A Unified Theory of Consumption and Travel[1]

by

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ABSTRACT

The microeconomic theory of demand explains the purchasing decisions of consumers ignoring that most consumption requires travel. Travel demand theory, strongly influenced by econometrics, has developed largely independently of microeconomic theory and ignores consumption decisions. I treat consumption and travel in a unified manner so that the consumer allocates his time and income among discretionary trips in order to purchase a variety of consumption goods. The focus is the complementariness and substitution between travel and consumption. I formulate a model in which the consumer decides – for a period of time – the frequency of the trips to make to each store and how much to buy on each trip. I then modify the model to examine how the consumer should mix separate trips to two stores, with chained trips in which he visits both stores. Under the preferences specified, travel cost saved by trip-chaining goes into more travel so that the total travel expenditure remains unchanged. I point out how the model developed can be generalized in a number of ways to improve its scope and applicability.

Keywords: Consumer theory, travel demand theory, trip-chaining, taste for variety.

JEL classification: D11, J29, R41

1. Introduction

The theory of the consumer in microeconomics ignores that most consumption cannot be realized without incurring travel or communication costs. Mainstream economics generally ignores spatial aspects of reality regardless of their importance. The economic theory of the allocation of time (Becker, 1965) overlooks explicit treatment of travel itself as an activity, but travel is intimately related to both consumption and the allocation of time among discretionary activities. The theory of travel demand within transportation science, on the other hand, has developed largely independently of standard microeconomic theory. Virtually all travel demand

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analysis has its roots in applied econometric techniques and especially in discrete choice models that, since the contribution of McFadden (1973), have advanced understanding of many aspects of travel.

Econometrics, more than microeconomic theory, is inspired by the study of real problems on which data are available and which are of policy interest. It is not surprising that the study of modal choice (e.g. the choice between auto and public transit) and of commuting have in one way or another dominated conventional travel demand analysis. Modal choice is important because of the competition between highways and transit systems for passengers and for resources. Commuting is important because it competes with leisure and work for the time of consumers. But both commuting and modal choice are narrow aspects of the total travel experience.

The travel demand problem can be and should be defined more generally by: *how many trips and what kind of trips to make over a period of time and to which destinations*? Commuting is generally (but not entirely) determined by prior decisions such as the choice of residence and of workplace. What remains is largely discretionary travel including trips for shopping, recreation, socializing, eating out and other activities. The demand for discretionary trips cannot be considered separately from the consumption benefits of such travel. From this perspective, standard consumer theory and travel demand should be inseparable. Although they have in fact evolved largely separately, in the present article I bring them together. The question defining the combined discretionary travel and consumption problem is: *how many trips and what kind of trips to make and to which destinations, and how much goods and services to purchase on those trips?*

Such a definition of travel theory appears natural and appropriate to the casual observer, but most urban economics and location theory models have ignored this way of looking at travel. For example, in conventional location theory it is assumed that the consumer travels to the nearest store or destination. In urban economics, it is standard to assume that the consumer commutes but non-standard in the extreme to assume that the consumer travels in any other way. Equilibrium models that combine both commuting and discretionary travel in order to analyze land use patterns and the location of employment are recent (Anas and Xu, 1999). The field of transportation analysis, on the other hand, has recognized the importance of the related problem of complex travel patterns and activity scheduling from a conceptual and empirical perspective. This has given rise to a rich and growing body of applied studies of travel activity patterns which take their cue directly from econometrics and not from microeconomic theory. See Ettema and Timmermans, (1997); Ben-Akiva and Bowman (1998) and Bhat and Koppelman (1999) for surveys.

According to Gordon and Richardson (2000), from 1969 to 1995 the average commuting time in the U.S. fell somewhat from 22 minutes to 20.7 minutes, presumably because decentralizing jobs improved the proximity between homes and workplaces, while the total vehicle miles traveled (VMT) grew much faster than the land area of the 65 largest urbanized areas. Some of the higher VMT may have come about because people traveled longer distances at higher speeds on their commutes, but most of it is likely to have come from a larger number of discretionary trips.

Table 1, borrowed from Nelson and Niles (2000), tells the story of the percentage change in vehicle travel and in trip lengths by purpose and per person from 1969-90 (and 1990-95).

Trip Purpose	Period Growth in Average Annual Vehicle Trips Per Capita	Period Growth in Average Annual VMT Per Capita	Period Growth in Average Vehicle Trip Length Per Capita
All purposes	+27 (+12)	+27 (+15)	+1 (+2)
To or from work	+1 (+22)	+20 (+33)	+17 (+8)
Shopping	+76 (+16)	+108 (+28)	+16 (+16)
Other family and personal business	+137 (+8)	+169 (+1)	+14 (-5)
Social and recreational	+15 (+7)	-1 (+11)	-10 (-5)
Other*	+27 (+36)	-32 (+68)	+1 (+23)

* Includes trips to school, church, doctor/dentist, and to drop off/pick up.
Source: 1995 NPTS

TABLE 1

The biggest percentage changes have occurred in "shopping" and in "other family and personal business trips". In addition to being the fastest growing, this category was also the single largest category of trips consisting of 23.6% of all trips in 1995. The authors also reported their other observations from the National Personal Transportation Survey: 3/4ths of person trips and 4/5ths of vehicle trips in the US are for non-work purposes and that non-work trips are a major travel reason even during peak periods.

The reason for the proliferation of non-work discretionary trips is easy to conjecture. First of all jobs and residences have decentralized, reducing the average distance between homes and employment concentrations to which non-work trips are made. Incomes have increased and as incomes increase the demand for product variety grows and consumers seek a larger diversity of opportunities to shop, purchase services and engage in recreation or leisure-related activities. As incomes increase, car ownership also increases and the availability of multiple private vehicles or of more persons with access to a private vehicle stimulates more travel and discretionary mobility.

To close the gap between theory and observation we need to develop models that treat the complementariness and substitution between consumption and travel, recognizing that while travel is necessary for consumption it also competes with consumption for income and time, that there are many alternative "shopping" destinations available to modern consumers, that such destinations are substitutes and that the degree of substitutability varies. A proper theory would have to take into account the important roles of income, the value of time which is related to income, and the substitutability among alternative trip destinations. It would treat properly both the income and substitution effects that arise when travel cost is reduced generally, for the commute or for a particular trip. Also, the complexity of travel has increased by the proliferation of *trip-chains* or multi-stop trips often called *tours* in the transportation field. These originate and end at home, at work or at some other starting point. The fact that consumers can combine different trips in a single tour or trip-chain is a major challenge for theory. We do not yet know whether chaining trips reduces overall travel miles or whether the ability to chain trips together frees up enough time that is in turn allocated in part or in whole to the making of more chained and unchained trips.

In section 2, I lay out a model of the consumption and travel activities of a single consumer based on a non-trivial extension of the Dixit-Stiglitz (1977) utility function subject to a non-convex budget set combining travel and consumption expenditures. I show how this model can be solved using a two-stage procedure and I analyze the most salient properties of the resulting demand equations. In section 3, I examine the model's solution for a simple hypothetical context in which stores are symmetrically located with respect to the consumer. In this simple setting I show how the consumer can optimally determine the choice set of stores that he visits. The analytics of trip chaining are considered in section 4, using an appropriate extension of the basic model of section 2. I show under what conditions a consumer will prefer to chain some of his trips and under what conditions trip chaining will not be practical. Additional extensions of the model are briefly sketched out in section 5. While in the basic model of section 2, the consumer is so variety-hungry that he visits all "stores" that are known to him, the extended model of section 4 shows how the range of the consumer's travel can be limited by his tastes and the travel costs and prices associated by the various stores. In section 5 I also briefly comment on how to extend the approach presented here to include the discretionary time spent in activities including shopping, and how to treat higher level decisions such as modal choice and location.

2. Consumption and Trip-making: Separate Trips

I view the consumer as having to travel to acquire goods and services to consume. This approach recognizes the basic complementariness between travel and consumption – ignored in both consumption theory and travel demand theory – but also treats the substitution between the cost or length of a trip and the quantity purchased per trip, as well as the substitution of trips and purchases among different trip destinations.

This perspective has wide applicability to a variety of travel contexts, but to keep a simple story in mind in this exposition, it is convenient to imagine a consumer who, over a period such as a month, a season or a year, makes separate visits to each of $i=1...I$ stores. These trips are denoted by the non-negative vector $\mathbf{n}\equiv(n_1,...,n_I)$. Although trips must be integers in reality, I will treat them as continuous non-negative variables for analytical purposes, commenting as needed on the types of complications that arise when trips are integers. During a particular visit to store *i*, the consumer buys quantity z_i. The non-negative vector $\mathbf{z}\equiv(z_1,...,z_I)$ denotes the quantities bought from each store per trip made.

