

Precautionary Saving and Aggregate Demand*

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Abstract

How do fluctuations in households' precautionary wealth contribute to the propagation of aggregate shocks? In this paper, we attempt to answer this question by formulating and estimating a tractable structural macroeconomic model of the business cycle with nominal frictions, unemployment and incomplete insurance against unemployment risk. We argue that, under those frictions, time-variations in precautionary wealth have two conflicting effects on output volatility: a stabilizing “aggregate supply” effect working through the supply of capital and potential output; and a destabilizing “aggregate demand effect” working through aggregate consumption and the output gap. We quantify these forces via a maximum-likelihood estimation of the structural parameters of the model, using as observables both aggregate and cross-sectional information (such as the extent of consumption insurance and the distributions of wealth and consumption across households). We find the impact of demand shocks on aggregates to be significantly altered by time-varying precautionary saving.

1 Introduction

How do fluctuations in households' precautionary wealth contribute to the propagation of aggregate shocks? In this paper, we attempt to answer this question by formulating and estimating a structural macroeconomic model in which the precautionary motive for holding wealth is operative and its quantitative impact on the business cycle can be evaluated. The model is designed to be sufficiently

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rich to incorporate the main theoretical channels through which changes in precautionary wealth may alter aggregate outcomes, but at the same time simple enough to allow for a simple representation of its equilibrium dynamics. This representation in turn makes it possible to incorporate a large number of state variables and structural shocks, and thereby to recover the deep parameters of the model by maximum likelihood.

On the theoretical front, our model combines three basic frictions: (i) nominal rigidities (in prices and wages), (ii) matching frictions in the labour market, and (iii) incomplete insurance and borrowing constraints. All three frictions are known, even in isolation, to capture important features of the business cycle. However, we argue that it is their interactions that matter for the aggregate impact of precautionary saving, because they give rise to a feedback loop between the latter and aggregate demand. More specifically, following an bad aggregate shock (of any type: policy, technology, preference...) that lowers aggregate demand, job creation is discouraged and unemployment risk persistently rises. Imperfectly insured households rationally respond to the rise in idiosyncratic income uncertainty by increasing precautionary wealth, thereby cutting consumption and depleting aggregate demand even more; this in turn magnifies the initial labour market contraction, further raises unemployment risk, and so on. Therefore, the endogenous response of households' precautionary wealth in an equilibrium where aggregate demand matters (due to nominal rigidities) and when unemployment risk is endogenous (thanks to labour market frictions) explains the potentially large effect of aggregate shocks on the equilibrium.

This “aggregate demand” effect of time-varying precautionary saving is, however, usually not the only one at work in incomplete-insurance economies. As is well understood at least since the work of Krusell and Smith (1998), time-varying precautionary savings also have “aggregate supply” effects working through investment and capital accumulation; more specifically, under imperfect insurance households are reluctant to dissave in bad times, so the precautionary motive tends to smooth fluctuations in investment and capital. In the absence of the aggregate demand effects described above (e.g., under full nominal flexibility), the supply effects necessarily dominates, ultimately causing output to be *less*, not more, volatile under incomplete markets than under complete markets.¹ The presence of (and competition between) the two effects implies that determining which effect dominates, and hence whether the precautionary motive ultimately makes the economy more or less volatile, essentially becomes an empirical question. Our theoretical assumptions allows for both possibilities, and it is the purpose of our modelling strategy to extract the answer to this question from the data.

From a methodological point of view, the contribution of this paper is to propose a simple an operational way to formulate incomplete-insurance models so that their solution dynamics admit a finite state-space representation. This representation is key in making the model amenable to structural

¹See, e.g., Krusell and Smith (1998).

estimation—and thereby in assessing the quantitative importance of the demand and supply effects just discussed—but it can be exploited for different reasons and in many other contexts. The achieved degree of tractability is in contrast with most earlier models with incomplete income insurance, which typically generate a full cross-sectional distribution of wealth due to the dependence of an agent’s wealth on the entire history of idiosyncratic shocks (Aiyagari, 1994; Krusell and Smith, 1998). The reduction of the dimensionality of the problem results from two underlying assumption. First, we assume that the debt limit faced by unemployed households is tighter than the natural limit, so that the households who remain unemployed are bound to face a binding debt limit in finite time.² Second, we assume a form of periodic reinsurance within a household, in the spirit of Lucas (1990). Namely, we assume that employed workers face perfect insurance within their own “representative family”, while unemployed workers are taken charged of by the public unemployment insurance scheme—assumed to provide less than full insurance. The latter assumption can be seen as the minimal departure from the representative-agent assumption, in that full insurance (and ultimately agents’ homogeneity) is allowed among a subset of the households (the employed) but not all households as in the representative-agent framework. But this minimal departure is sufficient to capture the essence of precautionary wealth holding, because imperfect insurance for the unemployed motivates buffer-stock saving by family members. The latter property, together with the limited wealth heterogeneity among the *unemployed* (due to the tight debt limit), imply that the cross-sectional wealth distribution has a finite number of states. Consequently, the solution dynamics of the model has a finite state space representation that can be exploited to recover the deep parameters of the model from suitably chosen time-series and cross-sectional data.

One important feature of our empirical strategy is the incorporation of cross-sectional information at the business-cycle frequency. Indeed, in as much as the theoretical model generates summary moments of the cross-sectional distribution of households, one can bring into the estimation of the model the empirical counterparts of these moments. In particular, one key feature of the model is that consumption is heterogenous and households can be grouped into consumption classes; the empirical counterpart of this is the consumption shares of the households by income quantiles, which can be measured at the business-cycle frequency (in the Consumption Expenditure Survey). We treat these shares as time series in our structural estimation, next the time series for inflation, labour market transition rates and macro aggregates.

²To summarize, under the “natural” debt limit, an agent never borrow more than he would be able to repay in the worst individual history—here an history with permanent unemployment. This can be shown to imply than households never borrow up to the limit for any *finite-length* unemployment spell. If the debt limit is tighter than the natural limits then there necessarily exists individual unemployment histories with unemployment spells of finite length in which the borrowing limit is binding (see, e.g., the technical appendix of Challe and Ragot, 2014, for further discussion of this point). We can then exploit the bindingness of the borrowing limit to reduce the dimensionality of the cross-sectional wealth distribution.

Once the joint distribution of the structural parameters of the model have been recovered, we ask the following question: how would our economy respond to the variety of aggregate shocks that we consider, were the precautionary motive to be inoperative? The answer to this question provides a measure of the importance of time-varying precautionary wealth in propagating those shocks. We consider two model variants wherein the precautionary motive is shut down: the representative-agent model—wherein households are perfectly insured and identical—, and the hand-to-mouth model—wherein households are perfectly insured but two household types coexist, including one (the impatient) facing a binding debt limit in every period. The comparison with these alternative benchmarks reveals the impact that precautionary saving has on the economy’s response to some of the structural shocks. For example, following a monetary policy (i.e., nominal interest rate) shock that temporarily boosts aggregate demand, the peak in aggregate consumption predicted by the precautionary saving model is about 20% larger than the peak implied by the hand-to-mouth model and about 35% larger than the peak implied by the representative-agent model.³ After this shocks, the aggregate demand effects on output dominate the aggregate supply effects, causing an output expansion about 10% larger than in the representative-agent model and 6% larger in the hand-to-mouth model (even though investment is *smoother* under time-varying precautionary savings, due to the aggregate supply effects discussed above). There is also significant amplification of the consumption to the job-separation shock, although the latter accounts for a small fraction of aggregate volatility in our sample. In contrast, there is little amplification (if any) after a permanent shock to total factor productivity. Overall, the general lesson that comes out of our analysis is that time-varying precautionary savings significantly affect the propagation of demand shocks (relative the comparable full-insurance economies), but do not significantly alter the impact of supply shocks. In as much as demand shocks account for a significant fraction of aggregate volatility at the short to medium frequencies, our analysis illustrates that time-varying precautionary saving does matter for the business cycle.

Related literature. Our analysis relates to several strands of the business cycle literature. Traditional “New Keynesian” business cycle analyses often emphasise the role of aggregate demand as a key driver of aggregate fluctuations (see, e.g., Woodford, 2003, Christiano et al. 2005, Smets and Wouters 2007; Gali, 2010). While the historical New Keynesian model has a perfectly competitive labour market, the model has recently been extended to incorporate labour market frictions and involuntary unemployment – see Langot and Chéron (2000), Walsh (2005), Faia (2008), Gertler et al. (2008), Trigari (2009), Herr and Maussner (2010) and Blanchard and Gali (2010)—see Gali (2011) for a survey. In this extended model a fall in firms’ sales does not mechanically manifest itself as a fall in firms’ hours demand schedule and the equilibrium real wage; rather it lowers the expected benefit from hiring a worker

³Since our model is linearised around its balanced growth path, the impact of a negative demand shock of this type is the opposite of a positive demand shock.

which, given the presence of hiring costs, leads to a slowing down of hiring flows. However, even in the extended model the maintained assumption that individuals' labour market transitions are fully insured within a "representative family" ensures that such transitions have little effect on households consumption choices; that is, aggregate demand affects idiosyncratic unemployment risk, but not the other way around as it does in our model. As a result, in that framework labour market frictions do not significantly contribute to exacerbate aggregate fluctuations.⁴ We use search frictions in the labour market as a way to generate involuntary unemployment and endogenously time-varying unemployment risk; as we show, once interacted with incomplete insurance, labour market frictions provide a powerful amplification mechanism for aggregate shocks, relative to an economy with nominal rigidities but no such frictions.

Krusell et al. (2011), Nakajima (2012) and more recently Kehoe et al. (2014) analyse incomplete-market, heterogenous-agent models with search frictions wherein the idiosyncratic income risk faced by the households is endogenised through firms' job creation policy. These models assume flexible prices and perfect competition in the goods market; consequently, monetary policy shocks are neutral and there is no specific role for aggregate demand in shaping aggregate fluctuations.

Other papers combine nominal frictions with incomplete insurance, but like Krusell and Smith (1998) treat labour market flows rates as exogenous constraints on total labour supply; this by construction rules out any feedback from the level of output to the extent of unemployment risk, which is a key amplification mechanism in our model. This class of models includes Guerrieri and Lorenzoni (2011), who study how a tightening of the borrowing constraint in a sticky-price economy may cause it to fall into a liquidity trap, as well as Oh and Reis (2012) and McKay and Reis (2013) who study the aggregate demand effects of a variety of fiscal and transfer policies. Finally, some papers such as Iacoviello (2005) and more recently Bilbiie et al. (2012) study hand-to-mouth economies with nominal rigidities, that is, economies where a borrowing limit is binding for impatient households but where none of the households face uninsured idiosyncratic risk. Our model collapses into the hand-to-mouth model when we allow workers to enjoy full insurance against unemployment risk (but maintain the assumptions of discount rate heterogeneity and borrowing constraints).

As far as we are, two papers consider the same frictions in goods, labour and asset markets as we do: Gornemann et al. (2012), and Ravn and Sterk (2013). There are important differences between these papers and ours, both in terms of focus and in terms of method. Gornemann et al. (2012) are essentially concerned with the redistributive impact of monetary policy shocks. They thus construct an heterogenous-agent model with a full cross-sectional wealth distribution and go on showing that an

⁴In Gali 's words, "Quantitatively realistic labor market frictions are likely to have, by themselves, a limited effect on the economy's equilibrium dynamics [...] When combined with a realistic Taylor-type rule, the introduction of price rigidities in a model with labor market frictions has a limited impact on the economy's equilibrium response to real shocks" (Gali, 2011, pp. 490-491).

increase in the policy rate raises income and wealth inequality, consistent with the empirical findings of Coibion and Gorodinchenko (2012). Ravn and Sterk (2013) study the ability of an exogenous shock to the job separation rate to explain the depth and length of the Great Recession. Their model illustrates the “aggregate demand/precautionary saving feedback loop” discussed above, but it has no capital and hence there is no competition between aggregate supply and demand effects. Additionally, we consider a variety of demand and supply shocks, and we estimate rather than calibrate our model.

