

# Dispersed Information over the Business Cycle: Optimal Fiscal and Monetary Policy \*

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## Abstract

We study how the heterogeneity of information impacts the efficiency of the business cycle and the design of optimal fiscal and monetary policy. We do so within a model that features a standard Dixit-Stiglitz demand structure, introduces dispersed private information about the underlying aggregate productivity, and allows this information to be imperfectly aggregated through certain prices and macroeconomic indicators. Our key findings are the following: **(i)** When information is exogenous to the agents' actions, the response of the economy to either fundamentals or noise is efficient along the flexible-price equilibrium. **(ii)** The endogeneity of learning renders the business cycle inefficient: there is too little learning and too much noise in the business cycle. **(iii)** Both state-contingent taxes and monetary policy can boost learning over the business cycle. **(iv)** Typically, this implies that the optimal tax is countercyclical, while the optimal monetary policy is less accommodative than what is consistent with replicating the flexible-price equilibrium. **(v)** Even if monetary policy were to replicate the flexible-price equilibrium, this would not mean targeting price stability. Rather, the optimal monetary policy has the nominal interest rate increase, and the price level fall, in response to a positive innovation in productivity.

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# 1 Introduction

Different people typically hold different expectations about the state and the prospect of the economy over the business cycle. For example, although they may agree that the economy is in a recession, their opinions may diverge greatly on how deep or how long the recession might be. This could be because they have been exposed to different local economic conditions or otherwise acquired different private information. Of course, such dispersed private information is likely to be aggregated through prices, macroeconomic statistics, and various market and non-market interactions. However, this aggregation is likely to be imperfect and slow, leaving firms and households with non-trivial heterogeneity in their expectations at the moment of crucial economic decisions.

This heterogeneity is evident in surveys of both consumer and professional forecasts of economic activity. To illustrate, Figure 1 depicts the times series of the cross-sectional means (dashed lines) and standard deviations (solid lines) of four types of forecasts: the consumers' forecasts of inflation over the next year, as taken from the University of Michigan Survey of Consumers; the corresponding forecasts of professional analysts, as taken from the Fed's Survey of Professional Forecasts (SPF); their forecasts of real consumption growth; and their forecasts of the growth rate of corporate profits. The standard deviation of the consumers' forecasts of inflation is about 4% in recent times, but it has been as high as 10% in more turbulent times. Not surprisingly, the corresponding SPF measure has been consistently lower, around 1%. However, even professional analysts exhibit significant heterogeneity when it comes to some other economic variables. For example, the standard deviation of the SPF forecasts of the growth rate of corporate profits is as high as 10%.<sup>1</sup>

Recent research has highlighted some of the *positive* implications of the dispersion of information for the business cycle. For example, when combined with complementarity in pricing decisions, the dispersion of information can lead to significant inertia in the response of the economy to the underlying fundamental shocks (e.g., Woodford, 2003b). Moreover, any common noise in information can generate aggregate fluctuations that are orthogonal to productivity shocks and that resemble the impact of demand or sentiment shocks (e.g., Lorenzoni, 2008a). However, it is unclear what are the *normative* implications of the dispersion of information for the business cycle. Is there any inefficiency in the response of the economy to the underlying fundamental and the underlying noise? And how should the dispersion of information affect the design of optimal fiscal and monetary policy over the business cycle?

A related question regards the desirability of policies that provide the market with more precise information by, say, improving the quality of macroeconomic data or the transparency of central bank communications. Using more abstract settings, recent work has shown that, if trading or informational interactions induce inefficiencies in the equilibrium use of information, more information may actually *decrease* welfare (e.g., Morris and Shin, 2002; Angeletos and Pavan, 2007; Amador and Weill, 2007, 2008). Is this possibility relevant within a canonical model of the business cycle?

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<sup>1</sup>The observed heterogeneity in opinions could also be because people are irrational, or because they have "agreed to disagree" as in the heterogeneous-priors literature. These possibilities will not be considered in this paper.

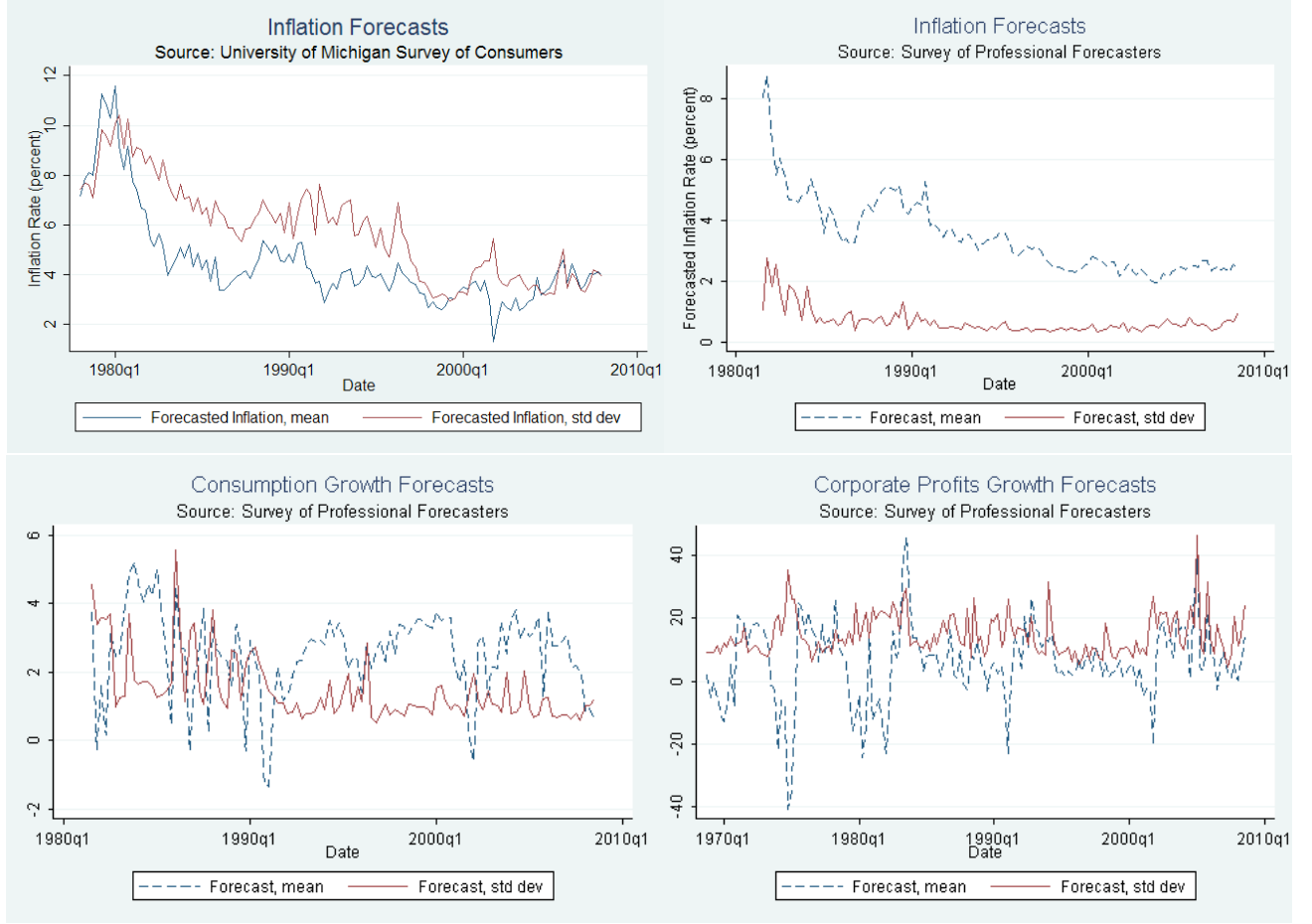


Figure 1: Heterogeneous expectations

In this paper we seek answers to these questions. Towards this goal, we consider a micro-founded business-cycle model that features a standard Dixit-Stiglitz structure as in Blanchard and Kiyotaki (1987) and that introduces dispersed information about the underlying aggregate productivity in a similar fashion as Lucas (1972) had done for money.<sup>2</sup> In particular, information is symmetric at the time of consumption choices—our economy admits a representative consumer—but is segmented across different “islands” at the time of certain production or pricing choices. This “geographical” segmentation of the information is the key friction in the model. However, we do allow information to be (partially) aggregated through certain prices, macroeconomic statistics, or other channels of social learning. Finally, we consider two versions of the model: the baseline version assumes flexible prices so as to isolate informational frictions from nominal frictions; an extension adds incomplete nominal adjustment so as to complete the analysis of optimal fiscal and monetary policies.

Our analysis then builds on three methodological blocks. First, we follow Angeletos and Pavan (2007, 2008) in studying an efficiency benchmark that abstracts from the details of policy

<sup>2</sup>Although we focus on productivity shocks, our analysis can also accommodate taste or monetary shocks.

instruments and identifies the best allocation among those that respect resource feasibility and the geographical segmentation of information. Second, we follow the Ramsey literature (e.g., Lucas and Stokey, 1983; Chari and Kehoe, 1999) in identifying the best allocation among the ones that can be implemented with simple fiscal- and monetary-policy instruments. Finally, we follow Woodford (2003a) in studying how optimal monetary policy relates to the benchmark of replicating flexible-price allocations. The combination of these approaches facilitates a sharp characterization of (i) the inefficiencies, if any, that the dispersion of information may cause over the business cycle; (ii) the power that different policies may have to influence the decentralized use of information over the business cycle; and (iii) the precise nature of the optimal fiscal and monetary policies.

**Preview of results.** Our first result (Theorem 1) is that the dispersion of information does not, by itself, cause any allocative inefficiency. In particular, as long as information is exogenous, the equilibrium business cycle is efficient when prices are flexible and there are no tax distortions. By implication, the aforementioned positive properties are not symptoms of inefficiency; the optimal fiscal policy is acyclical (it is the familiar fixed subsidy that offsets the monopolistic mark-up); and the optimal monetary policy only replicates the flexible-price allocations when prices are sticky.

This result extends an important lesson from the pertinent business-cycle literature. Under homogenous information, previous work has established that fluctuations driven by productivity or taste shocks are typically efficient in Dixit-Stiglitz economies when prices are flexible, or when monetary policy replicates the flexible-price equilibrium (see, e.g., Woodford, 2003a). Here, we establish that this important normative property of this class of models is robust to heterogeneous information as long as one abstracts from the endogeneity of this information.

Our second result (Theorem 2) is that, once the planner takes into account the endogeneity of learning, he finds it optimal to raise the sensitivity of allocations to local information relative to equilibrium. By our first result, doing so necessarily means some losses in allocative efficiency. However, it also means higher precision in the information contained in prices, macro data and other signals of economic activity, which explains why doing so is optimal. In other words, the efficient allocation trades off lower allocative efficiency for higher informational efficiency.

It is useful to note how the first result is instrumental to the second result. The literature on herding and social learning often abstracts from trading or other payoff interactions (e.g., Banerjee, 1992; Vives, 1997; Chari and Kehoe, 2003). When such interactions are absent, one can safely guess that the equilibrium features too little social learning—and too much herding—relative to what is efficient. However, one cannot presume this result more generally: if payoff interactions induce additional inefficiencies in the use of information, the equilibrium can feature either too little or too much social learning. Hence, it is only our first result that permits to pin down the informational inefficiency of the equilibrium for the particular class of economies considered in this paper.

We next study how this informational inefficiency impacts the design of optimal fiscal and monetary policies over the business cycle. Towards this goal, we first show that the contingency of taxes on macroeconomic outcomes give the government the power to manipulate the sensitivity

of equilibrium allocations to private information and thereby to enhance the equilibrium degree of social learning. The key insight is that, when firms expect taxes to fall with realized aggregate productivity, they find it optimal to react more strongly to any private information they might have about aggregate productivity, which in turn contributes to higher quality of information in prices and macro data. We further illustrate this power of state-contingent taxes by showing that, within our benchmark model, a simple linear state-contingent tax on firm output or household income suffices for implementing the efficient allocation as an equilibrium (Theorem 3).

We then proceed to study the extended version of our model, which has firms fixing nominal prices in stage 1, while information is dispersed, and making an additional employment/production choice in stage 2, thus permitting real output to respond to variations in aggregate demand (caused either by the realized productivity shock or by monetary policy). We show that the properties of efficient allocations for this extended model are similar to those of the baseline model: internalizing the informational externalities requires raising the sensitivity of local allocations to local information. We further show that monetary policy can have similar incentive effects as state-contingent taxes: by controlling how firms expect aggregate demand to react to the realized macroeconomic conditions, monetary policy can influence how firms react to their private information at the time of key production and pricing choices, and can thereby also influence the level of social learning.

Of course, this does not necessarily mean that it is optimal for the monetary authority to exercise this power: if state-contingent taxes can alone implement the efficient allocation as a flexible-price equilibrium, there is no reason for monetary policy to deviate from the principle of replicating the flexible-price allocations. However, because in our extended model firms engage in multiple production and pricing choices, each one of which may involve different informational externalities, full efficiency cannot be implemented merely with a tax that has a uniform incentive effect across all firm decisions. Rather, it requires multiple tax instruments, one for each relevant firm decision—a requirement that is likely to be too stringent for any practical purpose.

For this reason, we shift focus to the set of allocations that can be implemented with a single uniform tax instrument, namely a linear state-contingent tax either on firm output/employment or on household labor income/consumption, along with a state-contingent monetary policy. The best allocation within this set falls short of the efficient allocation. Nevertheless, it inherits all the key qualitative properties of the efficient allocation: it is once again optimal to trade off some allocative efficiency for better social learning. And because taxes alone can no more achieve full efficiency, monetary policy needs to assist fiscal policy in the aforementioned trade off.

This leads us to our fourth key result (Theorem 4): for the empirically relevant case where firm choices are strategic complements, the optimal monetary policy is less accommodative of the aggregate productivity shock than the one that replicates the flexible-price allocations. As for the optimal fiscal policy, this remains countercyclical, much alike in the baseline model.

We proceed to study the desirability of price stability (Theorem 5). In standard micro-founded new-Keynesian models, price stability is often synonymous to eliminating distortions in relative prices, replicating the flexible-price allocations, and stabilizing the efficiency gap (Goodfriend and

King, 1997, 2001; Rotemberg and Woodford, 1997; Woodford, 2002, 2003a; Galí, 2003). We show that this is not the case in our setting. In fact, we show that, when aggregate productivity is not common knowledge, a monetary policy that targets price stability implies a procyclical gap between the equilibrium and the efficient allocation and distorts both the allocative and the informational role of prices. We further show that the optimal monetary policy targets a negative correlation between innovations in the price level and innovations in aggregate productivity—or, equivalently, that the nominal interest must increase with positive innovations in aggregate productivity.

We conclude our analysis by revisiting the recent debate on the social value of information and the benefits of central bank transparency. As mentioned earlier, previous work has highlighted that providing the economy with more information could reduce equilibrium welfare because of inefficiencies that originate either from trading and other payoff interactions (Morris and Shin, 2002; Angeletos and Pavan, 2007) or from informational externalities (Amador and Weill, 2007, 2008). The former possibility is automatically ruled out by our first result (Theorem 1); our last result (Theorem 6) establishes that the second possibility is also ruled out once the fiscal and monetary policies are set optimally.

**Layout.** The paper is organized as follows. Section 2 discusses the relation of the paper to the pertinent literature. Section 3 introduces the baseline model. Section 4 studies equilibrium and efficiency with exogenous information. Section 5 considers the implications of social learning. Section 6 characterizes the optimal fiscal policy. Section 6 introduces the extended (monetary) model. Section 8 characterizes the optimal monetary policy. Section 9 studies the optimality of price stability and Section 10 the social value of information. Section 11 concludes.

## 2 Related literature

The macroeconomics literature on informational frictions has a long history, going back to Phelps (1971), Lucas (1972), Barro (1976), and Townsend (1983). Recently, this literature has been revived by Mankiw and Reis (2002), Morris and Shin (2002), Sims (2003), Woodford (2003b), and many others.<sup>3</sup> This paper contributes to this literature by studying the *normative* properties of business cycles with dispersed information; to the best of our knowledge, all the key normative results of this paper are novel to the literature.

Woodford (2003b) shows how the combination of dispersed information with complementarities in pricing decisions can induce significant inertia in the response of the price level to the underlying shocks in aggregate nominal demand, while Amato and Shin (2006) indicates how complementarities can also reduce the informational value of prices.<sup>4</sup> However, due to the lack of micro-foundations,

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<sup>3</sup>See, e.g., Amato and Shin (2006), Angeletos and Pavan (2004, 2007, 2008), Bacchetta and Wincoop (2005), Collard and Dellas (2005), Hellwig (2002, 2005), Lorenzoni (2008a,b), Mackowiak and Wiederholt (2007), Moscarini (2004), Reis (2006a, 2006b), Rodina (2008), Van Nieuwerburgh and Veldkamp (2008), and Veldkamp (2006).

<sup>4</sup>See also Hellwig (2002) for an extension of Woodford (2003b); Morris and Shin (2006) for a related point on the inertia of forward-looking expectations; and Bacchetta and Wincoop (2005) for how a similar mechanism might help

these papers do not address the normative content of these properties. Using a micro-founded model, Lorenzoni (2008a) shows how common noise in information regarding aggregate productivity can generate demand-like fluctuations, but also does not consider the normative properties of these fluctuations. Our contribution is, first, to show that these inertia and volatility effects are not *per se* symptoms of inefficiency; second, to identify fiscal and monetary policies that have the power to reduce this inertia and these noise-driven fluctuations, as well as to improve the informational content of prices; and third, to study when it is desirable to exercise this power.

Angeletos and Pavan (2007) highlights that the welfare implications of dispersed information can be understood only by comparing the equilibrium use of information with an appropriate efficiency benchmark. Angeletos and Pavan (2008) then shows how the contingencies of taxes on realized aggregate outcomes can control the decentralized use of information and thereby help improve efficiency. Both papers consider a flexible, but abstract, framework. This permits to identify several key principles that may guide welfare and policy analysis across a variety of applications, but does not address the normative properties of any particular application. The main contribution of the present paper is to characterize the normative properties of business cycles with dispersed information. A secondary contribution is to provide a concrete example of how the methodology of that earlier more abstract work can be adapted to fully micro-founded applications.<sup>5</sup>

Lorenzoni (2008b) studies optimal monetary policy in a micro-founded new-Keynesian economy that is similar to ours in that it features dispersed information about aggregate productivity (but rules out endogenous learning). That paper's main contribution is to show that, although monetary policy has the power to fully offset the aggregate fluctuations that are caused by noise, it is optimal to partly accommodate them. Our contribution is to shed further light on the nature of optimal monetary policy, first, by showing that monetary policy should only replicate the flexible-price allocations if information were exogenous; second, by showing that this means targeting a negative correlation between the price level and aggregate productivity rather than price stability; and third, by studying how optimal monetary policy is affected by the endogeneity of learning.

Hellwig (2005) studies the social value of information within a new-Keynesian economy in which firms have dispersed information about aggregate nominal demand; the latter is assumed to be exogenous but random. That paper makes an important contribution to the recent debate on the desirability of central-bank transparency that has followed Morris and Shin (2002). However, unlike our paper, it does not study the efficiency of the business cycle or the design of optimal monetary policy; it also rules out endogenous learning.<sup>6</sup>

Andersen (1986) is an early predecessor to this paper and all the aforementioned work. That paper studies a reduced-form monopolistic economy with dispersed information about nominal

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explain the exchange-rate determination puzzle.

<sup>5</sup>For other such examples, see Angeletos, Lorenzoni and Pavan (2007) and Lorenzoni (2008a).

<sup>6</sup>Incidentally, our results indicate that the finding in Hellwig (2005) that more precise private information reduces welfare originates in the sub-optimality of monetary policy rather than the primitives of the environment: in that paper, monetary policy is exogenous and different from the one that would replicate the flexible-price allocations.

shocks, much alike Woodford (2003b). Although it lacks a proper welfare analysis and rules out informational externalities, it does illustrate how feedback policy rules can affect the decentralized use of information. Related are also Weiss (1980) and King (1982). Following the tradition of Lucas (1972) and Barro (1976), those papers study how different monetary policy rules can affect the nominal uncertainty faced by the economy and thereby the information nominal prices convey about real (relative) prices. Our contribution, instead, concerns the incentive effects of monetary policy: by influencing incentives in the decentralized use of information, monetary policy, just as state-contingent taxes, has the power to influence the informativeness of any indicator of economic activity, whether nominal or real.

Finally, this paper contributes to the Ramsey literature (e.g., Lucas and Stokey, 1983; Chari, Christiano and Kehoe, 1994; Benigno and Woodford, 2004) by introducing dispersed private information about the aggregate shocks hitting the economy, by showing how state-contingent policies can impact the decentralized use of such information, and by studying how this dispersion of information impacts the design of optimal policies. In so doing, the paper builds a bridge between the Ramsey literature and the aforementioned macroeconomics literature on informational frictions. It also differentiates from the new-dynamic-public-finance literature, which focuses on private information about *idiosyncratic* rather than *aggregate* shocks.<sup>7</sup>

### 3 The baseline (RBC-like) model

There is a (unit-measure) continuum of households, or “families”, each consisting of a consumer and a continuum of workers. There is a continuum of “islands”, which define the boundaries of local labor markets as well as the “geography” of information: information is symmetric within an island, but asymmetric across islands. Each island is inhabited by a continuum of firms, which specialize in the production of differentiated commodities. Households are indexed by  $i \in I = [0, 1]$ ; islands by  $k \in K = [0, 1]$ ; firms and commodities by  $(j, k) \in J \times K$ ; and periods by  $t \in \{0, 1, 2, \dots\}$ .

Each period has two stages. In stage 1, each household sends a worker to each of the islands. Local labor markets then open, workers decide how much labor to supply, firms decide how much labor to demand, and local wages adjust so as to clear the local labor market. At this point, workers and firms in each island have perfect information regarding local productivity, but imperfect information regarding the productivities in other islands. After employment and production choices are sunk, workers return home and the economy transits to stage 2. At this point, all information that was previously dispersed becomes publicly known, and commodity markets open. Quantities are now pre-determined by the exogenous productivities and the endogenous employment choices made during stage 1, but prices adjust so as to clear product markets.

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<sup>7</sup>See Angeletos and Pavan (2008) for a more general analysis of the distinct policy implications of private information on aggregate shocks.

**Households.** The utility of household  $i$  is given by

$$u_i = \sum_{t=0}^{\infty} \beta^t \left[ U(C_{i,t}) - \int_k V(n_{ik,t}) dk \right] = \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \int_k \frac{n_{ik,t}^\epsilon}{\epsilon} dk \right]$$

where  $\gamma \geq 0$  parametrizes the income elasticity of labor supply (also, the coefficient of relative risk aversion),  $\epsilon \geq 1$  parameterizes the Frisch elasticity of labor supply,  $n_{ik,t}$  is the labor of the worker who gets located on island  $k$  during stage 1 of period  $t$ , and  $C_{i,t}$  is a CES aggregator of all the commodities that the household purchases and consumes during stage 2. In particular,

$$C_{i,t} = \left[ \int_{J \times K} c_{i,jk,t}^{\frac{\rho-1}{\rho}} d(j,k) \right]^{\frac{\rho}{\rho-1}}$$

where  $c_{i,jk,t}$  is the quantity household  $i$  consumes in period  $t$  of the commodity produced by firm  $j$  on island  $k$  and  $\rho > 1$  is the elasticity of substitution across commodities.

Households own equal shares of all firms in the economy. The budget constraint of household  $i$  is thus given by the following:

$$\int_{J \times K} p_{jk,t} c_{i,jk,t} d(j,k) + B_{i,t+1} \leq \int_{J \times K} \pi_{jk,t} d(j,k) + \int_K w_{k,t} n_{ik,t} dk + R_t B_{i,t},$$

where  $p_{jk,t}$  is the period- $t$  price of the commodity produced by firm  $j$  on island  $k$ ,  $\pi_{jk,t}$  is the period- $t$  profit of that firm,  $w_{k,t}$  is the period- $t$  wage on island  $k$ ,  $R_t$  is the period- $t$  nominal gross rate of return on the riskless bond, and  $B_{i,t}$  is the amount of bonds held in period  $t$ .

The objective of each household is simply to maximize expected utility subject to the budget and informational constraints faced by its members. Here, one should think of the worker-members of each family as solving a team problem: they share the same objective (family utility) but have different information sets when making their labor-supply choices. Formally, the household sends off during stage 1 its workers to different islands with bidding instructions on how to supply labor as a function of (i) the information that will be available to them at that stage and (ii) the wage that will prevail in their local labor market. In stage 2, the consumer-member collects all the income that the worker-member has collected and decides how much to consume in each of the commodities and how much to save (or borrow) in the riskless bond.

**Asset markets.** Asset markets operate in stage 2, when information is homogenous. This guarantees that asset prices do not convey any information. The sole role of the bond market in the model is then to price the risk-free rate. Moreover, because our economy admits a representative consumer, allowing households to trade risky assets in stage 2 would not affect any of the results.

**Firms.** The output of firm  $j$  on island  $k$  during period  $t$  is given by

$$q_{jk,t} = A_{k,t} (n_{jk,t})^\theta$$

where  $A_{k,t}$  is the productivity in island  $k$ ,  $n_{jk,t}$  is the firm's employment, and  $\theta \in (0,1)$  is a coefficient that parameterizes the degree of diminishing returns in production. The firm's realized profit is given by

$$\pi_{jk,t} = p_{jk,t} q_{jk,t} - w_{k,t} n_{jk,t}$$

Finally, the objective of the firm is to maximize its expectation of the representative consumer's valuation of its profit, namely, its expectation of  $U'(C_t)\pi_{jk,t}$ .<sup>8</sup>

**Labor and product markets.** Labor markets operate in stage 1, while product markets operate in stage 2. Because labor cannot move across islands, the clearing conditions for labor markets are as follows:

$$\int_J n_{jk,t} dj = \int_I n_{ik,t} di \quad \forall k$$

On the other hand, because commodities are traded beyond the geographical boundaries of islands, the clearing conditions for the product markets are as follows:

$$\int_I c_{i,jk,t} di = q_{jk,t} \quad \forall (j, k)$$

**Aggregate and idiosyncratic productivity shocks.** The island-specific productivities  $A_{k,t}$  are lognormally distributed in the cross-section of islands:

$$a_{k,t} \equiv \log A_{k,t} = \bar{a}_t + \xi_{k,t}$$

where  $\bar{a}_t$  denotes the underlying aggregate productivity shock and  $\xi_{k,t}$  is a purely idiosyncratic (i.e., island-specific) productivity shock. The aggregate shock is drawn from a Normal distribution with mean  $\mu_{A,t}$  and variance  $\sigma_{A,t}^2$ , while the idiosyncratic shock is drawn from a Normal distribution with mean 0 and variance  $\sigma_{\xi,t}^2$ . The variables  $\mu_{A,t}$ ,  $\sigma_{A,t}$  and  $\sigma_{\xi,t}$  must be common knowledge in period  $t$  but need not be deterministic: they could be arbitrary functions of the (public) history of past productivity shocks.<sup>9</sup> For future reference, we let  $\kappa_{A,t} \equiv \sigma_{A,t}^{-2}$  and  $\kappa_{\xi,t} \equiv \sigma_{\xi,t}^{-2}$ .

**Information.** Aggregate productivity is assumed to be common knowledge at stage 2, when all production materializes and consumption takes place, but not in stage 1, when the key employment and production choices are made. Rather, at this stage workers and firms in any given island get to know the productivity of their own island but not the productivities of other islands. Local productivity thus serves also as a valuable, but noisy, private signal of the distribution of productivities and informations in other islands.<sup>10</sup> In addition to this private signal, all firms and households observe an exogenous public signal of aggregate productivity:

$$y_t^a = \bar{a}_t + \varepsilon_t^{ya},$$

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<sup>8</sup>This objective can be justified in two ways. First, we could let the households trade stocks of all the firms in the financial market that operates during stage 2; firms would then make their employment and production choices in stage 1 so as to maximize their expected market valuation, where the latter is computed using the pricing Kernel that obtains in stage 2. Alternatively, we could let each family include a continuum of producers, each of whom is randomly located to a different island during stage 1 and gets to run one of these firms; producers would then make their employment and production choices in stage 1 so as to maximize their expectation of their families' valuation of the profit their firms will make.