The consumer's budget constraint should include travel expenditures as well as purchases at the stores. Suppose that the unit price of the goods sold at store *i* is p_i and

that the opportunity cost of traveling to and from the store is wt_i+c_i, where w is the consumer's wage rate or his value of time, t_i is the two-way travel time and c_i is the two-way monetary travel cost. The consumer has a time endowment H that he can allocate in part to earning a wage w, which requires commuting, and in part to discretionary travel which consists of travel to the stores. The consumer's two-way commute (from home to work and back) takes *T* hours and also incurs a monetary cost *C*. Hence, $wT+C$ is the consumer's full opportunity cost of the daily commuting travel. d is the number of commuting days over the year. In the present, I will assume that the consumer's residence and job locations are pre-determined. Hence, in the discussion that follows, the commute time, *T*, and its monetary cost, *C*, are treated as non-discretionary. They are, therefore, fixed costs. They have been predetermined by the choice of residence and workplace which I am not treating in the present.[2]

The budget constraint can now be written as (1a) below, with earned money income on the left side and all monetary expenditures related to commuting and shopping on the right side.

[2] In Anas and Xu (1999), a general equilibrium model of urban land use, a consumer chooses among the discrete combinations of workplace-residence pairs, while evaluating a number of continuous choices for each such discrete combination. Among these continuous choices are the frequencies of travel from the residence location to a variety of shopping destinations.

$$w\left(H - dT - \sum_{i=1}^{I} t_i n_i\right) = dC + \sum_{i=1}^{I} c_i n_i + \sum_{i=1}^{I} n_i p_i z_i \ , \tag{1a}$$

This can be rewritten by gathering all of the choice-related opportunity costs on the right side and the endowment income and all the fixed costs related to commuting on the left:

$$M \equiv wH - d(wT + C) = \sum_{i=1}^{I} \left(wt_i + c_i\right) n_i + \sum_{i=1}^{I} n_i p_i z_i \tag{1b}$$

In the (1b)-way of writing the budget, the left side is the consumer's potential income available for purchases and for travel, which is that income the consumer would have if he did not "shop" at all but were to spend all his time net of commuting earning a wage. From an analysis standpoint, (1b) is the more convenient way to write the budget although (1a) is more natural to look at. Note that the budget constraint is linear in product prices, travel costs and travel times but nonlinear in the choice variables $\mathbf{n} = \left(n_1, ..., n_I\right)$ and $\mathbf{z} = \left(z_1, ..., z_I\right)$. The budget constraint is homogeneous of degree zero in w, C, $\mathbf{c} = \left(c_1, ..., c_I\right)$ and $\mathbf{p} = \left(p_1, ..., p_I\right)$.

Property 1.*Non-convex budget set*: *The budget constraint may be written as* $\sum_{i=1}^{I} [\left(wt_i + c_i\right) n_i + n_i p_i z_i]$. *It follows that the set of affordable trip-consumption bundles* (**n**, **z**) *is not a convex set.*

The non-convexity of the budget set reflects a fundamental scale economy in shopping. The expenditure per trip made to store *i* consists of a fixed cost $wt_i + c_i$ which does not depend on quantity purchased and a variable cost $p_i z_i$ which increases with the quantity purchased per trip at the store, z_i.

Now suppose that the consumer's utility function for visiting all stores over the year is Dixit-Stiglitz (1977) C.E.S. as in (2a) below, with $\frac{1}{1-\rho}$ the elasticity of substitution.

$$U(\mathbf{n}, \mathbf{z}) = \left(\sum_{i=1}^{I} u_i(n_i, z_i)^{\rho}\right)^{\frac{1}{\rho}}, \ 0 < \rho < 1, \tag{2a}$$

where $u_i(n_i, z_i)$ is the sub-utility derived from n_i visits to store i ,quantity z_i being bought during each visit. I will assume that zero utility is derived unless something is bought, $u_i(n_i, 0) = 0$, and that each sub-utility function is strictly concave and increasing in z_i given $n_i > 0$. Hence, using the upper prime to denote a derivative with respect to z_i, $u_i'(n_i, z_i) > 0$ and $u_i''(n_i, z_i) < 0$ for

$n_i > 0$. In what follows, I assume that trips are "goods": $\frac{\partial u_i}{\partial n_i} > 0$ for each store *i*. I also assume that if a trip is not made to a store, then no utility can be derived from the store. Thus, $u_i(0, z_i) = 0$ for $z_i \geq 0$.[3]

I now consider a fundamental property of the utility function and a basic trade-off in the utility maximization problem.

Property 2. ***Extreme taste for shopping variety***: *An extreme taste for shopping variety exists because* $\frac{\partial U}{\partial u_i} = (\bullet)^{\frac{1}{\rho}-1} u_i(n_i, z_i)^{\rho-1}$ and $\lim_{u_i \to 0} \frac{\partial U}{\partial u_i} = +\infty$. *This means that if the consumer derives no utility from a particular store because he is not shopping there, he gets extremely desirous for what that store has to offer.*

Proof: By the assumption that $u_i(n_i, 0) = 0$, it follows that $\lim_{z_i \to 0} \frac{\partial U}{\partial z_i} = \frac{\partial U}{\partial u_i} \frac{\partial u_i}{\partial z_i} = +\infty$. Similarly, by $u_i(0, z_i) = 0$, $\lim_{n_i \to 0} \frac{\partial U}{\partial n_i} = \frac{\partial U}{\partial u_i} \frac{\partial u_i}{\partial n_i} = +\infty$ •

A consequence of this property is that if the consumer patronizes some stores and not others, he will seek to increase utility by shifting his time and money to stores that are not patronized. This taste for variety weakens to the point of vanishing as $\rho \to 1$ and the stores become viewed as perfect substitutes with an elasticity of substitution that tends to infinity. In this case, the consumer will normally patronize only one store. At the other extreme, as $\rho \to 0$, and the elasticity of substitution tends to unity, (2a) becomes Cobb-Douglas in the subutilities and all stores are considered essential.

Let us now specify the sub-utility as follows: $u_i(n_i, z_i) = n_i u_i(z_i)$ which means that the total utility derived from patronizing the store n_i times is the sum of the utility derived from the store each time it is patronized. Then, the utility function (2a) becomes:

$$U = \left(\sum_{i=1}^{I} \left(n_i u_i(z_i) \right)^{\rho} \right)^{1/\rho}, \ 0 < \rho < 1. \tag{2b}$$

The consumer maximizes the utility function (2b) subject to the budget constraint (1b) by choosing simultaneously the trip vector $\mathbf{n} = (n_1, ..., n_I)$, and the vector of quantity purchases per trip made, $\mathbf{z} = (z_1, ..., z_I)$. Let us suppose initially that the values of the trip vector $\mathbf{n}$ are

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[3] This does not rule out forms of shopping that do not involve trips (by mail, telephone, Internet etc.) because these may be considered as involving virtual trips with lower access costs than physical travel.

restricted so that each element can only be a non-negative integer. Then, we can identify the following property.

Property 3. ***Trade-off between travel cost and shopping variety*****:** *While the consumer likes to shop at all the stores to satisfy his taste for shopping variety, he saves transport cost from not visiting a particular store and can allocate this saved travel cost among the other stores to increase utility.*

Proof: There is no loss of generality in demonstrating this property with just two stores. Suppose that M is the consumer's potential income available for shopping travel and purchases. Suppose that $a_i \equiv wt_i + c_i$, $i = 1,2$. Assume $\min\left(2a_1 + a_2, a_1 + 2a_2\right) > M > a_1 + a_2$.

This means that the consumer has enough potential income to afford one visit to each store but not enough to afford a second visit to either one of the two stores. It is now easy to see that the consumer has three choices: a) Visit once and buy only from store 1; b) Visit once and buy only from store 2; c) Visit each store once and buy from each store. In each of these cases the income available for purchases is $M - a_1$ for **n** = (1,0), $M - a_2$ for **n** = (0,1) and $M - a_1 - a_2$ for **n** = (1,1). Given this constrained trip pattern, the consumer maximizes (2b) subject to (1b) treating z_1 and z_2 as the decision variables. The value of each case to the consumer is measured by the indirect utility in each case. These are $V(1) = \left(\dfrac{M - a_1}{p_1}\right)$, $V(2) = \left(\dfrac{M - a_2}{p_2}\right)$, and for the case of patronizing both stores,

$V(1\,\&\,2) = \left(p_1^{\frac{\alpha\rho}{\rho-1}} + p_2^{\frac{\alpha\rho}{\rho-1}}\right)^{\frac{1-\alpha\rho}{\alpha\rho}}(M - a_1 - a_2)$. By inspection or by plugging in values for the parameters verify that $\max\left(V(1), V(2)\right) > V(1\,\&\,2)$ is possible. This can occur, for example, when travel to the stores is sufficiently dear that it makes sense to sacrifice store variety in order to save the cost of travel to the more inaccessible store, allocating this saving towards more purchases in the nearer store. •

Although **n** must consist of integers, for convenience, I will hereafter follow the approximation and treat its elements as real numbers. This will be a good approximation provided the total trips made over the year are numerous so that any fractional trips to a particular store can be rounded off to the nearest integer without any significant loss of accuracy. Such a condition can be had by assuming that potential income, M, is sufficiently large relative to the purchase prices and the travel costs of each store. Both **n** and **z** can have only non-negative elements.[4]

Now consider how such a problem can be solved. To get a specific solution, I will assume that $u_i(z_i) = z_i^{\alpha}, 0 < \alpha < 1$. The majority might solve this problem by formulating a Lagrangian and then finding all of the first order conditions with respect to (**n**,**z**) as in a textbook. Instead of emulating this standard and dull approach, in this article we will consider two alternative two-stage solution procedures that give the same result as the textbook method and at once are more intuitively appealing and yield more insight than the textbook method.

