Plant Life Cycle and Aggregate Employment Dynamics[1]

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Abstract

Past empirical studies have repeatedly found the link between plant life cycle and aggregate employment dynamics: cross-section aggregate employment dynamics differ significantly by plant age. Interestingly, the dynamics of plant-level productivity distribution also display a strong age pattern. This paper develops a model of plant life cycle with demand fluctuations, to capture both of these empirical regularities. We model plants to differ by vintage, and an idiosyncratic component that is not directly observable, but can be learned over time. We show that this model, developed to match the observed dynamics of plant-level productivity distribution, introduces two driving forces for job flows: learning and creative destruction. The resulting job flows can match, both qualitatively and quantitatively, the differences between young and old plants in their job-flow magnitude and cyclical responses observed in the U.S. manufacturing sector.

Keywords: Plant life cycle; Employment dynamics; Heterogeneous Employers; Job creation; Job destruction; Productivity dynamics; Demand fluctuations.

JEL: E32, L16, C61

1 Introduction

With access to longitudinal business data bases becoming more available in the past 20 years, an assortment of studies have emerged exploring the micro foundations of aggregate employment dynamics. It has been documented that economies across time and regions are characterized by a large and pervasive process of job flows driven by continuous entry, exit, expansion, and contraction of businesses. Maybe surprisingly, the majority of this process turns out to occur within-industry.[1] In other words, labor is being reallocated across heterogeneous businesses within the same industry, rather than across different industries. Such findings point out that the traditional representative-agent paradigm does not hold in the real world, and has stimulated a series of theoretical work on models with heterogeneous businesses.[2]

While many factors contribute to business heterogeneity, this paper proposes a model that emphasizes plant life cycle. We are motivated by an age pattern that stands out in many empirical studies: job-flow magnitude declines with business age, even after controlling for other business characteristics.[3] Moreover, the cyclical dynamics of various job-flow margins display an age pattern as well. According to Davis and Haltiwanger (1998), in the U.S. manufacturing sector, job creation and job destruction are approximately equally volatile for young plants while, for old plants, job destruction appears much more volatile.

In this paper we present a model to incorporate the age pattern of job flows, emphasizing two dimensions in the productivity dynamics over a plant's life cycle. Intuitively, old plants are more experienced as they have survived long; but their technologies are usually outdated, or their products often flag in popularity. By contrast, young plants, while embodied with

[1] According to Davis and Haltiwanger (1998), about 87% of the excess reallocation takes place within each of the four-digit Standard Industry Classification (SIC) industries in the U.S. manufacturing sector. The dominance of the within-industry reallocation persists under even narrower industry definitions such as four-digit industries by specific regions.

[2] See, for example, Caballero and Hammour (1994), Mortensen and Pissaridies (1994), Gomes et al. (2001), and Barlevy (2002).

[3] This has been documented, for example, by Evans (1987), Dunne et. al. (1989), and Aw et al. (2001). Davis et al.(1996) summarize the age pattern of job flows to be surprisingly robust regardless of major differences among studies in measurement, country, sectoral coverage, and data.

the leading technology, tend to be inexperienced. If more productive plants hire more labor, then the productivity dynamics over a plant's life cycle should generate multiple margins to hire or fire labor, which, when interacted with the business cycle, give rise to an age pattern of aggregate employment dynamics.

To capture the above story, we build a model that combines passive learning in the spirit of Jovanovic (1982) with a vintage model by Aghion and Howitt (1992). In our model, plants differ in two productivity components: vintage and idiosyncratic productivity. The idiosyncratic productivity is not directly observable, but can be learnt over time. Young plants have newer vintages, but are unsure about their true idiosyncratic productivity. Old plants perceive themselves to have high idiosyncratic productivity, but are with older vintages. Two reallocation effects arise with this setup: a learning effect, under which a plant creates jobs when it learns its true idiosyncratic productivity to be high or destroys jobs when it learns that to be low, and a creative-destruction effect, under which entrants with new technology create jobs to enter and incumbents destroy jobs as they become technologically outdated over time and eventually exit the industry.

This model of plant life cycle can incorporate the observed age pattern of job flows. Because marginal learning diminishes as plants grow old, the learning-driven reallocation declines with plant age, so that young plants feature higher job-flow magnitude. Since the creative-destruction driven job destruction are concentrated at old plants, the cyclical dynamics in job destruction dominates for old plants. We calibrate our model to the job flow series from the U.S. manufacturing sector. Simulation of the calibrated model delivers, at least qualitatively, the observed age pattern in both the magnitude and the cyclical dynamics of job flows.

The age pattern of cyclical job flows has been explored by Campbell and Fisher (2004), who model adjustment cost proportional to the number of jobs created or destroyed. In their model, a plant currently adjusting employment is more likely to do so again in the immediate future. Since plants have to create jobs to enter, young plants adjust employment more frequently and therefore feature higher job-flow magnitude. However, their baseline model fails to deliver the high variance ratio of job destruction over job creation for old plants. Our model emphasizes the interaction of the learning effect with the creative destruction effect. The learning effect causes the job-flow magnitude to decline with plant age, and the creative destruction effect drives job destruction to be more responsive for old plants. However, our calibration exercises suggest that, to deliver the observed high variance ratio of job destruction over job creation for old plants, our model requires a creative destruction effect probably stronger than what the existent empirical evidence suggests. We discuss several possible extensions to improve the model's quantitative performance.

The paper proceeds as follows. Section 2 summarizes the age pattern in employment dynamics with data from the U.S. manufacturing sector. Section 3 presents the model. Section 4 analyzes the dynamics in the learning and creative destruction effects at the steady-state equilibrium. Section 5 calibrates the model to the data, and discusses results from the simulation exercises. We conclude in Section 6.

2 Plant Age and Reallocation Dynamics

Our data source is the gross job flow series for the US manufacturing sector compiled by Davis et. al. (1996).[4] Job flows are separated into two components: job creation as the number of jobs created at expanding and newly born plants, and job destruction as the number of jobs lost at declining and closing plants. The age dynamics of job creation and destruction are displayed in Table 1 and Figure 1. The sample covers quarterly job creation and destruction for plants of three different age categories from the second quarter of 1972 to the fourth quarter of 1998. Following Davis et al. (1996), we aggregate the first two categories that cover all plants younger than ten years into one category of young plants; accordingly, old plants are those that have remained in operation for ten years or more.

Table 1A shows that the *magnitude* of job flows declines with plant age. From 1972 to

[4] Original series end in year 1993. It was extended later to 1999.

1998, quarterly job creation rate averages 7.63% and job destruction rate averages 6.50% for young manufacturing plants; the corresponding statistics are 4.40% and 5.03% for old plants. Table 1A further decomposes job creation into that contributed by plant births (Cb) and that by continuing operating plants that are expanding (Cc); job destruction is divided into that contributed by plant deaths (Dd) and that by continuing operating plants that are contracting (Dc). Not surprisingly, Cb is much higher at young plants, as Cb by old plants mainly arises from a few plants that resume operation after temporary closing. Interestingly, Dd, Cc, and Dc are all higher for young plants: for example, young plants' Dd averages 1.47%, more than three times of that for old plants. In summary, during early years of operation, plants not only face higher chances of closing, as implied by higher Dd, but also experience much more turnover even if they manage to survive, as suggested by higher Cc and Dc.

Table 1B shows that plant age matters for the *cyclical dynamics* of job flows. For young plants, job creation and job destruction are equally volatile, as the ratio of the variance of job destruction over that of job creation equals 1.01. For old plants, however, job destruction is much more volatile than job creation with a variance ratio of 4.18. This age pattern persists even after excluding plant births and deaths: the ratio of the variance of Dc over that of Cc equals 3.36 for old plants but only 1.47 for young plants. In summary, Table 1B suggests that a plant features similar volatility in its job creation margin and its job destruction margin during early years of operation; however, as it grows older, it experiences more variation on the job destruction margin.

The influence of plant age on employment dynamics shown in Table 1 has been documented by many other authors. For example, Evans (1987) finds that firm growth decreases with firm age. Dunne et. al. (1989) report that employment growth rates and failure rates both decline with plant age. Similar patterns are documented by Aw et. al. (2001) for Taiwanese firms and by Roberts (1996) for Columbian firms. The asymmetry in the volaitities of job destruction versus job creation is discussed in Foote (1998). The influence of plant age on the cyclical dynamics of job flows is also reported by Faberman (2007) with the 1992-2000 micro data from the Business Employment Dynamics.

3 A Model of Learning and Creative Destruction

To incorporate the age pattern of employment dynamics documented in Section 2, we present a model of plant life cycle. Consider an industry where labor and capital are combined in fixed proportions to produce a single output. Each production unit is called a plant. Each plant consists of:

1. a group of machines that embody some vintage;
2. an unobservable idiosyncratic productivity;
3. a group of employees.

