}_4 \tilde{\sigma} \right) \tilde{u}_t + \left[\tilde{h}_2 + \tilde{h}_4 \tilde{\sigma} \left(1 - \alpha \right) \right] \tilde{u}_{t-1} + \left(\tilde{h}_3 + \tilde{h}_4 \tilde{\sigma} \alpha \right) E_t \tilde{u}_{t+1}. \tag{44}$$

This version accounts for the movements of the real rate of interest necessary to be consistent with the path of the unemployment gap and so accounts for the cost channel implications of movements in $\tilde{u}_t$.

For the calibrations discussed in the next subsection, $\tilde{h}_4 \tilde{\sigma}$ is small,[4] thus, the unemployment rate gap coefficients in (43) and (44) are very similar. In addition, the coefficients on $\tilde{u}_{t-1}$ and $E_t \tilde{u}_{t+1}$ are small relative to the coefficient on $\tilde{u}_t$ and these coef-

[4] It is equal to -0.0015.

ficients are relative insensitive to the parameter variations we consider. Thus, we focus on $\tilde{h}_1$ in (43).

4.2.1 Calibration

The baseline values for the model parameters are given in Table 1. All of these are standard in the literature. We impose the Hosios condition by setting $b = 1 - \xi$. By calibrating the steady-state job finding probability q and the replacement ratio $\phi \equiv w^u/w$ directly, we use steady-state conditions to solve for the job posting cost κ and the reservation wage w^u.[5] Given the parameters in Table 1, the remaining parameters and the steady-state values needed to obtain the log-linear approximation can be calculated.

Table 1: Parameter Values

Exogenous separation rate	ρ	0.1
Vacancy elasticity of matches	ξ	0.4
Workers' share of surplus	b	0.6
Replacement ratio	ϕ	0.4
Vacancy filling rate	q	0.7
Labor force	N	0.95
Discount factor	β	0.99
Relative risk aversion	σ	2
Markup	μ	1.2
Price adjustment probability	$1-\omega$	0.75

[5] To find κ and w^u, assume $w^u = \phi w$, where ϕ is the wage replacement rate. Then (12) and (20) can be written as

$$[1 - \phi(1-b)]\, w^u = \phi b \left[\frac{1}{\mu} + (1-\rho)\beta\kappa\theta\right]$$

$$\left\{[1 - \beta(1-\rho)]\, \eta^{-1}\bar{\theta}^{1-\xi} + b\beta(1-\rho)\bar{\theta}\right\} \kappa = (1-b)\left(\frac{1}{\mu} - w^u\right)$$

and these two equations can be jointly solved for κ and w^u. That is,

$$\begin{bmatrix} w^u \\ \kappa \end{bmatrix} = \begin{bmatrix} 1-\phi(1-b) & -\phi b(1-\rho)\beta\theta \\ 1-b & [1-\beta(1-\rho)]\, \eta^{-1}\bar{\theta}^{1-\xi} + b\beta(1-\rho)\bar{\theta} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\phi b}{\mu} \\ \frac{1-b}{\mu} \end{bmatrix}.$$

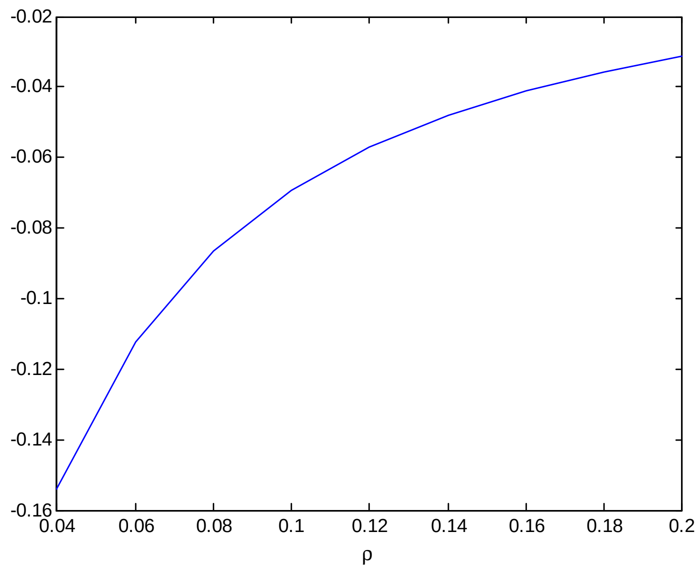

Figure 1: Effect of exogenous separation probability on the unemployment elasticity of inflation