First two-stage procedure: *Purchases conditional on a trip pattern*

Our first two-stage procedure is for the consumer to first determine a feasible trip vector **n**, such that $M\ (\mathbf{a},\mathbf{n}) \equiv M - \sum_{i=1}^{I} a_i n_i > 0$, where $\mathbf{a} = (a_1,...,a_I)$, and $a_i \equiv wt_i + c_i$. Then, the consumer maximizes the utility function (2b) with respect to **z** and subject to the budget constraint $\sum_{i=1}^{I} p_i n_i z_i = M - \sum_{i=1}^{I} a_i n_i$, taking **n** as given. Doing so determines the Marshallian demands for quantities purchased at each store, conditional on **n** and **p** and **a.** The solution is (3a), assuming as stated earlier, that $u_i(z_i) = z_i^{\alpha}, 0 < \alpha < 1$.

$$z_i(\mathbf{p},\mathbf{n}|\,M(\mathbf{a},\mathbf{n})) = \frac{p_i^{\frac{1}{\alpha\rho-1}} n_i^{\frac{1-\rho}{\alpha\rho-1}}}{\sum_{j=1}^{I} p_j^{\frac{\alpha\rho}{\alpha\rho-1}} n_j^{\frac{\rho(\alpha-1)}{\alpha\rho-1}}} (M - \sum_{k=1}^{I} a_k n_k)\,,\ i = 1,...,I\,. \qquad (3a)$$

In the next stage, the consumer plugs the above **z** into the utility function (2b) to get the following indirect utility which is conditional on **n**:

$$U(\mathbf{p},\mathbf{n}|\,M(\mathbf{a},\mathbf{n}))^{1/\alpha} = \left(\sum_{i=1}^{I} p_i^{\frac{\alpha\rho}{\alpha\rho-1}} n_i^{\frac{\rho(\alpha-1)}{\alpha\rho-1}} \right)^{\frac{1-\alpha\rho}{\alpha\rho}} \left(M - \sum_{i=1}^{I} a_i n_i \right). \qquad (3b)$$

This can now be maximized with respect to **n** to obtain the optimal trip pattern which is:

[4] In empirical study, the data obviously records trips as integers and consumers are observed to choose among all possible combinations of income-feasible trip patterns, **n**. Using the modern econometric techniques of estimation by simulation (e.g. Train, 2003), maximum likelihood estimates of probabilistic choices over such trip combinations are easy to obtain.

$$n_i(\mathbf{a},\mathbf{p}|(1-\alpha)M) = \frac{p_i^{-\frac{\alpha\rho}{1-\rho}} a_i^{\frac{\alpha\rho-1}{1-\rho}}}{\sum_{j=1}^{I} p_j^{-\frac{\alpha\rho}{1-\rho}} a_j^{\frac{(\alpha-1)\rho}{1-\rho}}}(1-\alpha)M \;,\; i=1,...,I. \tag{3c}$$

Second two-stage procedure: *trips and purchases conditional on store-specific expenditures*

Another intuitively appealing and conceptually simpler way to solve the problem and obtain exactly the **z** and **n** given by (3a) and (3c) is by means of an alternative two-stage procedure. I will refer to the stages as the *inner* and *outer* stages. I will initially approach this procedure using $u_i(z_i)$ but I will later specialize to $u_i(z_i) = z_i^{\alpha}, 0 < \alpha < 1$.

At the outer stage, the consumer optimally allocates positive expenditures $\mathbf{e} = (e_1,...,e_I)$ to each store, so that the allocation is feasible, summing up to the total income available for purchases and travel:

$$\sum_{i=1}^{I} e_i = wH - d(wT + C) \tag{4a}$$

At the inner stage, and for each store, the number of visits and the purchase per visit that maximizes the sub-utility from that store are found. This inner stage problem for the i^{th} store is:

$$Max_{n_i,z_i} \quad u_i^{\frac{1}{\rho}} \equiv n_i u_i(z_i) \tag{4b}$$

$$subject\ to \quad a_i n_i + n_i p_i z_i = e_i. \tag{4c}$$

The budget constraint (4c) is non-linear in the choice variables n_i and z_i. In fact the budget constraint is negatively sloped and strictly convex to the origin in (z_i, n_i)-space and defines a non-convex budget set (see Figure 1).

FIGURE 1 ABOUT HERE

This is seen directly from the first and second derivatives:

$$\frac{\partial n_i}{\partial z_i} = -e_i p_i \left(a_i + p_i z_i\right)^{-2} < 0\,, \tag{5a}$$

$$\frac{\partial^2 n_i}{\partial z_i^2} = 2 e_i p_i^2 \left(a_i + p_i z_i\right)^{-3} > 0\,. \tag{5b}$$

This observation reflects the basic non-convexity in traveling to a store. The cost of travel, per unit of the quantity purchased decreases if more is purchased per visit.

The next two noteworthy facts are about the indifference curves of the sub-utility

$n_i u_i(z_i)$. First, recall that $u_i(z_i)$ is strictly concave in z_i. Second, note that the indifference curve does not cut the axes of the (z_i, n_i) space.

Figure 1 puts together these facts. The fact that the budget constraint and the indifference curve are each strictly convex to the origin and the fact that the budget does but the indifference curves do not cut the axes imply that there is an interior solution. Corner solutions are not possible. Therefore, no matter how much or little income is allocated to store *i* in stage one, the consumer will choose to visit that store and to make a positive purchase.[5] This is clearly as it should be because, in a model without uncertainty, we do not want a solution where the store is visited a positive number of times but no purchases are made, or a solution in which purchases are made without visiting the store.[6]

I derive the first order conditions with respect to n_i and z_i, and divide one into the other. Doing so yields:

$$\frac{u_i(z_i)}{u_i'(z_i)} = \frac{a_i + p_i z_i}{p_i}. \qquad (6a)$$

The above says that the ratio of the marginal utility of a trip to the marginal utility of the good purchased on the trip is equal to the ratio of the cost of one trip (inclusive of the cost of purchases on the trip) to the price of the good that is purchased.

Dividing both sides with z_i, (6a) is rewritten as follows:

$$\frac{u_i(z_i)/z_i}{u_i'(z_i)} = 1 + \frac{a_i}{p_i z_i} \qquad (6b)$$

Note that the left side of (6b) is greater than unity by the concavity property. The condition says that for each store, the ratio of the average utility to the marginal utility of the quantity purchased is equal to one plus the ratio of the opportunity cost of the trip to the cost of purchases per trip at the destination. Yet another useful way to write (6b) is:

$$\frac{u_i(z_i)}{u_i'(z_i)} - z_i = \frac{a_i}{p_i}. \qquad (6c)$$

(6c) gathers on the left all the terms involving the quantity purchased on the trip, while on the right is the exogenous ratio of the marginal cost of a trip to the marginal cost of the quantity purchased. Note that the solution to (6c) gives the quantity demanded per trip, z_i^*, and plugging

[5] Of course there must be enough expenditure allocated to enable at least one visit. We are circumventing this problem in this theoretical treatment by ignoring the integer nature of trips – as explained earlier.
[6] Tele-shopping is not fundamentally different because the communication cost replaces the travel cost. The essential difference is that the communication cost is lower than the corresponding travel cost.

this into the budget one gets the number of trips demanded by the consumer, n_i^*. These two demands are *conditional* on knowing the expenditure e_i from the outer stage and are related to each other by the following property.

Property 4a. ***Conditional trip and quantity demands are trip-cost substitutes*****:** *Trips,* n_i *, and the quantity purchased on a trip,* z_i*, conditional on expenditure* e_i *are substitutes with respect to the opportunity cost of travel,* $a_i \equiv wt_i + c_i$ *.*

Proof: Cross-multiplying (6c) and totally differentiating the resulting equation with respect to z_i and a_i, we get: $\frac{dz_i}{da_i} = -\frac{u_i'(z_i)}{u_i''(z_i)(z_i + a_i)} > 0$. Then, totally differentiating the budget constraint (4c) and using $\frac{dz_i}{da_i}$ in it we get that $\frac{dn_i}{da_i} = -n_i \frac{1 - \frac{u_i'(z_i)p_i}{(a_i + p_i z_i)u_i''(z_i)}}{a_i + p_i z_i} < 0$. •

Property 4b. ***Conditional trip and quantity demands are price-substitutes*** **(*complements*)** ***if the quantity purchased is price-elastic*** **(*inelastic*)*.*** *Trips,* n_i *, and the quantity purchased on a trip,* z_i*, conditional on expenditure* e_i *are substitutes with respect to the product price* p_i *only if the elasticity of the demand for product* z_i*,* $\mathrm{E}_{z_i,p_i} < -1$.

