

# Private and Public Liquidity, Financial Intermediation, and Monetary Policy

Stephen D. Williamson  
Washington University in St. Louis  
Richmond Federal Reserve Bank  
St. Louis Federal Reserve Bank

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## **Abstract**

A model of monetary exchange with private financial intermediation is constructed. Claims on financial intermediaries of two types are traded in transactions: circulating notes and deposits. There can be a role for the government in supplying liquidity, and level changes in the money supply accomplished through open market operations can be nonneutral. A Friedman rule is suboptimal, due to costs of maintaining the stock of currency. The model is used to address some issues related to current monetary policy in the United States.

# 1 Introduction

Liquidity consists of a class of assets that are somehow useful in exchange. Some of these liquid assets are government liabilities. In the United States, for example, Federal Reserve notes serve as a medium of exchange in retail transactions, deposits with the Federal Reserve are used as a medium of exchange in inter-bank transactions, and Treasury bills play an important role as collateral in financial transactions. As well, there are liquid assets that are the liabilities of private financial intermediaries, or the products of these intermediaries. Banks issue deposit liabilities which can be traded using debit cards and checks, and securitized loans can be traded on financial markets or can serve as collateral in financial transactions. Further, in the past, for example in the United States prior to the Civil War, in Scotland in the early 19th century, and in Canada before 1935, private banks could issue small-denomination liabilities that circulated, sometimes alongside government-issued currency. To further complicate matters, private financial intermediaries that issue liquid liabilities also hold liquid government liabilities as assets.

Conventional wisdom holds that the main role of a central bank is to manage public liquidity in a manner that controls inflation, and influences the provision of private liquidity and credit. However, the mechanism by which central bank actions affect prices and quantities still appears to be poorly understood. As evidence for this, consider the dramatic actions taken by the Federal Reserve System during 2008 and 2009, and the reaction of economists to these actions. There is considerable disagreement about the implications and appropriateness of these actions, both inside and outside of the Federal Reserve System. How do these dramatic interventions matter for inflation and for real activity?

The main purpose of this paper is to build a model of public and private liquidity to answer some basic questions in monetary economics. What is the role of a central bank? What is a liquid asset, and what roles do privately-provided and publicly-provided liquid assets play in exchange? Do open market operations matter, and if so, why? How should we evaluate the recent actions of the Federal Reserve System?

The model here builds on the basic New Monetarist framework of Lagos and Wright (2005) (see also Williamson and Wright 2009). The model adds a financial intermediation sector, based on a costly state verification friction, as in Williamson (1987), and on risk-sharing, as in Diamond-Dybvig (1983). One attraction of this type of intermediation model is that it yields a rich structure of financial arrangements, including debt contracts and endogenous default premia. As well, the model yields coexistence of deposit claims and currency, and determines liquidity premia on tradeable assets, along the lines of Lagos (2008), Lester, Postlewaite, and Wright (2009), and Lagos and Rocheteau (2008).

The first step in the analysis is to consider equilibria without a government. In such an environment, private financial intermediaries issue two types of tradeable claims: circulating notes and deposits. These two types of intermediary liabilities exploit the available information technologies for making transactions.

Under some conditions, private financial intermediaries will not provide sufficient liquid assets in equilibrium to achieve efficiency in exchange, and the economy will have too much private financial intermediation in equilibrium. These are circumstances where the government can do better.

We model the government as having the ability to tax, to ban private circulating notes, and to issue two types of liabilities: nominal bonds and outside money. In the policy regime we study, fiscal policy governs the rate of growth in the total quantity of nominal government liabilities, while monetary policy determines the composition of the nominal debt. Under conditions such that the private sector fails to supply sufficient liquidity, there always exists a government policy which achieves an optimal allocation of resources. The government's outside assets and power to tax permit it to correct the "insufficient liquidity" problem, but the critical source of liquidity turns out to be bonds, rather than money.

In a regime where there is insufficient liquidity, a one-time open market purchase is not neutral. Holding constant the rate of growth in total nominal government debt, if there is a one-time increase in the ratio of money to bonds, this permanently reduces the real interest rate, and increases the quantity of lending - there is a type of "credit channel" effect of monetary policy. Thus, in this regime the central bank may view itself as a powerful institution with the ability to manipulate GDP at will. However, if the government were setting policy optimally, the central bank would not have this power - monetary policy is essentially neutral at the optimum.

There are at least two ways for the government to achieve optimality. First, it could ban circulating private notes, increase the total nominal debt at the appropriate rate, and provide a sufficiently large ratio of bonds to money to achieve efficiency. Second, it could issue sufficient government debt to achieve efficiency, and permit unrestricted issue of private circulating notes. Thus, in the model, central banking is not critical to achieving efficiency.

An important result is that a Friedman rule is not optimal. This follows from the fact that it is costly to maintain the stock of currency. That is, there are costs associated with replacing worn-out currency and designing the currency to prevent counterfeiting, that result in a deviation from the Friedman rule. Just as in Sanches and Williamson (2008), where we consider the role of theft as a cost of operating a monetary system, the higher are the costs of operating the monetary system, the larger will be the deviation from the Friedman rule.

An interesting application of the model is to current monetary policy in the United States, as the model permits us to study a wide range of central bank interventions. We first consider a monetary policy under which the nominal interest rate on government debt is zero temporarily, with the expectation that the policy will be "unwound" in the future. Given the zero nominal interest rate, if the central bank injects more outside money in the present than is necessary to achieve a zero nominal interest rate, then this outside money will be hoarded as bank reserves, with no effect on any prices or quantities. Essentially, there is a liquidity trap. However, the government could act to issue outside money in the present to finance private lending, using the returns on its portfolio to retire the

money in the future. If the government lends on the same terms as do private sector intermediaries, then its lending will simply displace an equal quantity of private lending, and the stock of outside money can increase by a large amount in the present with no effects on quantities and prices. However, if the government lends on better terms than does the private sector, this will reduce nominal interest rates on loans, and expand lending. In this case, however, the returns on the central bank portfolio are insufficient to unwind the monetary injection, and taxes must be levied to retire the additional outside money in the future. This causes a redistribution from taxpayers to borrowers.

The paper is organized as follows. The first chapter is a description of the model, while the second chapter contains the details of the intermediary structure. Then, Chapter 3 contains an analysis of equilibrium without government, while the Chapter 4 analysis concerns equilibrium with a government. Finally, Chapter 5 applies the model to issues concerning recent monetary policy in the United States.

## 2 The Model

The basic model is builds on Lagos-Wright (2005), with an information structure related to Sanches and Williamson (2009), and a financial intermediation sector similar to Williamson (1987). Time is indexed by  $t = 0, 1, 2, \dots$ , and there are two subperiods within each period that we denote day and night.

