Computing Optimal policy in a Timeless-Perspective: An application to a small-open economy

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Abstract

Since the contribution of Kydland and Prescott (1977), it is well known that the optimal Ramsey policy is time inconsistent. In a series of recent contributions, Woodford (2003) proposes a new methodology to circumvent this problem, namely the timeless perspective solution. However, one main limitation is that it is not yet empirically implementable. In this paper, we develop a new methodology to compute initial values of the Lagrange multipliers in order to implement the timeless-perspective solution. In so doing, we also provide a generalization of the Ramsey and timeless-perspective problems. We apply our results to a small-open economy model in Canada.

JEL Classifications:

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1 Introduction

Since the seminal contribution of Kydland and Prescott (1977, 1980), it is well known that the fully unconstrained optimal policy (the Ramsey problem) has the main drawback of being time inconsistent—without incentives or punitive schemes, the optimal policy chosen in the past or the contemporaneous period is no longer optimal if the social planner can re-optimize at a later date. To mitigate the pervasive effects of the optimal but time inconsistent Ramsey policy, Woodford (1999, 2003), Giannoni and Woodford (2002), Benigno and Woodford (2003, 2004a and 2004b) have developed the concept of optimal policy in a timeless perspective. This solution may be viewed as representing the continuation of the Ramsey policy an arbitrarily long period after the initial period when the policy was implemented. However, despite its attractive feature, one potential pitfall of the timeless-perspective approach is that it is not yet empirically implementable in the sense that estimates of the Lagrange multipliers of implementability constraints (namely the behavioral equations of the private sector) at the initial period are necessary and are obviously non observable. At the same time, for many practical policy issues, it could be interesting to obtain quantitative and normative predictions of the timeless-perspective policy in reasonably well-specified models. The purpose of this paper is to fill this gap in the literature and to provide for the first time, to our knowledge, a (Bayesian) methodology to estimate the solution suggested by Woodford (1999). This allows us to compare the outcomes of the Ramsey- and timeless perspective-based solutions.

Traditionally, the new science of monetary policy (see Clarida, Gali and Gertler, 1999) deals with the problem of optimal policy as follows. At the current period $t = t_0$, a social planner (say an independent central bank) seeks to determine the path of the nominal interest rate and maximizes an objective function, which reflects the social preferences of individual agents, subject to implementability constraints.[1] In this respect, two solutions have been widely studied : the commitment and discretion outcomes. In the former, as is shown by Kydland and Prescott (1977), the Ramsey (or commitment) solution is optimal but time-inconsistent. This stems from the fact that, in the contemporaneous period, the policy maker sets the new policy after that the private agents have formed their expectations, so that it can take advantage of this situation and has therefore an incentive to re-optimize in the future. From a technical point of view, time-inconsistency comes from the fact that the policymaker, when it re-optimizes at a later date, resets the Lagrange multipliers to zero, away from the values provided by running forward the solution function in the first optimization. Note also that the Ramsey problem is not recursive in the natural state variables. This further complicates the analysis substantially in the presence of stochastic shocks as in standard DSGE models. In contrast, the discretionary solution—the central bank re-optimizes at each period—is time consistent but can lead to extremely undesirable outcomes regarding, for instance, the conditional or unconditional welfare and the stabilization bias (see Walsh, 2003).

The timeless-perspective solution suggests that the policy maker gives up its first period advantage and sets the policy instruments in the first period as it will in the future. The policy maker therefore behaves as if the Ramsey optimal rule had been computed in the

[1] We assume that the objective function of the central bank cannot be different from those of society, as in the optimal targeting regimes literature (see Jensen, 2002; and Walsh, 2003).

remote past (stationary solution).[2] More formally, the timeless perspective solution is one way of dealing with a recursive Ramsey problem. Indeed, it is possible to formulate the Ramsey problem recursively by augmenting the set of natural state variables with a vector of co-state variables, which depends on the problem studied. As is pointed out by Marcet and Marimon (1998), solving this recursive problem leads to Markovian policy rules in the augmented set of states and the shocks for $t > t_0$.[3] The Ramsey equilibrium outcome thus depends on the values of the exogenous state variables at the initial period t_0. To deal with this dependence, Woodford (1999, 2003) proceeds by imposing that the Markovian policy rule, which is optimal from the standpoint of $t \geq t_0 + 1$, is also optimal at time t_0. In other words, this amounts to endogeneizing the initial values of the exogenous states. Therefore, the policy maker renounces to the possibility of setting the Lagrange multipliers to zero in the initial period and sets them by using the same recurrence as in other periods.

From a practical point of view, this temporal dependence complicates the derivation of policy implications. In particular, one cannot use directly the methodology proposed by Söderlind (1999) to estimate the Ramsey or discretionary policy rule. A further difficulty comes from the fact that usually not all variables of a DSGE model are observed and the shocks almost never. To circumvent these problems, we propose to proceed in practise as follows. First, we estimate the dynamics of the economy on the period prior to the switch to optimal policy. To do so, we need to specify an empirical policy rule simply aimed at describing the policy in effect at that time. This first step allows for computing the optimal policy under commitment and to derive smoothed values of the unobservable state variables during the pre-period of implementation of the timeless-perspective solution. Moreover, it permits to recompute an artificial series for the Lagrange multipliers and thus to obtain initial values of the Lagrange multipliers at the "announcement" of the timeless-perspective policy. This means that to close the model, a simple policy rule has to be specified and that its specification should be enough flexible and robust so that its estimation does not alter significantly the values of the Lagrange multipliers at the period of announcement of the timeless-perspective policy. In a second step, the last values of the smoothed state variables and artificial multipliers provide initial conditions required to compute the timeless-perspective policy after the end of the estimation period.

The rest of the paper is organized as follows. In section 2, we present the commitment and timeless-perspective solutions in a nonlinear framework. In section 3, our methodology is explained in the standard linear quadratic case. In section 4, we apply our results to a small-open economy model in Canada. Section 5 discusses about the robustness of our results, and especially how to account for model and parameter uncertainty to implement the timeless-perspective solution. The last section concludes.

[2] This is also equivalent to the interpretation that the policy maker restricts the set of admissible policies among which it must choose the best one.

[3] This solution was first suggested by Kydland and Prescott (1980).

2 Ramsey and timeless-perspective policies

In this section, we briefly present the Ramsey- and timeless-perspective solutions. The standard linear quadratic case is presented in Annex 1.

The Ramsey policymaker's objective is to maximize the conditional expected private sector welfare at time $t = 1$ subject to implementability constraints (namely the behavioral equations of the private sector and the dynamics of exogenous shocks). The dynamics of the economy is defined by

$$E_t f(y_{t+1}, y_t, y_{t-1}, u_t, \varepsilon_t) = 0 \tag{1}$$

where y and u are respectively the vector of endogenous variables, of instruments and ε is a vector of i.i.d shocks. By convention, ε_t is known at the beginning of the period t when agents take their decisions and y_0 is given. The objective function is defined as the expected discounted sum of utility

$$E_1 \sum_{t=1}^{\infty} \beta^{t-1} U(y_t). \tag{2}$$

Note that the set of instruments do not appear explicitly in the objective function, but it is easy to insert them by adding an auxiliary endogenous variables set equal to the instruments.

To solve the optimal policy problem, we first write the Lagrangian,

$$L = E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[U(y_t) - \mu_t f(y_{t+1}, y_t, y_{t-1}, u_t, \varepsilon_t) \right]. \tag{3}$$

Then, differentiating with respect to y_t, u_t and μ_t yields the following first-order conditions:

$$\begin{aligned} E_t \big[U_1(y_t) - \mu_t' f_2(y_{t+1}, y_t, y_{t-1}, u_t, \varepsilon_t) - \beta \mu_{t+1}' f_3(y_{t+2}, y_{t+1}, y_t, u_{t+1}, \varepsilon_{t+1}) & \qquad (4) \\ - \beta^{-1} \mu_{t-1}' f_1(y_t, y_{t-1}, y_{t-2}, u_{t-1}, \varepsilon_{t-1}) \big] &= 0 \\ E_t \big[\mu_t' f_4(y_{t+1}, y_t, y_{t-1}, u_t, \varepsilon_t) \big] &= 0 \qquad (5) \\ E_t \big[f(y_{t+1}, y_t, y_{t-1}, u_t, \varepsilon_t) \big] &= 0 \qquad (6) \end{aligned}$$

with $\mu_0 = 0$ and y_0 given.

