Learning from Prices: Central Bank Communication and Welfare*

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Abstract

We present a micro-founded economy with money where agents are uncertain about both an aggregate productivity parameter and the monetary aggregate. We show that when agents learn from the distribution of prices, an increase in public information about the monetary aggregate can reduce the information content of the price system and welfare.

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1 Introduction

There is widespread agreement among economists that transparency of monetary policy is beneficial. Among the reasons, it is argued that the introduction of a well defined and transparent rule to conduct monetary policy will remove uncertainty from the economy and achieve better price stability. Also, by committing to a rule, a Central Bank can manage expectations better, and the temptation to create inflation might be alleviated. Finally, by announcing its policy in advance, the Central Bank can be held accountable for its actions. All these are valid and well understood reasons in favor of transparency and indeed, several Central Banks have adopted explicit rules as a basis for their policy making. The main objective of this paper is to investigate the limits of the pro-transparency argument.

We consider a micro-founded economy with money composed of a large number of islands or locations populated by households who are uncertain about the value of an aggregate productivity parameter. To highlight the role of information, we assume that there is no mobility of goods or factors across islands. As is common in the rational expectations literature, information about the productivity parameter is initially dispersed among the households. This dispersed information gets aggregated in the nominal price distribution across locations, which we assume all households observe and learn from. However, the productivity parameter is not fully revealed by this observation as nominal prices are also affected by an unknown level of money supply.

When the Central Bank discloses information about the money supply, it has a direct beneficial effect: other things equal, households can extract more information about the productivity parameter from nominal prices. There is however, a countervailing equilibrium effect: after an announcement, households' decisions rely more on public information, and less on their own local knowledge. This change in behavior tends to reduce the endogenous informational content of prices. Indeed, as long as the Bank cannot provide complete information about the money supply, the distribution of nominal prices in the economy may end up aggregating less information about productivity, reducing households' knowledge and their welfare.

In particular, we show that the observation of the price distribution is equivalent to observing two distinct signals about productivity. The first is a public signal, and can be understood as extracting information from the price distribution about productivity based on the common public prior about the money supply. The second signal is local and obtained by extracting information from the prices based on the local information about the money supply. Thus, households learn from both public and local sources. The existence of the local source is a necessary condition for the negative effects of public information about the money supply: public information can be welfare reducing because it hinders the generation of local information.

The basic model is agnostic about the sources of the monetary disturbances and the reasons why the Central Bank does not posses complete information about them. In Section 3, we introduce two different monetary shocks in order to be explicit about what the monetary disturbance stands for. The first shock is the change in a narrow monetary aggregate which is observed (and controlled) perfectly by the Bank. The second shock is an aggregate velocity shock. It represents changes in general credit conditions in the economy which are imperfectly observed by the Central Bank and not known to the households. We show that publicly disclosing the narrow monetary aggregate is always welfare enhancing, but a public release of imperfect information regarding velocity can be welfare reducing, for the same reasons as in the basic model.

We show that multiple equilibria can occur in a minor generalization of the basic model. We discuss a novel role for monetary policy announcements: public information releases affect the degree of multiplicity. In particular, the release of a sufficiently precise public signal about the monetary aggregate eliminates the multiplicity. However, a mild release could instead generate it.

Several authors have studied Central Bank transparency based on different trade-offs. Moscarini (2007) adds a cheap talk stage in a Barro-Gordon monetary policy game and shows that more precise Central Bank information induces more transparent communication with the general public, and ultimately enhances credibility. Atkeson, Chari and Kehoe (2007) consider the trade-off between a tight policy instrument reducing the volatility of inflation around its target, and a transparent policy instrument allowing better public monitoring of policy action, thus improving equilibrium outcomes. Hoerova, Monnet and Temzelides (2007) setup a Lagos and Wright (2004) model where the Central Bank can only communicate by making money creation contingent on its signal: transparency is socially costly because it entails deviation from the Friedman rule.

More related to our work is Vives (1993), who formalized the learning externality that arises in large abstract economies where agents learn from noisy aggregates. He shows that reliance on new public information makes endogenous signals less precise, and slows down learning; although he did not study the social value of public information. Amato and Shin (2006) is another very related paper. They point out that the release of public information may reduce the informativeness of endogenous signals such as prices: the mechanism at work in our paper, and highlight that this could be welfare reducing. However, agents in their model are not learning from endogenous sources, and thus this reduction in informativeness does not affect the level of knowledge, nor welfare. Also closely related is Morris and Shin (2005), in which the authors argue that public information releases by a Central Bank can impair its ability to gather information in the future, by affecting the way information is aggregated in the economy. Importantly, without introducing payoff externalities, public information releases are welfare improving in this model as well. Our current paper shows that the mechanism highlighted by Amato and Shin (2006) and Morris and Shin (2005) can indeed cause reductions in welfare, while making it explicit in the context of a micro-founded model with money.

There are several papers that have argued that monetary transparency might be welfare reducing in a different context. Recently, Morris and Shin (2002) showed, in a beauty-contest game, that in the presence of payoff externalities, public information releases can reduce welfare (see the subsequent analysis of Angeletos and Pavan (2007)). However, as shown by Hellwig (2005) and Roca (2006) when the payoff externalities are micro-founded in a sticky-price economy à la Woodford (2002), the adverse welfare effect of transparency disappears (see also the results of Lorenzoni (2007) in a different micro-founded model). All of these papers abstract from endogenous learning, the key element emphasized in the current paper. Note that by assuming that goods and factors are immobile, we rule out payoff externalities from the start: the only connection among locations is informational.

The rest of the paper is organized as follows. Section 2 presents the basic set up of the model, defines and characterizes the unique linear equilibrium of the economy. The main results concerning the effects of public announcements about the monetary aggregate on information aggregation and welfare are analyzed in Subsection 2.2. Section 3 introduces velocity shocks and studies optimal announcements of narrow versus broad monetary aggregates. Section 4 provides an extension of the basic model where multiple equilibria can occur and public announcements interplay with the degree of multiplicity. Section 5 shows that opening a bond market does not affect our equilibrium. Section 6 concludes.

2 The Basic Model

Our basic model is a standard cash-in-advance model modified to allow for three features of interest. First, we introduce a real shock that affects the cost of producing which, initially, is imperfectly and differentially known by agents. Second, we allow for differential information among the agents regarding the monetary supply shocks that are affecting the economy (such heterogeneity is naturally generated by idiosyncratic initial money holdings). And finally, even though there is no trade across locations, every household observes the economy wide distribution of nominal prices, and learns from them.

Time is discrete. Although the model is essentially static, we let time be infinite so that money is valued. The economy is composed of a $[0, 1]$-continuum of islands or locations that will be affected by the same real shock. As in Townsend (1983), labor and goods do not flow between islands, but information does: the representative household of each island will observe, and learn, from the nominal prices prevailing in other locations.

Preferences and technology

In each island, there are competitive firms operating a linear technology, transforming one unit of labor into one unit of consumption good. Firms hire labor in a competitive local labor market and sell their output in a competitive local good market. Because labor and goods are immobile, the relative wage in terms of the consumption good is unity in all islands.