<sup>9</sup>For example, the special case where aggregate productivity follows a random walk could be nested by letting  $\mu_t = \bar{a}_{t-1}$  and letting  $\sigma_t$  be a constant.

<sup>10</sup>The assumption that firms and workers know their own productivities perfectly is inessential; all our results go through if we allow for uncertainty about local as well as aggregate productivity.

where  $\varepsilon_t^{ya} \sim \mathcal{N}(0, \sigma_{ya}^2)$  is noise. For future reference, we let  $\kappa_{ya} \equiv \sigma_{ya}^{-2}$ .

Finally, firms and households in each island observe two endogenous signals about the production activity that is taking place in other islands, one public and one private. In particular, letting

$$Q_t \equiv \left[ \int_{J \times K} q_{jk,t}^{\frac{\rho-1}{\rho}} d(j,k) \right]^{\frac{\rho}{\rho-1}}$$

measure aggregate output, the endogenous public signal is

$$y_t^q = \log Q_t + \varepsilon_t^{yq},$$

while the endogenous private signal is

$$x_{k,t}^q = \log Q_t + \varepsilon_{k,t}^{xq},$$

where  $\varepsilon_t^{yq} \sim \mathcal{N}(0, \sigma_{yq}^2)$  and  $\varepsilon_t^{xq} \sim \mathcal{N}(0, \sigma_{xq}^2)$  are noises, the first one common and the second one idiosyncratic. The first signal is meant to capture the macroeconomic data collected and released by various government agencies.<sup>11</sup> The second signal can be thought of as emerging from firms collecting data about product conditions in other islands; idiosyncratic measurement error could then justify with the idiosyncratic noise in these signals. In a richer version of the model, these private signals could also emerge from localized trading or other interactions; each island then gets to see the production levels of the islands with which it trades, but each island trades only with a few other islands, so that it is as if each island observes aggregate output with idiosyncratic noise. More generally, the signals  $y^q$  and  $x^q$  are meant to capture various channels of social learning, some of which could be global (public) and some of which could be local (private).

## 4 Equilibrium and efficiency with exogenous information

In this section we characterize equilibrium and efficient allocations abstracting from the endogeneity of information: we momentarily shut down the endogenous signals  $y_t^q$  and  $x_{k,t}^q$ . The information of a firm or a worker in island  $k$  during stage 1 then consists of the following: the local productivity  $a_{k,t}$ ; a sufficient statistic  $x_{k,t}$  for the private (local) information regarding the unknown aggregate productivity  $\bar{a}_t$ , whose precision we denote by  $\kappa_x$ ; and a sufficient statistic  $y_t$  for the public information regarding  $\bar{a}_t$ , whose precision we denote by  $\kappa_y$ . Here, the statistics  $x_t$  and  $y_t$  coincide, respectively, with the local productivity  $a_t$  and the public signal  $y_t^a$ ; their precisions are thus given by  $\kappa_x = \kappa_\xi$  and  $\kappa_y = \kappa_{ya}$ . However, we introduce the different notation for two reasons. First, to accommodate a straightforward extension that would have allowed for additional exogenous private

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<sup>11</sup>In the extended model (Section 7), we will introduce a signal of the aggregate price level in addition to the signal of aggregate output. Moreover, our analysis can readily accommodate a signal of aggregate employment. See Appendix B for a discussion of alternative signals.

and public signals about  $\bar{a}_t$ .<sup>12</sup> And second, to facilitate the transition to the next section, where these sufficient statistics will also include the information about  $\bar{a}_t$  contained in the endogenous signals about aggregate output.

With this notation, the “type” of an island is summarized in  $\omega_t \equiv (a_t, x_t, y_t)$ , while the “state of the world” is the distribution from which Nature makes independent draws for  $\omega_t$ , one for each island. We denote the c.d.f. of this distribution by  $\Omega_t : \mathbb{R}^3 \rightarrow [0, 1]$ . Adopting the usual convention for economies with a continuum of agents, we assume that  $\Omega_t$  is also the realized distribution of  $\omega_t$  in the cross-section of islands and that all aggregates are functions of  $\Omega_t$ . Note that the latter is unknown during stage 1: each island faces uncertainty about the distribution of productivities and information in the rest of the economy. However, because all idiosyncratic shocks are Gaussian with known variances, the mean productivity  $\bar{a}_t$ , along with the public signal  $y_t$ , is a sufficient statistic for  $\Omega_t$ . By implication, all aggregate variables can also be expressed as functions of  $(\bar{a}_t, y_t)$  and the task of forming expectations about the high-dimensional object  $\Omega_t$  reduces to the simpler task of forming expectations about  $\bar{a}_t$ .

Because of the symmetry of preferences across households, and the symmetry of technologies and information within each island, we can talk of a typical worker and firm for each island; that is, it is without any loss of generality to impose symmetry in the choices of workers and firms within each island. Finally, because each family sends workers to every island and receives profits from every firm in the economy, each family’s income is fully diversified during stage 2. This guarantees that our model admits a representative consumer and that no trading takes place in the bond market. Along with the absence of capital, this means that our economy is essentially static. To simplify the exposition, we thus set  $B_t = 0$  and drop the time index from all variables—with the understanding, of course, that everything stated henceforth applies to any period  $t$ .

## 4.1 Equilibrium

The labor supply of any worker, the labor demand and production level of any firm, and the wage that clears the labor market of any given island all have to be functions of the  $\omega$  of that particular island. On the other hand, the prices that clear the commodity markets in stage 2 can also depend on the entire  $\Omega$ . We thus define an equilibrium as follows.

**Definition 1.** *An equilibrium consists of an employment strategy  $n(\omega)$ , a production strategy  $q(\omega)$ , a wage function  $w(\omega)$ , an aggregate output function  $Q(\Omega)$ , a price function  $p(\omega, \Omega)$ , an aggregate price index  $P(\Omega)$ , and a consumption strategy  $c(p, P, Q)$ , such that the following are true:*

(i) *The quantity  $c(p, P, Q)$  is the representative consumer’s optimal demand for any commodity whose price is  $p$  when the aggregate price level is  $P$  and the aggregate output (income) is  $Q$ .*

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<sup>12</sup>In particular, suppose that each island observes  $S$  private signals:  $x_{k,t}^s = \bar{a}_t + \varepsilon_{k,t}^s$ , where  $\varepsilon_{k,t}^s \sim \mathcal{N}(0, 1/\kappa_s)$  is idiosyncratic noise and  $s \in \{1, \dots, S\}$ . Then, the sufficient statistic  $x_{k,t}$  no more coincides with  $a_{k,t}$ . Rather, it is now given by  $x_{k,t} = \frac{\kappa_\xi}{\kappa_x} a_{k,t} + \sum_s \frac{\kappa_s}{\kappa_x} x_{k,t}^s$ , and its precision is given by  $\kappa_x = \kappa_\xi + \sum_s \kappa_s$ . A similar argument applies for public information.

(ii) The price that clears the market for the product of the typical firm from island  $\omega$  is  $p(\omega, \Omega)$ ; the employment and output levels of that firm are, respectively,  $n(\omega)$  and  $q(\omega)$ , with  $q(a, x, y) = e^a n(a, x, y)^\theta$  for all  $(a, x, y)$ ; and the aggregate output and price indices are, respectively,

$$Q(\Omega) = \left[ \int q(\omega)^{\frac{\rho-1}{\rho}} d\Omega(\omega) \right]^{\frac{\rho}{\rho-1}} \quad \text{and} \quad P(\Omega) = \left[ \int p(\omega, \Omega)^{1-\rho} d\Omega(\omega) \right]^{\frac{1}{1-\rho}}$$

(iii) The quantities  $n(\omega)$  and  $q(\omega)$  are optimal from the perspective of the typical firm in island  $\omega$ , taking into account that firms in other islands are behaving according to the same strategies, that the local wage is given by  $w(\omega)$ , that prices will be determined in stage 2 so as to clear all product markets, that the representative consumer will behave according to consumption strategy  $c$ , and that aggregate income will be given by  $Q(\Omega)$ .

(iv) The local wage  $w(\omega)$  is such that the quantity  $n(\omega)$  is also the optimal labor supply of the typical worker in an island of type  $\omega$ .

This definition is a hybrid of a Walrasian equilibrium for the complete-information exchange economy that obtains in stage 2, once production choices are fixed, and a subgame-perfect equilibrium for the incomplete-information game played among different islands in stage 1.

Consider first stage 2. The optimal consumption of a commodity whose price is  $p$  satisfies the following familiar first-order condition:  $c = \left(\frac{p}{P}\right)^{-\rho} C$ . It follows that total nominal consumption expenditure reduces to  $PC$ . As for total nominal income, in equilibrium this is simply  $PQ$ . Since there is no asset trading, the budget constraint reduces to  $PC = PQ$ . It follows the equilibrium consumption strategy is given by the following:

$$c(p, P, Q) = \left(\frac{p}{P}\right)^{-\rho} Q$$

Equivalently, the inverse demand function faced by a firm is given by  $p = Pq^{-\frac{1}{\rho}}Q^{\frac{1}{\rho}}$ .

Consider now stage 1. Given that the marginal value of nominal income for the representative household is  $U'(Q)/P$ , the objective of the firm is simply its profit times  $U'(Q)/P$ . Using the above results, it follows that the production and employment choices of the typical firm on island  $\omega$  must maximize the following objective:

$$\mathbb{E} \left[ \frac{U'(Q(\Omega))}{P(\Omega)} \left( P(\Omega) Q(\Omega)^{\frac{1}{\rho}} q(\omega)^{1-\frac{1}{\rho}} - w(\omega) n(\omega) \right) \middle| \omega \right], \quad (1)$$

where  $q(\omega) = A(\omega)n(\omega)^\theta$  and where  $A(\omega)$  denotes the local productivity in island  $\omega$  (with  $A(\omega) = e^a$  for  $\omega = (a, x, y)$ ). Since  $1 > (1 - \frac{1}{\rho})\theta > 0$ , the above objective is a strictly concave function of  $n$ , which guarantees that the solution to the firm's problem is unique and that the corresponding first-order condition is both necessary and sufficient. Finally, the optimal labor supply of the typical worker on island  $\omega$  gives

$$n(\omega)^{\epsilon-1} = w(\omega) \mathbb{E} \left[ \frac{U'(Q(\Omega))}{P(\Omega)} \middle| \omega \right]$$

Solving the latter for  $w(\omega)$  and substituting the solution into the firm's first-order condition, we conclude that the equilibrium level of employment is pinned down by the following condition:

$$n(\omega)^{\epsilon-1} = \left( \frac{\rho-1}{\rho} \right) \mathbb{E} \left[ U'(Q(\Omega)) \left( \frac{q(\omega)}{Q(\Omega)} \right)^{-\frac{1}{\rho}} \left( \theta A(\omega) n(\omega)^{\theta-1} \right) \middle| \omega \right]. \quad (2)$$

This condition has a simple interpretation: it equates the private cost and benefit of effort in each island. To see this, note that the left-hand side is simply the marginal disutility of an extra unit of labor in island  $\omega$ ; as for the right-hand side,  $\frac{\rho-1}{\rho}$  is the reciprocal of the monopolistic mark-up,  $U'(Q(\Omega)) \left( \frac{q(\omega)}{Q(\Omega)} \right)^{-\frac{1}{\rho}}$  is the marginal utility of an extra unit of the typical commodity produced in island  $\omega$ , and  $\theta A(\omega) n(\omega)^{\theta-1}$  is the corresponding marginal product of labor.

If we use  $q(\omega) = A(\omega) n(\omega)^\theta$  to eliminate  $n(\omega)$  in the above condition, we infer that, in any equilibrium, the strategy  $q : \Omega \rightarrow \mathbb{R}$  is the fixed point to the following functional equation:

$$q(\omega)^{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} = \left( \frac{\rho-1}{\rho} \right) \theta A(\omega)^{\frac{\epsilon}{\theta}} \mathbb{E} \left[ Q(\Omega)^{\frac{1}{\rho} - \gamma} \middle| \omega \right] \quad \forall \omega, \quad (3)$$

with  $Q(\Omega) = \left[ \int q(\omega)^{\frac{\rho-1}{\rho}} d\Omega(\omega) \right]^{\frac{\rho}{\rho-1}} \forall \Omega$ . We can guess and verify that there is always a unique equilibrium in which  $q(\omega)$  is log-linear in  $\omega$ ; we can then use an independent argument to rule out the possibility that there exists equilibria in which  $q(\omega)$  is not log-linear. Letting

$$\beta \equiv \frac{\frac{\epsilon}{\theta}}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} > 1 \quad \text{and} \quad \alpha \equiv \frac{\frac{1}{\rho} - \gamma}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} < 1, \quad (4)$$

we thus reach the following complete characterization of the equilibrium.

**Proposition 1.** (i) *There exists a constant  $\zeta$  such that the equilibrium production strategy  $q : \Omega \rightarrow \mathbb{R}$  is the fixed point to the following functional equation:*

$$\log q(\omega) = \zeta + \beta \log A(\omega) + \alpha \mathbb{E} [\log Q(\Omega) | \omega] \quad \forall \omega, \quad (5)$$

(ii) *The equilibrium level of output in island  $\omega = (a, x, y)$  is given by*

$$\log q(\omega) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_y y \quad (6)$$

where

$$\varphi_a = \beta, \quad \varphi_x = \left\{ \frac{(1-\alpha)\kappa_x}{(1-\alpha)\kappa_x + \kappa_y + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta, \quad \varphi_y = \left\{ \frac{\kappa_y}{(1-\alpha)\kappa_x + \kappa_y + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta. \quad (7)$$

Part (i), which follows directly from (3) along with the log-normality of aggregate output, offers a game-theoretic interpretation. The general equilibrium of our economy reduces to the Bayes-Nash equilibrium of a particular incomplete-information game. The relevant “players” for this game are the different islands of our economy; their “actions” are the production levels in each island; their “types” are the local information sets; and their “best responses” are simply the one given

by condition (5). One then sees that the coefficient  $\beta$ , which is necessarily positive, determines the elasticity of local output to variations in local productivity, while the coefficient  $\alpha$ , which can be either positive or negative, determines the elasticity of local output to variations in (expected) aggregate output. The former can thus be interpreted as the sensitivity of best responses to the local fundamental, whereas the latter can be interpreted as the degree of strategic complementarity (if  $\alpha > 0$ ) or substitutability (if  $\alpha < 0$ ) in production choices.

Part (ii) then gives a closed-form solution of the equilibrium production strategy as a log-linear function of local productivity  $a$ , the local (private) information  $x$ , and the global (public) information  $y$ . It is then immediate that employment, consumption and all *relative* prices are also uniquely determined and log-normally determined. What remains indeterminate is the price level,  $P(\Omega)$ , simply because our economy has no monetary anchor; without serious loss of generality, we henceforth normalize  $P(\Omega) = 1$  for the remainder of the analysis of the benchmark model.<sup>13</sup>

A number of positive properties of the equilibrium are worth highlighting. First, whether production choices are complements or substitutes depends on two opposing effects. On the one hand, higher aggregate income implies a higher demand for the products of each island, which increases local returns. This “demand-side” effect—which is generic to the new-keynesian paradigm and more generally to any economy that features Dixit-Stiglitz preferences—is the source of complementarity. On the other hand, higher aggregate income implies a higher demand for leisure and hence a higher real wage, which decreases local returns. This “supply-side” effect—which is standard in the neo-classical paradigm—is the source of substitutability. For business-cycle frequencies, the empirically relevant case is most likely one where  $\alpha > 0$ ; if that were not the case, the anticipation of higher aggregate demand would lead to a reduction in employment, which seems counterfactual. However, we will not need to impose this restriction until the monetary extension of Section 7.<sup>14</sup>

Second, the dispersion of information is relevant for the business cycle when, and only when,  $\alpha \neq 0$ . Indeed, when  $\alpha = 0$ , local activity does not depend on expectations of aggregate activity. When instead  $\alpha \neq 0$ , these expectations become crucial. The idiosyncratic noise in local information then contributes to cross-sectional dispersion in employment, output and relative prices that cannot be justified by the heterogeneity of local productivities, while the common noise in public information contributes to aggregate fluctuations that are orthogonal to aggregate productivity.

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<sup>13</sup>If the nominal price level is random but nominal wages are indexed, then both the level and the variability of the price level are irrelevant for real allocations. The same is true if nominal wages are non-indexed but no agent has private information about the price level. This won’t be true, however, if wages are non-indexed and the price level co-varies with the realized productivity shock, in which case different agents will have different private information about the price level.

<sup>14</sup>The strength of the “demand-side” effect is determined by the elasticity of substitution across goods, which is parameterized by  $\rho$ . The strength of the “supply-side” effect is determined by the income elasticity of labor supply, which is parameterized by  $\gamma$ . This explains why in our model the sign of  $\alpha$  depends on  $\rho$  versus  $\gamma$ . Note, however, that an extension of the model that would introduce capital would reduce the strength of the supply-side effect; this is because labor income is a small fraction of total wealth when there is capital. Finally, note that because an increase in local productivity for given expected aggregate productivity entails only a substitution effect, local effort necessarily increases with local productivity, guaranteeing that  $\beta > 1$ .

Third, the stronger the complementarity  $\alpha$ , the bigger these noise-driven aggregate fluctuations (but also the lower the noise-driven cross-sectional dispersion). The intuition for this property is similar to the one in Morris and Shin (2002) and Angeletos and Pavan (2007, 2008): stronger complementarity induces agents to rely more on common sources of information—and hence the economy to react more to the noise in such information—simply because such information is a relatively better predictor of others activity.

Finally, for the same reason that stronger complementarity raises the sensitivity of equilibrium choices to public information, stronger complementarity also raises the anchoring effect of the common prior. It follows that a higher  $\alpha$  implies more inertia in the response of equilibrium employment and output to the underlying shifts in aggregate productivity.

From a certain perspective, the aforementioned noise-driven fluctuations resemble “demand” or “sentiment” shocks: they can feature positive co-movement in employment, output and wages; they are orthogonal to the underlying productivity of taste shocks; and they are closely related to shifts in expectations about aggregate (real) demand. Lorenzoni (2008a) further explores these positive implications with a new-Keynesian economy similar to the one that we introduce in Section 7: it shows, under certain monetary policy responses, how the common noise shock can generate the same type of output and price fluctuations as the demand shocks identified in standard VAR exercises. Woodford (2003b), on the other hand, explores the inertia effects of the (related) complementarity in pricing decisions: they consider a reduced-form new-Keynesian economy that features dispersed information about aggregate nominal demand shocks and show how stronger complementarity can lead to more sluggish response of prices to the underlying shocks.<sup>15</sup> Finally, Angeletos and La’O (2008) makes a first attempt at exploring the quantitative importance of these properties.

All the aforementioned properties are interesting from a purely positive perspective. However, their normative content is not obvious. Are the aforementioned noise-driven aggregate fluctuations, the potentially high inertia with respect to the underlying fundamentals, or the additional cross-sectional dispersion in employment and relative prices, symptoms of inefficiency? To address this normative question, one needs to understand whether the equilibrium use of information over the business cycle is optimal from a social perspective, which is what we do next.

## 4.2 Efficiency

The notion of constrained efficiency we adopt is similar to the one that Angeletos and Pavan (2007, 2008) used within a more abstract framework, here appropriately adapted to our business-cycle economy. This notion of constrained efficiency seeks to identify the best (resource-feasible) allocation among the ones that respect the geographical segmentation of information.

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<sup>15</sup>Although in our baseline model prices are flexible, it is straightforward to recast the results of Lorenzoni (2008a) and Woodford (2003b) within the monetary extension that we will introduce in Section 6: in that model, a positive noise shock can raise both output and prices (provided that monetary policy is sufficiently accommodative of the shock) and a higher complementarity can reduce the response of prices to the underlying fundamentals.

**Definition 2.** An efficient allocation is a collection of a production and an employment strategy for each firm on each island, an allocation of labor across workers on each island, and an allocation of consumption across households, that maximizes ex ante utility subject to the following constraints:

- (i) The resource constraint is satisfied for each commodity.
- (ii) Total labor supply in each island equals total employment in that island.
- (iii) Employment and production levels in each island are measurable with respect to the local and public information available to that island, not the local information of other islands.

The first two constraints impose resource feasibility. The third one defines the informational frictions faced by the planner: it imposes that the planner respects the geographical segmentation of information in the sense that he cannot transfer information from one island to another at the moment of employment and production choices.

Because of the concavity of preferences and technologies, efficiency dictates symmetry in consumption across households, as well as symmetry across firms and workers within any given island. Using these facts, the planner's problem can be reduced to the following.

**Planner's problem.** Choose a production strategy  $q : \Omega \rightarrow \mathbb{R}$  so as to maximize

$$\max_{q: \Omega \rightarrow \mathbb{R}} \int_{\Omega} \left[ U(Q(\Omega)) - \int_{\omega} \frac{1}{\epsilon} \left( \frac{q(\omega)}{A(\omega)} \right)^{\frac{\epsilon}{\theta}} d\Omega(\omega) \right] d\mathcal{F}(\Omega) \quad (8)$$

subject to

$$Q(\Omega) = \left[ \int q(\omega)^{\frac{\rho-1}{\rho}} d\Omega(\omega) \right]^{\frac{\rho}{\rho-1}} \quad \forall \Omega$$

where  $\mathcal{F}$  denotes the c.d.f. of  $\Omega$ .

This problem has a simple interpretation.  $U(Q(\Omega))$  is the utility of consumption for the representative household when the state of the world is  $\Omega$ ;  $\frac{1}{\epsilon} \left( \frac{q(\omega)}{A(\omega)} \right)^{\frac{\epsilon}{\theta}} = \frac{1}{\epsilon} n(\omega)^{\epsilon}$  is the marginal disutility of labor for the typical worker in island  $\omega$ ;  $\int \frac{1}{\epsilon} \left( \frac{q(\omega)}{A(\omega)} \right)^{\frac{\epsilon}{\theta}} d\Omega(\omega)$  is the overall disutility of labor for the representative household when the state of the world is  $\Omega$ ; finally, integrating over  $\Omega$  gives ex-ante utility.<sup>16</sup> The reduced-form objective in (8) is thus a functional that gives the level of welfare implied by any arbitrary production strategy  $q : \Omega \rightarrow \mathbb{R}$  that the planner dictates to the economy.

Because this problem is strictly concave, it has a unique solution and this solution is pinned down by the following first-order condition:<sup>17</sup>

$$q(\omega)^{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} = \theta A(\omega)^{\frac{\epsilon}{\theta}} \mathbb{E} \left[ Q(\Omega)^{\frac{1}{\rho} - \gamma} \middle| \omega \right] \quad \forall \omega \quad (9)$$

<sup>16</sup>Note that  $(\bar{a}, y)$  is a sufficient statistic for  $\Omega$ ; hence, the integral in this objective is simply integrating over the possible realizations for  $(\bar{a}, y)$ .

<sup>17</sup>Because of the continuum, the efficient allocation is determined only for *almost* every  $\omega$ . For expositional simplicity, we bypass the *almost* qualification throughout the paper.

To interpret this condition, we can use  $q(\omega) = A(\omega)n(\omega)^\theta$  to restate it as follows:

$$n(\omega)^{\epsilon-1} = \mathbb{E} \left[ U' (Q(\Omega)) \left( \frac{q(\omega)}{Q(\Omega)} \right)^{-\frac{1}{\rho}} \left( \theta A(\omega)n(\omega)^{\theta-1} \right) \middle| \omega \right]. \quad (10)$$

This simply means that the planner equates the social cost of employment in island  $\omega$  with the *local* expectation of the social value of the marginal product of that employment. Finally, thanks to the Gaussian information structure, we can guess and verify that the unique solution to the planner's problem is log-linear, which leads to the following characterization of the efficient allocation.

**Proposition 2.** (i) *There exists a constant  $\zeta' > \zeta$  such that the efficient production strategy  $q : \Omega \rightarrow \mathbb{R}$  is the fixed point to the following functional equation:*

$$\log q(\omega) = \zeta' + \beta \log A(\omega) + \alpha \mathbb{E} [\log Q(\Omega) | \omega] \quad \forall \omega, \quad (11)$$

with  $Q(\Omega) = \left[ \int q(\omega)^{\frac{\rho-1}{\rho}} d\Omega(\omega) \right]^{\frac{\rho}{\rho-1}} \forall \Omega$ , and with  $\beta$  and  $\alpha$  defined as in (4).

(ii) *The efficient level of output in island  $\omega = (a, x, y)$  is given by*

$$\log q(\omega) = \varphi_0^* + \varphi_a^* a + \varphi_x^* x + \varphi_y^* y,$$

where  $\varphi_0^*$  is higher than its equilibrium counterpart, but  $\varphi_a^*, \varphi_x^*$  and  $\varphi_y^*$  are the same.

Part (i) follows directly from the aforementioned first-order condition of the planner's problem. Like part (i) of Proposition 1, this result has an appealing game-theoretic interpretation: the efficient allocation of the economy is isomorphic to the Bayes-Nash equilibrium of an incomplete-information game among the “islands” of the economy.

Combined, these results imply that the micro-founded economy studied in this paper is (approximately) nested in the class of abstract games studied in Angeletos and Pavan (2007, 2008): there, the best responses that characterized the equilibrium and efficient allocations were linear; here, they are log-linear. This illustrates how the methodology of that earlier, more abstract work can be adapted to fully micro-founded applications. What the underlying micro-foundations in our business-cycle model do is to put specific restrictions on the structure of these best-response conditions. These restrictions, in turn, are essential for understanding the efficiency of the equilibrium in our model. Indeed, in the class of games studied in Angeletos and Pavan (2007, 2008), the best responses that characterize the equilibrium and the efficient allocations are arbitrary and could thus be completely different from one another. In contrast, in the business-cycle model studied in this paper, the underlying micro-foundations guarantee that the equilibrium and efficient best responses are identical, except for the different intercept due to the monopolistic mark-up. In particular, the equilibrium and the efficient best responses in our economy feature the same sensitivity to local fundamentals ( $\beta$ ) and the same degree of strategic complementarity ( $\alpha$ ).<sup>18</sup>

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<sup>18</sup>Note that this result rules out for our economy the type of effects emphasized by Morris and Shin (2002): the economy studied in that paper featured overreaction to public information only because the equilibrium degree of complementarity was higher than the efficient one. (See also the discussion in Section 10.)

This in turn explains part (ii): the efficient production strategy features the same sensitivity as the equilibrium strategy, not only to the local productivity, but also to any information—private or public—regarding aggregate productivity. By implication, *none* of the positive properties that we highlighted before are symptoms of inefficiency: the additional cross-sectional dispersion due to idiosyncratic noise, the demand-like fluctuations due to common noise, and the potential inertia in the response of the economy to the underlying aggregate productivity shock, are just right.

To further understand what drives these normative properties, compare condition (10), which characterizes the solution to the planner’s problem, with its equilibrium counterpart, condition (2). Clearly, the only discrepancy between the equilibrium and the efficient allocation is the monopolistic mark-up: the dispersion of information impacts equilibrium and efficient allocations in a completely symmetric way. Importantly, this property does not rely on the Gaussian information structure: as long as information is exogenous, the aforementioned conditions extend to arbitrary information structures. We can thus summarize the first key lesson of our analysis in the following.

**Theorem 1.** *When information is dispersed but exogenous, the business cycle is efficient: the reaction of equilibrium activity to the underlying productivities and the underlying noises, whether idiosyncratic or aggregate, is efficient.*

This result provides an important benchmark for the normative properties of business cycles under dispersed information. Assuming homogeneous information, previous work has highlighted that, within the Dixit-Stiglitz class of economies that underlies the new-Keynesian paradigm, fluctuations driven by productivity or taste shocks are typically efficient when prices are flexible or when monetary policy replicates the flexible-price equilibrium (e.g., Benigno and Woodford, 2005; Goodfriend and King, 1997, 2001; Rotemberg and Woodford, 1997; Woodford, 2002, 2003a). Our result here—along with the related result in the monetary extension of Section 7—establishes that this property extends to dispersed information, provided that one abstracts from informational externalities.