Specifically, at the initial period, $t = 1$, there holds

$$\begin{aligned} E_1 \big[U_1(y_1) - \mu_1' f_2(y_2, y_1, y_0, u_1, \varepsilon_1) - \beta \mu_2' f_3(y_3, y_2, y_1, u_2, \varepsilon_2) \big] &= 0 \qquad (7) \\ E_1 \big[\mu_1' f_4(y_2, y_1, y_0, u_1, \varepsilon_1) \big] &= 0 \\ E_1 \big[f(y_2, y_1, y_0, u_1, \varepsilon_0) \big] &= 0 \qquad (8) \end{aligned}$$

since $\beta^{-1} \mu_0' f_1(y_1, y_0, u_0, \varepsilon_0) = 0$.

The deterministic steady-state of the Ramsey problem is thus defined by the following n-uplet (y^*, u^*, μ^*) so that

$$\begin{aligned} U_1(y^*) - \mu^{*'} f_2(y^*, y^*, y^*, u^*, 0) - \beta \mu^{*'} f_3(y^*, y^*, y^*, u^*, 0) & \\ -\beta^{-1} \mu^{*'} f_1(y^*, y^*, y^*, u^*, 0) &= 0 \\ \mu^{*'} f_4(y^*, y^*, y^*, u^*, 0) &= 0 \\ f(y^*, y^*, y^*, u^*, 0) &= 0. \end{aligned}$$

Therefore, for a given vector y^*, one can use the first matrix equation above to obtain

$$\mu^{*\prime} = U_1(y^*) A^{-1}(y^*, y^*, y^*, u^*, 0)$$

where $A = \left[f_2(y^*, y^*, y^*, u^*, 0) - \beta f_3(y^*, y^*, y^*, u^*, 0) - \beta^{-1} f_1(y^*, y^*, y^*, u^*, 0)\right]$. Substituting this expression in the second equation

$$U_1(y^*) A^{-1}(y^*, y^*, y^*, u^*, 0) f_4(y^*, y^*, y^*, u^*, 0) \quad = \quad 0$$

and using $f(y^*, y^*, y^*, u^*, 0) = 0$ yield a system that has only y^* and u^* as unknowns.

Let denote $\widehat{x}$, the logarithm deviation of the variable x from its steady-state value, the logarithm approximation of the model around the steady-state of the Ramsey policy is

$$\begin{aligned} E_t\big[U_{11}\widehat{y}_t - \widehat{\mu}_t' f_2' - \beta\widehat{\mu}_{t+1}' f_3 - \beta^{-1}\widehat{\mu}_{t-1}' f_1 - (\mu^* \otimes I)\left[\beta f_{31}\widehat{y}_{t+2} + \beta^{-1} f_{13}\widehat{y}_{t-2} + \right. & \\ (f_{21} + \beta f_{32})\widehat{y}_{t+1} + (f_{23} + \beta^{-1} f_{12})\widehat{y}_{t-1} + (f_{22} + \beta f_{33} + & \\ \beta^{-1} f_{11})\widehat{y}_t + f_{24}\widehat{u}_t + \beta f_{34}\widehat{u}_{t+1} + \beta^{-1} f_{14}\widehat{u}_{t-1} + f_{25}\widehat{\varepsilon}_t + \beta f_{35}\widehat{\varepsilon}_{t+1} + \beta^{-1} f_{15}\widehat{\varepsilon}_{t-1}\big]\big] &= 0 \quad (9) \\ E_t\big[\mu_t' f_4 + \mu^*[f_{41}\widehat{y}_{t+1} + f_{42}\widehat{y}_t + f_{43}\widehat{y}_{t-1} + f_{44}\widehat{u}_t + f_{45}\widehat{\varepsilon}_t]\big] &= 0 \quad (10) \\ E_t\big[f_1\widehat{y}_{t+1} + f_2\widehat{y}_t + f_3\widehat{y}_{t-1} + f_4\widehat{u}_t + f_5\widehat{\varepsilon}_t\big] &= 0. \quad (11) \end{aligned}$$

where I is an identity matrix of suitable order, and f_{ij} indicates a matrix of second order derivatives corresponding to the i^{th} and the j^{th} argument of the function f.

This system can be interpreted as a new dynamic stochastic equilibrium model augmented by the Lagrange multipliers. In contrast to (1), the dynamics of the optimal policy problem can be characterized by leads and lags on one additional period.[4] More importantly, the second order partial derivatives of the economy dynamics are part of the first order approximation of the Ramsey problem as long as the steady-state value of the Lagrange multipliers is different from zero. Indeed, if $\mu^* = 0$, the following conditions are exactly the same as those of the linear quadratic stabilization policy problem:

$$\begin{aligned} E_t[U_{11}\widehat{y}_t - \widehat{\mu}_t' f_2 - \beta\widehat{\mu}_{t+1}' f_3 - \beta^{-1}\widehat{\mu}_{t-1}' f_1] &= 0 \\ E_t[f_1\widehat{y}_{t+1} + f_2\widehat{y}_t + f_3\widehat{y}_{t-1} + f_4\widehat{u}_t + f_5\widehat{\varepsilon}_t] &= 0 \qquad (12) \\ E_t[\widehat{\mu}_t' f_4] &= 0. \qquad (13) \end{aligned}$$

[4]This is the case for example if the utility function incorporates internal habit formation.

If this condition is not fulfilled, the linear quadratic approximation leads to a “spurious” optimal policy rule in the sense that we neglect some important terms in the objective function and thus in the derivation of the optimal Ramsey policy. In particular, if one does not include mechanisms so that to correct for price and wage mark-ups (as in Erceg, Levin and Henderson, 2000), the Jacobian matrix of the utility function with respect to the variables of interest is no longer null—which is a direct consequence of the previous condition— and the quadratic approximation of the objective function of the policymaker is no longer valid.[5]

The general solution takes the form, $\forall t \geq 1$,

$$\begin{bmatrix} y_t \\ u_t \\ \mu_t \end{bmatrix} = g(y_{t-1}, \mu_{t-1}, \varepsilon_t) \tag{14}$$

with $\mu_0 = 0$ and y_0 given.

From this solution, it is straightforward to see that, if the policy maker re-optimizes at a later date and thus resets the Lagrange multipliers to zero, the latter are far away from the values provided by running forward the solution function defined above—the so-called time-inconsistency problem of the Ramsey policy. In other words, the policy maker exploits an informational advantage in the sense that it knows ex-ante the decisions of the private sector before setting its instruments. In this respect, without incentive mechanisms, the optimal policy is not time-consistent. At the same time, in the presence of a commitment-based technology, one may doubt however, from a practical point of view, whether it is really desirable for the monetary authority to commit itself, once and for all to a time-dependent instrument rule. At least, two arguments can be invoked. First, a time-dependent rule is complex, and thus difficult to explain to the private sector. Second, the time-dependence makes the rule to privilege a particular time period—the date at which the policy happens to have been implemented. Hence commitment to the decision rule may not be optimal today whereas it was optimal from the point of view of society's interest at that time.

In the timeless-perspective solution proposed by Woodford (1999, 2003), the policy maker renounces the possibility of setting the Lagrange multipliers to zero if it re-optimizes at a later date. As is shown by Marcet and Marinon (1998), it is possible to formulate the Ramsey problem recursively. For $t > 1$, the private sector first-order conditions can be used to define a mapping from policy at time t to the competitive equilibrium at time t and the shadow prices

[5]The conditions under which the solution to the linear quadratic (LQ) approximation yields a correct linear approximation to the optimal policy problem are often restrictive. More specifically, standard conditions of differentiability of the objective function and the constraints are not sufficient. Based on our framework, it is straightforward to show that the “common” LQ is a correct local characterization of the optimal policy problem in the following cases: (i) the constraints are exactly linear, (ii) the Taylor expansion is done around a steady-state at which the gradient vector equals zero (see above), (iii) the exact first-order conditions for (Ramsey) optimal policy (using the nonlinear specification) are log-linearized (see King and Woolman, 2004; Khan, King and Woolman, 2003; Schmitt-Grohé and Uribe, 2004a and 2004b). Note that the latter method is correct as long as we assume small enough exogenous shocks.

or Lagrange multipliers. More specifically, the Lagrange multipliers can be defined as

$$\mu_t = g_\mu(y_{t-1}, \mu_{t-1}, \varepsilon_t)$$

This constraint can be included in the policy problem. It embeds the assumption of commitment since today's decisions are tied to past decisions. Therefore, the policy maker will select the value of μ that it wants to commit to since future policy makers will be constrained by the fact that they have to choose the policy that is optimal from the standpoint of the current period. The problem is that this procedure does not pin down initial values of the Lagrange multipliers since the policy chosen at time $t = 1$ depends on initial conditions and the solution for all future periods depends on these values. In this respect, adopting a timeless perspective policy means that the dependence on initial conditions is removed by imposing that the choice of the Lagrange multipliers μ_0 is governed by the same Markovian rule from period 1 onwards.

