Job Displacement Risk and the Cost of Business Cycles

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Tom Krebs*
Brown University†

Abstract

This paper analyzes the welfare costs of business cycles when workers face uninsurable job displacement risk. The paper uses a simple macroeconomic model with incomplete markets to show that cyclical variations in the long-term earnings losses of displaced workers can generate arbitrarily large cost of business cycles even if job displacement rates and the variance of individual income changes are constant over the cycle. In addition to the theoretical analysis, this paper also conducts a quantitative study of the cost of business cycles using empirical evidence on the long-term earnings losses of U.S. workers. The quantitative analysis suggests that cyclical variations in job displacement risk generate sizable cost of business cycles.

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†Department of Economics, 64 Waterman Street, Providence, RI 02912. E-mail: tom_krebs@brown.edu

I. Introduction

In a highly influential contribution, Lucas (1987) argues that standard macroeconomic theory implies that the welfare cost of business cycles is negligible. In other words, Lucas (1987) argues that from a welfare point of view, business cycle research and counter-cyclical stabilization policy are irrelevant. His argument is based on a representative-agent model with no production and "standard" preferences. More specifically, Lucas (1987) assumes that i) there is no uninsurable idiosyncratic risk (complete markets), ii) there is no link between business cycles and economic growth, and iii) preferences allow for a time-additive expected utility representation with moderate degree of (relative) risk aversion. In principle, any one of these three assumptions could be questioned, and an extensive literature has subsequently studied how weakening these assumptions could change the surprisingly strong conclusion drawn by Lucas (1987). In his recent survey, Lucas (2003) summarizes the findings of this literature in the following way: *"But I argue in the end that, based what we know now, it is unrealistic to hope for gains larger than a tenth of a percent from better countercyclical policy"*.

There is considerable empirical evidence that workers face a substantial amount of uninsurable idiosyncratic labor market risk.[1] This paper asks to what extent cyclical variations in uninsurable idiosyncratic labor market risk (incomplete markets) can generate non-negligible cost of business cycles. In accordance with the previous literature, this paper assumes that eliminating business cycles amounts to eliminating the variations in idiosyncratic labor market risk.[2] In contrast to the previous literature, however, this paper focuses on the long-term earnings losses of displaced workers, that is, income losses that persist even after the displaced worker is re-employed. An extensive empirical literature has shown that such earnings

[1] The empirical literature on idiosyncratic labor market risk is surveyed in Section V.

[2] Section II provides a detailed survey of the literature on the cost of business cycles when workers face cyclical variations in labor market risk.

losses are substantial. Moreover, there is also evidence that these long-term earnings losses display a strong cyclical component. In other words, job displacement leaves permanent scars that persist even after the displaced worker finds a new job, and this scarring effect is more pronounced for workers that are laid of during tight labor market conditions. This paper shows that cyclical variations in such long-term earnings losses of displaced workers are likely to generate sizable cost of business cycles.

The analysis conducted in this paper proceeds in two steps. First, using a simple macroeconomic model with incomplete markets, this paper shows that the cost of business cycles can be arbitrarily large even if job displacement rates and the variance of individual income shocks are constant over the cycle. In other words, cyclical variations in the long-term earnings losses can go a long way towards generating sizable cost of business cycles. Moreover, this is true even if, as suggested by the recent empirical literature, job displacement rates and the variance of individual income shocks have no cyclical component.[3] Intuitively, if job displacement is a rare event with devastating long-term consequences for workers that are laid off during recessions, then the cost of business cycles might be very large even if job displacement rates and second moments do not exhibit any cyclical variation.

In addition to the theoretical analysis, this paper also provides a quantitative analysis of the welfare effects of cyclical fluctuations in the long-term earnings losses of displaced workers. More specifically, this paper uses evidence about the long-term earnings losses of displaced U.S. workers obtained by the empirical literature to calibrate the model economy, and then computes the cost of business cycles for the calibrated version of the model. The quantitative analysis shows that realistic variations in the long-term earnings losses of displaced workers generate sizable welfare cost of business cycles. For example, in the baseline economy with log-utility preferences (degree of risk aversion of one), the welfare cost of busi-

[3]For recent evidence on job separation rates, see Hall (2005) and Shimer (2005). For the evidence on the second moments of the distribution of individual income shocks, see the discussion in Section II.

ness cycles for high-tenure workers is around half a percentage point of lifetime consumption, and this cost increases to more than one percentage point if we assume a degree of relative risk aversion of two. In contrast, for the same economy with no cyclical fluctuations in the earnings losses of displaced workers, the welfare cost of business cycles is nil. Moreover, the implied cyclical variations in the second moments of the distribution of individual income shocks are so small that a log-normal view of the world would suggest negligible cost of business cycles.

At this stage, it is worth pointing out that the results reported here do not imply that macroeconomic stabilization policy necessarily leads to sizable welfare gains. More specifically, there are many channels through which business cycles affect the mean of aggregate output, and this paper follows Lucas (1987) by disregarding any such link between business cycles and economic growth.[4] Moreover, in accordance with the previous literature, this paper uses a "black-box" approach to the elimination of business cycles in the sense that it does not explicitly model the interaction between counter-cyclical stabilization policy and the business cycle. More specifically, this paper follows recent contributions to the literature (Krebs 2003a, Krusell and Smith, 1999, Lucas, 2003) and uses the "integration principle" in order to remove cyclical variations in idiosyncratic risk. For the specific case considered in this paper, the integration principle implies that cycle-dependent earnings losses are replaced by cycle-independent earnings losses with the same mean. Despite these limitations, the current analysis seems well-suited for addressing the issue that is the topic of Lucas (2003), namely whether our current theoretical and empirical knowledge suffices to rule out substantial welfare cost of business cycles. As shown in this paper, sizable cost of busi-

[4]For example, the elimination of business cycles might affect the aggregate stock of physical capital (Krusell and Smith, 1999) or human capital (Krebs, 2003a) in the economy. Moreover, business cycles affect the re-allocation of factors across heterogenous production units, which in turn affects aggregate economic growth. See, for example, Haltiwanger (2000) for a recent survey of the theoretical and empirical literature on the growth effects of factor re-allocation.

ness cycles cannot be ruled out once we take into account the long-term earnings losses of displaced workers. In other words, any comprehensive welfare analysis of business cycles ought to take into account the cost of cyclical variations in the long-term earnings losses of displaced workers.

Finally, a comment regarding modeling strategy is in order. The objective of the current paper is to study the welfare effects of business cycles when workers face uninsurable job displacement risk. As it is well-known, equilibria of incomplete-market models with idiosyncratic risk and aggregate shocks are in general difficult to compute and (almost) impossible to analyze theoretically. In order to focus on the main economic issues, the analysis presented here is therefore based on a highly tractable incomplete-market model along the lines of Constantinides and Duffie (1996) and Krebs (2004). The model is simple enough to allow for the derivation of simple and transparent expressions for the welfare cost of business cycles, yet rich enough to establish a tight link between the theoretical earnings process and the empirical literature on job displacement risk.[5]

The paper is organized as follows. Section II surveys the previous literature. Section III develops the model that is used to discuss the effect of job displacement risk on the welfare cost of business cycles. Section IV derives a closed-form expression for the welfare cost of business cycles, and uses these expressions to prove the main theoretical result: cyclical fluctuations in job displacement risk may have an arbitrarily large effect on the cost of business cycles even if there is no employment risk and the second moments of the distribution of income shocks have no cyclical component (proposition 2). Section V provides an extensive review of the empirical literature on job displacement risk. The results of this

[5]An alternative approach would be to use preferences and estimates of the process of individual consumption risk to derive the welfare cost of business cycles. Although in general quite attractive, such an approach has the drawback that the empirical literature on cyclical variations in labor market risk has mainly focused on individual income risk (for an important exception, see Brav, Constantinides, and Gescy, 2002).

literature are used to calibrate the process of job displacement risk, and the calibrated model economy is then used to analyze the quantitative effect of cyclical fluctuations in job displacement risk on the cost of business cycles. Section VI concludes.

II. Previous Literature

We now briefly survey the literature on the cost of business cycles in models with incomplete markets. With the exception of Atkeson and Phelan (1994), the previous literature has focused on quantitative results derived from analyzing calibrated model economies. In other words, general results along the lines of the Proposition have so far been missing in the literature.