At each time $t \in \{1, 2, \ldots\}$ in island $i \in [0, 1]$, a representative household chooses his effort supply, L_{it}, consumption, C_{it}, and money balance M^d_{it}, in order to maximize

$$\mathrm{E}_{i1}\left[\sum_{t=1}^{\infty} \beta^{t-1}\Big(\log(C_{it}) - \Theta L_{it}\Big)\right], \tag{1}$$

where Θ represent an aggregate permanent effort cost, and subject to sequence of budget and cash-in-advance (CIA) constraints,

$$C_{it} + \frac{M^d_{it}}{P_{it}} \leq L_{it} + \frac{M^d_{it-1}}{P_{it}} \tag{2}$$

$$C_{it} \leq \frac{M^d_{it-1}}{P_{it}}. \tag{3}$$

where P_{it} denotes the nominal price level in island i at time t. The initial money balance of the representative household is $M^d_{i0} = M_i$.

Exogenous information about productivity

A key timing assumption of our model is that the cost of effort, Θ, is unknown as of time $t = 1$ but is revealed to everyone at $t = 2$. The goal of this timing assumption is to introduce a risky investment in the model: indeed, in the first period of our equilibrium, households will choose the amount L_{i1} of effort to put in their work, but the return on their investment will have a random component, $-\Theta L_{i1}$. While there are of course others and perhaps more standard models of risky investment, our timing assumption has the advantage of keeping the analysis tractable and transparent.

We assume that all households share the common and fully diffused prior that $\log(\Theta) \equiv \theta$ is normally distributed with a mean and precision of zero. (The diffuse prior assumption will be relaxed in Section 4). Households also observe a local signal about the effort cost:

$$\hat{\theta}_i = \theta + \varepsilon_{\theta i} \tag{4}$$

where $\varepsilon_{\theta i}$ is normally distributed with mean zero and precision ψ_θ.

Exogenous information about money

Households do not know the aggregate money supply. Instead, they share the common prior that the logarithm of the aggregate money supply is normally distributed with mean zero and precision Ψ_m. As is standard in the literature, the precision Ψ_m represents the amount of public information about the money supply. In particular, a public information release about the money supply will translate into an increase in Ψ_m.

In practice, why would households be uncertain about the money supply? First, although the Federal Reserve Board publishes weekly data on money aggregates, M1 and M2, the estimates are subsequently revised as depository institutions either report new data or revise the data they previously reported.[1] In addition, one may argue that even error-free measures of M1 and M2 remain noisy estimates of the "true" aggregate quantity of liquidity that enters the quantity equation and directly influences the price level. This aggregate quantity of liquidity may include some less liquid assets omitted in M1 and M2, and could be influenced by unobserved velocity shocks. In Section 3, we formalize this argument by adding velocity

[1] See the December 2006 Performance Evaluation of the Statistical Release about Money Stock Measures on the Federal Reserve Board website:
http://www.federalreserve.gov/releases/h6/perfeval2006.htm

shocks in the model.

The initial money endowment of the representative household of island i is

$$\log M_i \equiv \hat{m}_i = m + \varepsilon_{mi} \tag{5}$$

where ε_{mi} is normally distributed across islands with mean zero and precision ψ_m. The logarithm of the aggregate money supply is thus $\log M \equiv m + (2\psi_m)^{-1}$. Note that the initial money endowment provides information about the aggregate money supply.

In all the above, we assume that the random variables θ, m, $\varepsilon_{\theta i}$ and ε_{mi}, are all pairwise independent. In all what follows, we use lower-case ψ (upper-case Ψ) to denote the precisions of local (public) information information about aggregate variables.

Information from nominal prices

The only way the islands are connected is informationally: households observe the distribution of nominal prices of the entire economy when making their labor supply and consumption decisions. Let us denote by p_t the average logarithmic price across islands:

$$p_t = \int p_{it} di \tag{6}$$

As we will show later on, the average price level will become a sufficient statistic of the entire price distribution in equilibrium.

Market clearing conditions

We assume that goods and labor cannot flow across the islands, so that the goods market must clear locally

$$C_{it} = L_{it} \tag{7}$$

for all $i \in [0, 1]$ and $t \in \{1, 2, \ldots\}$. Also the money market is in equilibrium,

$$\int M_{it}^d di = M. \tag{8}$$

2.1 Equilibrium

An equilibrium is made up of a sequence of distributions of consumption, labor, and money holdings across islands, together with a distribution of prices in the economy such that, at each time,

1. given the information conveyed by the distribution of prices in the economy, and given the local price, households choose consumption, labor, and money holdings to maximize their expected utility;

2. every local good markets clear.

Before characterizing an equilibrium formally, it is convenient to first analyze the household's problem.

Solving the household problem

Consider the representative household of island $i \in [0,1]$ and let $\beta^{t-1}\lambda_{it}$ and $\beta^{t-1}\mu_{it}$ be the non-negative Lagrange multipliers of his budget constraint (2) and CIA constraint (3). Then, the first-order conditions for consumption, labor, and money balances are

$$\frac{1}{C_{it}} = \lambda_{it} + \mu_{it} \tag{9}$$

$$\mathrm{E}_{it}\left[\Theta\right] = \lambda_{it} \tag{10}$$

$$\frac{\lambda_{it}}{P_{it}} = \beta \mathrm{E}_{it}\left[\frac{\lambda_{it+1} + \mu_{it+1}}{P_{it+1}}\right] \tag{11}$$

where the expectation operator, $\mathrm{E}_{it}\left[\,\cdot\,\right]$, is conditional on all the information available to household i as of time t.

Anticipating that the CIA constraint binds at all times, $C_{it} = M^d_{it-1}/P_{it}$, it follows that $\lambda_{it} + \mu_{it} = P_{it}/M^d_{it-1}$. In addition, plugging the good market clearing condition (7) into the binding budget constraint (2), we obtain that

$$M^d_{it} = M^d_{it-1},$$

implying that a household's money holding must stay equal to his initial money endowment at each time, i.e. $M^d_{it} = M_i$. Now, together with equations (10) and (11), these manipulations show that

$$P_{it} = \beta^{-1} M_i \mathrm{E}_{it}\left[\Theta\right]. \tag{12}$$

Plugging this back into the binding CIA constraint, we then obtain an inverse relationship between consumption and the expected effort cost,

$$C_{it} = \beta \mathrm{E}_{it}\left[\Theta\right]^{-1} \tag{13}$$

Plugging this into equation (9), we obtain that $\mathrm{E}_i[\Theta]/\beta = \lambda_{it} + \mu_{it}$. Together with (10), this implies that $\mu_{it} = (1-\beta)E_{it}\left[\Theta\right]/\beta$ is strictly positive, confirming our guess that the CIA constraint is binding at all times.

Note that the households are concerned only with the cost parameter Θ. Because the goods markets clear locally and the local money supply is constant and known, uncertainty about the aggregate money supply does not directly affect the household's problem. However, as seen in equation (12), nominal prices in the economy combine local expectations about the cost parameter, and local money supplies. Thus, information about money will affect households' ability to extract information about Θ from observing the economy wide prices.

Equilibrium after the second period

From period $t = 2$ onwards, households know the exact realization of Θ. Hence we have from equation (12) that, in equilibrium

$$\begin{aligned} P_{it} &= \beta^{-1} M_i \Theta \\ C_{it} &= L_{it} = \beta \Theta^{-1}, \end{aligned}$$

for all $t \geq 2$. So quantities and prices are determined from $t = 2$ onwards.

Linear equilibrium in the first period

As we proceed to study the competitive equilibrium in the first period, and given that the economy is stationary from $t = 2$ onwards, we simplify notations by removing the time subscript from all first-period variables.