More broadly, there is a revival of interest in macroeconomic theory for the various ways in which “aggregate demand” and its spillover effects may matter for business cycles and “crises”. Our work clearly speaks to this strand of the literature, and notably the recent contributions of Rendhal (2014), Michaillat and Saez (2014a, 2014b), Beaudry and Portier (2014), Heathcote and Perri (2013) and Chamley (2014). Rendhal (2014) shows how the combination of rigid nominal wages, a zero lower bound on the nominal interest rate and labour market frictions may cause the economy to dive into a low liquidity trap when after being hit by a persistent negative shock, and how fiscal policy may have large multiplier effects in this context. Michaillat and Saez (2014a, 2014b) study a framework with matching frictions in both goods and labour markets and show demand shortages spill over from one market to the other. Beaudry and Portier (2014) show that when the economy has excessive capital then unemployment risk leads to precautionary saving and thereby a demand shortage, and study what the efficient fiscal policy is in this context. Their approach is closely related to Heathcote and Perri (2013), who show that the feedback loop between aggregate demand and precautionary savings may lead to multiple equilibria. We share with Beaudry and Portier (2014) and Heathcote and Perri (2014) the focus on the interactions between households’ asset wealth and unemployment risk, although the feedback loop we look at is different from theirs and is embedded in the standard New Keynesian framework. While Michaillat and Saez do study the impact of spillovers between labour and goods markets, in their is no idiosyncratic income risk to be self-insured against. Crucially, in contrast to all these papers we use our structural model not only for theoretical investigation, but also to evaluate the strength of the aggregate demand-precautionary saving spillovers from the data.

Finally, our empirical/quantitative approach is closely connected to that of McKay (2014), who shows how to estimate a variant of the Krusell and Smith (1998) model by maximum likelihood.⁵ Like him, we consider a framework with countercyclical variance of uninsured idiosyncratic labour income risk, and measure the impact of this risk by means a counterfactual full-insurance experiment. Our approach differs from his in at least two respects. First, we use a framework with both demand and supply effects, while he only has supply effects by construction. Second, our framework can accommodate a broader set of structural (demand and supply) shocks and allows us to systematically examine their impact on a variety of endogenous variables, including macro and monetary aggregates as

⁵provide details?

well as labour market transitions and policy variables. For some of these structural shocks we consider our results are consistent with McKay’s findings of a large effect of precautionary saving on the response of aggregate consumption.

The rest of the paper is organised as follows. Section 2 describes the model, from the behavior of the various types of agents to the definition of the recursive equilibrium. Section 3 shows how the structural assumptions we make about periodic reinsurance and the tightness of the borrowing constraint causes the dimension of the state space to collapse to a finite number. Section 4 describes our empirical strategy, and Section 5 illustrates the importance of precautionary savings for the propagation of some of the structural shocks. Section 6 offers some concluding remarks.

2 The model

The model introduces incomplete insurance against time-varying idiosyncratic unemployment risk into a quantitative “New Keynesian” business cycle model. There are two household types: “workers” and “firm owners”; both supply labor, but only firm owners supply actually own capital and supply capital services and own and run the firms. Workers can trade one-period nominal bonds subject to a borrowing constraint. Both types of households can be either employed or unemployed. Idiosyncratic unemployment risk is time-varying and it cannot be fully insured by workers due to incomplete markets and the borrowing constraint (as in, e.g., Krusell and Smith, 1998); this will motivate workers’ precautionary saving behavior.

The production side is composed of four types of firms, in the spirit of, e.g., Trigari (2009) or Heer and Maussner (2010). Competitive intermediate goods firms hire labour services from the labor intermediaries and capital services from firm owners. Labor intermediaries hire labor from households in a market with matching frictions as in Mortensen and Pissarides (1994) and transform it into labor services. This market generates the job transition rates that the households take as given when choosing how much to consume and save. Intermediate goods are used as inputs by wholesale goods firms, each of which is the monopolistic supplier of the differentiated good it produces but faces Calvo-type nominal frictions when setting nominal prices (e.g., Christiano et al., 2005; Smets and Wouters, 2007). Those firms sell goods to the competitive final good sector, which aggregates them into a single final good used for consumption, investment, utilization costs, and vacancy posting. The Central Bank determines the nominal interest rate via a Taylor-like rule.

The timing of events within a period is as follows (see Figure 1). Every period is divided into three stages: the “labour market transitions”, “production”, and “consumption and saving stages”. After the aggregate state is revealed to all households, in the first stage matches are first destroyed and then reformed, which allows separated workers to find a job within the period; thus, by the end of

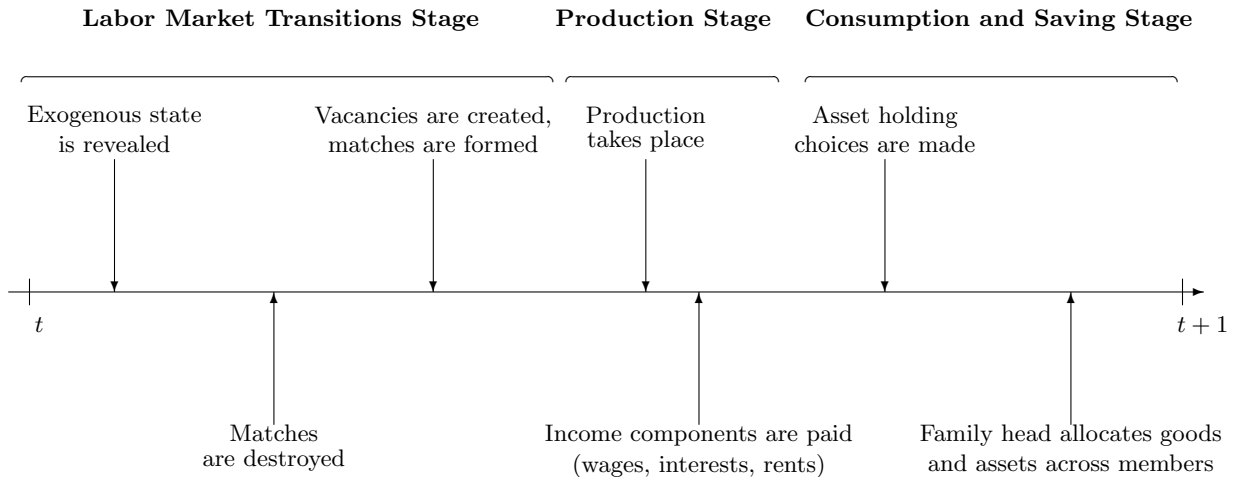


Figure 1: Model timeline within a period

this stage, all hiring decisions have been made. In the production stage production takes place and the aggregate income generated is shared between the agents in the following forms: net wages (for employed workers and employed firm owners), monopolistic rents (for firm owners) and unemployment benefits (for unemployed workers). Finally, households' consumption and savings (including their precautionary component) are determined in the last stage.

We present the model recursively and use primes to denote next period's values. We call the aggregate state X , a vector containing all the relevant aggregate variables in the model, whether exogenous or endogenous. We assume that the agents know the current value of X as well as its law of motion $X' = \Gamma(X, \epsilon')$, where ϵ' is the innovation to the *exogenous* state $\Phi \subset X$ (which is assumed to be first-order Markovian). Henceforth, all expectation operators are conditional on X , but for expositional clarity we summarize the content of X only in Section 2.5 below, after the presentation of the model has been completed. We first present the behavior of the households (Section 2.1), then that of the firms (Section 2.2), and finally turn to the market clearing conditions (2.4) and the definition of the equilibrium (Section 2.5). In what follows all current-period variables either belong to the aggregate state or are a function of it. To save on notation when presenting the model we will not always make explicit the dependence of some variables on the aggregate state. We will clarify this dependence when describing the equilibrium.

2.1 Households

There is a unit mass of households, each of whom is endowed with one unit of labour that is supplied inelastically during the production stage if the household is employed at the end of the labor market transitions stage. All households are subject to idiosyncratic changes in their employment status. Thus a share f of households who are unemployed at the beginning of the labor market transitions stage

will be employed at the end of the same stage. Also a share s of households who are employed at the beginning of labor market transitions stage will be unemployed at the end of the same stage. As we will see below, this idiosyncratic shocks to the employment status are time-varying.

Households are of two types. There is a measure $\Omega \in [0, 1)$ of workers (indexed by W henceforth) and a measure $1 - \Omega$ of firm owner (indexed by F). Households have instant utility function $u(c, \mathbf{c}) = u(c - h\mathbf{c})$, where c is consumption, \mathbf{c} is the level of consumption habits, $h \in (0, 1)$ a constant habit parameter, and where $u(\cdot)$ satisfies $u'(\cdot) > 0$, $u''(\cdot) < 0$. Habits are external and defined as follows. $c^W(\aleph)$ is the habit level of workers in the current period having been continuously unemployed for $\aleph \in N$ periods. It is assumed to be equal to the average consumption of workers having experienced the same number of consecutive periods of unemployment ($= \aleph$) in the previous period. For example, $c^W(0)$ is the habit level of currently employed workers and it is equal to the last period average consumption of employed workers. Similarly $c^W(1)$ is the consumption habit of an unemployed workers who was employed in the previous period and it is equal to the last period average consumption of those workers that just lost their job. This implies that all workers with the same \aleph share the same habit level while two workers with different \aleph s in general have different habit levels. Similarly, c^F is the consumption habit of firm owners in the current period and it is assumed to be equal to the average consumption of these households in the previous period; therefore all firm owners share the same habit level.

2.1.1 Workers

Let $\tilde{\boldsymbol{\mu}}(a, \aleph)$ denote the aggregate cross-sectional distribution of workers across the individual state vector (a, \aleph) at the beginning of the labour market transitions stage. That is, $\tilde{\boldsymbol{\mu}}(a, \aleph)$ is the share of workers with nominal bonds holdings less or equal to a and having experienced exactly $\aleph \geq 0$ consecutive periods of unemployment at the beginning of labour market transitions stage. It satisfies $\sum_{\aleph \geq 0} \int_a d\tilde{\boldsymbol{\mu}}(a, \aleph) = 1$. We let $\boldsymbol{\mu}(a, \aleph)$ characterize the same cross-sectional distribution at the end of labour market transitions stage.

We let $\tilde{\mathbf{n}}^W \equiv 1 - \sum_{\aleph \geq 1} \int_a d\tilde{\boldsymbol{\mu}}(a, \aleph)$ and $\mathbf{n}^W = f(1 - \tilde{\mathbf{n}}^W) + (1 - s)\tilde{\mathbf{n}}^W$ denote the workers' employment rate at the beginning and at the end of labour market transitions stage, respectively. Of course employment rates at the beginning and at the end of labour market transitions stage coincide with employment rates at the beginning and at the end of the period. Thus, we have $\mathbf{n}^W = \tilde{\mathbf{n}}^{W'}$, i.e., the employment rate at the end of the current period is equal to that at the beginning of the next period.