Therefore, the Lagrangian for the timeless-perspective policy can be written

$$L = E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[U(y_t) - \mu_t f(y_{t+1}, y_t, y_{t-1}, u_t, \varepsilon_t)\right] - \beta^{-1}\mu_0 y_1.$$

for μ_0 given.

The first order conditions for $t > 2$ are the same as those of the optimal Ramsey problem. For $t = 1$, there holds

$$\begin{aligned} E_1\big[U_1(y_1) - \mu_1^{'} f_2(y_2, y_1, y_0, u_1, \varepsilon_1) - \beta^{-1}\mu_0^{'} f_1(y_1, y_0, u_0, \varepsilon_0) - & \qquad \qquad (15) \\ \beta\mu_2^{'} f_3(y_3, y_2, y_1, u_2, \varepsilon_2)\big] &= 0 \\ E_1\big[\mu_1^{'} f_4(y_2, y_1, y_0, u_1, \varepsilon_1)\big] &= 0 \\ E_1\big[f(y_2, y_1, y_0, u_1, \varepsilon_0)\big] &= 0 \qquad (16) \end{aligned}$$

As we explain before, the time inconsistency of the Ramsey solution may arise because private agents take certain actions before the policy maker choose the optimal path of instruments. In particular, the private sector takes its decisions based on expectations of government policy while the policy maker takes its decisions based on given expectations of the private sector. This leads to an informational advantage for the policy maker and the situation can be compared to a traditional Stackelberg game (see Cohen and Michel, 1988). Indeed, by committing to follow a policy at the initial date, which corresponds to the stationary Markovian policy rule, the policy maker will influence sufficiently expectations of the private sector. Note that this means that the timeless-perspective solution is not a commitment free technology.

3 Estimating the initial value of the Lagrange multipliers to implement timeless perspective

The first order (log-linearized) conditions of the timeless perspective problem can be rewritten in a state-space representation as follows,

$$\hat{s}_t = M(\Theta)\,\hat{s}_{t-1} + P(\Theta)\,\hat{\eta}_t \tag{17}$$

$$\hat{x}_t = C(\Theta)\,\hat{s}_t \tag{18}$$

where $\hat{x}_t$ contains observable variables, and $\hat{s}_t$ contains state variables. It includes unobservable elements such that conditional expectations, natural variables or shock processes. Last, η_t is a vector of i.i.d. variables with zero mean and covariance matrix I_n.

In order to estimate system, we adopt the strategy proposed, among others, by Smets and Wouters (2005a and 2005b) and An and Schorfheide (2006), e.g. Bayesian econometrics.[6] A Kalman filter is used to estimate the system (17)–(18). The algorithm preliminary evaluates the number of explosive eigenvalues. Consequently, indeterminate models (that do not satisfy the Blanchard-Kahn conditions) are directly ruled out during the course of the estimation. For a given structural model $m \in M$ and a set of parameters Θ, we denote $\Gamma(\Theta|m)$ the prior distribution of Θ and $\mathcal{L}\left(X^T|\Theta, m\right)$ the likelihood function associated to the observable variables $X^T = \{\hat{x}_t\}_{t=1}^T$. Then, the posterior distribution of the parameter vector is proportional to the product of the likelihood function and the prior distribution of Θ,

$$\Gamma\left(\Theta|X^T, m\right) \propto \mathcal{L}\left(X^T|\Theta, m\right)\Gamma(\Theta|m). \tag{19}$$

This posterior distribution function can be evaluated using a Monte-Carlo Markov Chain (MCMC) sampling approach (see An and Schorfheide, 2006).

Obviously, the main problem here is that the Lagrange multipliers are not observable. Furthermore, they typically depend on unobservable state variables. Therefore, we propose to implement the timeless-perspective optimal policy by distinguishing the pre-period of implementation or period of estimation, $[t_0, t_1]$ and the "implementation" period of this policy $t = t_1 + 1, \cdots, T$.[7] Indeed, as is it common in empirical Bayes method, we initialize the Lagrange multipliers and some of the state variables by estimating the structural model of the economy. To do so, we need to close the model and to specify a simple monetary policy rule. This means that this specification must be enough flexible and robust so that results are not too sensitive to its specification. In practise, it is possible to control for such an effect by taking into account different simple monetary policy rules and by measuring uncertainty around parameters (see further).

Next, to capture initial values of the Lagrange multipliers when the timeless-perspective policy is implemented by the central bank, it is necessary to solve recursively the system estimated in the pre-period. Therefore, the initial values of the Lagrange multipliers are set to zero at

[6] Note that a FIML approach could be implemented to estimate the optimal policy in a timeless perspective.

[7] To some extent, the first period might be compared to a pre-sampling approach in Bayesian econometrics.

time t_0, $\lambda_{t_0} = 0$. The intuition is that if the estimation period is long enough, this initialization will have almost no effect on the value of the Lagrange multipliers at the end of the pre-period $t = t_1$ (stationary solution), and thus do not matter for the optimal policy in a timeless-perspective. Then solving the dynamics by standard techniques, it is straightforward to obtain the Lagrange multipliers at period $t = t_1$. An alternative is to follow the approach developed by Giannoni and Woodford (2002) in their appendix and extend by Juillard and Pelgrin (2007).[8] In this respect, we have all ingredients to determine the timeless-perspective recommendation.

To sum up, the proposed algorithm goes as follows:

- **Step 1**: Estimate the dynamics of the economy on the period prior to the switch to a timeless-perspective with an empirical policy rule simply aimed at describing the policy in effect at that time

- **Step 2**: Use the coefficients of the estimated model to compute optimal policy under commitment and compute smoothed values of the vector of unobservable state variables.

- **Step 3**: Recompute an artificial series for the Lagrange multipliers during the estimation period, setting initial values of Lagrange multipliers at the beginning of estimation period to zero

- **Step 4**: Compute optimal policy in a timeless-perspective after the end of the estimation sample. The last values of the Lagrange multipliers computed in the previous step and smoothed variables provide initial values.

In principles, this algorithm can be extended to the non linear case. As long as we can estimate the non linear model (see An and Schorfheide, 2006; Fernandez-Villaverde and Rubio-Ramirez, 2004) and the smoothed values of unobservable variables can be derived, the proposed algorithm is still valid. However, this is far from being an easy task, especially when the dimensionality of the problem increases since we have to consider the first order conditions and the dynamics of the economy. After deriving an higher order approximation of the model (say, a second order approximation) and computing the optimal Ramsey problem or timeless perspective problem, the estimation step can be computed by using nonlinear Kalman filter or particle filtering techniques, such as a bootstrapping filter, a non-gaussian particle filter, or a sequential non gaussian particle filter. We leave this issue for future extensions of this work.

[8]Both methods yield comparable results. For further details, see Juillard and Pelgrin (2007).

4 An application to a small open economy

In this section, we begin by presenting the small open economy DSGE model of Lubik and Schorfheide (2007). We then explain the Ramsey solution. Finally, we present the benchmark results.

4.1 A benchmark model

As an application of the methodology proposed in the last section, we consider a small open economy model for Canada. The model is the same as in Lubik and Schorfheide (2007).[9] The original model has a fully micro-founded core and is a generalization of the canonical New-Keynesian model for a closed economy developed by Rotemberg and Woodford (1997), Goodfriend and King (1997) and others. Note that, for our purpose, we only report here the reduced form.

The world economy is assumed to consist of two countries: a small home open economy and a large, approximately closed, foreign economy. The latter is only described by a set of AR(1) processes (see further) whereas the small-open economy is described by the following equations. The first equation describes a purely forward-looking pricing rule

$$\widetilde{\pi}_t = \beta\widetilde{\pi}_{t+1|t} + \alpha\beta E_t\Delta\widetilde{q}_{t+1} - \alpha\Delta\widetilde{q}_t + \frac{\kappa}{\tau+\alpha(2-\alpha)(1-\tau)}(\widetilde{y}_t - \widetilde{\bar{y}}_t) \tag{20}$$

where $v_{t+1|t}$, for any variable v, denotes $E_t v_{t+1}$, $\widetilde{\pi}_t$ is the annualized quarterly CPI inflation, $\widetilde{\bar{y}}_t = -\alpha(2-\alpha)\frac{1-\tau}{\tau}\widetilde{y}^*$ is potential output in the absence of nominal rigidities and when technology is non-stationary, $\widetilde{y}$ is the aggregate output, y_t^* is exogenous world output and $\widetilde{q}_t$ are the terms of trade, defined as the relative price of exports in terms of imports. Note that the terms of trade is in first difference since it is assumed that changes in relative prices affect inflation and thus the real interest rate through the definition of the consumption-based price index. The closed economy version of the model is obtained by setting $\alpha = 0$. It can be shown that this equation is obtained from the intertemporal minimization program of the firm and that the coefficient κ is a function of underlying structural parameters, such as labor supply, demand elasticities, and parameters measuring the degree of price stickiness.