4.2.2 Results

In this section, we explore the effects of the probability of exogenous separation, labor's share of the match surplus, and the job finding probability on the unemployment elasticity of inflation.

Figure 1 plots $\tilde{h}_1$ as a function of ρ. As ρ increases, the elasticity of employment (and unemployment) with respect to θ rises. With fewer matches surviving from one period to the next, the share of new matches in total employment increases, making employment more sensitive to labor market conditions. Conversely, a given change in unemployment is associated with a smaller change in θ and, consequentiality, in retail firm's marginal cost. Inflation becomes less sensitive to unemployment. In addition, the role of past labor market conditions falls as match duration declines, and this also reduces the impact of unemployment on expected future marginal cost and inflation.

Under Nash bargaining, the dynamics of unemployment and inflation are affected by the respective bargaining power of workers and firms. Figure **??** illustrates the impact of labor's share of the match surplus, b, on the responsiveness of inflation to unemployment.

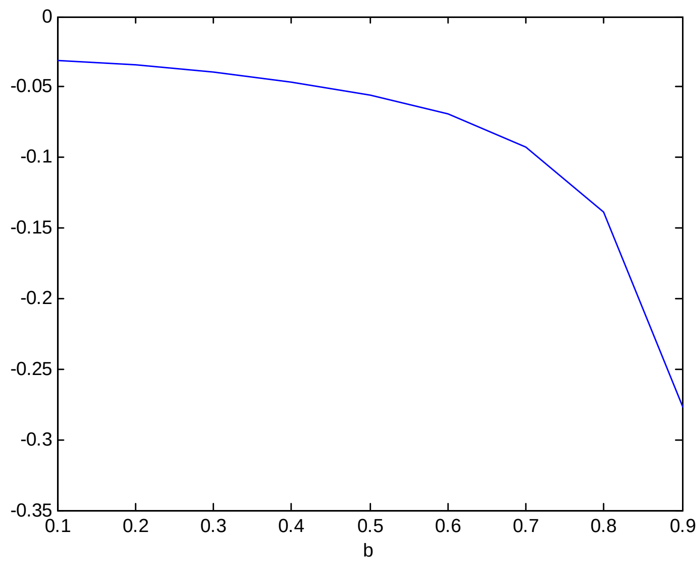

Figure 2: Effect of labor share on the unemployment elasticity of inflation

As labor's share of the surplus rises, the incentive to create new jobs falls. An expansion of output must be associated with a larger rise in the price of whole goods relative to retail goods if wholesale firms are to increase production. Thus, the marginal cost to the retail firms, and retail price inflation, becomes more responsive to unemployment movements as b increases.

The last exercise we examine is the impact of the probability of filling a job on the Phillips curve. In the baseline calibration, we set the steady-state probability of filling a vacancy equal to 0.7. In absolute value, the impact of unemployment on inflation declines with the steady-state value of $q(\theta)$. The steady-state value of a filled job falls as the steady-state probability of filling a vacancy rises. The effect a fall in the value of a filled job has on inflation can be inferred from (13) and (14). As $\kappa/q(\theta)$ becomes smaller, the marginal cost of labor to wholesale firms approaches the fixed opportunity wage w^u. In the extreme case with $\tau = w^u$, (13) implies that the price markup variable μ would be constant and equal to Z_t/w^u. This corresponds to the case of a perfectly elastic supply of labor to wholesale firms. A demand expansion leads to a fall in unemployment but no increase in the price of wholesale goods relative to retail goods. Thus, the marginal cost