The population consists of three types of economic agents: buyers, sellers, and entrepreneurs. There is a continuum of buyers with mass one, and each buyer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)],$$

Here,  $0 < \beta < 1$ ,  $H_t$  denotes the difference between labor supply and consumption during the day,  $x_t$  is consumption in the night, and  $u(\cdot)$  is a strictly increasing, strictly concave, and twice continuously differentiable function with  $u(0) = 0$ ,  $u'(0) = \infty$ , and with the property that there exists some  $\hat{q} > 0$  such that  $u(\hat{q}) - \hat{q} = 0$ . Define  $q^*$  by  $u'(q^*) = 1$ . There is a continuum of sellers with unit mass, and each seller has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t],$$

where  $X_t$  is consumption in the day and  $h_t$  is labor supply in the night. The production technology potentially available to buyers and sellers allows the production of one unit of the perishable consumption good for each unit of labor supply. Buyers can produce only in the day, and sellers only in the night, so we have one of the necessary ingredients for monetary exchange - a double coincidence problem.

During the day of each period, a continuum of entrepreneurs with mass  $\alpha$  is born, and each lives until the day of the following period. An entrepreneur has no endowment during his or her life. An entrepreneur born in the day of period  $t$  consumes only in the day of period  $t + 1$  and is risk neutral. An entrepreneur has access to an investment project. This project is indivisible and requires one unit of the consumption good in the day of period  $t$  to operate, and yields a return of  $w$  in the day of period  $t$ , where  $w$  is distributed according to the distribution function  $F(w)$ , with associated density function  $f(w)$ , which is strictly positive on  $[0, \bar{w}]$ , where  $\bar{w} > 0$ . Assume also that  $f(\cdot)$  is continuously differentiable. Investment project returns are independent across entrepreneurs. The return  $w$  is private information to the entrepreneur, but subject to costly state verification, whereby any other individual can bear a fixed cost and observe  $w$  ex post. The verification cost  $\gamma$  is entrepreneur-specific, and  $G(\gamma)$  denotes the distribution of verification costs across entrepreneurs, with  $\gamma \geq 0$ .

During the night, each buyer is matched at random with a seller. The seller in a match is not able to observe the buyer's history, and the seller will never have an opportunity to signal default on a credit arrangement, so the seller will not accept a personal IOU in exchange for goods. A fraction  $\rho$  of nighttime bilateral meetings are *not monitored*, in the sense that, if the buyer wants to acquire goods from the seller, he or she must have a claim to goods in the next day, where the claim is somehow documented in an object that the buyer carries. In what follows, these objects may be circulating liabilities issued by financial intermediaries or fiat money issued by the government. In each case, the issuer of the object must incur a cost to render the object recognizable for what it is. We will specify this in more detail in what follows. A fraction  $1 - \rho$  of buyers and sellers are in *monitored* meetings at night. In these meetings, though a credit transaction cannot take place between the buyer and the seller, a communication technology is costlessly available which permits the buyer to transfer ownership of a claim on a financial intermediary to the seller. Again, we will spell out more details in what follows.

An important feature of the environment is that, when production and consumption decisions are made during the day, buyers do not know whether they will be in a non-monitored or a monitored meeting with a seller in the following night. Each buyer learns this information, which then becomes public, at the end of the day. This will give rise to a Diamond-Dybvig (1983) risk-sharing role for financial intermediaries.

### 3 Financial Intermediation

We will assume that stochastic verification is not feasible. Assume that entrepreneurs are economic agents who are subject to full commitment. Then, as in Williamson (1987), an efficient lending arrangement is for individual entrepreneurs to act as perfectly-diversified financial intermediaries. Efficient and incentive-compatible loan contracts with entrepreneurs take the form of non-contingent debt. That is, the financial intermediary observes the verification

cost  $\gamma$  associated with the entrepreneur in the daytime of period  $t$  and offers him or her a contract that specifies a non-contingent payment  $R_t(\gamma)$  that the entrepreneur must make to the intermediary in the day of period  $t+1$ . If the entrepreneur cannot make the loan payment, then default occurs, the intermediary incurs the verification cost  $\gamma$ , observes the return  $w$ , and seizes it. As shown in Williamson (1987), the expected payoff to the intermediary from the loan contract, as a function of the non-contingent payment  $R_t(\gamma)$  and the verification cost  $\gamma$ , is then given by

$$\pi[R_t(\gamma), \gamma] = R_t(\gamma) - \gamma F[R_t(\gamma)] - \int_0^{R_t(\gamma)} F(w)dw \quad (1)$$

Then, we can interpret  $R_t(\gamma)$  as the gross real loan interest rate on a loan to an entrepreneur of type  $\gamma$ , and

$$D_t(\gamma) = \gamma F[R_t(\gamma)] + \int_0^{R_t(\gamma)} F(w)dw \quad (2)$$

is a default premium.

Since the financial intermediary is perfectly diversified (this requires only that it hold a positive mass of loans to entrepreneurs), it can guarantee a certain return per unit invested, which we denote  $r_t$ . In equilibrium, the payoff per unit invested will be the same for each loan made by the financial intermediary, so

$$r_t = R_t(\gamma) - \gamma F[R_t(\gamma)] - \int_0^{R_t(\gamma)} F(w)dw \quad (3)$$

for each entrepreneur who receives a loan. Differentiating the intermediary's expected payoff function in (1), we obtain

$$\pi_1[R_t(\gamma), \gamma] = 1 - \gamma f[R_t(\gamma)] - F[R_t(\gamma)],$$

$$\pi_{11}[R_t(\gamma), \gamma] = -\gamma f'[R_t(\gamma)] - f[R_t(\gamma)]$$

and assume that  $-\gamma f'(w) - f(w) < 0$  for all  $w \in [0, \bar{w}]$  and for all  $\gamma \geq 0$ . Then  $\pi[R_t(\gamma), \gamma]$  is strictly concave in  $R_t(\gamma)$  for  $R_t(\gamma) \in [0, \bar{w}]$  and attains a maximum for  $R_t(\gamma) = \hat{R}(\gamma) < \bar{w}$ , where

$$1 - \gamma f[\hat{R}(\gamma)] - F[\hat{R}(\gamma)] = 0. \quad (4)$$

In equilibrium, there is a marginal entrepreneur in each period  $t$ , with verification cost  $\gamma_t^*$  and facing the gross loan interest rate  $R_t^*$ , where, from (4),

$$1 - \gamma_t^* f(R_t^*) - F(R_t^*) = 0, \quad (5)$$

so that the gross loan interest rate faced by the marginal entrepreneur maximizes the expected return to the financial intermediary given the marginal entrepreneur's verification cost  $\gamma_t^*$ . Further, the financial intermediary earns an expected return  $r_t$  from lending to the marginal borrower, or from (3),

$$r_t = R_t^* - \gamma_t^* F(R_t^*) - \int_0^{R_t^*} F(w)dw \quad (6)$$

Then, each entrepreneur who receives a loan has  $\gamma \leq \gamma_t^*$ , and if  $\gamma < \gamma_t^*$  then  $R_t(\gamma) < \hat{R}(\gamma)$ . Entrepreneurs with  $\gamma > \gamma_t^*$  do not receive loans as, even if  $R_t(\gamma) = \hat{R}(\gamma)$  for one of these agents, so that the expected return to the intermediary is maximized, the intermediary will have an expected loss from the loan. That is, verification costs for the set of entrepreneurs with  $\gamma > \gamma_t^*$  are too high for lending to be profitable.

