

Competitive Search: A Test for Direction and Efficiency*

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Abstract

In this paper we estimate the canonical competitive search model of [Moen \(1997\)](#). In addition, likelihood ratio tests are developed to test several equilibrium conditions that differentiates competitive search from other types. The results fail to reject that workers direct their search, and to sub-markets or firms with a particular level of productivity, while posting (i.e. efficiency via [Hosios \(1990\)](#)) is rejected in three of the six industries considered.

Keywords: unemployment, wages, posting, directed search

JEL Codes: J0, J63, J64

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1 Introduction

Models with directed search state that workers who wait longer for a job are compensated with higher wages. In addition, firms post wages to maximize their profits. As a result of directed search with posting, the competitive search model delivers an efficient allocation of resources, i.e., the Hosios condition is satisfied. In the standard random search environment, however, unemployment is typically inefficient and therefore may be improved with policies such as a minimum wage, hiring subsidies, and so on. This paper tests the restrictions implied by the competitive search model and, in turn, its implications.

The competitive search model of [Moen \(1997\)](#) is estimated using maximum likelihood estimation (MLE). In particular, the likelihood function incorporates wage and unemployment duration data taken from the Current Population Survey (CPS), job vacancies from the Job Openings and Labor Turnover Survey (JOLTS), and Internal Revenue Service (IRS) tax data on corporate earnings. The estimation strategy follows the work of [Eckstein and Wolpin \(1995\)](#) and [Flinn \(2006\)](#) in terms of constructing the likelihood function. In addition, the estimation is performed on aggregate U.S. data as well as disaggregated by major industry classification.

We test the restrictions imposed by the competitive search model, or equilibrium conditions, using likelihood ratio tests. The likelihood ratio tests fail to reject that workers direct their search to sub-markets with a particular level of productivity; while posting (i.e. efficiency via [Hosios \(1990\)](#)) is rejected in three of the six industries considered as well as in the aggregate.

The paper also makes several other methodological contributions. First, earlier papers in this line of research, such as [Flinn and Heckman \(1982\)](#) make a parametric assumption on the distribution of productivity (almost universally a log-normal distribution) in order to identify the parameters of the model. We take a different route and use conditions from the directed search model with multiple markets, which introduces a new identifying restriction—that people are indifferent between searching in different markets. As a result, we estimate a non-parametric productivity distribution using a semi-parametric approach. Second, the Cobb-Douglas matching function, and, in particular the elasticity of the matching function, has been a difficult object to estimate because the arrival rate of jobs and the number of unemployed individuals is not enough to identify the elasticity. Of course, this is a key parameter if one wishes to test for efficiency in terms of the Hosios condition. We provide a new approach by introducing vacancy data from JOLTS that can

directly identify the elasticity of the matching function, without having to identify it indirectly from a restriction in the model or variation in the minimum wage as in [Flinn \(2006\)](#).

The findings are important for several reasons. First and foremost, we are testing whether efficiency exists, a property inherent in the competitive search literature as outlined in [Moen \(1997\)](#). The test is also critical to the bargaining literature as discussed in [Pissarides \(2000\)](#). If we fail to reject efficiency, then our results not only provide support for further research in the area but also have implications for policies aimed at distorting the bargaining power between workers and firms, as discussed in [Flinn \(2006\)](#).

Second, we test whether individuals direct their search to the best alternatives. The literature on directed search is inherently related to the matching process and thus we contribute to the investigation related to the form of the matching function as well as how it occurs (refer to [Rogerson, Shimer, and Wright \(2005\)](#) for surveys).

In relation to previous work on the estimation of search models, our work stands apart from [Flinn and Heckman \(1982\)](#), [Bowlus, Kiefer, and Neumann \(1995\)](#) and others who use the lowest wage (which is often trimmed) to estimate the reservation wage. Furthermore, our work has eliminated the necessity of assuming a parametric distribution of productivity and thus is free of misspecification regarding its form. In this regard, this paper is more similar to [Bontemps, Robin, and Berg \(2000\)](#) and [Postel-Vinay and Robin \(2002\)](#) but distinctly different from papers such as [Eckstein and Wolpin \(1995\)](#) or [Engelhardt and Fuller \(2009\)](#). Finally, we are the only paper, to our knowledge, that structurally estimates a competitive or directed search model; however, we note the careful calibration of [Kurt \(2008\)](#).

2 Model

We use the standard competitive search model based on [Moen \(1997\)](#) including the extension of heterogeneous workers. In order to understand the key features of the model, we briefly highlight them here, with little discussion. There is a continuum of risk neutral workers and firms with the number of workers normalized to one, who discount at rate r . The labor market is subject to search-matching frictions. The flow of hires is governed by an aggregate matching function, $x(u, v)$, where u is the measure of unemployed workers actively looking for jobs and v is the measure of vacant jobs. We assume a constant returns to scale (CRS) matching function and will focus on the

Cobb-Douglas form $x(v, u) = u^{1-\eta} v^\eta$.¹ Following the standard terminology, we define $\theta \equiv v/u$ as labor market tightness. Each vacancy is filled according to a Poisson process with arrival rate $\frac{x(U, V)}{V} \equiv q(\theta)$. Similarly, each unemployed worker finds a job according to a Poisson process with arrival rate $\frac{x(U, V)}{U} = \theta q(\theta) \equiv p(\theta)$. When a match occurs, the worker-firm pair begins producing immediately, generating output, y . Filled jobs receive negative idiosyncratic productivity shocks, with a Poisson arrival rate s , that render matches unprofitable.

One of the key components of competitive search is that workers direct their search to the most attractive opportunities. Some competitive search models assume firms post wages and workers direct their search to firms. Alternatively, [Moen \(1997\)](#) assumes the existence of sub-markets with firms entering and committing to pay a particular wage determined by a market-maker and workers direct their search to the most attractive sub-market. However, as noted in [Rogerson et al. \(2005\)](#), “...these approaches are equivalent in the sense that they give rise to the same equilibrium conditions (within the competitive search literature)” (p. 973). We follow [Moen \(1997\)](#) and present the results in terms of sub-markets.

In each sub-market unemployed workers and firms with a vacancy search for each other. The number of matches that occur in sub-market “ i ” is governed by the instantaneous matching function $x(u_i, v_i)$ where u_i is the mass of unemployed workers and v_i is the mass of vacant firms in the sub-market.

We assume workers are heterogeneous with respect to their value of leisure, z_j for $j \in \{1, \dots, m\}$. The Bellman equations for unemployed and employed states are given as:

$$rU_j = z_j + p(\theta_i)(E_{ij} - U_j) \quad (1)$$

$$rE_{ij} = w_{ij} + s(E_{ij} - U_j), \quad (2)$$

for market i and worker j , where w_{ij} is the wage paid to worker j in market i . [Eq. 1](#) states that the flow value of being unemployed, U_j , is equal to the value of leisure plus the probability of finding a job times the capital gain from switching states. Note that it is conditional on worker type but not on the sub-market because workers enter the sub-market with the highest expected income. Thus, all existing sub-markets must result in the same value of unemployment for a type-

¹Others could be considered such as those introduced in [Burdett, Shi, and Wright \(2001\)](#) or [Albrecht, Gautier, Tan, and Vroman \(2004\)](#). [Petrongolo and Pissarides \(2001\)](#) provides a broader summary.

j individual. Eq. 2 states that the flow value of being employed is given by the wage rate plus the probability of being separated times the capital loss associated with the change from employment to unemployment. Using the Bellman equations we can derive a relationship between the duration of unemployment and the wage. Specifically,

$$p(\theta_i) = \frac{rU_j - z_j}{w_{ij} - rU_j}(r + s). \quad (3)$$

In other words, if the wage is low, workers must find jobs at a relatively higher rate to make it beneficial to enter such a market. This condition is not present in the random search model and will be tested in Section 3. In particular, if this constraint holds for more than one $(p(\theta_i), w_{ij})$ pair, then it provides strong evidence that workers direct their search rather than search randomly. If not, workers might be searching randomly and the differences in arrival rates are due to differences in search intensity.