The aggregate demand equation is defined as follows:

$$\begin{aligned}\widetilde{y}_t = E_t\widetilde{y}_{t+1} - \big[\tau+\alpha(2-\alpha)(1-\tau)\big](\widetilde{R}_t - E_t\widetilde{\pi}_{t+1}) + \rho_a dA_t - \alpha\big[\tau+\alpha(2-\alpha)(1-\tau)\big]E_t\Delta\widetilde{q}_{t+1} \\ +\alpha(2-\alpha)\frac{1-\tau}{\tau}E_t\Delta\widetilde{y}^*_{t+1}\end{aligned} \tag{21}$$

where $0 < \alpha < 1$ is the import share, τ^{-1} the intertemporal substitution elasticity, and $\widetilde{R}$ is the nominal interest rate (policy instrument). Note that the equation also reduces to its closed economy variant when $\alpha = 0$. Finally, if $\tau = 1$, the world output does not matter in the demand equation and the trade balance is identically equal to zero.

[9] For further details of small-open versus closed economy in Canada, see Dib (2003).

The nominal exchange rate fulfills the purchasing power parity condition (PPP)

$$\widetilde{\pi}_t = \Delta\widetilde{e}_t + (1-\alpha)\Delta\widetilde{q}_t + \widetilde{\pi}^*_t \tag{22}$$

where $\widetilde{\pi}^*$ is a world inflation shock.

The monetary policy authority follows a Taylor-type policy rule that includes some foreign variables (nominal exchange rate depreciation)

$$\widetilde{R}_t = \rho\widetilde{R}_{t-1} + (1-\rho)\big[\psi_1\widetilde{\pi}_t + \psi_2(\widetilde{y}_t - \widetilde{\hat{y}}_t) + \psi_3\Delta\widetilde{e}_t\big] + \varepsilon^r_t \tag{23}$$

where ε^r_t is a monetary policy shock (non systematic component of monetary policy). Notice that the monetary policy rule does not include any lead terms of inflation or the domestic output gap. However, as we explain before, we check the robustness of our results by looking at alternative forms of monetary policy rules and it appears that the choice of the simple rule for the pre-sampling period does not affect significantly the results. We assume that the policy coefficients ψ_1, ψ_2 and ψ_3 are non-negative. In order to match the persistence in nominal interest rates, we include a smoothing term in the rule.[10] Finally, the dynamics of the term of trade is assumed to be described by an AR(1) process:

$$\Delta\widetilde{q}_t = \rho_q\Delta\widetilde{q}_{t-1} + \varepsilon^q_t \tag{24}$$

As is pointed out by Lubik and Schorfheide (2007), this specification is not fully consistent with the underlying structural model. However, it can be considered as a “semi-small open economy”.

All world variables, $\widetilde{y}^*_t$ and $\widetilde{\pi}^*_t$ are assumed to be AR(1) processes and not correlated together.[11]

4.2 The Ramsey solution

The monetary policy authority seeks to minimize a standard objective function (subject to the implementability constraints) which is quadratic in deviations of inflation, the output gap, and the first-difference of the nominal interest rate—due to smoothing motive—from their zero target levels[12]

$$min_{R_t} E_1 \sum_{t=1}^{\infty} \beta^{t-1}\frac{1}{2}\big[\omega_1\widetilde{\pi}^2_t + \omega_2\widetilde{x}^2_t + \omega_3(\widetilde{R}_t - \widetilde{R}_{t-1})^2\big] \tag{25}$$

where ω_2/ω_1 is the central bank weight on output stabilization relative to inflation stabilization. Note that we do not specify the second-order accurate approximation of the utility

[10] Indeterminacy is essentially driven by ψ_1 and ψ_3. In particular, it is straightforward to show that a generalized Taylor principle is given by $\psi_1 + \psi_3 > 1$.

[11] See Svensson (2000), Walsh (1998), Leiteimo and Söderström (2005), Clarida, Gali and Gertler (2001), and McCallum and Nelson (2000)

[12] Note that the objective function is not derived from first principles. Under certain regularity conditions, one alternative would be to use the non-linear dynamics of the economy and derive the relevant second-order approximation of the utility function. Since our model is a ”reduced-form”, we leave this issue for further research.

function, but rather an ad hoc objective function. However, for our purpose, it does not matter since we consider the reduced form model.

The Lagrangian of the optimal Ramsey problem can be summarized as follows:

$$L = E_1 \sum_{t=1}^{\infty} \beta^{t-1} \Big\{ \frac{1}{2} \big[\omega_1 \widetilde{\pi}_t^2 + \omega_2 \widetilde{x}_t^2 + \omega_3 \widetilde{R}_t^2 \big] + \lambda_{1t} \big[\widetilde{\pi}_t - \beta \widetilde{\pi}_{t+1|t} - \alpha\beta E_t \Delta \widetilde{q}_{t+1} + \alpha \Delta \widetilde{q}_t \quad (26)$$
$$- \frac{\kappa}{\tau + \alpha(2-\alpha)(1-\tau)} (\widetilde{y}_t - \widetilde{\bar{y}}_t) \big] + \lambda_{2t} \big[\widetilde{y}_t - E_t \widetilde{y}_{t+1} + \big[\tau + \alpha(2-\alpha)(1-\tau) \big] (\widetilde{R}_t - E_t \widetilde{\pi}_{t+1}) + \rho_a dA_t$$
$$+ \alpha \big[\tau + \alpha(2-\alpha)(1-\tau) \big] E_t \Delta \widetilde{q}_{t+1} - \alpha(2-\alpha) \frac{1-\tau}{\tau} E_t \Delta \widetilde{y}^*_{t+1} \big] \Big\}.$$

Using the law of iterated projections and differentiating with respect to x_t, π_t, R_t and the Lagrange multipliers, one obtains the first-order conditions. In the case of the timeless perspective solution, we set additional constraints on initial values of the Lagrange multipliers.[13]

4.3 Model estimation and reaction functions

We use observations on real output growth, inflation, nominal interest rates, exchange rate changes, and terms of trade changes in our empirical analysis. All data are seasonally adjusted and at quarterly frequencies for the period 1983:1-2004:4. All series are obtained from the OECD Analytical database (OECD ADB). We choose the same priors as the original authors.[14] Table 1 provides information about the prior distributions. Prior distributions are assumed to be independent.[15]

The Bayesian estimates of the structural parameters are given in Table 2.[16] In addition to 90% posterior probability intervals, we report posterior means as point estimates. The results are consistent with those of Lubik and Schorfheide (2006). Posterior and prior distributions are displayed in Figures 1 and 2.

[Insert Tables 1 and 2, and Figures 1 and 2 around here]

Using the posterior mode estimates of the parameters (except the ones from the policy function!), one obtains the optimal policy as well as the function reaction of the Lagrange mulipliers[17]:

$$\begin{aligned} \tilde{R}_t &= 0.10\tilde{R}_{t-1} - 0.05\lambda_{1,t-1} - 0.01\lambda_{2,t-1} - 0.15\tilde{z}_{t-1} - 0.01\Delta\tilde{q}_{t-1} + 0.08\tilde{y}^{\star}_{t-1} \\ &\quad -0.33\epsilon_{z,t} - 0.51\epsilon_{q,t} + 0.09\epsilon_{y^{\star},t} \end{aligned}$$

[13]See Annex 1 for the derivation of the linear quadratic problem.

[14]See further details on the choice of prior distributions in Lubik and Schorfheide (2007).

[15]We control for indeterminacy by checking that the pre-defined prior volume and the posterior volume are stable

[16]Our estimation assumes that the model is stable over the 1983-2004 sample. This might be contradicted by empirical evidence, especially after the implementation of the flexible inflation targeting regime at the beginning of the 90's. We leave this complication to future work.

[17]We discuss later on the choice of the posterior mode against the posterior mean, as well as how to account for uncertainty of the parameter distributions.

and the dynamics for the multipliers:

$$\begin{aligned}\lambda_{1,t} &= 0.40\tilde{R}_{t-1} + .41\lambda_{1,t-1} + 0.09\lambda_{2,t-1} + 0.73\tilde{z}_{t-1} + 0.09\Delta\tilde{q}_{t-1} - 0.18\tilde{y}^{\star}_{t-1} \\ &\quad +1.63\epsilon_{z,t} + 0.26\epsilon_{q,t} - 0.20\epsilon_{y^{\star},t} \\ \lambda_{2,t} &= 0.36\tilde{R}_{t-1} - 0.60\lambda_{1,t-1} + 0.15\lambda_{2,t-1} + 0.64\tilde{z}_{t-1} + 0.14\Delta\tilde{q}_{t-1} - 0.41\tilde{y}^{\star}_{t-1} \\ &\quad +1.41\epsilon_{z,t} + 0.40\epsilon_{q,t} - 1.58\epsilon_{y^{\star},t}\end{aligned}$$

4.4 Ramsey versus Timeless-perspective policy

To proceed in the spirit of the exercise, we assume that the switch to the optimal (Ramsey) policy should take place in the first quarter of 2005. In this context, if the monetary authorities want to pursue an optimal policy under commitment, they would set the value of the two multipliers to zero in the fourth quarter of 2004 and proceed from there. The time inconsistency stems from the possibility they have to reset the multipliers to zero in a future period and their commitment is about not doing it. On the other hand, if, in order to enhance their credibility, they decide not to take advantage of the possibility to set the multipliers to zero in the fourth quarter of 2004, they could adopt an optimal policy in a timeless perspective.