Then, the total quantity of loans extended by financial intermediaries during the day of period  $t$  is given by

$$L_t = \alpha G(\gamma_t^*). \quad (7)$$

Therefore, given the certain return on investment  $r_t$ , the loan quantity  $L_t$ , the verification cost of the marginal borrower,  $\gamma_t^*$ , and the gross loan interest rate faced by the marginal borrower  $R_t^*$ , are determined by (5), (6), and (7). It is straightforward to show that

$$\frac{dL_t}{dr_t} = -\frac{\alpha G'(\gamma_t^*)}{F(R_t^*)} < 0 \quad (8)$$

Thus, given an increase in the payoff per unit of lending by the financial intermediary, the quantity of lending by financial intermediaries must decline. This is because, from (3), the loan interest rate for each entrepreneur receiving a loan will increase, and it will be unprofitable for an intermediary to lend to a formerly marginal entrepreneur, once the payoff per unit of loans increases. Further, since the loan interest rate will be higher for each creditworthy entrepreneur when the deposit rate increases, from (2) each of these creditworthy entrepreneurs will be faced with a higher default premium.

Given (8), we can write  $L_t = L(r_t)$ , where  $L(\cdot)$  is a decreasing function. Though there is some rich detail in how entrepreneurs' investment projects are funded and the structure of loan interest rates and default premia across projects, fundamentally the mechanics of investment are similar to what we would obtain under standard production technology with a decreasing marginal product of capital and 100% depreciation.

## 4 Equilibrium Without Government Activity

To understand the role of a central bank in our model, it helps to see how this economy will perform in the absence of a government. Suppose then, that there is no central bank or fiscal authority. During the daytime of period  $t$ , buyers make production decisions and entrepreneurs make investment decisions, before buyers learn whether they are making non-monitored or monitored transactions during the night of period  $t$ . During the day, buyers produce, make deposits with financial intermediaries, and the intermediaries lend to entrepreneurs. The deposit contracts written by buyers with financial intermediaries provide for particular withdrawal privileges, to be specified below. After these actions are taken, each buyer learns whether he or she will be engaged in a non-monitored or a monitored transaction during the night.

If the buyer will meet a seller in the night in a non-monitored transaction, then (as part of the deposit contract previously written with the financial intermediary) the buyer withdraws *circulating notes* from the financial intermediary. Circulating notes are claims on the financial intermediary that are non-replicable, and any seller can verify that these claims are what they purport to be. For the intermediary, circulating notes are costly to issue, requiring  $\sigma$  units of goods for each unit of consumption goods the notes will trade for in nighttime exchange. This cost is incurred to encode on these notes a non-replicable message that indicates that the noteholder has a claim on the financial intermediary. If the buyer learns at the end of the day that he or she will be in a monitored meeting, then the buyer does not withdraw notes. When a buyer meets a seller in a monitored meeting, the buyer and seller are able to contact the bank and transfer deposit claims from the buyer to the seller. Note that this communication technology is not available in a non-monitored meeting.

Now, let  $x_t$  denote the quantity of goods deposited with the financial intermediary by each buyer in the daytime of period  $t$ ,  $y_{t+1}^n$  the payoff to each noteholder in the financial intermediary in the daytime of period  $t+1$ , and  $y_{t+1}^d$  the payoff for each depositor in period  $t+1$  who did not withdraw circulating notes. Assume that, in a bilateral meeting between a buyer and seller in the night, that the buyer makes a take-it-or-leave-it offer. Then, if all circulating notes are traded away by buyers in the night (as we will show is equilibrium behavior), the cost to the financial intermediary of issuing circulating notes is  $\sigma\beta y_{t+1}^n$ . This then implies that the zero-profit condition for the financial intermediary is

$$(x_t - \rho\sigma\beta y_{t+1}^n) r_t = \rho y_{t+1}^n + (1 - \rho) y_{t+1}^d. \quad (9)$$

In equation (9), the left-hand side is the payoff per unit invested in loans to entrepreneurs by the financial intermediary, which is total deposits minus the cost of note issue, all multiplied by the gross expected return per loan. On the right hand side are the total payouts by the intermediary to noteholders and deposit-holders respectively.

An equilibrium deposit contract  $(x_t, y_{t+1}^n, y_{t+1}^d)$  maximizes the expected utility of the buyer, and satisfies the zero-profit condition (9). Therefore, an equilibrium deposit contract solves

$$\max_{x_t, y_{t+1}^n, y_{t+1}^d} \left( -x_t + \rho u(\beta y_{t+1}^n) + (1 - \rho) \max_{\hat{y}_{t+1}^d \leq y_{t+1}^d} \{ u[\beta(y_{t+1}^d - \hat{y}_{t+1}^d)] + \beta \hat{y}_{t+1}^d \} \right)$$

subject to (9). In the optimization problem above,  $\hat{y}_{t+1}^d$  is the quantity of deposits not traded away by deposit-holders in monitored transactions during the night. As the demand for intermediary deposits would be infinite if  $r_t > \frac{1}{\beta}$ , in equilibrium we must have

$$r_t \leq \frac{1}{\beta},$$

which implies that in equilibrium  $(x_t, y_{t+1}^n, y_{t+1}^d)$  satisfies

$$u'(\beta y_{t+1}^n) = \frac{1 + \sigma r_t \beta}{r_t \beta}, \quad (10)$$



$$u'(\beta y_{t+1}^d) = \frac{1}{r_t \beta}, \text{ if } r_t < \frac{1}{\beta}, \quad (11)$$

$$y_{t+1}^d \geq \frac{q^*}{\beta}, \text{ if } r_t = \frac{1}{\beta}, \quad (12)$$

and (9).

Note that the solutions for  $y_{t+1}^n$  and  $y_{t+1}^d$  from (10)-(12) give the same consumption for buyers in monitored and non-monitored meetings as we would obtain in equilibrium if buyers knew in the daytime whether they would be in monitored or non-monitored meetings before consumption and production decisions are made. However, if that were the case, then production would be different in the daytime for monitored and non-monitored buyers. In the model we are considering here, the deposit contract implies that each buyer makes the same deposit, so that there is an element of Diamond/Dybvig-type insurance in the financial intermediation arrangement.