The second relationship used in comparing the two models comes from the efficiency condition, that occurs due to the decisions by firms of what wage to post. The competitive search model assumes firms draw a productivity from a discrete distribution F with support y_1, \dots, y_n . Once drawn, they choose a sub-market to enter that maximizes profits (maximization occurs because competition is assumed between market-makers). We refer to Moen (1997) for the proof, but the general result is

$$w_{ij} = \beta y_i - (1 - \beta)rU_j, \quad (4)$$

where β is the “bargaining” weight of the worker. Alternatively, the random search model with bargaining assumes workers search and at the time of a match draw a match specific productivity from a distribution, F , and split the surplus via Nash bargaining. However, it turns out that the way in which workers and firms split the surplus is identical to (4). The key difference is that posting results in efficiency, i.e., $\beta = \eta$. In other words, the bargaining weight equals the elasticity of the matching function. Therefore, the competitive search model with posting is “nested” within the random search model with bargaining and testing whether $\beta = \eta$ is a test of whether the labor market allocates resources efficiently.

The final equilibrium condition that differentiates the two models concerns worker flows. Specifically, the flow of workers into a particular employment state must equal the flow out. In

any model with heterogeneous arrival rates, this implies

$$\sum_i^n u_i p(\theta_i) = es, \quad (5)$$

where e is the mass of employed workers. However, competitive search makes a more restrictive assumption—that workers search in different markets that contain only one wage. Therefore, the flow condition is given as:

$$u_i p(\theta_i) = f_i es, \quad (6)$$

for all i where f_i is the probability mass function of F .

Although we do not provide a proof of existence nor characterize the equilibrium, we note Proposition 5 on p. 403 in [Moen \(1997\)](#). Specifically, it states two important results for our work:

1. “In any equilibrium with heterogeneous firms, the wage in all sub-markets joined by firms with productivity y_i is strictly greater than the wage in sub-markets joined by firms with productivity $y_j < y_i$ ”
2. “Workers with unemployment income i join sub-markets with a strictly higher wage than workers with unemployment income $z_j < z_i$.”

These two points are key because they imply a sub-market will contain only one type of worker and one type of productivity.

To sum up, the three equilibrium conditions (3), (4) and (5)/(6) distinguish the competitive search model. In addition, if transitions happen at a Poisson rate and sub-markets contain only one type of worker and productivity, then we are able to estimate and test these restrictions using duration and wage data taken from the CPS, job vacancies from JOLTS and income data from the IRS.

3 Estimation

Using the steady-state conditions enables the use of cross section data to estimate a dynamic model. We use MLE to identify the parameters, conditional on the data, and do so for aggregate U.S. data as well as each major industrial classification. In this section we describe the data sources, derive

the likelihood function, discuss identification and provide the estimates of the model’s parameters. With these estimates, we test whether the restrictive assumptions inherent in the competitive search model hold.

3.1 Data

We use the March, 2006 CPS to get the duration of unemployment, t , hourly wages, w , and major industry codes. To identify the surplus-splitting rule given by Eq. 4, we follow a similar methodology as Flinn (2006) who uses the McDonald’s Corporation Consolidated Statement of Income to measure the fraction of firm surplus going to workers. Instead, however, we use 2006 IRS data on corporate tax returns where we obtain a breakdown by industry of firm income and employee earnings.² We use that data to construct estimates by industry of labor’s share of net income, $\hat{\pi}$. We use JOLTS data for March, 2006 to capture the number of vacancies per labor force participant by industry, \hat{v} , which is used to identify the elasticity of the Cobb-Douglas matching function. Regarding the estimates by industry, we consider six out of the seven classifications available in JOLTS.³ Finally, note that the estimation method does not allow an independent estimate of r , therefore, we set $r = 0.05$ annually.

Table 1 provides the key descriptive statistics by industry. In addition, we have included other demographic information available in the CPS as well as the unemployment rate, \hat{u} .

3.2 Likelihood Function

The likelihood function primarily incorporates two types of data: the length of unemployment, t , and wages, w , where each enters independently. Unemployed workers contribute t to the likelihood function. That is, conditional on being unemployed and in market i , the rate a worker transitions to employment is $p(\theta_i) \equiv p_i$ and the duration of unemployment is distributed as:

$$f_u(t|u, i) = p_i \exp(-p_i t), \quad (7)$$

which is the survivor function divided by the average time unemployed. Conditional on being unemployed, duration is given by:

²See <http://www.irs.gov/taxstats/index.html> for more information.

³We exclude the “government” classification because of the lack of IRS data.

$$f_u(t|u) = \sum_{i=1}^n \frac{u_i}{u} f_u(t|u, i) \quad (8)$$

where n is the number of discrete markets. In other words, a worker's duration of unemployment in market i is $f_u(t|u, i)$ while the probability of being in market i is $\frac{u_i}{u}$.⁴ Given that the likelihood of being unemployed is $p(u) = u$, the contribution of an unemployed observation becomes

$$f(t, u) = \sum_{i=1}^n u_i p_i \exp(-p_i t), \quad (9)$$

where u_i and p_i will be estimated independently even though u_i is endogenous within the competitive search model and p_i is determined by workers and firms decisions. This approach is similar to that of [Eckstein and Wolpin \(1995\)](#).

The employed contribute the other part of the likelihood function. Following much of the literature, we incorporate measurement error in the wage data. Specifically, w_i is normally distributed with mean μ_i and standard deviation σ_w (which is assumed to be constant across all observations). As a result, the probability of being employed with a wage w in market i is written as

$$f_e(w|e, i) = \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \left(\frac{w - \mu_i}{\sigma_w} \right)^2 \right\}. \quad (10)$$

Adding measurement error serves to estimate the domain of the non-parametric productivity distribution, or μ_i for each market. The probability of observing a wage conditional on employment is given by

$$f_e(w|e) = \sum_{i=1}^n f_i f_e(w|e, i). \quad (11)$$

In words, a wage in market i is drawn with probability f_i and the wage is observed with some measurement error. Therefore, the contribution to the likelihood function from an employed worker is

$$f(w, e) = e \sum_{i=1}^n f_i \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \left(\frac{w - \mu_i}{\sigma_w} \right)^2 \right\}, \quad (12)$$

where the probability of being employed is $p(e) = e$.

Therefore, the likelihood function is

⁴At this point, an implicit assumption is being made about the directed search environment. Specifically, we have assumed markets have a one-to-one mapping to the domain of F for simplicity. However, this assumption is not restrictive as we fail to reject homogeneous workers and assume a well-behaved matching function.

$$\ln L(\psi) = \sum_{i \in e} \ln(f(w_i, e_i)) + \sum_{i \in u} \ln(f(t_i, u_i)), \quad (13)$$

where $\psi = \{u_1, \dots, u_n, p_1, \dots, p_n, f_1, \dots, f_n, \mu_1, \dots, \mu_n, \sigma_w, e, s\}$ and the identities and ergodic constraints

$$\sum_{i=1}^n u_i + e = 1, \quad (14)$$

$$\sum_{i=1}^n f_i = 1, \text{ and} \quad (15)$$

$$\sum_i^n u_i p(\theta_i) = es, \quad (16)$$

have been imposed.