Proof: Following the same procedure as in the proof of Property 4a, we can derive the following:

$\frac{dz_i}{dp_i} = \frac{u_i(z_i) - z_i u_i'(z_i)}{u_i''(a_i + p_i z_i)} < 0$ and $\frac{dn_i}{dp_i} = -\frac{1 - \mathrm{E}_{z_i,p_i}}{z_i(a_i + p_i z_i)} n_i <=> 0$ as $\left|\mathrm{E}_{z_i,p_i}\right| <=> 1$. •

Thus, as a trip to a store gets more (less) expensive (higher travel cost for whatever reason including distance), the consumer travels there fewer (more) times but buys a larger (lower) quantity on every trip. Similarly, as the price at a store gets lower (higher), the consumer buys more (less) at that store but travels there less (more) often.

We will now specialize to $u_i(z_i) = z_i^{\alpha}, 0 < \alpha < 1$. Then, the demand function for the quantity to purchase on a single trip is:

$$z(p_i, a_i) = \frac{\alpha}{1-\alpha} \frac{a_i}{p_i}, \tag{7a}$$

which is independent of the expenditure and homogeneous of degree zero in w, c_i, p_i. Cross multiplying, we see that the purchase expenditure per trip is proportional to the cost of the trip. If, for example, $\alpha = 1/2$, then the cost of the trip and the cost of the purchases on the trip are equal. If $\alpha > 1/2$ the cost of the purchases on the trip are higher than the trip's cost. If $\alpha < 1/2$ the trip's cost exceeds the cost of the purchases on the trip. Note also that the elasticity of $z_i(p_i, a_i)$ with respect to p_i is minus one. Therefore, according to Property 4b, the demand for trips is independent of p_i. This is indeed verified by plugging (7a) into the budget constraint (4c) and solving the demand for the number of trips,

$$n(a_i, e_i) = (1-\alpha)\frac{e_i}{a_i}\ , \qquad (7b)$$

which are homogeneous of degree zero in w, e_i, c_i. Cross multiplying, this shows that the expenditure on travel to destination *i* is a constant fraction, $1-\alpha$, of the expenditure allocated to that destination in the outer stage. Then, it follows immediately (and can also be verified from (7a) and (7b)) that the remaining fraction α of the expenditure allocated to destination *i* is spent on purchases there, so that $p_i n_i z_i = \alpha e_i$.

Substituting (7a) and (7b) into the sub-utility function for the i^{th} store, we get the indirect sub-utility which is homogeneous of degree zero in p_i, c_i, w, e_i:

$$v(p_i, a_i, e_i) = (1-\alpha)^{(1-\alpha)\rho}\alpha^{\alpha\rho} p_i^{-\alpha\rho} a_i^{(\alpha-1)\rho} e_i^{\rho} \qquad (7c)$$

We are primarily interested in how this indirect sub-utility depends on the expenditure, e_i, allocated to *i.* Since $0 < \rho < 1$, the indirect utility (7c) is strictly concave in e_i. I write it as $v_i(e_i) = A_i e_i^{\rho}$, where $A_i \equiv (1-\alpha)^{(1-\alpha)\rho}\alpha^{\alpha\rho} p_i^{-\alpha\rho} a_i^{(\alpha-1)\rho}$.

We can now move to the outer stage of the consumer's utility maximization:

$$Max_{\forall e_i} \quad U(e_1, ..., e_I) = \left(\sum_{i=1}^{I} A_i e_i^{\rho}\right)^{1/\rho}; \quad 0 < \rho < 1, \qquad (8a)$$

$$subject \quad to: \ \sum_{i=1}^{I} e_i = wH - d(wT + C). \qquad (8b)$$

As promised, the solution to this problem determines the optimal way the consumer should allocate his income among all the destinations, to be expended for trips and purchases at those destinations. (8a) is Dixit-Stiglitz C.E.S. The indifference curves are strictly concave to the origin and are tangent to all axes. The budget constraint (8b) is linear and symmetric in $\mathbf{e} = (e_1, ..., e_I)$ (see Figure 2). Therefore, the solution is strictly an interior one: the consumer will

FIGURE 2 ABOUT HERE

allocate some income to each and every store because he has an extreme taste for variety in spending at each store. He would rather spend even a small amount at each store than not visit any one store. Solving, we get the *Marshallian allocation of income* or the *"demand for expenditure*" at store *i*:

$$e_i^* = \frac{A_i^{\frac{1}{1-\rho}}}{\sum_{j=1}^{I} A_j^{\frac{1}{1-\rho}}}\left[wH - d(wT + C)\right]. \qquad \text{(8c)}$$

From (8c) it is directly verified by summation that the budget constraint is satisfied. It is revealing to rewrite the above as a proportion. Let $M \equiv wH - d(wT + C)$. Then, dividing (8c) by M and substituting for the A_i :

$$\pi_i^* \equiv \frac{e_i^*}{M} = \frac{A_i^{\frac{1}{1-\rho}}}{\sum_{j=1}^{I} A_j^{\frac{1}{1-\rho}}} = \frac{p_i^{-\frac{\alpha\rho}{1-\rho}} a_i^{\frac{(\alpha-1)\rho}{1-\rho}}}{\sum_{j=1}^{I} p_j^{-\frac{\alpha\rho}{1-\rho}} a_j^{\frac{(\alpha-1)\rho}{1-\rho}}}. \qquad \text{(8d)}$$

(8d) says that the proportion, π_i^* ,of income allocated to the i^{th} destination is given by the logit model but, in this case, the model is deterministic not the stochastically derived one of applied travel demand analysis, and the dependent variable is not a probability but a fraction of income. (8d) shows that *ceteris paribus* more income is allocated to more accessible destinations and less to destinations with a higher unit price. Notably, of course, the *independence of irrelevant alternatives property* of logit holds: the ratio of incomes allocated to two destinations is independent of changes in price or accessibility that can occur in any other destination. But obviously this does not hold because of any assumption about the distribution of random utilities since there aren't any in this deterministic model.

Property 5. *Strict gross substitution of store-expenditures*: *The expenditures optimally allocated to the alternative stores satisfy strict gross-substitution with respect to product prices at the stores as well as the opportunity cost of traveling to the stores.*

Proof: From (8d) one can calculate the own- and cross- elasticities of the expenditure allocated to a particular store *i*. These elasticities are:

$$\mathrm{E}_{e_i,p_i} = -\frac{\alpha\rho}{1-\rho}(1-\pi_i^*) < 0 \text{ and } \mathrm{E}_{e_i,a_i} = -\frac{(1-\alpha)\rho}{1-\rho}(1-\pi_i^*) < 0$$

$$\mathrm{E}_{e_i,p_{j,j\neq i}} = \frac{\alpha\rho}{1-\rho}\pi_j^* > 0 \text{ and } \mathrm{E}_{e_i,a_{j,j\neq i}} = \frac{(1-\alpha)\rho}{1-\rho}\pi_j^* > 0 \quad \bullet$$

These elasticities also reveal the special property that the expenditure allocated to a store is more (less) elastic and more (less) cross-elastic with respect to the price for the goods sold at the store rather than with respect to the opportunity cost of accessing the store if $\alpha > 1/2$ $(\alpha < 1/2)$.

We have completed both stages of the utility maximization analysis. Putting the two stages together, th

e full solution gives the *unconditional trip and quantity demands* n_i^*, z_i^* for i=1,…, I which are as follows:

$$z_i^* = \frac{\alpha}{1-\alpha}\frac{a_i}{p_i} \tag{9a}$$

and

$$n_i^* = (1-\alpha)\frac{p_i^{-\frac{\alpha\rho}{1-\rho}} a_i^{\frac{\alpha\rho-1}{1-\rho}}}{\sum_{j=1}^{I} p_j^{-\frac{\alpha\rho}{1-\rho}} a_j^{\frac{(\alpha-1)\rho}{1-\rho}}}\left[wH - d(wT+C)\right]. \tag{9b}$$

The exogenous variables are $\alpha, \rho, w, H, d, T, C$ and the vectors $\mathbf{p} = \left(p_1,..., p_I\right)$, $\mathbf{t} = \left(t_1,...,t_I\right)$ and $\mathbf{c} = \left(c_1,...,c_I\right)$. Both demands are homogeneous of degree zero in w, C, **c**, **p.** Note that (9b) is identical to (3c) derived earlier via the previous two-stage procedure.

Property 6. ***Unconditional quantity and trip demands are price-complements and trip-cost-substitutes***: *The consumer's optimally determined trips,* n_i *, and the quantity purchased on a trip,* z_i*, are complements with respect to the product price* p_i *and substitutes with respect to the opportunity cost of travel,* $a_i \equiv wt_i + c_i$.

Proof: The derivatives of z_i^* are $\frac{\partial z_i^*}{\partial p_i} = -\frac{\alpha}{1-\alpha} a_i p_i^{-2} < 0$, $\frac{\partial z_i^*}{\partial a_i} = \frac{\alpha}{1-\alpha} p_i^{-1} > 0$. To see the derivatives of n_i^*, I write it as $n_i^* = (1-\alpha) M \pi_i^* a_i^{-1}$. From this, we get

$\frac{\partial n_i^*}{\partial p_i} = -\frac{(1-\alpha)\alpha\rho}{1-\rho} M \frac{\pi_i^*(1-\pi_i^*)}{p_i a_i} < 0$, $\frac{\partial n_i^*}{\partial a_i} = -\frac{(1-\alpha)M}{a_i^2}\pi_i^*\left[1 + \frac{(1-\alpha)\rho}{1-\rho}(1-\pi_i^*)\right] < 0.$ •

From these partial derivatives we can see that if the unit product price increases, fewer trips are made to that store and less is purchased per trip. If the cost of traveling to a store increases, fewer trips are made to that store but more is purchased on each trip. It is also easy to see that the total quantity purchased from a store is $Q_i \equiv n_i^* z_i^* = \alpha M \pi_i^* p_i^{-1}$. It is easy to see using the foregoing that $\frac{\partial Q_i}{\partial p_i} < 0$ and $\frac{\partial Q_i}{\partial a_i} < 0$.