Every worker has subjective discount factor $\beta^W \in (0, 1)$. Employed workers earn the net labour income $(1 - \tau)w$, where w is the real wage (measured in units of the final good) and τ the social contribution rate. Unemployed workers earn the unemployment benefit $b^u e^z$, which is indexed by the economy's productivity shock e^z (to be specified later). The unemployment benefits are such that

$b^u < (1 - \tau)\mathbb{E}(w/e^z)$ (i.e., insurance is incomplete) and the following constraint holds:

$$\tau w \mathbf{n}^W = b^u e^z (1 - \mathbf{n}^W), \quad (1)$$

where \mathbb{E} stands for the unconditional expectations operator. Condition (1) states that the unemployment insurance scheme is balanced in every period.

As mentioned, workers can trade one-period nominal bonds (measured in units of the final good), subject to the borrowing constraint that the value of their holdings be less than $\underline{a}e^z$, where $\underline{a} \leq 0$ is a constant. We assume that $\underline{a} > -b^u \beta^F / (1 - \beta^F)$, where β^F is the subjective discount factor of firm owners (see below). This will ensure that $\underline{a}e^z$ is always tighter than the natural debt limit, a property that is a key in making our approach tractable.⁶

Unemployed workers Let $V^u(a^u, \aleph, X)$ denote the intertemporal utility of an unemployed worker at the beginning of the consumption-saving stage, with nominal bonds a^u and own unemployment duration $\aleph \geq 1$ when the aggregate state is X . The Bellman equation for this worker is:

$$V^u(a^u, \aleph, X) = \max_{a^{w'}, c^u} \left\{ e^{\varphi_c} u(c^u - h \mathbf{c}^W(\aleph)) + \beta^W \mathbb{E} \left[(1 - f') V^u(a^{w'}, \aleph + 1, X') + f' \frac{V^e(\tilde{\mu}', a^{e'}, X')}{n^{W'}} \right] \right\}, \quad (2)$$

where c^u is the worker's consumption, $V^e(\tilde{\mu}, a^e, X)$ is the intertemporal utility of a family of employed workers at the beginning of the consumption-saving stage with cross-sectional distribution of workers $\tilde{\mu}$ and per family member nominal bonds a^e when the aggregate state is X . φ_c is a preference shock that affects all households.⁷ Note that (i) because $V^e(\tilde{\mu}', a^{e'}, X')$ is the value function for the whole family, it must be divided by $n^{W'}$ to find its per family member analogue and (ii) because a worker is atomistic relative to the size of a family, the nominal bonds holding choice of an unemployed worker, $a^{w'}$, does not affect the value of the family.⁸

The unemployed worker solves (2) subject to:

$$a^{w'} + c^u = b^u e^z + (1 + r) a^u \quad (3)$$

and

$$a^{w'} \geq \underline{a} e^z, \quad (4)$$

where $r = (1 + \mathbf{R}_{-1}) / (1 + \pi) - 1$ is the real return on nominal bond holdings (measure in units of the final good today), \mathbf{R}_{-1} is the nominal interest rate on last period's bonds holdings and π is the final good

⁶Loosely speaking, the natural debt limit is the level of nominal debt that a worker would be able to repay for certain, i.e., even if this worker stayed unemployed forever (see, e.g., Aiyagari, 1994).

⁷The intertemporal utility of a family of employed workers will be described below in detail.

⁸Formally, we have $\partial V^e(\cdot) / \partial a^u = 0$.

current inflation. The solution to the problem of an unemployed worker is summarized by the optimal policy functions of nominal bonds and consumption, i.e., $a^u = g_a^u(a^u, \aleph, X)$ and $c^u = g_c^u(a^u, \aleph, X)$.

Employed workers. Our risk-sharing arrangement among employed workers is as follows: we assume that every employed worker (for whom $\aleph = 0$) belongs to a family, whose head pools resources and allocate consumption goods and nominal bonds so as to maximize the intertemporal utility of all the family members. Given preferences, this implies that all employed worker within a particular family will share the same level of nominal bonds and consumption. There are Ω such families, each of size 1. When an employed worker falls into unemployment, the workers leaves the family and receives the unemployment benefit $b^u e^z$. When an unemployed worker finds a job, the worker re-enters the same family as that which was left when the worker had fallen into unemployment. To fix terminology, we will say that workers who is currently employed “belong” to the family while a worker who is currently unemployed is “attached” to the family. Since all workers attached to a particular family enter unemployment with the same level of assets, they all have the same levels of bond holdings and consumption, conditional on a particular value of \aleph . Hence, all workers attached to a particular family (employed and unemployed) are identical conditional on a particular \aleph .

Let $\tilde{\mu}(a, \aleph)$ denote the family-level cross-sectional distribution of workers at the beginning of the labour market transition stage. This distribution gathers the workers that are part of the family at that moment (i.e., those for whom $\aleph = 0$) as well as those who are not currently in the family (i.e., for whom $\aleph > 0$) but are “attached” to it in the sense that they will return to the same family when they find a job. Hence, $\tilde{\mu}(a, \aleph)$ is the share of workers in or attached to a particular family with nominal bonds holdings less or equal to a and having experienced exactly $\aleph \geq 0$ consecutive periods of unemployment at the beginning of labour market transitions stage. It satisfies $\sum_{\aleph \geq 0} \int_a d\tilde{\mu}(a, \aleph) = 1$. We let the distribution $\mu(a, \aleph)$ to characterize the family-level cross-sectional distribution at the end of labour market transitions stage.

We let $\tilde{n}^W \equiv 1 - \sum_{\aleph \geq 1} \int_a d\tilde{\mu}(a, \aleph)$ and $n^W = f(1 - \tilde{n}^W) + (1 - s)\tilde{n}^W$ denote the family’s employment rate at the beginning and at the end of labour market transitions stage, respectively. Of course employment rates of a particular family at the beginning and at the end of labour market transitions stage coincide with employment rates of the same family at the beginning and at the end of the period. Thus, we have $n^W = \tilde{n}^{W'}$, i.e., the employment rate of a family at the end of the current period is equal to the employment rate of the same family at the beginning of the next period.

The labour market transition stage generates wealth flows into and out of the family. The wealth inflow comes from new family members who where unemployed at the beginning of the labour market transition stage but are employed at the end of that stage. This inflow is given by:

$$B = f \sum_{\aleph \geq 1} \int_a ad\tilde{\mu}(a, \aleph).$$

The wealth outflow comes from workers who were employed at the beginning of the labour market transition stage but are unemployed at the end of that stage. Since a^e denotes nominal bonds per family member and \tilde{n}^W is the size of the family, both at the beginning of the labor market transition stage, the wealth outflow is simply $s\tilde{n}^W a^e$. Then, the total wealth of the family at the end of the labour market transitions stage is given by:

$$A^e = (1 - s)\tilde{n}^W a^e + B. \quad (5)$$

Given these flows, the budget constraint of the family at the beginning of the consumption-saving stage is given by:

$$n^W (a^{e'} + c^e) = (1 - \tau)wn^W + (1 + r)A^e. \quad (6)$$

Finally, the borrowing constraint is

$$a^{e'} \geq \underline{a}e^z. \quad (7)$$

The intertemporal utility of a family of employed workers, at the end of labour market transitions stage, is given by:

$$V^e(\tilde{\mu}, a^e, X) = \max_{a^{e'}, c^e} \{e^{\varphi c} n^W u(c^e - hc^W(0)) + \beta^W \mathbb{E}[V^e(\tilde{\mu}', a^{e'}, X') + s'\tilde{n}^{W'} V^u(a^{e'}, 1, X')]\}. \quad (8)$$

The family head solves (8) subject to (5)–(7), and taking as given the laws of motions for $\tilde{\mu}$ and n^W (i.e., $\tilde{\mu}' = g_{\tilde{\mu}}(\tilde{\mu}, f, s, X)$ and $n^W = f(1 - \tilde{n}^W) + (1 - s)\tilde{n}^W$). Note that the family head values the potential utility loss that current members may experience in the next period if they fall into unemployment. There will be $s'\tilde{n}^{W'}$ such members, hence the corresponding weight before $V^u(a^{e'}, 1, X')$ in the Bellman equation (8).

We focus on a symmetric equilibrium, wherein the cross-sectional distribution of workers belonging or attached to a particular family, $\tilde{\mu}$, is identical across families. That is, the distribution of cross-sectional distributions over $(\tilde{\mu}, a^e)$ is degenerate, i.e., all families have the same cross-sectional distribution $\tilde{\mu}$ and per family member nominal bonds a^e at the beginning of the labor market transition stage. Since both are part of the aggregate state X , we abuse notation and write the optimal policy functions for nominal bonds and consumption as functions of X only, i.e., $a^{e'} = g_{a^e}(X)$ and $c^e = g_{c^e}(X)$.⁹

2.1.2 Firm owners

Firm owners are more patient than workers: their subjective discount factor is $\beta^E \in (\beta^W, 1)$. Besides earning either wages or unemployment benefit and participating in the bond market like the workers, firm owners own and rent out the capital stock and all the firms.

⁹A more detailed description of these two policy functions is left for the Appendix.

In a similar fashion as in the case of employed workers, our risk-sharing arrangement among firm owners is as follows: we assume that every firm owner belongs to a family, whose head pools resources and allocate consumption goods and assets so as to maximize the intertemporal utility of all the family members. There are $1 - \Omega$ such families, each of size 1. In this case, when an employed firm owner falls into unemployment, the firm owners does *not* leave the family. In other words, there is full insurance among firm owners (while in the worker case there was full insurance among employed workers only). Hence, all firm owners within a family (employed or unemployed) are identical.

The employment rates of every family of firms owners before and after labour market transitions stage are denoted by \tilde{n}^F and n^F , respectively. Just as with the workers we have $\tilde{n}^{F'} = n^F$ and $n^F = f(1 - \tilde{n}^F) + (1 - s)\tilde{n}^F$. As before, these are the individual variables. The corresponding aggregate variables are denoted $\tilde{\mathbf{n}}^{F'}$ and \mathbf{n}^F .

Since all the families are identical, we will refer to the “representative family” henceforth. The intertemporal utility of a representative family, at the end of labour market transitions stage, is given by:

$$V^F(\tilde{n}^F, k, a^F, i, X) = \max_{a^{F'}, c^F, i', u, k'} \{e^{\varphi^c} u(c^F - h\mathbf{c}^F) + \beta^F \mathbb{E}[V^F(\tilde{n}^{F'}, k', a^{F'}, i', X')]\},$$

where k and a^F are the family’s holdings of capital and nominal bonds at the beginning of the period, i its investment in the previous period (which enters the value function due to the presence of investment adjustment costs), c^F is consumption, i' its investment in the current period, and $u \in [0, 1]$ is the capital utilization rate (a choice variable for the typical firm owner). Firm owners face the budget constraint:

$$c^F + i' + a^{F'} = w^F n^F + [r_k u - \eta(u)]k + (1 + r)a^F + \Upsilon, \quad (9)$$

where w^F is the real wage earned by firm owners, r_k the real rental rate of capital services and Υ is the real rent generated by the firms and rebated to their owners as dividends (all of them measured on units of the final good).¹⁰ The function $\eta(u)k$ is the real cost that the utilization rate entails in unit of the final good, where $\eta(u)$ is such that $\eta(1) = 0$, $\partial\eta(u)/\partial u > 0$ and $\partial^2\eta(u)/\partial^2 u > 0$. Finally, the capital stock evolves as:

$$k' = (1 - \delta)k + e^{\varphi_i}(1 - S(i'/i))i', \quad (10)$$

where φ_i is an investment-specific shock and $S(\cdot)$ an investment adjustment cost function satisfying $\partial^2 S(\cdot)/\partial (i'/i)^2 > 0$ and $S(g_i) = \partial S(\cdot)/\partial (i'/i)|_{i'/i=g_i} = 0$, where g_i is the steady-state value of i'/i .