Borrowing from the literature on noisy Rational Expectations in financial markets (see, among many others, Grossman (1975) and Hellwig (1980)), we will look for linear equilibria,

Definition 1 (Linear Equilibrium). *A linear equilibrium is a cross sectional distribution of nominal prices P_i, consumption C_i, effort supplies L_i, and expectations about Θ, $\mathrm{E}_i[\Theta]$, such that*

i) conditional on the realization of (m,θ), the distribution of prices is log normal with constant dispersion and a mean parameter

$$p = a_0 + a_1\theta + a_2 m \ , \tag{14}$$

for some constants a_0, a_1 and a_2.

ii) households' expectations are rational; that is, after observing their private signals $(m_i, \hat{\theta}_i)$ and the distribution of nominal prices in the economy,

$$\mathrm{E}_i[\Theta] = \mathrm{E}\left[\Theta \,|\, \hat{\theta}_i, \hat{m}_i, p\right] \ ; \tag{15}$$

iii) households decisions are optimal and markets clear:

$$C_i = L_i = \beta \mathrm{E}_i[\Theta]^{-1} \tag{16}$$

$$P_i = \beta^{-1} M_i \mathrm{E}_i[\Theta] \, . \tag{17}$$

To understand our rational expectations condition (15), note the following: even though households observe the entire cross-sectional distribution of nominal prices, it is sufficient to condition expectations with respect to only one moment of the distribution, the average price level p. Indeed, in the equilibria we consider, the distribution of prices in the economy is log normal and hence is uniquely parameterized by its mean and its dispersion. Given the additional requirement in part (i), that the dispersion does not depend on the realization (m,θ), the mean parameter, p, thus conveys all the information embedded in the price distribution.

A unique linear equilibrium

We now proceed to construct an equilibrium and show that it is unique. We start by noting that the mean parameter p is informationally equivalent to observing a signal $\hat{z}$ such that $\hat{z} = \theta + m/\alpha$, where $\alpha = a_1/a_2$; given that a_0, a_1, and a_2 are constants. In addition, the following simple transformation of households' information set allows to determine an equilibrium,

Lemma 1. *The joint observation of* $\hat{\theta}_i = \theta + \varepsilon_{\theta i}$, $\hat{m}_i = m + \varepsilon_{mi}$ *and* $\hat{z} = \theta + m/\alpha$ *is equivalent to the joint observation of,*

$$\hat{\theta}_i = \theta + \varepsilon_{\theta i} \tag{18}$$

$$\hat{z}_i \equiv \hat{z} - \hat{m}_i/\alpha = \theta - \varepsilon_{mi}/\alpha \tag{19}$$

$$\hat{z} = \theta + m/\alpha. \tag{20}$$

Proof. This follows immediately by replacing $\hat{m}_i$ by $\hat{z}_i \equiv \hat{z} - \hat{m}_i/\alpha$ in a household's information set, while keeping the two other observations, $\hat{\theta}_i$ and $\hat{z}$, the same. □

The Lemma shows that observing the price level, p, generates two independent signals about about θ. There is first a *public signal* of precision $\alpha^2\Psi_m$, given by (20), which intuitively follows from interpreting the price level in light of the public information about the money supply. Second, the price also generates an *local* signal about θ of precision $\alpha^2\psi_m$, given by (19), which follows from interpreting the price in light of the local information, $\hat{m}_i$, about the money supply. The finding that the publicly observable price level also generates a local signal is the main insight of the Lemma, and will be a key driver of our results.

Because the three signals $(\hat{\theta}_i, \hat{z}_i, \hat{z})$ about θ have independent noises, and their precisions are ψ_θ, $\alpha^2\psi_m$, and $\alpha^2\Psi_m$ respectively, it follows that

Lemma 2. *In any linear equilibrium, households posterior beliefs about* Θ *are log normal with mean and variance parameters:*

$$\mathrm{E}_i\left[\theta\right] = \mathrm{E}\left[\theta \,|\, \hat{\theta}_i, \hat{m}_i, p\right] = \frac{\psi_\theta\,\hat{\theta}_i + \alpha^2\psi_m\,\hat{z}_i + \alpha^2\Psi_m\,\hat{z}}{\psi_\theta + \alpha^2\psi_m + \alpha^2\Psi_m} \tag{21}$$

$$\mathrm{var}_i\left[\theta\right] = \mathrm{var}\left[\theta \,|\, \hat{\theta}_i, \hat{m}_i, p\right] = \frac{1}{\psi_\theta + \alpha^2\psi_m + \alpha^2\Psi_m}, \tag{22}$$

where $\alpha = a_1/a_2$.

Given households' diffuse prior, equations (21) and (22) are the standard Bayesian updating formula for independent signals and normal distribution. In equation (21), the posterior belief about θ is a convex sum of the three signals, where the convex weights reflect the signals relative precisions. In equation (22), the posterior precision, $1/\mathrm{var}_i\,[\theta]$, is obtained by adding up the precisions of the three signals.

Now, from the logarithms of (17) it follows that

$$\log(P_i) = p_i = -\log\beta + \hat{m}_i + \mathrm{E}_i[\theta] + \frac{\mathrm{var}_i[\theta]}{2}. \tag{23}$$

Taking the cross-sectional average, we obtain the average log price is,

$$\begin{aligned} p &= \int p_i\,di = -\log\beta + \int \hat{m}_i\,di + \int \mathrm{E}_i\,[\theta]\ di + \frac{\mathrm{var}_i\,[\theta]}{2} \\ &= -\log\beta + \theta + \left(1 + \frac{\alpha\Psi_m}{\alpha^2\Psi_m + \alpha^2\psi_m + \psi_\theta}\right) m + \frac{\mathrm{var}_i\,[\theta]}{2}, \end{aligned}$$

where the second line follows from substituting the formula of equation (21) into the first line. Hence, our our linear guess that $p = a_0 + a_1\theta + a_2 m$ is verified for $a_0 = -\log\beta + \mathrm{var}_i[\theta]/2$, $a_1 = 1$, $a_2 = 1/\alpha$, and for some α solving the fixed-point equation

$$\begin{aligned} & 1/\alpha = 1 + \frac{\alpha\Psi_m}{\alpha^2\Psi_m + \alpha^2\psi_m + \psi_\theta} \\ \Leftrightarrow\quad & 1 = \alpha + \frac{\alpha^2\Psi_m}{\alpha^2\Psi_m + \alpha^2\psi_m + \psi_\theta} \\ \Leftrightarrow\quad & \alpha = H(\alpha) \equiv \left(1 + \frac{\alpha^2\Psi_m}{\alpha^2\psi_m + \psi_\theta}\right)^{-1}. \end{aligned} \tag{24}$$

The function $H(\alpha)$ is a positive and strictly decreasing function of α; with $H(0) = 1$ and $\lim_{\alpha\to\infty} H(\alpha) = \psi_m/(\Psi_m + \psi_m)$. Hence, it follows that:

Lemma 3. *There exists a unique solution $\alpha_\star$ to equation (24); and $\alpha_\star \in (\psi_m/(\Psi_m+\psi_m), 1)$.*

Note that $\alpha = \alpha_\star$ uniquely determines the cross-sectional distribution of log prices, which is normal with a constant dispersion, as can be seen from equation (23) after substituting in for equations (21) and (22). Also, $\alpha_\star$ determines the cross-sectional distribution of mean beliefs as implied by (21). Finally this determines a unique distribution for consumption and labor supplies, according to (16). Thus, we have shown,

Proposition 1. *There exists a unique linear equilibrium.*

Informational Efficiency and Welfare

Lemma 1 showed that the distribution of nominal prices generates two independent signals about productivity: a local signal with precision $\alpha^2\psi_m$, and a public signal with precision $\alpha^2\Psi_m$. Recall also that a household also receives an exogenous local signal about productivity, $\hat{\theta}_i$. Hence, the total amount of information gathered by a household from local sources has a precision of

$$\psi'_\theta \equiv \psi_\theta + \alpha^2\psi_m, \tag{25}$$

the sum of all local precisions. Since a household starts from a diffuse prior and receives no exogenous public signal about θ,

$$\Psi'_\theta \equiv \alpha^2\Psi_m, \tag{26}$$

measures the precision of the total amount of information it gathers from public sources. As long as ψ_m, Ψ_m and ψ_θ are finite, the equilibrium values of Ψ'_θ and ψ'_θ are also finite as α is bounded above by 1: the price distribution does not perfectly reveal the value of the productivity parameter.