Let A to be the leading technology. An exogneous technological progress drives A to grow over time at rate $\gamma > 0$. When entering the market, a new plant adopts the leading technology at the time, which becomes its vintage and remains constant throughout its life cycle. Let $A(a)$ to represent the leading technology a periods ago, so that $A(a)$ is also the vintage of a plant of age a: $A(a) = A(1+\gamma)^{-a}$.

At the time of entry, a plant is endowed with an idiosyncratic productivity θ. Many factors can contribute to θ: it can be the talent of the manager as in Lucas (1978), the location of the store, the organizational structure of the production process, or the plant's fitness to the embodied technology. The key assumption regarding θ is that its value, although fixed at the time of entry, is not directly observable.

Production takes place through a group of workers. Let n to be the number of employees. The output of a plant with vintage $A(a)$ and n employees in period t equals $A(a)x_t n_t^{\alpha}$, where $0 < \alpha < 1$. $x_t = \theta + \varepsilon_t$: the true idiosyncratic productivity θ covered by a random noise ε_t. The noise ε_t is an i.i.d. random draw from a fixed distribution that masks the influence of

θ on output. A plant knows its vintage and can directly observe output. Thus, it can infer the value of x. A plant uses its information on current and past x's to learn about θ.

3.1 "All-Or-Nothing" Learning

Plants are price takers and profit maximizers. They attempt to resolve the uncertainty about θ. The random component ε represents transitory factors that are independent of the idiosyncratic productivity θ. Assuming mean zero for ε, $E(x) = E(\theta) + E(\varepsilon) = E(\theta)$.

Given the knowledge of the distribution of ε, a sequence of observations on x allows a plant to learn about its θ. Although a continuum of potential values for θ is more realistic, for simplicity it is assumed that there are only two values: θ_g for a good plant and θ_b for a bad plant. Furthermore, ε is assumed to be distributed uniformly on $[-\omega, \omega]$. Therefore, a good plant will have x each period as a random draw from a uniform distribution over $[\theta_g - \omega, \theta_g + \omega]$, while the x of a bad plant is drawn from an uniform distribution over $[\theta_b - \omega, \theta_b + w]$. Finally, θ_g, θ_b and ω satisfy $0 < \theta_b - \omega < \theta_g - \omega < \theta_b + \omega < \theta_g + \omega$.

Pries (2004) shows that the above assumptions give rise to an "all-or-nothing" learning process. With an observation of x within $(\theta_b + \omega, \theta_g + \omega]$, the plant learns with certainty that it is a good plant; conversely, an observation of x within $[\theta_b - \omega, \theta_g - \omega)$ indicates that it is a bad plant. However, an x within $[\theta_g - \omega, \theta_b + \omega]$ does not reveal anything, since the probability of falling in this range as a good plant and that as a bad plant both equal $\frac{2\omega+\theta_b-\theta_g}{2\omega}$.

This all-or-nothing learning process simplifies the model considerably. Three values of θ^e determine that there are only three types of plants: those who know they are good with $\theta^e = \theta_g$, those who know they are bad with $\theta^e = \theta_b$, and those remain unsure about their true idiosyncratic productivity with $\theta^e = \theta_u$, the prior mean of θ. We call the third type "unsure plants".

Let φ to be the unconditional probability of $\theta = \theta_g$, and p to be the probability of true idiosyncratic productivity being revealed every period. According to the all-or-nothing learning process, $p = \frac{\theta_g - \theta_b}{2\omega}$. This setup generates the following plant life cycle: a flow of new plants enter the market as unsure; thereafter, every period they stay unsure with probability $1 - p$, learn that they are good with probability $p\varphi$, and learn that they are bad with probability $p(1 - \varphi)$. The evolution of θ^e from the time of entry is a Markov process with values $(\theta_g, \theta_u, \theta_b)$, an initial probability distribution $\begin{pmatrix} 0, & 1, & 0 \end{pmatrix}$,and a transition matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ p\varphi, & 1-p, & p(1-\varphi) \\ 0 & 0 & 1 \end{pmatrix}.$$

If plants were to live forever, eventually all uncertainty would be resolved, as the limiting probability distribution as a goes to ∞ is $\begin{pmatrix} \varphi, & 0, & (1-\varphi) \end{pmatrix}$. Because there is a continuum of plants, it is assumed that the law of large numbers applies, so that both φ and p are not only the probabilities but also the fraction of good plants and that of plants who learn their true idiosyncratic productivity each period. Hence, *ignoring plant exit for now*, the fractions of three plant groups in a cohort of age a is

$$\begin{pmatrix} \varphi\left[1-(1-p)^a\right], & (1-p)^a, & (1-\varphi)\left[1-(1-p)^a\right] \end{pmatrix}. \tag{1}$$

Figure 2 presents the evolution of plant distribution across three values of θ^e within a birth cohort. The horizontal axis depicts the cohort age over time. The densities of plants that are certain about their idiosyncratic productivity, whether good or bad, grow as a cohort ages. Moreover, the two "learning curves" (depicting the evolution of the densities of good plants and bad plants) are concave. This feature is the decreasing property of marginal learning with age in Jovanovic (1982), which, in my model, is reflected as that the marginal number of learners decreases with cohort age. The convenient feature of the all-or-nothing learning process is that, on the one hand, any single plant learns suddenly, which allows us to easily keep track of the cross-section distribution while, on the other hand, any cohort as a group of plants still learn gradually.

3.2 Plant Decisions and Industry Equilibrium

We consider a *recursive competitive (partial) equilibrium* definition, which includes as a key component as the law of motion of the aggregate state of the industry. The aggregate state is (F, D). F denotes the distribution of plants across vintages and expected idiosyncratic productivity. D is an exogenous demand parameter; it captures the aggregate condition and is fully observable. The law of motion for D, denoted H_D, is exogenous. The law of motion for F, denoted H_F, is endogeneous and defined as $F' = H_F(F, D)$. The element in F that measures the number of plants with belief θ^e and age a is denoted $f(\theta^e, a)$. H_F captures the influence of entry, exit, and learning based on the following sequence of events:

At the beginnin of a period, plants make entry or exit decision after observing the aggregate state; if deciding to enter or stay in operation, a plant adjusts its employment and produces; the equilibrium price is realized; plants observe their revenue and update their beliefs; then, another period begins.

3.2.1 The Employment Decision

A plant adjusts its employment to solve a static profit maximization problem. The wage rate is normalized as 1; θ^e is a plant's current belief of its true idiosyncratic productivity; P represents the equilibrium output price; Ψ denotes a fixed operation cost each period. A plant's optimal employment is

$$\begin{aligned} n(\theta^e, a) &= \arg\max_{n \geq 0} E\left[PA(a)xn^{\alpha} - n - \Psi\right] \\ &= \left[\alpha \frac{PA\theta^e}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}}, \end{aligned} \tag{2}$$

where θ^e can take on three values θ_g, θ_b, or θ_u. (2) suggests that plant-level employment depends positively on θ^e and P but negatively on a. Thus, a plant hires more employees if the output price is higher, the expected idiosyncratic productivity is higher, or its vintage is newer. Put differently, a plant creates jobs if the ouput prices rises or if it learns its idiosyncratic productivity to be high; it destroy jobs as its vintage becomes old.

The corresponding plant-level output is

$$q(\theta^e, a) = (\alpha P)^{\frac{\alpha}{1-\alpha}} \left(\frac{A\theta^e}{(1+\gamma)^a} \right)^{\frac{1}{1-\alpha}}. \tag{3}$$

And the corresponding profit is,

$$\pi(\theta^e, a) = (\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}) \left[\frac{PA\theta^e}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} - \Psi. \tag{4}$$

3.2.2 The Exit Decision

A plant exits if and only if its expected value of staying is below zero. Let $V(\theta^e, a; F, D)$ to be the expected value of staying in operation for one more period and optimizing afterward for a plant with age a and belief θ^e. Then V satisfies:

$$\begin{aligned} V(\theta^e, a; F, D) &= \pi(\theta^e, a; F, D) + \beta E\{\max[0, V(\theta^{e'}, a+1; F', D')] | \theta^e, F, D\} \\ \text{subject to} &: F' = H_F(F, D), D' = H_D(D) \end{aligned} \tag{5}$$

and the all-or-nothing learning that determines the evolution of θ^e.

To further simplify the model, it is assumed that the parameter values are such that $V(\theta_b, a; F, D) < 0$ for any a, F and D, so that the expected value of staying for plants with $\theta^e = \theta_b$ is always negative. Therefore, plants that know their true idiosyncratic productivity to be bad always exit.