We will consider the implications of informational externalities in the next section. Before that, we briefly comment on a second property upon which this result relies: the property that the dispersion of information does not cause any uninsurable idiosyncratic income risk. In our model, this property follows from the assumption of “big families”: because each family receives labor income and profits from all islands and consumes the products of all islands, it is completely diversified against the risk induced by the heterogeneity of information across different islands. Alternatively, we could have ensured that the dispersion of information does not cause idiosyncratic risk by allowing households to trade insurance contracts prior to stage 1. These tricks for eliminating the idiosyncratic income risk that could have been caused by idiosyncratic pricing or informational frictions is standard practice in business-cycle theory—and for a good reason. In our context, it permits us to isolate the imperfections society may face in the communication of information from the imperfections it may face in insurance possibilities; and, in so doing, it helps isolate the policy objectives we document in this paper from any social insurance/redistributive considerations.

## 5 Equilibrium and efficiency with endogenous information

In this section we turn to the characterization of equilibrium and efficient allocations with endogenous learning.

### 5.1 Equilibrium

The information available on island  $k$  is now given by  $(a_k, x_k^q, y^a, y^q)$ , where, recall,  $a_k$  is local productivity,  $y^a$  is the public signal of aggregate productivity, and  $x_k^q$  and  $y^q$  are the private and the public signal of aggregate output. For tractability, we need to focus on allocations that preserve the Gaussian structure of the information; effectively, this means that local output is restricted to be log-linear in the available signals. We can then summarize all the exogenous and endogenous signals in some (appropriately constructed) Gaussian sufficient statistics  $x$  and  $y$ . This permit us to continue identifying the type of an island with a triplet  $\omega = (a, x, y)$  and motivates the following definition of a rational-expectations equilibrium for our economy.

**Definition 3.** *An equilibrium consists of a collection of consumption, employment, output, wage, and price functions  $(c, p, q, n, w, Q, P)$ , along with endogenous signals  $(x^q, y^q)$  and sufficient statistics  $(x, y)$ , such that the following are true:*

(i) *Given the sufficient statistics  $x$  and  $y$ , the functions  $(c, p, q, n, w, Q, P)$  satisfy all the conditions of Definition 1.*

(ii) *The endogenous signals  $x^q$  and  $y^q$  are generated by the production strategy according to  $x^q = \log Q(\Omega) + \varepsilon^{xq}$  and  $y^q = \log Q(\Omega) + \varepsilon^{yq}$ , where  $\varepsilon^{xq}$  and  $\varepsilon^{yq}$  are the exogenous noises; and the variables  $x$  and  $y$  are sufficient statistics with respect to  $\Omega$  for, respectively,  $(a, x^q)$  and  $(y^a, y^q)$ .*

Condition (i) requires that the consumption allocations, the employment and production strategies, and the commodity prices are an equilibrium in the sense of Definition 1, taking the information structure as exogenous. Condition (ii) then requires that the information structure is itself consistent with the equilibrium production strategy. Taken together, these conditions impose a fixed-point relation between the equilibrium allocations and the equilibrium information structure, as standard in rational-expectations equilibria.

We can characterize this fixed point as follows. Let  $\kappa_x$  and  $\kappa_y$  denote the (now endogenous) precisions of the sufficient statistics  $x$  and  $y$ . Whatever these precisions are, the equilibrium production strategy is characterized by Proposition 1. In turn, this strategy induces certain values for the precisions  $\kappa_x$  and  $\kappa_y$ . The latter can be characterized as follows.

**Lemma 1.** *Take any log-linear production strategy of the form*

$$\log q(\omega) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_y y, \quad (12)$$

*for arbitrary  $(\varphi_0, \varphi_a, \varphi_x, \varphi_y)$ . The precisions of the sufficient statistics  $x$  and  $y$  generated by this strategy are given by*

$$\kappa_x = \sigma_\xi^{-2} + (\varphi_a + \varphi_x)^2 \sigma_{xq}^{-2} \quad \text{and} \quad \kappa_y = \sigma_{ya}^{-2} + (\varphi_a + \varphi_x)^2 \sigma_{yq}^{-2} \quad (13)$$

To understand this result, note first that the endogenous signals  $x^q$  and  $y^q$  about aggregate output can be transformed to simple Gaussian signals about the underlying aggregate productivity because aggregate output is itself a log-linear function of  $\bar{a}$ . Next, note that the sum  $\varphi_a + \varphi_x$  determines the sensitivity of aggregate output—as well as of any other aggregate variable such as aggregate employment or aggregate consumption—to the underlying aggregate productivity. But then note that, for any given exogenous noises, it is precisely this sensitivity that determines how much information the endogenous signals contain about aggregate productivity: the more sensitive is aggregate output to aggregate productivity, the more informative these signals. It follows that the sum  $\varphi_a + \varphi_x$  also determines the precisions  $\kappa_x$  and  $\kappa_y$  of the overall private and public information.

Combining this result with the characterization of the equilibrium strategy, we reach the following characterization of the entire equilibrium.

**Proposition 3.** (i) *There always exists an equilibrium in which the production strategy is as in (12), with the coefficients  $(\varphi_a, \varphi_x, \varphi_y)$  and the precisions  $(\kappa_x, \kappa_y)$  jointly solving the following system:*

$$\varphi_a = \beta \tag{14}$$

$$\varphi_x = \left\{ \frac{(1-\alpha)\kappa_x}{(1-\alpha)\kappa_x + \kappa_y + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta \tag{15}$$

$$\varphi_y = \left\{ \frac{\kappa_y}{(1-\alpha)\kappa_x + \kappa_y + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta \tag{16}$$

$$\kappa_x = \sigma_\xi^{-2} + (\varphi_a + \varphi_x)^2 \sigma_{xq}^{-2} \tag{17}$$

$$\kappa_y = \sigma_a^{-2} + (\varphi_a + \varphi_x)^2 \sigma_{yq}^{-2} \tag{18}$$

(ii)  $\alpha > 0$  and  $\frac{\sigma_{yq}^{-2}}{\sigma_A^{-2} + \sigma_a^{-2}} > \frac{\sigma_{xq}^{-2}}{\sigma_\xi^{-2}}$  suffice for the equilibrium to be unique.

For any given information structure there exists a unique strategy that can be an equilibrium (in the Bayes-Nash sense); and for any given strategy there is a unique information structure that can be generated by this strategy. Nevertheless, multiplicity can originate in the fixed-point relation between the two, as often the case with rational-expectations equilibria. This possibility is intriguing, but we will ignore because it is orthogonal to the goals of this paper. Part (ii) of the proposition reassures us that the equilibrium is unique at least when individual incentives to produce increase with expectations of aggregate demand (which is most likely the case empirically) and when the public sources of social learning are sufficiently precise relative to the private ones (which is less obvious but might also be empirically plausible). More generally, we know that there exist at most three equilibria and that the equilibria are ranked according to the precisions of information that they feature; we can then focus attention to the one with the highest precisions, which is also the one with the highest welfare.<sup>19</sup>

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<sup>19</sup>This selection is convenient but not strictly needed. In particular, the result we shall establish shortly that the equilibrium features inefficiently low sensitivity to local information holds no matter which equilibrium is selected. Similarly, the type of policies we consider in Section 6 can locally improve the efficiency of all equilibria at the same time. Finally, the optimal policy we identify in Theorem 3 can always implement the efficient allocation as an equilibrium; the only complication is that this might not be the unique equilibrium.

## 5.2 Efficiency

We now turn to efficient allocations. The key difference for equilibrium (as well as from the efficient allocation when information is exogenous) is that the planner internalizes how different allocations impact social learning. To preserve the Gaussian nature of the information structure, we must restrict attention to log-normal allocations and hence to log-linear production strategies as in (12). These observations motivate the following definition of efficient allocations for the case of endogenous information.

**Definition 4.** *An efficient allocation is a production strategy  $q$  and a collection of employment and consumption allocations, along with Gaussian endogenous signals  $(x^q, y^q)$  and sufficient statistics  $(x, y)$ , such that the following hold:*

(i) *The endogenous signals  $(x^q, y^q)$  are generated by the production strategy  $q$  and the variables  $x$  and  $y$  are sufficient statistics with respect to  $\Omega$  for  $(a, x^q)$  and  $(y^a, y^q)$ .*

(iii) *The production strategy and the associated employment and consumption allocations maximize ex-ante utility, subject to the feasibility and informational constraints described in Definition 2, taking into account the endogeneity of information described in part (i) above.*

This efficiency concept permits the planner to take into account how different allocations sustain different information structures. At the same time, it does not endow the planner with any communication channels in addition to those already available to the market: the planner is still prohibited from transferring information from one island to another in any way other than through the specific signals  $(x^q, y^q)$ . In this sense, this efficiency concept preserves the spirit of the one we used when information was exogenous.

Using the fact that efficiency imposes symmetry in consumption as well as symmetry across workers and firms within each island, we can obtain welfare as function of merely the production strategy and the precisions of information. Thus, take any coefficients  $(\varphi_0, \varphi_a, \varphi_x, \varphi_y)$  and any precisions  $(\kappa_x, \kappa_y)$  and write welfare as  $\mathcal{W}(\varphi_0, \varphi_a, \varphi_x, \varphi_y; \kappa_x, \kappa_y)$ ; a closed-form expression of this function is provided in the Appendix. Next, recall that the precisions induced by any given strategy are characterized by the conditions in Lemma 1; let  $\kappa_x(\varphi_a + \varphi_x)$  and  $\kappa_y(\varphi_a + \varphi_x)$  denote the functions defined by (13). We can then express the planner's problem as follows.

**Planner's problem.** *Choose a strategy  $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_y)$  and precisions  $\kappa = (\kappa_x, \kappa_y)$  so as to maximize  $\mathcal{W}(\varphi, \kappa)$  subject to the constraint that  $\kappa_x = \kappa_x(\varphi_a + \varphi_x)$  and  $\kappa_y = \kappa_y(\varphi_a + \varphi_x)$ .*

The solution to this problem is complicated by the fact that this problem is non-concave and that a closed-form solution for the efficient strategy does not exist. Nevertheless, because the precisions depend on the strategy only through the sum  $\varphi_a + \varphi_x$ , we can bypass these complications by splitting the planner's problem in two steps. The first step lets the planner optimize over the set of strategies subject to an additional constraint, namely that the sum  $\varphi_a + \varphi_x$  equals  $\bar{\varphi}$  for some arbitrary  $\bar{\varphi}$ . The second step then lets the planner optimize over  $\bar{\varphi}$  and over the precisions that are induced by it. The first-step problem is strictly concave and, in fact, its first-order conditions can be reduced

to a simple linear system. The solution to this problem leads to the conditions (19) and (20) below, which express the efficient strategy as a function of the Lagrange multiplier that corresponds to the aforementioned auxiliary constraint. The second step then permits us to prove the existence of an efficient allocation, to interpret the aforementioned Lagrange multiplier as the shadow value of the informational externality, and to complete the characterization of the efficient allocation.

**Proposition 4.** *An efficient strategy always exists. Moreover, for any efficient strategy, there exists a scalar  $\Delta > 0$  such that the efficient strategy is given by*

$$\log q(\omega) = \varphi_0^* + \varphi_a^* a + \varphi_x^* x + \varphi_y^* y,$$

with the coefficients  $(\varphi_a^*, \varphi_x^*, \varphi_y^*)$  and the associated precisions  $(\kappa_x^*, \kappa_y^*)$  satisfying the following:

$$\begin{aligned} \varphi_x^* &= \beta \\ \varphi_x^* &= \left\{ \frac{(1-\alpha)\kappa_x^*}{(1-\alpha)\kappa_x^* + \kappa_y^* + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta + \Delta \end{aligned} \quad (19)$$

$$\varphi_y^* = \left\{ \frac{\kappa_y^*}{(1-\alpha)\kappa_x^* + \kappa_y^* + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta - \frac{\kappa_y^*}{\kappa_A + \kappa_y^*} \Delta \quad (20)$$

$$\kappa_x^* = \sigma_\xi^{-2} + (\varphi_a^* + \varphi_x^*)^2 \sigma_{xq}^{-2}$$

$$\kappa_y^* = \sigma_{ya}^{-2} + (\varphi_a^* + \varphi_x^*)^2 \sigma_{yq}^{-2}$$

The scalar  $\Delta$  is a wedge that summarizes the impact of the informational externality on the efficient production strategy relative to the one that obtains in equilibrium—or, equivalently, relative to the one that would have maximized welfare for given information. Let us elaborate on this. Note that the precisions  $\kappa_x^*$  and  $\kappa_y^*$  obtained at the efficient allocation above are higher than those obtained at the equilibrium allocation of Proposition 3. If some divine power were to keep the precisions of available information constant at those higher levels and give the planner, then the allocation the planner would choose is the one obtained from Proposition 4 if we replace  $\Delta$  with 0 (which in turn coincides with the equilibrium one apart from the mark-up). Clearly, any deviation from this allocation involves a loss in *allocative* efficiency in the sense that it reduces welfare for given information. But it is only this sacrifice that permits the planner to sustain these higher precisions in the absence of the aforementioned divine power. What then justifies the sacrifice is precisely that these higher precisions contribute to higher welfare. In this sense, the planner trades off less *allocative* efficiency (i.e., less welfare for given information) for more *informational* efficiency (i.e., higher welfare via better information).

The property that  $\Delta$  is positive simply means that the efficient allocation features a higher sensitivity to local information than the equilibrium one. This is intuitive: starting from equilibrium, an increase in  $\varphi_x$  raises both  $\kappa_x$  and  $\kappa_y$ . However, the result is not self-evident for three reasons.

First, in general it is not obvious that, starting from equilibrium, a local increase in the precision of information increases welfare. For example, this is not the case in the economy of Morris and Shin (2002). However, the results of the previous section have already ruled out this complication

for our class of economies: if information had been exogenous, the equilibrium use of information would have been efficient, guaranteeing that equilibrium welfare would increase with any additional information, whether private or public.

Second, raising the precision of information does not come for free: it comes at the cost of raising the cross-sectional dispersion in output, employment and relative prices, which means less allocative efficiency for given information. However, because the equilibrium allocation maximizes allocative efficiency to start with, this cost is only second-order locally, while the welfare benefit of more information is first-order.

Finally, although this last argument establishes the direction of *local* welfare improvements, such local arguments are not necessarily informative about the position of the *global* maximum when the planner's problem fails to be concave, as it is the case here. In particular, a complication is that the planner can induce a high precision of the endogenous signals by picking, not only a sufficiently high  $\varphi_a + \varphi_x$ , but also a sufficiently negative  $\varphi_a + \varphi_x$ . This is simply because the informativeness of the endogenous signals depends only on the absolute value of the sensitivity of aggregate output to aggregate productivity, not on the sign of this sensitivity. We can nevertheless rule out negative values for  $\varphi_a + \varphi_x$  because they involve unnecessarily high allocative inefficiency: the planner can always achieve the same precision along with higher allocative efficiency by choosing the symmetrically positive value for  $\varphi_a + \varphi_x$ . This is because the value of  $\varphi_a + \varphi_x$  that maximizes allocative efficiency—i.e., the equilibrium one—is positive to start with and the welfare function is symmetric around this point. Along with the fact that any positive value for  $\varphi_a + \varphi_x$  that is lower than the equilibrium one is clearly suboptimal—for locally raising this value would have improved both allocative and informational efficiency.

We can thus summarize the second key lesson of our analysis in the following.

**Theorem 2.** *When information is endogenous, the business cycle is inefficient: the equilibrium degree of social learning is too low, the sensitivity of equilibrium allocations to private information is too low, and their sensitivity to public information is too high.*

The natural question, then, is whether there are simple policies that permit the government to improve the efficiency of social learning over the business cycle—a question that we address in the next section.

## 6 Optimal Fiscal Policy

Following the spirit of the Ramsey literature, we now shift attention to the set of allocations that can be implemented with a relatively simple set of tax instruments. However, unlike the Ramsey literature, we do not rule out lump-sum taxes, thus guaranteeing that the state-contingent taxes are not necessary for minimizing tax distortions. Rather, the effects of state-contingent taxes that we document in this section originate merely from the impact that the anticipation of the contingencies have on the decentralized use of information.

We consider any combination of the following tax instruments: a linear tax on firm sales, output, employment, or payroll; a linear tax on household labor income; or a linear tax on household consumption. These taxes are collected in stage 2 and can be contingent on the information that is publicly available at that state; for tractability, we require the tax rates to be log-normal. It is then straightforward to show that any combination of the aforementioned taxes reduces to a single tax wedge between the marginal return and the marginal cost of labor.

**Lemma 2.** *The equilibrium satisfies*

$$n(\omega)^{\epsilon-1} = \left( \frac{\rho-1}{\rho} \right) \mathbb{E} \left[ (1 - \tau(\Omega)) U' (Q(\Omega)) \left( \frac{q(\omega)}{Q(\Omega)} \right)^{-\frac{1}{\rho}} \left( \theta A(\omega) n(\omega)^{\theta-1} \right) \middle| \omega \right] \quad (21)$$

where  $\tau(\Omega)$  is the total tax wedge induced by the aforementioned taxes.

We henceforth represent the tax policy directly in terms of this wedge; we refer to the latter as *the tax*; and we let

$$-\log(1 - \tau(\Omega)) = \tau_0 + \tau_A \bar{a} + \tau_Q \log Q(\Omega) + \tau_y y, \quad (22)$$

where  $\tau_0, \tau_A, \tau_Q$ , and  $\tau_y$  are known scalars. The coefficients  $\tau_A, \tau_Q$ , and  $\tau_y$  determine the elasticities of the tax with respect to aggregate productivity, aggregate output, and the common belief about aggregate productivity; the coefficient  $\tau_0$  controls the mean level of the tax. Because  $(\bar{a}, y)$  is a sufficient statistic for the entire exogenous state  $\Omega$ , we can always write the tax as a function of  $(\bar{a}, y)$  alone. However, we favor implementations that express the tax as a function of aggregate productivity ( $\bar{a}$ ) and aggregate output ( $Q$ ) rather than the public signal ( $y$ ) for one simple reason: even though in the model  $y$  is common knowledge, in practice it seems hard to measure  $y$ . After all, the public signal is just a convenient modeling device that permits us to capture a variety of common sources of information that may be available to the market but not necessarily directly observed by the government. It is then reassuring that the contingency on  $y$  is strictly needed only in a degenerate case of no practical interest.

**Proposition 5.** *Consider any strategy as in (12). There exists a state-contingent tax policy as in (22) that implements this strategy as an equilibrium strategy if and only if  $\varphi_a = \beta$ . Moreover, as long as  $(\varphi_x, \varphi_y) \neq 0$ , this can be achieved with a policy that sets  $\tau_y = 0$ .*

This result highlights the power that the contingencies of taxes on macroeconomic outcomes may have in controlling the decentralized use of information, and thereby the reaction of the economy to both the fundamentals and the noise—an insight that extends beyond our model and that was first highlighted by Angeletos and Pavan (2008) within a more abstract framework. In the next section, we will show that, to the extent that prices are sticky, monetary policy can play a similar role, although it does not have as much power as state-cotigent taxes.

The simplest (and more robust) way to understand this result is by ignoring the details of specific implementations and instead looking at the correlations of the tax with underlying productivity and noise shocks. In particular, suppose the tax is negatively correlated with innovations in aggregate

productivity and positively correlated with the common noise; the government can induce these correlations by making the tax contingent either on  $(\bar{a}, y)$ , or on  $(\bar{a}, Q)$ , or perhaps in other ways. Either way, anticipating these correlations, firms will have an incentive during stage 1 to react more strongly to any information they may have about aggregate productivity and less strongly to any information they may have about the common noise. It follows that firms will unambiguously increase their response to their private sources of information; whether they will at the same time reduce their response to common information then depends simply on whether the positive correlation of the tax with the underlying common noise is sufficiently strong relative to its negative correlation with respect to aggregate productivity. This explains why state-contingent taxes permit the government to control freely both  $\varphi_x$  and  $\varphi_y$ , the sensitivities to local and public information.<sup>20</sup> The only sensitivity that the policy cannot affect is the one with respect to local productivity, namely the coefficient  $\varphi_a$ ; this is simply because the tax is contingent only on macroeconomic outcomes, not on idiosyncratic variables.

To further appreciate this result, suppose for a moment that the tax was restricted to be non-contingent. The government would then be able to control only the mean level of equilibrium activity, not its response to fundamentals and noise: formally, if  $\tau$  is invariant with the state of the economy, then the government has no control on either  $\varphi_x$  or  $\varphi_y$ . Also, if the government conditions the tax only on realized aggregate productivity  $\bar{a}$ , then it can control the overall sensitivity of equilibrium activity to any information about aggregate productivity, but cannot control the relative sensitivity to different types of information: formally, if  $\tau$  is a (log-linear) function of  $\bar{a}$  alone, then the government can freely control the sum  $\varphi_x + \varphi_y$ , but has no control over the ratio  $\varphi_x/\varphi_y$ . Therefore, it is only the contingency on aggregate output  $Q$  (or, more generally, the correlation of the tax with the common noise in  $y$ ) that permits the government to control the relative sensitivity to the two types of information.

We can now proceed to identify the best implementable allocation. Note that the above result keeps the information structure fixed. However, if the policy can induce any particular strategy, it can also induce any information structure that can be generated by this strategy (in the sense of Lemma 1), whenever the information structure is endogenous. Next, note that the set of implementable allocations is strictly smaller than the set of allocations that we allowed the planner to choose from when we studied the efficient allocation of the economy. However, the only restriction on implementable allocations is that  $\varphi_a = \beta$ , a condition that is satisfied for free at the efficient allocation. It follows that the best implementable allocation is simply the efficient one.<sup>21</sup>

Note then there are multiple combinations of the coefficients  $(\tau_a, \tau_Q, \tau_y)$  that induce the same incentives and hence implement the efficient allocation. However, if we restrict attention to our

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<sup>20</sup>To be precise, the argument made above only explains how taxes impact individual firm incentives holding given the behavior of other firms; that is, they explain they impact of the policy on best responses, not on equilibrium. However, the equilibrium could fail to inherit the comparative-static properties of best responses only when the degree of complementarity is too strong (namely  $\alpha > 1$ ), which is never the case here.

<sup>21</sup>As anticipated in the introduction, this won't be true in the extended model; but here it offers a useful benchmark.

preferred class of policies, which allow the tax rate to depend only on aggregate productivity and aggregate output, then there remains a unique policy within that class that implements the efficient allocation. Moreover, even without this particular selection, the value of the tax as a function of the aggregate productivity shock  $\bar{a}$  and the aggregate noise shock  $\varepsilon$  (where  $\varepsilon = y - \bar{a}$ ) is always uniquely determined along the equilibrium path. We can thus talk about *the* optimal tax, once again ignoring the details of the particular implementation. The cyclical properties of the optimal tax are then characterized below.

**Theorem 3.** (i) *With exogenous information, the optimal tax is invariant with the business cycle.*

(ii) *With endogenous learning, instead, the optimal tax is countercyclical in either of the following senses:  $\text{Corr}(\tau, \bar{a}) < 0$ ,  $\text{Corr}(\tau, \bar{a}|y) < 0$ , and  $\text{Corr}(\tau, Q) < 0$ . Moreover, the tax is positively correlated with the noise:  $\text{Corr}(\tau, \varepsilon) > 0$ .*

Part (i) is immediate given our earlier result that, when information is exogenous, the only wedge between the equilibrium and the efficient allocation is the fixed monopolistic mark-up. Part (ii) also follows directly from comparing equilibrium and efficient allocations. Recall that, when information is endogenous, efficiency dictates the government to raise  $\varphi_x$ , the sensitivity of economic activity to private information, so as to boost social learning; it then also dictates to lower  $\varphi_y$ , the sensitivity to public information downwards, so as to preserve allocative efficiency. How can the tax system provide the agents with the right incentives for these goals to materialize in equilibrium? For the agents to find it optimal to raise their response to their private information about aggregate productivity, it better be that they expect the tax to fall—and hence their net-of-tax returns to increase—with any positive innovations in aggregate productivity. And for them to find it optimal to decrease their response to public information, it better be that they expect the tax to increase with the public signal or, equivalently, with the noise in it. This explains why the optimal tax must be negatively correlated with  $\bar{a}$  and positively correlated with  $\varepsilon$  along the equilibrium.

Note that it is the combination of the two cyclical properties of the tax—its negative correlation with  $\bar{a}$  and its positive correlation with  $\varepsilon$ —that achieves full efficiency. However, it is only the negative correlation with  $\bar{a}$  that is the key instrument for increasing  $\varphi_x$  and thereby for boosting the aggregation of information over the business cycle. The positive correlation with the noise is instrumental only for reducing  $\varphi_y$ , which is necessary for counterbalancing the allocative inefficiency caused by the higher  $\varphi_x$  but is irrelevant for the efficiency of learning.

Also, as mentioned above, there are a variety of tax schedules as in (22) that can achieve these cyclical properties. Assuming that good news about productivity raises employment and output ( $\varphi_y^* > 0$ ), our preferred implementation would set  $\tau_y = 0$  along with  $\tau_Q < 0$  and  $\tau_A > 0$ . That is, firms should expect their taxes to increase during a boom that is not warranted by aggregate productivity, and to decrease when actual productivity is higher.

Finally, note that the mean value of the tax is negative, because the policy must always offset the monopolistic mark-up. However, this property is completely orthogonal to the cyclical properties of the optimal tax. To see this, we can consider a variant of our model that introduces perfect

substitutability among the commodities produced within any particular island (or “sector”), while preserving the imperfect substitutability across sectors. It is then easy to check that all the equilibrium, efficiency, and policy results continue to hold, with only one difference: the monopolistic distortion disappears. As a result, the optimal tax would now be exactly zero in the case of exogenous information, while it would preserve the same cyclical properties in the case of endogenous information.

## 7 The extended (New-Keynesian) model

In this section we introduce a New-Keynesian variant of our baseline model, which permit us to study how the dispersion of information interacts with nominal frictions and how it impacts optimal monetary policy.

We modify the baseline model in four dimensions. First, we introduce price rigidities. In particular, we assume that firms set nominal prices at the end of stage 1, while information is still dispersed, and cannot adjust them in stage 2, in response to the new information that becomes available at that stage.

Second, we let firms make a second employment choice in stage 2 and, accordingly, we let households make a second labor-supply choice in that stage; this permits real output to respond to the new information that becomes available during that stage as well as the monetary policy to have real effects. In particular, the output and consumption level of the typical commodity produced on island  $\omega$  is now given by

$$c(\omega, \Omega) = A(\omega)n(\omega)^{\theta_1}l(\omega, \Omega)^{\theta_2}$$

where  $l(\omega, \Omega)$  denotes the labor employed on island  $\omega$  during stage 2 and where  $\theta_1, \theta_2 > 0, \theta_1 + \theta_2 < 1$ . (Note that second-stage employment can depend on the information that becomes available at that stage, which explains why  $l$  is a function of  $\Omega$  and not just  $\omega$ .) Accordingly, the per-period utility of the representative household is given by

$$u = U(C(\Omega)) - \int \frac{1}{\epsilon_1} n(\omega)^{\epsilon_1} d\Omega(\omega) - \int \frac{1}{\epsilon_2} l(\omega, \Omega)^{\epsilon_2} d\Omega(\omega).$$

where  $\epsilon_1, \epsilon_2 > 1$ .<sup>22</sup> Finally, for future reference we let

$$q(\omega) \equiv A(\omega)n(\omega)^{\theta_1} \quad \text{and} \quad Q(\Omega) \equiv \left( \int q(\omega)^{\frac{(\rho-1)\epsilon_2}{\rho\epsilon_2 - (\rho-1)\theta_2}} d\Omega(\omega) \right)^{\frac{\rho\epsilon_2 - (\rho-1)\theta_2}{(\rho-1)\epsilon_2}}.$$

Third, we let firms and workers in each island observe signals of the (nominal) prices set by firms in other islands in addition to signals of the (real) production activity. In particular, we let all agents observe the following two public signals:  $y^q = \log Q(\Omega) + \varepsilon^{yq}$  and  $y^p = \log P(\Omega) + \varepsilon^{yp}$ , where

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<sup>22</sup>Here, we assume local labor markets for both stages, but our results easily extend if we assume a global labor market during stage 2; the “geographical” boundaries are important only for avoiding perfect information aggregation at stage 1.