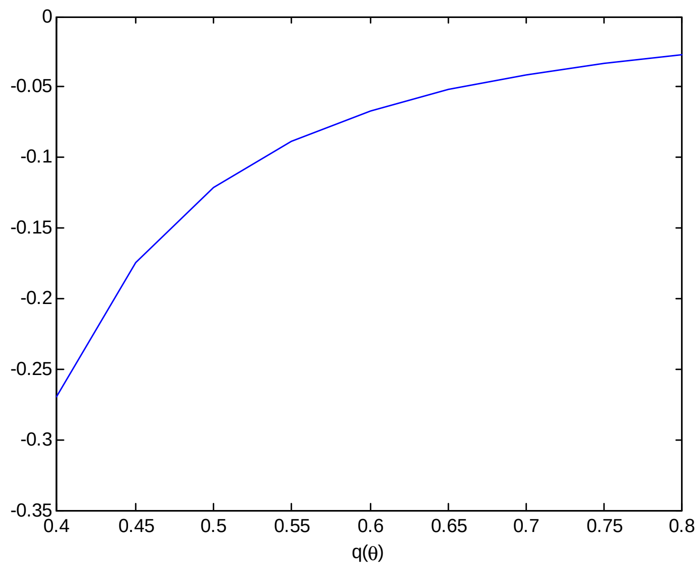

Figure 3: Effect of the job filling probability on the elasticity of inflation with respect to unemployment.

faced by retail firms would remain constant, as would inflation.

5 Monetary policy

The model consisting of (40) and (42) provides a convenient framework, consistent with labor market search frictions, that can be used to study optimal monetary policy. We assume the objective of the central bank is to minimize a standard quadratic loss function in inflation and the unemployment gap. This takes the form

$$E_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda \tilde{u}_{t+i}^2 \right). \tag{45}$$

This loss function has not been derived explicitly from the welfare of the representative agent in the model. However, inflation variability is costly because it generates an inefficient dispersion of relative prices across retail firms. Assume, as is standard in new Keynesian models, that fiscal policy employes a subsidy-tax policy that eliminates the distortion due to imperfect competition in the retail goods market. Then, if we calibrate the Nash bargaining parameter b such that it equals $1-\xi$ so that the matching process satisfies the Hosios condition, unemployment variability in the flexible-price equilibrium is efficient. In this case, the appropriate object for monetary policy is the unemployment gap.

As is well understood, in a standard new Keynesian model, the absence of explicit interest rate objectives in (45) means that the IS relationship does not impose any constraints on the central bank. Thus, optimal policy can be found by minimizing (45) subject only to the constraint implied by the inflation adjustment equation. The situation is slightly more complicated in the present case, since the real interest rate appears directly in the Phillips curve. However, it can be eliminated by using (40), yielding (44) as the relevant constraint on policy. Hence, the single equation constraint on the central bank's choice of $\tilde{u}_t$ is

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} - \left(\tilde{h}_1 + \tilde{h}_4 \tilde{\sigma}\right) \tilde{u}_t + \left[\tilde{h}_2 + \tilde{h}_4 \tilde{\sigma}\,(1-\alpha)\right] \tilde{u}_{t-1} \\ &\quad + \left(\tilde{h}_3 + \tilde{h}_4 \tilde{\sigma}\alpha\right) E_t \tilde{u}_{t+1} + e_t, \end{aligned} \tag{46}$$

where we have added an exogenous cost shock e_t.

Using (46), the unemployment gap can be treated as the policy instrument of the central bank. Let ϕ_t be the Lagrangian multiplier associated with the constraint (46). Under commitment, the first order conditions for the central bank's problem are

$$\pi_t + \phi_t = 0$$

$$\lambda \tilde{u}_t + \left(\tilde{h}_1 + \tilde{h}_4 \tilde{\sigma}\right) \phi_t - \beta \left[\tilde{h}_2 + \tilde{h}_4 \tilde{\sigma}\,(1-\alpha)\right] \phi_{t+1} = 0$$

and for all $i > 0$,

$$\pi_{t+i} + \phi_{t+i} - \phi_{t+i-1} = 0$$

$$\lambda \tilde{u}_{t+i} + \left(\tilde{h}_1 + \tilde{h}_4 \tilde{\sigma}\right) \phi_{t+i} - \beta \left[\tilde{h}_2 + \tilde{h}_4 \tilde{\sigma}\,(1-\alpha)\right] \phi_{t+i+1} - \beta^{-1} \left(\tilde{h}_3 + \tilde{h}_4 \tilde{\sigma}\alpha\right) \phi_{t+i-1} = 0.$$