According to (4), $\pi(\theta^e, a; F, D)$ decreases in a. It follows that, $V(\theta^e, a; F, D)$ also decreases in a holding other parameters constant. Therefore, there exists a maximum plant age $\overline{a}(\theta^e; F, D)$ for each pair of θ^e and (F, D), so that plants with θ^e and $a > \overline{a}(\theta^e; F, D)$ choose to exit.

3.2.3 The Entry Size

New plants are free to enter as long as they bear an entry cost c. c is modeled as a linear function of the entry size. According to the sequence of events, entry and exit take place after plants observe F, which updates F to F'. Let $f(\theta_u, 0)'$ to be the element of F' that measures the number of new entrants as plants with $\theta^e = \theta_u$ and $a = 0$:

$$c = c_0 + c_1 f(\theta_u, 0)', \ c > 0, c_1 \geq 0.$$

We let the entry cost to depend positively on the entry size to capture the idea that, for the industry as a whole, *fast* entry is costly and adjustment cannot take place instantaneously. This can arise from a limited amount of land available to build production sites or an upward-sloping supply curve for the industry's specific capital (Goolsbee, 1998). Entrants drive up the price of land or capital necessary for entry, so that entry becomes more costly as more plants enter.

With this setup, new plants keep entering as long as the expected value of entry is above the entry cost. As entry size rises, the entry cost is driven up until it reaches the expected value of entry:

$$V(\theta_u, 0; F, D) = c_0 + c_1 f(\theta_u, 0)'. \tag{6}$$

At this point, entry stops.

3.2.4 The Industry Equilibrium

Let $Q(F, D)$ to represent the equilibrium output, the recursive competitive industry equilibrium constitutes a law of motion H_F, a value function V, and a pricing function P such that:

1. V satisfies (5);

2. $F' = H_F(F, D)$ is generated by the appropriate summing up of plant entry, exit, and

learning. Let $f(\theta^e, a)'$ to denote the element of F' that measures the number of plants with belief θ^e and age a, and $f(\theta^e, a)$ to be that of F, $H_F(F, D)$ is such that:

$$f(\theta_u, a)' = \left\{ \begin{array}{c} \frac{V(\theta_u, a; F, D) - c_0}{c_1} \text{ for } a = 0 \\ (1-p) f(\theta_u, a-1) \text{ for } 1 \leq a \leq \overline{a}(\theta_u) \\ 0 \text{ for } a > \overline{a}(\theta_u) \end{array} \right\},$$

$$f(\theta_g, a)' = \left\{ \begin{array}{c} f(\theta_g, a-1) + p\varphi f(\theta_u, a-1) \text{ for } 1 \leq a \leq \overline{a}(\theta_g) \\ 0 \text{ for } a = 0 \text{ or } a > \overline{a}(\theta_g) \end{array} \right\},$$

where $\overline{a}(\theta_g)$ satisfies $V(\theta_g, \overline{a}(\theta_g); F, D) = 0$ and $\overline{a}(\theta_u)$ satisfies $V(\theta_u, \overline{a}(\theta_u); F, D) = 0$.

3. $Q(F, D)$ equals the sum of operating plants' output.[5]

4. $P(F, D)$ satisfies:

$$P(F, D) = \frac{D}{Q(F, D)}, \tag{7}$$

The key component of this definition, H_F, are characterized by three essential elements: the entry size $f(\theta_u, 0)'$, good plants' maximum age $\overline{a}(\theta_g)$, and the unsure plants' maximum age $\overline{a}(\theta_u)$. These three elements, together with the all-or-nothing learning, update F to F', which determines the equilibrium output and price by conditions 3 and 4 and serves as the aggregate state for the next period.

4 The Steady State: Plant Life Cycle and Job Flows

In the model described above, new plants embodied with the latest technology keep entering the industry; the employment level of incumbents grow or shrink, depending on what they learn and how fast the technology updates; those that learn there true idiosyncratic productivity as bad or those whose vintages become too old cease operation. Hence, plant entry,

[5] Although industry-level output should equal the sum of *realized* plant-level output, it can be shown that the expectation error and the random noise cancel out within each age cohort so that the sum of *expected* plant output equals the sum of *realized* output.

exit, and learning generates a reallocation process in which labor flows from bad plants to good plants, and from old vintages to new vintages. This reallocation process is driven by two forces: learning and creative destruction.

Before allowing for stochastic demand fluctuations, this section examines the link between plant life cycle and aggregate employment dynamics at the steady-state equilibrium when demand remains time invariant. The next section will turn to stochastic demand fluctuations and explore whether the intuition from this section carries over.

4.1 The Steady State

We define a steady state as a recursive competitive equilibrium with time-invariant aggregate states: D is *and is perceived* as time-invariant: $D = H_D(D)$; F is also time-invariant: $F = H_F(F, D)$. Since H_F is generated by entry, exit and learning, a steady state must feature time-invariant entry and exit for $F = H_F(F, D)$ to hold. Thus, it can be summarized by $\left\{f^{ss}(0, D), \bar{a}_u^{ss}(D), \bar{a}_g^{ss}(D)\right\}$: $f^{ss}(0, D)$ is the steady-state entry size when demand equals D, $\bar{a}_g^{ss}(D)$ is the maximum age for good plants, and $\bar{a}_u^{ss}(D)$ is the maximum age for unsure plants. Proposition 1 establishes the existence of a unique steady-state equilibrium.

Proposition 1 *With constant D, there exists a unique $\left\{f^{ss}(0, D), \bar{a}_u^{ss}(D), \bar{a}_g^{ss}(D)\right\}$ that satisfies conditions 1-4; moreover, the steady-state PA remains time-invariant: $PA = \overline{PA}^{ss}(D)$.*

See the appendix for proof. Proposition 1 suggests that, at a steady state, the plant distribution across vintages and idiosyncratic productivity stays time-invariant, so does the product of the price and the leading technology PA. Since A grows at γ, P must be declining at the same rate for the steady-state PA to be constant. Put intuitively, continuous entry and exit drives the leading technology A, as well as the industry output Q, to grow at γ. With constant D, growing Q causes P to decline at γ.

Figure 3 displays the steady-state evolution of plant distribution within a representative cohort, with the horizontal axis depicting the cohort age across time. Upon entry, the cohort contains unsure plants only. As it ages, bad plants keep learning and exiting. At a certain age, all unsure plants exit with their vintage too old to survive by staying unsure; but those plants who have leant their idiosyncratic productivity to be good stay. Afterward, the cohort contains good plants only; its size remains constant. Eventually, all plants exit because their vintage is too old even for good plants to survive.

Figure 3 can also be interpreted differently if we take the horizontal axis as cohort age *across section.* Note that, at a steady state, all cohorts enter at the same size and experience the same dynamics afterward. Thus, at any time point, different life stages of various birth cohorts overlap, giving rise to the steady-state plant distribution across ages and idiosyncratic productivity as shown in Figure 3. The entering cohort is of size $f^{ss}(0, D)$. Older cohorts are of smaller sizes, because plants who learnt their idiosyncratic productivity were bad have exited. No unsure plants are older than $\overline{a}_u^{ss}(D)$. No good plants are older than $\overline{a}_g^{ss}(D)$. Cohorts older than $\overline{a}_u^{ss}(D)$ contain no unsure plants, and thus are of the same size.

4.2 The Plant Life Cycle

To assess how plant life cycle impacts aggregate employment dynamics, we begin with the steady-state plant-level employment n^{ss}as a funcation of θ^e, a, and D. (2) can be written as:

$$\begin{aligned} n^{ss}(\theta^e, a; D) &= \left[\alpha\theta^e \frac{\overline{PA}(D)}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}}. \qquad (8) \\ \theta^e &= \theta_u \text{ or } \theta_g \\ a &\leq \overline{a}_u^{ss}(D) \text{ if } \theta^e = \theta_u; \\ a &\leq \overline{a}_g^{ss}(D) \text{ if } \theta^e = \theta_g \end{aligned}$$

According to (8), three forces affect the plant-level employment. D affects n^{ss} through its influence on $\overline{PA}$. θ^e affects n^{ss} positively: a plant creates jobs when learning its idiosyncratic productivity to be good and destroys jobs when learning that it is bad. a affects n^{ss}

negatively: a plant destroys jobs as it ages, and exits competely once it reaches $\overline{a}_g^{ss}(D)$ or $\overline{a}_g^{ss}(D)$. We call the impact of θ^e *the learning effect* and that of *a the creative destruction effect.*

Assuming $\frac{\theta_g}{1+\gamma} > \theta_u$, Figure 4 plots the steady-state plant life cycle driven by the learning and creative destruction effects together. The horizontal axis depicts plant age across time. The employment dynamics of three representative plants are presented: plant one stays unsure and exits at $\overline{a}_u^{ss}(D)$; plant two learns its true idiosyncratic productivity to be good at some point, stays in operation afterward, and exits at $\overline{a}_g^{ss}(D)$; plant three learns that its idiosyncratic productivity to be bad at the same age as plant two does, and exits immediately. Figure 4 shows that, before any plant learns, all three keep shrinking due to the creative destruction effect. At the age of learning, plant two creates but plant three destroys jobs. After learning takes place, plant two once again destroys jobs over time, and exits competely once she reaches $\overline{a}_g^{ss}(D)$.