$\varepsilon^{yq} \sim \mathcal{N}(0, \sigma_{yq}^2)$  and  $\varepsilon^{yp} \sim \mathcal{N}(0, \sigma_{yp}^2)$  are the respective noises. Note that the price signal is properly defined only conditional on having first defined what a price is, which is clear on equilibrium but not off equilibrium. We thus extend the definition of the price signal for arbitrary allocations so that it is always a signal of the shadow prices faced by the representative consumer. Finally, it would be straightforward to extend the analysis to situations where some of the output or price signals are private rather than public, as in the baseline model; here, the output and price signals are left as public only for expositional simplicity.

Finally, we let the monetary authority control nominal aggregate demand. In particular, we let

$$M(\Omega) \equiv P(\Omega)C(\Omega)$$

and assume that the monetary authority directly controls  $M(\Omega)$ . As in Woodford (2003a), this is equivalent to assuming that the monetary authority pays interest on money holdings and appropriately adjusts the nominal interest rate so as to induce the desired level of nominal spending. To simplify the exposition, we specify monetary policy directly in terms of the target level of nominal spending. For tractability, we then restrict attention to the following class of log-normal policy rules:

$$\log M(\Omega) = \lambda_0 + \lambda_A \bar{a} + \lambda_p \log P(\Omega), \quad (23)$$

where the coefficients  $\lambda_A$  and  $\lambda_p$  parameterize the degree to which monetary policy accommodates innovations in aggregate productivity and the aggregate price level.

Note that, once we have identified the optimal monetary policy in terms of a target rule for aggregate nominal demand, it is straightforward to translate that policy in terms of a target rule for the nominal interest rate—provided, of course, that we take a stand on the behavior of real interest rate under the flexible-price allocation. We will discuss such a translation in Section 9, when we examine the optimality of price stability.

## 8 Optimal Monetary Policy

In this section we characterize the optimal combination of fiscal and monetary policies for the extended model. Towards this goal, we start by revisiting the equilibrium and the efficient allocations that would have obtained if prices had been flexible and information had been exogenous.

### 8.1 Exogenous information

Following similar steps as in Section 4, it is easy to check that, in the absence of taxes, the following conditions are necessary and sufficient for equilibrium:

$$n(\omega)^{\epsilon_1-1} = \left( \frac{\rho-1}{\rho} \right) \mathbb{E} \left[ U'(C(\Omega)) \left( \frac{c(\omega, \Omega)}{C(\Omega)} \right)^{-\frac{1}{\rho}} \left( \theta_1 \frac{c(\omega, \Omega)}{n(\omega)} \right) \middle| \omega \right] \quad (24)$$

$$l(\omega, \Omega)^{\epsilon_2-1} = \left( \frac{\rho-1}{\rho} \right) (U'(C(\Omega)) \left( \frac{c(\omega, \Omega)}{C(\Omega)} \right)^{-\frac{1}{\rho}} \left( \theta_2 \frac{c(\omega, \Omega)}{l(\omega, \Omega)} \right)) \quad (25)$$

with

$$c(\omega, \Omega) = A(\omega)n(\omega)^{\theta_1}l(\omega, \Omega)^{\theta_2} \quad \text{and} \quad C(\Omega) = \left\{ \int c(\omega, \Omega)^{\frac{\rho-1}{\rho}} d\Omega(\omega) \right\}^{\frac{\rho}{\rho-1}}. \quad (26)$$

The interpretation of conditions (24) and (25) is similar to that of condition (2) in the baseline model: the left-hand side gives the marginal disutilities of first- and second-stage employment in island  $\omega$ , while the right-hand side gives the corresponding marginal products, multiplied by the marginal utility of the corresponding commodities and the monopolistic wedge (the reciprocal of the mark-up). The same conditions characterize the efficient allocation if we remove this wedge. We conclude that, as long as information is exogenous, the flexible-price equilibrium remains efficient in this model as in our baseline model.

The implication for monetary policy is then immediate: when information is exogenous, if there is a policy that induces the same allocations under sticky prices as the ones that obtain in the flexible-price equilibrium, then this policy is clearly optimal. It is straightforward to show that such a policy indeed exists, which leads to the result that we anticipated earlier on in the baseline model.

**Theorem 1'.** *When information is exogenous, the efficient allocation is implemented with an acyclical tax, which simply offsets the monopolistic mark-up, and a monetary policy that replicates the flexible-price equilibrium allocations.*

Given this result, the subsequent analysis will concentrate on the characterization of optimal monetary policy when information is endogenous. In Section 10, however, we will revisit the flexible-price benchmark to address whether targeting the flexible-price allocations is synonymous to targeting price stability, as it is typically the case in new-Keynesian models.

Finally, letting

$$q(\omega) \equiv A(\omega)n(\omega)^{\theta_1} \quad \text{and} \quad Q(\Omega) \equiv \left( \int q(\omega)^{\frac{(\rho-1)\epsilon_2}{\rho\epsilon_2 - (\rho-1)\theta_2}} d\Omega(\omega) \right)^{\frac{\rho\epsilon_2 - (\rho-1)\theta_2}{(\rho-1)\epsilon_2}},$$

we can show that Proposition 1 continues to hold intact, except that now the coefficients  $\beta$  and  $\alpha$  need to be redefined as follows:

$$\beta \equiv \frac{\frac{\epsilon_1}{\theta_1}}{\frac{\epsilon_1}{\theta_1} + (\rho-1)\frac{\epsilon_2}{\rho(\epsilon_2 - \theta_2) + \theta_2}} > 1 \quad \text{and} \quad \alpha \equiv \left( \frac{1}{\rho} - \gamma \right) \frac{\frac{\epsilon_2}{(\epsilon_2 - \theta_2) + \gamma\theta_2} \frac{\rho\epsilon_2}{\rho(\epsilon_2 - \theta_2) + \theta_2}}{\frac{\epsilon_1}{\theta_1} + (\rho-1)\frac{\epsilon_2}{\rho(\epsilon_2 - \theta_2) + \theta_2}}.$$

The interpretation and role of these coefficients for equilibrium behavior remains the same as in the baseline model. To simplify the exposition, we henceforth restrict attention to the case where production choices are strategic complements.

**Assumption.**  $\alpha > 0$ .

From an empirical perspective, this restriction is rather innocuous: it means that the expectation that aggregate demand will increase in the near future (i.e., in stage 2 in our model) in response to some shock (whether technological or monetary) leads to an increase in aggregate employment and production in the present (i.e., in stage 1). For our purposes, it helps sharpen the predictions about monetary policy.

## 8.2 Endogenous learning: efficient vs optimal implementable allocations

Consider the set of allocations that satisfy resource-feasibility, the informational restriction that stage-1 choices do not depend on the local information of other islands, and the usual symmetry across workers, firms and consumers; to simply, we henceforth call these allocations simply “feasible”. Under log-normality, the set of feasible allocations can then be reduced to a pair of functions, or production “strategies”, that characterize the employment and production taking place in each island  $\omega$  during stages 1 and 2:

$$\log q(\omega) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_y y \quad \text{and} \quad \log l(\omega, \Omega) = l_0 + l_A \bar{a} + l_a a + l_x x + l_y y, \quad (27)$$

for arbitrary coefficients  $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_y)$  and  $l = (l_0, l_A, l_a, l_x, l_y)$ .<sup>23</sup> When prices are sticky, it is natural to think of firms choosing  $q$  and  $p$ , rather than  $q$  and  $l$ ; the combination of the first-stage choices with the monetary policy then determines the second-stage employment outcomes. However, for the purpose of studying efficiency, it is more appropriate—or at least more convenient—to reason directly in terms of allocations, independently of whether prices are sticky or not.

Thus take any pair of strategies as in (27) and any pair of precisions  $\kappa = (\kappa_x, \kappa_y)$  for the sufficient statistics. We can calculate welfare as a (closed-form) function  $\mathcal{W}(\varphi, l, \kappa)$ .<sup>24</sup> Moreover, we can characterize the information structure induced by these strategies as follows.

**Lemma 3.** *Take any pair of strategies as in (27).*

(i) *The precision of the available public information is given by*

$$\kappa_y = \sigma_{ya}^{-2} + (\varphi_a + \varphi_x)^2 \sigma_{yq}^{-2} + \rho^{-2} (\varphi_a + \varphi_x + \theta_2 l_a + \theta_2 l_x)^2 \sigma_{yp}^{-2} \quad (28)$$

(ii) *Provided that  $\varphi_a + \varphi_x > 0$  and  $\varphi_a + \varphi_x + \theta_2 l_a + \theta_2 l_x > 0$ ,  $\kappa_y$  increases with both  $\varphi_x$  and  $l_x$ .*

The term  $\kappa_q \equiv (\varphi_a + \varphi_x)^2 \sigma_q^{-2}$  in condition (28) captures the learning through the quantity signal  $y^q$ , as in the baseline model, while the term  $\kappa_p \equiv \rho^{-2} ((\varphi_a + \varphi_x) + \theta_2 (l_a + l_x))^2 \sigma_{yp}^{-2}$  captures the learning through the price signal  $y^p$ . To understand this new term, note that  $c_a \equiv \varphi_a + \theta_2 l_a$  and  $c_x \equiv \varphi_x + \theta_2 l_x$  measure the sensitivities of the output (and consumption) level of commodity  $\omega$  to, respectively, the local productivity and local information. Multiplying those by the elasticity of the demand function, namely  $1/\rho$ , gives the corresponding sensitivities of the shadow prices faced by the representative consumer. It follows that the precision of the price signal is  $\kappa_p = (\frac{1}{\rho} c_a + \frac{1}{\rho} c_x)^2 \sigma_{yp}^{-2}$ , which explains the new term.

Using these observations, we can express the planner’s problem as follows.

**Planner’s problem.** *Choose a pair of strategies  $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_y)$  and  $l = (l_0, l_A, l_a, l_x, l_y)$  and a precision  $\kappa_y$  so as to maximize  $\mathcal{W}(\varphi, l; \kappa_x, \kappa_y)$  subject to (28).*

<sup>23</sup>Here it is important to note that the second-stage production allocations can depend on the realized aggregate productivity, simply because the latter has become public information at that stage.

<sup>24</sup>See the proof of Proposition 7 in the Appendix for details.

This problem can be solved in a similar manner as that of the baseline model.<sup>25</sup> It can then be shown that the qualitative properties of the efficient allocation are similar to those of the baseline model: internalizing the information externality tends to increase the sensitivity of allocations—now in both stages—to local information. Moreover, it can further be shown that there exist a sufficiently rich set of state-contingent taxes that can implement the efficient allocation as a flexible-price equilibrium. But then it also follows that, once these taxes are set in place, the optimal monetary policy is simply the one that replicates the flexible-price equilibrium.

This result has a pedagogical value: it highlights that, although learning necessarily introduces an inefficiency, it does not necessarily change the nature of optimal monetary policy. However, its practical value is questionable for the following reason. In general, it is no more possible to implement the efficient allocation merely with a state-contingent tax on output, profit, or total employment—or with any other tax that affects symmetrically the incentives in stages 1 and 2. Rather, it is now necessary that the government also uses a tax that is specific to either of the two stages, such as a differential tax on second-stage employment. But it is unlikely that the government can distinguish these two stages in practice—after all, these stages are very gross representations for the much richer information and price-setting dynamics that are likely to take place in reality.

For this reason, we find it more appropriate to consider a restricted planning problem, one that identifies the optimal allocation among the ones that can be implemented with the combination of a contingent linear tax on output and a contingent monetary policy.<sup>26</sup> The next lemma characterizes the set of such implementable allocations.

**Proposition 6.** *(i) When prices are sticky, a pair of strategies as in (27) can be implemented as an equilibrium with an appropriate combination of a contingent linear tax and a monetary policy if and only if the following conditions are satisfied.<sup>27</sup>*

$$\varphi_a = \beta \quad (29)$$

$$l_a = \frac{\epsilon_1}{\epsilon_2 \theta_1} (\varphi_a - 1) \quad (30)$$

$$l_x + l_A \frac{\kappa_x}{\kappa_A + \kappa_x + \kappa_y} = \frac{\epsilon_1}{\epsilon_2 \theta_1} \varphi_x \quad (31)$$

$$l_y + l_A \frac{\kappa_y}{\kappa_A + \kappa_x + \kappa_y} = \frac{\epsilon_1}{\epsilon_2 \theta_1} \varphi_y \quad (32)$$

*(ii) When instead prices are flexible, a pair of strategies as in (27) can be implemented if and only if the following condition is satisfied in addition to conditions (29)-(32):*

$$l_A = \frac{\epsilon_1}{\epsilon_2 \theta_1} \frac{\varphi_x}{\beta} \frac{\kappa_A + \kappa_x + \kappa_y}{\kappa_x}. \quad (33)$$

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<sup>25</sup>A detailed characterization of the efficient allocation is available upon request.

<sup>26</sup>Keep in mind that, as in the baseline model, a tax on firm output is equivalent to a tax on firm total employment or payroll, a tax on household labor income, or household consumption, or any other tax that has a uniform impact across the two employment choices. Therefore, the results we present here apply more generally to the tax wedge induced by any combination of these taxes.

<sup>27</sup>There is also a restriction between  $\varphi_0$  and  $l_0$ , which we omit because it is of no interest:  $\varphi_0$  and  $l_0$  are irrelevant for the business-cycle properties of allocations.

This lemma plays a similar role as the familiar “implementability” results in the Ramsey literature: it represents the optimal policy problem in terms of the allocations that are induced by the policy rather than the policy instruments themselves. Furthermore, it helps understand the extent to which the government can manipulate the decentralized use of information. Indeed, much alike in the baseline model, the government may always induce any  $\varphi_x$  and  $\varphi_y$  it may desire. This is true no matter whether prices are flexible or not: state-contingent taxes alone suffice for this. But when prices are flexible, monetary policy can also help manipulate the decentralized use of information.

The intuition for these results is simple. First, note that, whether prices are flexible or sticky, the absence of a differential tax on the two types of labor implies that the equilibrium necessarily equates the (expected) marginal rate of transformation between these two types of labor with the corresponding marginal rate of substitution in preferences. This explains why  $l_a, l_x$  and  $l_y$  are related to  $\varphi_a, \varphi_x$  and  $\varphi_y$  as in (29)-(32).<sup>28</sup> Next, note that when prices are flexible, once the taxes have been set to achieve the desired  $\varphi_x$  and  $\varphi_y$ , the government has no further control on  $l_A$ : the sensitivity of second-stage employment and production to the realized aggregate productivity is pinned down by equating the realized marginal returns and costs of stage-2 labor. It is this restriction that gives (33). But when prices are sticky, this restriction is no more present: by designing the extent to which monetary policy accomodates the realized productivity shock, the government can freely choose  $l_A$ . Of course, anticipating the monetary policy’s response to the realized productivity, firms adjust their own response to the information they have about aggregate productivity when they set their prices—which explains why  $l_x$  and  $l_y$  are negatively related to  $l_A$  in the way defined by conditions (31) and (32).

To see this more clearly, consider the equilibrium prices that are associated with any given implementable allocation. Using the facts that  $\log p(\omega) - \log P(\omega) = -\frac{1}{\rho}(\log c(\omega, \Omega) - \log C(\Omega))$  and  $\log c(\omega, \Omega) = \log q(\omega) + \theta_2 \log l(\omega, \Omega)$  along with (27), we infer that the equilibrium prices satisfy

$$\log p(\omega) = \psi_0 + \psi_a a + \psi_x x + \psi_y y,$$

where  $\psi_a$  and  $\psi_x$  are pinned down by

$$\psi_a = -\frac{1}{\rho}(\varphi_a + \theta_2 l_a) \quad \text{and} \quad \psi_x = -\frac{1}{\rho}(\varphi_x + \theta_2 l_x),$$

while  $\psi_y$  is indeterminate.<sup>29</sup> Saying that monetary policy can control  $l_x$ , the sensitivity of second-stage allocations to local information, is thus synonymous to saying that monetary policy can control  $\psi_x$ , the sensitivity of prices to local information. In particular, using (31), we get that  $\psi_x$  is

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<sup>28</sup>In particular, the equality of expected marginal rates of transformation and substitution gives  $\mathbb{E}[\log l(\omega, \Omega)] = \text{const} + \frac{\epsilon_1 \theta_2}{\epsilon_2 \theta_1} \log n(\omega)$ , where *const* includes second-order terms; using then  $\log q(\omega) = \log A(\omega) + \theta_1 \log n(\omega)$  and noting that this condition must be satisfied for every  $\omega = (a, x, y)$ , gives the constraints in part (i) of Proposition 6.

<sup>29</sup>That is, the sensitivities of local prices to local productivity and local information are pinned down by the elasticity of demand and the corresponding sensitivities of local output, while the sensitivity of prices to public information is indeterminate, simply because any component of monetary policy that is public information at the moment prices are set cannot have any real effect.

decreasing in  $l_A$  : the more firms expect monetary policy to accomodate the realized productivity shock, the higher their incentive to raise their prices in response to any private information they may have about aggregate productivity.

Turning back to the planner's problem, we can now use Proposition 6 to express it as follows.

**Optimal policy problem.** Choose strategy coefficients  $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_y)$  and  $l = (l_0, l_A, l_a, l_x, l_y)$  and a precision  $\kappa_y$  so as to maximize  $\mathcal{W}(\varphi, l; \kappa_x, \kappa_y)$  subject to (28) and (29)-(32).

Like standard Ramsey policy problems, this problem imposes certain implementability constraints on the set of allocations that the planner can choose; as explained before, these constraints are summarized in conditions (29)-(32). But unlike standard Ramsey exercises, the allocations have also been restricted to respect the dispersion of information that we take as a primitive; this restriction is already embedded in the assumption that the strategy  $q$  depends only on locally available information. Finally, the planner takes into account that different allocations induce different information structures according to condition (28).

Solving this problem is characterized in the following result.

**Proposition 7.** *There exist scalars  $\Delta_q > 0$  and  $\Delta_p > 0$  such that the following are true:*

(i) *The first-stage production strategy is given by*

$$\log q(\omega) = \varphi_0^* + \varphi_a^* a + \varphi_x^* x + \varphi_y^* y$$

where

$$\varphi_a^* = \beta \tag{34}$$

$$\varphi_x^* = \left\{ \frac{(1-\alpha)\kappa_x^*}{(1-\alpha)\kappa_x^* + \kappa_y^* + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta + (\Delta_q + \Delta_p) \tag{35}$$

$$\varphi_y^* = \left\{ \frac{\kappa_y^*}{(1-\alpha)\kappa_x^* + \kappa_y^* + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta - \frac{\kappa_y^*}{\kappa_A + \kappa_y^*} (\Delta_q + \Delta_p) \tag{36}$$

(ii) *The second-stage production strategy is given by*

$$\log l(\omega, \Omega) = l_0^* + l_A^* \bar{a} + l_a^* a + l_x^* x + l_y^* y$$

where

$$l_a^* = \hat{l}_a \tag{37}$$

$$l_x^* = \hat{l}_x + \left( \frac{\kappa_x}{\kappa_A + \kappa_x + \kappa_y^*} \right) (\lambda_q \Delta_q + \lambda_p \Delta_p) \tag{38}$$

$$l_y^* = \hat{l}_y + \left( \frac{\kappa_y^*}{\kappa_A + \kappa_x + \kappa_y^*} \right) (\lambda_q \Delta_q + \lambda_p \Delta_p) \tag{39}$$

$$l_A^* = \hat{l}_A - (\lambda_q \Delta_q + \lambda_p \Delta_p) \tag{40}$$

where  $(\hat{l}_a, \hat{l}_x, \hat{l}_y, \hat{l}_A)$  are the coefficients that would have obtained in the flexible-price equilibrium when taxes are such that the first-stage production strategy is the optimal one, and where  $(\lambda_q, \lambda_p)$  are positive scalars.

This result establishes that the impact of learning on the optimal implementable allocations is qualitatively very similar to the one in the baseline model. In particular, the scalars  $\Delta_q$  and  $\Delta_p$  are the Lagrange multipliers that measure the social value of increasing the precisions of, respectively, the quantity signal  $y^q$  and the price signal  $y^p$ . Using similar techniques as in the proof of Proposition 4, we can prove that both  $\Delta_q$  and  $\Delta_p$  are positive. Part (i) then shows that it is optimal to increase  $\varphi_x$  relative to the one that would characterize the flexible-price equilibrium allocations in the absence of taxes. By Proposition 6 we know that this increase in  $\varphi_x$  would have been associated with an increase in both  $l_x$  and  $l_A$  if monetary policy were replicating the flexible-price equilibrium. Part (ii) establishes that it is actually optimal to increase  $l_x$  further than that, which in turn is possible only by reducing  $l_A$ . We will further discuss how these results translate in terms of the optimal mixture of fiscal and monetary policies in the next subsection. Before doing that, we discuss what are the key forces that drive these results.

If the government could use a differential tax on the two types of labor in addition to the uniform output tax, then the government would have implemented the efficient allocation. The latter would feature the same qualitative properties, except for two differences. First, the precise values of  $\Delta_q$  and  $\Delta_p$  would be different. And second, whereas the optimal allocation features  $l_A^* + l_x^* < \hat{l}_1 + \hat{l}_x$  and  $l_y^* > \hat{l}_y$ , the efficient allocation would feature  $l_A^* + l_x^* = \hat{l}_A + \hat{l}_x$  and  $l_y^* = \hat{l}_y$ . To understand this, note that the planner wishes to increase  $l_x$  in order to induce more learning. This is true no matter whether a differential tax is available or not. The difference lies on what it takes to achieve this goal. Along the best implementable allocation, by the implementability constraints in Proposition 6 we know that increasing  $l_x$  is possible only by reducing  $l_A$  more than one-to-one, as well as that this reduction in  $l_A$  in turn necessitates an increase in  $l_y$ . But when a differential tax is available, these constraints are no longer binding. Rather, the planner can decrease  $l_A$  one-to-one with the desired increase in  $l_x$  so as to preserve allocative efficiency (in the sense of maintaining the efficient overall sensitivity of second-stage employment, and hence of output and consumption, to aggregate productivity). Moreover, there is clearly no reason to distort  $l_y$ .

Finally, if the government could use only output taxes and prices were flexible, then part (i) would continue to hold (with different values for  $\Delta_q$  and  $\Delta_p$ ), but of course now it would necessarily be the case that  $l_A^* = \hat{l}_A, l_x^* = \hat{l}_x, l_y^* = \hat{l}_y$ , simply because there would be no instrument that would permit the planner to do otherwise.

We conclude that the nature of the optimal allocation—and hence of the policies that implement it—is driven by, and only by, the combination of three properties of the environment: the presense of informational externalities, which makes it desirable to manipulate the decentralized use of information in a similar manner as in the baseline model; second, the fact the prices are sticky, which gives monetary policy the power to contribute to this goal along with the state-contingent tax; and finally, the absence of a differential tax, which makes it optimal for monetary policy to exercise this power, thus deviating from the principle of replicating the flexible-price equilibrium.

### 8.3 Optimal policy

We can now translate these results in terms of the policies that implement the optimal allocation.

**Theorem 4.** (i) *The optimal tax is countercyclical, as in the baseline model.*

(ii) *The optimal monetary policy is less accommodative of the productivity shock—in the sense that it induces aggregate income to react less to the productivity shock—than the policy that would have replicated the flexible-price allocations.*

To understand part (ii), recall from Proposition 7 that the optimal allocation satisfies  $l_a^* = \hat{l}_a$  and that  $l_A^* + l_x^* < \hat{l}_A + \hat{l}_x$  (where the hats indicate the values that would obtain in the flexible-price equilibrium in which taxes are such that the first-state strategy is optimal). Note then that the overall sensitivity of aggregate income to aggregate productivity along the optimal allocation is given by  $\varphi_a^* + \varphi_x^* + \theta_2(l_a^* + l_x^* + l_A^*)$ , while the corresponding sensitivity of the flexible-price equilibrium in which the first-state strategy coincides with the optimal one is given by  $\varphi_a^* + \varphi_x^* + \theta_2(\hat{l}_a + \hat{l}_x + \hat{l}_A)$ . It follows that aggregate income responds less to aggregate productivity in the optimal allocation than what would be consistent with the flexible-price equilibrium.

To understand part (i) suppose for a moment that monetary policy were replicating the flexible-price equilibrium. The result would then have followed from precisely the same reasoning as in the baseline model: a countercyclical tax would be necessary for inducing a higher  $\varphi_x$ . But now note the fact that the optimal monetary policy is less accommodative induces the opposite effect: other things equal, a less accommodative policy implies a lower incentive to react to local information about aggregate productivity. It follows that the optimal tax necessarily remains countercyclical; if it were not, the monetary policy would have induced a lower  $\varphi_x$ , which is a contradiction.

We conclude that the basic intuition for the countercyclicality of the tax is much alike the baseline model, while the basic intuition for the optimality of making monetary policy less accommodative of the productivity shock is that this induces more learning through the price signals.

## 9 On the suboptimality of price stability

The analysis so far has characterized the optimal monetary policy relative to a certain policy benchmark, namely the one where monetary policy replicates the flexible-price allocations. As mentioned in the Introduction, within the new-Keynesian paradigm replicating the flexible-price allocations is usually synonymous to targeting price stability. We will now show that this is not the case in our environment. We will further show that, whenever aggregate productivity is not common knowledge in the economy, targeting price stability can be detrimental for both allocative and informational efficiency.

To establish this result, we start with the case of exogenous information, for which we know that full efficiency is implemented with, and only with, an acyclical tax and a monetary policy that replicates the flexible-price allocations. In the Appendix we show that the associated equilibrium

prices must then satisfy the following condition:

$$\mathbb{E} [\log p(\omega) - \log P(\Omega)|\omega] = \text{const} - \nu \mathbb{E} [\log q(\omega) - \log Q(\Omega)|\omega] \quad (41)$$

for some coefficient  $\nu > 0$ . To interpret this condition, note that  $q(\omega)$  represents the effective productivity of stage-2 employment or, equivalently, the effective marginal cost of stage-2 production; this condition thus simply states that relative prices are proportional to relative marginal costs, which, clearly, is essential for efficiency.

Next, recall that the first-stage strategy is given as in Proposition 1 (but with the new definitions for  $\beta$  and  $\alpha$ ). Using this result in the above condition, we infer that the equilibrium prices must satisfy

$$\log p(\omega) = \text{const} + \psi_a a + \psi_x x + \psi_y y,$$

with  $\psi_a = -\nu\varphi_a$  and  $\psi_x = -\nu\varphi_x$ , while  $\psi_y$  remains indeterminate (because the dependence of prices on public information has no impact on real allocations). By implication, the aggregate price level must satisfy

$$\log P(\Omega) = \text{const} - \nu(\varphi_a + \varphi_x)\bar{a} + \psi_y y.$$

And since both  $\varphi_a + \varphi_x$  and  $\nu$  are positive, we conclude that it is necessary that the aggregate price level responds negatively to realized aggregate productivity.

**Theorem 5.** *When information about aggregate productivity is dispersed, replicating the no-tax flexible-price equilibrium allocations—and thereby achieving allocative efficiency—does not mean targeting price stability. Rather, it means targeting a certain negative correlation between the aggregate price level and aggregate productivity.*

This result offers an interesting contrast to the standard result in the pertinent literature that emphasizes the optimality of targeting price stability in economies with sticky prices when there are no “mark-up shocks” or other “wedges” that would render the business cycle inefficient under flexible prices.<sup>30</sup> Indeed, in standard micro-founded business-cycle models with sticky prices, whenever the response of the flexible-price equilibrium to aggregate productivity and taste shocks is efficient, monetary policy should only replicate the flexible-price allocations, which in turned is typically achieved by targeting price stability.<sup>31</sup> In our setting, the (no-tax) flexible-price equilibrium remains efficient as long as information is exogenous, and hence it also remains true that monetary policy should only replicate the flexible-price allocations. However, this no more means that monetary policy should target price stability.

