Reexamining the long-run properties of the real interest rate

Violetta Dalla*
Department of Economics, London School of Economics,
Houghton Street, London WC2A 2AE, UK

November 1, 2005

Abstract

In the empirical literature, stationary and nonstationary behavior has been reported for the U.S. real interest rate over different time periods. We examine its long-run properties, through estimation of the order of integration, and interpret the results in light of the Taylor rule for monetary policy. When concentrating on the post-October 1987 period, our analysis suggests that the business cycle component of the real interest rate dominates its trend. Under such a setup, we argue on the basis of a theoretical example and a simulation study that, in finite samples, the estimator of the order of integration recovers information on the strength of the cycle, rather than the trend. For that reason, we extract the cyclical part of the real interest rate, and obtain considerably lower estimates of the order of integration. We therefore conjecture that in the post-October 1987 period, the estimates of the order of integration for the real interest rate are uninformative, and that its degree of long-run dependence has been overestimated.

JEL Classification: C22, E32.
Keywords: Real rate of interest, order of integration, business cycles, local Whittle estimation.

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*I would like to thank my supervisor J. Hidalgo and L. Giraitis for helpful discussions and suggestions. Financial support is given by the Economic and Social Research (ESRC) grant R000239936. E-mail: v.dalla@lse.ac.uk, Tel: +44 (0)20 7955 7501.

1 Introduction

The empirical properties of the real interest rate have received a great deal of attention, as it is a key variable in the conduct of monetary policy and the savings or investment decisions of households and firms. Although economic theory routinely assumes that it is constant or fluctuates in a stationary way, subject to regime changes, nonstationary behavior for the U.S. real interest rate has been reported over particular time periods. The main purpose of this paper is to combine economic theory for the generating process of the real interest rate with an econometric methodology on quantifying the degree of long-run dependence, in order to uncover the causes of the apparent nonstationarity found in the empirical applications. Such an exercise might not only shed light on the gap between the assumptions made on the real interest rate in economic theory and the econometric results obtained so far, but will hopefully bring us a step closer to understanding the long-run statistical properties of this extensively studied process.

Initially, Fama (1975) employed techniques based on the sample autocorrelation function, and inferred that the ex post real interest rate remains approximately constant over the period 1953-1971. In a later study, covering the longer periods 1953-1979 and 1931-1952, Mishkin (1981) used the same methodology as Fama (1975), and rejected the constancy in the ex post real interest rate. Subsequent investigations by Rose (1988) provided evidence in support of $I(1)$ ex post real interest rate over various time periods spanning from 1892 to 1986. However, as the critical values of the Dickey-Fuller test employed by Rose (1988) had often been found misleading in finite samples, see Schwert (1987), Mishkin (1992) performed a similar analysis using critical values from Monte-Carlo simulations. For data spanning from 1953 to 1990 and for different subperiods, Mishkin (1992) inferred that the ex post real interest rate is $I(0)$, in all but the post-1982 period. More recently, by allowing for a noninteger order of integration, Phillips (1998) concluded that the nominal interest rate is integrated of a higher order than inflation and ex post real interest rate, for different subperiods over the 1934-1997 period. The outcome, which was clearly at odds with the Fisher equation, was reexamined later by Sun and Phillips (2004). With the availability of data on inflation forecasts from the Survey of Professional Forecasters, they found that the degree of integration of the ex ante real interest rate had been underestimated, and fell in the nonstationary region. To eliminate this negative bias, they proposed a modified version of the estimator, and furthermore suggested a testing procedure for the null hypothesis of equal order of integration. Using data for the period 1934-1999, they inferred that the three variables in the Fisher equation are integrated of the same order, which was found to fall in the nonstationary region.

In a parallel line of research, Garcia and Perron (1996) reanalyzed the data of Rose (1988) over the period 1961-1986, using regime shift techniques, and found that the ex post real interest is constant subject to regime shifts dating 1973 and 1979. Breaks in the process of the ex post real interest rate have also been reported by Huizinga and

Mishkin (1986), where the dates of the breaks were 1979 and 1982. On the other hand, the results of Judd and Rudebusch (1998) and Clarida, Gali, and Gertler (2000) suggest that the relationship of the ex post real interest rate and inflation shifted in 1979, while that of the ex post real interest rate and output gap g_t in 1987. The latter authors employed the Taylor (1993) rule for their analysis, which has been found to do a fairly accurate job in describing how monetary policy has been conducted.

In short, the empirical literature suggests that, even if one accounts for the possible breaks in the process of the real interest rate, nonstationary behavior cannot be excluded for the post-1982 period. We start by examining the order of integration d of the variables in the Fisher equation using the local Whittle estimator of Künsch (1987). This estimator has the advantage that it only requires specification only of the long-run dynamics of the process through the order of integration d. By allowing for a noninteger order of integration, various types of long-run behavior can be accounted for, including unit root nonstationarity. Estimates of d are of interest, since they enable us to understand if the process is stationary ($d < \frac{1}{2}$), or nonstationary ($d \geq \frac{1}{2}$). In addition, we are able to determine whether shocks to the process of interest are short-lived ($d = 0$), long-lived ($0 < d < 1$), or infinitely lived ($d \geq 1$). As the Taylor rule will play an important role in our analysis, the sample period is broken at 1979 and 1987 according to the results of Judd and Rudebusch (1998) and Clarida, Gali, and Gertler (2000). Our resulting local Whittle estimates of the order of integration are in line with the empirical observations in the literature, in that they suggest stationary behavior in the first two periods and nonstationary behavior in the latter.

However, when concentrating in the post-October 1987 period, and under the principles of the Taylor rule, our preliminary analysis suggests that for the nominal and real interest rates, the trend component is dominated by a cycle, with estimated period very close to the duration of the only business cycle recorded by National Bureau of Economic Research (NBER) in that period. Under such circumstances and when the sample size is not considerably bigger than the period of the cycle, a simple theoretical example and a Monte-Carlo study indicate that the local Whittle estimator recovers information on the strength of the cycle rather than the trend. To that end, we propose a modification of the narrow-band Frequency Domain Least Squares (FDLS) regression, found in Marinucci and Robinson (2001), to extract the cyclical component of the real interest rate. We perform FDLS regression of the real interest rate on output gap, and lower estimates of the order of integration arise for the residuals of this regression. Notice that such a method could be also of interest for the calculation of the natural real interest rate, see Williams (2003).

We thus conjecture that, in the post-1987 period, the estimates of the order of integration for the real and nominal interest rates are uninformative, and their degree of long-run dependence is overestimated. This helps to explain the invalidity of the long-run Fisher effect, and the lack of a cointegrating relationship between the nominal interest and inflation rates, reported in empirical applications. Although we work in the frequency domain where the decomposition into long-, medium-, and short-run comes more naturally, we should add that time domain estimates and test statistics for this sample period are likely to be biased towards a more nonstationary behavior, as a higher estimate of d is generally associated with a slower decay of the sample autocorrelation function.

In Section 2, we discuss integrated process $I(d)$, along with semiparametric estimation of the parameter d, and introduce a cyclical counterpart of $I(d)$ processes. The empirical analysis is performed in Section 3 in conjunction with the Appendix, which includes graphs not presented in this section. Section 4 considers a theoretical example and a Monte-Carlo study, while Section 5 concludes.

2 Integrated processes and estimation method

One of the most popular models for the level of a series is the *Autoregressive Integrated Moving Average* $ARIMA(p,d,q)$ of Box and Jenkins (1971) defined as

$$\phi(L)(1-L)^d x_t = \theta(L)\varepsilon_t, \tag{2.1}$$

where d is a non-negative integer, L is the lag operator, ε_t is a white noise sequence, and $\phi(L)$ and $\theta(L)$ are the autoregressive and moving average polynomials

$$\phi(L) = 1 - \phi_1 L - ... - \phi_p L^p, \quad \theta(L) = 1 + \theta_1 L + ... + \theta_q L^q,$$

respectively, all of whose zeros lie outside the unit circle and $\phi(L)$ and $\theta(L)$ have no zero in common.

Under this specification, when the data are differenced d times, the resulting process is a stationary and invertible $ARMA(p,q)$. The order of integration d is assumed to take integer values, the most common of which are 0 and 1. For $d = 0$ the process collapses to the $ARMA(p,q)$ one, so that x_t is covariance stationary sequence with exponentially decaying autocorrelation function, and any shock to x_t will die out fast (short-run mean reversion). On the other hand, when $d = 1$ the process x_t has a unit root and is nonstationary; shocks to x_t will now be infinitely lived (no mean reversion). Therefore, the oftenly used values of 0 and 1 constitute two rather extreme descriptions of long-run behavior. Allowing the order of integration d in (2.1) to take noninteger values results to more general long-run dependence structure. This class of processes was considered by Adenstedt (1974) and explored by Granger and Joyeux (1980), giving a *Fractional Autoregressive Integrated Moving Average* $FARIMA(p,d,q)$ model.

$I(d)$ processes

We say that the process x_t is integrated of order d, denoted as $x_t \sim I(d)$, if

$$(1-L)^d x_t = u_t, \tag{2.2}$$

where u_t is a covariance stationary process with absolutely summable autocorrelation function. The $ARIMA(p,d,q)$ and $FARIMA(p,d,q)$ models above are special cases of (2.2), if one assumes that u_t is an $ARMA(p,q)$ process, i.e. $\phi(L)u_t = \theta(L)\varepsilon_t$.

The condition $d < \frac{1}{2}$ ensures that the process x_t is covariance stationary. In particular, for $d = 0$ the autocorrelation function of x_t is absolutely summable, while for $0 < d < \frac{1}{2}$ it is not. Hence, for $0 < d < \frac{1}{2}$, x_t is covariance stationary persistent process. For $d \geq \frac{1}{2}$ the process x_t is nonstationary. Also, given a shock to the process of interest, the effect will eventually dissipate for $0 < d < 1$ (long-run mean reversion), but not for $d \geq 1$ (no mean reversion). Note that negative values of d are allowed and can arise in practice when the practitioner overdifferences the data, e.g. if $x_t \sim I(0.75)$ then $\Delta x_t \sim I(-0.25)$. Figure 1 below shows 100 simulated data from (2.2) with u_t a Gaussian i.i.d. sequence with mean 0 and variance 1. It is clear from Figure 1 that different values of d give rise to different long-run dependence structure and that the higher the value of d the less the process is mean reverting, while when d exceeds 1 the process in no longer mean reverting. Thus, we can regard d as a parameter that summarizes the long-run behavior of the process x_t and quantifies it's mean reversion. We now discuss estimation of the parameter d.

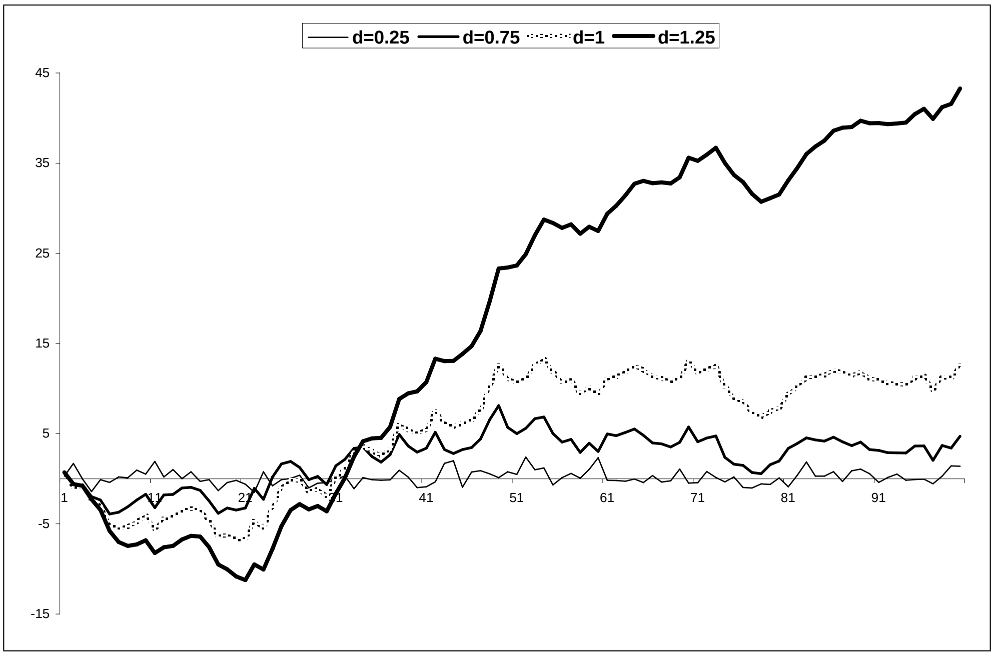

Figure 1: Simulated $I(d)$ data

Estimation method for $I(d)$ processes

There are two main approaches for the estimation of the differencing parameter d, parametric and semiparametric ones. The difference between the two methods is that the first requires the correct specification of u_t in (2.2) up to a finite vector of parameters, e.g. u_t follows an $ARMA(p,q)$ process, while for the latter one only the long-run dynamics of u_t have to modelled. Parametric estimation is superior to the semiparametric one if the correct model is chosen for u_t. However, even if u_t is an $ARMA(p,q)$ process, inconsistent estimates of d arise when the orders p and q are mispecified.