The difference between the first order conditions in the initial period and in subsequent periods reflects the dynamic inconsistency of the optimal commitment policy. In standard new Keynesian models, this inconsistency arises solely from the presence of expected future inflation in the Phillip curve. In the current set up, this effect is present, but a

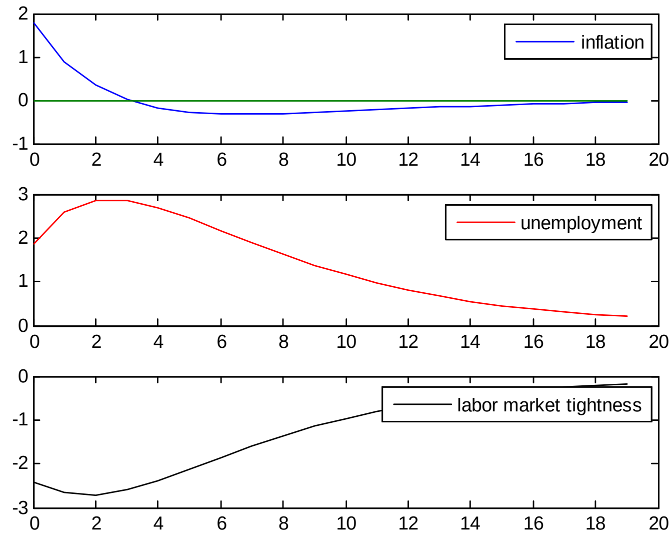

Figure 4: Response to a cost shock under the optimal (timeless) commitment policy

second source arises from the effect of expected future unemployment on current inflation.

Eliminating the Lagrangian multiplier, equilibrium under the optimal (timeless perspective) commitment policy is obtained as the joint solution to

$$\lambda\left(\tilde{u}_t - \tilde{u}_{t-1}\right) - \left(\tilde{h}_1 + \tilde{h}_4\tilde{\sigma}\right)\pi_t + \beta\left[\tilde{h}_2 + \tilde{h}_4\tilde{\sigma}\left(1-\alpha\right)\right]E_t\pi_{t+1} + \beta^{-1}\left(\tilde{h}_3 + \tilde{h}_4\tilde{\sigma}\alpha\right)\pi_{t-1} = 0 \tag{47}$$

and (44). Equation (47) is the optimal targeting rule in the presence of labor market frictions. Figure 4 shows the responses of inflation, unemployment and vacancies to a one unit, serially correlated cost shock under the optimal commitment policy.[6]

It is instructive to compare the response to a cost shock under the optimal timeless policy with the response under the optimal targeting rule derived in the standard new Keynesian model. This rule takes the form

$$\lambda\left(\tilde{x}_t - \tilde{x}_{t-1}\right) - \kappa\pi_t = 0.$$

In the standard model, $\tilde{x}_t = \tilde{n}_t$, so the rule expressed in terms of non-market hours $1 - N_t$

[6] The cost shock is $AR(1)$ with serially correlation coefficient 0.7, and $\lambda = 1/16$.