4.3 The Cohort Life Cycle

The cohort life cycle is given by aggregating the plant life cycle to the cohort level using the evolution of within-cohort plant distribution. Propositions 2 and 3 capture the dynamics of the learning and creative destruction effects.

Proposition 2 *Under the learning effect, the steady-state job creation and destruction of a cohort of age a,* $jc_l^{ss}(a)$ *and* $jd_l^{ss}(a)$*, both weakly decrease with cohort age.*

The magnitudes of $jc_l^{ss}(a)$ and $jd_l^{ss}(a)$ are determined by the fraction of plants who learn and how many of the learning plants are good or bad. Combining (1) with (8) gives

$$\begin{aligned} jc_l^{ss}(a) &= \frac{\varphi p (1-p)^a \left[\left(\frac{\theta_g}{1+\gamma} \right)^{\frac{1}{1-\alpha}} - \theta_u^{\frac{1}{1-\alpha}} \right]}{\theta_g^{\frac{1}{1-\alpha}} \varphi + (1-p)^a \left(\theta_u^{\frac{1}{1-\alpha}} - \theta_g^{\frac{1}{1-\alpha}} \varphi \right)} \text{ for } 0 < a \leq \overline{a}_u^{ss}(D); \qquad (9) \\ jd_l^{ss}(a) &= \frac{\theta_u^{\frac{1}{1-\alpha}} p (1-p)^a (1-\varphi)}{\theta_g^{\frac{1}{1-\alpha}} \varphi + (1-p)^a \left(\theta_u^{\frac{1}{1-\alpha}} - \theta_g^{\frac{1}{1-\alpha}} \varphi \right)} \text{ for } 0 < a \leq \overline{a}_u^{ss}(D); \\ jc_l^{ss}(a) &= jd_l^{ss}(a) = 0, otherwise \end{aligned}$$

Apparently, $jc_l^{ss}(a)$ and $jd_l^{ss}(a)$ both weakly decrease in a. This is due to the decreasing property of marginal learning captured by the all-or-nothing learning process. For $a > \overline{a}_u^{ss}(D)$, learning has stopped so that there are no learning-driven job creation or destruction. For $0 < a < \overline{a}_u^{ss}(D)$, the fraction of learning plants per period equals $\frac{p(1-p)^a}{(1-p)^a + \varphi[1-(1-p)^a]}$, out of which φ proportion creat jobs and $1-\varphi$ destroy jobs. Since $\frac{p(1-p)^a}{(1-p)^a + \varphi[1-(1-p)^a]}$ decreases in a, the learning-driven job creation and destruction also decrease in cohort age.

Proposition 3 *Under the creative destruction effect, the steady-state job destruction by aging plants $jd_{cd}^{ss}(a)$, defined as plants who do not learn or have learnt already, weakly increases with cohort age; moreover, $jd_{cd}^{ss}(a)$ rises in γ.*

According to Proposition 1, the steady-state PA remains time-invariant. Therefore, (8) suggests that n^{ss} declines as a grows as long as θ^e does not change. Put intuitively, plants who have learnt their idiosyncratic productivity to be good and those who remain unsure destroy jobs as they grow old; moreover, the higher γ is, the more jobs they destroy while they age. Thus, $jd_{cd}^{ss}(a)$ is determined by the within-cohort fraction of aging plants and how fast the technology updates. Combining (1) and (8) gives

$$
\begin{aligned}
jd_{cd}^{ss}(a) &= \left[1-(1+\gamma)^{-\frac{1}{1-\alpha}}\right]\left[\frac{\theta_g^{\frac{1}{1-\alpha}}\varphi\left[1-(1-p)^a\right]+\theta_u^{\frac{1}{1-\alpha}}(1-p)^{a+1}}{\theta_g^{\frac{1}{1-\alpha}}\varphi+(1-p)^a\left(\theta_u^{\frac{1}{1-\alpha}}-\theta_g^{\frac{1}{1-\alpha}}\varphi\right)}\right] \text{ for } 0\le a<\overline{a}_u^{ss}(D); \qquad (10)\\
jd_{cd}^{ss}(a) &= 1-(1+\gamma)^{-\frac{1}{1-\alpha}} \text{ for } \overline{a}_u^{ss}(D)<a<\overline{a}_g^{ss}(D).
\end{aligned}
$$

Apparently, $jd_{cd}^{ss}(a)$ weakly increases in a and is bigger with higher γ. As a cohort ages, the fraction of learning plants declines and that of aging plants rises. Hence, the creative-destruction driven job destruction by aging plants rises with cohort age.

Two remarks should be made. First, the creative destruction effect not only causes the job destruction by aging plants, but also drives the job creation by entry as well as the job destruction by exit at $\overline{a}_u^{ss}(D)$ and $\overline{a}_g^{ss}(D)$. The job creation by entry occurs before the plant life cycle starts, which does not contradict the intuition that the creative destruction effect strengtherns with plant age. However, the job destruction by exit at $\overline{a}_u^{ss}(D)$ and $\overline{a}_g^{ss}(D)$ adds to this intuition, as such exits occur only at older ages. Put intuitively, as a cohort ages, the creative destruction effect drives more and more plants to shrink; as this effect strengtherns over time, it eventually causes the entire cohort to exit.

Second, note that the learning and creative destruction effects interact. It is the decreasing property of the marginal learing effect that causes the creative destruction effect to strenthern. Moreover, according to (9), the magnitude of the learning-driven job creation equals $\left[\left(\frac{\theta_g}{1+\gamma}\right)^{\frac{1}{1-\alpha}}-\theta_u^{\frac{1}{1-\alpha}}\right]$. Put intuitively, when a plant learns its $\theta=\theta_g$, its vintage also grows one period older as captured by $\frac{\theta_g}{1+\gamma}$. This suggets that the creative destruction effect dampens the learning effect in driving the job creation.

Figure 5 displays the life cycle of a representative cohort. It presents the employment dynamics of an "average" plant, as the sum of a good plant employment weighted by good plants' fraction and an unsure plant's employment weighted by unsure plants' fraction. When the "average" plant is very young, it keeps creating jobs because the learning effect dominates the creative destruction effect. As it grows older, the learning effect weakens and the creative destruction effect strengtherns. Once the creative destruction effect dominates, its employment begins to decline. Its employment jumps at $\overline{a}_u^{ss}(D)$ due to the exit of all unsure plants. After $\overline{a}_u^{ss}(D)$, it keeps destroying jobs due to the creative destruction effect, and exits at $\overline{a}_g^{ss}(D)$.

4.4 Aggregate Employment Dynamics

The aggregate employment dynamics is given by aggregating the cohort life cycle to the industry level. It reflects the number of plants choosing to adjust employment and the magnitude of their adjustment. Demand has significant influences on both dimensions.

First, demand influences the magnitude of employment adjustment. According to (8), D affects n^{ss} through $\overline{PA}$. $\overline{PA}$ does not enter (9) or (10), because it remains constant at the steady state and thus cancels out. However, $\overline{PA}$ would fluctuate over time with stochastic demand variations. In that case, a decrease in PA amplifies job destruction but dampens job creation, and an increase in PA does the opposite. In extreme cases, an increase in PA may dominate the creative destruction effect, so that the aging plants create jobs even if their vintages grow older; or, a decrease in PA may dominate the learning effect, so that the learning plants destroy jobs even if they learn their idiosyncratic productivity to be good.

Second and more importantly, demand influences the number of plants that choose to adjust employment. Note that, from a pure accounting point of view, the industry features four job-flow margins: the entry margin, the learning margin, the aging margin, and the exit margin at $\overline{a}_u^{ss}(D)$ and $\overline{a}_g^{ss}(D)$. Demand affects the number of plants at all these margins through its influence on $f^{ss}(0, D)$, $\overline{a}_u^{ss}(D)$ and $\overline{a}_g^{ss}(D)$:

Proposition 4 *At a steady-state equilibrium, $f^{ss}(0, D)$, $\overline{a}_u^{ss}(D)$ and $\overline{a}_g^{ss}(D)$ all weakly increase in D.*

See the appendix for proof. Proposition 4 suggests that, if we compare two steady-state equilibria, one with a high demand and the other with a low demand, the low-demand steady state features less entry and younger exit ages. Intuitively, lower demand reduces the number of plants at the entry margin but raises that at the exit margin. Moreover, younger exit ages imply shorter plant life cycle, less time for unsure plants to learn, and thus smaller number of plants at the learning margin.