The representative family maximizes the intertemporal utility subject to (9)–(10) and taking as given the laws of motions for \tilde{n}^F and n^F (i.e., $\tilde{n}^{F'} = n^F$ and $n^F = f(1 - \tilde{n}^F) + (1 - s)\tilde{n}^F$). We focus on a symmetric equilibrium. Thus, all families of firm owners are identical. More specifically, the cross-sectional distribution of families of firm owners over the individual state vector (\tilde{n}^F, k, a^F, i) is

¹⁰Ownership of the firms here takes the form of a fully diversified portfolio of private equity. We could equivalently allow the profit flows to be traded (and hence priced) among firm owners.

degenerate. Since the individual state vector is part of the aggregate state X , we again abuse notation and write the optimal policy functions as functions of X only, i.e., $a^{F'} = g_{a^E}(X)$, $c^F = g_{c^F}(X)$, $i' = g_i(X)$, $u = g_u(X)$, and $k' = g_k(X)$.¹¹

Importantly, the homogeneity of firm owners implies that they share the same pricing kernel, hence the intertemporal decisions of all the firms (as described in the next section) are unambiguous. The pricing kernel is given by:

$$M^{F'} = \beta^E e^{\Delta\varphi'_c} \frac{u'(c^{F'} - hc^F)}{u'(c^F - hc^F)}, \quad (11)$$

where \mathbf{c}^F is an element of X , c^F a function of X , and where we have used the fact that $c^F = g_{c^F}(X) = \mathbf{c}^{F'} \in X'$ (i.e., current consumption determines next period's habit).

2.2 Firms

The economy has four production layers, with each type of firm forming a continuum of measure 1. A competitive final goods sector buys differentiated products from the wholesale sector and combines them into a single final good that is sold to the households (for consumption, investment, and capital utilization cost) and labour intermediaries (for job vacancy posting costs). Wholesale goods firms buy undifferentiated goods from intermediate goods firms, which they turn into differentiated goods sold to final goods firms; every firm in the wholesale good sector is a monopolistic supplier but faces nominal frictions a la Calvo (1985). Competitive intermediate goods firms use labour and capital services to produce the undifferentiated intermediate goods. The labour services used by these firms are sold by labour intermediaries, who hire labour from households in a labour market plagued with search frictions. Capital is hired from firm owners.

2.2.1 Final goods firms

The final good is produced by a continuum of identical and competitive firms that combine wholesale differentiated goods according to the production function:

$$y = \left(\int_0^1 y_\zeta^{(\theta-1)/\theta} d\zeta \right)^{\theta(\theta-1)}, \quad (12)$$

where $\theta > 1$ is the cross-partial elasticity of substitution between any two wholesale goods.

Let p_ζ denote the price of wholesale good ζ in terms of the final good. This price is taken as given by the final goods firms. The program of the representative final good producer is thus:

$$\max_{y, y_\zeta} y - \int_0^1 p_\zeta y_\zeta d\zeta, \quad (13)$$

¹¹A more detailed description of these four policy functions is left for the Appendix.

subject to (12). From the optimal choices of final good firms, one can deduce the demand function for the wholesale good firms, $\varsigma \in [0, 1]$:

$$y_\varsigma(p_\varsigma) = p_\varsigma^{-\theta} y, \quad (14)$$

where y is the total demand for final goods. The zero-profit condition for final good producers implies that:

$$\left(\int_0^1 p_\varsigma^{1-\theta} d\varsigma \right)^{1/(1-\theta)} = 1.$$

2.2.2 Wholesale goods firms

The wholesale sector is imperfectly competitive. Wholesale firm $\varsigma \in [0, 1]$ is the monopolistic supplier of the good it produces, which it does by means of a linear production function with a fixed cost:

$$y_\varsigma = x_\varsigma - \kappa_y e^z, \quad (15)$$

where x_ς is the quantity of intermediate goods used in production and where $\kappa_y e^z$ is the fixed cost measured in units of the intermediate goods. Firm ς ' current profit, measured in units of final good, is given by:

$$\Xi = (p_\varsigma - p_m) y_\varsigma(p_\varsigma) - p_m \kappa_y e^z,$$

where p_m is the price of intermediate goods in term of the final good (which is taken as given by wholesale goods firms).

Firm ς chooses p_ς to maximise the present discounted value of future profits, taking as given the demand curve (14). Following Calvo (1985), we assume that in every period every wholesale goods firm can be in one of the following two idiosyncratic states: either the firm can freely reoptimize its price, or it cannot and simply rescales the existing price according to the indexation rule:

$$p_\varsigma = \frac{(1 + \bar{\pi})^{1-\gamma} (1 + \pi_{-1})^\gamma}{1 + \pi} p_{\varsigma,-1}. \quad (16)$$

where $\gamma \in (0, 1)$ measures the degree of indexation to the most recently available final good inflation measure, π_{-1} is final good inflation in the previous period and $\bar{\pi}$ is steady state inflation.¹² The ex ante probability of each firm being able to reoptimize the price in the next period is constant and equal to $1 - \alpha \in [0, 1]$, irrespective of the time elapsed since the period in which the price of such a firm was last revised.

It follows from this price adjustment mechanism that the behaviour of a firm can be described by two Bellman equations, corresponding to the two idiosyncratic states in which the firm can be. The value of a firm that is allowed to reset its price is given by $\mathbb{W}^W(X)$ and only depends on the aggregate

¹²This indexation rule is simply the standard rule $P_\varsigma = (1 + \bar{\pi})^{1-\gamma} (1 + \pi_{-1})^\gamma P_{\varsigma,-1}$, written in the final good units (where P_ς are nominal prices).

state. The value of a firm not allowed to reset its selling price and with last period's price $p_{\varsigma,-1}$ is denoted as $\mathbb{V}^W(p_{\varsigma,-1}, X)$. The corresponding Bellman equations are:

$$\begin{aligned} \mathbb{W}^W(X) &= \max_{p_{\varsigma}} \Xi + \alpha \mathbb{E} [M^{E'} \mathbb{V}^W(p_{\varsigma}, X')] + (1 - \alpha) \mathbb{E} [M^{F'} \mathbb{W}^W(X')] \\ \text{and } \mathbb{V}^W(p_{\varsigma,-1}, X) &= \Xi + \alpha \mathbb{E} [M^{F'} \mathbb{V}^W(p_{\varsigma}, X')] + (1 - \alpha) \mathbb{E} [M^{F'} \mathbb{W}^W(X')], \end{aligned}$$

where, in $\mathbb{V}^W(\cdot)$, p_{ς} is given by (16) (i.e., is not the result of an optimisation by the firm).

We focus on a symmetric equilibrium wherein the solution to wholesale goods firms' problem is the optimal reset price common to all price resetting firms, and we denote the corresponding policy function $p^* = g_{p^*}(X)$. This optimal reset price, together with the Calvo price setting mechanism and the indexation rule (16), imply the following law of motion for inflation:

$$\pi = \frac{\alpha \left((1 + \bar{\pi})^{1-\gamma} (1 + \pi_{-1})^\gamma \right)}{(1 - (1 - \alpha)(p^*)^{1-\theta})^{1/(1-\theta)}} - 1 \quad (17)$$

that clearly implies that π is a function of X as p^* is.

The price-setting mechanism generates a non-degenerated cross-sectional distribution of prices, since the selling price of a firm not reoptimising its price depends on the time that has elapsed since the last time the price was set optimally. However, as we will see later, the price dispersion index $\Lambda \equiv \int_0^1 p_{\varsigma}^{-\theta} d\varsigma$ is a sufficient statistics to capture the relevant properties of the price distribution. The Calvo mechanism implies that Λ evolves according to the law of motion:

$$\Lambda = (1 - \alpha)(p^*)^{-\theta} + \alpha \left(\frac{(1 + \bar{\pi})^{1-\gamma} (1 + \pi_{-1})^\gamma}{1 + \pi} \right)^{-\theta} \Lambda_{-1}, \quad (18)$$

where Λ_{-1} is the value of the index in the previous period.

2.2.3 Intermediate goods firms

The intermediate good is produced by a continuum of identical and competitive firms. The representative intermediate goods firm produces with the technology $y_m = \check{k}^\phi (e^z \check{n})^{1-\phi}$, $\phi \in (0, 1)$, where \check{n} and \check{k} denote labour and capital services. It solves the following program:

$$\max_{\check{n}, \check{k}} \left\{ p_m \check{k}^\phi (e^z \check{n})^{1-\phi} - Q \check{n} - r_k \check{k} \right\}, \quad (19)$$

where Q is the unit real price of labour services, expressed in units of the final good. The solution (19) gives the optimal demands for factor services $\check{n} = g_{\check{n}}(X)$ and $\check{k} = g_{\check{k}}(X)$.

2.2.4 Labour intermediaries

Job values Labor services are sold to intermediate goods firms by labor intermediaries, which hire labor from households in a market with search frictions. Our timing convention about job destruction

and creation is as in Walsh (2005), Gali (2011) and many others. More specifically, at the beginning of the labour market transition stage, a fraction $\rho = \rho(\varphi_s)$ of existing employment relationships are destroyed, where φ_s is a job destruction shock. The workers who loose their job on that occasion enter the unemployment pool, where they join the workers who where already unemployed at the end of the previous period.

After job destruction has taken place, labour intermediaries post vacancies, at the unit cost $\kappa_v e^z$ in terms of the final goods. One employed worker provides one unit of labour services but a firm owner provides $\psi > 1$ units of labor services. We call this parameter the “skill premium”. If the labour market were competitive the wage paid by a labour intermediary to a firm owner would simply be $w^F = \psi w$. For simplicity we will assume that it is also the case here, despite the fact that the labour market is not competitive. We will assume (and verify numerically later on) that both w and w^F belong to the relevant bargaining sets, in the sense that when a household (either worker or firm owner) is matched with a labour intermediary then both extract a positive surplus from the match.

The values to the labour intermediary of a match with a worker and a firm owner are, respectively:

$$J^e = Q - w + \mathbb{E}[(1 - \rho')M^{F'}J^{e'}] \text{ and } J^F = \psi(Q - w) + \mathbb{E}[(1 - \rho')M^F J^{F'}]. \quad (20)$$

We note that $J^F = \psi J^e$, i.e. the value of a match with a firm owner is proportional to that with a worker (with a coefficient of proportionality equal to the skill premium). We assume that, when posting a vacancy, labour intermediaries cannot target a particular skill type. As a consequence, labour intermediaries adjust vacancies until the expected payoff on a posted vacancy be equal to its cost, i.e.,

$$\lambda[\Omega J^e + (1 - \Omega)J^F] = \kappa_v e^z, \quad (21)$$

where λ is the economywide vacancy-filling rate.