3.3 Identification

Note that [Eq. 13](#) is a discrete mixture of exponential and normal distributions. Therefore, the parameters p_i , u_i , f_i , μ_i for all i , σ_w , and e are identified up to the point that such a class of problems are identified. [Eq. 16](#) identifies s . Also, the wage distribution is estimated as a mixture of normals. However, the underlying distribution is non-parametric.

We add to [Eq. 13](#) by introducing two estimators to provide identification for η and β . Also, two equilibrium conditions are imposed to increase efficiency and provide identification of rU_j for all j . The introduction of these restrictions is similar to [Flinn and Heckman \(1982\)](#) or [Flinn \(2006\)](#), but differs significantly in the fact that we estimate the wage distribution semi-parametrically as well as the fact that the lowest wage (which is often trimmed) does not play a critical role in determining any of the parameters.

The first estimator introduces the JOLTS data and is necessary because the matching function is assumed to be Cobb-Douglas, i.e., $p(\theta_i) = x(u, v)/u = u^{1-\eta} v^\eta / u$. Therefore, we construct the estimator

$$\hat{v} = \sum_{i=1}^n v_i = \sum_{i=1}^n p_i^{1/\eta} u_i, \quad (17)$$

where \hat{v} is the vacancy rate, and the second equality arises from inserting the estimates p_i and u_i

into the definition of the matching function, $p_i = u_i^{1-\eta} v_i^\eta / u_i$, and solving for v_i . As one can see, this restriction allows for the identification of η given estimates of u_i and p_i from (13).

The second estimator introduces demand side data and is necessary to identify the bargaining parameter if the market is not assumed to be efficient, i.e., the relaxation of $\beta = \eta$. Therefore, we construct the estimator

$$\hat{\pi} = \frac{\sum_{i=1}^n f_i \mu_i}{\sum_{i=1}^n f_i y_i} = \frac{\sum_{i=1}^n f_i \mu_i}{\sum_{i=1}^n f_i \frac{\mu_i - (1-\beta)rU_j}{\beta}}, \quad (18)$$

where $\hat{\pi}$ is the labor share of net income, wages, and other employee benefits in industry i . The second equality arises from solving for y_i from the equilibrium condition $\mu_i = \beta y_i + (1-\beta)rU_j$. As one can see, this restriction allows for the identification of β given estimates of f_i and μ_i from (13) as well as rU_j for all j .

The first competitive search equilibrium condition implements (6). In other words, we constrain

$$u_i p_i = f_i (1-u) s, \quad (19)$$

for all i . This is done to improve efficiency of the estimates and to test for the presence of sub-markets. However, the rejection of this condition is a necessary, but not sufficient, condition for sub-markets. Therefore, its existence does not reject alternative assumptions such as workers and firms draw match specific productivity. However, it does provide evidence against it.

To identify rU_j for all j , we use an equilibrium condition from the directed search assumption and assume all worker types are indifferent between at least two sub-markets. As a result, there exist at least two restrictions

$$p_i = (r+s) \frac{rU_j - z_j}{\mu_i - rU_j} \quad (20)$$

for each type of worker. Therefore, identification of rU_j and z_j is possible given μ_i , p_i and s from (13), and r . Note that identification of rU_j is lost if (20) is rejected. Also, if workers choose from more than two sub-markets, then the parameters are over-identified and additional constraints, which represent additional markets, can be tested. If they were rejected, then we would need to introduce a parametric assumption on F to identify rU_j following [Eckstein and Wolpin \(1995\)](#). Under this strategy, the model and likelihood function would be left unchanged, although it would be based on a more restrictive (and ad hoc) assumption regarding the distribution of productivity. However, in the estimation we find that additional markets are not rejected and so proceed without

the additional parametric assumption.

3.4 Empirical Findings

We estimate the model for the aggregate and by industry following the approach of [van den Berg and Ridder \(1998\)](#) and others; an approach that assumes segmented markets. We have provided estimates for eight firm types as the measure of fit does not improve significantly with additional heterogeneity in productivity, i.e. a higher n .

We construct four specifications of the likelihood function to test the three distinct equilibrium conditions that differentiate the competitive search model. Each specification becomes more restrictive. Specifically, Specification A is the base case with restrictions (14)-(18), restrictions that are present in competitive search model as well as many others including random search. Specification B adds the constraint in (19) for all i in addition to (14)-(18). Specification C incorporates (20) for all i in addition to (14)-(19). We allow for only one z and rU , which we discuss below. Finally, Specification D is the most restrictive and incorporates the Hosios condition, $\beta = \eta$, in addition to (14)-(20). Therefore, Specification A versus B tests restriction (19), i.e., whether workers search in markets with a particular level of productivity. Specification B versus C tests restriction (20), i.e., whether the directed search assumption holds since it requires a negative relationship between wages and job finding rates. In addition, the comparison between Specifications B and C doubles as a sufficient condition to test for homogeneous workers as we introduce only one rU and z . Finally, Specification C versus D tests the restriction $\beta = \eta$ and whether the market is efficient.

Before going further, it is important to note that all the parameters of the competitive search model are identified in Specifications C & D. However, it is not true for Specifications A & B. We have allowed under-identification because of the discussion in [Section 3.3](#) on what it would take to completely identify it. To reiterate, rU is not identified in the first two specifications but could be by making a parametric assumption on F and using the lowest wage to determine rU . Therefore, the likelihood values and our results would change if we completely identified the model in Specification A. However, testing the restrictions of the competitive search model would be limited by an ad-hoc parametric assumption on the productivity distribution. Therefore, we have chosen the less restrictive semi-parametric approach to allow for a cleaner test of the competitive search restrictions rather than focusing on identification in the first two stages.

The log likelihood values, as well as the test results for each specification, are presented in Table 2. The estimates for Specifications C and D are presented in Table 3 and Table 5, respectively. Bootstrapped standard errors are given in separate tables, Table 4 and Table 6, immediately following the estimates for Specification C and D. The reported standard errors provide confidence bands at the 5th and 95th percentiles.

As shown, we fail to reject that workers search in sub-markets with singular levels of productivity for the aggregate as well as in all industries examined. In other words, we fail to reject Specification B for all the samples considered. As shown in the table, the test statistics are quite small. The primary reason is the test effectively restricts the proportion of people in each section of the wage distribution to have an equivalent proportion of people entering the unemployed state at a particular rate (the rates are restricted in the next specification). The test statistics are small because the data display only two to three different exit rates (depending upon the sample) while the number of markets is set to eight due to the heterogeneity in wages. Therefore, if one thinks of separating the two or three different exit rates (or specifically the proportion of people with each exit rate) into eight different bins (with a specific proportion of workers in each bin as defined by the wage distribution), then it becomes clear why the test lacks power and therefore does not reject the restriction.

The next test, B vs. C, restricts wages to have a particular relationship with the exit rates. Specifically, as the exit rate rises, the wage falls as workers are willing to trade off lower wages for less time unemployed. The test fails to reject this relationship by industry. It also fails to reject that workers are homogeneous within industries as we allow for only one z and rU . However, we note that the CPS does not allow for the construction of a joint distribution between wages and duration times, and therefore reduces the power of this test.