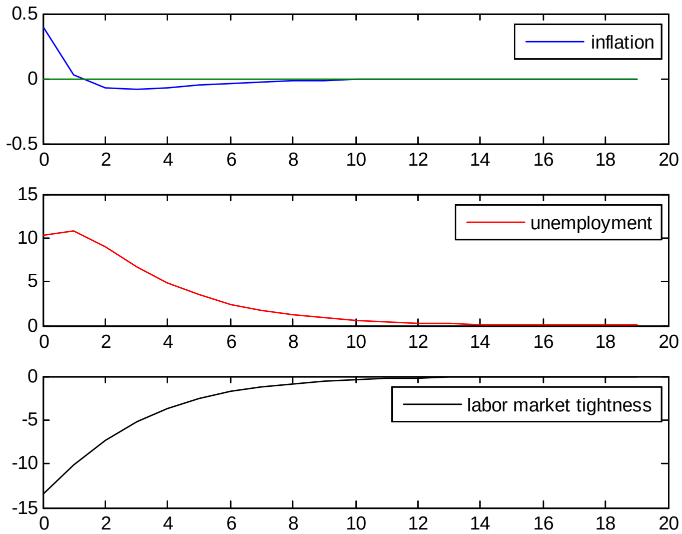

Figure 5: Responses to a cost shock under targeting rule (48)

becomes

$$\lambda\left(\tilde{u}_t - \tilde{u}_{t-1}\right) + (N/u)\kappa\pi_t = 0. \tag{48}$$

Figure 5 illustrates the responses to a serially correlated cost shock when policy rule (48) is employed in place of the optimal targeting rule. Less persistence is generated, and unemployment and labor market tightness are much more volatility.

An important property of our model is that we can use it to compare responses under different assumptions about the characteristics of the labor market. For example, Blanchard and Galí (2006) argue that $\rho = 0.04$ and $N = 0.9$ is appropriate for studying the European economy, rather than the values of $\rho = 0.1$ and $N = 0.95$ used for a calibration based on the US. These changes imply a significant difference in the probability a searching worker finds a job. Under the US calibration, this probability is 0.655; under the EU calibration, it is only 0.265. These differences translate in to an expected duration of unemployment of 4.6 months under the US calibration and 11.3 months under the EU calibration.

Figures 6 and 7 plot the responses to a serially uncorrelated cost shock for the US and EU calibrations respectively. Unemployment rises more under the EU calibration

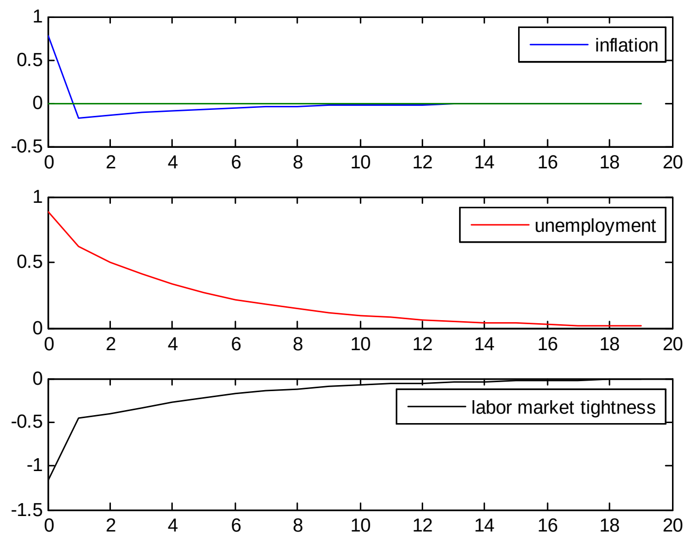

Figure 6: Response to a serially uncorrelated costs shock under optimal policy (US calibration)

but it also displays much less persistence than with the US calibration (recall that $\tilde{u}$ is the unemployment gap between actual unemployment deviations and the flexible-price equilibrium unemployment, both expressed as deviations from the steady state). Perhaps more interesting is the contrasting responses of inflation. Under the US calibration, we obtain the standard result that inflation becomes negative after the initial impact of the cost shock, thereby ensuring the price level is stationary. This result has motivated the study of price-level targeting under discretionary policy regimes (Vestin 2006). In contrast, inflation returns quickly to zero under the EU calibration but never turns negative. Thus, the price level is non-stationary under the optimal commitment policy.[7]

These results reflect the greater sensitivity of inflation to unemployment – current, lagged, and expected future unemployment – with the EU calibration. This can be seen

[7] In interpreting these comparisons, it is important to keep in mind that we have changed only the calibrations for ρ and N. In particular, the degree of nominal price stickiness and the job filling probability are assumed to be the same.