As we are concentrating on the long-term properties of x_t, we chose to estimate d by semiparametric methods in order to avoid the consequences of mispecifying the short- and medium-term dynamics of x_t. Various semiparametric estimators of d have been developed in the literature, among others the well known local Whittle estimator and log-periodogram estimator of Künsch (1987) and Geweke and Porter-Hudak (1983) respectively, the exact local Whittle estimator of Shimotsu and Phillips (2005) and the averaged periodogram of Robinson (1994). In the analysis to follow, we focus on the local Whittle (LW) estimator $\widehat{d}$ defined as the minimizer

$$\widehat{d} = \arg \min_{[-\frac{1}{2},\frac{1}{2}]} U_n(d) \tag{2.3}$$

of the objective function

$$U_n(d) = \log\left(\frac{1}{m}\sum_{j=1}^{m}\frac{I(\lambda_j)}{\lambda_j^{-2d}}\right) + \frac{1}{m}\sum_{j=1}^{m}\log\left(\lambda_j^{-2d}\right), \tag{2.4}$$

where $\lambda_j = \frac{2\pi j}{n}$, $j = 1,..,n$ denote the Fourier frequencies,

$$I(\lambda) = \frac{1}{2\pi n}\left|\sum_{t=1}^{n} x_t e^{it\lambda}\right|^2 \tag{2.5}$$

is the periodogram of the data $\{x_1,...,x_n\}$ and $m = m_n$ is the bandwidth parameter for which we assume throughout that

$$m \to \infty \quad \text{and} \quad m = o(n), \quad \text{as } n \to \infty. \tag{2.6}$$

The objective function (2.4) is a local approximation of the frequency domain analogue of the Gaussian maximum likelihood. Most of the semiparametric estimators of d are based in the frequency domain, recalling that the long-run is associated with the zero frequency. The objective function is local in the sense that only the m closest Fourier frequencies to zero are used, contrary to parametric methods that employ the full band of Fourier frequencies in $[0,\pi]$. Semiparametric methods are essentially based on a vanishingly small fraction of frequencies as the sample size increases, resulting to estimators that have slower rates of convergence compared to those based on a correct fully specified model. The objective function is based on the Gaussian maximum likelihood although Gaussianity of the process x_t is not required, as we now briefly discuss.

The asymptotic properties of the estimator $\widehat{d}$ are well established. In the case of Gaussian and linear sequences, Robinson (1995) showed the consistency of the estimator $\widehat{d}$ when $-\frac{1}{2} < d < \frac{1}{2}$. More recently, Dalla, Giraitis, and Hidalgo (2004) extended the result of Robinson (1995) to nonlinear models including signal plus noise process, nonlinear functions of a Gaussian process and stochastic volatility models. For Gaussian and linear processes, Velasco (1999) proved the consistency of $\widehat{d}$ when $\frac{1}{2} \leq d < 1$, and showed that a tapered version of the estimator is consistent for $d \geq 1$. A uniform approach to the estimation of d was introduced recently by Abadir, Distaso, and Giraitis (2005). The authors provide a modified version of $\widehat{d}$, referred to as the fully extended local Whittle (FELW), and for all $-\frac{3}{2} < d < \infty$ they show consistency of the FELW estimator for linear and signal plus noise type of processes.

In Section 3 below, we first estimate the order of integration d in the levels of the series using the LW estimator in (2.3). In case that the estimates hit the $\frac{1}{2}$ upper bound in (2.3), first differences of the data are taken. The order of integration d_Δ of the differenced data is estimated and for the levels of the data we calculate the estimated order of integration as $d = d_\Delta + 1$. The results are also compared with the FELW estimates, which do not require any transformation of the data, and no significant differences are found. From our empirical analysis, the bandwidth parameter m is chosen to take all values $[n^{0.5}], ..., [n^{0.8}]$, where $[.]$ denotes the integer part.

As it was mentioned above, our estimation method is based on a shrinking neighborhood of the zero frequency. However, for small sample sizes this neighborhood of the zero frequency is not small enough and could include information from the business cycle frequencies band. In the case that the cyclical components are strong enough, the estimator of the strength of the trend $\widehat{d}$ could retrieve information on the degree of persistence of the cycle instead, as a simple theoretical example and several Monte-Carlo experiments below suggest. Before we conclude this section we introduce the periodic analogue of $I(d)$ processes for the purpose of modelling cyclical behavior.

$\boldsymbol{I_\omega(d^\omega)}$ processes

When $d > 0$, the process (2.2) exhibits a strong acyclical (zero frequency) component. However, we can generalize the definition in (2.2) to allow for strong cyclical components. We say that the process x_t is integrated of order d^ω at the frequency ω, denoted as $x_t \sim I_\omega(d^\omega)$, if

$$(1 - 2\cos(\omega)L + L^2)^{d^\omega} x_t = u_t, \tag{2.7}$$

where u_t is a covariance stationary process with absolutely summable autocorrelation function, e.g. u_t is an $ARMA(p, q)$ process and model (2.7) is referred to as the *Gegenbauer Autoregressive Moving Average* $GARMA(p, d, q)$, see Gray, Zhang, and Woodward (1989). Notice that for $\omega = 0$ and $d^\omega = \frac{d}{2}$ the model (2.7) collapses to the (2.2) one. The process x_t is covariance stationary for $d^\omega < \frac{1}{2}$ when $\omega \in (0, \pi)$, and for $d^\omega < \frac{1}{4}$ when $\omega \in \{0, \pi\}$. Recall that the relationship between period T of the cycle and frequency ω of the cycle is that $T = \frac{2\pi}{\omega}$. If for example the data are quarterly and exhibit a strong seasonal component, then $\omega = \frac{\pi}{2}$, while if the data are quarterly and exhibit a strong cyclical component of 10 years, then $\omega = 0.15$. Figure 2 below shows 100 simulated data from (2.7) with $\omega = 0.15$ and where u_t is a Gaussian i.i.d. sequence with mean 0 and variance 1. It is evident from Figure 2 that the higher the value of d^ω the stronger the cyclical behavior is.

Also, notice the difference between the $I(d)$ data in Figure 1 and the $I_\omega(d^\omega)$ data in the figure below. In the simulation of the data, we have set $d^\omega = \frac{d}{2}$. The main difference between the two cases, is that we have $\omega = 0$ in Figure 1, while for Figure 2 we set $\omega = 0.15$.

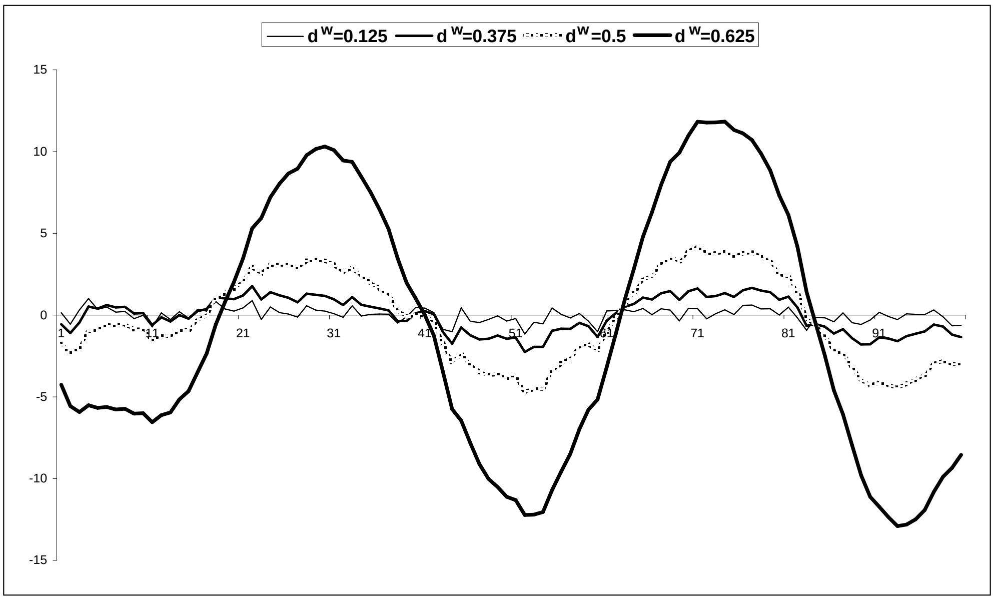

Figure 2: Simulated $I_\omega(d^\omega)$ data

3 Data and empirical results

The ex ante real interest rate r_t^e at time t on a given security maturing at time $t+1$ is defined from the Fisher (1930) equation

$$i_t = \pi_t^e + r_t^e, \tag{3.1}$$

where i_t is the nominal interest rate on the given security issued at period t, maturing in period $t+1$, and π_t^e is the expected inflation rate between periods t and $t+1$. The empirical analysis of the ex ante real rate of interest r_t^e is complicated by the factor that r_t^e is not directly measurable as expected inflation π_t^e is not observed. One way to overcome this problem is to obtain proxies for expected inflation from some survey, e.g. from the Survey of Professional Forecasters as in Sun and Phillips (2004).

The most common approach is to use data on the realized inflation rate π_t and calculate the ex post real interest rate r_t from

$$i_t = \pi_t + r_t. \tag{3.2}$$

Under rational expectations, the forecasting error $\varepsilon_t = \pi_t - \pi_t^e$ will be a martingale difference. In the estimation of the order of integration of the expected inflation rate π_t^e, ε_t is oftenly assumed to be integrated of order 0. In such a case, the ex post and ex ante real interest rates differ by an $I(0)$ process so that if r_t^e is an $I(d)$ process with some $d > 0$, then r_t is an $I(d)$ process as well. That is, the ex post and ex ante real interest rates have the same long-run properties and one can make inference on the long-term behavior of the unobserved r_t^e using data on the observed r_t. This line of reasoning has been adopted by various authors including Fama (1975), Mishkin (1981, 1992), Rose (1988) and Phillips (1998). However, as Nelson and Schwert (1977) and Sun and Phillips (2004) among others pointed out, when the variance of the forecasting error ε_t is large compared to the variance of the ex ante real interest rate r_t^e, the degree of persistence of the ex post real rate of interest r_t is masked by the short-run variability of its component ε_t, so that in finite samples the sample autocorrelation function and the estimates of d based on r_t are likely to be smaller than those based on r_t^e.