If this intuition carries over when demand fluctuates stochastically, then a drop in demand would cause less entry, suggesting a drop in job creation by young plants, and more exit, implying a rise in job destruction by old plants. Moreover, if demand varies stochastically, birth cohorts would start at different sizes, so that, at any time point, the size of a cohort of age a depends on the demand level a periods ago. Therefore, past demand variations determine current plant distribution and, consequently, the number of plants at the learning and aging margins. The next section explores quantitatively our model's implications on the aggregate employment dynamics with stochastic demand.

5 Quantitative Implications

This section applies numerical techniques to analyze a stochastic version of our model, in which demand follows a two-state Markov process with values $[D_h, D_l]$ and transition probability μ. Throughout this section, plants expect the current demand to persist for the next period with probability μ, and to change with probability $1 - \mu$.

We first calibrate our model to data presented in Section 2. Then we approximate value functions and the corresponding law of motion using the approach by Krusell and Smith (1998). With the approximated law of motion, we simulate the calibrated model and examine whether our model can deliver the observed age pattern of aggregate employment dynamics.

5.1 Baseline Calibration

Table 2 presents the parameter values in our baseline calibration. Some of the parameter values are pre-chosen. We allow a period to represent one quarter and set the quarterly discount factor $\beta = 0.99$. μ is chosen to equal 0.95 so that demand switches between a high level and a low level with a constant probability 0.05 per quarter. Ψ is set at 1 as it matters only as a scale for D. We set α, the elasticity of production with respect to labor input, to 0.66 as the share of return to labor.

Some other parameter values are chosen according to the existent empirical evidence. The elasticity of entry cost with respect to entry size, c_1, is chosen based on Goolsbee (1998), who estimates that a 10% increase in demand for equipment raises equipment price by 7.284% (page 143, Table VII). Accordingly, $c_1 = 0.7284$. The relative productivity of good and bad plants is chosen following Davis and Haltiwanger (1998), who assume a ratio of high-to-low productivity of 2.4 based on between-plant productivity differentials reported by Bartelsman and Doms (2000). We normalize productivity of bad plants as 1 and set productivity of good plants as 2.4. The technological pace, γ, determines the strength of the creative destruction effect. The technology pace, γ, is set according to Basu et al (2001), who estimate a quarterly technological pace of 0.0037 for durable manufacturing and a pace of 0.0027 for non-durable manufacturing. We set $\gamma = 0.003$.

The strength of the learning force is jointly determined by the prior probability of being a good plant (φ) and the quarterly pace of learning (p). Since young plants feature stronger learning effect, φ and p are calibrated to the observed job creation and destruction rates by young plants. Table 1 shows that, in the U.S. manufacturing sector, young plants' mean job creation rate equals 7.52% and their mean job destruction rate equals 6.56%. Let jc_y to denote young plants' job creation rate, jd_y their job destruction rate. Young plants are defined as those younger than 40 quarters as in Table 1. This sets the following two restrictions on φ and p: $jc\_y = 7.52\%$ and $jd\_y = 6.56\%$. With other parameter values given by Table 2, jc_y and jd_y are functions of φ and p only as $\overline{PA}(D)$ cancels out. Using a search algorithm, we find $p = 0.083$ and $\varphi = 0.017$.

The remaining parameters are high demand D_h, low demand D_l, and the fixed component of the entry cost c_0. Proposition 4 suggests that demand variations cause exit ages to vary, which is reflected in the data as fluctuations in job destruction. Thus, the values of D_h, D_l, and c_0 are chosen to match the observed moments of job destruction. From 1972 to 1993, the U.S. manufacturing job destruction rate fluctuates between 2.96% and 11.60% with a mean of 5.6%. This puts the following restrictions on our calibrated model.

First, its implied long-run job destruction rate must be around 5.6%. We let $\overline{a_g}$ and $\overline{a_u}$ to represent the maximum ages of good plants and unsure plants at the high-demand steady state and $\overline{a_g}'$ and $\overline{a_u}'$ represent the exit ages at the low-demand steady state. The steady-state job destruction rate implied by either pair, has to be around 5.6%.

Second, we match the peak in job destruction that occurs at the onset of a recession. Our model suggests that the jump in the job destruction rate at the beginning of a recession comes from the shift of exit margins to younger ages. We assume that when demand drops, the exit margins shift from $\overline{a_g}$ and $\overline{a_u}$ to $\overline{a_g}'$ and $\overline{a_u}'$ immediately, and the job destruction rate at this moment must not exceed 11.6%.[6]

Third, we match the trough in job destruction that occurs at the onset of a boom. Our model suggests that when demand goes up, the exit margins extend to older ages, so that for several subsequent periods job destruction comes only from the learning margin, implying a trough in the job destruction rate. To match the data, the job destruction rate at this moment has to be around 3%.

Additionally, $(\overline{a_g}, \overline{a_u})$ and $(\overline{a_g}', \overline{a_u}')$ must satisfy the steady state conditions on the gap between the exit ages of good and unsure plants. Using a search algorithm, we find that these conditions are satisfied for the following combination of parameter values: $\overline{a_g} = 78$, $\overline{a_u} = 64$, $\overline{a_g}' = 73$, $\overline{a_u}' = 59$, and $c_0 = 0.1587$. By applying these $\overline{a_g}$, $\overline{a_u}$, $\overline{a_g}'$ and $\overline{a_u}'$ to the

[6] As I have noted earlier, the calibration exercises suggest that when a negative aggregate demand shock strikes, the exit margins shift more than $\overline{a_g}'$ and $\overline{a_u}'$. The bigger shift implies a bigger jump in job destruction, This is why I require $neg_{\max}$ to lie below 11.60%. I experiment with different demand levels to find those that generate the closest fit.

steady state industry structure, we find $D_h = 925.98$ and $D_l = 873.90$.

5.2 Aggregate Employment Fluctuations

With all of the parameter values assigned, we approximate the value functions. Our key computational task is to map F, the plant distribution across ages and idiosyncratic productivity, given demand level D, into a set of value functions $V(\theta^e, a; F, D)$. Unfortunately, the endogenous state variable F is a high-dimensional object. The numerical solution of dynamic programming problems becomes increasingly difficult as the size of the state space increases. Our computational strategy follows Krusell and Smith (1998) by shrinking the state space into a limited set of variables and showing that these variables' laws of motion can approximate the equilibrium behavior of plants in the simulated time series. The appendix presents the details. The approximated laws of motion suggests that the dynamic system is globally stable: the industry structure eventually settles down with constant entry and exit along any sample path where the demand level is unchanging. With the corresponding decision rules and an initial plant distribution, we can then investigate the aggregate time series properties of employment fluctuations in the calibrated model.

5.2.1 Simulation Statistics from Baseline Calibration

We start with a random plant distribution across ages and idiosyncratic productivity, simulate the model for 5000 periods according to the approximated value functions, and discard the first 500 periods to investigate the property of the stationary region of the simulated time series. Table 3 reports the statistics of the simulated job flows under the baseline calibration.

Table 3 suggests that the calibrated model does deliver, at least qualitatively, the relationship between plant age and the job-flows properties: the means of job flow rates decline with plant age; and the relative volatilities of job destruction to creation rises with age; these patterns are evident even after excluding the contributions of plant birth and plant death.

However, two remarks should be made. Note that, in Table 3, our simulated magnitudes of old plants' job flows are significantly smaller than those in the data as shown in Table 1. For example, the mean job creation for old plants is 4.40% in the data, but it is only 0.55% in our simulation. Two factors contribute to this difference. First, our simulation produces a zero mean for old plants' job creation by plant birth because, by definition, newborn plants cannot be old. But the corresponding statistic in the data is 0.12%. This is because, in reality, some old plants cease operation temporarily and reopen later. Such temporary exit behavior is not captured in our model. Second, the all-or-nothing learning process, while greatly simplifying our analysis, imposes very little learning to old plants as many old cohorts contain no unsure plants at all. Due to such limited learning, old aging plants experience much less job creation and destruction in our calibrated model.

Moreover, the difference between the variance ratio of job destruction over job creation for old plants and that for young plants is rather small compared to that in the data. For example, this ratio rises from 1.20 for young plants to 1.85 for old plants, but the corresponding statistics in Table 1 are 1.01 and 3.75.