From equation (24), it follows that

$$\alpha = \left(1 + \frac{\Psi'_\theta}{\psi'_\theta}\right)^{-1} = \frac{\psi'_\theta}{\psi'_\theta + \Psi'_\theta}.$$

Lastly, the observation of nominal prices increases a household's precision by

$$\alpha^2\psi_m + \alpha^2\Psi_m = \psi'_\theta + \Psi'_\theta - \psi_\theta.$$

This increase constitutes a natural measure of the informational efficiency of nominal prices. An important result in our island economy is that an improvement in the informational efficiency of nominal prices, $\psi'_\theta + \Psi'_\theta - \psi_\theta$, unambiguously increases *ex ante* utilitarian welfare:

Lemma 4. *The ex ante welfare of any household in the equilibrium is given by*

$$-\frac{1}{2}(\psi'_\theta + \Psi'_\theta)^{-1} + \frac{\log\beta - \beta}{1-\beta} \tag{27}$$

Proof. The *ex ante* time-t flow welfare of a household is

$$\mathrm{E}_0[\log C_{it} - \Theta L_{it}] = \mathrm{E}_0\Big[\mathrm{E}_{it}[\log C_{it}] - \mathrm{E}_{it}[\Theta]L_{it}\Big] = -\mathrm{E}_0 \log \mathrm{E}_{it}[\Theta] + \log\beta - \beta$$

where we used that $C_{it} = L_{it} = \beta \mathrm{E}_{it}[\Theta]^{-1}$ together with the law of iterated expectations. From period 2 onwards, $\mathrm{E}_{it}\Theta = \Theta$. And we know that $\mathrm{E}_0[\log\Theta] = 0$ by the prior distribution assumption. So, $\mathrm{E}_0 \log \mathrm{E}_{it}[\Theta] = \mathrm{E}_0[\log\Theta] = 0$. In the first period, we have that $E_0 \log E_{i1}[\Theta] = E_0[E_{i1}\log\Theta] + \mathrm{var}_{i1}[\log\Theta]/2$. Using the law of iterated expectations and that $\mathrm{var}_{i1}[\log\Theta] = (\psi_\theta + \alpha^2\psi_m + \alpha^2\Psi_m)^{-1}$, and adding up through time, the result follows. □

Households' *ex ante* welfare goes up with the total precision of their first-period beliefs, $\psi'_\theta + \Psi'_\theta$. This simply means that households are better off if they know more about about θ when they make their labor supply decisions. Although intuitive, this result is not a forgone conclusion: indeed, an increase in the informational efficiency of prices does not generally improves welfare.[2]

2.2 The Welfare Impact of Public Communication

Having characterized the unique linear equilibrium of the economy, we proceed now to analyze the question of interest: what is the impact, if any, of public information releases about m? As it is standard in the literature, we will interpret public information releases about m as increases in the precision of the prior about m, Ψ_m. In what follow, we study in turn the impact of Ψ_m on local information, ψ'_θ, on public information, Ψ'_θ, and finally on the total information $\psi'_\theta + \Psi'_\theta$.

Public Communication and Local Precision

Our first result concerns the value of $\alpha_\star$, the equilibrium sensitivity of prices to local information.

Lemma 5. *The equilibrium value of $\alpha_\star$ is strictly decreasing in Ψ_m. It tends to 1 as Ψ_m goes to zero, and tends to zero as Ψ_m goes to infinity.*

Proof. From equation (24), we see that an increase in Ψ_m, reduces $H(\alpha)$. Given that $H(\alpha)$ is strictly decreasing, this implies that $\alpha_\star$ decreases. Given that $\alpha_\star$ decreases in Ψ_m and is

[2]See the first chapter of Brunermeier (2001) and the references therein.

bounded below by 0, $\alpha_\star$ converges to a finite limit as Ψ_m tends to infinity. Clearly, this limit cannot be positive, or else equation (24) cannot be satisfied for sufficiently high Ψ_m. Hence $\alpha_\star$ tends to 0 as Ψ_m tends to infinity. Alternative, given that $\alpha_\star$ is bounded above by 1, and decreases in Ψ_m, it follows that $\alpha_\star$ converges to a finite number as Ψ_m tends to zero, which from (24) implies that $\alpha_\star$ converges to 1. □

Hence, more public information about m will lower the sensitivity of the price to the local information. The intuition is straightforward: the more public information about m, the more weight households assign to their public signal, and the less weight they assign to their local signals when they make their labor supply decision. This lemma immediately implies our second result:

Proposition 2 (Public Crowds Out Private)**.** *The precision of the local information about θ, ψ'_θ, strictly decreases in Ψ_m.*

Proof. Recall that $\psi'_\theta = \psi_\theta + \alpha_\star^2 \psi_m$. Since $\alpha_\star$ is strictly decreasing, it follows that ψ'_θ decreases. □

According to the Proposition, public releases of information concerning m always reduce the amount of local information gathered about θ in the economy.

Public Communication and Public Precision

We now turn to the impact of Ψ_m on public knowledge, Ψ'_θ. From the formula that $\Psi'_\theta = \alpha_\star^2 \Psi_m$, one sees that an increase in Ψ_m has two opposite effects on Ψ'_θ. Holding $\alpha_\star$ constant, Ψ_m increases public knowledge. This is the intuitive direct beneficial effect of public information about m: when households know more about money, they can extract more information from nominal prices. There is, however, a countervailing equilibrium effect: following an increase in public information about m, households put less weight on their local knowledge, reducing the sensitivity $\alpha_\star$. The net effect depends on the size of the release:

Proposition 3. *The precision of the public knowledge about θ, Ψ'_θ is strictly increasing in Ψ_m whenever $\psi_m < 27\psi_\theta$. When $\psi_m > 27\psi_\theta$, there exist values $\underline{\Psi}_m < \overline{\Psi}_m$, such that Ψ'_θ is strictly decreasing in Ψ_m when $\Psi_m \in [\underline{\Psi}_m, \overline{\Psi}_m]$ and strictly increasing otherwise.*

Proof. We have that

$$\Psi'_\theta = \alpha_\star^2 \Psi_m = \frac{1-\alpha_\star}{\alpha_\star}(\psi_\theta + \alpha_\star^2 \psi_m).$$