The LW estimator $\widehat{d}$ is no exemption to this effect. Using quarterly data from the Survey of Professional Forecasters[1] we employ one-period ahead forecasts as a measure of expected inflation, as in Sun and Phillips (2004), and compare this series with two series for realized inflation calculated as the percent change in the price level between two subsequent quarters based on CPI and GDP deflator data[2]. Figure 11 in the Appendix plots the data, all in annual rates, for the period 1981Q4 to 2004Q4, when the data from the Survey of Professional Forecasters are available. It is clear from Figure 11 that the three series have the same long-run tendencies and that the measure of expected inflation is the least volatile, while the realized inflation based on CPI data is the most. One would then expect the estimates of the order of integration of the three series to be similar. However, as Figure 12 in the Appendix shows, the estimates of the order of integration are rather different, and they are higher the less volatile the associated forecasting error ε_t is. Therefore, when calculating the ex post real interest rate r_t we use the inflation rate based on the GDP deflator data since for this rate the estimates of d are less biased than those based on the CPI data. For reasons to become clear below, we study the long-run behavior of the real interest rate mainly in the post-October 1987 period. For this sample period, given the availability of data from the Survey of Professional Forecasters, we present estimation results for the ex ante real interest rate r_t^e using the expected inflation rate as calculated above.

[1]The Survey of Professional Forecasters is found at http://www.phil.frb.org/econ/spf/.

[2]CPI data are taken from U.S. Department of Labor: Bureau of Labor Statistics, Series I.D. CPIAUCSL, http://stats.bls.gov. GDP deflator data are taken from U.S. Department of Commerce: Bureau of Economic Analysis, Series I.D. GDPDEF, http://www.bea.gov.

The data are quarterly time series spanning the period 1960Q1-2004Q4. We use the average Federal Funds rate in the first month of each quarter, expressed in annual rates, as the nominal interest rate i_t[3]. The inflation rate π_t is calculated as the annualized rate of change of the GDP deflator between two subsequent quarters. Figure 3 below plots the data along with the resulting ex post real interest rate r_t.

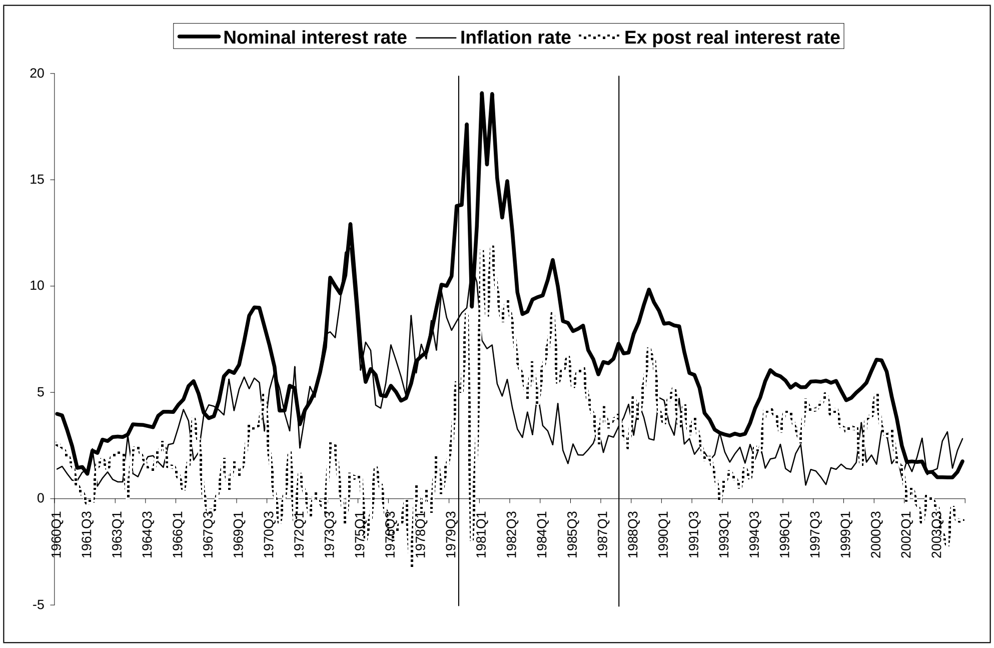

Figure 3: Data on nominal interest rate i_t, inflation rate π_t, and ex post real interest rate r_t for the period 1960Q1-2004Q4

Various authors have reported shifts in the process of the real interest rate. For example, Huizinga and Mishkin (1986) identified 1979 and 1982 as shift periods in the ex post real rate of interest r_t. Garcia and Perron (1996) on the other hand identified 1973 and 1982 as the dates of the shifts. The vertical lines in Figure 3 stand for the dates 1979Q3 and 1987Q3 when Paul Volcker and Alan Greenspan were appointed Chairman of the Board of Governors of the Federal Reserve System. The sample has been split into three subperiods: the pre-Volcker period (1960Q1-1979Q2), Volcker period (1979Q3-1987Q2) and Greenspan period (1987Q3-2004Q4). The results of Judd and Rudebusch (1998) and Clarida, Gali, and Gertler (2000) suggest that the relationship of nominal interest rates i_t and inflation π_t shifted with the appointment of Volcker as the Chairman of the Board of Governors of the Federal Reserve System. The authors also reported shifts in the relationship of nominal interest rate i_t and output gap g_t with the appointment of Greenspan as the Chairman of the Board of Governors of the Federal Reserve System. In particular, the results of Judd and Rudebusch (1998) suggest that the Volcker

[3]Federal Funds rate data are taken from Board of Governors of the Federal Reserve System, Series I.D. FEDFUNDS, http://www.federalreserve.gov.

period involved a response to the change in output gap g_t rather than its level as in the Greenspan period, while Clarida, Gali, and Gertler (2000) found in some cases an insignificant relationship between the nominal interest rate i_t and output gap g_t for the Volcker period, but a significant one for the Greenspan period.

Figures 13-18 in the Appendix plot the nominal interest i_t, inflation π_t and ex post real interest rate r_t data along with the estimates of their order of integration for the three subperiods. For the first two subsamples, the estimates of the order of integration of i_t and π_t are of similar magnitude and are both higher than that of r_t. On the contrary, for the last subperiod, the estimates of the order of integration of π_t is lower than that of both r_t and i_t. Also, notice that in the first two subperiods the estimates of the order of integration of r_t fall in the stationary region, while for the last subperiod in the nonstationary region. Similar observations can be made for the ex ante real interest rate r_t^e in the last subperiod, as Figures 19-20 in the Appendix show. Overall, the estimates of the differencing parameter of i_t tend to exceed that of both π_t and π_t^e.

It is rather surprising that for the Greenspan period the estimates of the differencing parameter of i_t, r_t and r_t^e exceed 1. A value of d greater or equal to 1 would impose no upper bound on the path of these processes, see Figure 1. However, it is clear from Figures 13 and 15 that i_t, r_t and r_t^e are not explosive series, so that our estimates of d are far from realistic. In addition, it is reasonable to assume that any shocks to these processes will eventually dissipate, so that one would anticipate estimates below 1. Notice also from Figure 15 that, the nominal interest i_t and expected inflation π_t^e rates seem to exhibit similar long-run tendencies. Then we would expect estimates of d for the nominal interest rate i_t to be on the same level as the estimates of d for expected inflation π_t^e, or lower if we take into account that i_t looks more volatile than π_t^e. However, the estimates of d are overall higher for i_t than those for π_t^e, raising doubts as to whether i_t can be cointegrated with π_t^e and therefore with π_t.

Here, we seek to provide an explanation for the instability of the estimates of the order of integration of the real interest rate over the three different sample periods. We start by considering a monetary policy reaction function in the lines of Taylor (1993), Judd and Rudebusch (1998) and Clarida, Gali, and Gertler (2000). According to the principles of the Taylor rule, the level of the nominal interest rate is a linear function of the gaps between expected inflation and output and their perspective target levels. Therefore, we can write

$$i_t = i^* + \beta(\pi_t^e - \pi^*) + \gamma g_t + \varepsilon_t,$$

where i^* and π^* are the target levels of the nominal interest rate and inflation, respectively, and ε_t is the error term capturing the short-run dynamics of the nominal interest rate i_t. Hence, the ex ante real interest rate r_t^e will be determined by

$$r_t^e = a + (\beta - 1)\pi_t^e + \gamma g_t + \varepsilon_t,$$

where $a = i^* - \beta\pi^*$.

Figure 4 plots the nominal interest rate i_t, expected inflation π_t^e, ex ante real interest rate r_t^e and output gap g_t over the period 1987Q3-2004Q4, where we have calculated output gap as the percent deviation between actual GDP and its corresponding target[4]. The shaded areas in Figure 4 correspond to recessions according to the NBER business cycle chronology. The duration of the only business cycle recorded in this period is 128 months, calculated either from peak to peak or from trough to trough. This duration corresponds to a period of approximately 42 quarters, giving a frequency $\omega = 0.15$ (recall that the relationship between period T and frequency ω is $\omega = \frac{2\pi}{T}$).

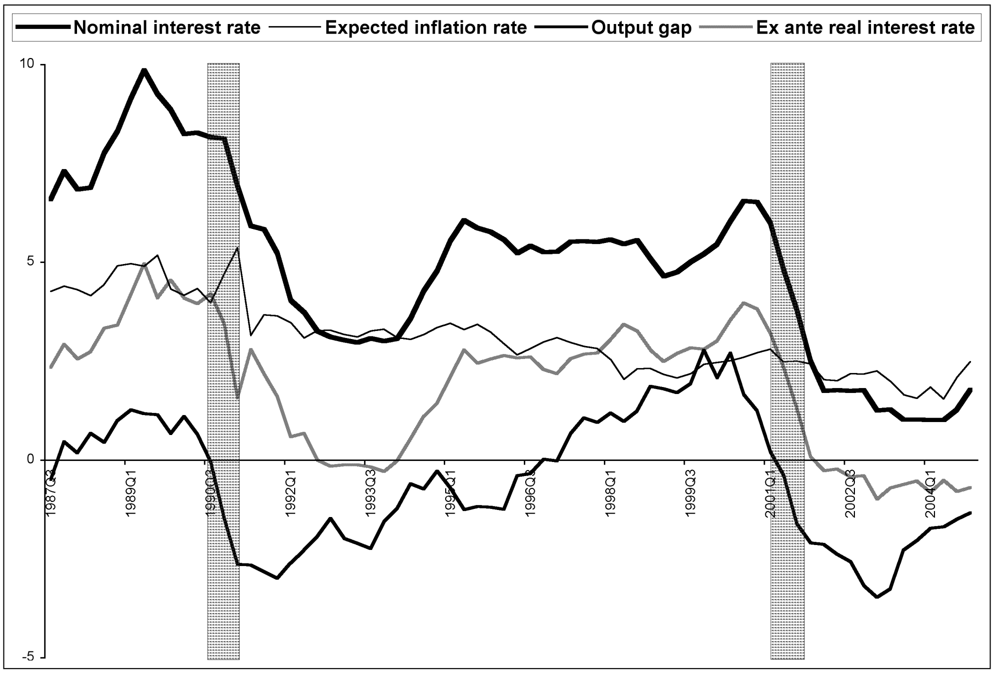

Figure 4: Data on nominal interest rate i_t, expected inflation rate π_t^e, ouput gap g_t, and ex ante real interest rate r_t^e for the period 1987Q3-2004Q4

Looking at Figure 4 we draw our attention to the cyclical behavior of the nominal i_t and ex ante real interest ratee r_t^e, which appears to resemble that of output gap g_t but does not seem to be followed by expected inflation π_t^e. This observation can also be made from Figure 21 in the Appendix of the sample autocorrelation function of the series. Notice in addition from Figure 3 above that, the cyclical pattern in the ex post real interest rate r_t is only evident in the post-October 1987 period. The different shape of the sample autocorrelation function of the ex post real interest rates r_t over the three subperiods, see Figure 22 in the Appendix, also suggest this. A closer look at Figure 4

[4]GDP data are taken from U.S. Department of Commerce: Bureau of Economic Analysis, Series I.D. GDPC96, http://www.bea.doc.gov. Potential GDP data are taken from U.S. Congress: Congressional Budget Office, Series I.D. GDPPOT, http://www.cbo.gov.

indicates that in this sample period, most of the variability of the nominal i_t and real interest rate r_t seems to be explained by the cyclical component rather than the trend. To support this argument, we consider estimating the frequency of the most persistent acyclical/cyclical component using the estimator of Yajima (1996). This estimator has been further explored by Hidalgo and Soulier (2004), where the latter authors suggested in addition a modification to improve its performance in small samples. Both the original estimator of Yajima (1996) and the modified version proposed by Hidalgo and Soulier (2004) recover $\widehat{\omega} = 0$ for π_t and π_t^e, and $\widehat{\omega} = 0.18$ for i_t, r_t, r_t^e and g_t. This would imply that most of the variability of π_t and π_t^e is explained by the trend, while for i_t, r_t, r_t^e and g_t by a cycle of estimated frequency $\widehat{\omega} = 0.18$. Notice that the estimated frequency $\widehat{\omega} = 0.18$ is almost the frequency of the latest business cycle recorded by NBER which is found to be $\omega = 0.15$, as discussed above.