Labour market flows Let $\tilde{\mathbf{n}} = \Omega \tilde{\mathbf{n}}^W + (1 - \Omega) \tilde{\mathbf{n}}^F$ denote the economywide employment rate before labor market transitions stage and $\mathbf{n} = \Omega \mathbf{n}^W + (1 - \Omega) \mathbf{n}^F$ the same rate after labour market transitions stage. These two definitions imply that $\tilde{\mathbf{n}}' = \mathbf{n}$. The unemployment pool is made of workers who were unemployed at the beginning of the labour market transitions stage (in number $1 - \tilde{\mathbf{n}}$) as well as workers who where employed at the beginning of the labour market transitions stage but loose their job during after the job destruction shock hits the economy (in number $\rho \tilde{\mathbf{n}}$). The matching technology produces job relationships using as input the unemployment pool and the aggregate number of vacancies v . The technology is assumed to have the form:

$$m = \bar{m} e^{\varphi_m} (1 - (1 - \rho) \tilde{\mathbf{n}})^\chi v^{1-\chi}, \quad (22)$$

where \bar{m} is scaling parameter, φ_m an matching productivity shock and $\chi \in (0, 1)$ the elasticity of match production with respect to the size of the unemployment pool. Accordingly, the economywide

job-finding and vacancy-filling rates are given by, respectively:

$$f = m / (1 - (1 - \rho)\tilde{\mathbf{n}}) \quad (23)$$

and

$$\lambda = m/v.$$

Using (22) and (23), we find:

$$f = (\bar{m}e^{\varphi m} (1 - (1 - \rho)\tilde{\mathbf{n}})^\chi v^{1-\chi}) / (1 - (1 - \rho)\tilde{\mathbf{n}}). \quad (24)$$

Note that under our timing convention the workers that are separated from their firm at the beginning of the period can be re-matched within the period (in which case they effectively do not change employment status). It follows that the period-to-period separation rate s is given by:

$$s = \rho(1 - f). \quad (25)$$

As usual, there are two equivalent ways of viewing labour market flows. From the point of view of the households, the dynamics of the aggregate employment rate is determined by the flows of job losers and job finders, i.e., $\mathbf{n} = f(1 - \tilde{\mathbf{n}}) + (1 - s)\tilde{\mathbf{n}}$. From the point of view of the labour intermediaries, it follows from the natural process of job destruction and the intensity of vacancy posting, i.e., $\mathbf{n} = (1 - \rho)\tilde{\mathbf{n}} + \lambda v$.

Wages The presence of labour market frictions implies that there exists a full bargaining set over which a labour intermediary and an employee find it mutually profitable to be matched. However, the theory does not pin down the specific way in which the match surplus is shared among the parties. We assume that there are some rigidities in nominal wage adjustment, with workers' nominal wage, that implies that the dynamics of the workers' real wage (in terms of the final good) is given by:

$$w = \left(\frac{\mathbf{w}_{-1}}{1 + \pi} \right)^{\gamma w} \left(\bar{w} e^{z + \varphi w} \left(\frac{\mathbf{n}}{\bar{n}} \right)^{\psi_n} \right)^{1 - \gamma w}. \quad (26)$$

As discussed above, we assume (and verify numerically in the quantitative implementation of the model) that w and $w^F (= \psi w)$ always lie inside the relevant bargaining sets.

2.3 Central Bank

We assume that the Central Bank sets the nominal interest rate R according to the following simple rule:

$$\log \left(\frac{1 + R}{1 + \bar{R}} \right) = \rho_R \log \left(\frac{1 + \mathbf{R}_{-1}}{1 + \bar{R}} \right) + (1 - \rho_R) \left[a_\pi \log \left(\frac{1 + \pi}{1 + \bar{\pi}} \right) + a_y \log \left(\frac{1 + g}{1 + \bar{g}} \right) \right] + \varphi_R \quad (27)$$

where \bar{R} is the steady state nominal wage, $\rho_R \in (0, 1)$ an interest rate smoothing parameter, (a_π, a_y) the reaction coefficients to inflation and output growth, φ_R an monetary policy shock and φ_π a shock to the inflation target. We let $g = y/\mathbf{y}_{-1} - 1$ to be the growth rate of final output and \bar{g} its mean. Also, \mathbf{y}_{-1} and \mathbf{R}_{-1} are the levels of output and the nominal interest rate in the previous period, both of which are part of the aggregate state X .

2.4 Market clearing

Labor services Recall from Section 2.1 we focus on a symmetric equilibrium so that families of both employed and firm owners are symmetric and that they all face the same labour market transition rates. This implies that:

$$\tilde{n}^W = \tilde{n}^F = \tilde{\mathbf{n}}^W = \tilde{\mathbf{n}}^F = \tilde{\mathbf{n}} \quad \text{and} \quad n^W = n^F = \mathbf{n}^W = \mathbf{n}^F = \mathbf{n}. \quad (28)$$

Hence, and because a matched firm owner provides ψ times more units of labour services than a worker, the total supply of labour services is $\Omega \mathbf{n}^W + (1 - \Omega)\psi \mathbf{n}^F = (\Omega + (1 - \Omega)\psi) \mathbf{n}$. It follows that clearing of the market for labor services requires:

$$(\Omega + (1 - \Omega)\psi) \mathbf{n} = \check{n}. \quad (29)$$

Finally, because we focus on a symmetric equilibrium and since the workers belonging or attached to a particular family form a continuum, by the law of large number the cross-sectional distributions of workers at the family and the aggregate levels are identical:

$$\tilde{\mu}(a, \aleph) = \check{\mu}(a, \aleph) \quad \text{and} \quad \mu(a, \aleph) = \boldsymbol{\mu}(a, \aleph). \quad (30)$$

Asset markets Recall that firm owners are symmetric and in measure $1 - \Omega$. Since each of them supplies uk units of capital services, the total supply of capital services is $(1 - \Omega)uk$. The market-clearing condition for capital services is thus:

$$(1 - \Omega)uk = \check{k}. \quad (31)$$

All the households may participate in the market for nominal bonds (subject to the debt limit), which are in zero net supply. Because of symmetry, at the end of the consumption and saving stage, every firm owners hold the same bond wealth $a^{F'}$, every employed worker the same bond wealth $a^{e'}$, while the bond wealth of unemployed only varies across values of \aleph , i.e., $a^{u'} = g_{a^u}(a^u, \aleph, X)$. Clearing of the market for bonds thus requires:

$$(1 - \Omega)a^{F'} + \Omega \mathbf{n} a^{e'} + \Omega \sum_{\aleph \geq 1} \int_a a^{u'} d\boldsymbol{\mu}(a, \aleph) = 0, \quad (32)$$

where again $\mu(a, \aleph)$ is the cross-sectional distribution of workers over (a, \aleph) after labour market transitions stage.

Goods markets The aggregate demand for final goods is made of total investment (by firm owners), the consumption of all types of households (firm owners and workers alike), as well as capital utilization costs (paid by firm owners) and vacancy costs (paid by the labour intermediaries). Clearing of the market for final goods requires that demand be equal to supply, i.e.,

$$(1 - \Omega)(c^F + i' + \eta(u)k) + \Omega \mathbf{n}^W c^e + \Omega \sum_{\aleph \geq 1} \int_a c^u d\mu(a, \aleph) + \kappa_v e^z v = y, \quad (33)$$

where, as it was the case with the market of nominal bonds, the demand side has this particular form because of symmetry of workers and firm owners.

The wholesale sector demands one unit of intermediate goods for any unit of wholesale goods. Hence the market-clearing condition for the intermediate good is:

$$\int_0^1 x_\zeta d\zeta = y_m = \check{k}^\phi (e^z \check{n})^{1-\phi}. \quad (34)$$

The total demand for wholesale goods by the final good sector is $\int_0^1 y_\zeta(X, p_\zeta) d\zeta = \Lambda y$. The total supply of intermediate goods is equal to $\int_0^1 x_\zeta d\zeta - \kappa_y e^z$. Hence, clearing of the market for wholesale goods requires, using (34):

$$\Lambda y = \check{k}^\phi (e^z \check{n})^{1-\phi} - \kappa_y e^z. \quad (35)$$

2.5 Aggregate state and equilibrium

We are now in a position to summarise the content of the aggregate state. Again, because we focus on a symmetric equilibrium where all cross-sectional, family-level distributions of workers are identical and equal to the aggregate distribution, the aggregate state is given by:

$$X = \{ \tilde{\mu}(\cdot), k, a^F, i, \mathbf{c}^F, \mathbf{c}^W(\aleph)_{\aleph \in \mathbb{N}}, a^e, \mathbf{R}_{-1}, \mathbf{\Lambda}_{-1}, \pi_{-1}, \mathbf{y}_{-1}, \mathbf{w}_{-1}, \Phi \}, \quad (36)$$

where

$$\Phi \equiv \{ z, \varphi_i, \varphi_c, \varphi_m, \varphi_s, \varphi_R, \varphi_w, \varphi_\pi \}$$

is the vector of exogenous state variables. Note that knowledge of the cross-sectional distribution $\tilde{\mu}(\cdot)$ allows one to compute the beginning of labor market transitions stage employment rate $\tilde{\mathbf{n}} = \tilde{\mathbf{n}}^W = \int_a d\tilde{\mu}(a, 0)$.

A *symmetric recursive equilibrium* is a set of value and policy functions, a set of prices, and labour market flows such that:

1. *Workers.* Given $r(X)$, $w(X)$, $\tau(X)$, $b^u e^z$, $\mathbf{c}^W(\aleph)_{\aleph \in \mathbb{N}}$, $f(X)$ and $s(X)$, the value function $V^u(a^u, \aleph, X)$, $V^e(\tilde{\mu}(\cdot), a^e, X)$ and associated policy functions $g_{a^u}(a^u, \aleph, X)$, $g_{c^u}(a^u, \aleph, X)$, $g_{a^e}(X)$, and $g_{c^e}(X)$ solve the workers' problems;

2. *Firm owners.* Given $r_k(X)$, $w^F(X)$, \mathbf{c}^F , $\Upsilon(X)$, $f(X)$ and $s(X)$ the value function $V^F(n^F, k, a^F, i, X)$ and associated policy functions $g_{a^F}(X)$, $g_{c^F}(X)$, $g_i(X)$, $g_u(X)$, and $g_k(X)$ solve the problem of a firm owner;
3. *Final goods firms.* Given p_ς , $\varsigma \in [0, 1]$, the demands for intermediate goods $y_\varsigma(p_\varsigma, X)$ is optimal from the point of view of final goods firms;
4. *Wholesale goods firms.* Given $p_m(X)$ and $M^F(X, X')$, the value functions $\mathbb{W}^W(X)$ and $\mathbb{V}^W(p_{\varsigma-1}, X)$ and the reset price $p^*(X)$ solve the problem of wholesale goods firms;
5. *Intermediate goods firms.* Given $p_m(X)$, $Q(X)$ and $r_k(X)$, the demand for labour and capital services $\check{n}(X)$ and $\check{k}(X)$ solve the problem of intermediate good firms;
6. *Labour intermediaries.* Given $J^e(X)$, $J^F(X)$, $M^F(X, X')$ and $\lambda(X)$, the free entry condition (21) determines the number of vacancies $v(X)$, and thereby the labour market transition rates $f(X)$ and $s(X)$ according to (24)–(25);
7. *Profits.* The profit function $\Upsilon(X)$ results from the optimal decision of the wholesale goods firms and the labour intermediaries.
8. *Wages, nominal interest rate, SDF and social contribution rate.* Firm owners' wage $w^F(X)$ is equal to $\psi w(X)$, where the worker's wage $w(X)$ is given by (26); The nominal interest rate $R(X)$ and the stochastic discount factor $M^F(X, X')$ are given by (27) and (11), respectively; the social contribution rate $\tau(X)$ adjusts so that (1) holds, given \mathbf{n}^W and $b^u e^z$;
9. *Market clearing.* The market clearing conditions (29), (31), (32), (33), and (35).
10. *Laws of motion.* Inflation $\pi(X)$ and price dispersion $\Lambda(X)$ evolve according (17) and (18). The law of motion of the cross-section distribution $\tilde{\boldsymbol{\mu}}(\cdot)$ is consistent with policy rules (only labor moves from $\tilde{\boldsymbol{\mu}}(\cdot)$ to $\boldsymbol{\mu}(\cdot)$):

$$\boldsymbol{\mu}(a, 0, X) = f(X) \sum_{\aleph \geq 1} \tilde{\boldsymbol{\mu}}(a, \aleph) + (1 - s(X))\tilde{\boldsymbol{\mu}}(a, 0), \quad \boldsymbol{\mu}(a, 1, X) = s(X)\tilde{\boldsymbol{\mu}}(a, 0),$$

$$\text{and, for } \aleph \geq 2, \quad \boldsymbol{\mu}(a, \aleph, X) = (1 - f(X))\tilde{\boldsymbol{\mu}}(a, \aleph - 1);$$