Therefore, as is explained in section 3, our chief task is to reconstruct an artificial series for the multipliers. We can compute them using the above difference equations provided we set them to zero before the beginning of the estimation sample, in the fourth quarter of 1982. This is done by using the smoothed value of the state variables as computed by the Kalman smoother on the basis of the posterior mode of the parameters. Finally, we can compute the optimal policy in a timeless-perspective given that the last values of the smoothed Lagrange multiplier variables (the last quarter, 2004:4) provide initial values for the Lagrange multipliers at the next period. Given the recursive nature of both the Ramsey policy (optimal policy under commitment) and the timeless perspective optimal policy, it is now possible to compute the value of the objective function at any time. It is in particular possible to compute it at the end of the estimation period—at the time of the potential shift to optimal policy.

Table 3 reports values of the objective function for the simple rule solution, and the Ramsey and timeless-perspective solutions.[18] In our benchmark case, the weights of the objective function are given by $\mu_1 = 1$, $\mu_2 = 1$, and $\mu_3 = 0.1$.[19] The second column yields the loss function if the monetary authority would keep following the monetary policy rule estimated for the previous period. The third column represents the value of the objective function if the monetary authority were to follow the timeless-perspective policy whereas the last column yields the corresponding value of the Ramsey solution (setting the multipliers to zero in the previous period). For instance, in the last period (2004:4), the loss function equals 57.99 (respectively 58.38) under commitment (respectively timeless perspective).[20]

[Insert Table 3 around here]

[18]Other methods to evaluate welfare of alternative policy rules are provided in Benigno and Woodford (2006).

[19]We conduct robustness analysis in Section 5.

[20]As usual, the loss difference must be interpreted with caution since there is no units per se.

Results show that implementing an optimal policy in a timeless-perspective, based on estimated model for the Canadian economy, is not so costless with respect to the Ramsey policy. Furthermore, there are some gains (in this example) for the monetary authority to consider the timeless-perspective solution rather than to follow the monetary policy rule estimated at the previous period (henceforth, MPR-based solution). However, the assumption of arbitrary weights as well as the specification of the preferences are questionable. Therefore, we gauge the robustness of our conclusions when the relative weights μ_2/μ_1 and μ_3/μ_1 are varied over a grid on the unit sphere. More specifically, Table 4 presents the welfare under the optimal Ramsey policy and the timeless-perspective when one assumes that the preferences of central banks leads to a flexible inflation targeting (no smoothing of the interest rate) as well as a higher or lesser motive for inflation stabilization.[21]

[Insert Table 4 around here]

Results are reported in Table 4. Overall, ceteris paribus, there is not so much difference between the timeless-perspective solution and the Ramsey solution. In contrast, the difference between the MPR-based solution and the timeless-perspective outcome depends on the central bank weight on output stabilization relative to inflation stabilization. This provides additional evidence on existing results in the literature.

5 Discussion

In this section, we present some robustness checks of our previous results regarding the choice of priors and the simple monetary policy rule used in the structural model. Finally, we proceed with a discussion of parameter and/or model uncertainty and how it can be incorporated into policy exercises based on timeless-perspective solutions.

5.1 Robustness

A first robustness check concerns the estimation of the model and especially the choice of the prior distributions. At this stage, rather than using the posterior distribution to capture parameter uncertainty, we ask what are the welfare implications of getting too informative priors. In this respect, we relax the priors on the policy parameters as well as the other reduced-form parameters. Following Lubik and Schorfheide (2007), we impose a uniform prior on the smoothing parameter, ρ, and make the prior on other parameters more diffuse. This leads to response coefficients of the Taylor rule which are higher than in the benchmark case. To assess the impact on the timeless-perspective policy, we look at the estimates of the initial values of the Lagrange multipliers before implementing the timeless-perspective policy as well as the welfare difference with the Ramsey policy. Overall, our results are robust and the welfare loss differs only slightly.[22]

A second robustness check concerns the specification of the monetary policy rule. In our methodology, since the estimation of the model rests on the specified monetary policy rule,

[21]Other results are available on request.

[22]Results are not reported here but are available on request.

the initial values of the Lagrange multipliers may critically depend on the chosen functional form. To some extent, this can be interpreted as a first source of model uncertainty.[23] Numerous studies have shown that simple rules not only perform well but are more robust to model uncertainty than complicated rules. For example, Levin et al. (1999) provide evidence that simple rules, which have a high degree of interest rate smoothing and that responds both to inflation and output gap deviations, perform nearly as well as more complicated rules in different models of the US economy.[24] In contrast, Côté et al. (2005) argue that simple monetary policy rules are not robust to model uncertainty in Canada—no single rule performs well in all models. In this context, we re-estimate the model under alternative monetary policy rules.[25] To do so, we first impose the following restrictions: (i) $\psi_3 = 0$, (ii) $\rho = 0$, (iii) $\rho = 0$ and $\psi_3 = 0$. Results are presented in Table 5. Not surprisingly, the smoothing parameter, ρ, has much larger effects on welfare than the coefficient of the real exchange rate in the policy function. Then, we consider the class of forecast-based rules (Batini and Haldane, 1999),

$$\widetilde{R}_t = \rho \widetilde{R}_{t-1} + (1-\rho)\psi_1 E\widetilde{\pi}_{t+1} + \varepsilon_t^r$$

and monetary policy rules with a speed limit effect (Walsh, 2004)[26],

$$\widetilde{R}_t = \rho \widetilde{R}_{t-1} + (1-\rho)\left[\psi_1 \widetilde{\pi}_t + \psi_2(\widetilde{y}_t - \widehat{\widetilde{y}}_t) + \psi_3 \Delta\widetilde{e}_t + \psi_4 \Delta\widetilde{y}_t\right] + \varepsilon_t^r.$$

In order to assess the impact on the timeless-perspective policy, we proceed as before and thus compare the initial values of the timeless-perspective Lagrange multipliers as well as the welfare difference with the Ramsey policy. Results are presented in Table 6.

[Insert Tables 5 and 6 around here]

Overall, results are fairly robust—the initial values of Lagrange multipliers remain relatively close to the benchmark case—with the exception of forecast-based rules. Moreover, monetary policy rules with a speed limit effect perform better than the forecast-based rules in terms of welfare analysis.

5.2 Parameter and model uncertainty

We now address the question of parameter and model uncertainty in the derivation of the timeless-perspective policy. Our methodology rests on the Bayesian estimation, which is undertaken in the face of uncertainty about the correct parameter values within a parametric model, the correct model within a range of possible models (say the space of models), and the correct space within a collection of space. In particular, until now, we assume that there is no source of uncertainty either in parameters or in the model specification. This leads to take (i) the posterior mode (or mean) estimates of the structural parameters as given when estimating shock series and (ii) the mean estimates of the structural shocks (using the Kalman

[23]Another source of uncertainty would be to consider the robustness of simple rules in various models or the performance of simple rules in different versions of the same model. We leave this issue for further research.

[24]For a complete discussion, see the NBER Business Cycles Series, volume 31.

[25]In all cases, we implicitly assume, for the sake of simplicity, that the target inflation is zero. This is obviously not the case in Canada where the monetary authority have started implementing a flexible inflation targeting regime since the beginning of the 90's.

[26]The study of super-inertial interest rate rules is left for future research.

filtering technique) as well as the Lagrange multipliers as given. In this context, allowing for the uncertainty by using the full posterior distribution for the structural parameters, the shocks and the Lagrange multipliers, would permit to derive for each draw a slightly different timeless-perspective optimal policy. At the same time, allowing for alternative models would also permit to account for model uncertainty and slightly different scenarios of the timeless perspective solution. In what follows, we follow the first road leaving important issues for future analysis.[27]

Hence, we assume that there is no model uncertainty and use the posterior distribution to capture the uncertainty inherent in any given model parameter. In this respect, each draw of the posterior distribution for the shocks and multipliers can be used to derive a timeless-perspective solution and then compute the welfare loss. Not surprisingly, results (Table 7) show that uncertainty matters for the evaluation of the welfare and especially if one would like to conduct counterfactual exercises in order to compare the performance of the timeless-perspective policy with respect to simple policy rules. However, this is not of the scope of this paper and we leave this issue for future work.