Finally, C vs. D shows that we reject efficiency for the aggregate but fail to reject that the segmented labor markets are efficient in half of the industries, specifically (i) construction, (ii) trade, transportation, & utilities, and (iii) Leisure & Hospitality. Alternatively, we reject efficiency in (i) manufacturing, (ii) Professional & Business Services, and (iii) Education & Health Services at the 1% level. The estimates of η and β under Specification C show why some are rejected while others are not. Specifically, the estimates for Specification C shows the bargaining power in the manufacturing sector is too low relative to the elasticity of the matching function. However, Education & Health Services shows the opposite. The bargaining parameter for Professional &

Business Services is close to the average, however it is too low compared to their relatively high elasticity of the matching function. In comparison to the related literature, the elasticity of the matching function is high relative to [Flinn \(2006\)](#) but similar to the calibration of [Shimer \(2005\)](#). The bargaining power parameter is relatively high, but not out of line with the related literature. The estimates for the non-parametric wage distribution are not surprising while the estimates of the arrival rates of jobs are disperse as previously found in [Eckstein and Wolpin \(1995\)](#) and [Engelhardt \(2009\)](#).

As further comparison, we plot in Figures 1-7 the estimates from each specification relative to the data. As shown, the estimates for the wage distribution and arrival rates of jobs fit the empirical distributions very closely. In [Table 2](#), it is shown we reject Specification C for B in the aggregate sample. However, graphically the comparison shows almost no difference. Alternatively, we reject efficiency, or Specification D for C, in several markets and the differences are visually distinguishable, especially for the Education & Health Services.

4 Conclusion

We estimate and test the restrictions of a competitive search model. The maximum likelihood estimation procedure uses micro-level wage and duration data to determine the relationship between wages and the arrival rates of jobs. In addition, the JOLTS vacancy data is used to estimate the elasticity of the matching function while IRS income data identifies the worker-firm surplus splitting wage rule.

The estimation is performed on aggregate U.S. data and also on data disaggregated by major industry classifications. The results fail to reject that workers direct their search to sub-markets or firms with a particular level of productivity; while posting (i.e. efficiency via [Hosios \(1990\)](#)) is rejected in three of the six industries considered as well as in the aggregate.

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Table 1: Descriptive Statistics

	Aggregate	Construction	Manufacturing	Trade, Transportation, & Utilities	Professional & Business Services	Education & Health Services	Leisure & Hospitality
	<u>CPS data</u>						
w	18.15 (12.12)	19.03 (9.91)	19.8 (12.09)	15.77 (10.61)	21.59 (15.33)	18.22 (11.68)	11.79 (9)
t	17.06 (21.79)	13.91 (16.33)	20.38 (23.91)	16.18 (21.99)	18.49 (22.74)	18.35 (24.73)	15.54 (20.6)
\hat{u}	0.043	0.075	0.041	0.045	0.056	0.023	0.073
	<u>JOLTS data</u>						
\hat{v}	0.031	0.015	0.023	0.025	0.053	0.023	0.045
	<u>IRS data</u>						
$\hat{\pi}$	0.515	0.474	0.46	0.644	0.61	0.765	0.714
	<u>Demographic Information from CPS</u>						
High School	0.884	0.789	0.862	0.876	0.912	0.948	0.74
College	0.295	0.108	0.233	0.18	0.428	0.457	0.142
Female	0.476	0.096	0.31	0.41	0.44	0.755	0.532
Non-white	0.156	0.09	0.15	0.149	0.162	0.176	0.198
Married	0.578	0.596	0.64	0.555	0.585	0.608	0.357
Age	41.23 (13.67)	39.95 (12.76)	42.5 (11.98)	40.54 (14.22)	41.48 (13.34)	42.66 (13.16)	34.01 (14.64)
N	68,639	5,472	7,683	13,437	6,814	14,524	5,865

Note: Standard deviations are in parenthesis, unemployment duration is weekly, wages are hourly, and those who did not report hours worked were excluded. Those reporting a $w > 100$ were compressed to $w = 100$, but otherwise the wage data was not trimmed. Also, wages are only available for roughly one-third of those employed due to the structure of the CPS questionnaire.

Table 2: Specification Tests

	Aggregate	Construction	Manufacturing	Trade, Transportation, & Utilities	Professional & Business Services	Education & Health Services	Leisure & Hospitality
<i>ln L</i> under Specification							
A	-60,518.4	-4,465.58	-7,379.08	-11,401.91	-5,987.13	-12,578.8	-5,248.58
B	-60,518.52	-4,465.64	-7,379.09	-11,401.96	-5,987.14	-12,578.91	-5,248.66
C	-60,529.59	-4,466.91	-7,380.22	-11,406.93	-5,988.11	-12,581.45	-5,249.97
D	-60,602.44	-4,467.08	-7,384.74	-11,406.96	-5,999.45	-12,592.31	-5,250.56
Test of sub-markets (A vs. B)							
LR test	0.24	0.1	0.03	0.09	0.02	0.23	0.15
p-value	1	1	1	1	1	1	1
Test of Homogeneous workers (B vs. C)							
LR test	22.12	2.54	2.25	9.95	1.94	5.07	2.62
p-value	0	0.86	0.9	0.13	0.92	0.54	0.85
Test of efficiency with homogeneous workers (C vs. D)							
LR test	145.71	0.34	9.05	0.05	22.69	21.73	1.19
p-value	0	0.56	0	0.83	0	0	0.28

Note: The number of restrictions and degrees of freedom used in the likelihood ratio test of sub-markets is equal to the number of firm types ($n = 8$) minus one as the identity, (14), is already imposed. Also, the degrees of freedom used in the likelihood ratio test of homogeneous workers is equal to the number of firm types minus two.

Table 3: Specification C - Competitive Search Estimates without Efficiency Constraint

	Aggregate	Construction	Manufacturing	Trade, Transportation, & Utilities	Professional & Business Serv.	Education & Health Services	Leisure & Hospitality
p_1	0.133	0.112	0.102	0.136	0.135	0.197	0.116
f_1	0.51	0.44	0.48	0.57	0.41	0.1	0.74
μ_1	10.11	11.29	11.51	9.49	10.13	10.75	8.01
p_2	0.078	0.08	0.064	0.078	0.088	0.187	0.066
f_2	0.25	0.28	0.26	0.24	0.24	0.47	0.16
μ_2	18.42	19.2	19.26	17.37	17.41	11.1	16.14
p_3	0.055	0.067	0.046	0.054	0.061	0.075	0.047
f_3	0.12	0.07	0.12	0.11	0.14	0.26	0.07
μ_3	26.81	24.44	27.6	25.58	26.2	21.02	23.95
p_4	0.041	0.058	0.036	0.038	0.046	0.045	0.033
f_4	0.07	0.15	0.08	0.05	0.1	0.11	0.02
μ_4	35.91	29.92	35.91	36.58	35.62	31.94	34.92
p_5	0.032	0.046	0.028	0.03	0.037	0.031	0.026
f_5	0.028	0.043	0.034	0.016	0.053	0.033	0.007
μ_5	46.49	39.19	46.78	46.79	46.03	44.35	44.71
p_6	0.026	0.036	0.023	0.024	0.029	0.024	0.021
f_6	0.015	0.012	0.018	0.01	0.041	0.013	0.008
μ_6	57.59	52.66	56.39	58.33	58.37	55.87	57.49
p_7	0.021	0.027	0.019	0.019	0.024	0.018	0.017
f_7	0.005	0.001	0.005	0.004	0.01	0.007	0.003
μ_7	71.85	72.13	70.41	72.72	72.55	71.62	71.94
p_8	0.016	0.02	0.014	0.014	0.018	0.013	0.012
f_8	0.002	0.001	0.002	0.001	0.005	0.001	0.001
μ_8	97.23	100	92.52	99.98	97.7	99.01	99.95
σ_w	3.34	3.02	3.28	2.97	3.15	3.82	2.59
β	0.536	0.566	0.481	0.659	0.643	0.711	0.754
η	0.716	0.598	0.721	0.655	0.884	0.605	0.781
s	0.004	0.007	0.003	0.004	0.004	0.002	0.007
rU	-1.56	-8.49	-1.74	-1.06	-3.24	4.47	-2.66
$\ln L$	-60,529.6	-4,466.9	-7,380.2	-11,406.9	-5,988.1	-12,581.4	-5,250.