Atkeson and Phelan (1994), Imrohoroglu (1989), and Krusell and Smith (1999,2002) all study models of worker unemployment, and assume that cyclical fluctuations in unemployment rates and unemployment durations are the only sources of cyclical variations in idiosyncratic labor market risk, but assume that the earnings of displaced workers fully recover after re-employment at a new job. Thus, they rule out by assumption the type of effect studied here. Gomes, Greenwood, and Rebelo (2001) extent the analysis of unemployment risk and allow for endogenous search. In this case, business cycles may have a positive effect on welfare (option value of search).

The papers by Gomes, Greenwood, and Rebelo (2001) and Krusell and Smith (1999,2002) employ versions of the neoclassical model with aggregate productivity shocks that generate cyclical fluctuations in the aggregate wage, and in this sense these papers take into account cyclical fluctuations in the income losses of displaced workers. However, in the calibrated model economies considered by these authors, the cyclical fluctuations in the aggregate wage have relatively low persistence and small amplitude, and therefore do not generate sizable cost of business cycles.[6]

Beaudry and DiNardo (2001) consider a model of unemployment in which the wage of new hires varies over the business cycle, and argue that these cyclical variations are much more persistent and stronger than the cyclical variations in the aggregate wage. In this sense, the analysis conducted in Beaudry and Pages (2001) is an important fore-runner to the current analysis. However, in contrast to Lucas (2003) and most of the work in the literature, Beaudry and Pages (2001) assume that the elimination of business cycles eliminates all idiosyncratic risk, which clearly violates the integration principle (9). Thus, it is difficult to judge to what extent the welfare cost reported in Beaudry and Pages (2001) are truly cost of business cycle.

Krebs (2003a) and Storesletten, Telmer, and Yaron (2001) discuss the cost of business cycles when individual income shocks are (log)-normally distributed and the variance of these shocks depends on business cycle conditions, but they do not condition on the job displacement event. According to the model analyzed in this paper, log-income changes are only normally distributed after we condition on the individual job displacement event (and business cycles conditions). In a certain sense, the models considered by Krebs (2003a) and Storesletten et al. (2001) are mis-specified. The subsequent analysis will show that this type of mis-specification can lead to a serious under-estimation of the cost of business cycles.

Both Krebs (2003a) and Storesletten et al. (2001) report sizable cost of business cycles for moderate degrees of risk aversion, but their results are based on unrealistically large variations in the second moment of individual income shocks. More specifically, both papers calibrate their models using the estimation results reported in the working-paper version of Storesletten, Telmer, and Yaron (2004), but these estimation results have been heavily

[6] Similarly, the cyclical fluctuations in unemployment durations considered by these authors introduce cyclical variations in the income losses of displaced workers, but these income losses do not persist beyond unemployment.

revised in the version of Storesletten, Telmer, and Yaron (2004) that was finally published. Moreover, Barlevy and Tsiddon (2004) have recently argued that the data do not support the view that income inequality (labor market risk) increases during recessions once long-run trends in earnings inequality are taken into account. Similarly, once we de-trend the estimates of the variance of permanent income changes reported in Meghir and Pistaferri (2004), we cannot reject the hypothesis that labor market risk is constant over the cycle (results are available on request). In short, the cost of business cycles reported in Krebs (2003a) and Storesletten et al. (2001) are based on early estimates of the cyclical variations in the variance of the persistent component of individual income changes, but recent evidence suggests that these early estimates have to be revised.

Finally, Krebs (2003b) and Rogerson and Schindler (2002) study the welfare cost of job displacement risk and also focus on the income losses of displaced workers. However, Krebs (2003b) and Rogerson and Schindler (2002) analyze the welfare gain from eliminating job displacement risk in an economy with constant job displacement risk, whereas the current paper studies the welfare gains from eliminating the *cyclical variations* in job displacement risk (keeping average job displacement risk constant).

III. Model

The model is an incomplete-market version of the Lucas asset pricing model (Lucas, 1978) similar to the one considered in Constantinides and Duffie (1996) and Krebs (2004).[7] It provides a formal approach to the intuitive idea that consumption equals permanent income. The model features ex-ante identical, long-lived households (workers) with homothetic pref-

[7]To simplify the analysis, the current paper does not allow for assets in positive net supply. See Constantinides and Duffie (1996) and Krebs (2004) for an extension of the model to this case. In the appendix, we also consider a version of the model with two groups of workers (low- and high-tenure workers) who face different degrees of job displacement risk.

erences that make consumption/saving choices in the face of uninsurable income shocks. Income shocks are permanent, which implies that self-insurance is an ineffective means to smooth out income fluctuations. Indeed, the economy is set up in a way so that in equilibrium households will not self-insure at all. That is, income shocks translate one-to-one into consumption changes (proposition 1). Notice that the result that permanent income shocks have large effects on consumption does not depend on the assumption that aggregate saving is zero, even though we will make it to simplify the analysis. For example, Krebs (2003a,2003b) considers a production economy with only permanent income shocks (log-income follows a random walk) and ex-ante identical households, and shows again that self-insurance is highly ineffective.[8] Deaton (1991) and Carroll (1997) provide a partial equilibrium analysis of the effect of permanent income shocks on consumption and saving, and also conclude that the main effect of permanent income shocks is to change consumption.

There is strong empirical evidence that individual labor income risk has a substantial permanent (or highly persistent) component,[9] and the empirical estimates of this permanent component will be used in the quantitative section (section IV) to calibrate the model economy. The same empirical literature also provides clear evidence in favor of a substantial transitory component of labor income risk, and in this sense the current model is not consistent with a certain dimension of the data. More specifically, the job displacement event has two effects on the earnings of a displaced worker. First, the worker goes through a period of unemployment with no earnings (the transitory effect). Second, the worker finds a new job, but receives a permanently lower wage (the permanent effect). In the current paper, we

[8]One implication of the random walk assumption is that the cross-sectional distributions of income and consumption diverge. However, Constantinides and Duffie (1996) show how to modify the model by introducing death probabilities so that a stationary distribution of income and consumption always exists. Indeed, they show that by choosing the death probabilities appropriately, the model can match any cross-sectional distribution of income and consumption.

[9]See, for example, Carroll and Samwick (1997), Jacobson, LaLonde, and Sullivan (1993), Meghir and Pistaferri (2004), Ruhm (1991), and Storesletten, Telmer, and Yaron (2004).

disregard the first effect, and only focus on the second effect. Given that the first effect has been extensively studied by the previous literature (Atkeson and Phelan (1994), Imrohoroglu (1989), Krusell and Smith (1999,2002)), this modeling choice seems appropriate.

III.1. Economy

Time is discrete and open ended. Labor income of worker i in period t is denoted by y_{it}. Labor income is random and defined by an initial level y_{i0} and the law of motion

$$y_{i,t+1} = (1+g)(1+\theta_{i,t+1})(1+\eta_{i,t+1})\, y_{it} \ , \tag{1}$$

where g is the (constant) aggregate growth rate of labor income and $\theta_{i,t+1}$ and $\eta_{i,t+1}$ describe shocks to the labor income of worker i.[10] We assume that for each i and t, the two random variable $\theta_{i,t+1}$ and $\eta_{i,t+1}$ are independently distributed. Further, we assume that the sequence of random variables $\{\theta_{it}\}$ is i.i.d. across workers and over time with log-normal distribution function:[11]

$$log(1+\theta_{i,t+1}) \sim N(-\sigma^2/2, \sigma^2) \ . \tag{2}$$

The sequence of random variables $\{\eta_{it}\}$ is also i.i.d. across workers, but not over time. More specifically, we assume that there is an aggregate state process $\{S_t\}$ that is i.i.d., and that the distribution of η_{it} depends on the aggregate state of the economy in the following way:

$$\eta_{i,t+1} = \begin{cases} -d_S & with \ \ probability \ \ p \ \ if \ \ S_{t+1} = S \\ \frac{pd_S}{1-p} & with \ \ probability \ \ (1-p) \ \ if \ \ S_{t+1} = S \ , \end{cases} \tag{3}$$

[10] Notice that we do not allow the growth rate g to vary over the business cycle. Thus, any welfare cost of business cycles reported here are solely due to cyclical variations in *idiosyncratic* labor market risk.

[11] Note that it is standard in the literature (Carroll (1997), Constantinides and Duffie (1996), and Storesletten, Telmer, and Yaron (2001)) to introduce the extra term $-\sigma^2/2$, which ensures that the mean of income growth is independent of σ^2. More precisely, specification (2) implies that $E[\theta_{i,t+1}] = 0$ and $var[\theta_{i,t+1}] = e^{\sigma^2} - 1$ using the standard formula for log-normal distributions. Thus, any increase in σ^2 increases $var[\theta_{i,t+1}]$, but leaves $E[\theta_{i,t+1}]$ unchanged.