Property 7. ***Effect of wage on changes in expenditures by store distance*****:** *Consider a sequence of many stores i=1…I, located at different distances from the consumer's home as shown in the middle panel of Figure 2, so that for any i,* $t_{i+1} > t_i$ *and* $c_{i+1} > c_i \Rightarrow a_{i+1} > a_i$.

(i) Substitution effect: *As the consumer's wage increases, keeping his total income constant, he will decrease his expenditures on far away stores while increasing his trips and expenditures on nearby stores.*

(ii) Income effect: *As the consumer' wage increases, his income increases and this causes the consumer to want to increase his expenditures in all the stores.*

Proof: From (8d), we can calculate that $\frac{\partial e_i}{\partial w} = [wH - d(wT + C)]\frac{\partial \pi_i^*}{\partial w} + \pi_i^*(H - dT)$. We can also calculate that $\frac{\partial \pi_i^*}{\partial w} = \frac{(\alpha - 1)\rho}{1-\rho}\pi_i^*\left[\frac{t_i}{a_i} - \sum_{j=1}^{I}\pi_j^*\left(\frac{t_j}{a_j}\right)\right]$. The first derivative shows substitution and income effects of the wage increase on store expenditures and the second is used to determine the sign of the substitution effect for each store, keeping income constant.

(i) The bracket is zero for an "average store" in the sense that the time cost of a trip to that store as a ratio of the total trip cost is average when weighted by expenditures. For such an "average store", the substitution effect is exactly zero. For stores that are closer to the consumer than average in the above sense, the bracket is negative and this gives a positive substitution, while for stores that are more distant than average in the above sense, the bracket is positive and the substitution negative. Thus, as the consumer's wage rises, income constant, he tilts his expenditures in favor of nearby stores (and away from far away stores) because his value of time rises.

(ii) The income effect of the wage increase as measured by $\pi_i^*(H - dT)$ is positive but falls with store distance since, *ceteris paribus*, the fraction of expenditure allocated to a store falls with distance as we have seen. •

Thus, if the income effect of the wage increase is not very large, the substitution effect will dominate and the consumer will spend more on travel and purchases on nearby and less on more distant stores as his wage rate increases and thus the opportunity cost of travel increases. But if the income effect is large enough, it is possible that the consumer will spend more traveling to and purchasing from all stores as his wage increases.

3. The Symmetrical Case

In order to gain insight into the economic properties of the model, it is worthwhile to examine the symmetrical case. Suppose that all stores are located symmetrically with respect to the consumer so that t and c are travel time and travel cost to any store, and p is the unit price at any store. The top panel of Figure 2 illustrates this by putting the consumer at the center of a circle and locating the stores on the circle's perimeter, with the consumer connected to each store by a radial road. In this hypothetical situation, using $a = wt + c,$ (9a) and (9b) simplify to

$$z^* = \frac{\alpha}{1-\alpha}\frac{a}{p}, \tag{10a}$$

and

$$n^* = (1-\alpha)\frac{wH - d(wT + C)}{Ia}. \tag{10b}$$

I can now substitute (10a) and (10b) into (2b) to get the indirect utility of the consumer in the case of symmetry. This indirect utility, homogeneous of degree zero in p, w, c and M is,

$$V(p,a,M,I) = (1-\alpha)^{1-\alpha}\alpha^{\alpha}p^{-\alpha}a^{-(1-\alpha)}MI^{\frac{1-\rho}{\rho}}. \tag{11}$$

This leads to the following observation.

Property 8. ***The marginal utility of a store***: *when the consumer is symmetrically situated with respect to the stores, then as the number of stores, I, increases remaining symmetric, the consumer's indirect utility is increasing and strictly convex, linear or strictly concave with respect to I according to whether the elasticity of substitution among stores,* $\frac{1}{1-\rho}$, *is less than, equal to or greater than 2.*

Proof: Note that (11) is increasing strictly convex, linear or strictly concave in I according to whether $\frac{1-\rho}{\rho} >=< 1,$ hence according to whether $\rho <=> \frac{1}{2}$ and, hence, according to whether $\frac{1}{1-\rho} <=> 2$ •

The intuition behind Property 9 is that if the elasticity of substitution among the stores is high enough (greater than 2) then they are sufficiently close to the case of perfect substitutes that the taste for variety weakens to give the effect of a positive but decreasing marginal utility for an additional store.

Property 9. ***Invariance of aggregates with respect to the number of alternatives***: *When the consumer is symmetrically situated with respect to all stores, then the total trips* (*TRIPS*) *made by* *the consumer, the consumer's total travel time (TTT), total travel cost (TTC) and the total quantity purchased on all trips (TPQ) all remain unchanged as the number of stores increases or decreases.*

Proof: Let us define $TRIPS = n^* I$ as the total number of trips. It is seen from (10b) that these trips are independent of the number of stores, *I*. The consumer likes variety. Therefore, any new stores that appear will be visited but the trips to the new stores come at the expense of visits to existing stores so that the total number of trips remains unchanged. The total travel time, $TTT = n^* It$, total travel cost, $TTC = n^* Ia$ and the total quantity purchased on all trips, $TQP = n^* z^* I$ are also independent of the number of available stores. Since, z^* , the quantity purchased per trip given by (10a) is also independent of the number of stores, as more stores are added the consumer spreads his trips and his total purchase quantity among all stores without changing the total number of his trips or the quantity he buys per trip. •

We can also see that the total number of trips increases as the wage rate, *w*, increases. Note that as the wage increases there are income and substitution effects on the total trips (Property 8). The income effect is that the consumer sees the total trips as a normal good and wants to make more trips as his wage and hence his income increases. At the same time as the wage increases, the time cost of a trip becomes more onerous because of the opportunity cost of traveling. As a result of this, each trip becomes more expensive and this creates a substitution effect in favor of making fewer trips. In the present model, however, the income effect dominates the substitution effect which can be ascertained by calculating that,

$$\frac{\partial TRIPS}{\partial w} = \frac{1-\alpha}{\alpha}(H - dT) > 0. \tag{12}$$

Property 10. *Optimal number of stores*: *If the elasticity of substitution among stores is greater than 2, and the consumer incurs an annual cost f for each store that he patronizes, then there exists an optimal number of stores* $I^* = (1-\rho)\frac{M}{f}$ *that uniquely maximizes the consumer's utility with respect to the number of stores, I.*

Proof: Including the annual cost incurred for all stores, the indirect utility (11) becomes, $V(M - fI, I) = const. \times (M - fI) I^{\frac{1-\rho}{\rho}}$. Maximizing this with respect to *I*, we get $I^* = (1-\rho)\frac{M}{f}$, and it can be shown that as long as $\frac{1}{1-\rho} > 2$ ($\rho > \frac{1}{2}$), $\frac{\partial^2 V}{\partial I^2} < 0$ for all *I* satisfying the second order condition for a maximum. •

The cost f can be interpreted as the opportunity cost of "getting to know" a store. Suppose that a consumer does not visit a store unless he knows it and that, in the symmetrical situation, it costs f to get to know a store. Once such a cost is incurred, the consumer knows a store perfectly and will patronize it. Under this assumption, the consumer will not want to patronize more than I^* stores (ignoring the integer nature) because the marginal utility of an additional store (Property 8) is lower than f the marginal cost of knowing it. Note that after incurring the annual cost of getting to know the optimal number of stores, the consumer has ρM of his potential income available and will allocate a fraction α of this to purchases and a fraction $1-\alpha$ to trips, which follows from section 2.

4. Trip Chaining: A Simple Case

I assumed that the consumer travels to each store on a separate trip. The model extends non-trivially to combined trips commonly known as "trip chains" or "tours". Because trip-chains can be complex, I will here examine only the simplest case of just two stores. These could be located on the circumference of the circle in the upper panel of Figure 2, on the line in the middle panel, or on the Euclidian plane shown in the bottom panel.

FIGURE 3 ABOUT HERE

The arrows in each pattern show the nature of a tour. If on the circle, the consumer travels along a radius to a store on the circumference, then along an arc to another store and back home along another radius. It is intuitively clear that if the arc distance between the two stores is large, then it is better not to trip chain than to trip chain. By trip chaining the consumer saves the cost of two one-way trips along a radius. By not trip-chaining, two round trips are made, one to each store, and the cost of a one-way travel along the circle arc is saved. Therefore, on the circle, trip chaining costs less in travel as long as the one-way cost of traveling along the arc is less than the two way cost of reaching the periphery.

In the case of the line, the consumer travels out from home to the most distant store and back but on his way stops and shops in a store of intermediate distance. In this case, the consumer saves the cost of a two-way trip to the nearest of the two stores. If *ceteris paribus* the two stores are very close to each other, the saving is big and there is a very strong incentive to trip chain. But if store 1 is very close to home, the travel cost saving becomes negligible.