#### 4.1 Case 1: Sufficient Liquidity

We will first construct an equilibrium where some buyers will choose to carry deposit claims from one day to the next. Confine attention to stationary equilibria where  $r_t = r$  for all  $t$ . Buyers will want to carry over deposit claims from a monitored nighttime trade to the next day if and only if  $r = \frac{1}{\beta}$  and consumption for buyers in a monitored trade is  $q^*$ , the surplus-maximizing quantity. In this equilibrium,  $y_{t+1}^n = \tilde{y}^n$  for all  $t$  and  $y_{t+1}^d = \tilde{y}^d$  for all  $t$ . From (10),  $\tilde{y}^n$  is determined by

$$u'(\beta \tilde{y}^n) = 1 + \sigma. \quad (13)$$

Therefore  $\beta \tilde{y}^n < q^*$ , so that the quantity traded in non-monitored meetings during the night is less than the surplus-maximizing quantity, due to the cost of note issue. As  $\sigma$  increases, consumption by buyers in nonmonitored trades falls, from (13). The total quantity of loans will be  $L\left(\frac{1}{\beta}\right)$ . Therefore, given the zero-profit condition for the financial intermediary, (9), we have

$$L\left(\frac{1}{\beta}\right) \frac{1}{\beta} = \rho \tilde{y}^n + (1 - \rho) \tilde{y}^d. \quad (14)$$

Then, from (12) and (14), in equilibrium we must have

$$L\left(\frac{1}{\beta}\right) \geq \beta \rho \tilde{y}^n + (1 - \rho) q^*. \quad (15)$$

Inequality (15) is what we mean by “sufficient liquidity.” This condition states that, when the real interest rate is equal to the rate of time preference, there is a sufficiently large quantity of liquid assets to support an efficient level of transactions in monitored trades, along with the associated equilibrium level of transactions in non-monitored trades.

We can restate inequality (15) in terms of the underlying loan contracts between financial intermediaries and entrepreneurs, using (5), (6), and (7). Inequality (15) becomes

$$\alpha G(\gamma^*) \geq \beta \rho \tilde{y}^n + (1 - \rho)q^*,$$

where the verification cost for the marginal entrepreneur,  $\gamma^*$ , and the gross loan interest rate faced by the marginal entrepreneur,  $R^*$  solve

$$1 - \gamma^* f(R^*) - F(R^*) = 0,$$

and

$$\frac{1}{\beta} = R^* - \gamma^* F(R^*) - \int_0^{R^*} F(w)dw.$$

Thus, whether there is sufficient liquidity or not will depend on the functions  $F(\cdot)$  and  $G(\cdot)$ . In general, if returns on investment projects are higher, say in the sense of first-order stochastic dominance for the distribution  $F(\cdot)$ , then the economy's capacity to produce private liquidity is greater. Also, if verification costs are lower across the population of entrepreneurs, i.e. if  $G(\cdot)$  shifts upward, then this will tend to increase private liquidity capacity.

## 4.2 Case 2: Insufficient Liquidity

Next, the case with insufficient liquidity arises if and only if

$$L\left(\frac{1}{\beta}\right) < \beta \rho \tilde{y}^n + (1 - \rho)q^*. \quad (16)$$

Here, buyers will trade away all notes and deposit claims in monitored trades and  $r < \frac{1}{\beta}$ . Then, from (11) and (12) we get

$$u'(\beta y^n) = \frac{1 + \sigma r \beta}{r \beta}, \quad (17)$$

$$u'(\beta y^d) = \frac{1}{r \beta}. \quad (18)$$

and the zero-profit condition for financial intermediaries (9) gives

$$L(r)r = \rho y^n + (1 - \rho)y^d \quad (19)$$

Then, equations (17)-(19) solve for  $y^n$ ,  $y^d$ , and  $r$ . Assume that

$$-\frac{cu''(c)}{u'(c)} \leq \frac{1}{1 + \sigma}, \text{ for } c > 0. \quad (20)$$

**Proposition 1** *If (20) and (16) hold, then there exists a unique equilibrium with insufficient liquidity with  $r < \frac{1}{\beta}$ .*

**Proof.** Define  $z^n \equiv \frac{y^n}{r}$  and  $z^d \equiv \frac{y^d}{r}$ , and rewrite (17)-(19) as

$$\frac{r}{1 + \sigma r \beta} u'(\beta r z^n) = \frac{1}{\beta}, \quad (21)$$

$$r u'(\beta r z^d) = \frac{1}{\beta}, \quad (22)$$

$$L(r) = \rho z^n + (1 - \rho) z^d. \quad (23)$$

Then, (21), (22) and (20) imply that, for  $r \leq \frac{1}{\beta}$ , we can write  $z^n = \psi^n(r)$  and  $z^d = \psi^d(r)$  with  $\psi^{n'}(r) > 0$ ,  $\psi^{d'}(r) > 0$ ,  $\psi^n(0) = \psi^d(0) = 0$ ,  $\psi^n(\frac{1}{\beta}) = \beta \hat{y}^n$ , and  $\psi^d(\frac{1}{\beta}) = q^*$ . From (23), the equilibrium level of  $r$  is determined by

$$L(r) = \rho \psi^n(r) + (1 - \rho) \psi^d(r). \quad (24)$$

Since the left-hand side of (24) is continuous and strictly decreasing and the right hand side of (24) is continuous and strictly increasing for  $r \in [0, \frac{1}{\beta}]$ , with  $L(0) > \rho \psi^n(0) + (1 - \rho) \psi^d(0)$ , and  $L(\frac{1}{\beta}) < \rho \psi^n(\frac{1}{\beta}) + (1 - \rho) \psi^d(\frac{1}{\beta}) = \beta \rho \hat{y}^n + (1 - \rho) q^*$  (from (16)), therefore there is a unique solution for  $r \in (0, \frac{1}{\beta})$ , and we also solve for unique  $z^n$  and  $z^d$ , and in turn for unique  $y^n$  and  $y^d$ . ■

In an equilibrium with insufficient liquidity, the economy does not have enough intertemporal productive capacity to support an efficient quantity of exchange in monitored transactions. From (17)-(19), and (20), an increase in the cost of note issue,  $\sigma$ , causes  $y^n$  to fall, so that buyers consume less in non-monitored exchanges during the night. As well, when  $\sigma$  increases,  $r$  increases in equilibrium, the quantity of lending falls, and  $y^d$  increases, so that buyers consume more in monitored exchanges in the night.

## 5 Equilibrium with a Government

Thus far, we have shown how this economy performs without a government. Financial intermediaries act to efficiently channel investment funds to entrepreneurs, and the liabilities of financial intermediaries, which can be interpreted as circulating currency and bank deposits, are exchanged in decentralized transactions. Financial intermediaries not only perform a delegated monitoring role, also act to insure provide insurance against the need for liquid assets in different types of decentralized transactions. Without a government, there is a limit to the quantity of liquid assets that the private sector can supply for use in transactions. This limit is determined by the quantity of investment projects, verification costs, and the costs of providing circulating notes.

In this section, we introduce a government, so that we can address questions related to public vs. private provision of liquidity. As we will see, both interest-bearing and non-interest-bearing government liabilities will play important roles, with fiscal policy determining the total stock of government liabilities

and monetary policy determining the mix of interest-bearing vs. non-interest-bearing liabilities.