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<sup>30</sup>See, e.g., Benigno and Woodford (2005), Clarida, Galí and Gertler (1999), Galí, (2003), Goodfriend and King (1997, 2001), Khan, King, and Wolman (2003), Rotemberg and Woodford (1997), Woodford (2003a, 2005).

<sup>31</sup>The optimality of replicating flexible-price allocations is quite robust within this class of models as long as there are taxes that eliminate the monopolistic distortion. See, though, Adão, Correia and Telles (2003), Benigno and Woodford (2005), and Woodford (2003a) for some important qualifications regarding the optimality of price stability in the absence of such taxes.

What causes price stability to be suboptimal in our class of economies is the presence of dispersed private information regarding aggregate productivity. To see this, suppose for a moment that the realized level of aggregate productivity was common knowledge in the economy. Then, the response of monetary policy to aggregate productivity would have been irrelevant: in equilibrium, all nominal prices move one-to-one with any component of the monetary policy that is common knowledge, ensuring that real allocations are unaffected by this component of the monetary policy. Efficiency would thus put no restriction whatsoever on the response of either monetary policy or the aggregate price level to aggregate productivity. By implication, if aggregate productivity had been common knowledge, targeting price stability would have been without any loss of optimality (although not strictly necessary).

The same logic explains more generally why efficiency puts no restriction on the response of nominal prices and monetary policy to any public signal about aggregate productivity. In particular, consider the case where aggregate productivity is not known at stage 1 (when prices are set), but suppose that all agents share the same information about aggregate productivity. Then, the response of prices and monetary policy to this information would still remain indeterminate and, by implication, targeting price stability would still have been without any loss of optimality.

Consider then the case where agents have dispersed information about aggregate productivity. Efficiency requires that quantities and relative prices are sensitive to private information, whether this regards about local productivity or aggregate productivity. In particular, we have proved that the efficient allocation features  $\varphi_a + \varphi_x > 0$ , which means that firms with higher private information (higher  $a + x$ ) must produce more output and, by implication, face lower relative prices. When nominal prices are flexible, this is automatically satisfied in equilibrium, no matter what the nominal price level is, and monetary policy is irrelevant. But when nominal prices are sticky, monetary policy must sustain the right relative prices. Suppose, towards a contradiction, that the central bank targets price stability, thus ensuring that the nominal price level is invariant to the realized aggregate productivity shock  $\bar{a}$  for any given public information  $y$  (i.e.,  $\psi_a + \psi_x = 0$ ). For this to be the case, it must be that in equilibrium individual nominal prices are invariant to private information (i.e., that  $\log p(\omega)$  is invariant to  $a + x$ ). But this would also mean that relative prices are invariant to differences in private information, which contradicts efficiency. In other words, targeting price stability introduces—rather than eliminates—distortions in relative prices.

This argument explains why the nominal price level cannot be invariant to the realized aggregate productivity shock. To understand further why it must actually decrease with it, note that the *relative* price of firm  $j$  relative to firm  $j'$  can fall with the private information of the former only if its *nominal* price also falls; this is simply because the nominal price of  $j'$  cannot depend on the private information of  $j$  (by the very fact that this information is private to  $j$ ). But if the nominal price of each firm is a negative function of the firm's private information, then the cross-sectional average of nominal prices will also be a negative function of the cross-sectional average of private informations—which means that the price level must be a negative function of the aggregate productivity shock.

It is easy to translate this result in terms of a target rule for the nominal interest rate. To do this, we only have to take a stand on what are the stochastic properties of the real interest rate along the flexible-price allocation. Following the pertinent literature, suppose that aggregate productivity is a random walk, which guarantees that the real interest rate in the flexible-price allocation is constant at some “natural” level. Targeting price stability is then synonymous to keeping the nominal interest rate constant (on equilibrium). In contrast, replicating the flexible-price allocations requires the price level to fall with any positive innovation in aggregate productivity, which in turn can be achieved only by raising nominal interest rates—i.e., by contracting monetary policy—in response to such a positive innovation in aggregate productivity.

Finally, note that the distortionary effects of targeting price stability do not mean only an inefficiency in the *level* of aggregate real output (as, for example, it is the case with a constant positive inflation in the standard paradigm), but also an inefficiency in the *response* of aggregate real output to the aggregate productivity shock. This follows directly from the fact that the optimal monetary policy is less accommodative of the productivity shock than the one that sustains price stability: a monetary policy that targets price stability induces real output to overreact to the productivity shock relative to what is efficient.

The preceding discussion explained why targeting price stability is inefficient from a purely allocative perspective, abstracting from the potential endogeneity of information. However, taking into account the endogeneity of learning only reinforces this conclusion. Indeed, we have shown that the informational externality implies a higher  $\varphi_x$ . By itself, this would contribute to a more negative correlation between the price level and aggregate productivity should monetary policy replicate the flexible-price allocation. But we have also shown that (for the empirically relevant case in which  $\alpha > 0$ ) the optimal monetary policy is now less accommodative of the productivity shock, which contributes to an even more negative correlation. Basically, this is due to the fact that improving the precision of the endogenous price signal requires that local prices react more strongly (i.e., even more negatively) to local information. We conclude that targeting price stability can be detrimental, not only for allocative efficiency, but also for informational efficiency.

## 10 Social value of information and central bank transparency

Recently, an influential paper by Morris and Shin (2002) has provoked a debate on the merits of central bank transparency and, more generally, on the social value of any additional information that the government may be able to provide the market with.<sup>32</sup> We now discuss what are the implications of our results for this issue.

To start with, abstract from the endogeneity of information and imagine that the government has the option to decrease the noise in the exogenous public signal about aggregate productivity. The class of economies that we have studied in this paper (and that is the backbone of recent

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<sup>32</sup>For some of the contributions to this debate, see Amador and Weill (2007, 2008), Angeletos and Pavan (2004, 2007), Cornand and Heinemann (2008), Hellwig (2005), Svensson (2006), and Woodford (2005).

business-cycle theory) features strategic complementarity in the production and pricing choices of firms, much alike the model in Morris and Shin (2002). However, it is important to remember that what drives the result in Morris and Shin (2002) that more public information can reduce welfare is not per se the presence of strategic complementarity but rather the assumption that this complementarity is not warranted from a social perspective: the efficient allocation in their model is assumed to feature not such complementarity.<sup>33</sup> In contrast, our results have established that, for the class of micro-founded business-cycle economies that are of interest here, the degree of complementarity featured in equilibrium is exactly the same as the one featured in the efficient allocation (unless, of course, tax distortions or suboptimal monetary policies drive a wedge between the two). More generally, if we abstract from the endogeneity of information, our results guarantee that the equilibrium use of information is efficient as long as (i) prices are flexible or monetary policy replicates the flexible-price allocations and (ii) there are no tax distortions. But then it follows that equilibrium welfare necessarily increases with any additional information, whether public or private. This is simply because, whenever the equilibrium is efficient, it coincides with the solution to a single-agent decision problem, namely that of the planner; it then follows directly by Blackwell's theorem that more information cannot be harmful.<sup>34</sup>

Next, consider the case where information is endogenous and imagine that the government has the option to decrease either the noise in the exogenous public signal about aggregate productivity or the measurement errors in the endogenous macroeconomic statistics. One then needs to address the possibility raised by Amador and Weill (2007, 2008): providing more precise public information could induce agents to reduce the sensitivity of their actions to private information, thus reducing the efficiency of social learning and hence also reducing welfare. Whether this possibility emerges or not in our economy depends on the fiscal and monetary policies followed by the government. If monetary policy replicates the flexible-price allocations and there are no taxes, then this possibility may well occur; this is simply because the equilibrium then fails to internalize the informational externalities. If, on the other hand, there are policies that restore full efficiency in the equilibrium of information, then one can rule out this possibility and once again guarantee that welfare increases with any additional information.

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<sup>33</sup>This general point has been stressed before also in Angeletos and Pavan (2004, 2007). However, that earlier more abstract work did not examine what is the relation between equilibrium and efficient allocations for the class of business-cycle economies that are of interest here and therefore did not address what is the social value of information for this particular class of economies.

<sup>34</sup>This observation also suggests that the finding in Hellwig (2005) that welfare can decrease with the precision of private information must be driven by the suboptimality of monetary policy in that paper. In particular, that paper studies an economy that features dispersed information about money supply (as in Lucas, 1972) rather than aggregate productivity or tastes (as in our paper). Although the model of that paper is not identical to ours, one can easily adapt our results to that model. Because there is no endogenous learning in that model, one can guarantee that the equilibrium use of information would have been efficient, and hence that any information would have been welfare-improving, if monetary policy were replicating the flexible-price allocations. However, monetary policy is exogenously fixed in that paper and fails to replicate flexible-price allocations—which explains why the equilibrium use of information is inefficient, thus opening the door to the possibility that more information may reduce welfare.

Within our baseline model, a simple linear tax sufficed for this purpose. However, in our extended model, restoring full efficiency required an implausibly rich set of state-contingent taxes. The question of interest, therefore, is whether the simpler set of policies we have allowed suffices for welfare to increase with public information even though it does not suffice for restoring full efficiency. The answer to this question is affirmative.

**Theorem 6.** *The optimal fiscal and monetary policies guarantee that welfare increases with the precision of either the exogenous public information or the endogenous output and price signals.*

Although it is hard to know how robust this particular result might be to alternative restrictions on the available policy instruments or richer versions of the model, it does provide an important benchmark, highlighting two more general properties. First, the desirability of raising central bank transparency, of improving the quality of macro data, or of otherwise providing the market with more information cannot be addressed without also addressing the optimality of fiscal and monetary policies. And second, the type of policy responses we have identified in this paper help agents internalize their informational externalities and, in so doing, help guarantee that the provision of more information will increase welfare.

## 11 Concluding remarks

This paper made a first attempt to study the normative properties of business cycles when information regarding aggregate productivity and demand conditions is dispersed and only imperfectly aggregated through prices and macro data.

We first showed that the dispersion of information per se need not cause any inefficiency: as long as information is exogenous to the actions of the agents, the equilibrium remains efficient in the absence of nominal or tax distortions. By implication, the optimal fiscal policy is acyclical and the optimal monetary policy simply replicates the flexible-price allocations. This result established that an important lesson from the pertinent literature, the one about the efficiency of flexible-price allocations, is robust to the introduction of dispersed information provided that one abstracts from the potential endogeneity of this information. But then we also showed that another closely related lesson, the one about the optimality of price stability, is not robust.

In particular, we showed that, whenever aggregate productivity is not common knowledge, targeting price stability does not replicate the flexible-price allocations. Rather, it leads to distortions in relative prices as well as to inefficient fluctuations in the “output gap” (the distance between the equilibrium and the efficient business cycle). We then further showed that replicating the flexible-price allocations means that the central bank should target a negative correlation between innovations in the price level and innovations in aggregate productivity—or, equivalently, that nominal interest rates must increase with any positive innovation in aggregate productivity.

We next highlighted that the efficiency of flexible-price allocations is compromised by the endogeneity of information: a benevolent planner would have like to induce firms and households

to react more strongly to their private information so as to improve the endogenous aggregation of information through prices, macro data, and other sources of social learning. We proceeded to show that both state-contingent taxes and monetary policy have the power to provide the market with the right incentives for this goal to materialize. Typically, this means (i) that taxes should be countercyclical and (ii) that nominal interest rate should be even more procyclical than what would have been optimal in the absence of informational externalities. The basic intuition for the former property is that it helps improve the informational content of real output (or other quantity) signals. The basic intuition for the latter property is that it helps improve the informational content of nominal prices. Conversely, if the central bank were to target price stability, it would now distort, not only the allocative role of prices, but also their informational role.

We finally showed that the policies we identified in this paper help guarantee that welfare will increase with more central-bank transparency, with improvements in the technology of aggregating information, or with any other source of information.

Although the analysis focused on productivity shocks, the analysis can easily accommodate shocks to the marginal utility of consumption or the marginal disutility of labor, as well as exogenous monetary shocks.<sup>35</sup> In fact, our result that the equilibrium business cycle is efficient as long as information is exogenous and prices are flexible (or monetary policy replicates the flexible-price allocations), extends to arbitrary stochastic processes for the productivity, taste and monetary shocks and to arbitrary information structures, as well as to arbitrary functional forms for the utility of consumption, the disutility of labor, and the production technology of firms. As with the case of homogenous information, the efficiency of the flexible-price business cycle is hardwired in the Dixit-Stiglitz class of economies that is the backbone of the new-Keynesian paradigm: it relies merely on the fact that the monopolistic distortion is invariant with the business cycle.

If the monopolistic mark-up varies with the business cycle (e.g., Rotemberg and Woodford, 1991, 1999), then the flexible-price equilibrium ceases to be efficient even in the absence of information frictions. Provided that the government can offset the monopolistic distortion with an appropriate subsidy (or other regulatory policies), then all our results go through—one only has to re-interpret the cyclical properties of the optimal tax that we documented in Theorems 3 and 4 as statements about the cyclical properties of the gap between the optimal tax and the one that offsets the mark-up shock. More complicated, and probably more interesting, is the case when the government cannot use taxes to offset the mark-up shocks, or the case when the government cannot tell apart mark-up shocks from productivity shocks. In the former case, a deviation from price stability would have been optimal even with homogenous information. In the latter case, by affecting incentives in decentralized use of information, policy could affect, not only how much the market learns about the underlying shocks, but also how the government itself learns about them. How these possibilities affect the optimal design of monetary policy is left for future work.

There are a number of other (often overlapping) directions for future work. One is to address the

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<sup>35</sup>By the latter we mean the case where aggregate nominal demand is given by  $\log P(\Omega) + \log C(\Omega) = \log M(\Omega) + v$ , where  $M(\Omega)$  is the component that is controlled by the monetary authority and  $v$  is the exogenous monetary shock.

quantitative importance of the dispersion of information for the business cycle; a first attempt in this direction is made in Angeletos and La'O (2008). Another is to study how information frictions may impact the allocation and accumulation of capital as well as to allow financial markets to play a more central role in aggregating information.<sup>36</sup> A third is to give up the convenience of the representative household so as to study how the heterogeneity of consumer expectations may interact with income heterogeneity. A fourth is to allow for richer dynamics in social learning over the business cycle or to endogenize the collection of information.<sup>37</sup> We hope that the framework and the results of this paper offer a useful starting point for further studying the business-cycle implications of dispersed information.

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<sup>36</sup>Recall that our model abstracts from capital and let financial markets operate only when information is common. In Appendix B we discuss how the analysis can accommodate a signal that may mimic the informational role of financial prices: a signal of the cross-sectional average expectation of the marginal utility of consumption. Some interesting novel issues emerge because this signal depends on an aggregate of opinions (beliefs) rather than simply an aggregate of allocations (choices). Yet, provided that the informativeness of this signal increases with the sensitivity of allocations to local information, all our results go through. Far less obvious, however, are the implications of informational frictions for the allocation and the accumulation of capital over the business cycle.

<sup>37</sup>Allowing for rich learning dynamics would permit the model to produce richer dynamics in the response of output and prices to the underlying shocks, but would also bring in the complications associated with “forecasting the forecasts of others” highlighted by Townsend (1983): the cross-sectional distribution of beliefs in any given period depends not only on the current shocks but also the entire history of any past shocks that are not common knowledge, which makes the dimensionality of the relevant state variables to explode. Previous work (e.g., Woodford, 2003b, Amato and Shin, 2006) have avoided these complications only by ruling out any form of endogenous learning altogether, while the present paper avoided them only by assuming that all shocks become common knowledge by the end of each period. Finally, see Hellwig and Veldkamp (2008) for some recent work on endogenizing the collection of information in environments with strategic complementarity or substitutability; clearly, the policies we have identified here can help control incentives, not only in the use, but also in the collection of information.

## Appendix A: proofs

**Proof of Proposition 1 (equilibrium with exogenous info).** *Part (i).* Suppose that, conditional on  $\omega$ ,  $Q(\Omega)$  is log-normal, with variance independent of  $\omega$ ; that this is true follows from part (ii), which we will prove next. Taking the log of condition (3) and using the log-normality of  $Q$ , we infer that the equilibrium production strategy must satisfy

$$\begin{aligned} \left(\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1\right) \log q(\omega) &= \log \theta \left(\frac{\rho-1}{\rho}\right) + \frac{\epsilon}{\theta} \log A(\omega) + \left(\frac{1}{\rho} - \gamma\right) \mathbb{E}[\log Q(\Omega) | \omega] \\ &\quad + \frac{1}{2} \left(\frac{1}{\rho} - \gamma\right)^2 \text{Var}[\log Q(\Omega) | \omega] \end{aligned} \quad (42)$$

where  $\text{Var}[\log Q(\Omega) | \omega] = \text{Var}[\log Q(\Omega)]$ . Condition (5) then follows by letting

$$\zeta \equiv \frac{1}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} \left\{ \log \theta + \log \left(\frac{\rho-1}{\rho}\right) + \frac{1}{2} \left(\frac{1}{\rho} - \gamma\right)^2 \text{Var}[\log Q(\Omega)] \right\}.$$

*Part (ii).* Suppose the equilibrium production strategy takes a log-linear form:  $q(a, x, y) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_y y$ , for some coefficients  $(\varphi_0, \varphi_a, \varphi_x, \varphi_y)$ . Aggregate output is then given by

$$\log Q(\Omega) = \varphi'_0 + (\varphi_a + \varphi_x) \bar{a} + \varphi_y y \quad (43)$$

where  $\varphi'_0 \equiv \varphi_0 + \left(\frac{\rho-1}{\rho}\right) \frac{(\varphi_a + \varphi_x)^2}{2} \sigma_x^2$ . It follows that  $Q(\Omega)$  is indeed log-normal, with

$$\mathbb{E}[\log Q(\Omega) | \omega] = \varphi'_0 + (\varphi_a + \varphi_x) \mathbb{E}[\bar{a} | \omega] + \varphi_y y \quad (44)$$

$$\text{Var}[\log Q(\Omega) | \omega] = (\varphi_a + \varphi_x)^2 \sigma_0^2 \quad (45)$$

where  $\mathbb{E}[\bar{a} | \omega] = \frac{\kappa_A}{\kappa_A + \kappa_x + \kappa_y} \mu + \frac{\kappa_x}{\kappa_A + \kappa_x + \kappa_y} x + \frac{\kappa_y}{\kappa_A + \kappa_x + \kappa_y} y$ . Substituting these expressions into (5) gives

$$\begin{aligned} \log q(\omega) &= \zeta + \beta a + \\ &\quad + \alpha \left\{ \varphi'_0 + \varphi_y y + (\varphi_a + \varphi_x) \left( \frac{\kappa_A}{\kappa_A + \kappa_x + \kappa_y} \mu + \frac{\kappa_x}{\kappa_A + \kappa_x + \kappa_y} x + \frac{\kappa_y}{\kappa_A + \kappa_x + \kappa_y} y \right) \right\} \end{aligned}$$

For this to coincide with  $\log q(a, x, y) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_y y$  for every  $(a, x, y)$ , it is necessary and sufficient that the coefficients  $(\varphi_0, \varphi_a, \varphi_x, \varphi_y)$  solve the following system:

$$\begin{aligned} \varphi_0 &= \zeta + \alpha \left\{ \varphi'_0 + (\varphi_a + \varphi_x) \frac{\kappa_A}{\kappa_A + \kappa_x + \kappa_y} \mu \right\} \\ \varphi_a &= \beta \\ \varphi_x &= \alpha (\varphi_a + \varphi_x) \left( \frac{\kappa_x}{\kappa_A + \kappa_x + \kappa_y} \right) \\ \varphi_y &= \alpha \varphi_y + \alpha (\varphi_a + \varphi_x) \left( \frac{\kappa_y}{\kappa_A + \kappa_x + \kappa_y} \right) \end{aligned}$$

The unique solution to this system for  $(\varphi_a, \varphi_x, \varphi_y)$  is the one given in the proposition;  $\varphi_0$  is then uniquely determined from the first equation of this system along with the definitions of  $(\zeta, \varphi'_0)$  and the expression for  $\text{Var}[\log Q]$  in (45).

**Proof of Proposition 2 (efficiency with exogenous info).** *Part (i).* The planner's problem is strictly convex, guaranteeing that its solution is unique and is pinned down by its first-order conditions. The Lagrangian of this problem can be written as

$$\begin{aligned}\Lambda = & \int_{\Omega} \left[ U(Q(\Omega)) - \int_{\omega} \frac{1}{\epsilon} e^{-\frac{\epsilon}{\theta} a} q(\omega)^{\frac{\epsilon}{\theta}} d\Omega(\omega) \right] d\mathcal{F}(\Omega) \\ & + \int_{\Omega} \lambda(\Omega) \left[ Q(\Omega)^{\frac{\rho-1}{\rho}} - \int_{\omega} q(\omega)^{\frac{\rho-1}{\rho}} d\Omega(\omega) \right] d\mathcal{F}(\Omega)\end{aligned}$$

The first-order conditions with respect to  $Q(\Omega)$  and  $q(\omega)$  are given by the following:

$$U'(Q(\Omega)) + \lambda(\Omega) \left( \frac{\rho-1}{\rho} \right) Q(\Omega)^{-\frac{1}{\rho}} = 0 \quad \text{for almost all } \Omega \quad (46)$$

$$\int_{\Omega} \left[ -\frac{1}{\theta} e^{-\frac{\epsilon}{\theta} a} q(\omega)^{\frac{\epsilon}{\theta}-1} - \lambda(\Omega) \left( \frac{\rho-1}{\rho} \right) q(\omega)^{-\frac{1}{\rho}} \right] dP(\Omega|\omega) = 0 \quad \text{for almost all } \omega \quad (47)$$

where  $P(\Omega|\omega)$  denotes the posterior about  $\Omega$  (or, equivalently, about  $\bar{a}$  and  $y$ ) given  $\omega$ . Restating condition (46) as  $\lambda(\Omega) \left( \frac{\rho-1}{\rho} \right) = -U'(Q(\Omega)) Q(\Omega)^{\frac{1}{\rho}}$  and substituting into condition (47) gives condition (9). Assuming log-normality of  $Q(\Omega)$  in equation (9) and letting

$$\zeta' \equiv \frac{1}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} \left[ \log \theta + \frac{1}{2} \left( \frac{1}{\rho} - \gamma \right)^2 \text{Var}[\log Q(\Omega)] \right]$$

gives condition (11). That  $\zeta' > \zeta$  then follows from part (ii).

*Part (ii).* This part follows from similar steps as in the proof of part (ii) of Proposition 1. In particular, note that the only (potential) difference between condition (11) and its equilibrium counterpart, namely condition (5), is the constant  $\zeta'$ . It follows that the efficient coefficients  $(\varphi_0^*, \varphi_a^*, \varphi_x^*, \varphi_y^*)$  must solve the same system as their equilibrium counterparts, replacing  $\zeta$  with  $\zeta'$ . It is then immediate that  $(\varphi_a^*, \varphi_x^*, \varphi_y^*) = (\varphi_a, \varphi_x, \varphi_y)$ . But then  $\text{Var}[\log Q]$  is also the same in the equilibrium and the efficient allocation, from which it follows that  $\zeta' = \zeta - \frac{1}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} \log \frac{\rho-1}{\rho} > \zeta$  (since  $\frac{\rho-1}{\rho} < 1$ ) and hence  $\varphi_0^* > \varphi_0$ .

**Proof of Theorem 1 (efficient business cycle).** This follows from Propositions 1 and 2.

**Proof of Lemma 1 (precisions of endogenous signals).** *Parts (i) and (ii).* Take a log-linear strategy of the form  $q(a, x, y) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_y y$ , for arbitrary coefficients  $(\varphi_0, \varphi_a, \varphi_x, \varphi_y)$ . The endogenous public signal is then given by

$$y^q = \log Q(\Omega) + \varepsilon^{yq}$$

where

$$\log Q(\Omega) = \varphi_0' + \varphi_a a + \varphi_x x + \varphi_y y$$

is the log of aggregate output. It follows that the signal  $y^q$  can be transformed into an unbiased Gaussian signal  $\tilde{y}^q$  about aggregate productivity, defined as follows:

$$\tilde{y}^q \equiv \frac{y^q - \varphi'_0 - \varphi_y y}{\varphi_a + \varphi_x} = \bar{a} + \tilde{\varepsilon}^{yq} \quad (48)$$

where  $\tilde{\varepsilon}^{yq} \equiv \varepsilon^{yq}/(\varphi_a + \varphi_x)$ . The precision of this signal is

$$\kappa_{yq} \equiv \frac{1}{Var(\tilde{\varepsilon}^{yq})} = (\varphi_a + \varphi_x)^2 \sigma_{yq}^{-2}.$$

Standard Bayesian updating then implies that the sufficient statistic  $y$  of available public information is given by a weighted average of the exogenous productivity signal  $y^a$  and the (normalized) endogenous output signal  $\tilde{y}^q$ :

$$y = \frac{\kappa_{ya}}{\kappa_y} y^a + \frac{\kappa_{yq}}{\kappa_y} \tilde{y}^q,$$

where  $\kappa_{ya}$  and  $\kappa_{yq}$  are the precisions of these two signals, while  $\kappa_y = \kappa_{ya} + \kappa_{yq}$  is the overall precision of the sufficient statistic  $y$ .

The analysis of the private signal  $x^q$  is similar: it can be transformed to an unbiased signal with precision  $\kappa_{xq} = (\varphi_a + \varphi_x)^2 \sigma_{xq}^{-2}$ .

**Proof of Proposition 3 (equilibrium with learning).** The characterization of the equilibrium in part (i) follows from combining the characterization of the equilibrium strategy in part (ii) of Proposition 1 with the characterization of the equilibrium precision in Lemma 1. What remains is to study the existence and determinacy of the entire equilibrium, that is, of the fixed point between the equilibrium strategy and the equilibrium precisions. Let  $\bar{\varphi} \equiv \varphi_a + \varphi_x$ . From conditions (14) and (15), we get  $\bar{\varphi}$  as a function of  $\kappa_x$  and  $\kappa_y$ , while from (17) and (18) we get  $\kappa_x$  and  $\kappa_y$  as functions of  $\bar{\varphi}$ . We can thus reduce the aforementioned fixed-point relation between the equilibrium strategy and the equilibrium precisions to the following simple fixed-point problem for  $\bar{\varphi}$ :

$$\bar{\varphi} = F(\bar{\varphi})$$

where

$$F(\bar{\varphi}) \equiv \beta \frac{(\kappa_A + \kappa_\xi + \kappa_{ya}) + (\sigma_{xq}^{-2} + \sigma_{yq}^{-2}) \bar{\varphi}^2}{(\kappa_A + (1 - \alpha)\kappa_\xi + \kappa_{ya}) + ((1 - \alpha)\sigma_{xq}^{-2} + \sigma_{yq}^{-2}) \bar{\varphi}^2}.$$

Note that  $F$  takes only positive values, is continuous, and satisfies  $F(0) > 0$  and  $\lim_{\bar{\varphi} \rightarrow \infty} F(\bar{\varphi}) < \infty$  (which also means that it is bounded). It follows that there always exists a fixed point and any such fixed point is positive. Moreover, because we can always rewrite  $\bar{\varphi} = F(\bar{\varphi})$  as a cubic, we know there can exist at most three solutions. Finally, note that

$$F'(\bar{\varphi}) \equiv -\beta \frac{2\bar{\varphi}\alpha(\kappa_A + \kappa_{ya})\kappa_\xi \left( \frac{\sigma_{yq}^{-2}}{\kappa_A + \kappa_{ya}} - \frac{\sigma_{xq}^{-2}}{\kappa_\xi} \right)}{\left( (\kappa_A + (1 - \alpha)\kappa_\xi + \kappa_{ya}) + ((1 - \alpha)\sigma_{xq}^{-2} + \sigma_{yq}^{-2}) \bar{\varphi}^2 \right)^2},$$

so that  $\alpha > 0$  and  $\frac{\sigma_{yq}^{-2}}{\kappa_A + \sigma_a^{-2}} > \frac{\sigma_{xq}^{-2}}{\sigma_\xi^{-2}}$  suffice for  $F$  to be decreasing, and hence also for the fixed point to be unique.