Table 4: 5% & 95% Confidence Bands for Specification C Estimates

	Aggregate	Construction	Manufacturing	Trade, Transportation, & Utilities	Professional & Business Serv.	Education & Health Services	Leisure & Hospitality
p_1	(0.12,0.15)	(0.09,0.14)	(0.05,0.13)	(0.12,0.16)	(0.09,0.2)	(0.15,0.2)	(0.1,0.14)
f_1	(0.49,0.53)	(0.35,0.48)	(0.46,0.51)	(0.53,0.6)	(0.38,0.45)	(0.1,0.14)	(0.68,0.76)
μ_1	(9.9,10.3)	(10.4,11.6)	(11.2,11.7)	(9.1,9.7)	(9.8,10.4)	(10.6,10.8)	(7.6,8.2)
p_2	(0.07,0.08)	(0.07,0.09)	(0.04,0.08)	(0.07,0.09)	(0.07,0.11)	(0.15,0.18)	(0.06,0.08)
f_2	(0.24,0.27)	(0.01,0.32)	(0.23,0.29)	(0.22,0.27)	(0.2,0.27)	(0.42,0.48)	(0.14,0.19)
μ_2	(17.9,19)	(16.4,19.6)	(19,19.4)	(16.2,18.2)	(17.1,17.6)	(11,11.2)	(13.9,17.1)
p_3	(0.05,0.06)	(0.06,0.08)	(0.04,0.05)	(0.05,0.06)	(0.05,0.07)	(0.07,0.08)	(0.04,0.06)
f_3	(0.11,0.13)	(0.05,0.29)	(0.1,0.14)	(0.09,0.13)	(0.12,0.17)	(0.24,0.28)	(0.05,0.1)
μ_3	(26,27.7)	(20,28.6)	(26.4,28.6)	(23.6,26.7)	(25.2,27.2)	(20.1,21.6)	(20.3,25.5)
p_4	(0.04,0.04)	(0.05,0.07)	(0.03,0.04)	(0.03,0.04)	(0.04,0.05)	(0.04,0.05)	(0.03,0.04)
f_4	(0.06,0.07)	(0.05,0.18)	(0.06,0.09)	(0.04,0.06)	(0.08,0.12)	(0.1,0.12)	(0.01,0.03)
μ_4	(35.2,36.8)	(28.7,35.7)	(34.8,37.2)	(32.8,37.8)	(34.4,36.8)	(29.5,32.9)	(27.4,37.1)
p_5	(0.03,0.03)	(0.04,0.06)	(0.03,0.04)	(0.03,0.03)	(0.03,0.04)	(0.03,0.04)	(0.02,0.03)
f_5	(0.02,0.03)	(0.02,0.06)	(0.03,0.04)	(0.01,0.03)	(0.04,0.07)	(0.03,0.05)	(0,0.02)
μ_5	(45.8,47.2)	(37.3,42.6)	(45.2,48.3)	(42,48.7)	(45,47)	(39.1,46.3)	(37.8,49.2)
p_6	(0.02,0.03)	(0.03,0.05)	(0.02,0.04)	(0.02,0.03)	(0.03,0.03)	(0.02,0.03)	(0.02,0.02)
f_6	(0.01,0.02)	(0.01,0.02)	(0.01,0.02)	(0.01,0.01)	(0.03,0.05)	(0.01,0.02)	(0,0.01)
μ_6	(57.1,58.2)	(48.3,54.9)	(54.9,57.5)	(56.1,59.9)	(57.6,59.2)	(51.2,57.9)	(53.1,60.6)
p_7	(0.02,0.02)	(0.01,0.04)	(0.02,0.03)	(0.02,0.02)	(0.02,0.03)	(0.02,0.02)	(0.01,0.02)
f_7	(0,0.01)	(0,0.01)	(0,0.01)	(0,0.01)	(0.01,0.02)	(0.01,0.01)	(0,0.01)
μ_7	(71.2,72.6)	(54,95.3)	(69,71.7)	(70.9,74.2)	(71.9,73.3)	(70.2,72.7)	(62,76.9)
p_8	(0.01,0.02)	(0.01,0.03)	(0.01,0.03)	(0.01,0.02)	(0.02,0.02)	(0.01,0.02)	(0.01,0.02)
f_8	(0,0)	(0,0)	(0,0)	(0,0)	(0,0.01)	(0,0)	(0,0)
μ_8	(95.5,98.7)	(58.2,100.1)	(86.9,98)	(100,100)	(95.1,100)	(97.4,100)	(72.6,100)
σ_w	(3.26,3.41)	(2.61,3.19)	(3.09,3.4)	(2.77,3.09)	(2.94,3.35)	(3.75,3.83)	(2.3,2.68)
β	(0.52,0.56)	(0.49,0.74)	(0.43,0.82)	(0.63,0.7)	(0.59,0.72)	(0.71,0.74)	(0.71,0.8)
η	(0.7,0.73)	(0.57,0.62)	(0.68,0.82)	(0.63,0.68)	(0.83,0.94)	(0.59,0.66)	(0.74,0.82)
s	(0,0)	(0.01,0.01)	(0,0)	(0,0)	(0,0.01)	(0,0)	(0.01,0.01)
rU	(-3.3,-0.1)	(-40.4,-1.2)	(-86.5,2.4)	(-4.5,1)	(-14.3,1.7)	(2.2,4.7)	(-7.6,0.2)

Note: Bootstrapping with 500 draws were used to determine the intervals.

Table 5: Specification D - Competitive Search Estimates with Efficiency Constraint

	Aggregate	Construction	Manufacturing	Trade, Transportation, & Utilities	Professional & Business Services	Education & Health Services	Leisure & Hospitality
p_1	0.073	0.104	0.056	0.136	0.059	0.133	0.117
f_1	0.51	0.44	0.48	0.57	0.42	0.18	0.74
μ_1	10.09	11.29	11.5	9.49	10.15	11.14	8.01
p_2	0.064	0.079	0.051	0.077	0.058	0.132	0.07
f_2	0.25	0.28	0.26	0.24	0.24	0.4	0.16
μ_2	18.41	19.2	19.26	17.37	17.66	11.19	16.14
p_3	0.056	0.068	0.046	0.053	0.057	0.052	0.051
f_3	0.12	0.07	0.12	0.11	0.14	0.26	0.07
μ_3	26.8	24.32	27.6	25.58	26.4	21.19	23.95
p_4	0.05	0.059	0.043	0.038	0.056	0.032	0.036
f_4	0.07	0.15	0.08	0.05	0.1	0.11	0.02
μ_4	35.9	29.89	35.91	36.58	35.69	32.09	34.92
p_5	0.044	0.049	0.038	0.03	0.054	0.022	0.029
f_5	0.028	0.043	0.034	0.016	0.052	0.031	0.008
μ_5	46.49	39.18	46.78	46.79	46.06	44.53	44.72
p_6	0.04	0.039	0.035	0.024	0.053	0.017	0.023
f_6	0.015	0.012	0.018	0.01	0.039	0.012	0.009
μ_6	57.6	52.67	56.4	58.33	58.38	55.97	57.52
p_7	0.035	0.03	0.032	0.019	0.052	0.013	0.019
f_7	0.006	0.001	0.005	0.004	0.01	0.007	0.003
μ_7	71.87	72.15	70.42	72.71	72.56	71.59	71.97
p_8	0.029	0.023	0.027	0.014	0.049	0.009	0.014
f_8	0.002	0.001	0.002	0.001	0.005	0.001	0.001
μ_8	97.27	100.03	92.55	99.98	97.75	98.98	99.98
σ_w	3.33	3.02	3.28	2.97	3.16	3.84	2.59
$\eta = \beta$	0.789	0.604	0.786	0.656	0.97	0.708	0.772
s	0.003	0.006	0.002	0.004	0.003	0.002	0.007
rU	-45.78	-13.2	-65.86	-0.85	-424.93	4.64	-4.2
$\ln L$	-60,602.4	-4,467.1	-7,384.7	-11,407.	-5,999.5	-12,592.3	-5,250.6