Note that

$$\frac{\partial \Psi'_\theta}{\partial \Psi_m} = \frac{\partial \alpha_\star}{\partial \Psi_m} \times \frac{\alpha_\star^2 \psi_m - 2\alpha_\star^3 \psi_m - \psi_\theta}{\alpha_\star^2} \equiv \frac{\partial \alpha_\star}{\partial \Psi_m} \times \frac{F(\alpha_\star)}{\alpha_\star^2}.$$

Given that $\alpha_\star$ is strictly decreasing in Ψ_m, the derivative of Ψ'_θ with respect to Ψ_m depends on the sign of $F(\alpha)$. One see that $F(0)$ and $F(1) < 0$. In addition, one can show that $F(\alpha)$ is hump-shaped and achieves its maximum at $\alpha = 1/3$. The maximum value, $F(1/3)$, is negative if $\psi_m < 27\psi_\theta$, which immediately implies that $F(\alpha)$ is always negative. If $\psi_m > 27\psi_\theta$, then $F(\alpha)$ has two distinct positive roots, $\underline{\alpha}$ and $\bar{\alpha}$ such that $F(\alpha) > 0$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, and non-positive otherwise. Given that $\alpha_\star$ is a strictly decreasing function of Ψ_m mapping $[0, \infty]$ onto $[0, 1]$, the result follows. □

Public Communication and Welfare

The final result in this section answers the question that concerns us the most: what is the effect of public information about the money supply on the total knowledge of households and welfare? As can be already inferred from Propositions 2 and 3, an increase in Ψ_m can indeed reduce the total amount known by agents in the economy, $\psi'_\theta + \Psi'_\theta$, because it can reduce both ψ'_θ and Ψ'_θ at the same time. The Proposition below provides the complete characterization:

Proposition 4. *Total precision, $\psi'_\theta + \Psi'_\theta$, and welfare are U-shaped functions of Ψ_m achieving their minimum at:*

$$\Psi_m^{\min} \equiv \max\left\{ 2\psi_m \left(\sqrt{\frac{\psi_m}{\psi_\theta}} - 1 \right), 0 \right\}.$$

Also, total precision $\psi'_\theta + \Psi'_\theta$ tends to infinity as Ψ_m goes to infinity.

Proof. Equation (24) shows that $\alpha_\star = H(\alpha_\star) = (\psi_\theta + \alpha_\star^2 \psi_m)/(\psi_\theta + \alpha_\star^2 \psi_m + \alpha_\star^2 \Psi_m)$, implying that total precision is

$$\psi'_\theta + \Psi'_\theta = \psi_\theta + \alpha_\star^2 \psi_m + \alpha_\star^2 \Psi_m = \frac{\psi_\theta + \alpha_\star^2 \psi_m}{\alpha_\star} = \frac{\psi_\theta}{\alpha_\star} + \alpha_\star \psi_m.$$

Taking derivative with respect to Ψ_m, we obtain the derivatives to total precision with respect to Ψ_m:

$$\frac{\partial\left(\psi'_\theta + \Psi'_\theta\right)}{\partial \Psi_m} = \frac{\partial \alpha_\star}{\partial \Psi_m} \times \left(-\frac{\psi_\theta}{\alpha_\star^2} + \psi_m\right).$$

Keeping in mind that $\partial\alpha_\star/\partial\Psi_m < 0$, we find that total precision increases in Ψ_m if and only if $\alpha_\star < \tilde{\alpha}$, where $\tilde{\alpha} = (\psi_\theta/\psi_m)^{1/2}$. Now, since $\alpha - H(\alpha)$ is strictly increasing and $\alpha_\star - H(\alpha_\star) = 0$, we obtain that $\alpha_\star < \tilde{\alpha}$ if and only if $\tilde{\alpha} > H(\tilde{\alpha})$. The result follows after plugging the value of $\tilde{\alpha}$ in this inequality. $\square$

Proposition 4 is the main result of this paper. It summarizes the impact of public announcements on the informational efficiency of the price distribution and welfare. It tells us that an increase in public information about m, up to some finite precision Ψ_m, will reduce welfare if and only if $\Psi_m < \Psi_m^{\min}$. In particular, this condition holds if the local information ψ_m about the aggregate money supply is sufficiently precise, or when the local information ψ_θ about the productivity parameter is sufficiently small. On the other hand, because $\Psi_m^{\min}$ is finite, it immediately follows that there always exists a sufficiently strong public release that is welfare enhancing. Clearly, whether or not such a strong announcement is possible depends on the information available to the monetary authority about the aggregate money supply. Note also that, if ψ_θ is sufficiently small or ψ_m is sufficiently large, then $\Psi_m^{\min}$ can be made arbitrarily large. This implies that, for any finite information release, there exists a small enough ψ_θ or a large enough ψ_m such that the release ends up welfare reducing.

Another implication of the U shape welfare function is that the Central Bank optimal communication is bang-bang: the Bank should either be completely transparent and release all of its information, or being completely opaque and release no information at all. Thus partial releases of information are never optimal. It is important to highlight, of course, that this bang-bang result relies on the fact that there are no other forces favoring transparency in the model.

Figure 1 shows utilitarian welfare as a function of Ψ_m, for alternative choices of exogenous parameters. As shown in the figure, $\Psi_m^{\min}$ can be equal to zero when ψ_m is small enough: then, an increase in public information is always welfare improving. Hence, an important necessary condition for public information about m to be welfare decreasing is that $\psi_m > 0$: the distribution of price must generate local information. Amador and Weill (2006) have obtained a similar result in an abstract model of information diffusion: local learning from other actions is necessary for public information to be welfare reducing.

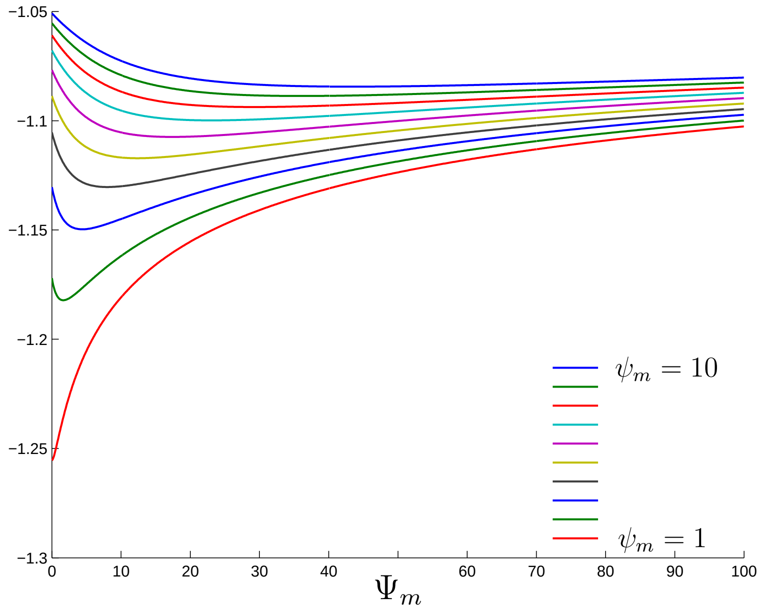

Figure 1: Utilitarian welfare as a function of Ψ_m, for alternative choices of ψ_m.

2.3 Related results from the literature

Perhaps the best known related result is that of Morris and Shin (2002) who have shown, in the context of beauty contest game, that public information can reduce welfare. Although reminiscent of their result, ours does not arise from any payoff externalities but instead from the endogenous aggregation of information through prices.