5.2.2 Strength of the Learning and Creative Destruction Effects: Additional Calibration

We further explore the impact of the relative strength of the learning and creative destruction effects on the relative volatility of job destruction and job creation. In the baseline calibration, we set the quarterly technological pace γ to 0.003 according to Basu et al (2001). An alternative estimate of γ is provided by Caballero and Hammour (1994), who assume a quarterly technological pace of 0.007 in their model of creative destruction to match the observed U.S. manufacturing job flows. To explore how technological pace impacts simulation results, we re-calibrate our model assuming $\gamma = 0.007$, approximate the corresponding laws of motion, and simulate the recalibrated model. The re-calibration results are presented in Table 4; key results from the baseline calibration are also included for comparison.

Table 5 shows that, with $\gamma = 0.007$, our model needs a slower learning pace ($p = 0.075$)

and a higher prior probability of being good ($\varphi = 0.030$) to match the observed magnitude of young plants' job flows. Simulations using the new calibration generates much bigger difference between the relative volatility of destruction and creation for young plants and that for old plants: 1.51 and 3.57, compared to 1.20 and 1.85 in the baseline calibration.

This result can be explained as follows. A a lower p and a higher φ imply a weaker learning effect: plants not only learn slowlier, but are less likely to learn that they are bad. But a faster technological pace implies that the creative destruction effect is strong: old plants become more technologically disadvantaged. Note that, in Table 5, the employment share of old plants falls with $\gamma = 0.007$, because plants live shorter life cycles in general with faster technological pace. With stronger creative destruction effect but weaker learning effect, the exit margins driven by the creative destruction effect becomes much more responsive on the one hand, so that the variance of old plants' job destruction rises; but the learning-driven job creation becomes less responsive on the other hand, so that the variance of all plants' job creation falls. The two effects together drive up the variance ratio of job destruction and job creation for old plants.

6 Conclusion

We propose a model of plant life cycle that can generate the observed age pattern in aggregate employment dynamics. Our model combines the learning effect with the creative destruction effect. Under the learning effect, business cycles generate symmetric responses in job creation and job destruction while, under the creative destruction effect, it is the job destruction that are more responsive. The key ingredient of our model is that the marginal learning decreases in plant age. Since, the learning effect weakens but the creative destruction effect strengtherns as plants age, the relative volatility of job destruction over job creation rises.

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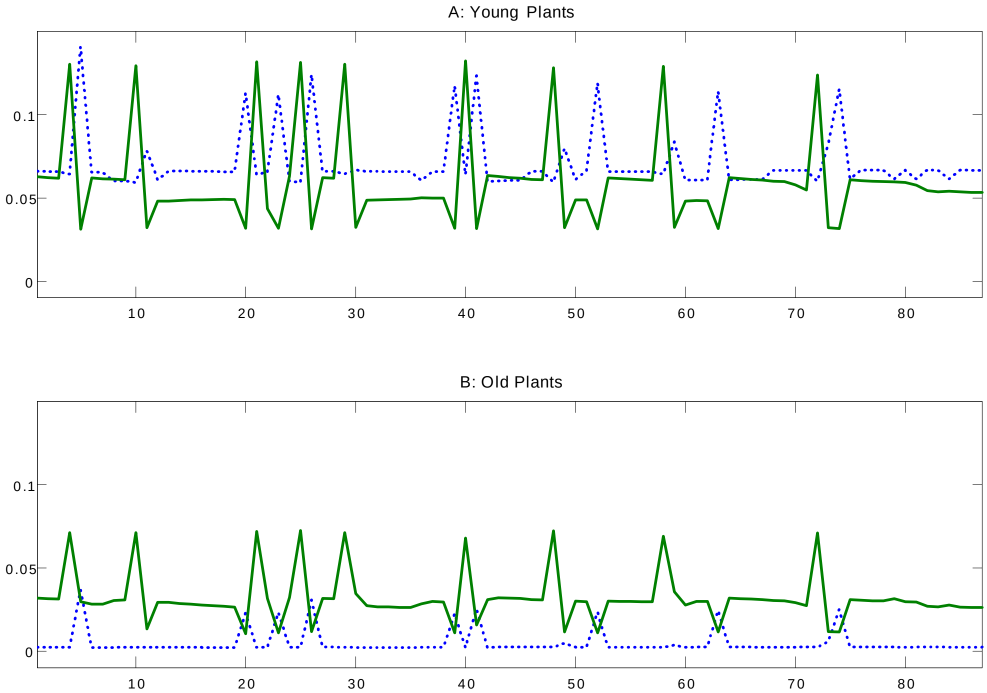

Figure 1: Job flows at young and old plants, 1972:2 – 1988:4. Dashed lines represent the job creation series; solid lines represent job destruction.

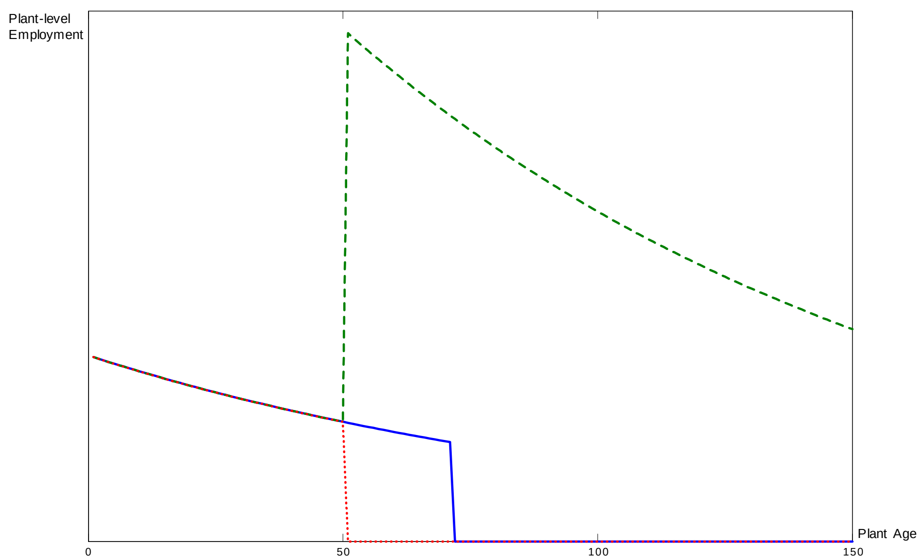

Figure 2: Employment Dynamics at Three Representative Plants: solid line plots the employment dynamics at an unsure plant that never learns and exit after 71 periods; dotted line plots those at a plant that learns it is a bad plant after 50 periods; dashed line plots those at a plant that learns it is a good plant after 50 periods and exit after 150 periods. This graph is generated by assuming $\theta_g = 2.4$, $\theta_b = 1$, $p = 0.05, \varphi = 0.5$,and $\gamma = 0.003$. The steady-state solution corresponding to a demand level of 10000 for the exit age of good plants is 150, for the exit age of an unsure plant is 71.

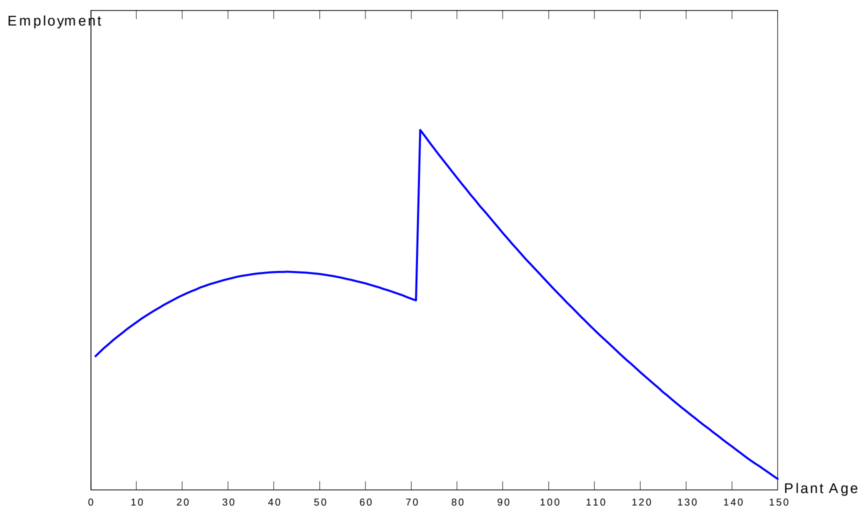

Figure 3: Average Plant-level Employment of A Birth Cohort: average plant-level employment equals the cohort-level employment divided by the number/measure of plants in this cohort; the steady-state parameter values and solutions are the same as in Figure 3.