We will assume that the government in our model has the power to tax, and the power to prohibit the private issue of circulating notes, but otherwise has available the same technologies as does the private sector. Each buyer receives a lump-sum transfer  $\tau_t$  in real terms, from the government during the daytime of period  $t$ . The government issues two liabilities. The first is fiat currency, which is perfectly divisible and, just as is the case for privately issued circulating notes, is costly to maintain. To prevent counterfeiting and to encode the information that makes it clear that the currency was issued by the government requires that  $\sigma$  units of goods be absorbed in the daytime of period  $t$  for each unit of goods the currency trades for at night. The second government liability is a one-period nominal bond. Nominal bonds are issued during the day of period  $t$ , with each selling for one unit of currency and paying off  $z_{t+1}$  units of currency in the day of period  $t+1$ . Nominal bonds are accounting balances held with the government that can be transferred using the same communications technology discussed previously in the context of nighttime monitored transactions. Thus, government bonds cannot be traded in non-monitored transactions, for the same reasons that deposits cannot be traded in these transactions.

Letting  $\phi_t$  denote the price of currency in terms of goods in the daytime of period  $t$ , we can then write the government budget constraint as

$$\phi_t [M_t - M_{t-1}] + \phi_t B_t = \sigma q_t + \phi_t z_t B_{t-1} + \tau_t, \quad (25)$$

for  $t = 1, 2, \dots$ . Here,  $M_t$  denotes the money stock in period  $t$ ,  $B_t$  is the stock of nominal bonds, and  $q_t$  is the real value of the money stock in exchange during the night of period  $t$ . The left-hand side of (25) is the revenue from money creation and bond issue, while the right-hand side is the cost of maintaining the stock of money, plus total payouts by the government to retire outstanding bonds, plus transfers. In period 0,

$$\phi_0 M_0 + \phi_0 B_0 = \sigma q_0 + \tau_0, \quad (26)$$

so that private agents are endowed with no outside assets at the first date.

At this stage, we will assume that private sector intermediaries are prohibited from issuing circulating notes in this regime, so that it is not possible to use private note issue to finance lending to entrepreneurs or the purchase of government bonds. However, private intermediaries can hold government bonds and issue deposits. In equilibrium, it will be irrelevant whether buyers hold government bonds directly or the deposits of institutions that intermediate government bonds, so for convenience we will assume that all government bonds intermediated, with intermediary deposits backed by loans to entrepreneurs and nominal government bonds.

As in the economy without a government, during the day each buyer produces and makes a deposit with the financial intermediary. Here, the intermediary holds a portfolio of nominal government bonds, loans to entrepreneurs, and currency. Buyers who learn that they will meet sellers during the night in

non-monitored trades withdraw currency from the financial intermediary at the end of the day. In equilibrium, the intermediary must be indifferent between holding nominal government bonds and loans to entrepreneurs, so

$$r_t = \frac{\phi_{t+1} z_{t+1}}{\phi_t}.$$

Further, in equilibrium bonds and loans to entrepreneurs will dominate currency in rate of return, since deposit claims cannot be traded in non-monitored exchanges at night, but currency can be exchanged in all transactions. As well, no gross rate of return can exceed  $\frac{1}{\beta}$ , since this would represent an arbitrage opportunity. Therefore, in equilibrium,

$$\frac{\phi_{t+1}}{\phi_t} \leq r_t \leq \frac{1}{\beta}. \quad (27)$$

Now, given (27), and since there is no aggregate uncertainty, the financial intermediary will hold only enough currency to satisfy the withdrawal demands of buyers in non-monitored trades. Let  $m_t$  denote the real quantity of money balances acquired in the daytime of period  $t$  by the financial intermediary, per depositor. As well, let  $d_t$  denote the real quantity of government bonds and loans to entrepreneurs acquired by the intermediary, per depositor. It is then straightforward to show, in a manner similar to the analysis in the previous section, that  $m_t$  solves

$$\left( \frac{\beta \phi_{t+1}}{\phi_t} \right) u' \left( \frac{\beta \phi_{t+1}}{\phi_t} m_t \right) = 1, \quad (28)$$

$d_t$  solves

$$\beta r_t u'(\beta r_t d_t) = 1, \text{ if } r_t < \frac{1}{\beta}, \quad (29)$$

or

$$d_t \geq q^*, \text{ if } r_t = \frac{1}{\beta}. \quad (30)$$

In equilibrium each buyer deposits  $m_t + d_t$  with the financial intermediary in the daytime of period  $t$ . Thus, as in the regime without a government, the financial intermediary provides Diamond-Dybvig-type insurance. However, in contrast to the Diamond-Dybvig (1983) setup, the bank here insures against the state where the buyer is in a monitored trade.

In equilibrium, all assets must be willingly held. Therefore,

$$(1 - \rho) d_t = L(r_t) + \phi_t B_t, \quad (31)$$

and

$$\rho m_t = \phi_t M_t. \quad (32)$$

Finally, the government budget constraints (25) and (26) must hold.

## 5.1 Government Policy

In this subsection, we set up a policy regime in which the total nominal government debt grows at a constant rate, determined by fiscal policy, and monetary policy consists of determining the composition of the total outstanding debt. In the daytime of period 0, the government begins by transferring  $M_0$  units of fiat money and  $B_0$  nominal bonds in equal amounts to buyers, with the real value of the transfer, from (26), determined by the equilibrium price of money,  $\phi_0$ . Then, assume that in the daytime of each period  $t = 1, 2, 3, \dots$ , the government levies taxes on buyers to pay the interest on the government bonds that come due, and to pay the costs of maintaining the currency stock. Further, during the daytime of periods  $t = 1, 2, 3, \dots$ , the government gives a nominal transfer of  $M_t - M_{t-1}$  units of money and  $B_t - B_{t-1}$  nominal bonds to each buyer. Then, it is straightforward to verify that the government budget constraint (25) holds for each  $t = 1, 2, 3, \dots$ .

Now, let  $\mu$  denote the constant gross rate of growth of currency outstanding and nominal bonds, with  $M_t = \mu M_{t-1}$  and  $B_t = \mu B_{t-1}$  for  $t = 1, 2, 3, \dots$ . This implies that the ratio of the currency stock to the stock of bonds is constant over time. For convenience, let  $\delta$  denote the fraction of nominal debt outstanding that consists of currency, i.e.

$$M_t = \delta(M_t + B_t) \quad (33)$$

for all  $t$ , where  $0 \leq \delta \leq 1$ . We can then interpret  $\mu$  as being determined by fiscal policy, and  $\delta$  by monetary policy, though in the model there is nothing that distinguishes the fiscal authority from the monetary authority.