As before, note that knowledge of the cross-sectional distribution $\boldsymbol{\mu}(\cdot)$ allows one to compute the end-of-labor market transitions stage employment rate $\mathbf{n} = \mathbf{n}^W = \int_a d\boldsymbol{\mu}(a, 0, X)$. The law of motion of the distribution $\tilde{\boldsymbol{\mu}}'(\cdot)$ is consistent with policy rules (only assets moves from $\boldsymbol{\mu}(\cdot)$ to $\tilde{\boldsymbol{\mu}}'(\cdot)$)

$$\tilde{\boldsymbol{\mu}}'(a', 0, X) = 1_{a'=g_{a^e}(X)}\boldsymbol{\mu}(a, 0, X) \quad \text{and} \quad \tilde{\boldsymbol{\mu}}'(a', \aleph, X) = \int_a 1_{a'=g_{a^u}(a, \aleph, X)} d\boldsymbol{\mu}(a, \aleph, X).$$

11. *Individual and aggregate laws of motions coincide.* The law of motion for the family cross-sectional distribution $g_{\tilde{\mu}}(\tilde{\mu}(\cdot), f, s, X)$ that the head of the family of workers takes as given when solving his problem coincides with the law of motion for the aggregate cross-sectional distribution.
12. *Habits.* Tomorrow's habit level of a particular household type is equal to the average consumption of this type today, i.e.,

$$\mathbf{c}^{F'} = \mathbf{c}^F \text{ and } \mathbf{c}^{W'}(\aleph) = \int_a g_{c^u}(a, \aleph, X) d\mu(a, \aleph).$$

3 Equilibrium dynamics

We now show how our risk-sharing arrangement allows us to simplify cross-sectional distribution $\tilde{\mu}(\cdot)$ and the computation of the equilibrium. We first show that the support of $\tilde{\mu}(\cdot)$ is finite. Then, we construct an equilibrium under the conjecture that all unemployed workers by the end of the labor market transitions stage have the same level of nominal bonds. Finally, we provide sufficient conditions for this conjecture to hold. In the empirical section, we will check that these conditions are satisfied at all times under our estimated distribution of parameters and history of structural shocks.

3.1 A characterization of cross-sectional wealth dispersion

From the analysis above, a worker who was employed at the beginning of the labor market transitions stage and falls into unemployment by the end of the same stage has nominal bond holdings equal to $a^u = a^e$ when entering the consumption and saving stage. Because in expectations unemployment benefits are strictly lower than wage income, this newly unemployed worker will start decumulating assets within her first visit to the consumption and saving stage in order to partly insulate consumption from the fall in income. Despite this gradual decumulation, our framework is tractable thanks to the following property.

Proposition 1 *The cross-sectional distribution $\tilde{\mu}(a, \aleph)$ has a finite number of wealth states (i.e. the support of a is finite).*

This property follows directly and generically from our assumptions that (i) employed workers pool income and assets; and (ii) the borrowing limit $\underline{a}e^z$ is always tighter than the natural limit. The first assumption ensures that all employed workers share the same end-of-period wealth, hence all workers falling into unemployment at the end of the labor transition stage that were employed at the beginning of such stage have identical nominal bonds holding. Then, because they start symmetric and all face the same prices and labour market transition rates, they stay symmetric thereafter (i.e., they

all decumulate at the same pace whilst remaining unemployed). The second assumption implies that unemployed workers eventually hit a binding borrowing constraint in finite time; hence the number of wealth levels that one must keep track of is itself finite.

While the proposition is helpful as a general characterisation of the wealth distribution, it does not tell us how many wealth states the distribution has. The latter is determined by the number periods of consecutive unemployment it takes before a worker reaches the borrowing limit and hence depends on, first, the amount of bond holdings at the start of the unemployment spell; and second, the speed at which asset decumulation takes place. Both are functions of the deep parameters of the model, and one can always set them to generate equilibria with as many wealth states as one wishes without losing tractability.¹³ However, the data imposes some discipline on this figure. For example, the amount of wealth that workers hold (hence the initial bond wealth of a worker falling into unemployment) must be consistent with the broad features of the empirical cross-sectional wealth distribution. Similarly, the job-market transition rates, a key determinant of both initial bond holdings and the pace of asset decumulation, must be consistent with their empirical counterparts. In the remainder of the paper we assume that workers face the borrowing limit (i.e., they exhaust their bond wealth) after exactly one period of unemployment. In Section 4 we will show that the data corroborates this assumption. It is straightforward to construct equilibria with more gradual asset decumulation (i.e. taking more than one period after unemployment to hit the borrowing limit), but the data favors our specification.

3.2 The two-wealth state equilibrium: construction and existence conditions

The assumption that workers face the borrowing limit after exactly one period of unemployment implies that the support of a in the cross-sectional distribution $\tilde{\mu}(a, \aleph)$ has two points. The reader should note that we are also assuming that a^e , the amount of nominal assets that a employed worker holds, is above the borrowing limit. If that was not the case, the support of a in the cross-sectional distribution $\tilde{\mu}(a, \aleph)$ will only have one point.¹⁴ In what follows we first construct the “two wealth state equilibrium” and then provide a set of sufficient conditions for its existence (namely, the fact that employed workers never face a binding debt limit, while all unemployed workers do).

¹³For example, a high average job-loss rate will induce high buffer stock saving for workers falling into unemployment, which will tend to increase the length of asset decumulation. On the contrary, a high average job-finding rate makes the idiosyncratic income shock associated with unemployment more transitory, which will increase the speed of asset decumulation (and hence reduce its length).

¹⁴The situation where the cross-sectional distribution of wealth among the workers has a single point correspond to the “hand-to-mouth” economy, which we use in Section 5 in our counterfactual experiments.

3.2.1 Construction

We first note first that all employed workers share the same individual level of nominal bonds nominal bonds holding level, a^e , and the same individual consumption level, c^e , by the end of consumption-saving stage. When the wealth of a family of employed workers is A^e and the implied per capita bond is a^e (both after labor market transitions stage), the consumption of every employed worker (“ e worker” thereafter) is given by:

$$c^e = (1 - \tau)w + (1 + r)A^e/n - a^e$$

Since all workers that are unemployed by the end of the market transitions stage hit the borrowing constraint by the end of the period, regardless of how many periods they have been unemployed, the nominal bond holding of any unemployed workers by the end of the period is:

$$a^{u'} = \underline{a}e^z,$$

Since the consumption level of any household depends on both beginning-of-period and end-of-period wealth, there can be only two possible types of unemployed workers. First, those who were employed at the beginning of the labor market transitions stage (“ eu workers” thereafter) and have wage income b^ue^z , asset income $(1 + r)a^e$ and end-of-period wealth $\underline{a}e^z$. Hence their consumption is:

$$c^{eu} = b^ue^z + (1 + r)a^e - \underline{a}e^z.$$

Second, those who were unemployed at the beginning of the labor market transitions stage (“ uu workers” thereafter) and have wage income b^ue^z , asset income $(1 + r)\underline{a}e^z$ and end-of-period wealth $\underline{a}e^z$. Hence their consumption is:

$$c^{uu} = b^ue^z + (1 + r)\underline{a}e^z - \underline{a}e^z = b^ue^z + r\underline{a}e^z.$$

While the e workers are a fraction \mathbf{n} of the workers by the end of the labor transitions stage, eu workers and uu workers are in proportion $\mathbf{n}^{eu} = s\tilde{\mathbf{n}}$ and $\mathbf{n}^{uu} = 1 - \mathbf{n} - \mathbf{n}^{eu} = (1 - f)(1 - \tilde{\mathbf{n}})$, respectively, by the end of the same stage.

To summarize, under the assumption that workers face the borrowing limit after exactly one period of unemployment (i.e., the limit is binding for all unemployed households) there are only three worker types, i.e., e , eu , and uu workers. Each type has consumption levels c^e , c^{eu} and c^{uu} after the consumption and saving stage and they are in proportions \mathbf{n} , \mathbf{n}^{eu} , and \mathbf{n}^{uu} after the labor market transitions stage, respectively. There are two possible wealth levels, a^e and $\underline{a}e^z$. While a^e determines the amount of nominal bonds in the hands of each e worker, both eu and uu workers have $\underline{a}e^z$ of nominal bonds.

It is also the case that, because there are only two types of unemployed workers, there are only two relevant habit levels for them: $\mathbf{c}^W(1)$ and $\mathbf{c}^W(2)$, which satisfy, respectively, $\mathbf{c}^{W'}(1) = c^{eu}$ and $\mathbf{c}^{W'}(\aleph) = \mathbf{c}^{W'}(2) = c^{uu}$ for all $\aleph \geq 2$.

The rest of the endogenous variables that are determined in equilibrium naturally follow from this simple cross-sectional distribution. For example, the wealth inflow into a family of employed workers at the beginning of the consumption and saving stage comes from workers who transit from unemployment to employment during the labor market transition stage and it is equal to $B = f(1 - \tilde{\mathbf{n}})\underline{a}e^z$ because all unemployed workers have bonds holdings equal to $\underline{a}e^z$.

3.2.2 Existence conditions

In order to construct the two wealth state equilibrium, we have assumed that all unemployed workers face a binding borrowing limit while none of the employed do. We now present a set of sufficient conditions that can be used to check whether this is the case in equilibrium. A first set of conditions is that all unemployed workers in our equilibrium (that is, both eu and uu workers) face a binding borrowing limit. Formally, their optimal asset holding condition must be corner, i.e.,

$$\mathbb{E} [M^{ju} (1 + r')] < 1, \quad j = e, u, \quad (37)$$

where M^{ju} is the SDF of j workers, $j = e, u$, which is given by:

$$M^{ju} = \beta^W e^{\Delta\varphi'_c} \frac{(1 - f') u'(c^{uu} - hc^{uu}) + f' u'(c^{eu} - hc^e)}{u'(c^{ju} - hc^W(\mathbf{s}))}$$

for $j = e, u$, with $\mathbf{s} = 1$ if $j = e$ and $\mathbf{s} = 2$ if $j = u$.

The third conditions is that employed workers do not face not to face a binding borrowing limit.. Formally, their optimal asset holding condition must be interior, i.e.,

$$\mathbb{E} [M^{e'} (1 + r')] = 1. \quad (38)$$

where $M^{e'}$ is the SDF of e workers, which is given by:

$$M^{e'} = \beta^W e^{\Delta\varphi'_c} \frac{(1 - s') u'(c^{e'} - hc^e) + s' u'(c^{eu'} - hc^{eu})}{u'(c^e - hc^W(0))}.$$

In the quantitative evaluation of the model we will make sure that conditions (37)–(38) hold in every period, given the estimated deep parameters of the model and the realized histories of structural aggregate shocks that we extract from our sample.