**Proof of Proposition 4 (efficiency with learning).** *Part (i).* Take an arbitrary log-linear strategy of the form  $q(a, x, y) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_y y$ . For any coefficients  $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_y)$  and any precisions  $\kappa = (\kappa_x, \kappa_y)$ , the implied level of welfare (ex-ante utility) can be expressed as follows:

$$\mathbb{E}u = \mathcal{W}(\varphi; \kappa) \equiv \frac{1}{1-\gamma} \exp V_c(\varphi; \kappa) - \frac{1}{\epsilon} \exp V_n(\varphi; \kappa),$$

where

$$\begin{aligned} V_c(\varphi; \kappa) &\equiv (1-\gamma)(\varphi_0 + (\varphi_a + \varphi_x + \varphi_y)\mu) \\ &\quad + \frac{1}{2}(1-\gamma) \left( \frac{\rho-1}{\rho} \right) \left[ \frac{\varphi_a^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2 \frac{\varphi_a \varphi_x}{\kappa_x} \right] \\ &\quad + \frac{1}{2}(1-\gamma)^2 \left[ \frac{\varphi_y^2}{\kappa_y} + \frac{(\varphi_a + \varphi_x + \varphi_y)^2}{\kappa_A} \right] \\ V_n(\varphi; \kappa) &\equiv \frac{\epsilon}{\theta} (\varphi_0 + (\varphi_a + \varphi_x + \varphi_y - 1)\mu) \\ &\quad + \frac{1}{2} \frac{\epsilon^2}{\theta^2} \left[ \frac{(\varphi_a - 1)^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2 \frac{(\varphi_a - 1)\varphi_x}{\kappa_x} + \frac{\varphi_y^2}{\kappa_y} + \frac{(\varphi_a + \varphi_x + \varphi_y - 1)^2}{\kappa_A} \right] \end{aligned}$$

Recall from Lemma 1 that any given strategy induces a  $\kappa_x$  and a  $\kappa_y$  as functions of  $\varphi_a + \varphi_x$ ; let  $\kappa_x(\varphi_a + \varphi_x)$  and  $\kappa_y(\varphi_a + \varphi_x)$  denotes these functions. We can then express the planner's problem as follows:

**Planner's problem.** Choose  $\varphi = (\varphi_0, \varphi_a, \varphi_x, \varphi_y)$  and  $(\kappa_x, \kappa_y)$  so as to maximize  $\mathcal{W}(\varphi, \kappa)$  subject to  $\kappa_x = \kappa_x(\varphi_a + \varphi_x)$  and  $\kappa_y = \kappa_y(\varphi_a + \varphi_x)$ .

To solve this problem, we proceed in two steps. The first step is to characterize the strategy that is optimal subject to the constraint that the sum  $\varphi_a + \varphi_x$  is kept constant at some  $\bar{\varphi} \in \mathbb{R}$  and accordingly the precisions  $\kappa_x$  and  $\kappa_y$  are kept constant at  $\kappa_x = \kappa_x(\bar{\varphi})$  and  $\kappa_y = \kappa_y(\bar{\varphi})$ . The second step is to optimize over the sum  $\bar{\varphi}$  and the precision  $\kappa_x$  and  $\kappa_y$  subject to the constraint that  $\kappa_x = \kappa_x(\bar{\varphi})$  and  $\kappa_y = \kappa_y(\bar{\varphi})$ . The first step permits us to characterize the efficient allocation as a function of the Lagrange multiplier associated with the constraint  $\varphi_a + \varphi_x = \bar{\varphi}$ . The second step permits us to interpret this Lagrange multiplier as the shadow value of the informational externality, as well as to prove the existence of an efficient allocation and to complete its characterization by showing that this multiplier is strictly positive.

Thus consider the first step. Fix some  $\bar{\varphi} \in \mathbb{R}$ , let  $\kappa = (\kappa_x(\bar{\varphi}), \kappa_y(\bar{\varphi}))$ , and consider the following constrained problem:

**Auxiliary problem 1.** Choose  $\varphi$  so as to maximize  $\mathcal{W}(\varphi, \kappa)$  subject to  $\varphi_a + \varphi_x = \bar{\varphi}$ .

Note that  $\mathcal{W}$  is differentiable in  $\varphi$  for fixed  $\kappa$ . Let  $\tilde{\eta}$  denote the Lagrange multiplier for the

constraint  $\varphi_a + \varphi_x = \bar{\varphi}$ . The first-order conditions for this problem are then the following:

$$\begin{aligned}\varphi_0 & : 0 = \frac{\partial \mathcal{W}}{\partial \varphi_0} \\ \varphi_a & : 0 = \frac{\partial \mathcal{W}}{\partial \varphi_a} + \tilde{\eta} \\ \varphi_x & : 0 = \frac{\partial \mathcal{W}}{\partial \varphi_x} + \tilde{\eta} \\ \varphi_y & : 0 = \frac{\partial \mathcal{W}}{\partial \varphi_y}\end{aligned}$$

Using the characterization of  $\mathcal{W}$ , the first of these conditions reduces to the following:

$$\varphi_0 : 0 = \exp V_c(\varphi, \kappa) - \frac{1}{\theta} \exp V_n(\varphi, \kappa).$$

This guarantees that  $V_c = V_n - \log \theta$  at the efficient allocation and gives  $\varphi_0$  as a function of  $\varphi_a, \varphi_x, \varphi_y, \kappa_x, \kappa_y$  and exogenous parameters. Let  $V \equiv V_c = V_n - \log \theta$  and let  $\eta \equiv e^{-V} \tilde{\eta}$ . The rest of the first-order conditions reduce to the following:

$$\begin{aligned}\varphi_a : 0 &= \left( \frac{\rho-1}{\rho} \right) \frac{\varphi_a}{\kappa_\xi} + \left( \frac{\rho-1}{\rho} \right) \frac{\varphi_x}{\kappa_x} + (1-\gamma) \frac{(\varphi_a + \varphi_x + \varphi_y)}{\kappa_A} \\ &\quad - \frac{\epsilon}{\theta} \frac{(\varphi_a - 1)}{\kappa_\xi} - \frac{\epsilon}{\theta} \frac{\varphi_x}{\kappa_x} - \frac{\epsilon}{\theta} \frac{(\varphi_a + \varphi_x + \varphi_y - 1)}{\kappa_A} + \eta \\ \varphi_x : 0 &= \left( \frac{\rho-1}{\rho} \right) \frac{\varphi_x}{\kappa_x} + \left( \frac{\rho-1}{\rho} \right) \frac{\varphi_a}{\kappa_x} + (1-\gamma) \frac{(\varphi_a + \varphi_x + \varphi_y)}{\kappa_A} \\ &\quad - \frac{\epsilon}{\theta} \frac{\varphi_x}{\kappa_x} - \frac{\epsilon}{\theta} \frac{(\varphi_a - 1)}{\kappa_x} - \frac{\epsilon}{\theta} \frac{(\varphi_a + \varphi_x + \varphi_y - 1)}{\kappa_A} + \eta \\ \varphi_y : 0 &= (1-\gamma) \frac{\varphi_y}{\kappa_y} + (1-\gamma) \frac{(\varphi_a + \varphi_x + \varphi_y)}{\kappa_A} - \frac{\epsilon}{\theta} \frac{\varphi_y}{\kappa_y} - \frac{\epsilon}{\theta} \frac{(\varphi_a + \varphi_x + \varphi_y - 1)}{\kappa_A}\end{aligned}$$

For fixed  $\eta$ , this is a linear system of three equations in the three coefficients  $\varphi_a, \varphi_x$  and  $\varphi_y$ . Subtracting the first equation from the second, we obtain

$$\varphi_a^{**} = \frac{\frac{\epsilon}{\theta}}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} \equiv \beta.$$

We can then solve the remaining two equations for  $\varphi_x$  and  $\varphi_y$  as follows:

$$\begin{aligned}\varphi_x^{**} &= \left\{ \frac{(1-\alpha)\kappa_x}{(1-\alpha)\kappa_x + \kappa_y + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta + \frac{1}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} \left\{ \frac{\kappa_x(\kappa_y + \kappa_A)}{(1-\alpha)\kappa_x + \kappa_y + \kappa_A} \right\} \eta \\ \varphi_y^{**} &= \left\{ \frac{\kappa_y}{(1-\alpha)\kappa_x + \kappa_y + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta - \frac{1}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} \left\{ \frac{\kappa_x \kappa_y}{(1-\alpha)\kappa_x + \kappa_y + \kappa_A} \right\} \eta\end{aligned}$$

Letting

$$\Delta \equiv \frac{1}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} \left( \frac{\kappa_x(\kappa_y + \kappa_A)}{(1-\alpha)\kappa_x + \kappa_y + \kappa_A} \right) \eta,$$

gives conditions (19) and (20). Finally, note that  $\Delta$  is just a rescaling of the Lagrange multiplier  $\eta$ , so we can think of  $\Delta$  itself as the relevant Lagrange multiplier. Using then the above results along with the constraint  $\varphi_a + \varphi_x = \bar{\varphi}$ , we can express  $\Delta$  (or equivalently  $\eta$ ) as follows:

$$\Delta = \bar{\varphi} - \left\{ \frac{\kappa_x + \kappa_y + \kappa_A}{(1 - \alpha)\kappa_x + \kappa_y + \kappa_A} \right\} \beta. \quad (49)$$

Using this into conditions (19) and (20), we can obtain the optimal coefficients as functions of the sum  $\bar{\varphi}$  and the precisions  $\kappa_x$  and  $\kappa_y$ . Let  $\varphi(\bar{\varphi}, \kappa)$  denote this solution; for the rest of this proof, whenever we write  $\varphi$ , we mean  $\varphi = \varphi(\bar{\varphi}, \kappa)$ .

We can then express the level of welfare obtained at this solution also as function of the sum  $\bar{\varphi}$  and the precisions  $\kappa_x$  and  $\kappa_y$ . In particular, using the FOC with respect to  $\varphi_0$ , we get that

$$\mathcal{W}(\varphi, \kappa) = \left( \frac{\frac{\epsilon}{\theta} - 1 + \gamma}{1 - \gamma} \frac{\theta}{\epsilon} \right) \exp V_c(\varphi, \kappa) \quad (50)$$

Since  $\frac{\epsilon}{\theta} - 1 + \gamma > 0$  and  $\frac{\epsilon}{\theta} > 0$ , we can consider the following monotone transformation of welfare:

$$\mathcal{TW}(\varphi, \kappa) \equiv \frac{1}{1 - \gamma} V_c(\varphi, \kappa).$$

Using then the characterization of the efficient coefficients, we conclude that

$$\mathcal{TW}(\varphi(\bar{\varphi}, \kappa), \kappa) = W(\bar{\varphi}, \kappa) \equiv A(\kappa) - B(\kappa) (\bar{\varphi} - f(\kappa))^2 \quad (51)$$

where

$$B(\kappa) \equiv \frac{\epsilon}{2\theta(1 - \alpha)} \frac{\kappa_0 + (1 - \alpha)\kappa_x + \kappa_y}{\kappa_x(\kappa_A + \kappa_y)} > 0$$

and

$$f(\kappa) \equiv \frac{\kappa_A + \kappa_x + \kappa_y}{(1 - \alpha)\kappa_x + \kappa_y + \kappa_A} \beta = \arg \max_{\bar{\varphi}} W(\bar{\varphi}, \kappa) = \arg \max_{\bar{\varphi}} \mathcal{W}(\varphi(\bar{\varphi}, \kappa), \kappa).$$

(The precise value of  $A(\kappa)$  has no particular interest, so it is omitted.) This result has a simple interpretation. Note that  $f(\kappa)$  identifies the sum  $\bar{\varphi} = \varphi_a + \varphi_x$  that would have been efficient had information been exogenous (equivalently,  $\varphi(f(\kappa), \kappa)$  are simply the coefficients of the efficient allocation when  $\Delta = 0$ ). Hence, (51) expresses welfare as a monotone transformation of the quadratic distance between any value  $\bar{\varphi}$  that the planner may choose and the one that would have been optimal from a purely allocative perspective. Clearly, the only reason that the efficient  $\bar{\varphi}$  may differ from  $f(\kappa)$  is the informational externality.

We now proceed to the second step, namely that of optimizing over the sum  $\bar{\varphi} = \varphi_a + \varphi_x$  and the induced precisions  $\kappa_x = \kappa_x(\bar{\varphi})$  and  $\kappa_y = \kappa_y(\bar{\varphi})$ . Letting

$$\bar{W}(\bar{\varphi}) \equiv W(\bar{\varphi}, \kappa(\bar{\varphi})),$$

the planner's problem reduces to the following unidimensional problem:

**Auxiliary problem 2.** Choose  $\bar{\varphi} \in \mathbb{R}$  so as to maximize  $\bar{W}(\bar{\varphi})$ .

First, note that, because  $f(\kappa) > 0$ , it is necessarily the case that, for any given  $\kappa$ ,  $W(\bar{\varphi}, \kappa) > W(-\bar{\varphi}, \kappa)$  whenever  $\bar{\varphi} > 0$ . And because  $\kappa(\bar{\varphi}) = \kappa(-\bar{\varphi})$ , it is immediate that  $\bar{W}(\bar{\varphi}) > \bar{W}(-\bar{\varphi})$  whenever  $\bar{\varphi} > 0$ , which means that it is never optimal to choose  $\bar{\varphi} < 0$ .

Next, we can show that

$$\frac{\partial W}{\partial \kappa_y} = \frac{\epsilon}{2\theta \kappa_y^2} \varphi_y(\bar{\varphi}, \kappa)^2 = \frac{\epsilon(\beta - (1 - \alpha)\bar{\varphi})^2}{2\theta(1 - \alpha)^2(\kappa_A + \kappa_y)^2}.$$

Along with the fact that  $\kappa_y$  is a quadratic function of  $\bar{\varphi}$ , this guarantees that

$$\frac{\partial W}{\partial \kappa_y} \frac{\partial \kappa_y}{\partial \bar{\varphi}} \rightarrow 0 \quad \text{as} \quad \bar{\varphi} \rightarrow \infty.$$

In words, the social value of a marginal increase in the precision  $\kappa_y$  of public information vanishes as this precision goes to infinity. A similar result holds for private information:

$$\frac{\partial W}{\partial \kappa_x} = \frac{\epsilon}{2\theta(1 - \alpha)\kappa_x^2} \varphi_x(\bar{\varphi}, \kappa)^2 = \frac{\epsilon(\beta - \bar{\varphi})^2}{2\theta(1 - \alpha)\kappa_x^2}$$

and hence

$$\frac{\partial W}{\partial \kappa_x} \frac{\partial \kappa_x}{\partial \bar{\varphi}} \rightarrow 0 \quad \text{as} \quad \bar{\varphi} \rightarrow \infty.$$

At the same time, because

$$\frac{\partial W}{\partial \bar{\varphi}} = -2B(\kappa)(\bar{\varphi} - f(\kappa))$$

and because  $B(\kappa) \rightarrow \frac{\epsilon}{2\theta(1 - \alpha)\kappa_x} > 0$  and  $f(\kappa) \rightarrow \beta$  as  $\kappa_y \rightarrow \infty$ , we have that

$$\frac{\partial W}{\partial \bar{\varphi}} \rightarrow -\infty \quad \text{as} \quad \bar{\varphi} \rightarrow \infty.$$

Combining, we conclude that

$$\frac{\partial \bar{W}(\bar{\varphi})}{\partial \bar{\varphi}} \rightarrow -\infty \quad \text{as} \quad \bar{\varphi} \rightarrow \infty.$$

Along with the facts that  $\bar{W}(\bar{\varphi})$  is continuous in  $\bar{\varphi}$  and that it is without loss of optimality to restrict  $\bar{\varphi} \in [0, \infty)$ , this guarantees the existence of a solution to auxiliary problem 2 (and hence the existence of an efficient allocation).

Let  $\bar{\varphi}^* \geq 0$  denote any such a solution. Since  $\bar{W}$  is differentiable, this solution must satisfy  $\frac{\partial \bar{W}}{\partial \bar{\varphi}} = 0$ . Using the definition of  $\bar{W}$ , this is equivalent to

$$\frac{\partial W}{\partial \bar{\varphi}} + \frac{\partial W}{\partial \kappa_y} \frac{\partial \kappa_y}{\partial \bar{\varphi}} + \frac{\partial W}{\partial \kappa_x} \frac{\partial \kappa_x}{\partial \bar{\varphi}} = 0. \quad (52)$$

Note that the second and the third term are always non-negative. Whenever  $0 \leq \bar{\varphi} < f(\kappa)$ , the first term is strictly positive, so that the sum is also strictly positive; this rules out  $\bar{\varphi}^* \in [0, f(\kappa))$ . Moreover, when  $\bar{\varphi} = f(\kappa)$ , the first term is zero, but now the other two terms are strictly positive, so that the sum is also strictly positive; this rules out  $\bar{\varphi}^* = f(\kappa)$ . It follows that  $\bar{\varphi}^* > f(\kappa)$  necessarily.

From (49) and the definition of  $f(\kappa)$ , we have that, at the efficient allocation,  $\Delta = \bar{\varphi}^* - f(\kappa)$ . It follows that  $\Delta > 0$ , as claimed in the proposition.

Finally, that  $\Delta$  (or equivalently  $\eta$ ) represents the shadow value of the informational externality follows directly from the envelope condition of auxiliary problem 1, namely  $\frac{\partial W}{\partial \bar{\varphi}} = -\eta$ , along with the first-order condition of auxiliary problem 2, namely condition (52). Indeed, combining these two conditions gives

$$\eta = \frac{\partial W}{\partial \kappa_y} \frac{\partial \kappa_y}{\partial \bar{\varphi}} + \frac{\partial W}{\partial \kappa_x} \frac{\partial \kappa_x}{\partial \bar{\varphi}},$$

which means that the Lagrange multiplier measures the social value of increasing the precision of available public information by increasing the sensitivity of allocations to local information.

**Proof of Theorem 2 (inefficient business cycle).** Follows directly from Propositions 3 and 4, in particular from the property that  $\Delta > 0$ .

**Proof of Lemma 2 (tax wedge).** We consider a combination of the following tax instruments: a linear tax  $\tau^R(\Omega)$  on firm revenue, a linear tax  $\tau^L(\Omega)$  on household labor income, and a linear tax  $\tau^C(\Omega)$  on household consumption (a sales tax that is uniform across commodities). To guarantee the existence of an equilibrium where the allocations are log-normal, these taxes are assumed to be log-linear functions of the  $(\bar{a}, Q, y)$ :

$$\begin{aligned} -\log(1 - \tau^R(\Omega)) &= \tau_0^R + \tau_A^R \bar{a} + \tau_Q^R \log Q(\Omega) + \tau_y^R y, \\ -\log(1 - \tau^L(\Omega)) &= \tau_0^L + \tau_A^L \bar{a} + \tau_Q^L \log Q(\Omega) + \tau_y^L y, \\ \log(1 + \tau^C(\Omega)) &= \tau_0^C + \tau_A^C \bar{a} + \tau_Q^C \log Q(\Omega) + \tau_y^C y. \end{aligned}$$

Given these taxes, the firm's realized net-of-tax profits are given by

$$\pi(\omega, \Omega) = (1 - \tau^R(\Omega)) p(\omega, \Omega) q(\omega) - w(\omega) n(\omega),$$

while the budget constraint of the household is given by

$$(1 + \tau^C(\Omega)) \int p(\omega, \Omega) c(\omega) d\Omega(\omega) = \int \pi(\omega, \Omega) d\Omega(\omega) + (1 - \tau^L(\Omega)) \int w(\omega) n(\omega) d\Omega(\omega) + T(\Omega)$$

where  $T(\Omega)$  is a lump-sum transfer or tax. (By the government budget, the latter is equal to the revenue from all the taxes.) It follows that the optimal labor supply of the typical worker on island  $\omega$  is given by

$$n(\omega)^{\epsilon-1} = w(\omega) \mathbb{E} \left[ (1 - \tau^L(\Omega)) \frac{U'(C(\Omega))}{(1 + \tau^C(\Omega)) P(\Omega)} \middle| \omega \right],$$

while the consumer's stochastic discount factor is given by  $\frac{U'(Q(\Omega))}{(1 + \tau^C(\Omega)) P(\Omega)}$ . The firm's objective is thus given by

$$\mathbb{E} \left[ \frac{U'(Q(\Omega))}{(1 + \tau^C(\Omega)) P(\Omega)} \left( (1 - \tau^R(\Omega)) P(\Omega) Q(\Omega)^{1/\rho} q(\omega)^{1-1/\rho} - w(\omega) n(\omega) \right) \middle| \omega \right].$$

Taking the FOC for the firm's problem, substituting the equilibrium wage, and guessing that the taxes and the allocations are jointly log-normal (which they are in the equilibrium we construct in the main text), we conclude that the equilibrium level of employment is pinned down by the following condition:

$$n(\omega)^{\epsilon-1} = \left( \frac{\rho-1}{\rho} \right) \mathbb{E} \left[ \frac{\chi (1 - \tau^R(\Omega)) (1 - \tau^L(\Omega))}{1 + \tau^C(\Omega)} U'(Q(\Omega)) \left( \frac{q(\omega)}{Q(\Omega)} \right)^{-\frac{1}{\rho}} \left( \theta A(\omega) n(\omega)^{\theta-1} \right) \middle| \omega \right].$$

where  $\chi$  is a constant that depends on second-order terms. The result then follows by defining the tax wedge as

$$1 - \tau(\Omega) \equiv \frac{\chi (1 - \tau^R(\Omega)) (1 - \tau^L(\Omega))}{1 + \tau^C(\Omega)}.$$

Equivalently, the tax wedge is given by (22) with  $\tau_0 \equiv -\log \chi + \tau_0^R + \tau_0^C + \tau_0^L$ ,  $\tau_A \equiv \tau_A^R + \tau_A^C + \tau_A^L$ ,  $\tau_Q \equiv \tau_Q^R + \tau_Q^C + \tau_Q^L$ , and  $\tau_y \equiv \tau_y^R + \tau_y^C + \tau_y^L$ .

**Proof of Proposition 5 (implementable strategies).** Following the same steps as in Section 4.1, we can show that, given the information structure, the equilibrium strategy must solve the following fixed point:

$$q(\omega)^{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} = \left( \frac{\rho-1}{\rho} \right) \theta A(\omega)^{\frac{\epsilon}{\theta}} \mathbb{E} \left[ \exp(-\tau_0 - \tau_A \bar{a} - \tau_y y) Q(\Omega)^{\frac{1}{\rho} - \gamma - \tau_Q} \middle| \omega \right] \quad (53)$$

with  $Q(\Omega) = \left[ \int q(\omega)^{\frac{\rho-1}{\rho}} d\Omega(\omega) \right]^{\frac{\rho}{\rho-1}}$ . It follows that the equilibrium strategy is given by

$$\log q(\omega) = \hat{\varphi}_0(\tau) + \hat{\varphi}_a(\tau) a + \hat{\varphi}_x(\tau) x + \hat{\varphi}_y(\tau) y \quad (54)$$

where

$$\hat{\varphi}_a(\tau) = \beta \quad (55)$$

$$\hat{\varphi}_x(\tau) = \left( 1 - \frac{\theta}{\epsilon \hat{\alpha}} \tau_A \right) \left( \frac{(1 - \hat{\alpha}) \kappa_x}{(1 - \hat{\alpha}) \kappa_x + \kappa_y + \kappa_A} \right) \frac{\hat{\alpha}}{1 - \hat{\alpha}} \beta \quad (56)$$

$$\hat{\varphi}_y(\tau) = \frac{1}{1 - \hat{\alpha}} \left( \frac{\kappa_y}{\kappa_x} \hat{\varphi}_x(\tau) - \frac{\beta \theta}{\epsilon} \tau_y \right) \quad (57)$$

$$\hat{\varphi}_0(\tau) = \frac{1}{\frac{\epsilon}{\theta} + \gamma - 1 + \tau_Q} \left[ -\tau_0 + \left( -\tau_A + \left( \frac{1}{\rho} - \gamma - \tau_Q \right) (\hat{\varphi}_a + \hat{\varphi}_x) \right) \frac{\kappa_A}{\kappa_A + \kappa_x + \kappa_y} \mu \right] \quad (58)$$

$$+ \left( \frac{1}{\rho} - \gamma - \tau_Q \right) \left( \frac{\rho-1}{\rho} \right) \frac{(\hat{\varphi}_a + \hat{\varphi}_x)^2}{2} \sigma_x^2 + \frac{1}{2} \left( \frac{1}{\rho} - \gamma - \tau_Q \right)^2 (\hat{\varphi}_a + \hat{\varphi}_x)^2 \sigma_0^2 \quad (59)$$

$$+ \frac{1}{2} \tau_A^2 \sigma_0^2 - \tau_A \left( \frac{1}{\rho} - \gamma - \tau_Q \right) (\hat{\varphi}_a + \hat{\varphi}_x) \sigma_0^2 + \log \left( \theta \frac{\rho-1}{\rho} \right) \quad (60)$$

and where

$$\hat{\alpha} = \hat{\alpha}(\tau_Q) \equiv \frac{\frac{1}{\rho} - \gamma - \tau_Q}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} = \alpha - \frac{\tau_Q}{\frac{\epsilon}{\theta} + \frac{1}{\rho} - 1} \quad (61)$$

represents the equilibrium degree of complementarity induced by the policy.

We now prove the claims in the lemma. Pick an arbitrary strategy for which  $\varphi_a = \beta$  and let  $(\varphi_0^\#, \varphi_x^\#, \varphi_y^\#)$  denote the remaining coefficients. Set  $\tau_Q = 0$ , in which case  $\hat{\alpha} = \alpha$ . From condition (56), there is a unique value for  $\tau_A$  that induces  $\hat{\varphi}_x(\tau) = \varphi_x^\#$ ; this is given by

$$\tau_A = \frac{\epsilon}{\theta} \left\{ \alpha - \varphi_x^\# \frac{(1 - \alpha)\kappa_x + \kappa_y + \kappa_A}{\beta\kappa_x} \right\}. \quad (62)$$

From (57), there is then a unique value for  $\tau_y$  that induces  $\hat{\varphi}_y(\tau) = \varphi_y^\#$ ; this is given by

$$\tau_y = \frac{\epsilon}{\beta\theta} \left\{ \frac{\kappa_y}{\kappa_x} \varphi_x^\# - (1 - \alpha)\varphi_y^\# \right\}. \quad (63)$$

Combining these two results, we have a unique pair  $(\tau_A, \tau_y)$  that induces the desired  $(\varphi_x^\#, \varphi_y^\#)$ . But then from (58) there is also a unique  $\tau_0$  that induces  $\hat{\varphi}_0(\tau) = \varphi_0^\#$ . This proves the desired strategy can always be implemented with a policy that allows  $\tau_y \neq 0$ .

Now restrict  $\tau_y = 0$  and suppose  $\varphi_x^\#, \varphi_y^\# \neq 0$ . From condition (61), there always exists a unique  $\tau_Q$  such that

$$(1 - \hat{\alpha}(\tau_Q)) \frac{\kappa_x}{\kappa_y} = \frac{\varphi_x^\#}{\varphi_y^\#}$$

and hence such that  $\hat{\varphi}_x(\tau) / \hat{\varphi}_y(\tau) = \varphi_x^\# / \varphi_y^\#$ . Given this  $\tau_Q$ , condition (56) gives a unique  $\tau_A$  such that  $\hat{\varphi}_x(\tau) = \varphi_x^\#$ , which along with previous result gives also  $\hat{\varphi}_y(\tau) = \varphi_y^\#$ . Finally, given these values for  $(\tau_Q, \tau_A)$ , there is then a unique  $\tau_0$  that ensures  $\hat{\varphi}_0(\tau) = \varphi_0^\#$ . This proves that, as long as  $\varphi_x^\#, \varphi_y^\# \neq 0$ , the policy can be contingent only on aggregate productivity and aggregate output.