### 5.1.1 Equilibrium with Sufficient Liquidity

We will confine attention to stationary equilibria with valued fiat currency, which implies that

$$\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu} \quad (34)$$

for all  $t$ . As in the case with no government, if there is sufficient liquidity available, then in monitored exchange during the night an efficient quantity  $q^*$  is traded, and the gross return on loans and bonds is  $r = \frac{1}{\beta}$ . From (28) and (34), we get

$$\frac{\beta}{\mu} u' \left( \frac{\beta}{\mu} \hat{m} \right) = 1, \quad (35)$$

which solves for the real quantity of currency per buyer in non-monitored trades,  $\hat{m}$ . Then, from (31), (32), and (33), in equilibrium,

$$d = \frac{L(\frac{1}{\beta})}{1 - \rho} + \frac{(1 - \delta)\rho\hat{m}}{\delta(1 - \rho)} \quad (36)$$

Therefore, from (30) and (??), there is sufficient liquidity if and only if

$$L(\frac{1}{\beta}) + \frac{(1 - \delta)\rho\hat{m}}{\delta} \geq (1 - \rho)q^*,$$

where the left-hand side is the quantity of lending to entrepreneurs at the gross real return  $\frac{1}{\beta}$ , plus the equilibrium real quantity of government debt, and the right-hand side is the total quantity of goods traded in efficient exchanges in monitored meetings at night. Here,  $\hat{m}$  is determined by (35). Thus, the real quantity of bonds is determined by the demand for currency in non-monitored transactions, from (35), and the government policy rule (33). We can rewrite the above inequality as

$$\frac{1 - \delta}{\delta} \geq \frac{(1 - \rho)q^* - L(\frac{1}{\beta})}{\rho\hat{m}}. \quad (37)$$

**Proposition 2** *For any  $\mu \geq \beta$  there exists a  $\delta^* > 0$  such that, if  $\delta \leq \delta^*$ , then there exists an equilibrium with a government with sufficient liquidity.*

**Proof.** Given any  $\mu \geq \beta$ , (35) determines  $\hat{m}$ . Then there exists an equilibrium with sufficient liquidity if and only if (37) holds. But (37) holds if and only if  $\delta \leq \delta^*$ , where

$$\delta^* = 1, \text{ if } q^*(1 - \rho) - L(\frac{1}{\beta}) \leq 0,$$

$$\delta^* = \frac{\rho\hat{m}}{\rho\hat{m} + (1 - \rho)q^* - L(\frac{1}{\beta})}, \text{ if } (1 - \rho)q^* - L(\frac{1}{\beta}) > 0.$$

■

Thus, for any feasible rate of growth for the total nominal government debt, there exists a critical ratio of money to nominal government bonds such that, if the ratio of money to bonds is less than this critical level, an equilibrium with sufficient liquidity exists. From (35), the real quantity of currency is determined by the rate of growth in total government liabilities, which also determines the inflation rate. Then, monetary policy, which has no effect on the real quantity of money, determines the real quantity of bonds, which in part backs financial intermediary deposits. Thus, it is the real quantity of bonds that is the key source of publicly provided liquidity influenced by monetary policy. The result above states that it is always possible to find a monetary policy that results in sufficient provision of public liquidity that there is efficient trade in monitored transactions.

### 5.1.2 Equilibrium with Insufficient Liquidity

Next, look for an equilibrium in which there is insufficient liquidity to support efficient exchange in monitored meetings during the night. In this equilibrium,  $r < \frac{1}{\beta}$ , and from (29) the quantity of deposit claims held at the end of the day by each buyer who will trade in a monitored meeting at night,  $d$ , is determined by

$$\beta ru'(\beta rd) = 1. \quad (38)$$

As in the case with sufficient liquidity, the real quantity of currency held by each buyer at the end of the day who will trade in a non-monitored meeting,

$m$ , is determined by

$$\frac{\beta}{\mu} u' \left( \frac{\beta}{\mu} m \right) = 1. \quad (39)$$

Also, from (31), (32), and (33), we have

$$d = \frac{L(r)}{1-\rho} + \frac{(1-\delta)\rho m}{\delta(1-\rho)} \quad (40)$$

Equations (38)-(40) solve for  $d$ ,  $m$ , and  $r$ .

**Proposition 3** *If*

$$-\frac{cu''(c)}{u'(c)} \leq 1, \text{ for } c > 0, \quad (41)$$

*and*

$$\frac{1-\delta}{\delta} < \frac{(1-\rho)q^* - L(\frac{1}{\beta})}{\rho\tilde{m}}, \quad (42)$$

*where  $\tilde{m}$  solves*

$$\frac{\beta}{\mu} u' \left( \frac{\beta}{\mu} \tilde{m} \right) = 1, \quad (43)$$

*then an equilibrium with a government and insufficient liquidity exists and it is unique.*

**Proof.** Given  $\mu$ , equation (39) solves for the real quantity of currency which we denote  $\tilde{m}$ . Then, from (38), if (41) holds then  $d = \omega(r)$ , where  $\omega(\cdot)$  is an increasing and continuous function. Then from (40), we have

$$\omega(r) = \frac{L(r)}{1-\rho} + \frac{(1-\delta)\rho\tilde{m}}{\delta(1-\rho)}. \quad (44)$$

Now, let  $\psi(r)$  denote the right-hand side of (44), which is a decreasing and continuous function of  $r$ . We then have  $\omega(0) = 0$ ,  $\omega(\frac{1}{\beta}) = q^*$ ,  $\psi(0) = \alpha$ , and  $\psi(\frac{1}{\beta}) = \frac{L(\frac{1}{\beta})}{1-\rho} + \frac{(1-\delta)\rho\tilde{m}}{\delta(1-\rho)}$ . Then, since  $\psi(0) > \omega(0)$ , if  $\psi(\frac{1}{\beta}) < \omega(\frac{1}{\beta})$ , then a unique solution for  $r$  exists with  $r < \frac{1}{\beta}$ . Thus, the condition we need is

$$\frac{L(\frac{1}{\beta})}{1-\rho} + \frac{(1-\delta)\rho\tilde{m}}{\delta(1-\rho)} < q^*,$$

which can be manipulated to give (42). Once we have a unique solution for  $r$ , we solve uniquely for  $d$  from  $d = \omega(r)$ . ■

Now, given (41), an increase in  $\mu$  results in a decrease in  $m$ , from (39). Then, equation (44) implies that  $r$  decreases, and so the quantity of lending to entrepreneurs,  $L(r)$ , increases. Then, (38) implies that  $d$  falls. Thus, consumption falls for all buyers during the night. Thus, an increase in the growth rate in the total nominal stock of government liabilities, when there is insufficient liquidity, acts to reduce the real quantity of money, and by implication the real quantity



of government bonds in equilibrium, thus reducing the real stock of liquid assets. The real interest rate falls, and lending increases, thus increasing the stock of privately-supplied liquid assets, but this does not completely mitigate the loss of publicly-supplied liquid assets.

An increase in  $\delta$  is interpreted as a monetary policy action, in that this increases the ratio of money to bonds, holding constant the growth rate in the total stock of government liabilities. This is essentially a one-time open-market purchase of bonds. From (44), this reduces  $r$ , so that  $L(r)$ , the quantity of lending to entrepreneurs, increases. From (39),  $m$  remains unaffected, so that the price level increases in proportion to the increase in the money stock. Consumption for buyers in non-monitored trades in the night is unaffected, but consumption for buyers in monitored trades must fall.