On the Euclidian plane, assume that all points can be traveled to "as the crow flies". The consumer would travel from his home to store 1, then to store 2 and then back home. Then the saving can be viewed as being similar to what happens on the circle. What is saved is the cost of a one-way trip to each store. The consumer's travel cost is reduced if the one-way cost of travel between stores 1 and 2 is less than his the sum of the one-way cost of traveling to each store.

Proposition 1.*Trip-chaining inequality*: *The consumer can increase his potential income available for purchases and trips provided that* $\frac{a_1}{2}+\frac{a_2}{2}\geq s$ *where* a_i *is the cost of two-way travel from the consumer's home location to store* I = 1,2, *and s is the one-way cost of travel on the arc* (*or distance*) *between the two stores. In the case of the two stores being located on the line, the saving from trip chaining is never negative.*

Proof: The cost of separately traveling once to each store and back is a_1+a_2, while the cost of chaining the two stores in a single tour is $\frac{a_1}{2}+\frac{a_2}{2}+s$. Therefore, trip chaining creates a saving as long as $\frac{a_1}{2}+\frac{a_2}{2}+s\leq a_1+a_2 \quad \Rightarrow \quad \frac{a_1}{2}+\frac{a_2}{2}\geq s$. In the case of the linearly arranged stores, $s=\frac{a_2-a_1}{2}$ and the inequality always holds •

In each of the above geometries, the travel cost savings from trip-chaining identified in Proposition 1 should be balanced against the loss of utility from committing to visit each of the two stores the same number of times. Since such an equality in the number of trips is not optimal in general, it does not make sense for the consumer to so restrict himself and, in general, the consumer can supplement chained-trips with additional unchained trips.

Let us now set up a utility maximizing analysis of trip-chaining for the linear situation in the middle panel of Figure 2. It readily generalizes to any geometry. Store two is farther away from the consumer than store 1 so that $t_2>t_1$ and $c_2>c_1$. Hence, $a_2>a_1$. If the consumer makes only one trip-chain stopping at each store, then his total travel time and cost are equal to what he would have incurred if he had visited only store two. Thus by trip-chaining, the cost of a second trip to store 1 (the nearer store) is saved.

The problem of utility maximization for this simple case can be stated as follows, where I am using an upper $\sim$ to distinguish between the n in this section and those of sections 2 and 3. I do not use a ~ on the z since how much one buys at a store does not depend on whether one got there on a chained or unchained trip:

$$Max_{\tilde{n},\tilde{n}_1,\tilde{n}_2,z_1,z_2} \quad U=\left((\tilde{n}+\tilde{n}_1)^{\rho}u_1(z_1)^{\rho}+(\tilde{n}+\tilde{n}_2)u_2(z_2)^{\rho}\right)^{1/\rho}$$

subject to: (13a)

$$\tilde{n}(a_2+p_1z_1+p_2z_2)+\tilde{n}_1(a_1+p_1z_1)+\tilde{n}_2(a_2+p_2z_2)=M$$

where $\tilde{n}$ are the chained trips and $\tilde{n}_1, \tilde{n}_2$ are additional separate trips to each store. The other symbols are as defined earlier. Adding $\tilde{n}a_1$ to both sides, I get:

$$\left(\tilde{n}+\tilde{n}_1\right)(a_1+p_1z_1)+\left(\tilde{n}+\tilde{n}_2\right)(a_2+p_2z_2)=M+\tilde{n}a_1, \qquad (13b)$$

which clearly shows on the right side, the economic cost saving due to the potential trip chaining.

This problem can be solved correctly in three stages which easily generalizes to stores located on the circle or the plane. In the innermost stage, I treat $\tilde{n}$, the number of trip chains, as a parameter, recognizing that its ultimate optimal value could be zero or positive. Given $\tilde{n}$, in the innermost stage, I also take the allocation of expenditure to each store as given and following a procedure that is similar to that of section 2, I solve for the $\tilde{n}_1, \tilde{n}_2$ which are optimal conditional on $\tilde{n}$ and e_1, e_2 respectively. In the middle stage, the problem is evaluated using the functions $\tilde{n}_1(\tilde{n}, e_1), \tilde{n}_2(\tilde{n}, e_2)$ implied by the inner stage and, similar to section 2, the expenditures are then allocated optimally among the stores conditional on $\tilde{n}$. Finally, in the third stage, the consumer optimizes with respect to $\tilde{n}$. Note that the problem can include corner solutions. One is the solution with $\tilde{n}=0$ and $\tilde{n}_1, \tilde{n}_2 > 0$ which means that there are no trip chains and the two stores are visited in separate trips. This corresponds to the outcome where the separate trips strategy assumed in section 2 wins over any mixed strategy involving trip chains. This will definitely be the case when the trip-chaining inequality of Proposition 1does not offer an advantage in favor of chaining trips. A second corner solution can occur so that all trips are chains: $\tilde{n}>0, \tilde{n}_1=\tilde{n}_2=0$. For this to be the case, it is necessary but not sufficient that the trip chaining inequality holods. Finally, there can also be solutions where $\tilde{n}>0$ and $\tilde{n}_1=0,\ \tilde{n}_2>0$ or where $\tilde{n}_2>0,\ \tilde{n}_1=0$.

Before looking the above problem which allows for mixing of chained and unchained trips, I will first consider only the two extremes which is helpful for our intuition. Suppose that the consumer is comparing whether to chain all his trips to the two stores ($\tilde{n}_i=\tilde{n}_2=0$) or not to chain them at all ($\tilde{n}=0$). The latter case was analyzed in section 2, and what I need now is to evaluate the indirect utility (optimized (2b)) from section 2, for the case of just two stores. Making the required substitutions, this indirect utility function is:

$$V(a_1,a_2,p_1,p_2,M \mid \tilde{n}=0)=\alpha^{\alpha}(1-\alpha)^{1-\alpha}\left(p_1^{\frac{\alpha\rho}{\rho-1}}a_1^{\frac{(\alpha-1)\rho}{1-\rho}}+p_2^{\frac{\alpha\rho}{\rho-1}}a_2^{\frac{(\alpha-1)\rho}{1-\rho}}\right)^{\frac{1-\rho}{\rho}}M \qquad (14)$$

The case of purely trip-chaining ($\tilde{n}_i=\tilde{n}_2=0$) requires solving the following problem:

$$Max_{\tilde{n},z_1,z_2} \quad U=\tilde{n}\left(u_1(z_1)^{\rho}+u_2(z_2)^{\rho}\right)^{\frac{1}{\rho}}$$

$$subject\ to:\ \tilde{n}a_2 + \tilde{n}p_1 z_1 + \tilde{n}p_2 z_2 = M \ . \tag{15}$$

It helps to recognize that (15) can also be written as $p_1 z_1 + p_2 z_2 = \frac{M}{\tilde{n}} - a_2$. Now maximizing utility with respect to this budget conditional on $\tilde{n}$ will give the quantities that should be purchased from each store during the chained trip. Using the sub-utility function $u_i(z_i) = z_i^{\alpha}$, these conditional Marshallian demands are:

$$z_i = \frac{p_i^{\frac{1}{\alpha\rho-1}}}{p_1^{\frac{\alpha\rho}{\alpha\rho-1}} + p_2^{\frac{\alpha\rho}{\alpha\rho-1}}} I(\tilde{n}); i = 1,2 \ , \tag{16a}$$

They can also be obtianed from (3a) by setting **n** = (1,1), where $I(\tilde{n}) \equiv \frac{M}{\tilde{n}} - a_2$. Now, the inner stage conditional indirect utility is:

$$V(\tilde{n}, I(\tilde{n}), p_1, p_2 \mid a_2, \tilde{n}_1 = \tilde{n}_2 = 0) = \tilde{n} I(\tilde{n})^{\alpha} \left(p_1^{\frac{\alpha\rho}{\alpha\rho-1}} + p_2^{\frac{\alpha\rho}{\alpha\rho-1}} \right)^{\frac{1-\alpha\rho}{\rho}} \tag{16b}$$

In the outer stage, this is maximized with respect to $\tilde{n}$, the number of chained trips to get:

$$\tilde{n}^* = (1-\alpha)\frac{wH - d(wT + C)}{a_2}. \tag{16c}$$

Now evaluating (16b), the conditional indirect utility of purely trip-chaining, by using (16c), we get:

$$V(a_2, p_1, p_2, M \mid \tilde{n}_1 = \tilde{n}_2 = 0) = \alpha^{\alpha}(1-\alpha)^{(1-\alpha)} a_2^{\alpha-1} \left(p_1^{\frac{\alpha\rho}{\alpha\rho-1}} + p_2^{\frac{\alpha\rho}{\alpha\rho-1}} \right)^{\frac{1-\alpha\rho}{\rho}} M. \tag{16d}$$