Table 6: 5% & 95% Confidence Bands for Specification D Estimates

	Aggregate	Construction	Manufacturing	Trade, Transportation, & Utilities	Professional & Business Services	Education & Health Services	Leisure & Hospitality
p_1	(0.07,0.08)	(0.09,0.12)	(0.05,0.06)	(0.12,0.16)	(0.06,0.2)	(0.12,0.15)	(0.1,0.14)
f_1	(0.49,0.57)	(0.33,0.48)	(0.45,0.51)	(0.53,0.6)	(0.38,0.6)	(0.1,0.53)	(0.68,0.76)
μ_1	(9.9,10.8)	(10.3,11.6)	(11.4,11.6)	(9.1,9.7)	(9.7,12.3)	(10.8,11.5)	(7.7,8.2)
p_2	(0.06,0.07)	(0.07,0.09)	(0.04,0.06)	(0.07,0.08)	(0.06,0.13)	(0.12,0.15)	(0.06,0.08)
f_2	(0.02,0.27)	(0.14,0.32)	(0.23,0.29)	(0.22,0.27)	(0.01,0.27)	(0.03,0.51)	(0.14,0.19)
μ_2	(11.5,19.7)	(15.2,20)	(18.4,19.9)	(16.2,18.2)	(16.1,23.4)	(10.8,11.6)	(14.1,17.2)
p_3	(0.05,0.06)	(0.06,0.08)	(0.04,0.05)	(0.05,0.06)	(0.06,0.09)	(0.05,0.06)	(0.05,0.06)
f_3	(0.09,0.24)	(0.04,0.25)	(0.1,0.15)	(0.09,0.13)	(0,0.2)	(0.23,0.28)	(0.05,0.09)
μ_3	(20.1,27.7)	(20.1,28.8)	(26.1,28.9)	(23.7,26.9)	(20.7,34.6)	(19.9,22.1)	(20.6,25.5)
p_4	(0.05,0.06)	(0.05,0.06)	(0.04,0.05)	(0.04,0.04)	(0.05,0.07)	(0.03,0.04)	(0.03,0.04)
f_4	(0.06,0.11)	(0.04,0.18)	(0.06,0.09)	(0.04,0.06)	(0,0.14)	(0.09,0.12)	(0.01,0.03)
μ_4	(28.5,36.6)	(28.8,36.1)	(34.7,37.2)	(32.7,37.9)	(26,45.9)	(29,33.2)	(27.8,37.5)
p_5	(0.04,0.05)	(0.04,0.05)	(0.04,0.04)	(0.03,0.03)	(0.05,0.06)	(0.02,0.03)	(0.03,0.03)
f_5	(0.03,0.06)	(0.01,0.06)	(0.02,0.04)	(0.01,0.03)	(0.01,0.11)	(0.02,0.05)	(0,0.02)
μ_5	(36.6,47.1)	(37.6,43.1)	(45.5,48.2)	(42.1,48.8)	(34.6,58.5)	(37.2,46.5)	(37.9,49.3)
p_6	(0.04,0.04)	(0.04,0.04)	(0.03,0.04)	(0.02,0.02)	(0.05,0.06)	(0.02,0.02)	(0.02,0.03)
f_6	(0.01,0.03)	(0.01,0.02)	(0.01,0.02)	(0.01,0.01)	(0.01,0.06)	(0.01,0.03)	(0,0.01)
μ_6	(46.6,58.1)	(49.1,54.9)	(55,57.6)	(55.9,59.9)	(45.1,72.8)	(46.9,58.1)	(53.3,61)
p_7	(0.03,0.04)	(0.01,0.04)	(0.03,0.03)	(0.02,0.02)	(0.05,0.05)	(0.01,0.02)	(0.02,0.02)
f_7	(0,0.02)	(0,0)	(0,0.01)	(0,0.01)	(0,0.05)	(0,0.01)	(0,0.01)
μ_7	(57.8,72.5)	(55.4,95.3)	(69,71.6)	(70.8,74.5)	(58.1,97.2)	(57,72.7)	(63.8,76.9)
p_8	(0.03,0.03)	(0.01,0.03)	(0.03,0.03)	(0.01,0.01)	(0.01,0.05)	(0.01,0.01)	(0.01,0.02)
f_8	(0,0.01)	(0,0)	(0,0)	(0,0)	(0,0.02)	(0,0.01)	(0,0)
μ_8	(77.9,98.8)	(75.1,100.2)	(87.1,99.1)	(100,100)	(77,100)	(76,100)	(99.9,100)
σ_w	(3.3,3.9)	(2.6,3.2)	(3.1,3.3)	(2.8,3.1)	(3,4.5)	(3.6,4.3)	(2.3,2.7)
$\eta = \beta$	(0.77,0.8)	(0.57,0.63)	(0.75,0.82)	(0.63,0.68)	(0.7,0.98)	(0.69,0.72)	(0.73,0.81)
s	(0,0)	(0.01,0.01)	(0,0)	(0,0)	(0,0.01)	(0,0)	(0.01,0.01)
rU	(-51.8,-40.1)	(-17,-9.5)	(-89.5,-50.7)	(-2.9,0.7)	(-555.4,-10.6)	(3.6,5.6)	(-8.8,-0.9)

Note: Bootstrapping with 500 draws were used to determine the intervals.