Thus, monetary policy is not neutral, given insufficient liquidity. A one-time open market purchase, which produces a level increase in the money stock, reduces the real quantity of government-supplied liquid assets, the real interest rate falls, and there is an increase in lending. This is a key result, and appears to be a unique property that this model does not share with alternative frameworks for analyzing monetary policy. In the insufficient liquidity case, monetary policy is not neutral, but the nonneutrality does not work through conventional means, as what we typically consider to be “liquidity” - the stock of money - is invariant in real terms to the change in monetary policy. Again, the key publicly-provided liquid asset is the stock of government bonds, and monetary policy actions affect the real value of this stock, causing changes in the privately-provided liquidity and the quantity of private lending.

### 5.1.3 Optimal Government Policy

In this section we want to determine, within the class of policies under consideration, which ones are optimal. Suppose first that we look for a policy that maximizes the total net surplus in nighttime trading, minus the total cost per period of maintaining the money stock. Then, letting  $q$  and  $s$  denote, respectively, the quantity of output traded during the night in nonmonitored and monitored trades, this quantity is given by

$$W = \rho [u(q) - q] + (1 - \rho)[u(s) - s] - \sigma \rho q$$

Then, suppose that we choose  $q$  and  $s$  to maximize  $W$ . This gives  $q = \hat{q}$  and  $s = q^*$  where  $\hat{q}$  solves

$$u'(\hat{q}) = 1 + \sigma \tag{45}$$

From (35) and (37), this outcome can be supported as an equilibrium with

$$\mu = \beta(1 + \sigma) \tag{46}$$

and with  $\delta$  set to satisfy

$$\frac{1 - \delta}{\delta} \geq \frac{(1 - \rho)q^* - L\left(\frac{1}{\beta}\right)}{\rho(1 + \sigma)\hat{q}}. \tag{47}$$

Thus, the above outcome can be achieved with a unique growth rate for total nominal government debt. This optimal growth rate is larger than the Friedman-rule money growth rate, with the gap between the optimal money growth rate and the Friedman rule rate increasing with the cost of maintaining the stock of currency. The optimal nominal interest rate on bonds is  $\sigma$ , and equation (46) implies that, in equilibrium at the optimum, the inflation tax is just sufficient to finance the costs of maintaining the stock of currency. Given the optimal money growth rate, (47) states that  $\delta$ , the ratio of money to total nominal government debt, must be sufficiently small so that the stock of liquid assets backing bank deposits is sufficiently large, in real terms.

Given the government policy specified by (46) and (47), the financial intermediation arrangement we have specified allocates resources efficiently between entrepreneurs and other agents (under the restriction of pure verification strategies). Therefore, this policy is one that maximizes the sum of utilities across agents in a stationary equilibrium, within the class of policies under consideration.

Now, note that, if (15) holds, so that there is sufficient liquidity in the absence of government, that the optimal government policy achieves the same equilibrium allocation as in the case without a government, where all liquidity is supplied by private financial intermediaries. Government policy can improve on what the private sector can accomplish, only if private intermediation supplies an insufficient quantity of liquidity to support efficient exchange. Thus, in this environment, government-provided liquidity need not be essential.

However, if (16) holds, then the ability of the private sector to provide liquidity is sufficiently limited that the government can improve matters. It is important to note, though, that even in these circumstances a central bank is not necessary. An optimum could either be achieved with a prohibition on the issue of private circulating notes and an appropriate monetary policy with central banking, or by having the government supply the appropriate quantity of government debt while permitting unrestricted issue of private circulating notes. As well, if private circulating notes are banned and the government is behaving optimally, so that (47) holds, then at the margin monetary policy is irrelevant, in that changes in  $\delta$  serve only to change the price level and affect no real quantities. Monetary policy can matter in a way that appears consistent with conventional central-banking wisdom, in that an open market purchase reduces the real interest rate and increases private lending. However, monetary policy matters in this fashion only when the government is behaving suboptimally.

## 6 U.S. Monetary Policy and the Financial Crisis

There are two unusual features associated with recent monetary policy in the United States. First, the nominal interest rate on short-term government debt has been essentially zero since late 2008, which has never been the case in the history of the Federal Reserve System. For monetary economists, there is nothing unusual about a zero nominal market interest rate. Indeed, we are accus-

tomed to working with models in which a Friedman rule achieves efficiency, and does this by driving the nominal interest rate to zero in all states of the world. However, it seems clear that the U.S. economy is not currently in a permanent Friedman rule monetary regime. The Federal Reserve System appears intent on increasing the nominal interest rate on Treasury bills above zero sometime in the future, though the Fed has made no commitment as to when this will happen. Second, the Fed has engaged in some unusual types of lending, for example to non-bank financial intermediaries, and has accumulated some unusual assets (for a central bank), including commercial paper and mortgage-backed securities. These types of interventions are certainly not common currency for monetary economists, and are not the types of experiments we typically consider in the context of monetary models.

The purpose of this section is to first show what it takes, in our model, for the central bank to support an equilibrium where the nominal interest rate is temporarily zero. Then we will study the effects of an expanded role for central bank financial intermediation when the nominal interest rate is zero.

First, assume that the intention of the government is to pursue a policy in periods  $t = 1, 2, 3, \dots$ , with  $\mu = \hat{\mu}$  and  $\delta = \hat{\delta}$ . This policy may be suboptimal, but for the experiments we consider here we will treat it as fixed, and think about the effects of central bank intervention in period 0. Given the policy in periods  $t = 1, 2, 3, \dots$ , whether the economy has sufficient liquidity or not, we have  $m = \hat{m}$  determined by (35) given  $\mu = \hat{\mu}$ . Therefore, the price of money in period 1 is given by

$$\phi_1 = \frac{\rho \hat{m}}{M_1}, \quad (48)$$

given take-it-or-leave-it offers by the buyer and market clearing during the day. Further, so long as  $\hat{\mu} > \beta$ , we have  $\hat{m} < q^*$ .

Now, what policy could achieve a zero nominal interest rate on nominal government bonds in period 0? If the nominal interest rate is zero in period 0, this implies that the efficient quantity of output  $q^*$  is traded in all nighttime exchanges in period 0. This then implies that  $r_0 = \frac{1}{\beta}$ , and the gross rate of return on currency is also  $\frac{1}{\beta}$  in period 0, so

$$\phi_0 = \beta \phi_1 = \frac{\beta \rho \hat{m}}{M_1},$$

from (48). Then, since  $q^*$  is the quantity of goods exchanged in the night of period 0 for  $\frac{M_0}{\rho}$  units of money, the quantity of money required to support a zero nominal interest rate in period 0, from market clearing, is

$$M_0 = \frac{q^* M_1}{\beta \hat{m}} > M_1,$$

where the inequality follows from  $\hat{m} \leq q^*$  and  $\beta < 1$ . Thus, the money supply in period 0 must be larger than the money stock at  $t = 1$ , and sufficiently large as to produce a deflation at the rate of time preference between period 0 and period 1. This serves, as in standard Friedman rule results, to equate rates of return

among the alternative assets in the model - deposits, government bonds, and currency. The deflation is achieved by “undoing” the period 0 monetary policy in period 1. Note that the zero-nominal-interest-rate policy is not optimal here, given the suboptimality of the Friedman rule.