The consumer will prefer to trip-chain than not to trip-chain whenever, given the access costs, the store prices and the other parameters, (16d) is greater than (14). An important consideration here is the proximity of the two stores in terms of travel cost. Assume that store 1 is closer to the consumer than is store 2. Then, $a_1 \le a_2$. The inequality that tells us whether trip-chaining is preferred to not trip-chaining, (16d) > (14), can be written as:

$$\frac{a_2}{a_1} < \left(\left[1 + \left(\frac{p_2}{p_1} \right)^{\frac{\alpha\rho}{\alpha\rho-1}} \right]^{\frac{1-\alpha\rho}{1-\rho}} - \left(\frac{p_2}{p_1} \right)^{\frac{\alpha\rho}{\rho-1}} \right)^{\frac{1-\rho}{(1-\alpha)\rho}} . \tag{17}$$

Note first that if store 1 is located next to the consumer's home, then $a_1 = 0$ and the consumer will not trip chain because the left side shoots out to infinity. The consumer will prefer to make separate trips to store 1 and adjust the number of these trips optimally as in section 2, since it costs nothing to make such trips. Suppose, next, that the two stores are located at the same place away from the consumer's residence. Then the left side is unity. If the prices are also equal, then the right side becomes $\left(2^{\frac{1-\alpha\rho}{1-\rho}} - 1\right)^{\frac{1-\rho}{(1-\alpha)\rho}}$ >1. In this case the consumer will always chain all trips than not chain at all. Next consider the situation where the price ratio $\frac{p_2}{p_1} > 1$. If this is sufficiently bigger than unity, the consumer will want to travel only a few times to store 2 and more times to store 1 and this effect is stronger the closer is ρ to one and, hence, the closer the two goods are to perfect substitutes. In this situation, trip-chaining all trips is highly undesirable.

Let us now return to the utility maximization problem (13a) where the mixed choice of trip chaining some trips and supplementing them with separate trips is not ruled out. It is easy to prove the following general theorem.

Theorem 1. *Trip-chaining dominance*: *Provided the inequality of* Proposition 1 *is satisfied, a consumer can always improve his utility by trip chaining some or all trips. Given that* n_1^*, n_2^* *are the optimal trips in the case of separate trip-making , the trip-chaining solution will be such that* $\tilde{n}^* \geq \min(n_1^*, n_2^*)$ *and supplemental separate trips will satisfy* $\tilde{n}_1^* \tilde{n}_2^* = 0$ (*one or both are zero*).

Proof: Suppose that I initially force the consumer not to trip chain and using the utility maximization problem of section 2, I calculate the optimal separate trips to be $n_1^*, n_2^* > 0$. Since the fundamental trip chaining inequality holds (see Proposition 1), the consumer can now choose $\tilde{n}^* = \min(n_1^*, n_2^*)$ and $\tilde{n}_i^* = n_i^* - \min(n_1^*, n_2^*)$ for $i = 1, 2$. Doing so saves expenditure without reducing the utility achieved by the earlier $n_1^*, n_2^* > 0$ in the case of separate trips. This saved expenditure can be allocated to making more separate trips or to making more chained trips. Clearly, however, if the expenditure were to be entirely or partly allocated to making more trips to the store with $\tilde{n}_i^* = 0$, then repetition of the argument shows that additional savings will accrue by chaining such a trip with a trip to the other store. (The reader is reminded that we are ignoring the integer nature of trips) •

There are important questions pertaining to trip-chaining that can be illuminated with the type of behaviorally consistent theoretical model I have developed here. Trip-chaining is favored socially because of the presumption that it reduces total travel miles by combining trips and shortening trip distances. There is, however, an *income-and-time effect* from deciding to chain trips compared to not chaining them. If planners rearrange the distribution of stores and thus successfully induce more trip-chaining, they cause consumers to save income and time but they also increase the attractiveness of travel and could induce more trip chains, more and shorter separate trips and more travel on aggregate. In the present model, it is easy to see that the total cost of travel remains unchanged when the consumer chains trips than when he makes separate trips. To see this, I can verify the following equality by substituting in from (16c) for $\tilde{n}$ and for the separate trips n_1^*, n_2^* from (9b):

$$\tilde{n}^* a_2 = n_1^* a_1 + n_2^* a_2 = (1-\alpha)M \ . \tag{18}$$

Since the consumer saves time and money by trip-chaining, it is clearly the case that, in the present model, those savings go into more trip-chains (and more store visits). So while the consumer benefits there is no reduction in aggregate travel expenditures. This result is of potential consequence to planners who are concerned about total travel expenditures, vehicle miles traveled and emissions from personal travel. However, the result is not as dismal as it appears. If I were to extend the model to include time allocation to leisure (defined as home activities that do not involve travel), then some of the time savings from trip-chaining would be allocated to such leisure and so that would tend to work toward some reduction in total travel expenditures. There is, however, a substitution effect that comes from chaining trips that induces more trips and more trip chaining. Therefore, it is unclear whether the consequences of total travel (that can be defined in dollars, minutes, miles or total emissions) increases or decreases and whether theoretically consistent models with more general functional forms than the one used here would reveal a different result when tested. This is an open question that can benefit from empirical scrutiny.

5. Extensions

1. *Store-specific effects*: In the foregoing I ignored that the consumer may feel differently about different stores. But this is easy to take care of by assuming that the utility from the i th store is $u_i(z_i) = \theta_i z_i^{\alpha_i}$ with $0 < \alpha_i < 1$ and $\theta_i > 0$ parameters that can be used to "calibrate" the relative importance to the consumer of the various stores and the goods sold there.

2. *The limited range of travel*: Another important extension is obtained by modifying the utility function so that the consumer does not want to visit all the stores that are available and, furthermore, is less likely on a *ceteris paribus* basis to visit more distant stores. I already pointed out the existence of corner solutions involving limited travel, namely patronizing a subset of all stores, because trips must be integer valued. Here, I discuss an alternative way of obtaining zero visits to certain available stores. To achieve this, (2b) can be modified to the following:

$$U = \left(\sum_{i=1}^{I} \left(n_i u_i(z_i) + \eta \right)^{\rho} \right)^{1/\rho}, \ 0 < \rho < 1, \ \ \eta > 0. \tag{19a}$$

The new parameter, η, plays two roles. First, note that if the consumer does not visit the *i* th store, he still derives a constant sub-utility η^{ρ} from the store. This is a bit awkward but it may be interpreted as a level of satisfaction from merely knowing that store *i* exists! (I do feel an excitement when a new restaurant opens in my area, though it may take me a long time to patronize it.) The second and more useful and important role is that η makes it possible to obtain a corner solution. To see this clearly, recall the two-stage solution procedure. The inner stage is unchanged since η is a constant and, hence, does not affect the choice of (n_i, z_i) given that expenditure e_i is allocated to store *i*. Therefore, the stage-one indirect utility function now takes the form:

$$U(e_1, ..., e_I) = \left(\sum_{i=1}^{I} \left(A_i \theta_i e_i + \eta \right)^{\rho} \right)^{1/\rho} \tag{19b}$$

In the outer stage, (19b) is maximized with respect to the expenditure vector, **e**. Now the stores with the relatively low $A_i \theta_i$ could be allocated zero expenditure at the utility maximum because the marginal utility of store *i* does not shoot out to infinity if the expenditure allocated to store *i* is zero. Since, as we saw in section 2, A_i decreases with distance from the consumer and with the price charged at the store, the consumer will allocate less expenditure to the more distant stores or to the more expensive stores, *ceteris paribus*. Thus with the utility (19b), the active choice set of the consumer (the subset of the *I* stores actually visited) is endogenously determined without requiring as I did in section 3 that there is a cost to getting to know a store. As a further extension, η can be different for each store or store type.

3. *Activities*: I purposely ignored the fact that purchasing at stores takes time in addition to costing money. Such time expenditures should be important. For example, searching for a parking space, or spending time in a store are clearly important considerations and the presence of such time costs cause consumers to trade off between distant and nearby stores, saving travel time in order to afford more time allocation at stores. The cost of searching for a parking space etc. should be included in t_i and may be viewed as not affording any utility. But the time spent inside a store may be a good or a bad. A good because checking out the quality of merchandise and services can be enjoyable, a bad because sitting in the dentist's chair is rarely enjoyable. The model easily extends to such cases, though it has non-trivial consequences. First, the store sub-utility function of a single trip can be modified to $u_i(z_i, \tau_i)$ where τ_i is "time in the store". Some of this may be fixed (non-discretionary) to the consumer, while some of it, for certain types of stores, is discretionary. If it is a bad, the consumer will limit the time to the fixed portion of τ_i, but if discretionary the consumer will choose that part of τ_i optimally. In the case of separate trips, the budget constraint now modifies to $\sum_{i=1}^{I} n_i a_i + w\sum_{i=1}^{I} n_i(\tau_{0i} + \tau_i) + \sum_{i=1}^{I} p_i n_i z_i - M = 0$, where τ_{0i} is the time-in-the-store that the consumer cannot control and τ_i is the discretionary part that enters the utility function as a good. The most interesting aspect of such an extension is the specification of the sub-utility $u_i(z_i, \tau_i)$. This is important because we may not know much about how the consumer trades off time and purchases in the store. But we do know that time can be spent at stores just enjoying the experience and buying nothing. Hence, $u_i(0, \tau_i) > 0$ should not be objectionable. Similarly, joy driving could be of non-negligible importance.