We found that for i_t, r_t, r_t^e and g_t the business cycle dominates the trend of these series, in the sense that most of the variability is explained by the cycle rather than the trend. The sample size is 70 and the cycle has period 42. Given any finite sample, the long-run has duration equal to the sample size. Hence, for the time period considered, the "long-run" has duration less than double of that of the cycle. Notice that $\omega = 0.15$ corresponds approximately to just the second Fourier frequency λ_2 used in the estimation procedure (2.3). According to our findings in the next section, the LW estimator $\widehat{d}$ could be subject to significant positive bias in data where the cycle dominates the trend and whose sample size is not big enough to distinguish between the two. Therefore, one cannot exclude that the high value of the estimates of d for i_t, r_t and r_t^e during the period 1987Q3-2004Q4 is misleading and is due to the strong cyclical behavior of these series rather than a strong trend.

To further explore this conjecture, we consider extracting the cyclical component of the ex ante real interest rate r_t^e. As the ex ante real interest rate r_t^e and output gap g_t seem to co-move, possibly with a lag, over the cycle, see Figure 4 above, we propose regressing the ex ante real interest rate r_t^e on output gap g_t over the business cycle frequencies. If the estimated order of integration of the residuals of this regression is found to be lower than that of the ex ante real interest rate r_t^e, we have further evidence to support our conjecture that the observed high estimated value of the order of integration of r_t^e is due to the presence of a strong cycle rather than a strong trend.

We introduce a modification of the narrow-band frequency domain least squares (FDLS) estimator initially suggested by Robinson (1994) and further explored in Marinucci and Robinson (2001). For generic sequences x_t and y_t suppose that

$$y_t = \alpha + \gamma x_t + \varepsilon_t, \quad t = 1, .., n. \tag{3.3}$$

Denote $w_x(\lambda) = \sum_{t=1}^{n} x_t e^{it\lambda}$, $w_y(\lambda) = \sum_{t=1}^{n} y_t e^{it\lambda}$, $I_{x,y}(\lambda) = w_x(\lambda) w_y^*(\lambda)$ and $I_{x,x}(\lambda) = w_x(\lambda) w_x^*(\lambda)$, where $*$ indicates complex conjugation. The FDLS estimator of γ in (3.3) is given by

$$\widehat{\gamma}_{m'} = \frac{\sum\limits_{j=s+1,..,s+m'} I_{x,y}(\lambda_j)}{\sum\limits_{j=s+1,..,s+m'} I_{x,x}(\lambda_j)}, \tag{3.4}$$

where $\lambda_j = \frac{2\pi j}{n}$, $j = 1, .., n$ are the Fourier frequencies, s is such that λ_s is the closest Fourier frequency to ω, and $m' = m'_n$ is the bandwidth parameter for which it is assumed that

$$m' \to \infty \quad \text{and} \quad m' = o(n), \quad \text{as } n \to \infty. \tag{3.5}$$

Robinson (1994) introduced the estimator (3.4) with $s = 0$ ($\omega = 0$) for the purposes of cointegration analysis. The consistency of the estimator has been shown by Robinson (1994) and Marinucci and Robinson (2001), where their main assumption is that x_t and y_t are $I(d)$ processes, while the error term ε_t follows an $I(d_\varepsilon)$ process with $d > d_\varepsilon$. We should add that for consistency of the estimator (3.4) orthogonality of the regressors and the errors is not required even in the stationary case. The estimation procedure is carried out over a degenerate band of frequencies around zero, i.e. long-run information is being used. In our case, $s = 2$ ($\omega = 0.15$) so that a shrinking band of frequencies around the frequency of the dominant cycle are used. If one was to assume that x_t and y_t are $I_\omega(d^\omega)$ processes, while the error term ε_t is an $I_\omega(d^\omega_\varepsilon)$ process with $d^\omega > d^\omega_\varepsilon$, the proof of consistency of the estimator (3.4) for the case $s \neq 0$ should be along the same lines as for the case $s = 0$.

We first calculate $corr(r^e_{t+k}, g_t)$ for all $k = -4, ..., 4$ and find the maximum value

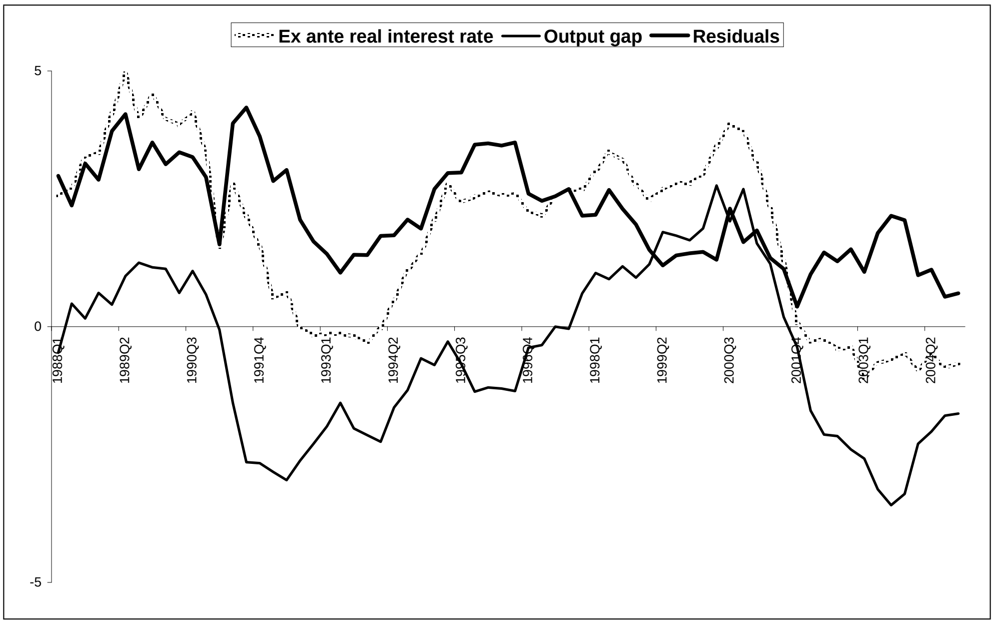

Figure 5: Data on ex ante real interest rate r^e_t, ouput gap g_{t-2}, and the residuals of the FDLS regression r^e_t on g_{t-2} for the period 1987Q3-2004Q4

for $k = 2$. Hence, we set r_t^e as the dependent variable and g_{t-2} as the regressor in (3.3). We calculate the estimates of $\widehat{\gamma}_{m'}$ for all $m' = 2, ..., 14$, where we set $s = 2$. The estimates $\widehat{\gamma}_{m'}$ are rather stable and vary in the interval $[0.79, 0.84]$. We calculate the mean of these values as the estimate of γ, which is found to be equal to 0.8. Figure 5 above plots the residuals from this regression along with the ex ante real interest rate r_t^e and output gap g_{t-2}. Notice that for the residuals the cyclical component is subdued, but not completely removed. This amounts probably to the fact that the relationship between output gap g_t and the ex ante real interest rate r_t^e is not symmetric over recoveries and recessions. As the ex ante real interest rate r_t^e and the residuals seem to have the same long-run tendency, one would expect similar estimates of the order of integration for these two series. On the contrary, we obtain estimates of d for the residuals much smaller than those of the ex ante real interest rate r_t^e, see Figure 6 below. Also, the estimates of d for the residuals are below 1 and are slightly lower than those of expected inflation π_t^e. The results further supports our conjecture that the observed high value of d for the ex ante real interest rate r_t^e is not due to a strong trend but rather due to a strong cyclical behavior.

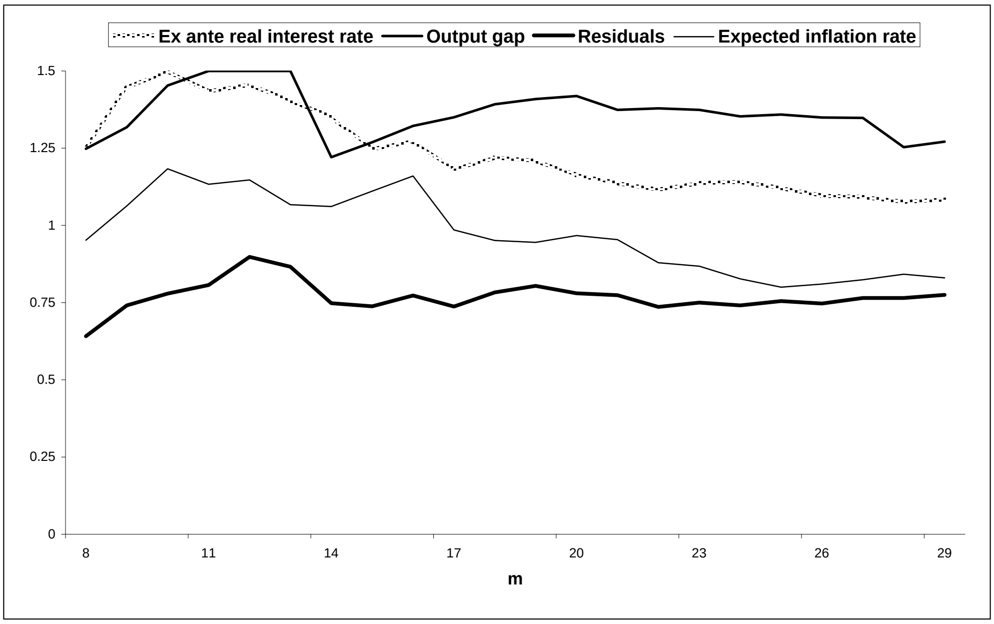

Figure 6: Estimates of d for the data in Figure 5 including that of expected inflation rate π_t^e

We should add that the same analysis has been performed for the post-1982 period. The results are similar, but not as strong and clear as in the post-October 1987 period. This is expected as for the post-1982 period the trend component of the ex ante real interest rate r_t^e seems to be stronger than in the post-October 1987 as the ex ante real interest rate r_t^e never exceeds the value of 5 in the post-October 1987 sample but reaches values of 6.5 in the post-1982 sample.

4 A theoretical example and Monte-Carlo Simulations

Our preliminary analysis in the previous section suggested that the nominal and real interest rates along with output gap exhibit a cyclical component of frequency ω that dominates the trend. The period of the cycle is almost half the sample size, so that $\omega \approx \lambda_2 < \lambda_m$ for all m's considered in practical applications. Consequently, the cycle of frequency ω is included in the calculation of our objective function (2.4), as in our attempt to estimate the trend we use information from all frequencies $\lambda_1, ..., \lambda_m$. Then, one would expect that the inclusion of the cycle will have little effect on the estimation of d when the cycle is weaker than the trend, but substantial one when the cycle dominates the trend. Below we present a theoretical example and Monte-Carlo simulations that support this view. We should add here that if the sample size n and the bandwidth parameter m is such that $\lambda_m < \omega$, then the results of Robinson (1995) suggest that $\widehat{d} \xrightarrow{p} d$, as $n \to \infty$, whether or not the cycle dominates the trend.