Now, suppose that the government is unhappy with what occurs at a zero nominal interest rate. For example, suppose that the government thinks that financial intermediaries are not lending enough, and it injects more than  $M_0$  units of outside money in period 0, hoping that intermediaries will lend this to entrepreneurs. Of course, the problem is that financial intermediaries have exhausted all available profit opportunities, and it will be optimal for private agents to simply hold any money in excess of  $M_0$  until period 1, when it is taxed away. Thus, money is hoarded for one period - in fact it is optimal for the extra outside money to be held as reserves by banks. Indeed, with a small cost of theft, the unique outcome is for all of the extra cash to be held as reserves. Given the outcomes here, we might want to call this a “liquidity trap.”

Next, suppose that the government, frustrated in its attempts to get private intermediaries to lend more, decides that it will have to take on this intermediation role for itself. For simplicity, suppose that the central bank issues  $M_0$  units of fiat money in period 0 so as to achieve a zero nominal interest rate, then issues  $\tilde{M}$  additional units of money in order to finance loans to entrepreneurs. These loans are made on the same terms that private intermediaries are willing to offer. That is, we assume that the central bank is as efficient as private intermediaries, facing the same verification costs as the private sector, and that it writes efficient debt contracts. If the central bank takes on as much loans as it can with an expected payoff of  $\frac{1}{\beta}$ , then it will have issued a total nominal quantity of money in period 0 equal to

$$M_0 + \tilde{M} = M_0 + L \left( \frac{1}{\beta} \right) \frac{1}{\phi_0}.$$

Lending to entrepreneurs will now have become unprofitable for private financial intermediaries, the stock of money will have expanded by a large amount in period 0, but all prices and quantities (at all dates) will be identical to what they were in the absence of central bank lending to entrepreneurs. In period 1, the extra outside money  $\tilde{M}$  is retired using the returns on the central bank’s portfolio. This government lending program accomplished nothing, other than to displace an equal quantity of private lending. However, the program also had no effect on prices, in spite of a large increase in the stock of outside money. The key to this result is that the extra money that was injected was fully backed by private loans, and was retired in the future.

Now, note that, even in the equilibrium with a zero nominal interest rate on deposits and government bonds, there are some interest rates that are positive. In particular, from (2), an entrepreneur with verification cost  $\gamma$  will face a nominal interest rate

$$\gamma F[R_t(\gamma)] + \int_0^{R_t(\gamma)} F(w) dw > 0,$$

which is equal to the entrepreneur's default premium, given his or her gross real loan interest rate  $R_t(\gamma)$ . Now, suppose that the central bank is even more ambitious, and decides it will lend to entrepreneurs on better terms than would private intermediaries, with the goal of reducing loan interest rates and increasing lending to entrepreneurs. This program will achieve its goal. If the central bank makes loans to entrepreneurs implying a real gross return to the central bank of  $r_0 < \frac{1}{\beta}$ , then this will increase the quantity of loans to  $L(r_0) > L\left(\frac{1}{\beta}\right)$  in period 0, and will reduce loan interest rates, conditional on borrower type. However, now the return on the central bank's portfolio is insufficient in period 1 to retire the money issued in period 0 to make the loans.

There are now two possibilities. First, the government can tax buyers in period 1 in order to carry out its commitment to its policy from  $t = 1$  on. This then implies that there is a redistribution of wealth. Buyers lose from the central bank's lending policy, while entrepreneurs gain. Second, the extra outside money issued in period 0 could remain in circulation from  $t = 1$  on. This would then imply that the zero nominal interest rate policy in period 0 is infeasible.

This policy experiment illustrates some key features of the current policy dilemma for the Federal Reserve System in the US. While the central bank may feel that it is somehow taking up the slack for private financial intermediaries that are reluctant to lend, this is an endogenous outcome. In the absence of the Fed's lending programs, the private sector would be lending more. Further, to the extent that the Fed is lending on generous terms, or investing in assets of questionable quality, its ability to retire money in the future is impaired, and the Fed could find itself unable to keep the nominal interest rate at zero or to curb the resulting inflation.

## 7 Conclusion

The model constructed here contains an explicit role for liquid assets in retail transactions. Under some circumstances, given the available information technology, what is required for making transactions is an asset like currency. However, currency is costly to produce, as counterfeiting must be deterred, the currency stock wears out over time, and information must be encoded in the currency concerning what it is a claim to. In our model, the private sector and the government possess the same currency technology, and so circulating notes can in principle be issued by the private sector. There may be circumstances in the model where a sophisticated information technology is available allowing decentralized transactions to be executed using deposit claims on financial intermediaries. These transactions correspond to retail payments using debit cards and checks. Deposit claims on financial intermediaries are backed by loans to entrepreneurs and by government bonds, which are account balances with the government. Thus, the underlying assets which are the source of liquidity for deposit claims are private loans and government bonds.

It may be the case that the economy has sufficient intertemporal productive

capacity for supplying liquid assets. In this case, efficiency can be achieved without a central bank. Otherwise, there is a role for the government in augmenting the supply of liquid assets. One way to achieve efficiency when the private sector cannot supply sufficient liquidity is for the government to ban the issue of private circulating notes, and then conduct monetary and fiscal policy appropriately. Alternatively, the government could supply the appropriate quantity of government bonds, and permit unrestricted issue of private circulating notes by financial intermediaries.

Monetary policy can be non-neutral, in that a one-time open market purchase can reduce the real interest rate permanently and increase private lending. However, this nonneutrality occurs at the margin only if the government is behaving suboptimally. At the optimum, the costs of currency issue imply a deviation from the Friedman rule at the optimum.

Our model delivers results that are favorable to a private role for the issue of circulating currency. The consensus view appears to be that it is appropriate for the government to have a monopoly in the issue of small-denomination circulating liabilities. Indeed, even Milton Friedman (Friedman 1960) reasoned that there were market failures unique to the market for currency that made government monopolization welfare-improving. However, there is ample theoretical and empirical support for the relative efficiency of private money systems. On the theoretical side, Cavalcanti and Wallace (1999), Champ, Smith, and Williamson (1996), Cavalcanti, Erosa, and Temzelides (1999), and Williamson (1999), have all constructed models showing how private money systems can work well. On the empirical side, Champ, Smith, and Williamson (1996) shows how the Canadian private money system that existed before 1935 acted to provide an elastic currency, and Smith and Weber (1999) study the similar behavior of the Suffolk banking system during the free banking era in the United States. Thus, it is not far-fetched to propose that a private money system could be efficient.

Our results concerning recent monetary policy in the United States are both encouraging and discouraging. On the encouraging side, it is certainly possible that there be large increases in the stock of outside money which do not produce inflation. This can be due either to a liquidity trap phenomenon or because the increase in the money stock is backed by private lending. In either case, prices do not change as a result of the monetary expansion because the additional outside money is retired in the future, either through taxation or by using the returns on the central bank's portfolio. On the discouraging side, when the nominal interest rate is zero, it is not possible to engineer additional increases in private lending through "monetary easing," unless the central bank lends on better terms than does the private sector. But in that case, this results in a redistribution of wealth from taxpayers to borrowers.

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