The random variable η_{it} is the cyclical component of labor income risk, and we interpret this component as describing job displacement risk. The number d_S is the long-term income loss of a worker who is displaced when the aggregate state is S, and the number p is the corresponding displacement probability. To emphasize our focus on cyclical variations in long-term earnings losses, we have assumed that job displacement rates are constant over the cycle. For simplicity, we have also assumed that all displaced workers experience the same income loss d_S. However, our analysis easily extends to the case in which earnings losses of displaced workers are log-normally distributed with mean d_S and constant variance. Notice that we have also assumed that the worker gains income $\frac{p}{1-p}d_S$ if he is not displaced in order to ensure that the random variable η_{it} has mean zero. Finally, the i.i.d. assumption means that income *changes* associated with the displacement event are unpredictable, which implies that the corresponding income losses are permanent.

The income losses of displaced workers, and in particular its permanent component, have been extensively studied by the empirical literature (see Section V.1. for a discussion of the empirical literature). This literature has shown that such earnings losses are substantial, and that they exhibit a strong cyclical component. In other words, job displacement leaves permanent scars that persist even after the displaced worker finds a new job, and this scarring effect is more pronounced for workers that are laid of during a tight labor market. Although a detailed modeling the reasons for this scarring effect of job displacement is beyond the scope of the current paper, one can conjecture that there are at least two reasons behind the cyclical dependence of the earnings losses. First, workers' skills depreciate during periods of unemployment, and this depreciation effect is more pronounced during recessions when unemployment durations are long. Second, displaced workers may lose occupation-specific human capital when they are forced to accept jobs in a new occupation, and these losses are likely to increase during recessions when job offers are rare.

The random variable θ_{it} is the a-cyclical component of labor income risk, and we interpret it as containing any labor income risk beyond job displacement risk. To relate this variable to the empirical literature, let us take logs in equation (1):

$$log y_{i,t+1} = log y_{it} + log(1+g) + log(1+\theta_{i,t+1}) + log(1+\eta_{i,t+1}) \; . \tag{4}$$

Equation (4) says log-labor income approximately follows a random walk with drift and heteroscedastic error term $\epsilon_{i,t+1} = log(1+\theta_{i,t+1}) + log(1+\eta_{i,t+1})$, which is yet another way of saying that income shocks are permanent. An extensive empirical literature has estimated the parameters of the permanent component of income shocks under the log-normal distribution assumption[12] and the estimates obtained by this literature can be used to find a value of $var((log\, y_{i,t+1} - log\, y_{it}))$, and therefore indirectly a value for $var(log(1+\theta_{i,t+1})) = \sigma^2$ in (2). Notice, however, that even though two recent contributions by Meghir and Pistaferri (2004) and Storesletten et al. (2004) have allowed the variance of log-income changes to depend on the aggregate state S, this literature (in contrast to the literature on job displacement mentioned above) has not conditioned their estimates on the displacement event and has not taken into account any deviations from the log-normal distribution assumption.[13] Clearly, if the specification (4) is correct, then the error term $\epsilon_{i,t+1} = log(1+\theta_{i,t+1}) + log(1+\eta_{i,t+1})$ is not normally distributed (indeed, it is the mixture of two normal distributions with different means), which implies that one of the identifying assumptions of this literature is violated.

Each worker begins life with no initial financial wealth. Workers have the opportunity to borrow and lend (dissave and save) at the risk-free rate r_t. There are no insurance markets

[12]See, for example, Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Storesletten, et al. (2004). Notice that even though Storesletten, et al. (2004) specify the permanent component to be AR(1), they estimate an autocorrelation coefficient close to one (the random walk case).

[13]Geweke and Keane (2000) allow for deviations from the log-normal distribution assumption. In contrast to Meghir and Pistaferri (2004) and Storesletten et al. (2004), however, they do not condition their estimates of the labor income parameters on the aggregate state of the economy and they do not decompose labor income risk into a transitory and a permanent component.

for idiosyncratic labor income risk. In other words, there are no assets with payoffs that, conditional on the aggregate state S, are correlated with either θ_{it} or η_{it}. Thus, the sequential budget constraint of worker i reads:[14]

$$\begin{aligned} a_{i,t+1} &= (1+r_t)a_{it} + y_{it} - c_{it} \\ a_{i,t+1} &\geq -M \ , \ \ a_{i0} = 0 \ . \end{aligned} \tag{5}$$

Here c_{it} denotes consumption of household i in period t and a_{it} his asset holdings (wealth excluding current interest payments) at the beginning of period t. The real number M represents an explicit debt constraint that rules out Ponzi schemes.

Workers have identical preferences that allow for a time-additive expected utility representation:

$$U(\{c_{it}\}) = E\left[\sum_{t=0}^{\infty} \beta^t u(c_{it})\right] \ . \tag{6}$$

Moreover, we assume that the one-period utility function, u, is given by $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, for $\gamma \neq 1$ and $u(c) = log c$ for $\gamma = 1$. That is, we assume that preferences exhibit constant degree of relative risk aversion γ. For given interest rate, households choose a consumption-saving plan that maximizes (6) subject to the budget constraint (5).

In Appendix 1, we derive an equilibrium for the economy outlined so far. In this equilibrium, the interest rate is constant, $r_t = r$, and equilibrium welfare can be written as

$$\begin{aligned} U &= \frac{y_{i0}^{1-\gamma}}{(1-\gamma)\left(1-\beta\,(1+g)^{1-\gamma}\, e^{\frac{1}{2}\gamma(\gamma-1)\sigma^2/2} \sum_S \pi_S \left[p(1-d_S)^{1-\gamma} + (1-p)\left(1-\frac{pd_S}{1-p}\right)^{1-\gamma}\right]\right)} \\ U &= \frac{1}{1-\beta} log\, y_{i0} \\ &+ \frac{\beta}{(1-\beta)^2}\left(log(1+g) - \sigma^2/2 + \sum_S \pi_S \left[p \log(1-d_S) + (1-p) log\left(1 + pd_S/(1-p)\right)\right]\right) \ . \end{aligned} \tag{7}$$

[14] Notice that the analysis remains unchanged if we assume that agents have the opportunity to trade assets whose payoffs depend on the aggregate state S (Krebs, 2004). Our assumption that there are no insurance markets means that one should interpret y_{it} as income after transfer payments from the government.

IV. Cost of Business Cycles: Qualitative Analysis

In this section, we use the welfare formula (7) to discuss the cost of business cycles. We first discuss how to eliminate business cycles. After this discussion, we prove the main theoretical result (proposition).

IV.1. Eliminating Business Cycles

We begin this section with a discussion of how the elimination of business cycles affects the nature of idiosyncratic risk. In our model, the elimination of business cycles amounts to moving from an economy with fluctuations in the aggregate state S (the economy with business cycles) to an economy with constant S (the economy without business cycles). In other words, we are moving from an economy with income risk defined by the S-independent distribution of income shocks θ_i and S-dependent distribution of income shocks η_i to an economy with income risk defined by the S-independent distributions of income shocks $\bar{\theta}_i$ and $\bar{\eta}_i$. The question that arises is how to find the distributions of $\bar{\theta}_i$ and $\bar{\eta}_i$ given the distributions of θ_i and η_i. Following Lucas (1987) and the subsequent literature, we will answer this question without an explicit model of the interaction between macroeconomic stabilization policy and the business cycle.