4 Time-varying precautionary savings

We can now present the main properties of the model concerning time-varying precautionary savings. If the sufficient conditions stated above hold, the optimal bond holdings decision of a family head of employed workers must be such that:

$$u'(c^e - hc^W(0)) = \mathbb{E} \left[\beta^W e^{\Delta\varphi'_c} [(1 - s') u'(c^{e'} - hc^e) + s' u'(c^{eu'} - hc^{eu})] (1 + r') \right]$$

The left hand side is the marginal cost of one unit saved per capita. The right hand side is the marginal gain, which depends on the next period employment status. More specifically, while a currently employed worker enjoys marginal utility $u'(c^e)$, it may either stay employed in the next period –with probability $1 - s'$ – and hence enjoy marginal utility $u'(c^{e'})$, or fall into unemployment –with probability s' – and enjoy marginal utility $u'(c^{eu})$. Under incomplete consumption insurance, the latter is greater than the former, which motivates precautionary asset holdings by the family in excess of the borrowing limit (despite the fact that workers are impatient relative to employers). It is useful to write this Euler equation as $\mathbb{E}[M^{e'}(1 + r')] = 1$, where $M^{e'}$ is the SDF of employed workers:

$$M^{e'} = \beta^W e^{\Delta\varphi'_c} \frac{(1 - s')u'(c^{e'}) + s'u'(c^{eu}) - hc^W(0)}{u'(c^e - hc^W(0))}, \quad (39)$$

and where we have used the fact that $c^{W'}(0) = c^e$ and $c^{W'}(1) = c^{eu}$.

Further intuition about how uninsured idiosyncratic risk affects the asset demand of workers can be obtained by means of an approximate version of (39). Suppose for clarity that $h = 0$. Then, for s' small we have:

$$\hat{M}^{e'} \simeq \underbrace{\Delta\varphi'_c - \sigma \left(\frac{c^{e'} - c^e}{c^e} \right)}_{\text{full-insurance SDF}} + \underbrace{\sigma \mathbb{E} \left[\frac{c^{e'} - c^{eu}}{c^e} \right] s'}_{\text{impact of incomplete insurance}}.$$

The term above the first horizontal curly bracket in the right hand side of the equation correspond to the usual (log-linear) SDF under complete markets. The last term in the equation is a correction to this SDF coming from the fact that markets are incomplete and hence workers are imperfectly insured. More specifically, the term $\mathbb{E}[(c^{e'} - c^{eu})/c^e]$ is the mean (i.e., steady-state) consumption growth differential between a worker who stays employed from the current to the next period and a worker who falls into unemployment in the next period. s' is the probability of the latter event occurring. The product of the two is positive whenever unemployment insurance is incomplete (so that $\mathbb{E}[c^{e'} - c^{eu}] > 0$) and there is a possibility of falling into unemployment in the next period (i.e., s'). It is apparent that in the latter case an increase in unemployment risk translates *at the first order* into a greater SDF, i.e., a greater desire to save.

5 Empirical analysis

In this section, we expound in detail how the model is taken to the data. We first outline the solution method and our Bayesian empirical strategy. We then described the data used to estimate the model. We then describe the calibration restrictions imposed at the onset and the prior choice for the remaining parameters together with posterior estimates. We then check that the equilibrium conditions are ex post met in our sample. Finally, we report a conditional variance decomposition exercise, highlighting which shocks account for the largest portion of aggregate variance at various horizons.

5.1 Solution method and empirical strategy

Before proceeding, we must specify the functional forms adopted for the utilization cost function $\eta(\cdot)$ and the investment adjustment cost function $S(\cdot)$. We assume:

$$\eta(u) = \frac{\bar{r}_k}{\nu_u} e^{\nu_u(u-1)-1} \quad \text{and} \quad S\left(\frac{i'}{i}\right) = \frac{\nu_i}{2} \left(\frac{i'}{i} - g_i\right)^2, \quad \text{with } \nu_u, \nu_i > 0.$$

Here \bar{r}_k is the steady-state value of the rental rate of capital r_k , ν_u is the curvature of the utilization cost, and ν_i is the curvature of the investment adjustment cost function. These function forms ensure that in a steady state with $i/\mathbf{i}_{-1} = g_i$ and $u = 1$, both the adjustment costs and the utilization costs vanish.

Moreover, we impose the following functional form for the exogenous job destruction rate:

$$\rho(\varphi_s) = \frac{1}{1 + e^{\bar{\rho} - \varphi_s}},$$

where $\bar{\rho}$ is a constant that pins down the steady-state value of ρ . With this functional form, we ensure that ρ varies only in the compact set $[0, 1]$.

In order to simplify the presentation of the empirical analysis we are going to index variables by a time subscript, where t will denote current variables and $t - 1$ lagged ones. Also, before, taking the model to the data, we first induce stationarity by normalizing the model by e^{zt} and we linearize the resulting system in the neighborhood of the resulting steady state.¹⁵ Then, let \hat{X}_t denote the vector collecting the deviation from steady state of the normalized state variables and let ϵ_t denote the vector collecting the innovations of the structural shocks. The law of motion of \hat{X}_t is of the form

$$\hat{X}_t = \mathbf{F}(\boldsymbol{\vartheta})\hat{X}_{t-1} + \mathbf{G}(\boldsymbol{\vartheta})\epsilon_t, \quad (40)$$

where $\boldsymbol{\vartheta}$ is the vector of model's parameters. The matrices $\mathbf{F}(\boldsymbol{\vartheta})$ and $\mathbf{G}(\boldsymbol{\vartheta})$ are functions of the model's parameters.

The stochastic shocks considered at the estimation stage are the following. We first consider $\varphi_{h,t}$, for $h \in \{c, i, w, s, p, R\}$. They are assumed to follow AR(1) processes of the form:

$$\varphi_{h,t} = \rho_h \varphi_{h,t-1} + \sigma_h \epsilon_{h,t}, \quad \epsilon_{h,t} \sim \mathcal{N}iid(0, 1).$$

Second, the technology shock z_t follows a random walk:

$$z_t = \mu_z + z_{t-1} + \sigma_z \epsilon_{z,t}, \quad \epsilon_{z,t} \sim \mathcal{N}iid(0, 1).$$

Finally, we append a serially correlated measurement error to the logged share of consumption of the 60 percent poorest in total consumption. This measurement error is meant to capture the statistical discrepancies between CEX and NIPA data (see Heathcote et al., 2010).

¹⁵See Appendix for further details on the procedure used to induce stationarity.

We use eight variables as observable in estimation, namely: The growth rate of consumption $\Delta \log(c_t)$, the growth rate of total investment $\Delta \log(\tilde{i}_t)$, the logged share of consumption of the 60 percent poorest in total consumption $\log(c_{60,t}/c_t)$, inflation π_t , the nominal interest rate R_t , wage inflation $\Delta \log(W_t)$, the separation rate s_t , and the job-finding rate f_t . Total investment \tilde{i}_t is defined as the sum of actual investment i_t and utilization and vacancy posting costs, so that:

$$\tilde{i}_t = i_t + \eta(v_t)k_{t-1} + \kappa_v v_t.$$

The measurement equation is

$$\begin{pmatrix} \Delta \log(c_t) \\ \Delta \log(\tilde{i}_t) \\ \log(c_{60,t}/c_t) \\ \pi_t \\ R_t \\ \Delta \log(W_t) \\ s_t \\ f_t \end{pmatrix} = \mathbf{M}(\boldsymbol{\vartheta}) + \mathbf{H}(\boldsymbol{\vartheta})\hat{X}_{t-1} + \mathbf{J}(\boldsymbol{\vartheta})\epsilon_t. \quad (41)$$

We use $\mathbf{M}(\boldsymbol{\vartheta})$ to denote the vector of means of observed variables. We follow the Bayesian approach to estimate the model's parameters. Based the state-space representation for the dynamic system, the Kalman filter is then used (i) to evaluate the likelihood of the observed variables at any value of $\boldsymbol{\vartheta}$ and (ii) to form the posterior distribution by combining the likelihood function with a joint density characterizing some prior beliefs.

Given the specification of the model, the posterior distribution cannot be recovered analytically but we may numerically draw from it, using a Monte-Carlo Markov Chain (MCMC) sampling approach. More specifically, we rely on the Metropolis-Hastings algorithm to obtain a random draw of size 1,000,000 from the posterior distribution of the parameters.

5.2 Data

The data used for estimation come from the Bureau of Economic Analysis, the Federal Reserve Bank of St. Louis' FRED II database, and from the Bureau of Labor Statistics website. Private investment is defined as the sum of gross private domestic investment (GPDI) and personal consumption expenditures on durable goods (PCDG). The resulting series is deflated by the implicit GDP deflator (GDPDEF). Consumption is defined as the sum of personal consumption expenditures on nondurable goods and services (PCNDS) and government consumption expenditures. The resulting series is also deflated by the implicit GDP deflator. Output is simply defined as the sum of consumption and investment. Consumption of the 60 percent poorest is computed as in Heathcote et al. (2010). Basically,

consumption is defined as the sum of consumption of non durables and services from CEX data, and is then sorted by income levels. All these series are converted to per-capita terms by dividing them by the civilian population, age 16 and over (CNP16OV).

For the labor-market transition probabilities, we proceed as follows. First, we compute monthly job-finding probabilities using CPS data on unemployment and short-run unemployment, using the two-state approach of Shimer (2005, 2012). (As suggested by Shimer, 2012, the short-run unemployment series is made homogenous over the entire sample by multiplying the raw series by 1.1 from 1994M1 onwards). We then compute monthly separation probabilities as residuals from a monthly flow equation similar to equation (1). Using these two series, we construct transition matrices across employment statuses for every month in the sample, and then multiply those matrices over the three consecutive months of each quarter to obtain quarterly transition probabilities (this naturally implies that cyclical fluctuations in the quarter-to-quarter separation rate partly reflect changes in the underlying monthly job-finding probability). All the series are seasonally adjusted except for population. Our sample runs from 1985Q1 to 2007Q1.

5.3 Calibrated parameters

The vector of parameters ϑ is split into two subvectors ϑ_1 and ϑ_2 . The first one,

$$\vartheta_1 = (\sigma, \delta, \theta, \chi, \bar{m}, \bar{\pi}, \mu_z, \beta^F, \beta^W, \phi, \underline{a}, b^u, \kappa_v, \bar{\rho}, \kappa_y, \psi, \bar{w}),$$

contains parameters calibrated prior to estimation. Typically, these are parameters difficult to estimate in our framework or parameters tied to a steady-state restriction imposed ex ante. The remaining parameters, contained in ϑ_2 , are estimated (see next section for details).

A number of parameters in ϑ_1 are calibrated outright. We impose the CRRRA parameter $\sigma = 2$. The depreciation rate $\delta = 0.015$ implies a 6 percent annual depreciation of physical capital. We choose θ such that the markup is 20 percent, as is now standard in the literature. The elasticity of the matching function with respect to vacancies is set 0.5, so that $\chi = 0.5$. This value is also standard. Finally, the parameters \bar{m} is normalized to 1.

The remaining parameters in ϑ_1 are tied to steady-state restrictions that we impose ex ante. The steady-state inflation rate $\bar{\pi}$ is set to match the average value of inflation over the sample. The growth rate of technical progress μ_z is set to match the average growth of output over the sample. The subjective discount factor of employers $\beta^F = 0.9985$ is set so that, given the restriction on μ_z and $\bar{\pi}$, the steady-state nominal real interest rate matches its average empirical counterpart. The subjective discount factor of workers $\beta^W = 0.9835$ is set so that the ratio of individual consumptions c^{eu}/c^e is 80 percent, meaning that upon reaching unemployment, workers lose 20 percent of consumption compared with workers who stay employed. We pin down ϕ so that the labor share in income is 64

percent. We restrict borrowing by setting $\underline{a} = 0$. The parameter b^u is set so as to impose a replacement rate of 50 percent. We set κ_v so that the share of vacancy costs in output is 1 percent. The parameter $\bar{\rho}$ is pinned down by imposing that the steady-state value of s coincides with its empirical counterpart. The parameter κ_y is set so that steady-state monopolistic profits are zero. We set the skill premium parameter ψ so as to match the average share of income of the 60 percent poorest in total income, as backed out from CEX data. This procedure underscores a clear advantage of our setup, in that it allows us to make explicit contact with micro data at the calibration stage. Given the preceding restrictions, we select \bar{w} to match the average value of f .