Morris and Shin (2002) emphasize that releases of public information are beneficial when the precision of the private beliefs about the aggregate state of the economy is sufficiently small. In our economy where the aggregate state has two dimensions, m and θ, this result does not always hold: while more local information about θ does indeed decreases the range where public announcements are welfare increasing as it does in Morris and Shin (2002), more precise local information about m causes the opposite effect.

In Svensson's 2006 critique of Morris and Shin (2002), it is proposed that a conservative benchmark of how likely it is that public information is welfare reducing, is when the precision of the public and the private signals are the same. Indeed, in practice, one would expect the monetary authority to know at least as much about its own policy than the general public. When imposing this restriction in Morris and Shin (2002)'s model, Svensson (2006) finds that public information is welfare increasing: he concludes that Morris and Shin (2002) are, in fact, pro-transparency. In our multidimensional economy, we interpret Svensson (2006)'s restriction as letting $\Psi_m = \psi_m$. A public release of central-bank information up to Ψ_m will then reduce welfare if $\psi_m = \Psi_m < \Psi_m^{\min}$, which is equivalent to $\psi_m > 9/4\psi_\theta$. If this condition is satisfied, then, public information about m decreases welfare even though the precision of the public and the local signals about m are the same.

3 Narrow versus Broad Monetary Aggregates

The objective of the exposition so far has been to clarify the forces at play and to gain an understanding of the mechanism through which public information can be welfare reducing. For that reason, so far we have been imprecise about what monetary aggregate M stands for. In reality, Central Banks collect information concerning several monetary aggregates: from the monetary base, to M1 and other more general credit conditions. In this section we analyze the optimal announcement policy of a monetary authority who has perfect information about a narrow measure of money (as for example the monetary base) and imperfect information about a broader measure. We show that the Central Bank will find it optimal to disclose publicly the narrow monetary aggregate, while the release of public but imprecise information about the broader aggregate can be welfare reducing.

3.1 The Model

Our distinction between narrow and broad monetary aggregate is based on velocity shocks: in our model, the price level in every island not only depends on the local money endowment M_i, but also on some level of velocity, V_i. Precisely, in the equilibrium to be described, the amount of liquidity that enters the quantity equation is not M_i but the product M_iV_i, which we interpret as a broad monetary measure. We assume that the monetary authority has complete information about the distribution of money endowments, M_i. This represent disaggregated information that the Central Bank collects about narrow measures of money, such as the monetary base.[3] On the other hand, we assume that the Central Bank has imperfect information about the velocity shocks, V_i. These represent, for instance, partial information about overall credit conditions.

[3] See http://www.federalreserve.gov/releases/h6/perfeval2006.htm for a description of the disaggregated information collected by the Fed.

The model goes as follows. The representative household in island i faces a CIA constraint of the form:

$$C_{it} \leq V_i \left(\frac{M^d_{it-1} + T_{it}}{P_{it}} \right), \tag{28}$$

where T_{it} are nominal transfers made by the government to household i at the beginning of period t, and V_i represent a local velocity parameter. A standard interpretation of this velocity parameter is that a household can use credit to purchase a fraction $(1 - 1/V_i)$ his consumption goods, but has to purchase the rest of the goods with cash. According to this interpretation, V_i is positively related to the amount of inside money created in island i. The budget constraint of a household is

$$C_{it} + \frac{M^d_{it}}{P_{it}} \leq L_{it} + \frac{M^d_{it-1} + T_{it}}{P_{it}}. \tag{29}$$

For tractability, we assume that the Central Bank follows the Friedman rule of shrinking the money supply at rate β.[4] Precisely we assume that at time t, households in island i have to pay a lump sum tax $T_{it} = -(1-\beta)\beta^{t-1}M_i$. Since the Bank knows the distribution of money endowment, it has sufficient information to implement this policy. Note that this policy is optimal in this economy from period 2 onwards.[5]

Finally, the market clearing conditions are,

$$C_{it} = L_{it} \tag{30}$$

$$\int M^d_{it} di = \beta^t M \tag{31}$$

where $M = \int M_i di$, the initial aggregate money stock.

Households' Information

Households start with the same information about Θ and M as before. Regarding the velocity parameter, we assume that $\log V_i \equiv \hat{v}_i = v + \varepsilon_{vi}$, where ε_{vi} is normally distributed with a mean of zero and a precision ψ_v. In addition, households share the common prior

[4] It is possible to solve for an equilibrium under the assumption that money is constant. We would need to modify the distributional assumptions on the velocity shocks to guarantee that the model remains log normal. These changes would not affect the bottom-line of the results that follow. The details of this exercise are available from the authors upon request.

[5] Whether the Friedman rule is optimal in period 1 remains to be shown. The difficulty relates to footnote 4: changes in the monetary rule, require changes in distributional assumptions for the model to remain log-normal.

that the average velocity, v, is normally distributed with a mean of zero and a precision Ψ_v.[6] Households do not observe the aggregate values m nor v which are assumed to be uncorrelated. They observe, however, the distribution of prices in the economy.

As before, information releases by the Central Bank boil down to increases in the public precisions Ψ_m and Ψ_v. Namely, if the Central Bank releases its perfect information about m, then it increases the corresponding public precision, Ψ_m, towards infinity. The monetary authority, however, does not have full information regarding the velocity shock: it can only increase Ψ_v up to some finite amount.

Equilibrium Equations

Optimization by the households, together with the market clearing conditions, implies that in equilibrium,

$$L_{it} = C_{it} = E_{it}[\Theta]^{-1} \tag{32}$$

$$P_{it} = \beta^t E_{it}[\Theta] V_i M_i \tag{33}$$

$$M_{it}^d = \beta^t M_i. \tag{34}$$

Once Θ is revealed in period 2, the equilibrium is deterministic, so the only interesting behavior occurs in period 1.

Linear equilibrium in the first period

The definition of equilibrium is the same as in Definition 1, except that we now require that the average log price be,

$$p = a_0 + a_1\theta + a_2 m + a_3 v \ ; \tag{35}$$

that the conditional expectations include v_i as part of the information sets; and that equations (16) and (17) are replaced by equations (32) and (33).

We now proceed to construct an equilibrium, and show that it is unique. Let us define $\alpha \equiv a_1/a_2$ and $\delta \equiv a_1/a_3$. Then observing the price p, is equivalent in this case to observing a signal $\hat{z} = \theta + m/\alpha + v/\delta$. The household's posterior belief after observing $\hat{z}$ can be

[6]Since V_i is log normally distributed in the cross section, V_i can be smaller than 1, implying that the faction of credit good, $1 - 1/V_i$, may be less than zero. However, by choosing the mean of V_i large enough, we can make this happen with arbitrarily small probability.

calculated with the projection formula:

$$\mathrm{E}_i[\theta] = \widetilde{\mathrm{E}}_i[\theta] + \frac{\widetilde{\mathrm{cov}}_i(\theta,\hat{z})}{\widetilde{\mathrm{var}}_i(\theta,\hat{z})}\left(\hat{z} - \widetilde{\mathrm{E}}_i[\hat{z}]\right), \tag{36}$$

where the tilde notation ("$\sim$") is used to denote prior beliefs conditional on θ_i, $\hat{m}_i$, and $\hat{v}_i$: $\widetilde{\mathrm{E}}_i[\theta] = \hat{\theta}_i$, $\widetilde{\mathrm{cov}}_i(\theta,\hat{z}) = 1/\psi_\theta$, and

$$\begin{aligned}\widetilde{\mathrm{E}}_i[\hat{z}] &= \hat{\theta}_i + \frac{\psi_m}{\alpha\left(\psi_m + \Psi_m\right)}\hat{m}_i + \frac{\psi_v}{\delta\left(\psi_v + \Psi_v\right)}\hat{v}_i \\ \widetilde{\mathrm{var}}_i(\hat{z}) &= \frac{1}{\psi_\theta} + \frac{1}{\alpha^2\left(\psi_m + \Psi_m\right)} + \frac{1}{\delta^2\left(\psi_v + \Psi_v\right)}.\end{aligned}$$