4. *Higher level decisions*: Commuting was treated as non-discretionary in this article. However, it is to an extent discretionary in the long run, because consumers can change residence or job location and thus indirectly affect their commuting costs. In making residence-workplace choices, consumers consider, among other things, the bundle of discretionary trips that a particular residence-workplace arrangement allows. Also, the commute itself can be chained with other discretionary trips. As well, locating in some neighborhoods may allow better trip chaining or, more generally, higher accessibility to stores. Economizing on the commute is one of the major drivers of the observed growth in discretionary travel. Using a simpler model of discretionary travel, I have elsewhere (Anas and Xu, 1999) examined the effects of commuting arrangements and of trip dispersion on urban spatial structure at the metropolitan level, which goes against the grain of standard urban economics. Boarnet and Sarmiento (1998) have posed the question of whether land use patterns can be manipulated by planners to significantly influence travel behavior. Being able to answer the question requires a microeconomic theory of how consumers travel, trip-chain and perform activities.

5. *Uncertainty and dynamic adjustment*: It would be interesting to extend the approach by introducing uncertainty, sequencing of trips over different days and dynamic adjustment in the consumer's travel and purchasing decisions as new information about stores, prices and travel costs becomes revealed to the consumer through time.

6. *Econometrics:* To make the model empirically applicable requires adding a stochastic structure to it. In the present paper, I purposely did not go into these aspects. Using the most modern developments in the estimation of complex choice models by simulation (see Train, 2003), econometricians have enormous power and can easily generalize what is proposed here both to deal with the integer nature of trips in real data and with the specification of error terms that are specific to the traveler preferences or to the store attributes.

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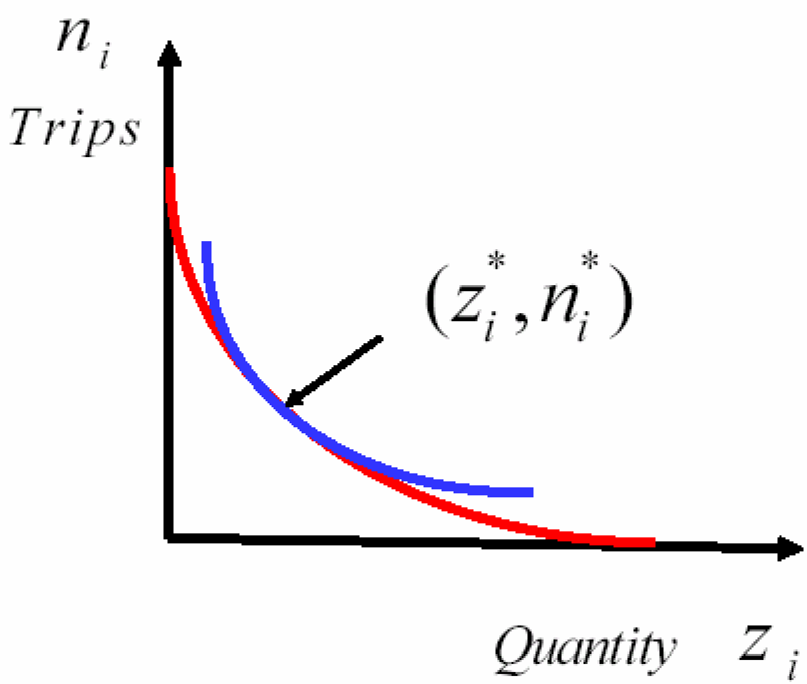

FIGURE 1

Inner stage problem

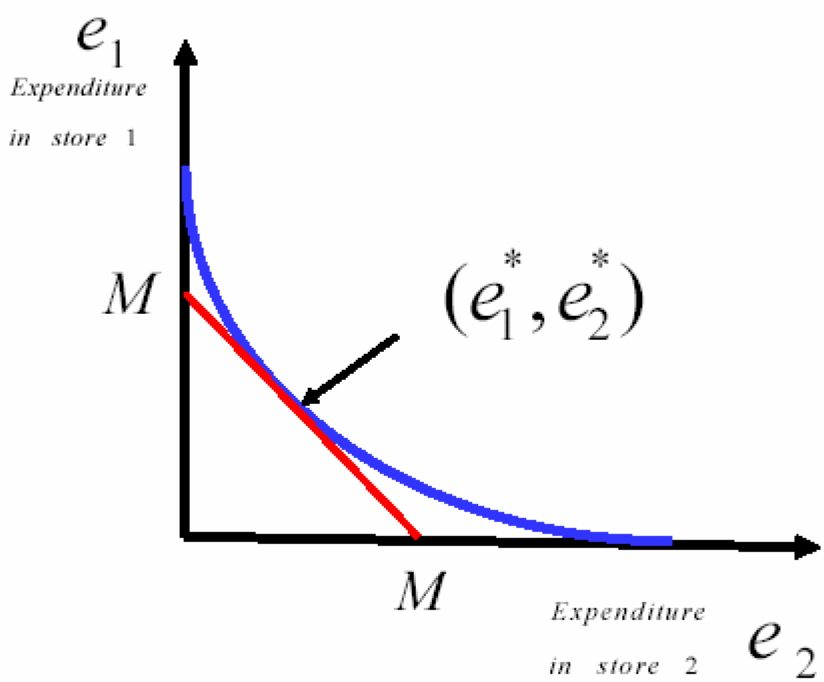

FIGURE 2

The outer stage problem

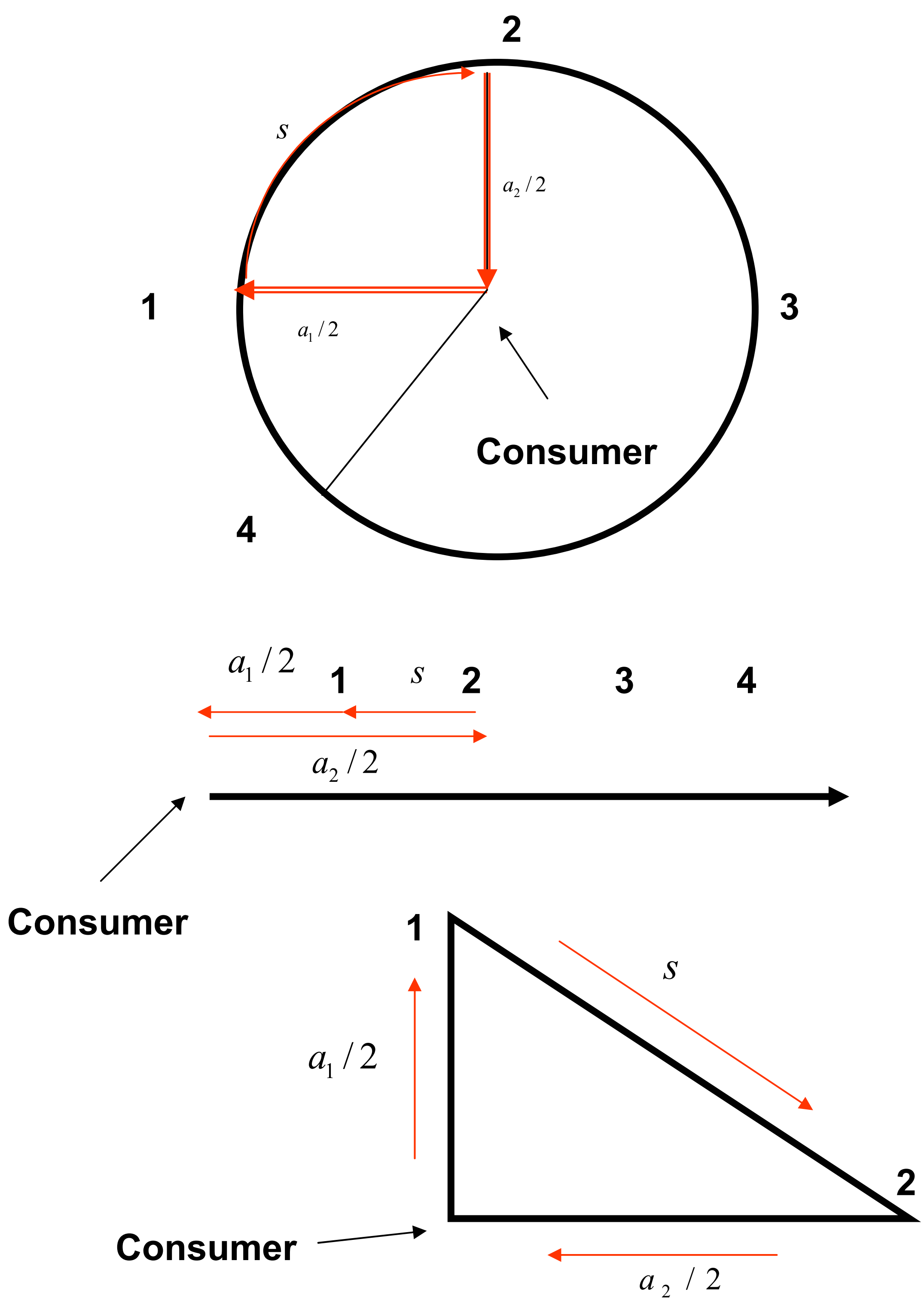

FIGURE 3: Trip Chains