For economies without uninsurable idiosyncratic risk (complete markets), Lucas (1987) postulates that the elimination of business cycles amounts to replacing all S-dependent economic variables by their mean value. That is, we take the expectations over S. Extending this approach to economies with uninsurable idiosyncratic risk (incomplete markets), we postulate that eliminating business cycles means that we replace all S-dependent economic variables by their expected value with respect to S conditional on the idiosyncratic state of an individual worker. This "integration principle" has been used by several previous authors (Krebs (2003a), Krusell and Smith (1999,2002), and Lucas (2003)). For economic variables that have no S-dependence, the integration principle implies that the elimination of business cycles has no effect. Thus, for the a-cyclical component of individual income shocks we have $\bar{\theta}_i = \theta_i$, and therefore

$$log(1 + \bar{\theta}_i) \sim N\left(-\sigma^2/2, \sigma^2\right) \ . \tag{8}$$

For the cyclical component of income shocks, η_i, the integration principle reads:

$$\bar{\eta}_i = E\left[\eta_i | s_i\right] \ , \tag{9}$$

where $s_i = 0$ if worker i is not displaced and $s_i = 1$ if worker i is displaced. Taking the expectations in (9) yields:

$$\bar{\eta}_i = \begin{cases} -\bar{d} \quad with \ \ probability \ \bar{p} \\ \frac{\bar{p}\bar{d}}{1-p} \quad with \ \ probability \ \ (1-p) \end{cases} , \tag{10}$$

where the earnings losses of displaced workers in the economy without business cycles are given by

$$\bar{d} = \sum_S \pi_S d_S \ . \tag{11}$$

IV.2. Cost of Business Cycles

Equation (10) shows how the elimination of business cycles affect the labor income process. We can use this information in conjunction with our welfare formula (7) to calculate the welfare cost of business cycles. More precisely, we define the welfare cost of business cycles as the number Δ that solves

$$E_0\left[\sum_{t=0}^{\infty} \beta^t u\left((1+\Delta)c_{it}\right)\right] = E_0\left[\sum_{t=0}^{\infty} \beta^t u(\bar{c}_{it})\right] \ , \tag{12}$$

where c_{it} is consumption in the economy with business cycles and $\bar{c}_{it}$ is consumption in the economy without business cycles. That is, we define the welfare cost of business cycles as the percentage of consumption in each date-event that workers have to receive in order to be fully compensated for the cyclical variations in labor income risk. Using the definition

(12) in conjunction with the welfare formula (7), we find the following formula for the cost of business cycles:

$$\Delta = \left(\frac{1-\beta\,(1+g)^{1-\gamma}\,E\left[(1+\theta_i)^{1-\gamma}(1+\eta_i)^{1-\gamma}\right]}{1-\beta\,(1+g)^{1-\gamma}\,E\left[(1+\theta_i)^{1-\gamma}(1+\bar{\eta}_i)^{1-\gamma}\right]}\right)^{\frac{1}{1-\gamma}} - 1 \quad if \;\; \gamma \neq 1 \tag{13}$$

$$\Delta = \frac{\beta}{1-\beta}\left(E\left[log\,(1+\bar{\eta}_i)\right] - E\left[log\,(1+\eta_i)\right]\right) \quad if \;\; \gamma = 1\;.$$

We can evaluate the expectations in (13) and find:

$$\Delta = \left(\frac{1-\beta\,(1+g)^{1-\gamma}\,e^{\frac{1}{2}\gamma(\gamma-1)\sigma^2}\sum_S \pi_S\left[p(1-d_S)^{1-\gamma}+(1-p)\left(1+\frac{pd_S}{1-p}\right)^{1-\gamma}\right]}{1-\beta\,(1+g)^{1-\gamma}\,e^{\frac{1}{2}\gamma(\gamma-1)\sigma^2}\left[p(1-\bar{d})^{1-\gamma}+(1-p)\left(1+\frac{p\bar{d}}{1-p}\right)^{1-\gamma}\right]}\right)^{\frac{1}{1-\gamma}} - 1$$

$$\begin{aligned}\Delta &= \frac{\beta}{1-\beta}\left(plog(1-\bar{d})+(1-p)log\left(1+p\bar{d}/(1-p)\right)\right) \\ &- \frac{\beta}{1-\beta}\left(\sum_S \pi_S\left[plog(1-d_S)+(1-p)log\left(1+pd_S/(1-p)\right)\right]\right)\;.\end{aligned} \tag{14}$$

Several facts about (14) are noteworthy. First, the cost of business cycles is the same for all worker. This is a result of the joint assumption of homothetic preferences and permanent income shocks with a distribution that is independent of workers' characteristics. Second, the welfare cost of business cycles is non-negative: $\Delta \geq 0$. This fact immediately follows from the concavity of the utility function in conjunction with the fact that η_i is a mean-preserving spread of $\bar{\eta}_i$. Thus, cyclical variations in idiosyncratic labor income risk never decrease the welfare cost of business cycles.

Inspection of (14) also suggests that we have $\Delta \to \infty$ when $d_S \to 1$ for some S if $\gamma \geq 1$. That is, for high enough degree of risk aversion, the cost of business cycles becomes arbitrarily large when the income losses of displaced workers during certain macroeconomic conditions (recessions) become arbitrarily large. In the appendix, we show that this result still holds even if job displacement rates and the second moments of the distribution of income changes are (almost) constant. The trick here is to take simultaneously the two limits $d_L \to 1$ and

$p \to 0$, where d_L is the long-term earnings loss of workers who become displaced during times of low aggregate economic activity. The Appendix show that we can take these limits in a way so that the cost of business cycles is growing without bounds, but the second moments of the distribution of income shocks is (almost) not affected. In short, we have the following proposition:

Proposition Suppose the degree of relative risk aversion is large enough: $\gamma \geq 1$. Then there is a process of job displacement risk (with constant job displacement rates) so that i) the cost of business cycles is arbitrarily large and ii) the second moments of the distribution of individual income shocks are (almost) constant over the business cycle. More precisely, denote the second moment of the distribution of individual income shocks by $var\,(y_{i,t+1}/y_{it}|S_{t+1}) = \sigma_y^2(S_{t+1})$ and assume $\gamma \geq 1$.[15] For any real numbers $\epsilon > 0$ and $\bar{\Delta} > 0$, we can find long-term earnings losses of displaced workers, d_S, so that i) the implied cost of business cycles is $\Delta = \bar{\Delta}$ and ii) the implied second moments of the distribution of income shocks satisfy $|\sigma_y^2(S) - \sigma_y^2(S')| < \epsilon$ for all S, S'.

V. Cost of Business Cycles: Quantitative Analysis

In this section, we analyze the quantitative importance of the main theoretical result derived in the previous Section (proposition). To this end, we first discuss the calibration of the model economy (section V.1), and then report the quantitative results (section V.2). The economy we consider in this quantitative part is an extension of the basic model discussed and analyzed in sections III and IV. It features two types of workers, high-tenure and low-tenure workers (low- and high-risk workers), who face different degrees of job displacement risk. The details of the theoretical model with two types of workers are discussed in the

[15] The proposition still holds if we consider the variance of log-income changes, $\sigma^2_{log\,y} \doteq var\,(logy_{i,t+1} - logy_{it}|S_{t+1})$, but for this version of the proposition the condition $\gamma > 1$ is required.

Appendix.

V.1. Calibration

Following the previous literature (Imrohoroglu, 1989, Krebs, 2003a, Krusell and Smith, 1999, and Storesletten et al., 2001) we assume now that there are two aggregate states, $S = L, H$, corresponding to low economic activity (economic contraction) and high economic activity (economic expansion). We further follow the literature and disregard any asymmetry in the business cycle, that is, we assume that on average both aggregate states have the same likelihood of occurences. In our setting without persistence in the aggregate state process, this means that $\pi_L = \pi_H = .5$. We choose the period length to be one year to be consistent with the empirical work on labor market risk (see below). Thus, the choice of $\pi_L = \pi_H = .5$ implies an average duration of both good and bad times of two years, which is also the value considered in Imrohoroglu (1989), Krebs (2003a), and Krusell and Smith (1999,2002). We choose an average growth rate of labor income of $g = .02$.

For the model with two types of workers (the appendix), the process of job displacement risk is defined by the parameters p and d_{sS}, where $S = L, H$ (economic contraction and economic expansion) and $s = l, h$ (low- and high-tenure workers). We choose these parameters so that the model economy matches the first moment of the job displacement rate and the first and second moments of the long-term earnings losses of the corresponding groups of U.S. displaced workers that have been estimated by the empirical literature. That is, we try to match previous estimates of i) the average probability of job displacement, ii) the average long-term earnings loss of displaced workers, and iii) the cyclical variations in the long-term income loss of displaced workers. We now turn to a discussion of the estimates of these moments by the empirical literature.

Earnings Losses of Displaced Workers

There are many studies of the long-term consequences of job displacement for U.S. workers. One of the most detailed studies is Jacobsen et al. (1993), who use longitudinal data on the earnings of high-tenure workers (workers with at least six years of tenure) in Pennsylvania from 1974 to 1986 to estimate the earnings losses of displaced workers. In their restricted sample, they confine attention to workers that are separated from distressed firms, where they define a distressed firm as a firm that experienced an employment contraction of at least 30%. For these workers, they find a very large drop in earnings the year following the displacement event (around 50%). Moreover, and more importantly from the point of view of the current paper, these income losses have a sizable component that is highly persistent. More specifically, even 6 years after separation the earnings of the displaced workers is 25% below the earnings of workers with similar characteristics that have not been displaced. [16] The result that high-tenure, displaced workers experience a permanent earnings loss of about 25% is also consistent with other estimates in the empirical literature. For example, Topel (1990) analyzes individual earnings data from the Panel Study of Income Dynamics (PSID) and estimates that displaced workers with at least 10 years of seniority prior to displacement suffer a permanent income loss of 25%.[17] In a similar vein, Kambourov and Manovskii (2002) find that ten years of occupational tenure increase wages by at least 19 percent.