4.1 Theoretical example

We consider a simple trend/cycle decomposition for x_t :

$$x_t = \mu_t + c_t + \eta_t \tag{4.1}$$

where μ_t is the trend, c_t is the cycle and η_t is the short-run noise. We assume that μ_t follows an $I(d)$ process with $d > 0$, while c_t is taken to be an $I_\omega(d^\omega)$ process with $d^\omega > 0$ and $\omega \neq 0$, where u_t in both (2.2) and (2.7) is taken to be a Gaussian white noise sequence. The process η_t is assumed to be a Gaussian white noise. $I(d)$ are commonly used to model the long-run. On the other hand, the modelling of the cycle as an $I_\omega(d^\omega)$ process is questionable, as it imposes periodic and symmetric cyclical behavior, while the business cycle is asymmetric and recurrent but not periodic. However, as this example is an illustration of the possible problem rather than a unified theory, we hope that such parameterization will suffice for our purposes. The assumption of Gaussianity and i.i.d. short-run noise is made for simplicity as the consistency of the estimator $\widehat{d}$ has been examined for linear processes and for a wide class of nonlinear models, see Robinson (1995) and Dalla, Giraitis, and Hidalgo (2004).

We further assume that the processes μ_t, c_t and η_t are stationary processes with well defined spectral densities f_μ, f_c and f_η, although our results are likely to follow for the nonstationary case, if one considers the FELW estimator of Abadir, Distaso, and Giraitis (2005). For the proof of the proposition below, we use results from Dalla, Giraitis, and Hidalgo (2004). In their signal plus noise decomposition, the dependence structure between the signal and the noise is not restricted. For simplicity we consider the case that μ_t, c_t and η_t in (4.1) are uncorrelated from each other, although our results are likely to hold for more general situations. Finally, we should add that we treat ω as known, as it seems to be in our empirical analysis. In any case, according to the results of Hidalgo and Soulier (2004), we expect the estimation of the order of integration to be unaffected by whether the true or estimated value of ω is being used or not.

Proposition 1 *Let the processe x_t be such that $x_t = \mu_t + c_t + \eta_t$, where μ_t is an $I(d)$ Gaussian process, c_t is an $I_\omega(d^\omega)$ Gaussian process, η_t is Gaussian white noise sequence, and the processes μ_t, c_t and η_t are uncorrelated from each other. If furthermore, $0 < d < \frac{1}{2}$, $0 < d^\omega < \frac{1}{4}$ with*

$$2d^\omega > d,$$

and ω is such that

$$\omega/\lambda_m \to 0, \quad \text{as } n \to \infty,$$

then

$$\widehat{d} \xrightarrow{p} 2d^\omega, \quad \text{as } n \to \infty. \tag{4.2}$$

Proof. We provide a heuristic argument for the proof of (4.2).

It can be easily shown that the spectral densities f_μ, f_c and f_η satisfy

$$\begin{aligned} f_\mu(\lambda) &= |\lambda|^{-2d} g_\mu(\lambda), \quad \text{for } -\pi < \lambda \leq \pi, \\ f_c(\lambda) &= \left|\lambda^2 - \omega^2\right|^{-2d_\omega} g_c(\lambda), \quad \text{for } -\pi < \lambda \leq \pi, \\ f_\eta(\lambda) &= \frac{1}{2\pi}, \quad \text{for } -\pi < \lambda \leq \pi, \end{aligned}$$

for some functions g_μ and g_c such that $0 < g_\mu(\lambda), g_c(\lambda) < \infty$ for all $-\pi < \lambda \leq \pi$.

Since the processes μ_t, c_t and η_t are uncorrelated from each other, it follows that the spectal density f_x of x_t satisfies

$$\begin{aligned} f_x(\lambda) &= f_\mu(\lambda) + f_c(\lambda) + f_\eta(\lambda) \\ &= |\lambda|^{-2d} g_\mu(\lambda) + \left|\lambda^2 - \omega^2\right|^{-2d_\omega} g_c(\lambda) + \frac{1}{2\pi}. \end{aligned}$$

If ω were equal to 0, then the spectral density f_x would satisfy

$$f_x(\lambda) = c_{x,0} \, |\lambda|^{-4d_\omega}, \quad \text{as } \lambda \to 0, \tag{4.3}$$

since $2d^\omega > d$.
Since $\{x_t\}$ is a Gaussian process, the results of Dalla, Giraitis, and Hidalgo (2004) would then imply that

$$\widehat{d} \xrightarrow{p} 2d^\omega, \quad \text{as } n \to \infty.$$

We assume that $\omega/\lambda_m \to 0$, as $n \to \infty$. Then expression (4.3) holds as before, and using similar arguments as in the case $\omega = 0$, (4.2) would follow. ■

It is clear from Proposition 1, than when $2d^{\omega} > d$, the estimator of the strength of the trend (as measured by d) is fooled into estimating twice the strength of the cycle (as given by d^{ω}). Hence, under such circumstances, positive bias in the estimation of d is likely to arise in finite samples. The bias will be bigger, the larger the difference between d and $2d^{\omega}$ is. Notice that when $d \geq 2d^{\omega}$, we conjecture that the results of Dalla, Giraitis, and Hidalgo (2004) apply so that $\widehat{d} \xrightarrow{p} d$, as $n \to \infty$.

4.2 Monte-Carlo simulations

To illustrate our theoretical findings, we carry out a series of Monte-Carlo experiments. $5,000$ replications of sample size $n = 128$ were generated by truncating at $5,000$ terms the $MA(\infty)$ representations of $I(d)$ and $I_{\omega}(d^{\omega})$ models,

$$y_t \sim I(d) : (1 - L)^d y_t = u_t$$

and

$$z_t \sim I_{\omega}(d^{\omega}) : (1 - 2\cos(\omega)L + L^2)^{d^{\omega}} z_t = u_t^{\omega}.$$

We set $\omega = 0.15$ according to our findings in Section 3. The terms u_t and u_t^{ω} are drawn as Gaussian white noises independent from each other, so that any $I(d)$ process will be independent of any $I_{\omega}(d^{\omega})$ one. Throughout our theoretical results we consider $I_{\omega}(d^{\omega})$ processes for modelling cyclical behavior. Such series oscillate in the long-run. To investigate whether short-lived cyclical behavior at frequency $\omega = 0.15$ affects the finite-sample properties of the estimator, we also generate stationary $AR(2)$ processes with appropriate autoregressive parameters ϕ_1 and ϕ_2. As long as $\cos\omega = \frac{\phi_1(\phi_2 - 1)}{4\phi_2}$ and $\phi_2 < 0$, the simulated series will have a damped oscillatory form with frequency as required. The innovations in the $AR(2)$ processes are Gaussian white noise and are independent of the error term in the $I(d)$ process. All $I(d)$, $I_{\omega}(d^{\omega})$ and $AR(2)$ processes are taken to have variance 1.

The bandwidth parameter m covers all values from $[n^{0.5}]$ to $[n^{0.8}]$. Notice that for all these m we have $\omega << \lambda_m$. Our main purpose is to show how neglecting cyclical effects of frequencies of this type can give rise to substantial finite-sample bias in the local Whittle estimator. We should add that its standard deviation is very similar across all models considered here and is therefore not reported. Also, the results are presented only for sample size $n = 128$. We have conducted the experiments for $n = 256$ as well. The graphs that follow look very similar in both cases, and as expected the bias and standard deviation are reduced with the increase in the sample size. To conclude this section and understand our findings in the time domain, we graph the sample autocorrelation function of simulated data following some of the models below.

Experiment 1

We start by considering the effect of an additive persistent cyclical process $I_{\omega}(d^{\omega})$ to the persistent aperiodic process $I(d)$. According to the theoretical results, the estimator $\widehat{d_x}$

should estimate d when $d > 2d^{\omega}$ and $2d^{\omega}$ when $d < 2d^{\omega}$. Since our models are of the signal plus noise type, we expect that the estimates are going to be subject to negative bias as the signal and the noise processes are by construction independent, see Dalla, Giraitis, and Hidalgo (2004). We analyze the following models:

$$\begin{aligned} &Model\ 1 : X \sim I(0.25) \\ &Model\ 2 : X \sim I(0.25) + I_{0.15}(0.05) \\ &Model\ 3 : X \sim I(0.25) + I_{0.15}(0.125) \\ &Model\ 4 : X \sim I(0.25) + I_{0.15}(0.2) \end{aligned}$$

The graph below shows the finite-sample values of the estimator $\widehat{d_x}$ across the bandwidth parameter m.

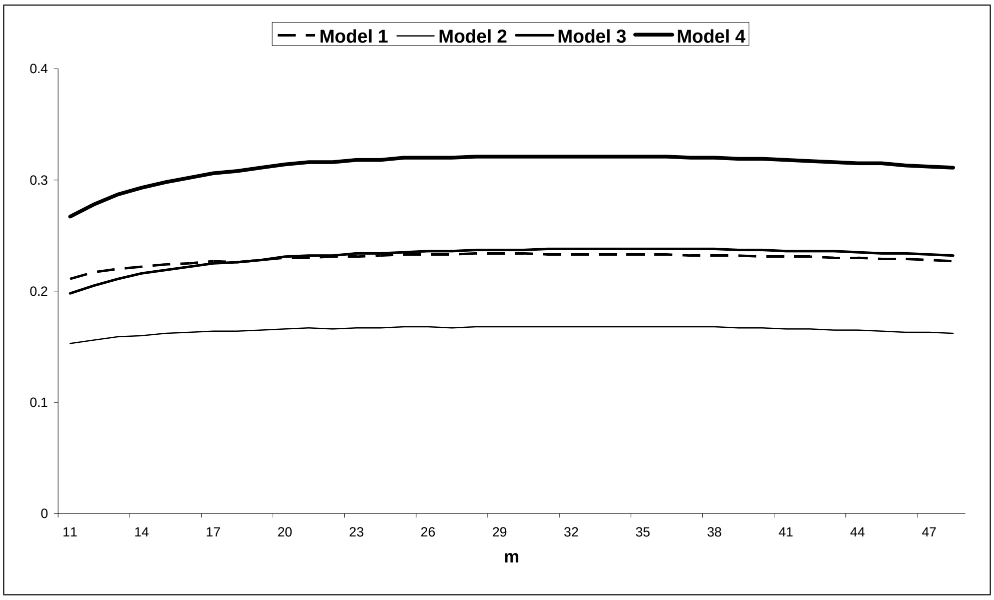

Figure 7: Estimates of d in Experiment 1

Model 1 is our benchmark model and for which the results are in line with those of Robinson (1995). In *Model* 2 the cyclical component is not strong as $d > 2d^{\omega}$. The estimates underestimate $d = 0.25$ since $I_{0.15}(0.05)$ acts as an additive noise to the signal $I(0.25)$. In *Model* 4 the role of the processes is reversed. As the cycle dominates, $d < 2d^{\omega}$, the process $I_{0.15}(0.2)$ becomes the signal. The estimator will then try to estimate 2×0.2 instead of 0.25. Notice also that the additive noise, now associated with $I(0.25)$, produces negative bias which probably explains why the estimates under *Model* 4 fluctuate around 0.3 instead of 2×0.2. The two processes under *Model* 3 have persistent components of the same magnitude. Neither of the series act as an additive noise and so the results are almost the same as under *Model* 1. Finally, we notice that the estimates are rather stable over m.

Experiment 2
In this experiment, we consider a scenario similar to the previous one but allowing for an additive cyclical component of an $AR(2)$ type. We chose a set of values for the $AR(2)$ parameters that guarantee stationarity and pseudocyclical behavior at frequency $\omega = 0.15$. We consider the following cases:

$$\begin{aligned}
&Model\ 1 : X \sim I(0.25) \\
&Model\ 5 : X \sim I(0.25) + AR(2) \text{ with } \phi_1 = 0.359 \text{ and } \phi_2 = -0.1 \\
&Model\ 6 : X \sim I(0.25) + AR(2) \text{ with } \phi_1 = 0.659 \text{ and } \phi_2 = -0.2 \\
&Model\ 7 : X \sim I(0.25) + AR(2) \text{ with } \phi_1 = 0.913 \text{ and } \phi_2 = -0.3
\end{aligned}$$

The following graph presents the finite-sample results for the estimator $\widehat{d}_x$.