The posterior variance is equal to

$$\mathrm{var}_i[\theta] = \widetilde{\mathrm{var}}_i[\theta]\left(1 - \frac{\widetilde{\mathrm{cov}}_i(\theta,\hat{z})^2}{\widetilde{\mathrm{var}}_i[\theta]\widetilde{\mathrm{var}}_i[\hat{z}]}\right),$$

the prior variance, multiplied by one minus the R^2 of the regression of θ on $\hat{z}$. Note that the island prices are

$$\log(P_i) = p_i = -\log\beta + \hat{m}_i + \hat{v}_i + \mathrm{E}_i[\theta] + \frac{\mathrm{var}_i[\theta]}{2},$$

which averaging in the cross-section implies

$$p = \int p_i di = -\log\beta + m + v + \frac{\widetilde{\mathrm{cov}}_i(\theta,\hat{z})}{\widetilde{\mathrm{var}}_i(\hat{z})}\left(\hat{z} - \int \widetilde{\mathrm{E}}_j[\hat{z}]\,dj\right) + \frac{\mathrm{var}_i[\theta]}{2}.$$

Plugging the formula for the prior moments and matching unknown coefficients, we find that:

$$1-\alpha = \frac{\Psi_m}{\psi_m + \Psi_m}\frac{1}{\psi_\theta}\left(\frac{1}{\psi_\theta} + \frac{1}{\delta^2(\psi_v + \Psi_v)} + \frac{1}{\alpha^2(\psi_m + \Psi_m)}\right)^{-1} \tag{37}$$

$$1-\delta = \frac{\Psi_v}{\psi_v + \Psi_v}\frac{1}{\psi_\theta}\left(\frac{1}{\psi_\theta} + \frac{1}{\delta^2(\psi_v + \Psi_v)} + \frac{1}{\alpha^2(\psi_m + \Psi_m)}\right)^{-1}, \tag{38}$$

and that the posterior precision of the beliefs is:

$$\mathrm{var}_i[\theta]^{-1} \equiv \Pi = \psi_\theta + \left(\frac{1}{\delta^2(\psi_v + \Psi_v)} + \frac{1}{\alpha^2(\psi_m + \Psi_m)}\right)^{-1} \tag{39}$$

Dividing equation (37) by equation (38), we can express α as a function of δ:

$$\alpha = 1 - (1-\delta)\,\frac{\Psi_m/(\psi_m+\Psi_m)}{\Psi_v/(\psi_v+\Psi_v)} \equiv h(\delta),$$

which, after plugging back into equation (38), gives:

$$1-\delta = \frac{\Psi_v}{\psi_v+\Psi_v}\frac{1}{\psi_\theta}\left(\frac{1}{\psi_\theta}+\frac{1}{\delta^2(\psi_v+\Psi_v)}+\frac{1}{h(\delta)^2(\psi_m+\Psi_m)}\right)^{-1} \equiv T(\delta) \tag{40}$$

The right-hand side of this equation is strictly increasing in δ because $h(\delta)$ is a strictly increasing function of δ, and the left-hand side is, on the other hand, a strictly decreasing one. Since the left-hand side is strictly greater than the right-hand side when $\delta = 0$, and strictly smaller when $\delta = 1$, it follows that there exists a unique value $\delta_\star \in (0,1)$ solving this equation:

Proposition 5. *There exists a unique linear equilibrium, with* $\delta_\star \in (0,1)$.

3.2 Optimal Communication

Given that the Central Bank has full knowledge of the narrow monetary aggregate M, one first question of interest is whether it should make this information public. Repeating the same steps as in the previous subsection, we can show that, if the central bank discloses the value of M and $\Psi_m = \infty$, then the posterior precision is

$$\mathrm{var}_i[\theta]^{-1} \equiv \Pi^{\mathrm{D}} = \psi_\theta + (\delta_\star^{\mathrm{D}})^2(\psi_v+\Psi_v)\ , \tag{41}$$

and $\delta_\star^{\mathrm{D}}$ is the unique solution of:

$$1-\delta = \frac{\Psi_v}{\psi_v+\Psi_v}\frac{1}{\psi_\theta}\left(\frac{1}{\psi_\theta}+\frac{1}{\delta^2(\psi_v+\Psi_v)}\right)^{-1} \equiv T^{\mathrm{D}}(\delta). \tag{42}$$

These formula can be obtained directly by setting $\Psi_m = \infty$ in (39) and (40). Note that $T^{\mathrm{D}}(\delta) \geq T(\delta)$, and hence $T^{\mathrm{D}}(\delta_\star) \geq T(\delta_\star) = 1-\delta_\star$. Given that $T^{\mathrm{D}}(\delta)$ is increasing, this implies that $\delta_\star^{\mathrm{D}} \leq \delta_\star$. Now note that

$$1 \leq \frac{1-\delta_\star^D}{1-\delta_\star} = \frac{T^{\mathrm{D}}(\delta_\star^{\mathrm{D}})}{T(\delta_\star)} = \frac{1/\psi_\theta + (\Pi-\psi_\theta)^{-1}}{1/\psi_\theta + (\Pi^{\mathrm{D}}-\psi_\theta)^{-1}},$$

so that $\Pi^{\mathrm{D}} \geq \Pi$. Hence, full disclosure of M enhances the precision of households' beliefs.

Proposition 6. *Full public disclosure of the narrow monetary aggregate M increases the precision of households' beliefs about θ, and increases welfare.*

Once the value of M has been made public, would the monetary authority find it optimal to release any information she has collected about the aggregate credit conditions, as given by v? We already know the answer to this question: as can be seen directly, once $\Psi_m = \infty$, the model becomes equivalent to our basic model, except that α is now replaced by δ, ψ_m and Ψ_m by ψ_v and Ψ_v. A version of Proposition 4 hence holds, which we state for completeness,

Proposition 7. *Under full disclosure of M, total precision about Θ, and welfare, are U-shape functions of Ψ_v achieving a minimum at*

$$\Psi_v^{\min} = \max\left\{2\psi_v\left(\sqrt{\frac{\psi_v}{\psi_\theta}} - 1\right), 0\right\}$$

The Central Bank in this economy will find itself in situations where it would not want to disclose the knowledge she has about the aggregate credit conditions. At the same time, her precise knowledge about the narrow monetary aggregate (cash in this case) can be safely announced to the public. The case rests on the implicit assumption that the narrow monetary aggregate is known with certainty, while broader measures are not. Recently,in March 2006, the Federal Reserve Board discontinued the reporting of M3. The main reason given for such a decision was that M3 is a very noisy measure, conveying little information, and hence its publication did not justify the costs of collecting the information. The present model warns us that, above and beyond collection costs, publishing such a noisy measure can be welfare damaging.