A. Means						
Plant type	$E(Cb)$	$E(Cc)$	$\mathbf{E(C)}$	$E(Dd)$	$E(Dc)$	$\mathbf{E(D)}$
all	0.39	4.60	4.99	0.61	4.70	5.31
young	1.65	5.98	**7.63**	1.27	5.23	**6.50**
old	0.12	4.27	**4.40**	0.45	4.58	**5.03**

B. Variance ratio of job destruction to job creation		
plant type	$\boldsymbol{\sigma}(\mathbf{D})^2/\boldsymbol{\sigma}(\mathbf{C})^2$	$\sigma(Dc)^2/\sigma(Cc)^2$
all	**3.27**	2.99
young	**1.01**	1.47
old	**3.75**	3.36

Table 1: Quarterly gross job flows from plant birth, plant death, and continuing operating plants in the US manufacturing sector: 1973 II to 1988 IV. Young plants are defined as those younger than 40 quarters. Cb denotes job creation from plant birth, Dd job destruction from plant death, Cc and Dc job creation and destruction from continuing operating plants.ong old plants, although old plants are those older than 40 quarters. C and D represent gross job creation and destruction. C=Cc+Cb, D=Dd+Dc. All numbers are in percentage points.

Calibrated Parameters	**value**
productivity of bad plants: θ_b	1
productivity of good plants: θ_g	2.4
quarterly technological pace: γ	0.003
quarterly discount factor: β	0.99
the elasticity of production w.r.t. labor: α	0.66
entry cost parameter: c_0	0.158
entry cost parameter: c_1	0.728
persistence rate of demand: μ	0.95
prior probability of being a good plant: φ	0.017
quarterly pace of learning: p	0.083
Operation cost per period: Ψ	1
High Demand: D_h	925.98
Low Demand: D_l	873.90

Table 2: Base-line Parameterization of the Model

A. Means						
Plant type	$E(Cb)$	$E(Cc)$	$\mathbf{E(C)}$	$E(Dd)$	$E(Dc)$	$\mathbf{E(D)}$
all	3.07	**1.59**	4.66	2.40	**2.83**	5.22
young	5.06	**2.26**	7.32	3.00	**2.21**	5.21
old	0	**0.55**	0.55	1.45	**3.25**	4.65

B. Variance ratio of job destruction to job creation		
plant type	$\boldsymbol{\sigma}(\mathbf{D})^2/\boldsymbol{\sigma}(\mathbf{C})^2$	$\sigma(Dc)^2/\sigma(Cc)^2$
all	1.21	1.28
young	1.20	1.24
old	1.85	1.58

Table 3: simulated job flows from plant birth, plant death, and continuing operating plants under the baseline calibration, Cb is job creation by plant entry; Dd is job destruction by plant death; Cc and Dc are job creation and destruction from continuing operating plants. C and D represent gross job creation and destruction. C=Cc+Cb, D=Dd+Dc. All numbers are in percentage points.

A. Additional Calibration $p = 0.075$, $\varphi = 0.030$, $\gamma = 0.007$				
Plant type	$Mean(C)$	$Mean(D)$	$Emp.Share$	$\frac{Var(D)}{Var(C)}$
all	4.66	5.22	100	1.66
young	7.31	6.20	75.4	1.51
old	0.55	3.70	24.6	3.57
B. Baseline Calibration $p = 0.083$, $\varphi = 0.017$, $\gamma = 0.003$				
Plant type	$Mean(C)$	$Mean(D)$	$Emp.Share$	$\frac{Var(D)}{Var(C)}$
all	4.33	4.27	100	1.22
young	7.66	6.36	65.8	1.20
old	0.62	2.22	34.2	1.85

Table 4: Job-flow Statistics from Baseline Calibration and Additional Calibration.

Appendix

Proof. According to steady-state demand condition,

$$D = (PA)^{\frac{1}{1-\alpha}} \left\{ \begin{array}{c} \sum_{a=0}^{\overline{a}_u^{ss}(D)} f^{ss}(\theta_u, a; D)\, \alpha^{\frac{\alpha}{1-\alpha}} \left[\frac{\theta_u}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} + \\ \sum_{a=0}^{\overline{a}_g^{ss}(D)} f^{ss}(\theta_g, a; D)\, \alpha^{\frac{\alpha}{1-\alpha}} \left[\frac{\theta_g}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} \end{array} \right\}. \tag{8}$$

, where $f^{ss}(\theta^e, a; D)$ is the steady-state measure of plants with age a and the expected idiosyncratic productivity θ^e. More specifically,

$$\begin{aligned} f^{ss}(\theta_u, a; D) &= f^{ss}(0, D)(1-p)^a \\ f^{ss}(\theta_g, a; D) &= f^{ss}(0, D)\,\varphi\,[1-(1-p)^a] \end{aligned} \tag{9}$$

By definition, a steady state features time-invariant distribution of plants across a and θ^e. This implies that PA has to be time-invariant for (8) to hold.

In addition to demand condition as (8), $f^{ss}(0, D)$, $\overline{a}_u^{ss}(D)$ and $\overline{a}_g^{ss}(D)$ have to satisfy the following conditions. The exit condition for a good plant is:

$$(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}) \left[\frac{PA\theta_g}{(1+\gamma)^{\overline{a}_g^{ss}(D)}}\right]^{\frac{1}{1-\alpha}} - \Psi = 0 \tag{10}$$

The exit condition for an unsure plant is

$$\begin{aligned} 0 = {} & (\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}) \left[\frac{PA\theta_u}{(1+\gamma)^{\overline{a}_u^{ss}(D)}}\right]^{\frac{1}{1-\alpha}} - \Psi + \\ & p\varphi \sum_{a=\overline{a}_u^{ss}(D)+1}^{\overline{a}_g^{ss}(D)} \beta^{a-\overline{a}_u^{ss}(D)} \left\{ (\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}) \left[\frac{PA\theta_g}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} - \Psi \right\}; \end{aligned} \tag{11}$$

The free entry condition is:

$$c_0 + c_1 f^{ss}(0,D) = \sum_{a=0}^{\overline{a}_u^{ss}(D)} \beta^a (1-p)^a \left\{ \left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right) \left[\frac{PA\theta_u}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} - \Psi \right\} + \sum_{a=0}^{\overline{a}_g^{ss}(D)} \beta^a \varphi \left[1-(1-p)^a\right] \left\{ \left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right) \left[\frac{PA\theta_g}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} - \Psi \right\}. \quad (12)$$

Furthermore, (10) suggests:

$$(PA)^{\frac{1}{1-\alpha}} = \frac{\Psi}{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)} \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g}\right]^{\frac{1}{1-\alpha}} \quad (13)$$

Plugging (13) and (9) into (8) gives

$$D = \frac{\Psi}{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)} \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g}\right]^{\frac{1}{1-\alpha}} f^{ss}(0,D) \left\{ \begin{array}{c} \sum_{a=0}^{\overline{a}_u^{ss}(D)} (1-p)^a \alpha^{\frac{\alpha}{1-\alpha}} \left[\frac{\theta_u}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} + \\ \sum_{a=0}^{\overline{a}_g^{ss}(D)} \varphi\left[1-(1-p)^a\right] \alpha^{\frac{\alpha}{1-\alpha}} \left[\frac{\theta_g}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} \end{array} \right\}. \quad (14)$$

Plugging (13) into (11) gives

$$\frac{1-\beta^{\overline{a}_g^{ss}(D)-\overline{a}_u^{ss}(D)+1}}{1-\beta} = \left(\frac{\theta_u}{\theta_g}\right)^{\frac{1}{1-\alpha}} (1+\gamma)^{\frac{\overline{a}_g^{ss}(D)-\overline{a}_u^{ss}(D)}{1-\alpha}} + p\varphi \frac{\frac{\beta}{(1+\gamma)^{\frac{1}{1-\alpha}}}}{1-\frac{\beta}{(1+\gamma)^{\frac{1}{1-\alpha}}}} \left[(1+\gamma)^{\frac{\overline{a}_g^{ss}(D)-\overline{a}_u^{ss}(D)}{1-\alpha}} - \beta^{\overline{a}_g^{ss}(D)-\overline{a}_u^{ss}(D)}\right] \quad (15)$$

Notice that D does not enter (15), so that, as long as (15) determines an unique value for $\overline{a}_g^{ss}(D) - \overline{a}_u^{ss}(D)$, (14) and (12) (with (13) plugged in) would jointly determine $\overline{a}_g^{ss}(D)$ and $f^{ss}(0,D)$ with $\overline{a}_u^{ss}(D) = \overline{a}_g^{ss}(D) - \left(\overline{a}_g^{ss}(D) - \overline{a}_u^{ss}(D)\right)$. It turns out that, for (15) to reveal a unique solution for $\overline{a}_g^{ss}(D) - \overline{a}_u^{ss}(D)$, it requires that $\theta_u < \theta_g$, which holds by definition. This proves Proposition 1. ■