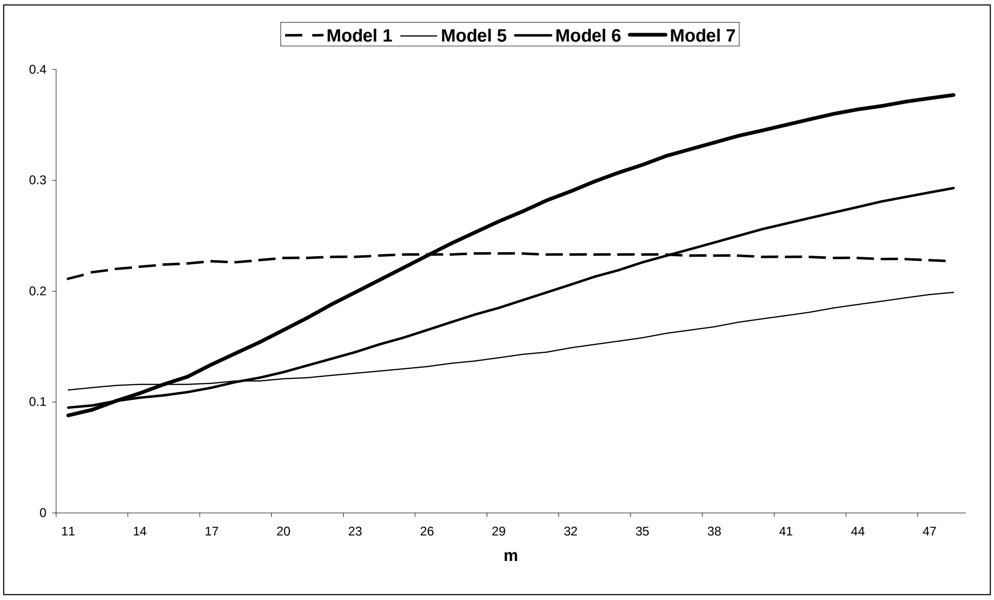

Figure 8: Estimates of d in Experiment 2

Model 1 is again our benchmark model and it is clear from Figure 8 that the additive cyclical $AR(2)$ process produces some element of finite-sample bias. For small bandwidth parameter m, the estimator underestimates $d = 0.25$. However, for large m we observe positive bias when the $AR(2)$ parameters are high enough, that is under *Models* 6 and 7.

Experiment 3
We now turn to the problem of applying the estimator $\widehat{d}$ to data that do not follow an $I(d)$ process, but rather an $I_\omega(d^\omega)$ one. From our theoretical results we expect that in finite samples, $\widehat{d}_x$ will estimate $2d^\omega$ instead of 0. We examine the following models:

$$\begin{aligned}
&Model\ 8 : X \sim I(0) \\
&Model\ 9 : X \sim I_{0.15}(0.05)
\end{aligned}$$

$$Model\ 10 : X \sim I_{0.15}(0.125)$$
$$Model\ 11 : X \sim I_{0.15}(0.2)$$

The graph that follows illustrates the finite-sample estimates.

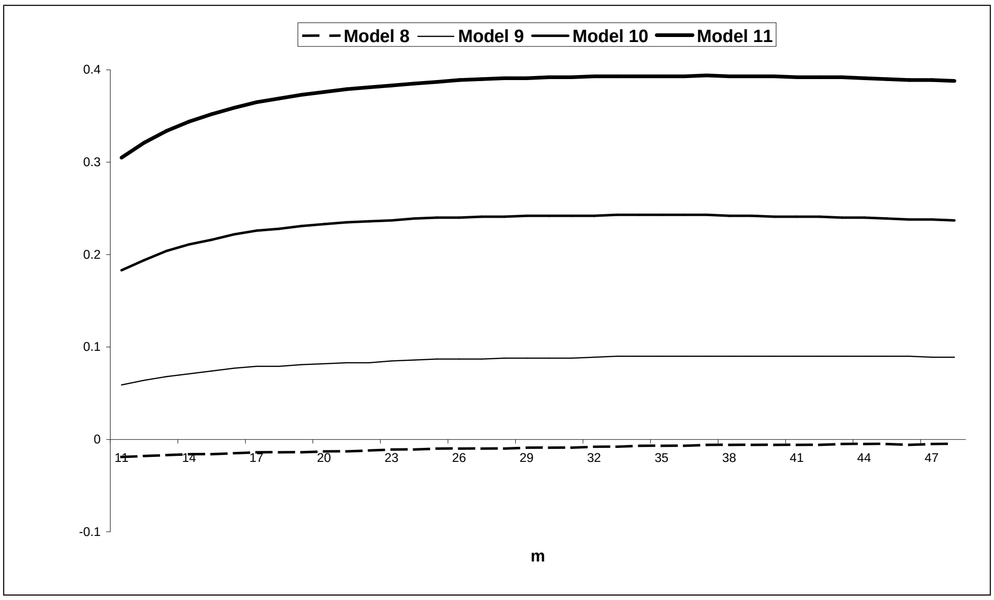

Figure 9: Estimates of d in Experiment 3

Model 8 is now our benchmark model and for which the results are in line with those of Robinson (1995). Our expectations are confirmed since comparing the results from *Models* $9-11$ to that of *Model* 8, it is evident that the estimates are just below $2d^{\omega}$ in each case. We also notice that the estimates are stable over the bandwidth parameter m.

Experiment 4

In this last experiment we repeat our Experiment 3 with a pseudocyclical $AR(2)$ process instead of an $I_{\omega}(d^{\omega})$ one. A set of values for the $AR(2)$ parameters are chosen so that stationarity and pseudocyclical behavior at frequency $\omega = 0.15$ are guaranteed. We consider the following models:

$$Model\ 8 : X \sim I(0)$$
$$Model\ 12 : X \sim AR(2) \text{ with } \phi_1 = 0.359 \text{ and } \phi_2 = -0.1$$
$$Model\ 13 : X \sim AR(2) \text{ with } \phi_1 = 0.659 \text{ and } \phi_2 = -0.2$$
$$Model\ 14 : X \sim AR(2) \text{ with } \phi_1 = 0.913 \text{ and } \phi_2 = -0.3$$

The following graph shows the finite-sample values of the estimator $\widehat{d}_x$.

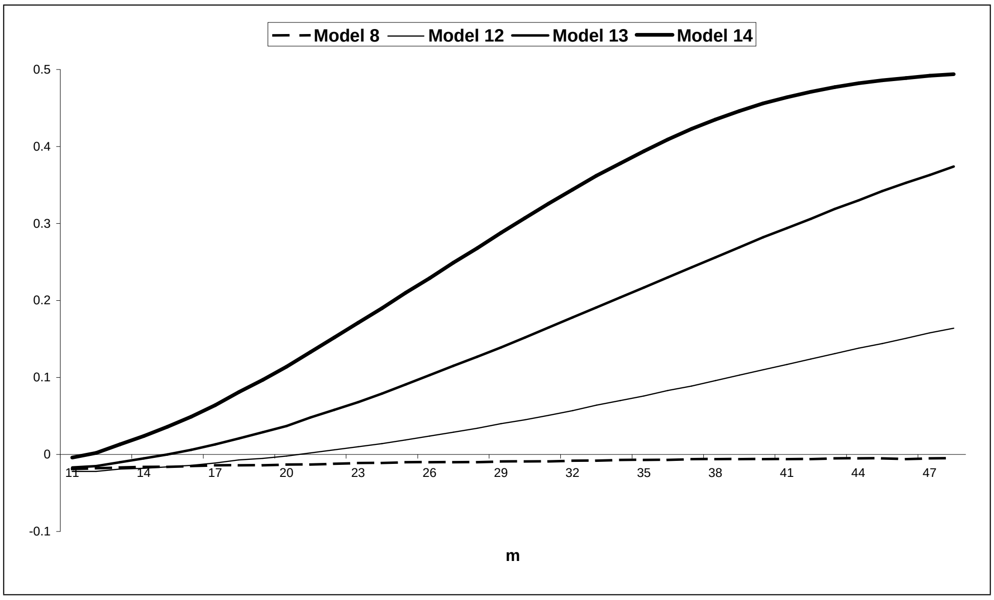

Figure 10: Estimates of d in Experiment 4

Comparing the results of *Models* $12 - 14$ to that of *Model* 8, it is clear that in finite samples, the estimator $\widehat{d_x}$ suffers from substantial positive bias. The bias is stronger the higher the $AR(2)$ coefficients are and the bigger the bandwidth parameter is.

5 Conclusions

In this paper, we attempt to explain the strong degree of nonstationarity that has been reported for the U.S.A. real interest rate in the recent time period. Through an extensive empirical analysis we find that, for the post-October 1987 period, the trend of the nominal and real interest rates is dominated by the business cycle, although this is not the case for inflation. As the length of the sample is not big enough to distinguish between the cycle and the trend, we show that in case of a symmetric cycle, estimates of the order of integration might recover twice the strength of the cycle rather than that of the trend. Furthermore, we provide a simple narrow-band regression method for extracting cyclical components. We observe a substantial reduction of the order of integration in the residuals from the regression of the real interest rate on output gap, which further supports that the degree of integration for the real interest rate has been overestimated. This method might also be of interest for the calculation of the natural real rate of interest, which we hope to examine in a future paper. Although our analysis is based only on estimation of the order of integration, we believe that the results are strong enough to let us conjecture that the degree of stationarity/nonstationarity of the nominal and real interest rates has been overestimated. This would help explain why estimated order of integration for the nominal and real interest rates are found to exceed that of expected inflation, and why the long-run Fisher effect and the cointegrating relationship between nominal interest and inflation rates have failed to be established.

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Appendix

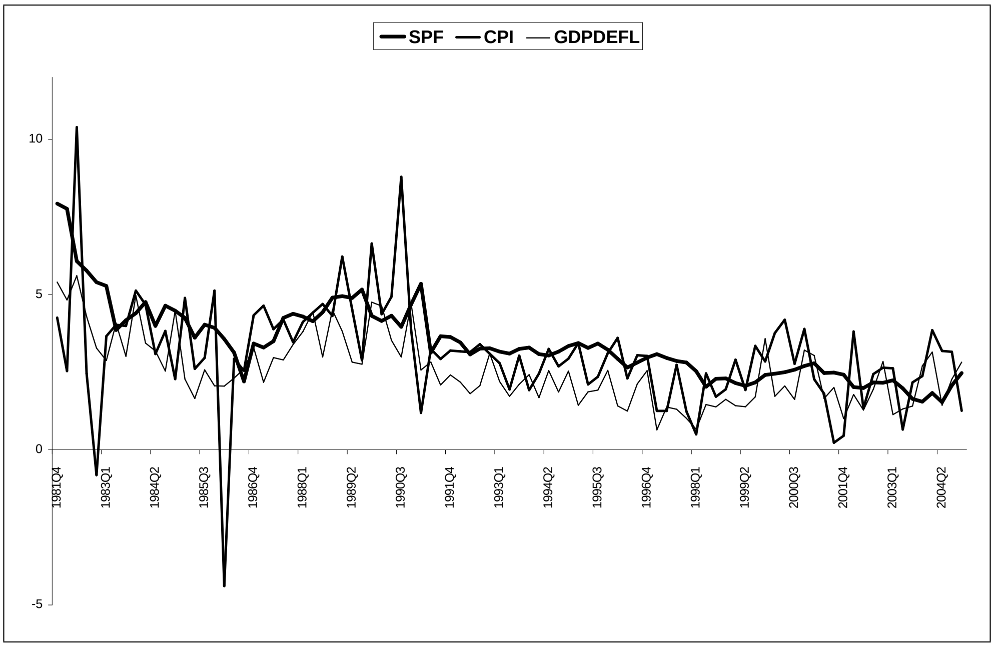

Figure 11: One expected inflation and two realized inflation series: SPF is expected inflation from the Survey of Professional Forecasters, CPI is inflation calculated from the Consumer Price Index, GDPDEFL is inflation calculated from the GDP deflator