Ruhm (1991) is another well-known study of the earnings losses of displaced workers. He uses PSID data to estimate these losses, and finds that the long-term earnings losses are

[16] Jacobson et al. (1993) also consider a larger sample of workers that consists of all separated workers including those who quit their job and those who get laid off due to "slack work". For workers who quit, we expect smaller earnings losses than before, and for workers who are laid off because of "slack work", we would expect higher earnings losses (Gibbon and Katz, 1991). Jacobson et al (1993) find that the average earnings losses of all separated workers are smaller than the earnings losses of workers that are displaced because of mass lay-offs, which suggest that the "quit-effect" dominates.

[17] Topel (1990) and others in the literature interpret these results as a confirmation of a human capital theory in which workers invest in firm-specific human capital, but recent work suggest that these losses are mainly due to the destruction of industry-specific (Neal, 1995) or occupation-specific (Kambourov and Manovskii, 2002) human capital. For a theoretical model of job separation with match-specific human capital, see Jovanovic (1979).

substantial ($10-13\%$), but significantly lower than the 25% found by Jacobson et al. (1993). Clearly, the two studies rely on different data sets, and this could be one reason for the difference in results. However, it is more likely that the divergence in results is mainly driven by the fact that Jacobson et al (1993) confine attention to high-tenure workers, whereas Ruhm (1991) does not split his sample. Clearly, any theory of match-specific or occupation-/industry-specific human capital would suggest that high-tenure workers experience large earnings losses upon displacement. This implication finds support in the study by Stevens (1997), who finds significant and large difference between the earnings losses of low- and high-tenure workers. However, even though there is a stark difference between the earnings losses of the two types of workers, it seems to be the case that even low-tenure workers experience substantial long-term losses as a result of job displacement. For example, Lorie and Farlie (2003) report that the young adult workers with only three years of labor market experience suffer a permanent earnings loss of around 10 percent, a loss that is mainly due to forgone earnings growth. Finally, we note that Farber (1997) uses evidence from the Displaced Workers Survey (DWS) and estimates earnings losses of around 15 percent for a sample that again includes both types of workers.

Based on the results described above, we choose a value of 25% for the average earnings losses of hight-tenure workers. That is, we choose the parameters d_{hL} and d_{hH} so that the restriction $.5d_{hL} + .5d_{hH} = .25$ is satisfied. Notice that both Krebs (2003b) and Rogerson and Schindler (2002) use a similar value in their studies of the welfare consequences of job displacement risk. For low-tenure workers, we assume an average earnings loss of 12%, which yields $.5d_{lL} = .5d_{lH} = .12\%$.

Another important issue is the degree to which the long-term earnings losses of displaced workers depend on cyclical conditions. Jacobson et al (1993) find strong evidence in favor of the view that these earnings losses vary over the business cycle, again for their sample of high-tenure workers. More specifically, they define the cyclical labor market condition by the unemployment rate and the deviation from trend employment, and estimate that workers who become displaced during the worst cyclical conditions experience a permanent income loss of 37%, whereas this income loss is only 13% for those workers who experience job displacement during the best cyclical conditions (see table 2 and the corresponding discussion in Jacobson et al., 1993). In other words, the spread is $37\% - 13\% = 24\%$.[18] Clearly, focusing only on the most extreme cyclical conditions is overstating the cyclicality of income losses, but these estimates indicate a very strong cyclical component. Weinberg (2001) considers a sample of displaced workers of all tenure-levels (low- and high-tenure workers), and also finds a very strong cyclical component of the long-term earnings losses. More specifically, he finds that a one-standard deviation increase in industry growth increases post-displacement wages by 4%.[19] Thus, the spread between good and bad cyclical conditions is 8% if we focus on one-standard deviations from the mean and 16% if we consider two-standard deviations from the mean.

Work by Solon, Barsky, and Parker (1994), Beaudry and DiNardo (1991), and Bils (1985) provides additional evidence in favor of the view that earnings losses of displaced workers display a strong cyclical component. More specifically, Barsky and Solon (1989) and Bils (1985) show that after controlling for selectivity bias, aggregate wages decrease by approximately $1 - 1.5\%$ in response to a one percent increase in the unemployment rate. Moreover, and more importantly for the current paper, Beaudry and DiNardo (1991) show that wages

[18]Note that empirical result regarding the cyclical variation of d reported by Jacobson et al. (1993) are derived exploiting differences across local labor markets. Put differently, the result is mainly based on cross-sectional variation, and the time-series inference drawn in this paper is therefore somewhat tentative.

[19]Weinberg (2001) exploits growth-rate differences across industries to identify the relationship between cyclical conditions and earnings losses of displaced workers. Thus, as in the case of Jacobson et al (1993), the business cycle implications of Weinberg's finding are not clear-cut. Moreover, Weinberg (2001) investigates low-frequency shocks to industry growth, whereas the current paper is concerned with short-term fluctuations.

of new hires decrease on average by approximately $3 - 4.5\%$ for every percent increase in the unemployment rate. Thus, assuming a spread of the unemployment rate of 5 percent between economic contractions and economic expansions, the last finding implies a variation of income losses over the cycle of somewhere between 15% and 22.5%.[20] Clearly, this type of evidence is silent about the persistence of income losses, but taken together with the evidence reported in Jacobson et al. (1993) they make a strong case in favor of the view that the permanent earnings losses of displaced workers have a large cyclical component.

Finally, we note that Keane and Wolpin (1997) estimate that the skill-level of white-collar workers depreciates by 30% for each year of unemployment. Combined with the fact that unemployment durations increase during recessions (Blanchard and Diamond, 1990), this finding provides additional evidence for the view that cyclical conditions affect the long-term losses of displaced workers.[21] Notice that this effect goes beyond the transitory effect of unemployment on earnings due to forgone wage payments.

Guided by the above evidence, we assume that for high-tenure workers, the difference in the earnings looses of displaced workers between booms and recessions is 16%. That is, we require that $d_{hL} - d_{hH} = .16$. Combined with the condition $.5d_{hL} + .5d_{hH}$, this yields $d_{hL} = .33$ and $d_{hH} = .17$. For low-tenure workers, we assume a spread of $d_{lL} - d_{lH} = .06$, which combined with the restriction $.5d_{lL} = .5d_{lH} = .12$ yields $d_{lL} = .15$ and $d_{lH} = .09$.

Job Displacement Rates

To complete the calibration of the job displacement process, we need to assign values

[20] Notice that a 5 percent spread in the unemployment rate is consistent with the fluctuations in the U.S. unemployment rate over the last 30 years. Note also that Imrohoroglu (1989) and Krusell and Smith (1999,2002) use a spread in the unemployment rate that is even larger.

[21] Ljungqvist and Sargent (1998) and Pissarides (1992) are two examples of papers in the macroeconomic literature that heavily rely on this skill-depreciation effect of unemployment.

for the job displacement probability p. Jacobson et al (1993) report that in their sample of high-tenure workers, the fraction of workers that experience at least one job displacement event due to mass layoffs is equal to 28 percent over a time span of 13 years (see table 1 in Jacobson et al (1993)). Assuming that the job displacement event is i.i.d. over time, this means that the probability of experiencing job displacement within a particular year is equal to $1 - (1 - .28)^{1/13} = .025$. An annual job displacement probability of 2.5%, however, is likely to be an underestimate of the actual job displacement probability for the following reasons. First, the sample of workers displaced by mass-layoffs constructed by Jacobson et al. (1993) excludes many workers that ought to be counted as displaced workers, namely all those workers who lost their jobs for "exogenous reasons" but worked for firms that did not contract by at least 30 percent . Second, Jacobson et al. (1993) eliminate all displaced workers who subsequently report no earnings, which in their sample is a very large fraction of the total sample. The job displacement rates reported by Farber (1997), which are estimated using the DWS data, avoid these shortcomings, and are in this sense more representative. For male workers of age 35-44, Farber (1997) reports an average job displacement rate of .0384. This is in accordance with the job displacement rates reported by Stevens (1997) using the PSID data. Finally, we note that both the job displacement rates reported in the DWS and the PSID are likely to be under-estimates of the true job displacement probabilities because of recall bias (Topel, 1991). Guided by these results of the empirical literature, we choose an annual job displacement rate of $p = .04$. Note that standard measures of *total* rates of job loss are much larger and lie somewhere in between a *quarterly* rate of $2 - 10\%$ (Hall, 1995).