**Proof of Theorem 3 (optimal tax).** That the efficient allocation can always be implemented follows directly from Lemma 2. Part (i) is obvious; thus consider part (ii). Without any loss of generality, set  $\tau_Q = 0$  and express the tax as a function of  $(\bar{a}, y)$  alone. From conditions (62) and (63) in the proof of Proposition 5, we have that the optimal tax satisfies

$$\begin{aligned} \tau_A^* &= \frac{\epsilon}{\theta} \left( \alpha - \varphi_x^* \frac{(1 - \alpha)\kappa_x + \kappa_y + \kappa_A}{\beta\kappa_x} \right) \\ \tau_y^* &= \frac{\epsilon}{\beta\theta} \left( \frac{\kappa_y}{\kappa_x} \varphi_x^* - (1 - \alpha)\varphi_y^* \right) \end{aligned}$$

Using the characterization of  $\varphi_x^*$  and  $\varphi_y^*$  from Proposition 4, we get

$$\tau_A^* = -\lambda\Delta \quad \text{and} \quad \tau_y^* = \frac{\kappa_y}{\kappa_A + \kappa_y} \lambda\Delta, \quad (64)$$

where

$$\lambda \equiv \frac{\epsilon}{\beta\theta} \frac{(1 - \alpha)\kappa_x + \kappa_y + \kappa_A}{\kappa_x} > 0.$$

It follows that  $\Delta > 0$  is both necessary and sufficient for every one of the following properties:  $\tau_A^* < 0$ ,  $\tau_A^* + \tau_y^* < 0$  and  $\tau_y^* > 0$ .

To interpret this result, note first that  $y = \bar{a} + \varepsilon$ , where  $\varepsilon$  is noise. The property that  $\tau_A < 0$  means that tax is negatively correlated with aggregate productivity for given common belief  $y$ ; that is, it is negatively correlated with the surprise component in realized aggregate productivity. At the same time, the property that  $\tau_a + \tau_y < 0$  means that the tax is negatively correlated with aggregate productivity for given noise  $\varepsilon$ ; that is, the overall effect of the productivity shock is also negative. Next, the property that  $\tau_y > 0$  means that the tax is positively correlated with the noise. Finally, to understand the overall cyclical behavior of the optimal tax, consider the covariance between the (log) tax and (log) output. Since

$$-\log(1 - \tau(\Omega)) = \tau_0^* + (\tau_A^* + \tau_y^*)\bar{a} + \tau_y^*\varepsilon \quad \text{and} \quad \log Q(\Omega) = \varphi_0^* + (\varphi_a^* + \varphi_x^* + \varphi_y^*)\bar{a} + \varphi_y^*\varepsilon,$$

their covariance is given by

$$Cov(-\log(1 - \tau), \log Q) = (\tau_A^* + \tau_y^*)(\varphi_a^* + \varphi_x^* + \varphi_y^*)Var(\bar{a}) + \tau_y^*\varphi_y^*Var(\varepsilon)$$

Using the fact that  $Var(\bar{a}) = 1/\kappa_A$  and  $Var(\varepsilon) = 1/\kappa_y$  and rearranging, we get

$$Cov(-\log(1 - \tau), \log Q) = (\tau_A^* + \tau_y^*)(\varphi_a^* + \varphi_x^*)\frac{1}{\kappa_A} + \left\{ \tau_A^*\frac{1}{\kappa_A} + \tau_y^*\frac{\kappa_0 + \kappa_y}{\kappa_A\kappa_y} \right\} \varphi_y^*.$$

By (64), the last term is necessarily zero. Next, note that  $\varphi_a^* + \varphi_x^*$  is necessarily positive, while  $\tau_A^* + \tau_y^*$  is necessarily negative. We conclude that the tax is negatively correlated with aggregate output.

**Proof of Theorem 1' (optimal policies with exogenous information).** This follows directly from the discussion in the main text.

**Proof of Lemma 3 (precision of price signal).** *Part (i).* From the consumer's optimal demand, we have that the (shadow) prices must satisfy

$$-\rho(\log p(\omega) - \log P(\Omega)) = (\log c(\omega, \Omega) - \log C(\Omega))$$

where

$$\begin{aligned} \log p(\omega) &= \text{const} + \psi_a a + \psi_x x + \psi_y y \\ \log P(\Omega) &= \text{const} + \psi_a \bar{a} + \psi_x \bar{x} + \psi_y y \\ \log c(\omega, \Omega) &= \text{const} + (\varphi_a + \theta_2 l_a) a + (\varphi_x + \theta_2 l_x) x + (\varphi_y + \theta_2 l_y) y + \theta_2 l_A \bar{a} \\ \log C(\Omega) &= \text{const} + (\varphi_a + \theta_2 l_a) \bar{a} + (\varphi_x + \theta_2 l_x) \bar{x} + (\varphi_y + \theta_2 l_y) y + \theta_2 l_A \bar{a} \end{aligned}$$

It follows that the following must hold for all  $(a, x, y, \bar{a})$ :

$$-\rho(\psi_a a + \psi_x x - (\psi_a + \psi_x)\bar{a}) = (\varphi_a + \theta_2 l_a) a + (\varphi_x + \theta_2 l_x) x - (\varphi_a + \theta_2 l_a + \varphi_x + \theta_2 l_x)\bar{a},$$

which is true if and only if

$$\psi_a = -\frac{1}{\rho}(\varphi_a + \theta_2 l_a) \quad (65)$$

$$\psi_x = -\frac{1}{\rho}(\varphi_x + \theta_2 l_x) \quad (66)$$

Finally, note that the observation of  $y^p = \log P(\Omega) + \varepsilon^p$  is equivalent to the observation of the unbiased Gaussian signal

$$\tilde{y}^p \equiv \frac{y^p - \text{const} - \psi_y y}{\psi_a + \psi_x} = \bar{a} + \tilde{\varepsilon}_p,$$

where  $\tilde{\varepsilon}_p = \varepsilon_p / (\psi_a + \psi_x)$ . We conclude that the precision of the price signal is given by

$$\kappa_{yp} = (\psi_a + \psi_x)^2 \sigma_p^{-2} = \frac{1}{\rho^2}(\varphi_a + \varphi_x + \theta_2(l_a + l_x))^2 \sigma_p^{-2},$$

which together with the fact that  $\kappa_y = \sigma_a^{-2} + \kappa_{yq} + \kappa_{yp}$  and the characterization of  $\kappa_{yq}$  from Lemma 1 gives the result.

*Part (ii).* This is immediate from condition (28).

**Proof of Proposition 6 (set of implementable allocations).** In equilibrium,  $l(\omega, \Omega)$  adjusts in stage 2 so as to satisfy the the consumer's demand:

$$\frac{p(\omega)}{P(\Omega)} = \left( \frac{q(\omega) l(\omega, \Omega)^{\theta_2}}{C(\Omega)} \right)^{-\frac{1}{\rho}} \quad (67)$$

Solving for  $l(\omega, \Omega)$  and substituting into the firm's objective, the latter reduces to the following:

$$\mathbb{E} \left[ \frac{U'(C(\Omega))}{P(\Omega)} \left( (1 - \tau(\Omega)) C(\Omega) p^{1-\rho} P(\Omega)^\rho - w_2(\omega) \left( \frac{p}{P(\Omega)} \right)^{-\frac{\rho}{\theta_2}} \left( \frac{C(\Omega)}{q} \right)^{\frac{1}{\theta_2}} - w_1(\omega) \left( \frac{q}{e^a} \right)^{\frac{1}{\theta_1}} \right) \middle| \omega \right].$$

Note that this objective is strictly concave in  $p^{1-\rho}$  and  $q^{1/\theta_1}$ , which guarantees that the FOCs are both necessary and sufficient and that they uniquely pin down the solution to the firm's problem for given wages. Next, note that the equilibrium wages satisfy

$$n(\omega)^{\epsilon_1-1} = w_1(\omega) \mathbb{E} \left[ \frac{U'(C(\Omega))}{P(\Omega)} \middle| \omega \right] \quad \text{and} \quad l(\omega)^{\epsilon_2-1} = w_2(\omega) \mathbb{E} \left[ \frac{U'(C(\Omega))}{P(\Omega)} \middle| \omega \right]$$

Solving these conditions for  $w_1(\omega)$  and  $w_2(\omega)$  and substituting the solutions into the first-order conditions for the firm's problem gives us the following two conditions for the equilibrium price and production choices taken in stage 1:

$$p(\omega)^{1-\rho+\frac{\rho\epsilon_2}{\theta_2}} = \frac{\mathbb{E} \left[ P(\Omega)^{\frac{\rho\epsilon_2}{\theta_2}} C(\Omega)^{\frac{\epsilon_2}{\theta_2}} q(\omega)^{-\frac{\epsilon_2}{\theta_2}} \middle| \omega \right]}{\theta_2 \mathbb{E} \left[ \left( \frac{\rho-1}{\rho} \right) (1 - \tau(\Omega)) C(\Omega)^{1-\gamma} P(\Omega)^{\rho-1} \middle| \omega \right]} \quad (68)$$

$$q(\omega)^{\frac{\epsilon_1}{\theta_1}} = p(\omega)^{1-\rho} e^{\frac{\epsilon_1}{\theta_1} a} \theta_1 \mathbb{E} \left[ \left( \frac{\rho-1}{\rho} \right) (1 - \tau(\Omega)) C(\Omega)^{1-\gamma} P(\Omega)^{\rho-1} \middle| \omega \right] \quad (69)$$

Using (67), we can restate these conditions in terms of allocations alone as follows:

$$\begin{aligned}
0 &= n(\omega)^{\epsilon_1-1} - \mathbb{E} \left[ \left( \frac{\rho-1}{\rho} \right) (1-\tau(\Omega)) U'(C(\Omega)) \left( \frac{c(\omega, \Omega)}{C(\Omega)} \right)^{-\frac{1}{\rho}} \left( \theta_1 \frac{c(\omega, \Omega)}{n(\omega)} \right) \middle| \omega \right] \\
0 &= \mathbb{E} \left[ l(\omega, \Omega) \left\{ l(\omega, \Omega)^{\epsilon_2-1} - \left( \frac{\rho-1}{\rho} \right) (1-\tau(\Omega)) U'(C(\Omega)) \left( \frac{c(\omega, \Omega)}{C(\Omega)} \right)^{-\frac{1}{\rho}} \left( \theta_2 \frac{c(\omega, \Omega)}{l(\omega, \Omega)} \right) \right\} \middle| \omega \right]
\end{aligned}$$

These conditions are similar to those that characterize the flexible-price allocations, namely conditions (24) and (25) in the main text, except for one difference: while with flexible prices the marginal costs and returns of stage-2 employment must be equated state-by-state, where they have to do only in expectation (in a sense that condition (25) makes precise).

Rearranging these conditions, we get

$$\mathbb{E} [l(\omega, \Omega)^{\epsilon_2} | \omega] = \frac{\theta_2}{\theta_1} e^{-\frac{\epsilon_1}{\theta_1} a} q(\omega)^{\frac{\epsilon_1}{\theta_1}} \quad (70)$$

$$q(\omega)^{\frac{\epsilon_1}{\theta_1} + \frac{1}{\rho} - 1} = e^{\frac{\epsilon_1}{\theta_1} a} \theta_1 \left( \frac{\rho-1}{\rho} \right) \mathbb{E} \left[ (1-\tau) C(\Omega)^{\frac{1}{\rho} - \gamma} l(\omega, \Omega)^{\theta_2 \left( \frac{\rho-1}{\rho} \right)} \middle| \omega \right] \quad (71)$$

The first condition equates the (expected) marginal rates of transformation and substitution between  $l$  and  $n$ . We conclude that a set of allocations, prices and policies constitute an equilibrium if and only if the following hold: (i) the allocations and the tax policy satisfy conditions (70) and (71) along with the resource constraint

$$C(\Omega) = \left[ \int \left( q(\omega) l(\omega)^{\theta_2} \right)^{\frac{\rho-1}{\rho}} d\Omega(\omega) \right]^{\frac{\rho}{\rho-1}}; \quad (72)$$

(ii) the nominal prices satisfy condition (67); and (iii) the monetary policy satisfies

$$M(\Omega) = P(\Omega) C(\Omega). \quad (73)$$

We now seek to translate conditions (67)-(73) in terms of the relevant coefficients that parameterize the allocations, prices and policy under a log-normal specification. Thus let

$$\begin{aligned}
\log q(\omega) &= \text{const} + \varphi_a a + \varphi_x x + \varphi_y y \\
\log l(\omega, \Omega) &= \text{const} + l_A \bar{a} + l_a a + l_x x + l_y y \\
\log C(\Omega) &= \text{const} + c_A \bar{a} + c_y y \\
\log p(\omega) &= \text{const} + \psi_a a + \psi_x x + \psi_y y \\
\log(1-\tau(\Omega)) &= \text{const} - \tau_A \bar{a} - \tau_y y \\
\log M(\Omega) &= \text{const} + \lambda_A \bar{a} + \lambda_y y
\end{aligned}$$

for some coefficients  $(\varphi_a, \varphi_x, \dots, \lambda_A, \lambda_y)$ . (To simplify the derivations, we have expressed the policies as functions of  $\bar{a}$  and  $y$ ; except for degenerate cases, can always translate these contingencies in terms of contingencies on  $\bar{a}$  and  $C$  or  $P$ .) Note that the resource constraint (72) is satisfied if and only if

$$c_A = (\varphi_a + \varphi_x) + \theta_2(l_a + l_x + l_A) \quad (74)$$

$$c_y = \varphi_y + \theta_2 l_y \quad (75)$$

while the nominal-demand condition (73) is satisfied if and only if

$$\lambda_A = c_A + (\psi_a + \psi_x) \quad (76)$$

$$\lambda_y = c_y + \psi_y \quad (77)$$

Next, we can rewrite the consumer's demand function as

$$-\rho(\log p(\omega) - \log P(\Omega)) = (\log c(\omega, \Omega) - \log C(\Omega))$$

where

$$\log c(\omega) = \log q(\omega) + \theta_2 \log l(\omega) = \text{const} + (\varphi_a + \theta_2 l_a)a + (\varphi_x + \theta_2 l_x)x + (\varphi_y + \theta_2 l_y)y + \theta_2 l_A \bar{a}$$

It follows that the following must hold for all  $(a, x, y, \bar{a})$  :

$$-\rho(\psi_a a + \psi_x x - (\psi_a + \psi_x)\bar{a}) = (\varphi_a + \theta_2 l_a)a + (\varphi_x + \theta_2 l_x)x - (\varphi_a + \theta_2 l_a + \varphi_x + \theta_2 l_x)\bar{a}.$$

This is true if and only if

$$\psi_a = -\frac{1}{\rho}(\varphi_a + \theta_2 l_a) \quad \text{and} \quad \psi_x = -\frac{1}{\rho}(\varphi_x + \theta_2 l_x)$$

Finally, note that conditions (70) and (71) may be rewritten as follows:

$$\mathbb{E}[\log l(\omega, \Omega) | \omega] = \text{const} + \frac{\epsilon_1}{\epsilon_2 \theta_1} (\log q(\omega) - a) \quad (78)$$

$$\log q(\omega) = \text{const} + \beta a - k(\tau_A \mathbb{E}[\bar{a} | \omega] + \tau_y y) + \frac{\alpha}{\chi} \mathbb{E}[\log C(\Omega) | \omega] \quad (79)$$

where

$$\beta \equiv \frac{\frac{\epsilon_1}{\theta_1}}{\frac{\epsilon_1}{\theta_1} - (\rho - 1)\nu} > 1, \quad \alpha \equiv \left(\frac{1}{\rho} - \gamma\right) \frac{\rho\nu\chi}{\frac{\epsilon_1}{\theta_1} - (\rho - 1)\nu},$$

$$\nu \equiv \frac{\epsilon_2}{\rho(\epsilon_2 - \theta_2) + \theta_2} > \frac{1}{\rho}, \quad \chi \equiv \frac{\epsilon_2}{(\epsilon_2 - \theta_2) + \gamma\theta_2} > 0, \quad k \equiv \nu\rho\frac{\theta_1}{\epsilon_1}\beta > 0.$$

Clearly, condition (78) holds for all  $\omega$  if and only if

$$l_a = \frac{\epsilon_1}{\epsilon_2 \theta_1} (\varphi_a - 1) \quad (80)$$

$$l_x = \frac{\epsilon_1}{\epsilon_2 \theta_1} \varphi_x - l_A \frac{\kappa_x}{\kappa} \quad (81)$$

$$l_y = \frac{\epsilon_1}{\epsilon_2 \theta_1} \varphi_y - l_A \frac{\kappa_y}{\kappa} \quad (82)$$

while condition (79) holds for all  $\omega$  if and only if

$$\varphi_a = \beta \quad (83)$$

$$\varphi_x = -k\tau_A \frac{\kappa_x}{\kappa} + \frac{\alpha}{\chi} c_A \frac{\kappa_x}{\kappa} \quad (84)$$

$$\varphi_y = -k\left(\tau_A \frac{\kappa_y}{\kappa} + \tau_y\right) + \frac{\alpha}{\chi} \left(c_A \frac{\kappa_y}{\kappa} + c_y\right) \quad (85)$$

where  $c_A$  and  $c_y$  are given by (74)-(75).

Note that conditions (80) through (83) give the implementability constraints stated in the proposition, completing the proof of the necessity of these conditions for an allocation to be part of an equilibrium. We next prove sufficiency.

Pick arbitrary  $(\varphi_x, \varphi_y, l_A)$  and let  $(\varphi_a, l_a, l_x, l_y)$  satisfy conditions (80) through (83). Note that there is a unique  $(\varphi_a, l_a, l_x, l_y)$  that has this property for any given  $(\varphi_x, \varphi_y, l_A)$ . Next, pick an arbitrary  $\psi_y$  and let  $(c_A, c_y, \psi_a, \psi_x)$  be determined as in (74)-(66). Next, let  $(\tau_A, \tau_y)$  be the unique solution to (84)-(85); for future reference, this solution is given by

$$\tau_A = \frac{1}{\chi k} \left\{ \alpha c_A - \chi \frac{\kappa}{\kappa_x} \varphi_x \right\} \quad (86)$$

$$\tau_y = \frac{1}{\chi k} \left\{ \alpha c_y - \chi \left( \varphi_y - \varphi_x \frac{\kappa_y}{\kappa_x} \right) \right\} \quad (87)$$

where  $\chi k > 0$ . Finally, set  $(\lambda_A, \lambda_y)$  as in (76)-(77). By construction, the allocations, prices and policies defined in this way constitute an equilibrium, which completes the sufficiency argument.

*Part (ii).* The proof of this part is similar to that of part (i), except for one key difference: now the marginal costs and returns of second-state employment must be equated state-by-state, not just on expectation. It is this additional restriction that pins down  $l_A$ . (A detailed derivation is available upon request.)

**Proof of Proposition 7 (optimal implementable allocation).** *Parts (i) and (ii).* Take any allocation in which  $\log q(\omega) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_y y$  and  $\log l(\omega) = l_0 + l_A \bar{a} + l_a a + l_x x + l_y y$ . Welfare (ex-ante utility) is then given by

$$\mathcal{W}(\varphi, l, \kappa) = \frac{1}{1 - \gamma} \exp V_c(\varphi, l, \kappa) - \frac{1}{\epsilon_2} \exp V_l(\varphi, l, \kappa) - \frac{1}{\epsilon_1} \exp V_n(\varphi, l, \kappa)$$

where  $\varphi = (\varphi_a, \varphi_x, \varphi_y)$ ,  $l = (l_a, l_x, l_y, l_A)$ , and  $\kappa = (\kappa_x, \kappa_y)$ , and where

$$\begin{aligned} V_c(\varphi, l, \kappa) &\equiv (1 - \gamma) (\varphi_0 + \theta_2 l_0 + [\theta_2 l_A + (\varphi_a + \theta_2 l_a) + (\varphi_x + \theta_2 l_x) + (\varphi_y + \theta_2 l_y)] \mu) \\ &\quad + \frac{1}{2} (1 - \gamma) \left( \frac{\rho - 1}{\rho} \right) \left[ \frac{(\varphi_a + \theta_2 l_a)^2}{\kappa_\xi} + \frac{(\varphi_x + \theta_2 l_x)^2}{\kappa_x} + 2 \frac{(\varphi_a + \theta_2 l_a)(\varphi_x + \theta_2 l_x)}{\kappa_x} \right] \\ &\quad + \frac{1}{2} (1 - \gamma)^2 \left[ \frac{(\varphi_y + \theta_2 l_y)^2}{\kappa_y} + \frac{(\theta_2 l_A + (\varphi_a + \theta_2 l_a) + (\varphi_x + \theta_2 l_x) + (\varphi_y + \theta_2 l_y))^2}{\kappa_A} \right] \\ V_l(\varphi, l, \kappa) &\equiv \epsilon_2 (l_0 + (l_A + l_a + l_x + l_y) \mu) + \frac{1}{2} \epsilon_2^2 \left[ \left( \frac{l_a^2}{\kappa_\xi} + \frac{l_x^2}{\kappa_x} + 2 \frac{l_a l_x}{\kappa_x} \right) + \frac{l_y^2}{\kappa_y} + \frac{(l_A + l_a + l_x + l_y)^2}{\kappa_A} \right] \\ V_n(\varphi, l, \kappa) &\equiv \frac{\epsilon_1}{\theta_1} (\varphi_0 + (\varphi_a + \varphi_x + \varphi_y - 1) \mu) \\ &\quad + \frac{1}{2} \frac{\epsilon_1^2}{\theta_1^2} \left[ \frac{(\varphi_a - 1)^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2 \frac{(\varphi_a - 1) \varphi_x}{\kappa_x} + \frac{\varphi_y^2}{\kappa_y} + \frac{(\varphi_a + \varphi_x + \varphi_y - 1)^2}{\kappa_A} \right] \end{aligned}$$

We henceforth consider a relaxed problem, where we ignore the constraint on  $\varphi_a$  imposed by (29); it will turn out that the solution to this relaxed problem satisfies this constraint, which means that the solution to the relaxed problem is also the solution to our initial problem.

The first-order conditions of the (relaxed) problem with respect to  $\varphi_0$  and  $l_0$  give

$$\begin{aligned}\varphi_0 &: 0 = \exp V_c - \frac{1}{\theta_1} \exp V_n \\ l_0 &: 0 = \theta_2 \exp V_c - \exp V_l\end{aligned}$$

Hence, at the optimal allocation,  $\exp V \equiv \exp V_c = \frac{1}{\theta_1} \exp V_n = \frac{1}{\theta_2} \exp V_l > 0$ . Let the Lagrange multipliers on the implementability constraints (29)-(32) be, respectively,  $e^V \mu_a, e^V \mu_x$ , and  $e^V \mu_y$ . Next, as in the proof of Proposition 4, we can represent the informational externalities by two Lagrange multipliers, one for the sum  $\varphi_a + \varphi_x$ , which determines  $\kappa_q$ , the precision of the output signal; and another for the sum  $\varphi_a + \varphi_x + \theta_2(l_a + l_x)$ , which determines  $\kappa_p$ , the precision of the price signal. Let these multipliers be, respectively,  $e^V \eta_q$  and  $e^V \eta_p$ . We can then state the rest of the first-order conditions of the optimal policy problem as follows.

First, the conditions for the stage-1 strategy are the following:

$$\begin{aligned}\varphi_a : 0 &= \left( \frac{\rho - 1}{\rho} \right) \left( \frac{(\varphi_a + \theta_2 l_a)}{\kappa_\xi} + \frac{(\varphi_x + \theta_2 l_x)}{\kappa_x} \right) \\ &\quad + (1 - \gamma) \frac{(\theta_2 l_A + (\varphi_a + \theta_2 l_a) + (\varphi_x + \theta_2 l_x) + (\varphi_y + \theta_2 l_y))}{\kappa_A} \\ &\quad - \frac{\epsilon_1}{\theta_1} \left[ \frac{(\varphi_a - 1)}{\kappa_\xi} + \frac{\varphi_x}{\kappa_x} + \frac{(\varphi_a + \varphi_x + \varphi_y - 1)}{\kappa_A} \right] + \eta_q + \eta_p - \frac{\epsilon_1}{\epsilon_2 \theta_1} \mu_a \\ \varphi_x : 0 &= \left( \frac{\rho - 1}{\rho} \right) \left( \frac{(\varphi_x + \theta_2 l_x)}{\kappa_x} + \frac{(\varphi_a + \theta_2 l_a)}{\kappa_x} \right) \\ &\quad + (1 - \gamma) \frac{(\theta_2 l_A + (\varphi_a + \theta_2 l_a) + (\varphi_x + \theta_2 l_x) + (\varphi_y + \theta_2 l_y))}{\kappa_A} \\ &\quad - \frac{\epsilon_1}{\theta_1} \left[ \frac{\varphi_x}{\kappa_x} + \frac{(\varphi_a - 1)}{\kappa_x} + \frac{(\varphi_a + \varphi_x + \varphi_y - 1)}{\kappa_A} \right] + \eta_q + \eta_p - \frac{\epsilon_1}{\epsilon_2 \theta_1} \mu_x \\ \varphi_y : 0 &= (1 - \gamma) \left[ \frac{(\varphi_y + \theta_2 l_y)}{\kappa_y} + \frac{(\theta_2 l_A + (\varphi_a + \theta_2 l_a) + (\varphi_x + \theta_2 l_x) + (\varphi_y + \theta_2 l_y))}{\kappa_A} \right] \\ &\quad - \frac{\epsilon_1}{\theta_1} \left[ \frac{\varphi_y}{\kappa_y} + \frac{(\varphi_a + \varphi_x + \varphi_y - 1)}{\kappa_A} \right] - \frac{\epsilon_1}{\epsilon_2 \theta_1} \mu_y\end{aligned}$$

And second, the conditions for the stage-2 strategy are the following:

$$\begin{aligned}
l_a : \quad 0 &= \left( \frac{\rho-1}{\rho} \right) \left( \frac{(\varphi_a + \theta_2 l_a)}{\kappa_\xi} + \frac{(\varphi_x + \theta_2 l_x)}{\kappa_x} \right) \\
&\quad + (1-\gamma) \left[ \frac{(\theta_2 l_A + (\varphi_a + \theta_2 l_a) + (\varphi_x + \theta_2 l_x) + (\varphi_y + \theta_2 l_y))}{\kappa_A} \right] \\
&\quad - \epsilon_2 \left[ \frac{l_a}{\kappa_\xi} + \frac{l_x}{\kappa_x} + \frac{(l_A + l_a + l_x + l_y)}{\kappa_A} \right] + \eta_p + \frac{\mu_a}{\theta_2} \\
l_x : \quad 0 &= \left( \frac{\rho-1}{\rho} \right) \left( \frac{(\varphi_x + \theta_2 l_x)}{\kappa_x} + \frac{(\varphi_a + \theta_2 l_a)}{\kappa_x} \right) \\
&\quad + (1-\gamma) \left[ \frac{(\theta_2 l_A + (\varphi_a + \theta_2 l_a) + (\varphi_x + \theta_2 l_x) + (\varphi_y + \theta_2 l_y))}{\kappa_A} \right] \\
&\quad - \epsilon_2 \left[ \frac{l_x}{\kappa_x} + \frac{l_a}{\kappa_x} + \frac{(l_A + l_a + l_x + l_y)}{\kappa_A} \right] + \eta_p + \frac{\mu_x}{\theta_2} \\
l_y : \quad 0 &= (1-\gamma) \left[ \frac{(\varphi_y + \theta_2 l_y)}{\kappa_y} + \frac{(\theta_2 l_A + (\varphi_a + \theta_2 l_a) + (\varphi_x + \theta_2 l_x) + (\varphi_y + \theta_2 l_y))}{\kappa_A} \right] \\
&\quad - \epsilon_2 \left[ \frac{l_y}{\kappa_y} + \frac{(l_A + l_a + l_x + l_y)}{\kappa_A} \right] + \frac{\mu_y}{\theta_2} \\
l_A : \quad 0 &= (1-\gamma) \left[ \frac{(\theta_2 l_A + (\varphi_a + \theta_2 l_a) + (\varphi_x + \theta_2 l_x) + (\varphi_y + \theta_2 l_y))}{\kappa_A} \right] - \epsilon_2 \left[ \frac{(l_A + l_a + l_x + l_y)}{\kappa_A} \right] \\
&\quad + \frac{\kappa_x \mu_x}{\kappa \theta_2} + \frac{\kappa_y \mu_y}{\kappa \theta_2}
\end{aligned}$$