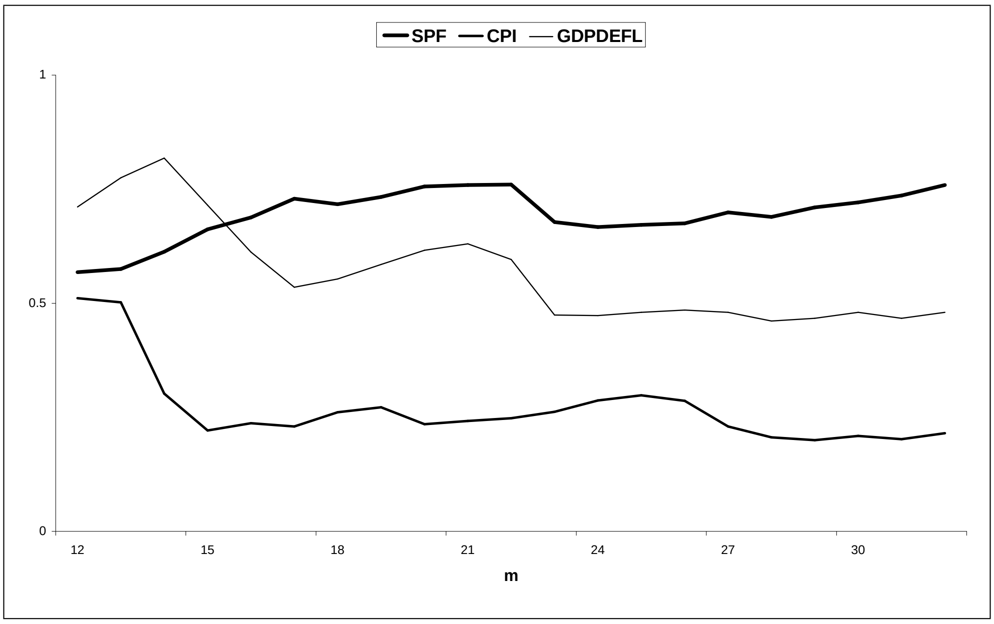

Figure 12: Estimates of d for the series in Figure 11

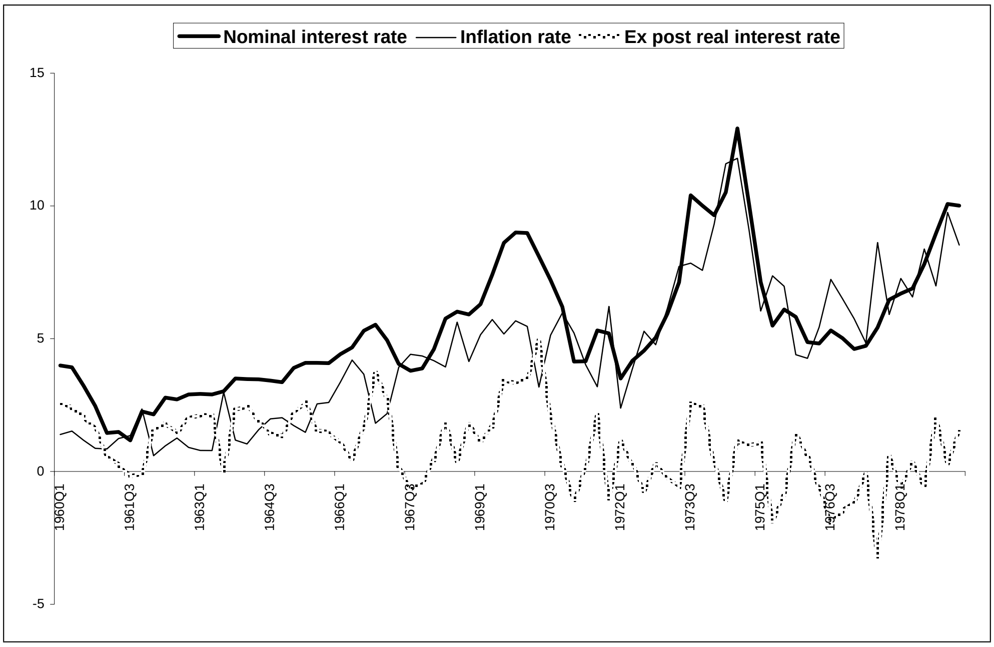

Figure 13: Data on nominal interest rate i_t, inflation rate π_t, and ex post real interest rate r_t for the period 1960Q1-1979Q2

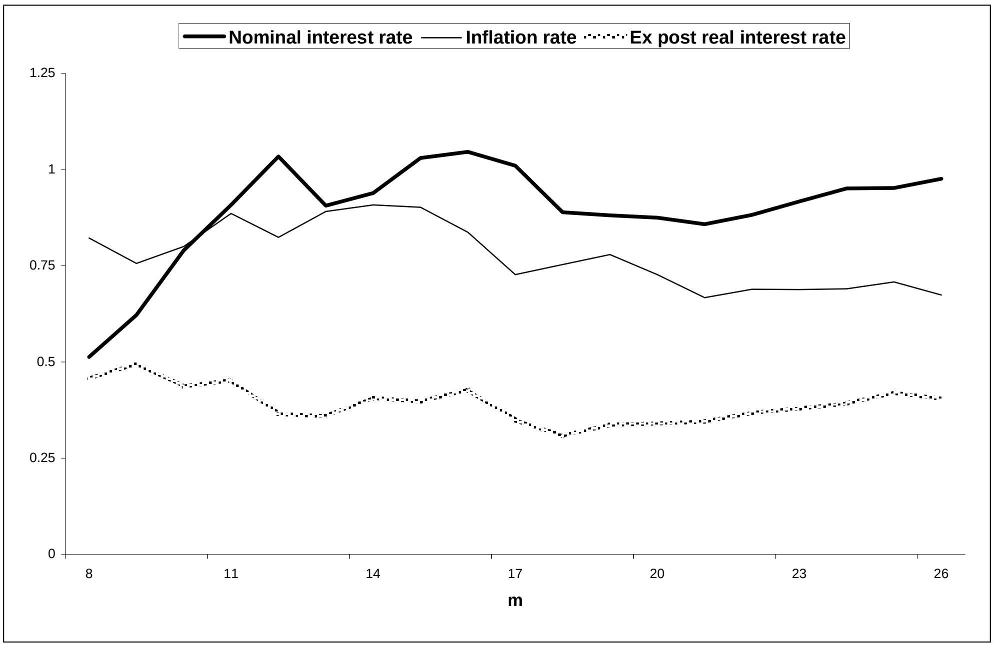

Figure 14: Estimates of d for the data in Figure 13

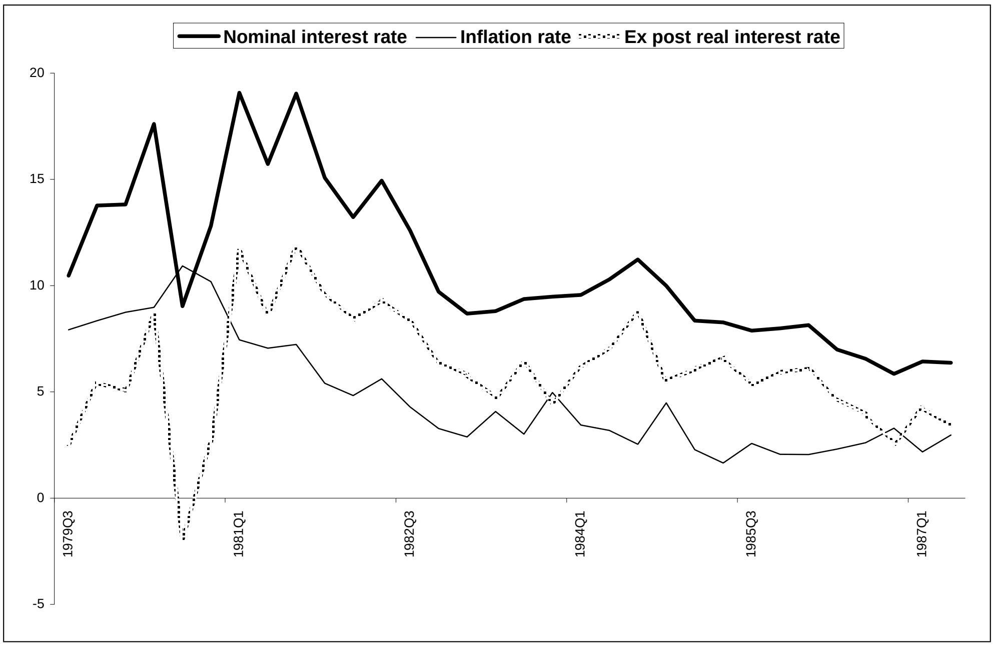

Figure 15: Data on nominal interest rate i_t, inflation rate π_t, and ex post real interest rate r_t for the period 1979Q3-1987Q2

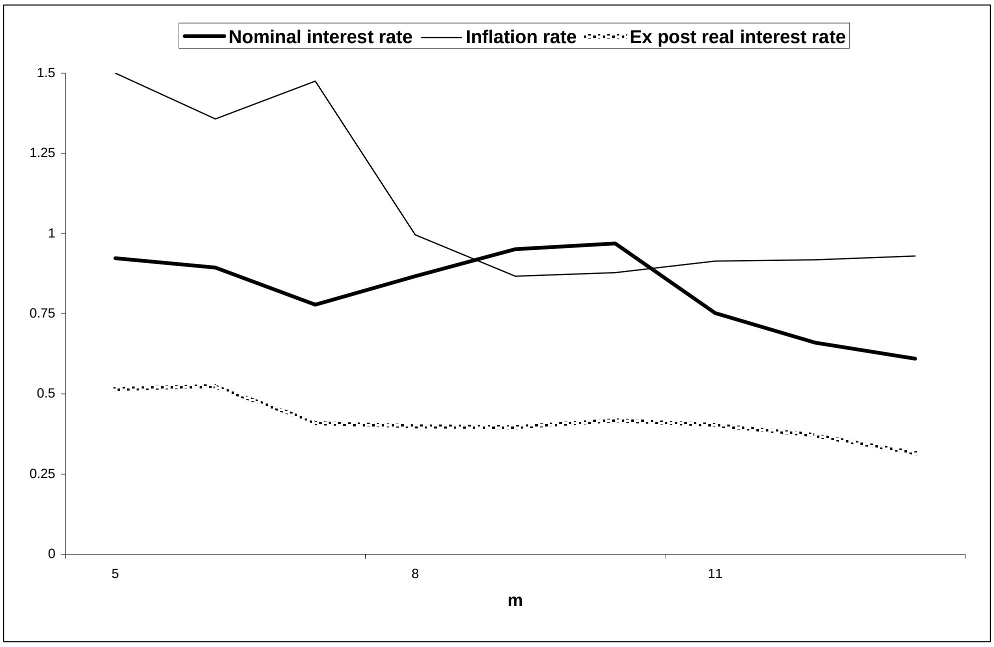

Figure 16: Estimates of d for the data in Figure 15

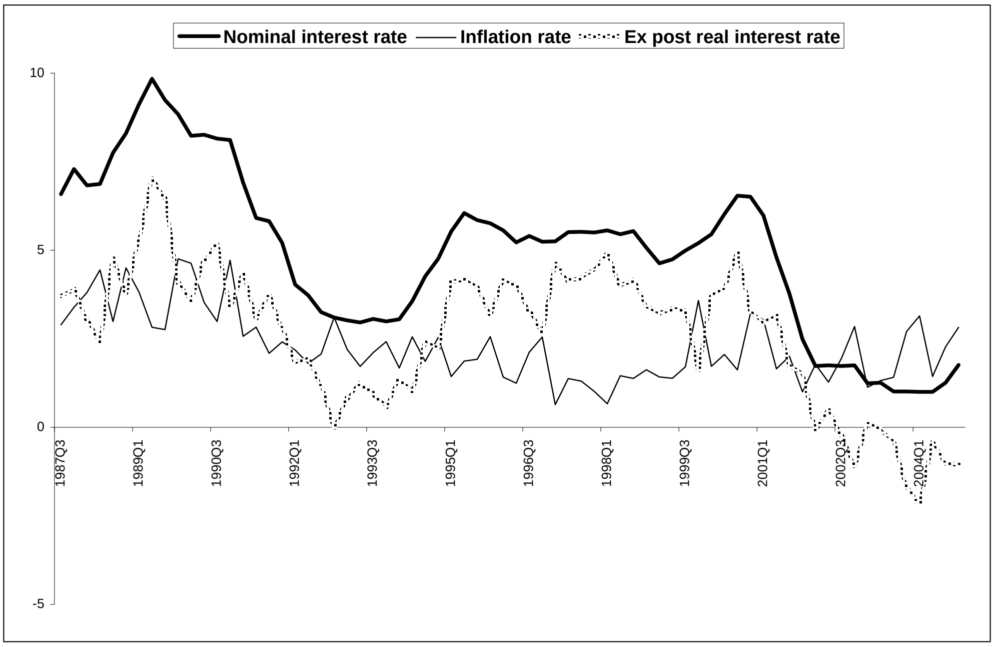

Figure 17: Data on nominal interest rate i_t, inflation rate π_t, and ex post real interest rate r_t for the period 1987Q3-2004Q4