Other Earnings Risk

To complete the calibration of the labor income process, it remains to find a value for the variance $var\left(log(1 + \theta_{i,t+1})\right) = \sigma^2$. To find this value, notice first that the average variance of log-income changes is

$$\begin{aligned}\sigma^2_{log\,y} &\doteq \sum_S \pi_S\, var\left((log\,y_{i,t+1} - log\,y_{it})|S\right) \\ &= \sigma^2 + \sum_S \pi_S\, var\left(log(1+\eta_{i,t+1})|S\right)\ .\end{aligned} \tag{15}$$

As argued before (see equation 4), the variance term $\sigma^2_{log\,y}$ has been estimated by an extensive empirical literature. More specifically, the empirical literature on labor income risk has often used a random walk specification for the permanent component of labor income (Carroll and Samwick, 1997, Gottschalk and Moffitt, 1994, and Meghir and Pistaferri, 2004), or has used an AR(1) specification and then estimated a serial correlation coefficient close to one (Storesletten et al., 2004). Thus, this literature provide a direct estimate of $\sigma^2_{log\,y}$. All the empirical studies use annual income data drawn from the PSID data set, and estimate an average variance of permanent income shocks between .022 and .033 for the pooled household/worker sample (a standard deviation between .15 and .18). In this paper, we calibrate the income process so that the implied value for $\sigma^2_{log\,y}$ is equal to .022 (a standard deviation of .15). Using the already assigned values for the displacement parameters p and d_S, equation (21) pins down σ^2. Using our previous values, we find $\sigma^2 = .0207$ (a standard deviation of $\sigma = .1438$) using $\eta_{i,t+1}$ of high-tenure workers in (23).

Preference Parameters

We choose the preference parameters as follows. We follow the bulk of the business cycle literature and choose an annual discount factor of $\beta = .96$. In comparison, Imrohoroglu (1989) also chooses $\beta = .96$, but Storesletten et al. (2001) pick $\beta = .95$. Krusell and Smith (1999,2002) assume a stochastic discount factor with a mean of .95. For the degree of relative risk aversion, the standard choice in the macroeconomic literature is $\gamma = 1$ (log-utility), and this is also our choice for the baseline model. However, we also report the results for $\gamma = 1.5$ and $\gamma = 2$. In comparison, Krusell and Smith (1999,2002) use log-utility, Storesletten et al. (2001) focus on $\gamma = 4$, and Imrohoroglu considers the two cases $\gamma = 1.5$ and $\gamma = 6$.

Summary of Parameter Choices

To sum up, for the baseline economy we choose $g = .02$, $\pi_L = \pi_H = .5$, $\sigma^2 = .0207$, $p = .04$, $q = .0312$, $d_{lL} = .15$, $d_{lH} = .09$, $d_{hL} = .33$, $d_{hH} = .17$, $\beta = .96$, and $\gamma = 1$. However, we also calculate the welfare cost for higher degrees of risk aversion $\gamma = 1.5$ and $\gamma = 2$.

V.2. Results

Tables 1 and 2 present the results of our quantitative study. The first column shows the cost of business cycles for different preferences parameters if business cycles are eliminated according to equation (19). If $\beta = .96$ and $\gamma = 1$ (log-utility), which is the standard assumption in the macroeconomic literature, we have a welfare cost of business cycles that is equal to $\Delta = .571\%$ for high-tenure workers and $\Delta = .303\%$ for low-tenure workers. In comparison, for the same preference parameters, Lucas (1987) finds $\Delta = .05\%$ using a representative-agent model. Thus, the cost of business cycles found here is roughly one order of magnitude larger than the ones found in Lucas (1987). Notice also that in the current model there are no fluctuations in aggregate income, so that the cost of business cycles is nil when markets are complete.

The second column shows the cost of business cycles when, incorrectly according to the current paper, the researcher assumes that log-income changes are normally distributed. More precisely, according to the model, the standard deviation of log-income changes, $\sigma^2_{log\,y}$, varies between $\sigma_{log\,y}(L) = .1086$ and $\sigma_{log\,y}(H) = .09862$ for high-tenure workers and $\sigma_{log\,y}(L) = .09713$ and $\sigma_{log\,y}(H) = .09524$ for low-tenure workers. In the fourth column of tables 1 and 2, we consider a labor income process that has, conditional on the aggregate state S, a symmetric two-state support, and choose the support so that we match the given S-dependent standard deviations.[22] The results reported in the fourth column show that

[22] This is the two-state approximation of the normal distribution using the Gauss-Hermite quadrature (Judd, 1998). The welfare results barely change if a four-state approximation is used, which suggests that

the cost of business cycles becomes negligible once the log-normal approach is taken. Put differently, the cyclical variation in income risk as measure by variations in $\sigma_{log\, y}(S)$ are so small that the implied cost of business cycles become negligible. More generally, if the true distribution is not symmetric, then assuming symmetry as in Krebs (2003a) and Storesletten et al. (2001) can lead to a serious under-estimation of the cost of business cycles.

Tables 1 and 2 also show how the cost of business cycles changes when the degree of risk aversion is varied. More specifically, we choose $\gamma = 1.5$ and $\gamma = 2$. The cost of business cycles is increasing and convex in γ . For example, for high-tenure workers the cost of business cycles is .571% if $\gamma = 1$, .887% if $\gamma = 1.5$, and 1.37% if $\gamma = 2$. The convexity of the cost of business cycles as a function of γ has been emphasized by Storesletten et al. (2001).

VI. Conclusion

This paper analyzed the welfare costs of business cycles when workers face uninsurable job displacement risk that has a cyclical component. Using a simple macroeconomic model with incomplete markets, this paper showed that cyclical variations in the long-term earnings losses of displaced workers can generate arbitrarily large cost of business cycles even if job displacement rates and the second moments of the distribution of individual income changes are constant over the cycle. In addition to the theoretical analysis, this paper also conducted a quantitative study of the cost of business cycles using empirical evidence about the long-term earnings losses of displaced U.S. workers. The quantitative analysis suggests that the cost of business cycles due to the cyclical variations in job displacement risk is quite substantial.

In this paper, the process of long-term earnings losses of displaced workers was taken as

the two-state approximation is already very good.

given. Clearly, a more detailed analysis of the economic forces behind the cyclical variations in long-term earnings losses would yield important new insights into the cost of business cycles. Such an analysis would require one to model explicitly the search and saving behavior of unemployed, risk averse workers in a setting with idiosyncratic and aggregate shocks. Moreover, to fully capture the welfare effect discussed here, it seems essential to incorporate skill depreciation during unemployment and/or occupation-specific human capital.[23] We leave such an analysis for future research.

[23] Most papers in the search literature have assumed risk-neutral workers and have neglected skill depreciation or occupation-specific human capital (see Rogerson, Shimer, and Wright (2004) for a recent survey of the literature). Kambourov and Manovskii (2004) and Rogerson (2005) have recently proposed interesting extensions of the search model of Prescott and Lucas (1974) allowing for occupation-specific human capital, but they confine attention to risk-neutral workers and steady state analysis.

Appendix 1

In this appendix, we construct the equilibrium and derive the welfare expression ( ). Notice first that the Euler equations associated with the consumption-saving problem of worker i read

$$c_{it}^{-\gamma} = \beta(1 + r_{t+1})E[c_{i,t+1}^{-\gamma}|F_{it}] \ , \tag{A1}$$

where F_{it} represents the information that is available to household i in period t. In the following, we assume that F_{it} contains any variable that has been realized up to time t. In particular, it contains θ_{it}, η_{it}, and S_t. The Euler equation (A1) says that the marginal utility cost of saving one more unit of the good is equal to the expected marginal utility gain of doing so. In equilibrium, the asset (bond) market must clear. In a closed economy exchange model, this means aggregate saving is zero:[24]

$$\sum_i a_{it} = 0 \ . \tag{A2}$$

Suppose the interest rate is constant and given by:

$$1 + r = \left(\beta\, E\left[\left(\frac{y_{i,t+1}}{y_{it}}\right)^{-\gamma}\right]\right)^{-1} \ . \tag{A3}$$

Evaluating the expectations in (A3) using (2) and (3) yields:

$$r \ = \ \left(\beta\,(1+g)^{-\gamma}\, e^{\frac{1}{2}\gamma(\gamma+1)\sigma^2} \sum_S \pi_S \left[p(1-d_S)^{-\gamma} + (1-p)\left(1 + \frac{pd_S}{1-p}\right)^{-\gamma}\right]\right)^{-1} \ . \tag{A4}$$

Given this interest rate, the Euler equation (A1) is satisfied if workers consume all their income: $c_{it} = y_{it}$.[25] If $c_{it} = y_{it}$, then $a_{it} = 0$ (budget constraint). In Krebs (2004) it is

[24] The notation suggests that there are a finite number of households, but all propositions in this paper remain valid for the case of a continuum of households.