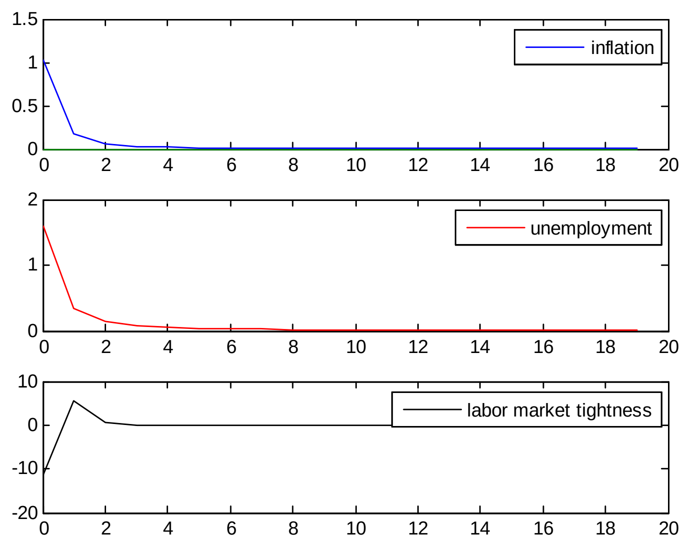

Figure 7: Response to a serially uncorrelated costs shock under optimal policy (EU calibration)

by comparing the implied Phillips curves under the alternative calibrations:

$$\text{EU: } \pi_t = \beta E_t \pi_{t+1} - 0.114\tilde{u}_t + 0.096\tilde{u}_{t-1} + 0.091 E_t \tilde{u}_{t+1}$$

$$\text{US: } \pi_t = \beta E_t \pi_{t+1} - 0.069\tilde{u}_t + 0.008\tilde{u}_{t-1} + 0.001 E_t \tilde{u}_{t+1}$$

With inflation more responsive to unemployment, the optimal policy calls for a stronger response to a cost shock, leading to a sharper initial rise in unemployment. This difference can be seen by comparing the optimal targeting rules for the two calibrations:

$$\text{EU: } u_t = u_{t-1} + 1.823\pi_t - 1.464\pi_{t-1} - 1.515 E_t \pi_{t+1}$$

$$\text{US: } u_t = u_{t-1} + 1.105\pi_t - 0.115\pi_{t-1} - 0.128 E_t \pi_{t+1}$$

6 Conclusions

To be added.

7 Appendix

7.1 Wage determination

Consider a comparison of the outcomes from the worker in making a match versus not making one. The value of the match is the wage plus the expected value of entering the following period with a job: $V_t^m = w_t + \beta E_t (\lambda_{t+1}/\lambda_t) V_{t+1}^E$. In turn,

$$V_{t+1}^E = [1 - \rho + \rho \theta_{t+1} q(\theta_{t+1})] V_{t+1}^m + \rho [1 - \theta_{t+1} q(\theta_{t+1})] V_{t+1}^n,$$

since an employed worker survives the exogenous separation process and remains in a match with probably $1 - \rho$, becomes unemployed with probability ρ but immediately finds another job with probability $\theta_{t+1} q(\theta_{t+1})$, or becomes unemployed with probability ρ but does not find a new match.

The value of not making a match is the alternative wage plus the expected value of entering the following period unemployed: $V_t^n = w^u + \beta E_t (\lambda_{t+1}.\lambda_t) V_{t+1}^u$. The value of being unemployed is

$$V_{t+1}^u = \theta_{t+1} q(\theta_{t+1}) V_{t+1}^m + [1 - \theta_{t+1} q(\theta_{t+1})] V_{t+1}^n.$$

Combining these results,

$$\begin{aligned} V_t^s &\equiv V_t^m - V_t^n = (w_t - w^u) + \beta E_t (\lambda_{t+1}/\lambda_t) \left(V_{t+1}^E - V_{t+1}^u \right) \\ &= (w_t - w^u) + \beta (1 - \rho) E_t (\lambda_{t+1}/\lambda_t) [1 - \theta_{t+1} q(\theta_{t+1})] V_{t+1}^s \end{aligned}$$

which is (11) of the text.