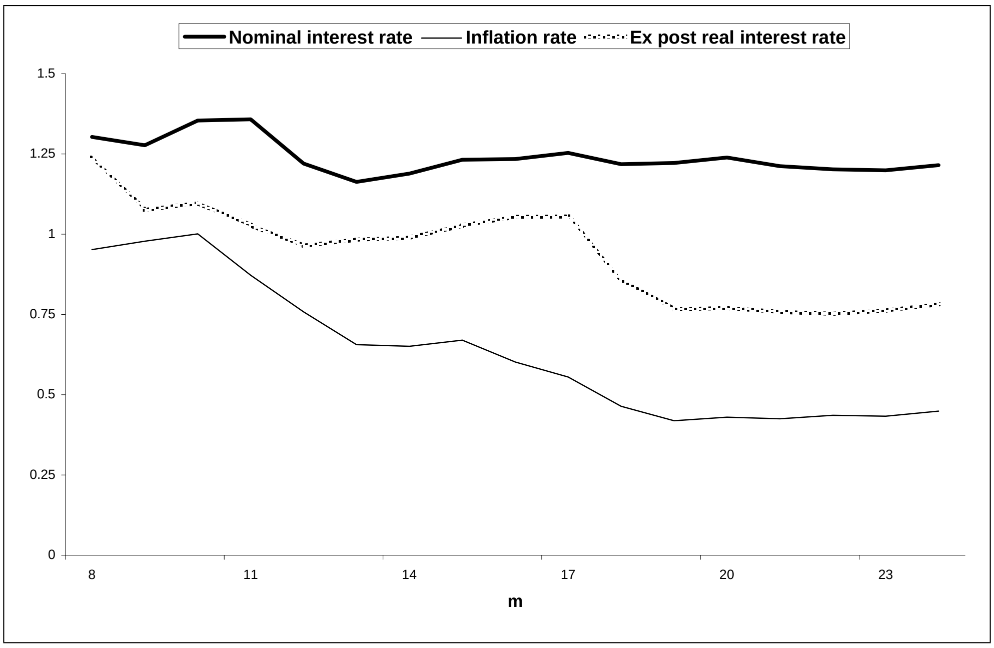

Figure 18: Estimates of d for the data in Figure 17

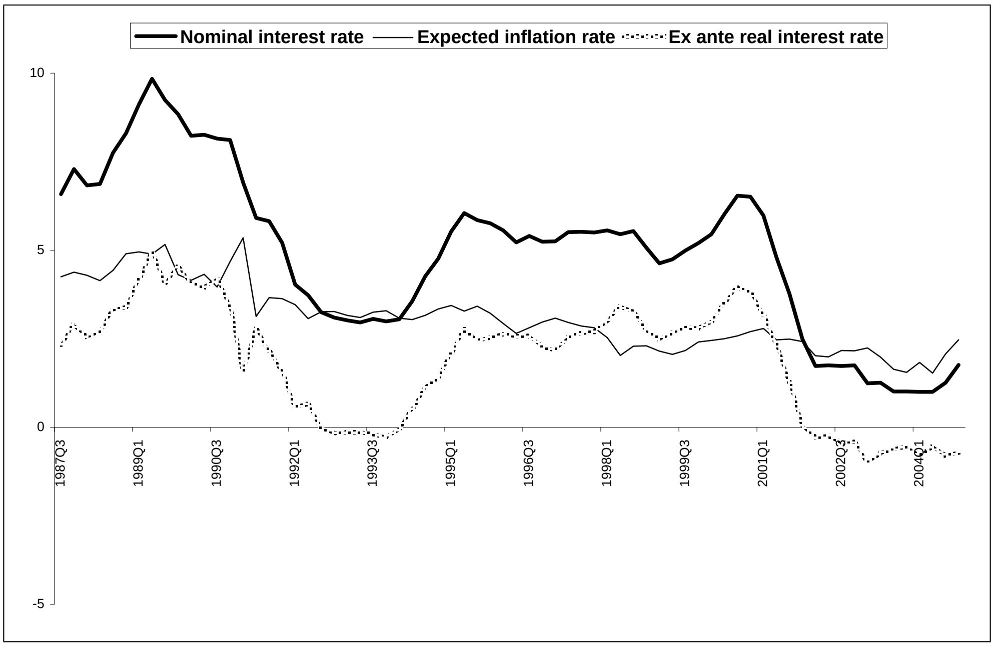

Figure 19: Data on nominal interest rate i_t, expected inflation rate π_t^e, and ex ante real interest rate r_t^e for the period 1987Q3-2004Q4

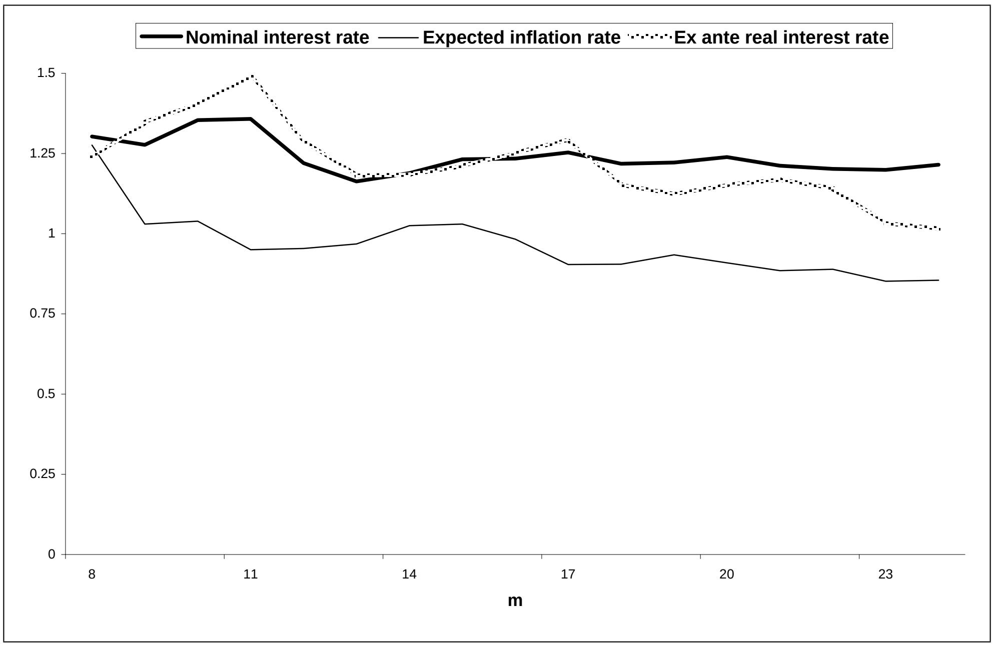

Figure 20: Estimates of d for the data in Figure 19

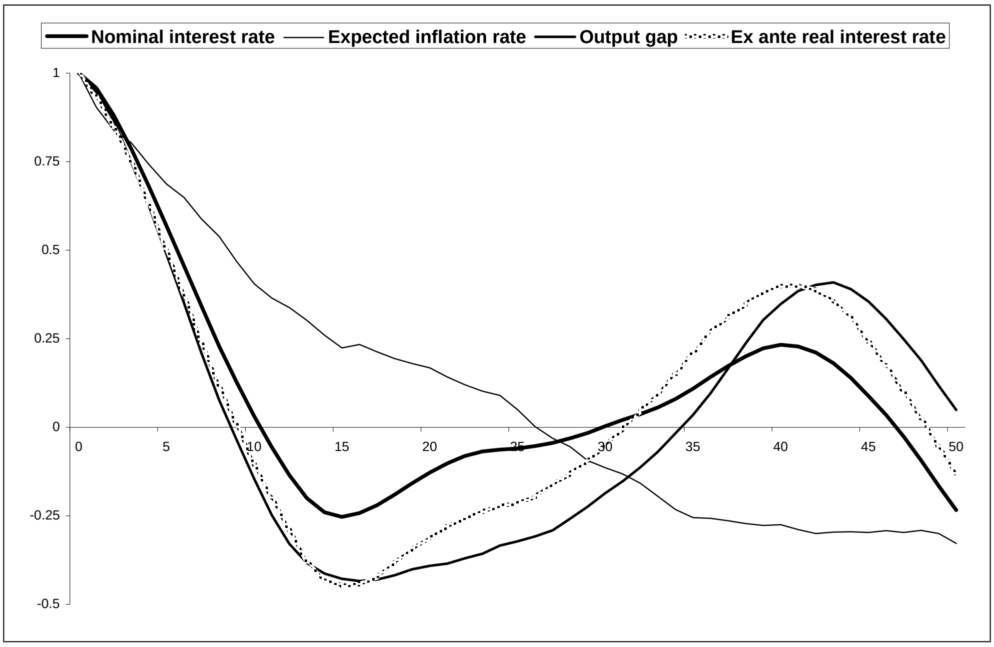

Figure 21: Sample autocorrelation function of data on nominal interest rate i_t, expected inflation rate π_t^e, output gap g_t and ex ante real interest rate r_t^e for the period 1987Q3-2004Q4

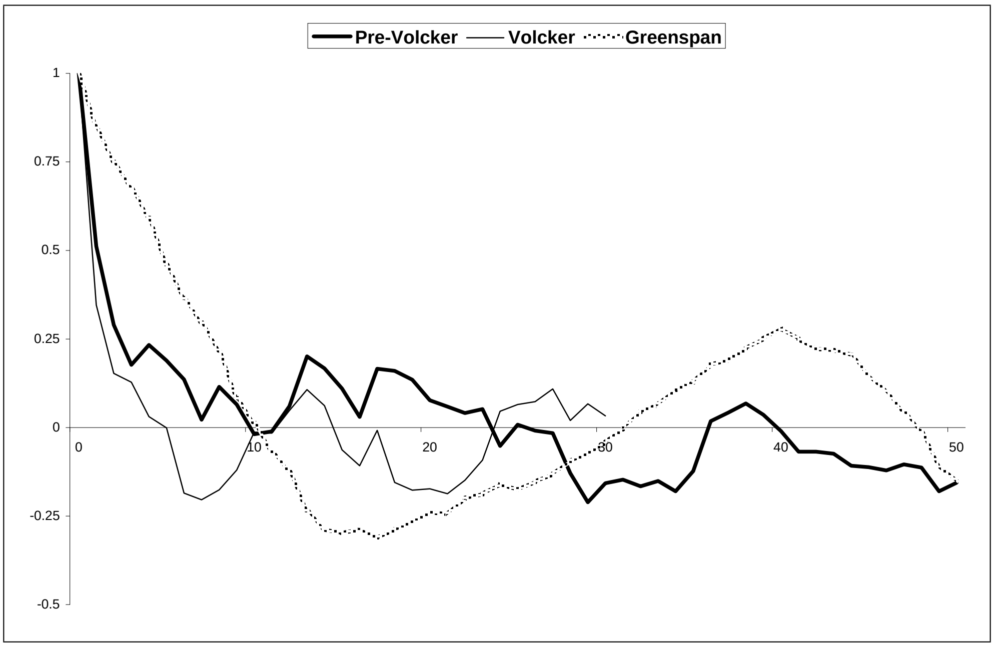

Figure 22: Sample autocorrelation function of the ex post real interest rate r_t for the three subperiods