[25] Notice that here we use the fact that θ_{it} and η_{it} do not predict future idiosyncratic shocks to income. That is, we have used the fact that (θ_{it}, η_{it}) and $(\theta_{i,t+1}, \eta_{i,t+1})$ are uncorrelated. Without this assumption, the Euler equation (A1) would not hold at $a_{it} = 0$ and $c_{it} = y_{it}$ for the interest rate (A3).

shown that the consumption-saving plan $c_{it} = y_{it}$ and $a_{it} = 0$ also satisfies a corresponding transversality condition if the interest rate is given by (A4). Moreover, expected lifetime utility is finite if the following condition is satisfied:

$$\begin{aligned} &\beta E\left[\left((1+g)^{1-\gamma}(1+\theta_{i,t+1})(1+\eta_{i,t+1})\right)^{1-\gamma}\right] \qquad (A5) \\ &= \beta\,(1+g)^{1-\gamma}\, e^{\frac{1}{2}\gamma(\gamma-1)\sigma^2} \sum_S \pi_S \left(p(1-d_S)^{1-\gamma} + (1-p)\left(1+\frac{pd_S}{1-p}\right)^{1-\gamma}\right) \\ &< 1 \end{aligned}$$

In short, the plan $c_{it} = y_{it}$ and $a_{it} = 0$ maximizes expected lifetime utility. Clearly, the choice $a_{it} = 0$ also satisfies the market clearing condition (A2). Hence, we have found an equilibrium.

The result that equilibrium consumption equals income,$c_{it} = y_{it}$, means that equilibrium welfare (expected lifetime utility), U, is given by:

$$U \doteq E\left[\sum_{t=0}^{\infty} \beta^t u(c_{it})\right] = E\left[\sum_{t=0}^{\infty} \beta^t u(y_{it})\right] \qquad (A6)$$

Using the preference specification (6) and the definition of the income process (1)-(3), direct calculation yields the following formula for equilibrium welfare:

$$U = \frac{y_{i0}^{1-\gamma}}{(1-\gamma)\left(1-\beta E\left[((1+g)^{1-\gamma}(1+\theta_i)(1+\eta_i))^{1-\gamma}\right]\right)} \quad if \;\; \gamma \neq 1 \qquad (A7)$$

$$U = \frac{1}{1-\beta} log\, y_{i0} + \frac{\beta}{(1-\beta)^2} E\left[log\left((1+g)(1+\theta_i)(1+\eta_i)\right)\right] \quad if \;\; \gamma = 1 \;\; .$$

Using the distributional assumption (2) and (3), we can evaluate the expectations in (A7) and find the welfare formula (7).

Appendix 2

In this appendix, we prove the proposition. We have

$$var\left[y_{i,t+1}/y_{it}|S_{t+1}\right] = var\left[\,(1+g)(1+\theta_{i,t+1})(1+\eta_{i,t+1})|S_{t+1}\right] \qquad (A8)$$

$$= (1+g)^2 var\left[\theta_{i,t+1} + \eta_{i,t+1} + \theta_{i,t+1}\eta_{i,t+1}|S_{t+1}\right],$$

$$= (1+g)^2 \left(var\left[\theta_{i,t+1}|S_{t+1}\right] + var\left[\eta_{i,t+1}|S_{1+1}\right] + var\left[\theta_{i,t+1}\eta_{i,t+1}|S_{t+1}\right]\right)$$

$$= (1+g)^2 \left(var\left[\theta_{i,t+1}|S_{t+1}\right] + var\left[\eta_{i,t+1}|S_{1+1}\right] + var\left[\theta_{i,t+1}|S_{t+1}\right] var\left[\eta_{i,t+1}|S_{t+1}\right]\right) ,$$

where the third and fourth lines follow from $cov\left[\theta_{i,t+1}, \eta_{i,t+1}|S_{t+1}\right] = 0$ and $E\left[\theta_{i,t+1}|S_{t+1}\right] = E\left[\eta_{i,t+1}|S_{t+1}\right] = 0$.

To simplify the notation, we consider the case of two aggregate states: $S = L, H$. We further assume $d_H = 0$. That is, if $S_{t+1} = H$, then $\eta_{i,t+1} = 0$. Thus, for $S_{t+1} = H$ we have:

$$var\left[y_{i,t+1}/y_{it}|H\right] = (1+g)^2 e^{\sigma^2} - 1 \,. \tag{A9}$$

On the other hand, for $S_{t+1} = L$ we have:

$$var\left[y_{i,t+1}/y_{it}|L\right] = (1+g)^2 \left[e^{\sigma^2} - 1 + \frac{pd^2}{1-p} + \left(e^{\sigma^2} - 1\right)\frac{pd^2}{1-p}\right] \,. \tag{A10}$$

Take any $\epsilon > 0$ and d with $0 < d < 1$, and choose $p = \frac{\epsilon}{\epsilon + d^2(1+g)^2 e^{\sigma^2}}$. Clearly, by construction we have $0 < p < 1$ and

$$\begin{aligned} var\left[y_{i,t+1}/y_{it}|L\right] - var\left[y_{i,t+1}/y_{it}|H\right] &= (1+g)^2 \frac{pd^2}{1-p} e^{\sigma^2} \\ &= \epsilon \,. \end{aligned} \tag{A11}$$

That is, the choice of $p = \frac{\epsilon}{\epsilon + d^2(1+g)^2 e^{\sigma^2}}$ ensures that the cyclical variation in the second moments of the distribution of income growth rates is always equal to ϵ, and therefore arbitrarily small. We now show that for any number $\bar{\Delta} > 0$ we can always find a number d with $0 < d < 1$ (and a corresponding number $p = \frac{\epsilon}{\epsilon + d^2(1+g)^2 e^{\sigma^2}}$) so that the implied welfare cost of business cycles is equal to $\bar{\Delta}$.

To simplify notation, introduce $c = (1+g)^2 e^{\sigma^2} > 0$, so that $p = \frac{\epsilon}{\epsilon + c\, d^2}$. Consider the cost of business cycles (13) for $\gamma > 1$ (the argument for the log-utility case is the same). Notice first that the expectations term in (13) can be written as

$$\beta E\left[(1+\theta_{i,t+1})^{1-\gamma}(1+\eta_{i,t+1})^{1-\gamma}\right] = \beta e^{-\frac{1}{2}\gamma(1-\gamma)\sigma^2} f(\epsilon, d) \tag{A12}$$

and

$$\beta E\left[(1+\theta_{i,t+1})^{1-\gamma}(1+\bar{\eta}_{i,t+1})^{1-\gamma}\right] = \beta e^{-\frac{1}{2}\gamma(1-\gamma)\sigma^2}\,\bar{f}(\epsilon,d)\ ,$$

where we introduced

$$f(\epsilon,d) = \pi_L\left(\frac{\epsilon}{\epsilon+cd^2}(1-d)^{1-\gamma} + \left(1-\frac{\epsilon}{\epsilon+cd^2}\right)\left(1+\frac{\frac{\epsilon}{\epsilon+cd^2}d}{1-\frac{\epsilon}{\epsilon+cd^2}}\right)^{1-\gamma}\right) + \pi_H \qquad (A13)$$

and

$$\bar{f}(\epsilon,d) = \frac{\epsilon\pi_L}{\epsilon+cd^2}(1-d\pi_L)^{1-\gamma} + \left(1-\frac{\epsilon\pi_L}{\epsilon+cd^2}\right)\left(1+\frac{\frac{\epsilon\pi_L}{\epsilon+cd^2}d\pi_L}{1-\frac{\epsilon\pi_L}{\epsilon+cd^2}}\right)^{1-\gamma} \ .$$

From the fact that η_i is a mean-preserving spread of $\bar{\eta}_i$, it follows immediately that

$$\bar{f}(\epsilon,d) < f(\epsilon,d) \qquad (A14)$$

for any $\gamma > 1$. Further, for any $\epsilon > 0$, there is a number d with $0 < d < 1$ so that

$$f(\epsilon,d) = \left(\beta e^{-\frac{1}{2}\gamma(1-\gamma)\sigma^2}\right)^{-1} \ . \qquad (A15)$$

The last equation (A15) follows from the continuity of f in conjunction with $f(\epsilon,0) = \pi_L + \pi_H = 1$, $\lim_{d\to -1} f(\epsilon,d) = +\infty$, and the fact that the right-hand-side of (A15) is strictly greater than one by assumption (7). Using (A12)-(A15) in the welfare expression (13) yields the desired result, namely the existence of a d and p so that (A11) is satisfied and the cost of business cycles is equal to $\bar{\Delta}$.