7.2 The social planner's problem

Using the function form of the matching function, the social planner's problem can be written as

$$\begin{aligned} &\max_{C,\, N,\, u,\, \theta} E_t \sum_{i=0}^{\infty} \beta^i \left\{ \left(\frac{C_{t+i}^{1-\sigma}}{1-\sigma} \right) + \lambda_{t+i} \left[Z_{t+i} N_{t+i} - \kappa s_{t+i} \theta_{t+i} + w_{t+i}^u (1 - \eta \theta_{t+i}^{\xi}) u_{t+i} - C_{t+i} \right] \right. \\ &\left. + \psi_{t+i} \left[(1-\rho) N_{t+i-1} + \eta \theta_{t+i}^{\xi} u_{t+i} - N_{t+i} \right] + \phi_{t+i} \left[s_{t+i} - 1 + (1-\rho) N_{t+i-1} \right] \right\}. \end{aligned}$$

First order conditions are

$$C\colon\ C_t^{-\sigma} - \lambda_t = 0;$$

$$\theta\colon\ -\lambda_t \left(\kappa + w_t^u \xi \eta \theta_t^{\xi-1}\right) u_t + \psi_t \xi \eta \theta_t^{\xi-1} s_t = 0;$$

$$u\colon\ \lambda_t \left[w^u(1-\eta\theta_t^{\xi}) - \kappa\theta_t\right] + \psi_t \eta \theta_t^{\xi} + \phi_t = 0;$$

$$N\colon\ \lambda_t Z_t - \psi_t + (1-\rho)\beta E_t \left(\psi_{t+1} + \phi_{t+1}\right) = 0.$$

The second of these first order conditions implies

$$\frac{\psi_t}{\lambda_t} = \left(\frac{\kappa}{\xi\eta}\theta_t^{1-\xi} + w_t^u\right),$$

while the third then implies

$$\begin{aligned}
\frac{\phi_t}{\lambda_t} &= -\left[w^u(1-\eta\theta_t^{\xi}) - \kappa\theta_t\right] - \frac{\psi_t}{\lambda_t}\eta\theta_t^{\xi} \\
&= -\left[w^u(1-\eta\theta_t^{\xi}) - \kappa\theta_t\right] - \left(\frac{\kappa}{\xi\eta}\theta_t^{1-\xi} + w_t^u\right)\eta\theta_t^{\xi} \\
&= \left(\frac{\xi-1}{\xi}\right)\kappa\theta_t - w^u.
\end{aligned}$$

The fourth first order condition then becomes

$$\begin{aligned}
Z_t &= \frac{\psi_t}{\lambda_t} - (1-\rho)\beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{\psi_{t+1}+\phi_{t+1}}{\lambda_{t+1}}\right) \\
&= \left(\frac{\kappa}{\xi\eta}\theta_t^{1-\xi} + w_t^u\right) - (1-\rho)\beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left[\frac{\kappa}{\xi\eta}\theta_{t+1}^{1-\xi} + \left(\frac{\xi-1}{\xi}\right)\kappa\theta_{t+1}\right].
\end{aligned}$$

Rearranging this condition for efficiency and noting that $\eta\theta_t^{\xi-1} = 1/q(\theta_t)$ yields

$$\begin{aligned}
Z_t = {} & w^u + \frac{1}{\xi}\frac{\kappa}{q(\theta_t)} - \frac{1}{\xi}(1-\rho)\beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right)\frac{\kappa}{q(\theta_{t+1})} \\
& +(1-\rho)\left(\frac{1-\xi}{\xi}\right)\beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right)\kappa\theta_{t+1}.
\end{aligned}$$

which is (23) of the text.

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