[Insert Table 7 around here]

6 Conclusion

The main purpose of this paper was to provide a new methodology in order to estimate the initial values of the Lagrange multipliers and thus implement the timeless-perspective solution proposed by Woodford (2003). We proceed as follows. In a first step, we estimate the dynamics of the economy on the period prior to the switch to a timeless-perspective with an empirical policy rule simply aimed at describing the policy in effect at that time. In a second step, we use the coefficients of the estimated model to compute optimal policy under commitment and compute smoothed values of the vector of unobservable state variables. In a third step, we recompute an artificial series for the Lagrange multipliers during the estimation period, setting initial values of Lagrange multipliers at the beginning of estimation period to zero. In the fourth step, we compute the optimal policy in a timeless-perspective after the end of the estimation sample. The last values of the Lagrange multipliers computed in the previous step and smoothed variables provide initial values. We use our new approach to estimate and to compare the Ramsey solution and the timeless-perspective solution in a small open economy model for Canada.

Two key issues are the sensitivity of the results to the choice of the monetary policy rule in the estimation period and thus the determination of the Lagrange multipliers before the implementation of the timeless-perspective policy. Following Giannoni and Woodford (2002), Juillard and Pelgrin (2007) show how to eliminate Lagrange multipliers in certain cases and thus to circumvent this first issue. A second concern is to account for model and/or parameter uncertainty in our methodology. We propose a first pass to parameter uncertainty. However,

[27] For more details, see Juillard and Pelgrin (2007).

much has to be done and we leave the issue of uncertainty for future research and extensions of this paper.

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Annex 1: The linear quadratic case

We discuss now the particular case where the objective function is quadratic and the dynamics are linear. Note that particular care must be taken, when the linear dynamics are in fact a linear approximation of a nonlinear model (see Benigno and Woodford, 2003). We generalize here the presentation of Woodford (2003).

Let assume that the dynamics of the economy is defined by

$$A_+E_ty_{t+1} + A_0y_t + A_-y_{t-1} + Bu_t + Ce_t = 0$$

where, as before y_t is the vector of endogenous variables, u_t the vector of instruments and e_t, a vector of zero–mean shocks uncorrelated with past values.

This can be rewritten as follows

$$\begin{bmatrix} A_+ & 0 \end{bmatrix} \begin{bmatrix} E_ty_{t+1} \\ E_tu_{t+1} \end{bmatrix} + \begin{bmatrix} A_0 & B \end{bmatrix} \begin{bmatrix} y_t \\ u_t \end{bmatrix} + \begin{bmatrix} A_- & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ u_{t-1} \end{bmatrix} + Ce_t = 0$$

The intertemporal loss function is

$$\frac{1}{2}E_1 \sum_{t=1}^{\infty} \beta^{t-1} z_t' W z_t$$

where $z_t = \begin{pmatrix} y_t' & u_t' \end{pmatrix}'$, $z_t'Wz_t$ is the instantaneous objective function and W is given by

$$W = \begin{pmatrix} W_{yy} & W_{yu} \\ W_{yu}' & W_{uu} \end{pmatrix}.$$

The Ramsey policy is based on the resolution of the following Lagrangian

$$\begin{aligned} L \;=\; & E_1 \sum_{t=1}^{\infty} \beta^{t-1} \Big[\frac{1}{2}\left(y_t'W_{yy}y_t + 2y_t'W_{yu}u_t + u_t'W_{uu}u_t\right) \\ & + \lambda_t' \left(A_+E_ty_{t+1} + A_0y_t + A_-y_{t-1} + Bu_t + Ce_t\right) \Big] \end{aligned}$$

or equivalently,

$$L = E_1 \sum_{t=1}^{\infty} \beta^{t-1} \Big[\frac{1}{2} z_t'Wz_t + \lambda_t' \Big(\begin{bmatrix} A_+ & 0 \end{bmatrix} E_tz_{t+1} + \begin{bmatrix} A_0 & B \end{bmatrix} z_t + \begin{bmatrix} A_- & 0 \end{bmatrix} z_{t-1} \Big) \Big]$$

The first-order conditions with respect to the predetermined, non-predetermined and instrument variables are given by (for $t > 0$):

$$
\begin{aligned}
\frac{\partial L}{\partial y_1} &= W_{yy}y_1 + W_{yu}u_1 + A_0'\lambda_1 + \beta A_-' E_1(\lambda_2) \\
&= 0 \\
\frac{\partial L}{\partial y_t} &= W_{yy}y_t + W_{yu}u_t + \beta^{-1}A_+'\lambda_{t-1} + A_0'\lambda_t + \beta A_-' E_t(\lambda_{t+1}) \qquad t = 2, \ldots \\
&= 0 \\
\frac{\partial L}{\partial u_t} &= W_{uy}y_t + W_{uu}u_t + B'\lambda_t \qquad t = 1, \ldots \\
&= 0 \\
\frac{\partial L}{\partial \lambda_t} &= A_+E_t y_{t+1} + A_0 y_t + A_- y_{t-1} + Bu_t + Ce_t \\
&= 0
\end{aligned}
$$

or equivalently (using the matrix notation)

$$
Wz_t + \beta^{-1}\begin{bmatrix} A_+' \\ 0' \end{bmatrix}\lambda_{t-1} + \begin{bmatrix} A_0' \\ B' \end{bmatrix}\lambda_t + \beta\begin{bmatrix} A_-' \\ 0' \end{bmatrix}E_t\lambda_{t+1} = 0
$$

with $\lambda_0 = 0$, y_0 given, and

$$
\begin{bmatrix} A_+ & 0 \end{bmatrix} E_t z_{t+1} + \begin{bmatrix} A_0 & B \end{bmatrix} z_t + \begin{bmatrix} A_- & 0 \end{bmatrix} z_{t-1} + Ce_t = 0.
$$

Therefore, as we explain before, we obtain a new dynamic stochastic equilibrium model augmented by the Lagrange multipliers. The first order conditions can be rewritten as follows

$$
\begin{bmatrix}
A_+ & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \beta A_-' & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I & 0 & 0 \\
0 & 0 & 0 & 0 & I & 0 \\
0 & 0 & 0 & 0 & 0 & I
\end{bmatrix}
\begin{bmatrix}
E_t y_{t+1} \\ E_t u_{t+1} \\ E_t \lambda_{t+1} \\ y_t \\ u_t \\ \lambda_t
\end{bmatrix}
+
\begin{bmatrix}
A_0 & B & 0 & A_- & 0 & 0 \\
W_{yy} & W_{yu} & A_0' & 0 & 0 & \beta^{-1}A_+' \\
W_{uy} & W_{uu} & B' & 0 & 0 & 0 \\
I & 0 & 0 & 0 & 0 & 0 \\
0 & I & 0 & 0 & 0 & 0 \\
0 & 0 & I & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
y_t \\ u_t \\ \lambda_t \\ y_{t-1} \\ u_{t-1} \\ \lambda_{t-1}
\end{bmatrix}
+
\begin{bmatrix}
Ce_t \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
= 0
$$

so that

$$
\Gamma_0 Y_t + \Gamma_1 Y_{t-1} + \Gamma_2 \eta_t = 0
$$

where $Y_t = \begin{pmatrix} E_t y_{t+1}' & E_t u_{t+1}' & E_t \lambda_{t+1}' & y_t' & u_t' & \lambda_t' \end{pmatrix}'$ and $\eta_t = \begin{pmatrix} e_t' & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$.

This is indeed just a larger linear rational expectation model that can be solved by usual techniques. For example, using the real, reordered, generalized Schur decomposition suggested by Klein (1997) and Sims (2004).

In contrast, the timeless perspective solution rests on the following Lagrangian,

$$
L = E_1 \sum_{t=1}^{\infty} \beta^{t-1}\Big[z_t'Wz_t + \lambda_t'\Big(\begin{bmatrix} A_+ & 0 \end{bmatrix} E_t z_{t+1} + \begin{bmatrix} A_0 & B \end{bmatrix} z_t + \begin{bmatrix} A_- & 0 \end{bmatrix} z_{t-1}\Big) + \beta^{-1}\lambda_0'(z_0 - \overline{z}_0)\Big]
$$

where $z_0 = \begin{pmatrix} y_0' & 0 \end{pmatrix}$.