Proof. Proposition 2. Plugging (13) into (12) gives

$$c_0 + c_1 f^{ss}(0, D) = \sum_{a=0}^{\overline{a}_u^{ss}(D)} \beta^a (1-p)^a \Psi \left\{ \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} \left[\frac{\theta_u}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} - 1 \right\} + \sum_{a=0}^{\overline{a}_g^{ss}(D)} \beta^a \varphi \left[1-(1-p)^a\right] \Psi \left\{ \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} \left[\frac{\theta_g}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} - 1 \right\} \quad (16)$$

which suggests

$$f^{ss}(0, D) = \left(\begin{array}{c} \sum_{a=0}^{\overline{a}_u^{ss}(D)} \beta^a (1-p)^a \Psi \left\{ \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} \left[\frac{\theta_u}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} - 1 \right\} + \\ \sum_{a=0}^{\overline{a}_g^{ss}(D)} \beta^a \varphi \left[1-(1-p)^a\right] \Psi \left\{ \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} \left[\frac{\theta_g}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} - 1 \right\} - c_0 \end{array} \right) / c_1. \quad (17)$$

Combining (14) and (17) gives

$$D = \frac{\Psi}{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)} \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} \left\{ \begin{array}{c} \sum_{a=0}^{\overline{a}_u^{ss}(D)} (1-p)^a \alpha^{\frac{\alpha}{1-\alpha}} \left[\frac{\theta_u}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} + \\ \sum_{a=0}^{\overline{a}_g^{ss}(D)} \varphi \left[1-(1-p)^a\right] \alpha^{\frac{\alpha}{1-\alpha}} \left[\frac{\theta_g}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} \end{array} \right\} \cdot \left(\begin{array}{c} \sum_{a=0}^{\overline{a}_u^{ss}(D)} \beta^a (1-p)^a \Psi \left\{ \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} \left[\frac{\theta_u}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} - 1 \right\} + \\ \sum_{a=0}^{\overline{a}_g^{ss}(D)} \beta^a \varphi \left[1-(1-p)^a\right] \Psi \left\{ \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} \left[\frac{\theta_g}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} - 1 \right\} - c_0 \end{array} \right) / c_1. \quad (18)$$

where $\overline{a}_u^{ss}(D) = \overline{a}_g^{ss}(D) - \left(\overline{a}_g^{ss}(D) - \overline{a}_u^{ss}(D)\right)$ with $\left(\overline{a}_g^{ss}(D) - \overline{a}_u^{ss}(D)\right)$ determined by (15) independently. Apparently, the right-hand side of (18) increases monotonically in $\overline{a}_g^{ss}(D)$. This implies that higher D leads to higher $\overline{a}_g^{ss}(D)$ and $\overline{a}_u^{ss}(D)$. Moreover, the right-hand side of (17) also increases monotonically in $\overline{a}_g^{ss}(D)$, which suggests that, by causing higher $\overline{a}_g^{ss}(D)$, higher D will also give higher $f^{ss}(0, D)$. This proves Proposition 3. ■

6.1 Approximating Value Functions with Krusell & Smith (1998) Approach

The key computational task is to map F, the plant distribution across ages and idiosyncratic productivity, given demand level D, into a set of value functions $V(\theta^e, a; F, D)$. To make the state space tractable, we define a variable X such that:

$$X(F) = \sum_a \sum_{\theta^e} f(\theta^e, a) \, .q(\theta^e, a) \tag{19}$$

where $f(\theta^e, a)$, as a component of F, measures the mass of plants with expected idiosyncratic productivity θ^e and age a. Apparently,

$$P(F, D) \cdot A = \frac{D}{X(F')} = \frac{D}{X(H(F, D))}. \tag{20}$$

F' is the updated plant distribution after entry and exit and $F' = H(F, D)$; $P(F, D)$ is the equilibrium price in a period with initial aggregate state (F, D). Plugging (20) into (3) gives

$$\pi(a, \theta; F, D) = (\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}) \left[\frac{D}{X(H(F, D))}\right]^{\frac{1}{1-\alpha}} \left[\frac{\theta^e}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} - \Psi. \tag{21}$$

Thus, the aggregate state (F, D) and its law of motion help plants to predict future profitability by suggesting sequences of X's from today onward under different paths of demand realizations. The question then is: what is the plant's critical level of knowledge of F that allows it to predict the sequence of $X's$ over time? Although plants would ideally have full information about F, this is not computationally feasible. Therefore we need to find an information set Ω that delivers a good approximation of plants' equilibrium behavior, yet is small enough to reduce the computational difficulty.

We look for an Ω through the following procedure. In step 1, we choose a candidate Ω. In step 2, we postulate perceived laws of motion for all members of Ω, denoted H_Ω, such that $\Omega' = H_\Omega(\Omega, D)$. In step 3, given H_Ω, we calculate plants' value functions on a grid of

Ω	$\{X\}$
H_Ω	$H_x(X, D_h)$: $\log X' = 0.0415 + 0.9937 \log X$ $H_x(X, D_l)$: $\log X' = 0.0495 + 0.9924 \log X$
R^2	for D_h: 0.9996 for D_l: 0.9989
standard forecast error	for D_h: $4.4 \cdot 10^{-7}\%$ for D_l : $4.7 \cdot 10^{-7}\%$
maximum forecast error	for D_h: $1.76 \cdot 10^{-6}\%$ for D_l: $1.78 \cdot 10^{-6}\%$
Den Haan & Marcet test statistic (χ^2_7)	0.4228

Table 5: The Estimated Laws of Motion and Measures of Fit

points in the state space of Ω applying value function iteration, and obtain the corresponding industry-level decision rules – entry sizes and exit ages across aggregate states. In step 4, given such decision rules and an initial plant distribution. We simulate the behavior of a continuum of plants along a random path of demand realizations, and derive the implied aggregate behavior — a time series of Ω. In step 5, we use the stationary region of the simulated series to estimate the *implied* laws of motion and compare them with the *perceived* H_Ω; if different, we update H_Ω, return to step 3 and continue until convergence. In step 6, once H_Ω converges, we evaluate the fit of H_Ω in terms of tracking the aggregate behavior. If the fit is satisfactory, we stop; if not, we return to step 1, make plants more knowledgeable by expanding Ω, and repeat the procedure.

We start with $\Omega = \{X\}$ — plants observe X instead of F. We further assume that plants perceive the sequence of future coming $X's$ as depending on nothing more than the current observed X and the state of demand. The perceived law of motion for X is denoted H_x so that $X' = H_x(X, D)$. We then apply the procedure described above and simulate the behavior of a continuum of plants over 10000 periods. The results are presented in Table 5.

As shown in Table 5, the estimated H_x is log-linear. The fit of H_x is quite good, as suggested by the high R^2, the low standard forecast error, and the low maximum forecast error. The good fit when $\Omega = \{X\}$ implies that plants perceiving these simple laws of motion make only small mistakes in forecasting future prices. To explore the extent to which the

Ω	$\{X, \sigma_a\}$
H_Ω	booms ($\log X$): $\log X' = -1.0406 + 0.9954 \log X + 0.1262\sigma_a$ booms(σ_a): $\sigma_a' = 0.2785 - 0.0068 \log X + 0.9754\sigma_a$ recessions($\log X$): $\log X' = -1.0371 + 0.9963 \log X + 0.8988\sigma_a$ recessions(σ_a): $\sigma_a' = 0.2775 - 0.0065 \log X + 0.9751\sigma_a$
R^2	booms ($\log X$): 0.9999 recessions($\log X$): 0.99999 booms (σ_a): 0.9989 recessions(σ_a): 0.9990
standard forecast error	booms ($\log X$): $1.1 \cdot 10^{-8}\%$ recessions($\log X$): $1.2 \cdot 10^{-8}\%$ booms (σ_a): $6.4 \cdot 10^{-9}\%$ recessions(σ_a):$6.25 \cdot 10^{-9}\%$
maximum forecast error	booms ($\log X$): $4.87 \cdot 10^{-8}\%$ recessions($\log X$):$5.05 \cdot 10^{-8}\%$ booms (σ_a):$1.48 \cdot 10^{-8}\%$ recessions(σ_a):$1.51 \cdot 10^{-8}\%$
Den Haan & Marcet test statistic (χ^2_7)	0.4375

Table 6: The Estimated Laws of Motion with two moments and Measures of Fit

forecast error can be explained by variables other than X, we implement the Den Haan and Marcet (1994) test using instruments $[1, X, \mu_a, \sigma_a, \gamma_a, \kappa_a, r_u]$, where μ_a, σ_a, γ_a, κ_a,r_u are the mean, standard deviation, skewness, and kurtosis of the age distribution of plants, and the fraction of unsure plants, respectively. The test statistic is 0.4228, well below the critical value at the 1% level. This suggests that given the estimated laws of motion, we do not find much additional forecasting power contained in other variables. Nevertheless, we expand Ω further to include σ_a, the standard deviation of the age distribution of firms. The results when $\Omega = \{X, \sigma_a\}$ are shown in Table 6.