Figure 1: Aggregate

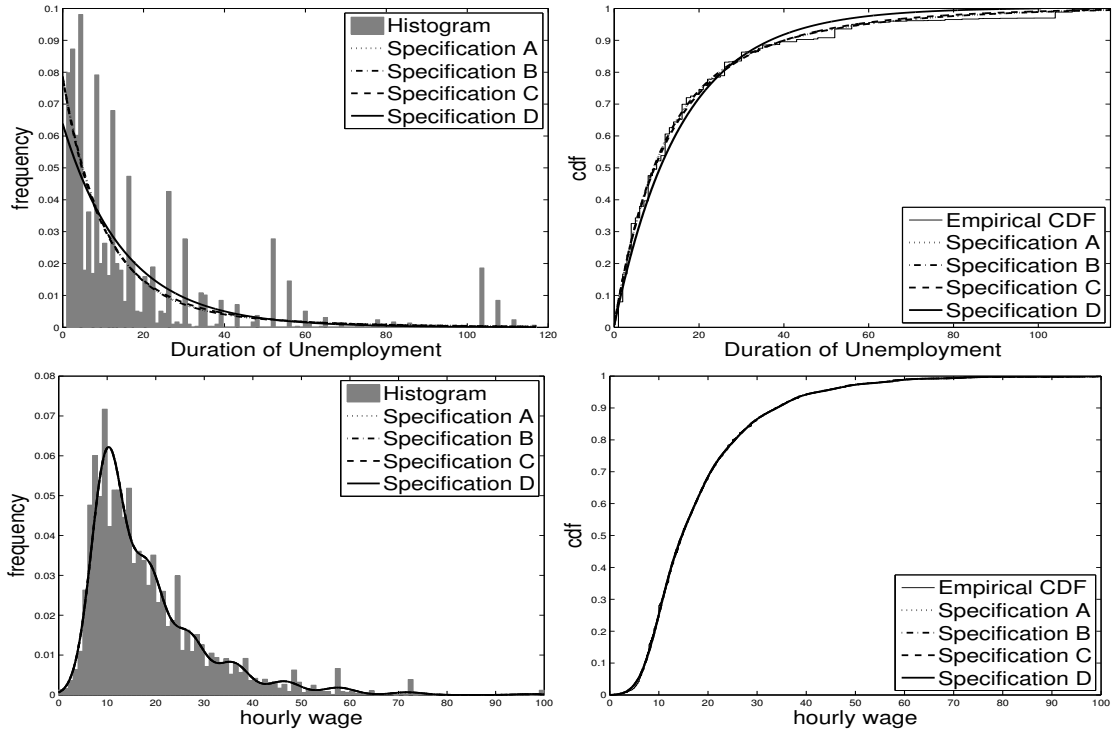


Figure 2: Construction

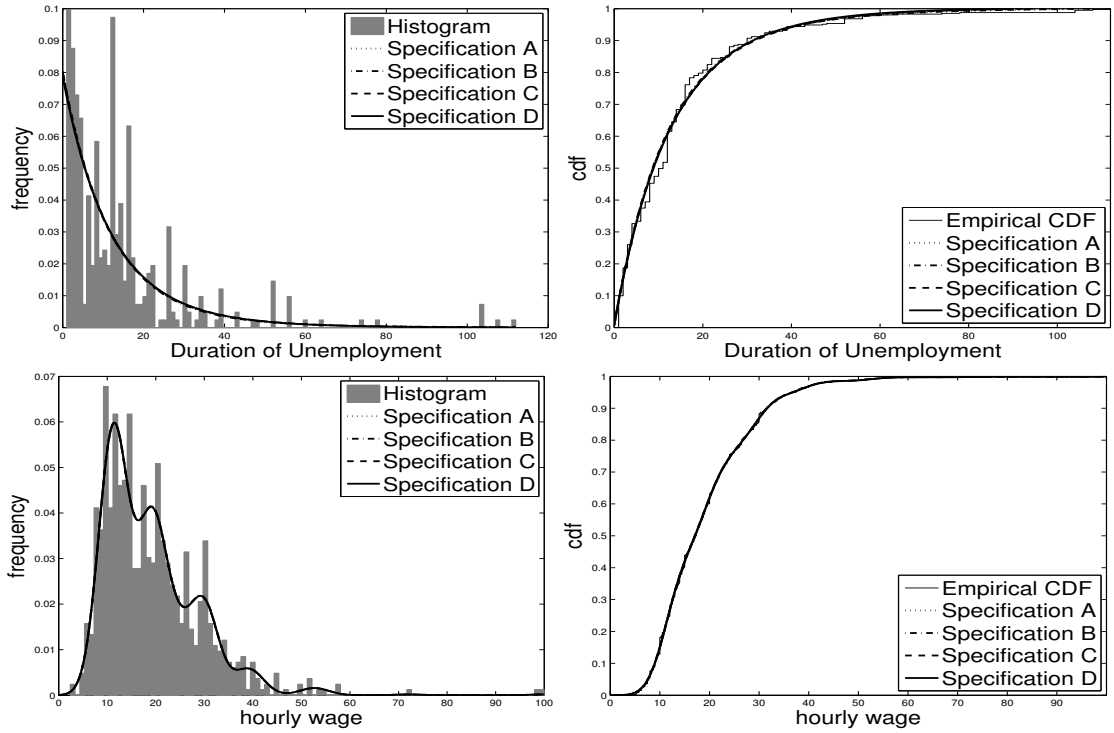


Figure 3: Manufacturing

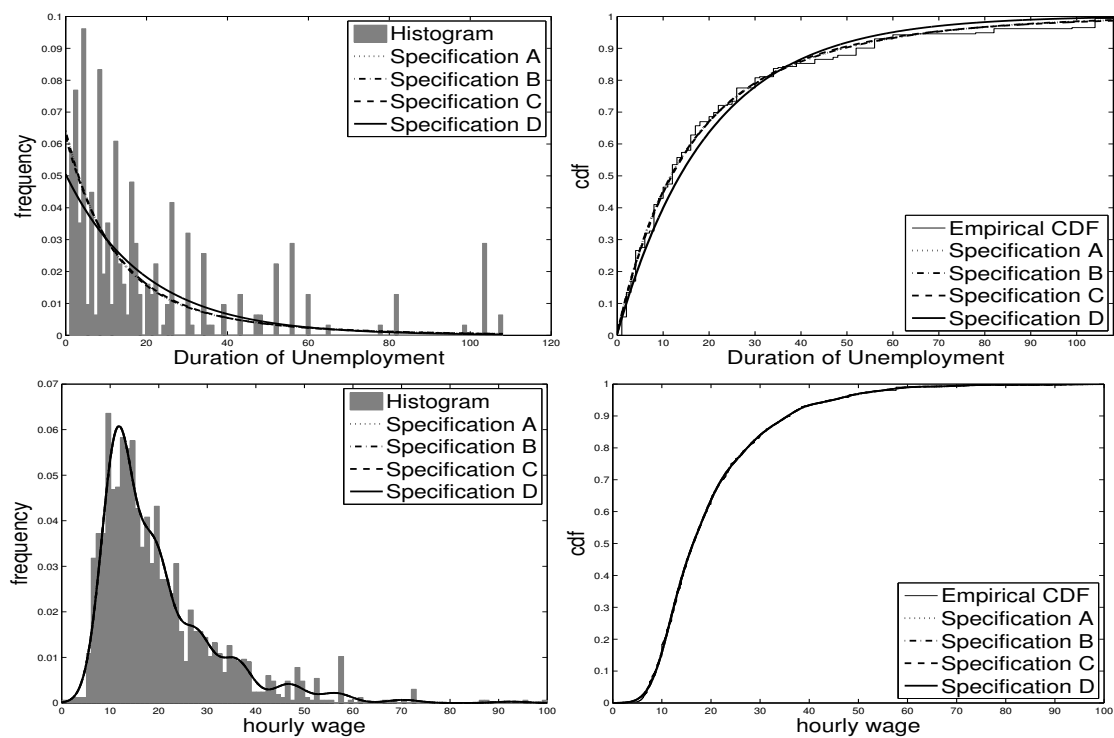


Figure 4: Trade, Transportation, & Utilities