Hence, the first-order conditions are:

$$W z_t + \beta^{-1} \begin{bmatrix} A_+' \\ 0' \end{bmatrix} \lambda_{t-1} + \begin{bmatrix} A_0' \\ B' \end{bmatrix} \lambda_t + \beta \begin{bmatrix} A_-' \\ 0' \end{bmatrix} E_t \lambda_{t+1} = 0$$

with $\lambda_0 \neq 0$, y_0 given, and

$$\begin{bmatrix} A_+ & 0 \end{bmatrix} E_t z_{t+1} + \begin{bmatrix} A_0 & B \end{bmatrix} z_t + \begin{bmatrix} A_- & 0 \end{bmatrix} z_{t-1} + C e_t = 0.$$

The resolution is the same as in the Ramsey policy problem. It is important to note, however, that the initial values of the Lagrange multipliers (of the non-predetermined variables) are a complex function of the past dynamics of the previous system and the initial values of the non-predetermined variables, the non-predetermined variables of z_0 are chosen so that (i) there are function of the predetermined variables at the initial period and (ii) there are the solution to the optimization problem with the additional constraints $z_0 = \overline{z}_0$ for $t > 0$. In order to estimate the timeless-perspective solution, the initial values of Lagrange multipliers (of the non-predetermined variables) become crucial. In particular, the methodology proposed by Söderlind (1999), among others, is no longer directly applicable.

Table 1: Priors and posterior mode

Parameters		Prior		Posterior	
	Density	Mean	S.D.	Mode	S.E.
ψ_1	Gamma	1.50	0.50	2.17	0.31
ψ_2	Gamma	0.25	0.13	0.14	0.08
ψ_3	Gamma	0.25	0.13	0.19	0.06
ρ_R	Beta	0.50	0.20	0.75	0.04
α	Beta	0.30	0.10	0.13	0.04
r	Gamma	2.50	1.00	2.09	0.91
κ	Gamma	0.50	0.25	0.55	0.16
τ	Beta	0.50	0.20	0.40	0.06
ρ_q	Beta	0.40	0.20	0.38	0.10
ρ_z	Beta	0.50	0.20	0.47	0.03
$\rho_{y^\star}$	Beta	0.80	0.10	0.90	0.02
$\rho_{\pi^\star}$	Beta	0.70	0.15	0.35	0.08
σ_R	InvGamma	1.25	∞	0.38	0.04
σ_q	InvGamma	2.51	∞	1.17	0.09
σ_A	InvGamma	1.25	∞	0.70	0.08
$\sigma_{y^\star}$	InvGamma	1.25	∞	0.87	0.25
$\sigma_{\pi^\star}$	InvGamma	1.88	∞	1.71	0.13

Table 2: Posterior mean and confidence intervals

Parameters		Mean		Confidence intervals	
	Density	Prior	Posterior	lower bound	upper bound
ψ_1	Gamma	1.50	2.354	1.820	2.974
ψ_2	Gamma	0.25	0.191	0.062	0.338
ψ_3	Gamma	0.25	0.208	0.111	0.320
ρ_R	Beta	0.50	0.738	0.679	0.807
α	Beta	0.30	0.151	0.091	0.228
r	Gamma	2.50	2.504	0.983	4.066
κ	Gamma	0.50	0.661	0.373	0.892
τ	Beta	0.50	0.429	0.329	0.514
ρ_q	Beta	0.40	0.380	0.232	0.527
ρ_z	Beta	0.50	0.479	0.424	0.539
$\rho_{y^\star}$	Beta	0.80	0.898	0.859	0.941
$\rho_{\pi^\star}$	Beta	0.70	0.348	0.217	0.467
σ_R	InvGamma	1.25	0.404	0.338	0.475
σ_q	InvGamma	2.51	1.191	1.041	1.329
σ_A	InvGamma	1.25	0.711	0.564	0.844
$\sigma_{y^\star}$	InvGamma	1.25	1.168	0.601	1.567
$\sigma_{\pi^\star}$	InvGamma	1.88	1.747	1.526	1.944

Table 3: Welfare analysis

	Discretion	TP	R
2002:3	137.1129	39.4213	39.3871
2002:4	148.5232	48.5585	48.4410
2003:1	141.5273	43.0367	42.8850
2003:2	184.0804	65.9945	65.6551
2003:3	174.5672	60.4436	59.8473
2003:4	174.1757	60.0793	59.6310
2004:1	166.1597	55.4961	55.0981
2004:2	155.1082	46.0195	45.8393
2004:3	167.5169	55.4237	55.2588
2004:4	171.3004	58.3831	57.9918

Table 4: Alternative specifications of the loss function

	Discretion	Timeless	Ramsey	Discretion	Timeless	Ramsey
	$\mu_1 = 1, \mu_2 = 1, \mu_3 = 0.1$			$\mu_1 = 1, \mu_2 = 1, \mu_3 = 0$		
2002:3	137.1129	39.4213	39.3871	132.6372	35.4474	35.3534
2002:4	148.5232	48.5585	48.4410	144.2159	44.8281	44.5783
2003:1	141.5273	43.0367	42.8850	137.2102	39.0838	38.8544
2003:2	184.0804	65.9945	65.6551	180.0675	62.5312	62.0423
2003:3	174.5672	60.4436	59.8473	170.5730	56.5075	55.9290
2003:4	174.1757	60.0793	59.6310	170.2322	56.2558	55.8558
2004:1	166.1597	55.4961	55.0981	162.1750	51.4989	51.0866
2004:2	155.1082	46.0195	45.8393	151.0413	41.7793	41.5446
2004:3	167.5169	55.4237	55.2588	163.6731	51.6833	51.4091
2004:4	171.3004	58.3831	57.9918	167.5645	54.7148	54.2502
	$\mu_1 = 0.25, \mu_2 = 1, \mu_3 = 0.1$			$\mu_1 = 0.25, \mu_2 = 1, \mu_3 = 0$		
2002:3	63.5599	37.7245	37.5771	59.0842	34.2570	33.8746
2002:4	72.9280	46.7902	46.4806	68.6207	43.6639	42.8453
2003:1	67.6000	41.4071	41.0468	63.2829	38.1191	37.3412
2003:2	94.1631	64.0047	63.2247	90.1502	61.2217	59.6005
2003:3	87.4702	58.8532	57.6150	83.4760	55.6823	53.7940
2003:4	87.2386	58.4822	57.4345	83.2951	55.2097	53.7597
2004:1	81.9140	54.0043	53.0364	77.9293	50.6148	49.2055
2004:2	72.4927	44.4608	43.9438	68.4258	40.8193	39.9655
2004:3	82.3698	53.7148	53.2117	78.5261	50.5407	49.5549
2004:4	85.4408	56.7381	55.8878	81.7050	53.8026	52.2834
	$\mu_1 = 2, \mu_2 = 1, \mu_3 = 0.1$			$\mu_1 = 2, \mu_2 = 1, \mu_3 = 0$		
2002:3	235.1836	43.9899	43.9815	230.7079	35.7029	35.6561
2002:4	249.3168	53.0419	53.0288	245.0095	45.0603	44.9294
2003:	240.0970	47.4430	47.4016	235.7799	39.2792	39.1606
2003:2	303.9701	70.7667	70.7325	299.9572	62.7949	62.5402
2003:3	290.6964	64.9605	64.7116	286.7023	56.6634	56.3612
2003:4	290.0918	64.5461	64.4435	286.1483	56.4830	56.2797
2004:1	278.4873	59.8916	59.7684	274.5026	51.6791	51.4656
2004:2	265.2622	50.5025	50.4592	261.1953	41.9833	41.8639
2004: 3	281.0463	59.8813	59.8747	277.2025	51.9216	51.7815
2004: 4	285.7798	62.7055	62.5954	282.0439	54.8888	54.6458