4 Transparency and Multiplicity

So far we have analyzed our model under the assumption of a completely diffuse prior about Θ. In this section we relax this, and show that, under some conditions, the model admits multiple equilibria. This gives rise to a novel effect of monetary policy announcements: they have an impact on equilibrium determinacy. Namely, a sufficiently strong release of public information about the monetary aggregate eliminates multiplicity. An important caveat is that, a mild release might instead create it.

Accordingly, suppose then that the prior about Θ is normal with a mean $\bar{\theta}$, and some positive precision Ψ_θ. Starting again from the guess that observing the distribution of prices is equivalent to observing $\hat{z} = \theta + m/\alpha$, and keeping the same notations as before, we find that a household's posterior belief about θ has mean parameter

$$\mathrm{E}_i[\theta] = \frac{\psi_\theta \hat{\theta}_i + \Psi_\theta \bar{\theta} + \alpha^2 \psi_m \hat{z}_i + \alpha^2 \Psi_m \hat{z}}{\psi_\theta + \Psi_\theta + \alpha^2 \psi_m + \alpha^2 \Psi_m}. \tag{43}$$

Following a similar sequence of calculation as in Section 2.1, we can derive a new fixed-point equation,

$$\alpha = \left(1 + \frac{\alpha^2 \Psi_m + \Psi_\theta}{\alpha^2 \psi_m + \psi_\theta}\right)^{-1}, \tag{44}$$

which is equivalent to equation (24) when $\Psi_\theta = 0$. The key difference with the $\Psi_\theta = 0$ case is that the right-hand side of the new fixed-point equation (44) it is not necessarily decreasing in α: in particular, there may exist several solutions. Let us rewrite equation (44) as,

$$G(\alpha) \equiv \alpha^3(\psi_m + \Psi_m) - \alpha^2 \psi_m + \alpha(\psi_\theta + \Psi_\theta) - \psi_\theta = 0, \tag{45}$$

and the following holds:

Proposition 8. *There can be multiple equilibria. In particular, for all $(\psi_m, \psi_\theta, \Psi_\theta)$ in the interior of some non-empty set Σ, there exist $\Psi_m^{(1)} < \Psi_m^{(2)}$ such that*

1. *there are three equilibria if $\Psi_m \in [\Psi_m^{(1)}, \Psi_m^{(2)}]$,*
2. *there is a unique equilibrium otherwise.*

On the other hand if $(\psi_m, \psi_\theta, \Psi_\theta)$ does not belong to Σ, then there is a unique equilibrium for all Ψ_m.

The Proposition tells us that a sufficiently large increase in Ψ_m will lead to a unique equilibrium. On the other hand, for a non-empty set Σ of parameter values $(\psi_m, \psi_\theta, \Psi_\theta)$, it is possible that an increase in Ψ_m will move the economy from a unique equilibrium towards multiplicity.

Figure 2 demonstrates graphically how increases in Ψ_m move the economy from uniqueness to multiplicity, and eventually back to uniqueness. The figure shows the function $G(\alpha)$ for different values of Ψ_m. Equilibria are zeros of the function.

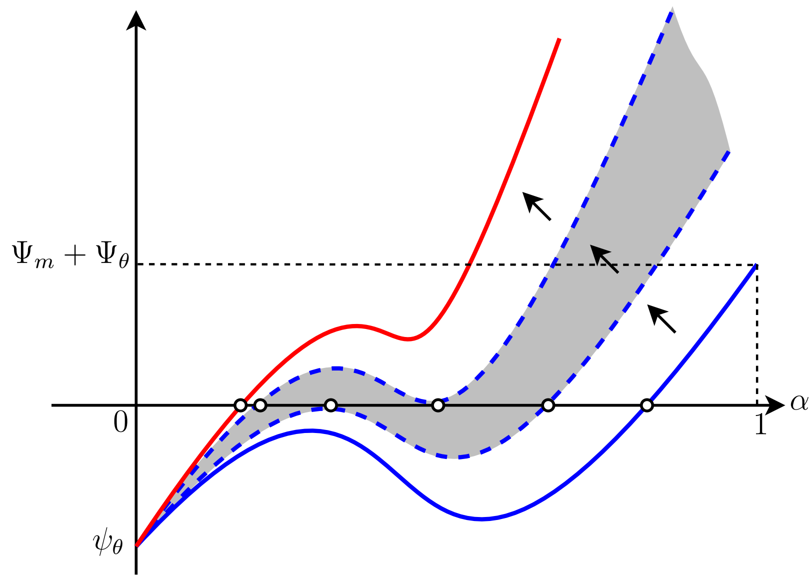

Figure 2: The figure plots the function $G(\alpha)$ for increasing values of Ψ_m, as denoted by the direction of the arrows. The shaded area is the area of multiplicity.

5 A Bond Market: No Trade and No Information

A familiar way in which an economy aggregates dispersed private information is through financial markets. One might wonder then how robust the results regarding the social value of public announcements that we have obtained are to the introduction of a financial market where households from different locations can interact. To answer this question, we introduce what we believe is a natural financial market in our economy: households are allowed to trade a nominal bond in zero net supply. Our main result is that the equilibrium nominal interest rate in that market does not provide any new information to the households, and that the allocation obtained by a competitive equilibrium when the bond market is closed remains the allocation of a competitive equilibrium once it is opened.

Thus, suppose that any household at period t can buy a bond that pays a unit of the currency in the following period, $t+1$, and let us denote by Q_t its nominal price. The budget constraint of the household i in period t is now given by

$$C_{it} + \frac{M_{it}^d}{P_{it}} + \frac{B_{it}}{P_{it}} Q_t \leq L_{it} + \frac{M_{it-1}^d}{P_{it}} + \frac{B_{it-1}}{P_{it}}, \tag{46}$$

where B_{it} are the amount of the bond held by household i in period t. As before, the household is subject to a cash-in-advance constraint.

The bond market clearing condition is, at all times:

$$\int B_{it}\, di = 0. \tag{47}$$

In an equilibrium, from the Euler equation of household i we obtain that

$$Q_t = \beta \mathrm{E}_{it}\left[\frac{u'(C_{it+1})}{u'(C_{it})}\frac{P_{it}}{P_{it+1}}\right].$$

We now check that the allocation without a nominal bond market remains an equilibrium once the nominal bond market opens. Because the CIA constraint binds at all time, $P_{it}C_{it} = M_i$, and hence,

$$Q_t = \beta \mathrm{E}_{it}\left[\frac{C_{it}P_{it}}{C_{it+1}P_{it+1}}\right] = \beta,$$

which is the same for all agents. Note as well that the price of the bond does not reveal any information: it is just equal to the discount factor. Thus any equilibrium allocation when the bond market is closed remains an equilibrium when the bond market is open with $B_{it} = 0$, and $Q_t = \beta$.

6 Conclusions

We have characterized the conditions under which announcements by the Central Bank about the state of the monetary aggregate reduce the informativeness of prices about real shocks and may actually lower welfare. Although we focus in the case where households observe nominal prices, we think it is reasonable to conjecture that a similar outcome will prevail in the presence of financial markets that also aggregate dispersed information in the economy.

Our model is basically static (the infinite horizon was just necessary for money to have value). However, similar techniques as the ones here developed may prove useful in studying the dynamic effects of information releases,[7] and also in answering the timing question: *when* should the Central Bank make announcements. This is all left for future research.

[7] Amador and Weill (2006) have analyzed a dynamic version of a more abstract model.

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