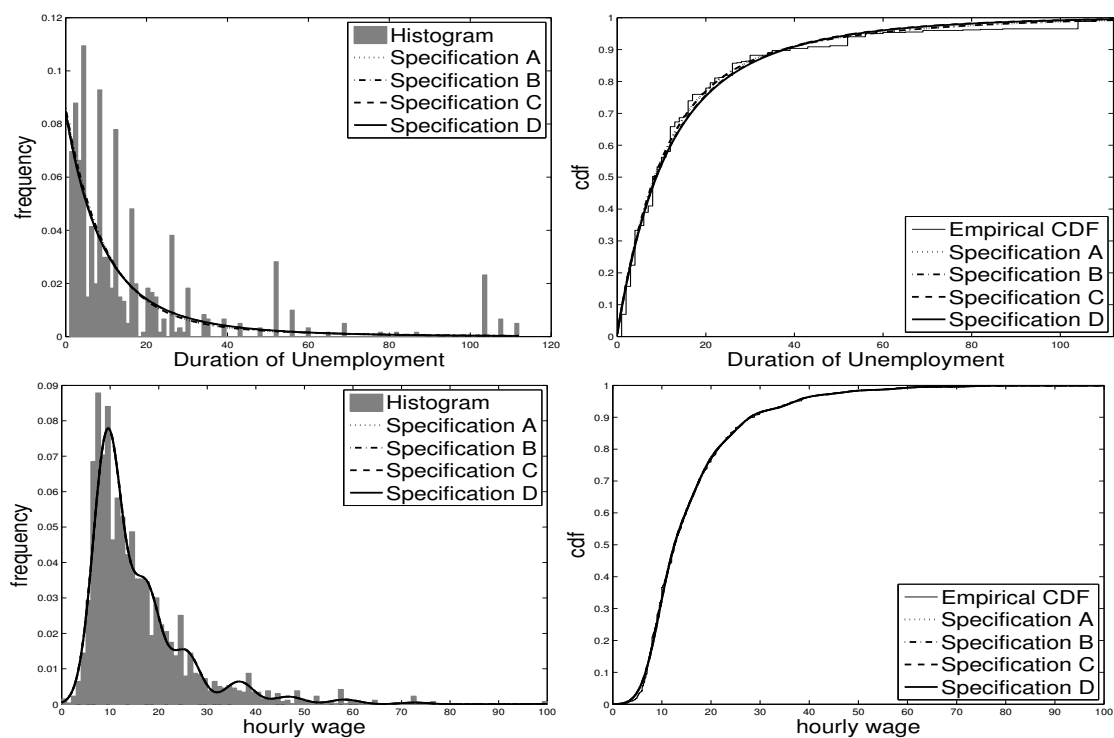


Figure 5: Professional & Business Services

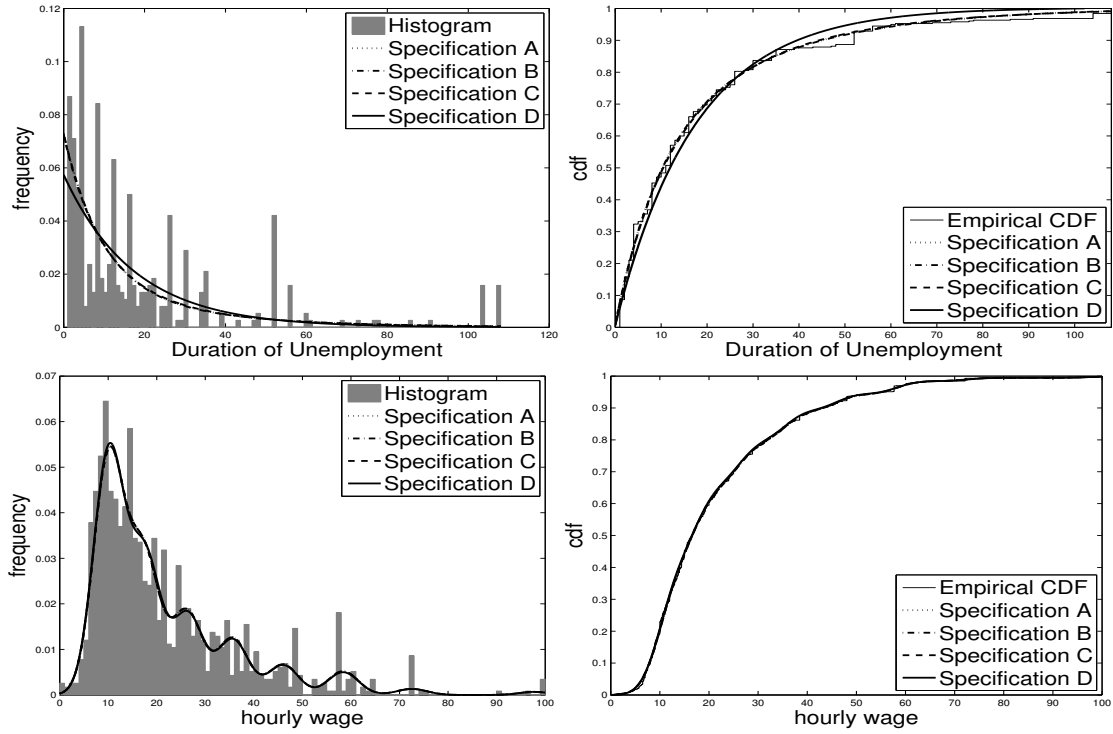


Figure 6: Education & Health Services

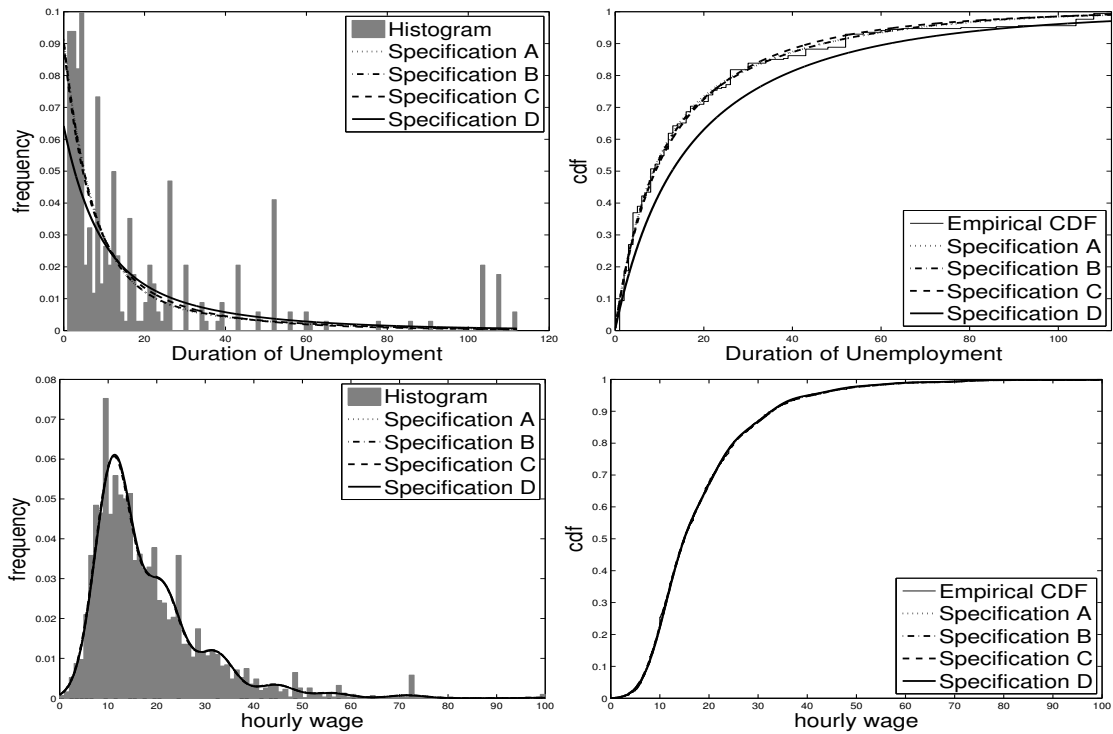


Figure 7: Leisure & Hospitality