For any given  $\eta_q$  and  $\eta_p$ , the combination of these seven FOCs with the three implementability constraints (30)-(32) defines a linear system of 10 equations in 10 unknowns, the allocation coefficients  $(\varphi_a, \varphi_x, \varphi_y)$  and  $(l_A, l_a, l_x, l_y)$  and the implementability multipliers  $(\mu_a, \mu_x, \mu_y)$ . The solution to this system gives the following results: For the stage-1 allocation, we get

$$\varphi_a^* = \beta \quad (88)$$

$$\varphi_x^* = \left\{ \frac{(1-\alpha)\kappa_x^*}{(1-\alpha)\kappa_x^* + \kappa_y^* + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta + \delta_q \eta_q + \delta_p \eta_p \quad (89)$$

$$\varphi_y^* = \left\{ \frac{\kappa_y^*}{(1-\alpha)\kappa_x^* + \kappa_y^* + \kappa_A} \right\} \frac{\alpha}{1-\alpha} \beta - \frac{\kappa_y^*}{\kappa_A + \kappa_y^*} (\delta_q \eta_q + \delta_p \eta_p) \quad (90)$$

where

$$\begin{aligned}
\delta_q &\equiv \frac{\theta_1 \kappa_x (\kappa_A + \kappa_y) (\beta (\varepsilon_2 \theta_1 + \varepsilon_1 \theta_2) (\kappa_A + \kappa_x + \kappa_y) - \varepsilon_1 \theta_2 (\kappa_A + (1-\alpha) \kappa_x + \kappa_y))}{\varepsilon_1 (\varepsilon_2 \theta_1 + \varepsilon_1 \theta_2) (\kappa_A + \kappa_x + \kappa_y) (\kappa_A + (1-\alpha) \kappa_x + \kappa_y)} > 0 \\
\delta_p &\equiv \frac{(\beta \varepsilon_2 \theta_1 + \varepsilon_1 \theta_2 + \beta \varepsilon_1 \theta_2) \kappa_x (\kappa_A + \kappa_y)}{\varepsilon_1 \varepsilon_2 (\kappa_A + (1-\alpha) \kappa_x + \kappa_y)} > 0
\end{aligned}$$

For the stage-2 allocation, we get

$$\begin{aligned}
l_A^* &= \hat{l}_A - (\zeta_q \eta_q + \zeta_p \eta_p) \\
l_a^* &= \hat{l}_a \\
l_x^* &= \hat{l}_x + \left( \frac{\kappa_x}{\kappa_A + \kappa_x + \kappa_y^*} \right) (\lambda_q \delta_q \eta_q + \lambda_p \delta_p \eta_p) \\
l_y^* &= \hat{l}_y + \left( \frac{\kappa_y^*}{\kappa_0 + \kappa_x + \kappa_y^*} \right) (\lambda_q \delta_q \eta_q + \lambda_p \delta_p \eta_p)
\end{aligned}$$

where

$$\begin{aligned}
\hat{l}_A &\equiv \frac{(\kappa_A + \kappa_x + \kappa_y) \varphi_x^* \varepsilon_1}{\beta \kappa_x \varepsilon_2 \theta_1} \\
\hat{l}_a &\equiv \frac{\varepsilon_1}{\varepsilon_2 \theta_1} (\varphi_a^* - 1) \\
\hat{l}_x &\equiv \frac{\varepsilon_1}{\varepsilon_2 \theta_1} \varphi_x^* - \hat{l}_A \frac{\kappa_x}{\kappa} \\
\hat{l}_y &\equiv \frac{\varepsilon_1}{\varepsilon_2 \theta_1} \varphi_y^* - \hat{l}_A \frac{\kappa_y}{\kappa}
\end{aligned}$$

and where

$$\begin{aligned}
\lambda_q &\equiv \frac{(\beta \varepsilon_2 \theta_1 + (\beta - 1) \varepsilon_1 \theta_2) (\kappa_A + \kappa_y)}{\beta \varepsilon_2 (\varepsilon_2 \theta_1 + \varepsilon_1 \theta_2)} > 0 \\
\lambda_p &\equiv \frac{(\beta \varepsilon_2 \theta_1 + (\beta - 1) \varepsilon_1 \theta_2) (\kappa_A + \kappa_x + \kappa_y)}{\beta \varepsilon_2^2 \theta_1} > 0
\end{aligned}$$

Note that, by Proposition 6,  $(\hat{l}_A, \hat{l}_a, \hat{l}_x, \hat{l}_y)$  identifies the stage-2 allocation that would obtain in the (unique) flexible-price equilibrium in which the stage-1 allocation is given by (88)-(90). Finally, for the implementability multipliers, we get

$$\mu_a = \mu_x = \frac{\varepsilon_2 \theta_1}{\varepsilon_1 + \varepsilon_2 \theta_1} \eta_q + \frac{\varepsilon_2 \theta_1 (1 - \theta_2)}{\varepsilon_1 + \varepsilon_2 \theta_1} \eta_p \quad \text{and} \quad \mu_y = 0$$

Letting

$$\Delta_q \equiv \eta_q \delta_q \quad \text{and} \quad \Delta_p \equiv \eta_p \delta_p$$

completes the proof of all the conditions in the proposition.

What remains is to show that  $\Delta_q$  and  $\Delta_p$  (or, equivalently,  $\eta_q$  and  $\eta_p$ ) are positive. Towards this goal, first note that

$$\begin{aligned}
\eta_q &= e^{-V} \frac{\partial \mathcal{W}}{\partial \kappa_y} \frac{\partial \kappa_y}{\partial \kappa_q} \frac{\partial \kappa_q}{\partial \varphi_x} \\
\eta_p &= e^{-V} \frac{\partial \mathcal{W}}{\partial \kappa_y} \frac{\partial \kappa_y}{\partial \kappa_p} \frac{\partial \kappa_p}{\partial \varphi_x}
\end{aligned}$$

Next, note that  $\frac{\partial \kappa_y}{\partial \kappa_q} = \frac{\partial \kappa_y}{\partial \kappa_p} = 1$  and

$$\frac{\partial \mathcal{W}}{\partial \kappa_y} = -(1 - \gamma) \exp V_c \frac{(\varphi_y + \theta_2 l_y)^2}{\kappa_y^2} + \frac{\varepsilon_2}{\theta_2^2} \exp V_l \frac{(\theta_2 l_y)^2}{\kappa_y^2} + \frac{\varepsilon_1}{\theta_1^2} \exp V_n \frac{\varphi_y^2}{\kappa_y}. \quad (91)$$

At the optimal allocation, we know that  $\exp V \equiv \exp V_c = \frac{1}{\theta_1} \exp V_n = \frac{1}{\theta_2} \exp V_l$ , as well as that  $\mu_y = 0$  and  $\mu_a = \mu_x$ . Using the first fact, we get

$$\frac{\partial \mathcal{W}}{\partial \kappa_y} = \frac{e^V}{\kappa_y^2} \left\{ (\gamma - 1) (\varphi_y + \theta_2 l_y)^2 + \frac{\epsilon_2}{\theta_2} (\theta_2 l_y)^2 + \frac{\epsilon_1}{\theta_1} \varphi_y^2 \right\}.$$

Using the second fact along with the FOCs with respect to  $(l_a, l_x, l_y, l_A)$  and the implementability constraint for  $\varphi_x$ , we can express  $l_y$  as a function of  $\varphi_y$  and  $\mu_x$ :

$$l_y = \frac{1}{(\epsilon_2 - \theta_2) + \gamma \theta_2} \left\{ (1 - \gamma) \varphi_y - \frac{\kappa_x \kappa_y}{\kappa_A + \kappa_x + \kappa_y} \frac{\mu_x}{\theta_2} \right\}$$

It follows that

$$\frac{\partial \mathcal{W}}{\partial \kappa_y} = \frac{e^V}{\beta} \left\{ (1 - \alpha) \frac{\epsilon_1}{\theta_1} \frac{\varphi_y^2}{\kappa_y^2} + \frac{(\beta - 1 + \alpha) \epsilon_1 \theta_2 + \beta \epsilon_2 \theta_1}{\epsilon_2^2 \theta_1 \theta_2 (\kappa_0 + \kappa_x + \kappa_y)^2} \mu_x^2 \kappa_x^2 \right\}.$$

Since  $\beta > 1$  necessarily, it is immediate that  $\alpha \geq 0$  (which we have assumed) suffices for  $\frac{\partial \mathcal{W}}{\partial \kappa_y} > 0$ . Finally, recall that  $\frac{\partial \kappa_q}{\partial \varphi_x} > 0$  if and only if  $\varphi_a + \varphi_x > 0$ , while  $\frac{\partial \kappa_p}{\partial \varphi_x} > 0$  if and only if  $\varphi_a + \varphi_x + \theta_2 l_a + \theta_2 l_x > 0$ . Combining these result, we conclude that  $\Delta_q > 0$  and  $\Delta_p > 0$  if and only if the optimal allocation satisfies  $\varphi_a + \varphi_x > 0$  and  $\varphi_a + \varphi_x + \theta_2 l_a + \theta_2 l_x > 0$ .

To prove this, we proceed in a similar fashion as in Proposition 4. Let

$$\bar{v} \equiv \begin{bmatrix} \varphi_a + \varphi_x \\ \varphi_a + \varphi_x + \theta_2 l_a + \theta_2 l_x \end{bmatrix} \quad \text{and} \quad v(\kappa) \equiv \begin{bmatrix} \frac{\kappa_A + \kappa_x + \kappa_y}{(1 - \alpha) \kappa_x + \kappa_y + \kappa_A} \beta \\ \frac{\kappa_A + \kappa_x + \kappa_y}{(1 - \alpha) \kappa_x + \kappa_y + \kappa_A} \beta \left( 1 + \frac{\epsilon_1 \theta_2}{\epsilon_2 \theta_1} \right) \end{bmatrix}$$

As in Proposition 4,  $\bar{v} - v(\kappa)$  is the distance between any value  $\bar{v}$  that the planner may choose and the one that would have been optimal from a purely allocative perspective. Moreover, welfare can be expressed as (a monotone transformation of) a quadratic form of this distance. In particular, using the FOCs with respect to  $\varphi_0$  and  $l_0$ , we get that welfare is given by

$$\mathcal{W}(\varphi, l, \kappa) = \frac{\left( \frac{\theta_1}{\epsilon_1} + \frac{\theta_2}{\epsilon_2} \right)^{-1} - 1 + \gamma}{1 - \gamma} \left( \frac{\theta_1}{\epsilon_1} + \frac{\theta_2}{\epsilon_2} \right) \exp V_c(\varphi, l, \kappa)$$

Since  $\left( \frac{\theta_1}{\epsilon_1} + \frac{\theta_2}{\epsilon_2} \right)^{-1} - 1 + \gamma > 0$  and  $\frac{\theta_1}{\epsilon_1} + \frac{\theta_2}{\epsilon_2} > 0$ , we can once again consider the following monotone transformation of welfare:

$$\mathcal{TW}(\varphi, l, \kappa) \equiv \frac{1}{1 - \gamma} V_c(\varphi, l, \kappa).$$

Using then the characterization of the optimal coefficients  $(\varphi, l)$  as functions of  $\bar{v}$ , we conclude that

$$\mathcal{TW}(\varphi, l, \kappa) = W(\bar{v}, \kappa) \equiv A(\kappa) - (\bar{v} - v(\kappa))' B(\kappa) (\bar{v} - v(\kappa)), \quad (92)$$

where  $A(\kappa)$  is scalar that identifies the level welfare attained when  $\bar{v} = v(\kappa)$ , while  $B(\kappa)$  is a 2-by-2 matrix that identifies the Hessian of the (transformed) welfare function  $W$ . The latter is given by

the following:

$$\begin{aligned}
B(\kappa) &\equiv \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \\
b_{11} &\equiv -\frac{(\varepsilon_2\theta_1 + \varepsilon_1\theta_2)(\kappa_A + \kappa_x + \kappa_y)((1-\alpha)\varepsilon_1\theta_2\kappa_x + \beta(\varepsilon_2\theta_1 + \varepsilon_1\theta_2)(\kappa_A + \kappa_y))}{(1-\alpha)\varepsilon_2\theta_1^2\theta_2\kappa_x^2(\kappa_A + \kappa_y)} \\
b_{22} &\equiv -\frac{\beta\varepsilon_2(\beta(\varepsilon_2\theta_1 + \varepsilon_1\theta_2)(\kappa_A + \kappa_x + \kappa_y) - \varepsilon_1\theta_2(\kappa_0 + (1-\alpha)\kappa_x + \kappa_y))}{(1-\alpha)\theta_2\kappa_x^2(\beta\varepsilon_2\theta_1 + (\beta-1)\varepsilon_1\theta_2)} \\
b_{12} &\equiv b_{21} \equiv \frac{\beta(\varepsilon_2\theta_1 + \varepsilon_1\theta_2)(\kappa_A + \kappa_x + \kappa_y)}{2(1-\alpha)\theta_1\theta_2\kappa_x^2}
\end{aligned}$$

Note that  $b_{11} < 0$  and that the determinant of  $B(\kappa)$  is positive:

$$\begin{aligned}
\det(B) &= b_{11}b_{22} - b_{12}b_{21} \\
&= \frac{\beta(\varepsilon_2\theta_1 + \varepsilon_1\theta_2)(\kappa_A + \kappa_x + \kappa_y)}{4(1-\alpha)^2\theta_1^2\theta_2^2\kappa_x^4(\beta\varepsilon_2\theta_1 + (\beta-1)\varepsilon_1\theta_2)(\kappa_A + \kappa_y)} \times \\
&\quad \times \left[ 4\alpha\varepsilon_1\theta_2\kappa_x((1-\alpha)\varepsilon_1\theta_2\kappa_x + \beta(\varepsilon_2\theta_1 + \varepsilon_1\theta_2)(\kappa_A + \kappa_y)) + \right. \\
&\quad \left. + (\beta\varepsilon_2\theta_1 + (\beta-1)\varepsilon_1\theta_2)(\kappa_A + \kappa_x + \kappa_y)(4(1-\alpha)\varepsilon_1\theta_2\kappa_x + 3\beta(\varepsilon_2\theta_1 + \varepsilon_1\theta_2)(\kappa_A + \kappa_y)) \right] \\
&> 0.
\end{aligned}$$

It follows that the matrix  $B(\kappa)$  is negative definite and, hence, the aforementioned quadratic form for welfare in (92) is also negative definite. The same type of arguments as in Proposition (4) then imply that the optimal  $\bar{v}$  is positive, and indeed higher than  $v(\kappa)$ , which in turn guarantees that  $\Delta_q > 0$  and  $\Delta_p > 0$ .

**Proof of Theorem 4 (optimal monetary policy).** We prove the result in reverse order.

*Part (ii).* At the optimal allocation, aggregate consumption is given by

$$\log C(\Omega) = \text{const} + c_A^*\bar{a} + c_y^*y = \text{const} + (c_A^* + c_y^*)\bar{a} + c_y^*\varepsilon$$

where  $c_A^* = (\varphi_a^* + \varphi_x^*) + \theta_2(l_a^* + l_x^* + l_A^*)$  and  $c_y^* = \varphi_y^* + \theta_2l_y^*$ . If monetary policy were replicating the flexible-price allocations, then aggregate consumption would be given by

$$\log C(\Omega) = \text{const} + \hat{c}_A\bar{a} + \hat{c}_yy = \text{const} + (\hat{c}_A + \hat{c}_y)\bar{a} + \hat{c}_y\varepsilon$$

where  $\hat{c}_A = (\varphi_a^* + \varphi_x^*) + \theta_2(\hat{l}_a + \hat{l}_x + \hat{l}_A)$  and  $\hat{c}_y = \varphi_y^* + \theta_2\hat{l}_y$ . From part (ii) of Proposition 7, it is immediate that  $c_A^* < \hat{c}_A$  and  $c_A^* + c_y^* < \hat{c}_A + \hat{c}_y$ . The first inequality means that if one fixes the common belief about aggregate productivity (namely  $y$ ) and considers the surprise component in the realization of aggregate productivity (namely  $\bar{a} - y$ ), then the response of aggregate consumption to this surprise component is lower in the optimal allocation than in the flexible-price allocation. The second inequality means that if one considers the overall response of consumption to the aggregate productivity keeping the noise ( $\varepsilon$ ), then this is also lower. So, no matter which way one sees it, the

optimal monetary policy is less accommodative of the productivity shock than what would have been consistent with replicating the flexible-price allocations.

*Part (i).* Write the tax in terms of the productivity and the noise shocks as

$$-\log(1 - \tau) = \tau_0^* + \tau_A^* \bar{a} + \tau_\varepsilon^* \varepsilon$$

We seek to prove that  $\tau_A^* < 0$ . Re-expressing the tax as a function of the productivity shock and the sufficient statistic of the public information gives

$$-\log(1 - \tau) = \tau_0 + \tau_A \bar{a} + \tau_y y$$

where  $\tau_A \equiv \tau_A^* - \tau_\varepsilon^*$  and  $\tau_y = \tau_\varepsilon^*$ . From conditions (86) and (87) in the proof of Proposition 6, we know that the optimal tax satisfies

$$\begin{aligned}\tau_A^* &= \frac{1}{\chi k} \left\{ \alpha c_A^* - \chi \frac{\kappa}{\kappa_x} \varphi_x^* \right\} \\ \tau_y^* &= \frac{1}{\chi k} \left\{ \alpha c_y^* - \chi \left( \varphi_y^* - \varphi_x^* \frac{\kappa_y}{\kappa_x} \right) \right\}\end{aligned}$$

where  $c_A^* = (\varphi_a^* + \varphi_x^*) + \theta_2(l_a^* + l_x^* + l_A^*)$  and  $c_y^* = \varphi_y^* + \theta_2 l_y^*$ . Let

$$\begin{aligned}\hat{\tau}_A &\equiv \frac{1}{\chi k} \left\{ \alpha \hat{c}_A - \chi \frac{\kappa}{\kappa_x} \varphi_x^* \right\} \\ \hat{\tau}_y &\equiv \frac{1}{\chi k} \left\{ \alpha \hat{c}_y - \chi \left( \varphi_y^* - \varphi_x^* \frac{\kappa_y}{\kappa_x} \right) \right\}\end{aligned}$$

where  $\hat{c}_A \equiv (\varphi_a^* + \varphi_x^*) + \theta_2(\hat{l}_a + \hat{l}_x + \hat{l}_A)$  and  $\hat{c}_y \equiv \varphi_y^* + \theta_2 \hat{l}_y$ ; this identifies the tax policy that would be required in order to implement the optimal stage-1 sensitivities,  $\varphi_x^*$  and  $\varphi_y^*$ , if monetary policy were replicating the flexible-price allocations associated with these stage-1 sensitivities. It is easy to verify that

$$\hat{\tau}_A = -\lambda \Delta \quad \text{and} \quad \hat{\tau}_y = -\lambda \frac{\kappa_y}{\kappa_A + \kappa_y} \Delta$$

where  $\Delta \equiv \Delta_q + \Delta_p$  and  $\lambda \equiv \frac{(\kappa_A + (1-\alpha)\kappa_x + \kappa_y)\epsilon_1 \epsilon_2}{\kappa_x(\beta \epsilon_2 \theta_1 + (\beta - 1 + \alpha)\epsilon_1 \theta_2)} > 0$ . This result is identical to the result in the baseline model, except for the different value of the coefficient  $\lambda$ . Once again,  $\Delta > 0$  is necessary and sufficient for  $\hat{\tau}_A < 0$  and  $\hat{\tau}_A + \hat{\tau}_y < 0$ ; that is, the tax that would have implemented the optimal stage-1 choices if monetary policy replicated the flexible-price allocation is countercyclical, much alike in the baseline model. But now note that the optimal tax satisfies

$$\begin{aligned}\tau_A^* &= \hat{\tau}_A + \frac{1}{\chi k} \alpha (c_A^* - \hat{c}_A) \\ \tau_y^* &= \hat{\tau}_y + \frac{1}{\chi k} \alpha (c_y^* - \hat{c}_y)\end{aligned}$$

By assumption,  $\alpha > 0$ , while by part (ii) of Proposition 7,  $c_A^* < \hat{c}_A$  and  $c_A^* + c_y^* < \hat{c}_A + \hat{c}_y$ . It follows that  $\tau_A^* < \hat{\tau}_A$  and  $\tau_A^* + \tau_y^* < \hat{\tau}_A + \hat{\tau}_y$ , which means that the optimal tax is even more countercyclical.

**Proof of Condition (41) and Theorem 5 (targeting price stability).** Using the fact that allocations and prices are log-normal in equilibrium, condition (68) can be written as follows:

$$\left(1 - \rho + \frac{\rho\epsilon_2}{\theta_2}\right) (\log p - \mathbb{E}[\log P|\omega]) = \left(\frac{\epsilon_2 - (1 - \gamma)\theta_2}{\theta_2}\right) \mathbb{E}[\log C|\omega] - \frac{\epsilon_2}{\theta_2} \log q + \text{const.}$$

Combining this with the property of the flexible-price equilibrium that  $\log C(\Omega) = \text{const} + \chi \log Q(\Omega)$  where  $\chi \equiv \frac{\epsilon_2}{\epsilon_2 - (1 - \gamma)\theta_2}$ , we find that the equilibrium prices must satisfy

$$\mathbb{E}[\log p(\omega) - \log P(\Omega)|\omega] = \text{const} - \nu \mathbb{E}[\log q(\omega) - \log Q(\Omega)|\omega] \quad (93)$$

with  $\nu \equiv \frac{\epsilon_2}{\rho\epsilon_2 - (\rho - 1)\theta_2}$ .

**Proof of Theorem 6 (welfare effects of information).** From the proof of Proposition 7 we already know that, at the optimal allocation, welfare necessarily increases with the precision  $\kappa_y$  of the sufficient statistic of all available public information. The result then follows immediately from the fact that  $\kappa_y$  itself decreases with either the variance of the noise in the exogenous public signal ( $\sigma_{ya}^2$ ) or the variances of the noises in the endogenous output and price signals ( $\sigma_{yq}^2$  and  $\sigma_{yp}^2$ ).

## Appendix B: other endogenous signals

The entire analysis has allowed endogenous signals only about output and prices. We now discuss how the results are affected by other types of endogenous signals. For simplicity, we focus on the baseline model; similar points apply to the extended model.

First, consider a signal of (the log of) aggregate employment; this signal may represent the employment data published by the BLS, although the discussion here does not rely on whether this signal is private or public. To understand the precision of this signal, note that local employment is given by

$$\log n(\omega) = \frac{1}{\theta} (\log q(\omega) - \log A(\omega)) = \frac{1}{\theta} (\varphi_0 + (\varphi_a - 1)a + \varphi_x x + \varphi_y y).$$

It follows that the sensitivity of aggregate employment to aggregate productivity, and hence the precision of the employment signal, is determined by  $\varphi_a + \varphi_x - 1$ . Depending on the value of  $\alpha$ , the equilibrium value of  $\varphi_a + \varphi_x - 1$  can be either positive (meaning that employment reacts positively to innovations in aggregate productivity) or negative (meaning the opposite). If this value is positive, raising  $\varphi_x$  improves the precision of both the output signals and the employment signal. As a result, the qualitative properties of the efficient allocation are not affected at all. If instead this value is negative, raising  $\varphi_x$  and (thereby bringing  $\varphi_a + \varphi_x - 1$  closer to zero) could reduce the precision of the employment signal; conversely, reducing  $\varphi_x$  (and thereby making  $\varphi_a + \varphi_x - 1$  more negative) could increase the precision of the employment signal. As a result, we cannot rule out the possibility that the efficient allocation features a negative  $\Delta$ , which would mean that better learning and more efficiency are achieved with lower sensitivity to local information. However, this

possibility rests on employment being negatively correlated with productivity and output, which is counterfactual. Indeed, within our model we can rule out this possibility by imposing  $\alpha \geq 0$ . Recall that this restriction simply means that individual incentives to produce increase with expected aggregate demand—an empirically plausible restriction that proved useful also when we studied optimal monetary policy.

Next, consider a signal of (the log of) the mean wage in the economy. This raises a conceptual issue. While the output and employment signals are well defined for arbitrary allocations, the wage signal is well defined only on equilibrium. To study efficiency, we need to extend the definition of this signal off the equilibrium, as a signal on the shadow wage associated with any arbitrary allocation. But which shadow wage? The one defined by the firms' marginal revenue of labor, or the one defined by the workers' marginal rate of substitution between consumption and leisure? Ultimately, the choice may depend on what implementations one has in mind, namely on what the market wage will be in those implementations.

Whichever choice one makes, the following issue arises. While the precisions of the output, price, and employment signals depend only on the production strategy dictated by the planner, the precision of the wage signal depends also on local expectations of aggregate productivity. This is either because the firms' shadow wage depends on their expectations of aggregate demand, or because the workers' shadow wage depends on their expectations of the marginal utility of consumption. In effect, the wage signal is partly an indicator of aggregate activity and partly a survey of opinions.

But now note that the social learning that can obtain from aggregating different local opinions depends on how sensitive these opinions are to local (private) information, which in turn depends on how precise the available private and public informations are, which in turn depend on social learning. This feedback mechanism opens the door to the possibility that multiple information structures can be consistent with the *same* strategy.<sup>38</sup> This in turn introduces discontinuities in the planner's problem, which complicates that characterization of the efficient allocation and the optimal policy. We can nevertheless bypass this complication by showing that the planner's problem remains upper-hemicontinuous. This is because, whenever there are multiple precisions associated with the same allocation, the planner always chooses the one that leads to higher welfare.

A second complication that emerges is the following: it is now possible that an attempt to increase the precision of public information backfires by reducing the sensitivity of local opinions to local information, and thereby reducing the precision of the wage signal (or of any other signal that aggregates opinions). As a result, although we can still show that Propositions 2 and 4 continue to hold for some  $\Delta$ , we cannot rule out the possibility that  $\Delta$  is negative.

Finally, consider a signal of the average expectation of either aggregate output or the marginal utility of aggregate consumption. This signal could mimic the role of the aggregation of information

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<sup>38</sup>To understand this more clearly, suppose the wage signal is private. When the precision of this signal is higher, then the precision of the private sufficient statistic is also higher. But then local opinions become more sensitive to private information, which in turn makes the wage signal more informative. It is this positive feedback that can sustain multiple precisions for the same strategy.

through financial markets. Clearly, raising the sensitivity of allocations to local information raises the sensitivity of both aggregate output and the marginal utility of consumption to the underlying aggregate productivity, which in turn contributes to a higher precision for the aforementioned signal. However, because this signal is an average of opinions, it is subject to exactly the issues as the signal of the average wage discussed above.

Combined, these observations qualify the conditions under which the cyclical properties of the optimal policies identified in Theorems 3 and 4 hold: these properties hold if and only if boosting social learning is achieved by raising the sensitivity of local activity to local information, a property that was necessarily true for the information structures we considered in the main text but does not have to be true for more general information structures. Indeed, these observations qualify more generally a key insight of the pertinent literature on herding and social learning (e.g., Banerjee, 1992; Vives, 1997): depending on the precise micro-foundations of social learning, the failure to internalize informational externalities could mean either too low or too high sensitivity to private information. That being said, we share with the pertinent literature the conviction that an increase in the sensitivity of economic activity to private information is most likely to boost social learning. That is, we think that  $\Delta > 0$  is an excellent benchmark.

Finally, note that, even when this is not the case (i.e., when  $\Delta < 0$ ), the type of state-contingent policies we have identified in this paper can still boost social learning and can still improve the efficiency of the business cycle; only the cyclical properties of the optimal policies are now reversed. The precise nature of the optimal policies may thus be sensitive to the details of the sources of learning, but the essence of our results is not.

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