Appendix 3

In this appendix, we extent the model discussed in section III to the case in which there are two different types of workers, low-tenure workers and high-tenure workers, who face different degrees of displacement risk. We further allow for an endogenous choice of hours worked (labor supply).

Suppose that income of worker i in period t is $y_{it} = wh_{it}l_{it}$, where w is the common wage rate per efficiency unit of labor, h_{it} is the human capital stock of worker i, and l_{it} is the number of hours worked. Suppose further that human capital evolves according to

$$h_{i,t+1} = (1+g)(1+\theta_{i,t+1})(1+\eta_{i,t+1})\, h_{it}\ , \tag{A16}$$

where θ_{it} is defined as before, but η_{it} is now given by

$$\eta^{s}_{i,t+1} = \begin{cases} -d_{sS'} & \textit{with probability } \ p \ \textit{ if } \ s_{it} = s \ \textit{ and } \ S_{t+1} = S \\ \frac{pd_{sS}}{1-p} & \textit{with probability } \ (1-p) \ \textit{ if } \ S_{t+1} = S\ , \end{cases} \tag{A17}$$

where $s = l$ if worker i is a low tenure worker in period t and $s = h$ if he is a high-tenure worker in period t. Clearly, high-tenure workers become low-tenure workers whenever they are displaced. That is, the transition probability of moving from $s_{it} = h$ to $s_{i,t+1} = l$ if $S_{t+1} = S$ is equal to p. To keep the model tractable, we assume that low-tenure become high-tenure workers in a stochastic fashion. That is, we assume that the probability of a low-tenure worker becoming a high-tenure worker is the same for all low-tenure workers and equal to q.

As in the basic model, we assume that workers can save at a risk-free rate r. However, in contrast to the previous analysis, we assume that workers cannot borrow. Thus, the modified budget constraint reads

$$a_{i,t+1} = (1+r)a_{it} + y_{it} - c_{it} \tag{A18}$$

$$a_{i,t+1} \geq 0\ \ ,\ \ a_{i0} = 0$$

Finally, preference over lifetime consumption still allow for a time-additive expected utility representation, but now the one-period utility function is given by

$$u(c,l) = \frac{c^{1-\gamma}}{1-\gamma} v(1-l)\ . \tag{A19}$$

We define an equilibrium as in Section III. It is again straightforward to show that $c_{it} = y_{it}$

and $a_{it} = 0$ is an equilibrium allocation if the equilibrium interest rates are given by

$$1 + r \;=\; \left(\beta\, E\left[\left(\frac{y_{i,t+1}}{y_{it}}\right)^{-\gamma} |s_{it} = h\right]\right)^{-1} \qquad (A20)$$

$$= \left(\beta\,(1+g)^{-\gamma}\, e^{\frac{1}{2}\gamma(\gamma+1)\sigma^2} \sum_S \pi_S \left[p(1-d_{hS})^{-\gamma} + (1-p)\left(1 + \frac{pd_{hS}}{1-p}\right)^{-\gamma}\right]\right)^{-1},$$

where we assumed that the high-tenure workers are the high-risk workers.

Given the allocation $c_{it} = y_{it}$, we can use the specification of the labor income process and preferences to compute welfare (expected lifetime utility) for each group of workers. To simplify notation, set $v(1-l^*) = 1$. Notice first that welfare of low- and high-tenure workers has to satisfy the following recursive equation:

$$\begin{aligned} V_h(y_i) \;&=\; \frac{y_i^{1-\gamma}}{1-\gamma} + \beta \sum_S \pi_S\, p\, E\left[V_l\left((1+g)(1+\theta_i)(1-d_{hS})y_i\right)\right] \qquad (A21) \\ &+ \beta \sum_S \pi_S\,(1-p)\, E\left[V_h\left((1+g)(1+\theta_i)(1+\frac{pd_{hS}}{1-p})\, y_i\right)\right] \end{aligned}$$

and

$$\begin{aligned} V_l(y_i) \;&=\; \frac{y_i^{1-\gamma}}{1-\gamma} + \beta \sum_S \pi_S\, p\, E\left[V_l\left((1+g)(1+\theta_i)(1-d_{lS})y_i\right)\right] \\ &+ \beta \sum_S \pi_S\,(1-p)(1-q)\, E\left[V_l\left((1+g)(1+\theta_i)(1+\frac{pd_{lS}}{1-p})\, y_i\right)\right] \\ &+ \beta \sum_S \pi_S\,(1-p)q\, E\left[V_h\left((1+g)(1+\theta_i)(1+\frac{pd_{lS}}{1-p})\, y_i\right)\right] \end{aligned}$$

Equation (A21) can be directly solved by the methods of undetermined coefficients. That is, it is straightforward that $V_s(y_i) = a_s \frac{y^{1-\gamma}}{1-\gamma}$ solves (A21) if and only of the coefficients a_s solve the linear equations system:

$$\left(1 - \beta c \sum_S \pi_S\,(1-p)\left(1 + \frac{pd_{hS}}{1-p}\right)^{1-\gamma}\right) a_h \;-\; \left(\beta c \sum_S \pi_S\, p(1-d_{hS})^{1-\gamma}\right)\; a_l \;=\; 1$$

$$\left(1 - \beta c \sum_S \pi_S\, \left(p(1-d_{lS})^{1-\gamma} + (1-p)(1-q)\left(1 + \frac{pd_{lS}}{1-p}\right)^{1-\gamma}\right)\right) a_l \qquad (A22)$$

$$- \left(\beta c \sum_S \pi_S \, (1-p) q \left(1 + \frac{p d_{lS}}{1-p} \right)^{1-\gamma} \right) \; a_h \; = \; 1 \; ,$$

where the constant c is given by:

$$c \;=\; (1+g)^{1-\gamma} e^{\frac{1}{2}\gamma(\gamma-1)\sigma^2} \; . \tag{A23}$$

Repeating the argument made in section IV, the cost of business cycles for a worker of type s becomes:

$$\Delta_s \;=\; \left(\frac{\bar{a}_s}{a_s} \right)^{\frac{1}{1-\gamma}} - 1 \; , \tag{A16}$$

where (a_l, a_h) is the solutions to (A22) using state-dependent job displacement parameters p_S and d_S (economy with business cycles) and $(\bar{a}_l, \bar{a}_h)$ is the solution to (A22) using state-independent job displacement parameters $\bar{p}$ and $\bar{d}$ (economy without business cycles.

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Table I. Cost of Business Cycles: High-Tenure Workers[26]

	Baseline	log-normal	complete markets
$\beta = .96$, $\gamma = 1$	.571 %	.001 %	0 %
$\beta = .96$, $\gamma = 1.5$	.887 %	.002 %	0 %
$\beta = .96$, $\gamma = 2$	1.370 %	.004 %	0 %

Table II. Cost of Business Cycles: Low-Tenure Workers[27]

	Baseline	log-normal	complete markets
$\beta = .96$, $\gamma = 1$	.303 %	.001 %	0 %
$\beta = .96$, $\gamma = 1.5$	.461 %	.002 %	0 %
$\beta = .96$, $\gamma = 2$	.743 %	.004 %	0 %

[26] Cost of business cycles are expressed as percentage of lifetime consumption. Job displacement process of high-tenure workers. The first column (baseline) assumes a displacement probability of $p = .04$ and corresponding income losses of $d_L = .33$ and $d_H = .17$.

[27] The first column (baseline) assumes a displacement probability of $p = .04$ and corresponding income losses of $d_L = .15$ and $d_H = .09$.