Table 5: Restrictions on the monetary policy rule

	Discretion	Timeless	Ramsey	Discretion	Timeless	Ramsey	Discretion	Timeless	Ramsey
	$\psi_3 = 0$			$\rho = 0$			$\rho = 0$ and $\psi_3 = 0$		
	$\mu_1 = 1, \mu_2 = 1, \mu_3 = 0.1$								
2002:3	157.6513	43.4373	43.4276	142.8404	49.7218	49.6613	168.2971	52.4383	52.3286
2002:4	170.8226	52.7046	52.6560	151.5820	57.3238	57.3117	181.7785	62.4856	62.4680
2003:1	159.0465	43.2789	43.1932	140.1260	50.9136	50.9041	158.6541	50.1795	50.1540
2003:2	193.5289	61.4235	61.3475	195.7092	84.9234	84.9111	204.5408	77.1364	77.1250
2003:3	191.1069	61.9896	61.6981	184.2269	77.2045	77.1322	203.1520	78.0949	78.0744
2003:4	186.1819	59.4116	59.1946	184.8293	76.4684	76.4632	197.3977	74.9532	74.9499
2004:1	183.2146	59.4389	59.2104	172.0051	68.9089	68.9008	191.9586	73.5009	73.4977
2004:2	170.6470	50.6045	50.4794	156.6011	59.2370	59.2215	175.9647	65.2398	65.2237
2004:3	178.4584	56.0751	56.0117	166.0624	66.3362	66.3127	179.4974	67.9756	67.9481
2004:4	178.3622	56.5608	56.3877	171.6509	71.1609	71.1520	177.4604	68.3540	68.3496
	$\mu_1 = 0.25, \mu_2 = 1, \mu_3 = 0.1$								
2002:3	69.6656	39.4217	39.2482	79.5004	47.1028	47.0638	88.0866	49.9721	49.8933
2002:4	79.4749	48.5873	48.2328	87.0988	54.7667	54.7598	98.6631	60.0723	60.0648
2003:1	69.8637	39.3442	39.0052	79.5613	48.4030	48.3986	84.1064	47.7622	47.7485
2003:2	90.8104	57.1138	56.5765	119.7100	82.4844	82.3399	115.5227	74.7826	74.7576
2003:3	90.7194	57.9387	56.9495	110.3897	74.8478	74.6151	115.6083	75.7925	75.6922
2003:4	87.6004	55.4839	54.5926	110.0415	74.0986	74.0055	111.6943	72.6553	72.6257
2004:1	87.1536	55.5858	54.6577	100.8059	66.5338	66.4680	109.0848	71.2037	71.1709
2004:2	77.9507	46.7535	46.1661	89.7414	56.7754	56.7716	98.8618	62.8980	62.8940
2004:3	83.5360	52.1258	51.6111	97.3505	63.9738	63.9685	101.6352	65.7071	65.7025
2004:4	83.8119	52.7084	51.9963	102.4081	68.9136	68.8492	101.4051	66.1669	66.1512
	$\mu_1 = 0.25, \mu_2 = 1, \mu_3 = 0$								
2002:3	65.5229	36.0588	35.6126	65.3104	43.9441	43.8948	73.8306	46.9418	46.8989
2002:4	75.4873	45.5600	44.6244	73.4777	51.8720	51.7324	85.0334	57.3365	57.1872
2003:1	65.8091	36.0126	35.2396	66.0518	45.4407	45.3563	69.8198	44.7424	44.6887
2003:2	87.0618	54.2316	52.9516	105.5296	80.0990	79.6402	101.9176	72.4001	72.1203
2003:3	86.9557	54.9160	53.1071	96.7174	72.0874	71.7550	102.2630	73.1727	72.9076
2003:4	83.8664	52.2458	50.8962	96.5328	71.4812	71.2288	98.5661	70.0516	69.8775
2004:1	83.3971	52.3112	50.8078	87.4847	63.7099	63.5028	96.0669	68.4803	68.2913
2004:2	74.1106	43.2087	42.2093	76.1089	53.6359	53.5504	85.7365	59.9236	59.8299
2004:3	79.8580	48.9033	47.9245	84.2839	61.2294	61.0717	88.9377	62.9775	62.8496
2004:4	80.2237	49.6946	48.3289	89.6452	66.3840	66.1363	88.9803	63.6099	63.4412

Table 6: Alternative specifications of the monetary policy rule

	Discretion	Timeless	Ramsey	Discretion	Timeless	Ramsey	Discretion	Timeless	Ramsey
	$\mu_1 = 1, \mu_2 = 1, \mu_3 = 0.1$			$\mu_1 = 0.25, \mu_2 = 1, \mu_3 = 0.1$			$\mu_1 = 0.25, \mu_2 = 1, \mu_3 = 0$		
	Forecast-based rule								
2002:3	157.6513	43.4373	43.4276	142.8404	49.7218	49.6613	168.2971	52.4383	52.3286
2002:4	170.8226	52.7046	52.6560	151.5820	57.3238	57.3117	181.7785	62.4856	62.4680
2003:1	159.0465	43.2789	43.1932	140.1260	50.9136	50.9041	158.6541	50.1795	50.1540
2003:2	193.5289	61.4235	61.3475	195.7092	84.9234	84.9111	204.5408	77.1364	77.1250
2003:3	191.1069	61.9896	61.6981	184.2269	77.2045	77.1322	203.1520	78.0949	78.0744
2003:4	186.1819	59.4116	59.1946	184.8293	76.4684	76.4632	197.3977	74.9532	74.9499
2004:1	183.2146	59.4389	59.2104	172.0051	68.9089	68.9008	191.9586	73.5009	73.4977
2004:2	170.6470	50.6045	50.4794	156.6011	59.2370	59.2215	175.9647	65.2398	65.2237
2004:3	178.4584	56.0751	56.0117	166.0624	66.3362	66.3127	179.4974	67.9756	67.9481
2004:4	178.3622	56.5608	56.3877	171.6509	71.1609	71.1520	177.4604	68.3540	68.3496
	Speed limit rule								
2002:3	137.4146	40.4915	40.4800	62.4427	36.5607	36.3715	58.0493	33.3488	32.9142
2002:4	148.2259	49.2018	49.1480	71.3647	45.2275	44.8520	67.1457	42.3407	41.4384
2003:1	142.1537	44.1524	44.0458	66.6931	40.3586	39.9306	62.4662	37.3261	36.4505
2003:2	184.5763	67.1059	66.9290	92.8987	62.7162	61.8034	88.9781	60.1675	58.4004
2003:3	173.9868	60.9081	60.4296	85.6145	57.0566	55.6623	81.7066	54.1625	52.0751
2003:4	174.4525	60.9326	60.6243	85.8803	57.1095	55.9110	82.0225	54.0823	52.4603
2004:1	165.7538	55.8834	55.5829	80.1342	52.2275	51.1238	76.2294	49.0978	47.5278
2004:2	155.2998	46.5950	46.4670	71.0315	42.8538	42.2479	67.0386	39.4882	38.5235
2004:3	168.1372	56.3015	56.2252	81.1886	52.3901	51.7883	77.4214	49.4507	48.3538
2004:4	171.8549	59.1668	58.8994	84.2066	55.3639	54.3796	80.5498	52.6870	50.9961

Table 7: Welfare analysis of the timeless-perspective policy under uncertainty

	Lower bound	TP	Upper bound
2002:3	36.1607	39.4213	43.5384
2002:4	43.1532	48.5585	54.3410
2003:1	37.1661	43.0367	48.1950
2003:2	59.1704	65.9945	71.1465
2003:3	58.4372	60.4436	64.1583
2003:4	56.8750	60.0793	65.2251
2004:1	49.1681	55.4961	59.2781
2004:2	41.2941	46.0195	50.0028
2004:3	50.0420	55.4237	59.7613
2004:4	53.0045	58.3831	64.1568

Figure 1: Prior and Posterior Distributions

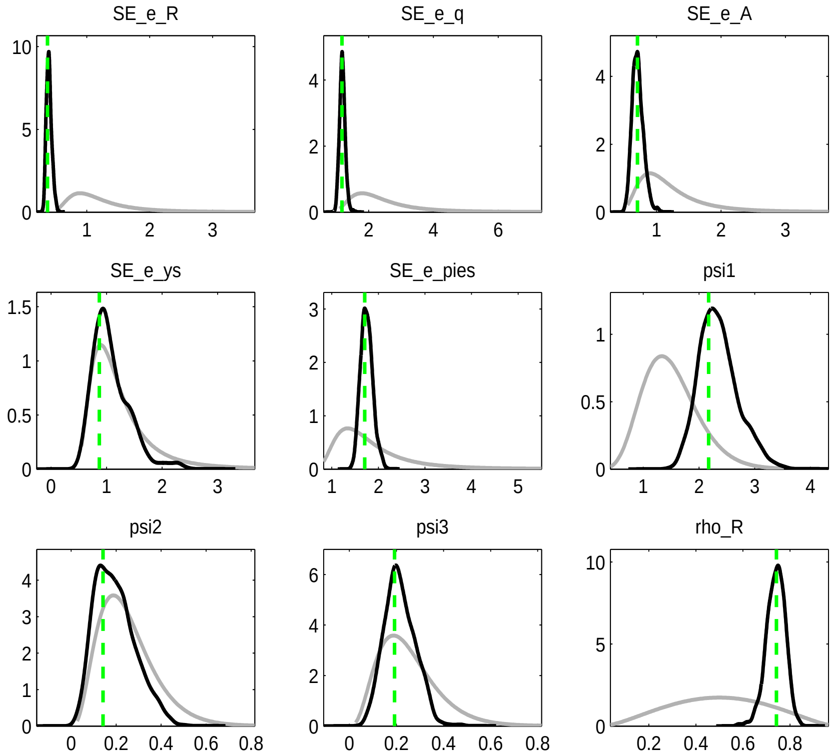

Figure 2: Prior and Posterior Distributions