It is important to acknowledge that because most of the steady-state restrictions also involve the estimated parameters in $\boldsymbol{\vartheta}_2$, the restricted parameters will be functions of the posterior draws. More precisely, for each possible draw of $\boldsymbol{\vartheta}_2$, we readjust the parameters in $\boldsymbol{\vartheta}_1$ to meet all the steady-state restrictions listed above.

5.4 Estimated parameters

The remaining parameters contained in $\boldsymbol{\vartheta}_2$ are estimated. They are listed in table 1, together with information on their prior and posterior distributions.

Our choice of priors is standard. We impose Beta distributions for all the parameters the theoretical support of which is the compact $[0,1]$. We use Gamma distributions for positive parameters. Finally, we use Inverse Gamma distributions for the standard errors of shocks.

All the parameters governing the serial correlation of structural shocks have priors centered on 0.75, with standard deviation 0.10. The only exception is ρ_R which has a prior distribution centered on 0.20, reflecting our prior belief that this shock has a lower degree of serial correlation than the other shocks. All the standard errors of shocks have priors centered on 1, with standard deviation set to 2. This choice is standard in the literature. The curvature of the utilization cost ν_u has a prior mean of 1, with standard deviation equal to 0.5. This is similar to the prior selected by Christiano et al. (2014). The curvature of investment adjustment costs has a prior mean equal to 3, with standard deviation set to 0.5. The degree of habit formation has a prior centred on 0.5, with standard deviation set to 0.1, as in Christiano et al. (2014). The degree of price stickiness α , the degree of price indexation γ_p , and the degree of wage indexation γ_w all have prior distributions centered on 0.75, with standard deviation equal to 0.1. The sensitivity of the real wage to employment ψ_n has a prior centered on 1, with standard deviation equal to 0.5.

The parameters governing the degree of interest-rate smoothing ρ has a prior centered on 0.75, with standard deviation 0.1. The degree of responsiveness to inflation in the Taylor-like rule a_π has a prior mean of 1.5, with standard deviation equal to 0.15. The degree of responsiveness to economic activity has a prior mean of 0.13, with standard deviation of 0.05. These values are standard in the literature

Table 1: Estimation Results

Parameter	Prior shape	Prior Mean	Prior s.d.	Post. Mean	Low	High
ν_u	Gamma	1.00	0.50	0.80	0.25	1.33
ν_i	Gamma	3.00	0.50	2.66	1.84	3.48
h	Beta	0.50	0.10	0.69	0.58	0.80
α	Beta	0.75	0.10	0.73	0.67	0.79
γ_p	Beta	0.75	0.10	0.59	0.38	0.79
γ_w	Beta	0.75	0.10	0.89	0.85	0.94
ψ_n	Gamma	1.00	0.50	2.46	1.56	3.35
ρ	Beta	0.75	0.10	0.79	0.75	0.84
a_π	Gamma	1.50	0.15	1.87	1.62	2.12
a_y	Gamma	0.13	0.05	0.16	0.07	0.25
ρ_{me}	Beta	0.75	0.10	0.99	0.99	1.00
ρ_p	Beta	0.75	0.10	0.82	0.73	0.90
ρ_c	Beta	0.75	0.10	0.82	0.71	0.93
ρ_w	Beta	0.75	0.10	0.65	0.48	0.82
ρ_i	Beta	0.75	0.10	0.65	0.51	0.79
ρ_s	Beta	0.75	0.10	0.91	0.87	0.96
ρ_R	Beta	0.20	0.10	0.45	0.31	0.59
σ_{me}	Inverted Gamma	1.00	2.00	0.43	0.38	0.49
σ_c	Inverted Gamma	1.00	2.00	3.61	2.53	4.72
σ_w	Inverted Gamma	1.00	2.00	0.31	0.27	0.35
σ_i	Inverted Gamma	1.00	2.00	3.00	2.05	3.89
σ_p	Inverted Gamma	1.00	2.00	1.45	0.92	1.98
σ_z	Inverted Gamma	1.00	2.00	1.06	0.93	1.19
σ_R	Inverted Gamma	1.00	2.00	0.14	0.12	0.16
σ_s	Inverted Gamma	1.00	2.00	9.58	8.37	10.74

Note: Low and High stand for the lower and upper boundaries of the 90 percent HPD interval, respectively.

and correspond broadly to the values found in Taylor (1993).

Next, we comment on the estimation results. For each parameter, table 1 reports the posterior mean together with the bounds of the 90 percent Highest Posterior Density interval (HPD, labeled “low” and “high”).

The posterior mean of curvature of the utilization cost ν_u is 0.8, slightly different from the prior mean. The data also bring about useful information on the curvature of investment adjustment costs, with a posterior mean for ν_i close to 2.7. In spite of a relatively tight prior, the estimation outcome yields a high degree of habit formation, with a posterior mean for h close to 0.7. The degree of price stickiness α has a posterior mean equal to 0.73, implying an average price duration of almost 4 quarters. The posterior means of the degrees of price and wage indexation are equal to 0.6 and 0.9, respectively. The value for γ_p is in the range of previous estimates obtained in the literature. The value for γ_w is quite high, suggesting a relatively strong degree of nominal wage stickiness. In turn, this is partly

compensated by a posterior mean degree of responsiveness of wages to employment twice as large as its prior mean. Similarly, the posterior mean of the degree of interest rate smoothing is somewhat higher than its prior mean, with ρ close to 0.8. The responsiveness of the nominal interest rate has a posterior mean of 1.9, higher than its prior mean. This is suggestive of strong reaction to inflation on the part of the monetary policy authority, consistent with the historical record over the post Volcker era. The posterior mean of the responsiveness to economic activity is hardly different from the prior mean, suggesting that the data are close to silent when it comes to this parameter.

5.5 Empirical results

5.5.1 Equilibrium reduction conditions

Before proceeding, we first have to check that the equilibrium reduction conditions are met over our sample.¹⁶ Recall that for our equilibrium reduction to hold, it must be the case that conditions 37 and 38, i.e. (i) $\mathbb{E}[M^{eu}(1+r')] < 1$ and $\mathbb{E}[M^{uu}(1+r')] < 1$, and (ii) $\mathbb{E}[M^{e'}(1+r')] = 1$.

Focusing on the first set of conditions, one sees that $\mathbb{E}[M^{eu}(1+r')] < 1$ equivalently rewrites

$$u'(c^{eu} - hc^{eu}) - \mathbb{E}[(1 - f')u'(c^{uu} - hc^{uu}) + f'(u'(c^{e'} - hc^{e'}))](1 + r') > 0, \quad (42)$$

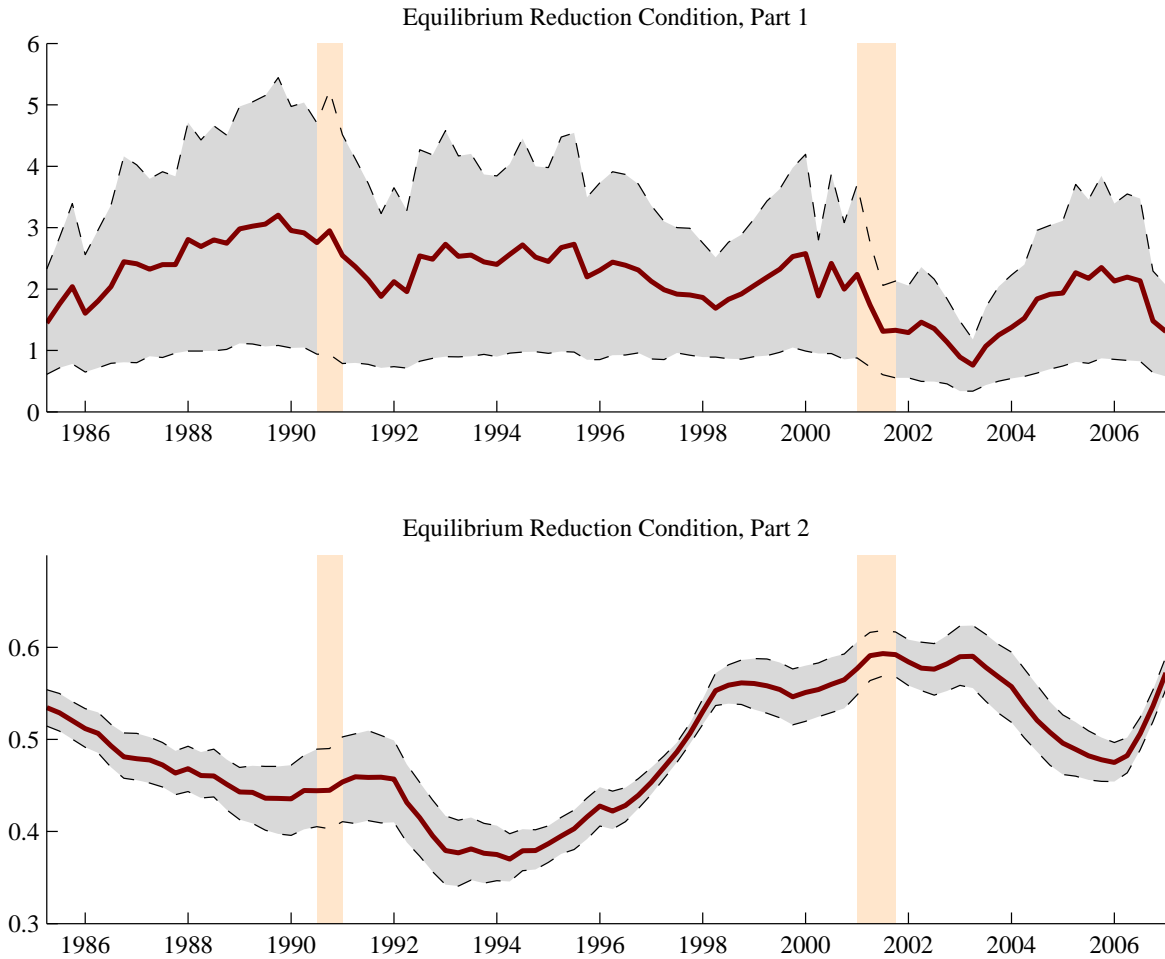
where it is understood that in our empirical evaluation, we focus on a version of this inequality involving appropriately detrended variables. Notice that since $u'(\cdot)$ is strictly decreasing and $c^{eu} > c^{uu}$, if the above condition holds, then $\mathbb{E}[M^{uu}(1+r')] < 1$ holds as well. The second condition stipulates that $\mathbb{E}[M^{e'}(1+r')] = 1$. This is equivalent to having $A_t^e > 0$.

That these conditions hold in our estimation is illustrated in figure 2. The top subplot reports the posterior mean values of the (appropriately detrended) left-hand side of the above inequality over the estimation sample (thick red line), together with the associated 90 percent HPD interval (the grey area delineated by the thin, black dashed lines). The bottom subplot reports the posterior mean values of the detrended version of A_t^e over the estimation sample (thick red line), together with the associated 90 percent HPD interval (the grey area delineated by the thin, black dashed lines). All these values are obtained by running the Kalman smoother for each MCMC draw from the posterior distribution of the estimated parameters. Finally, both subplot contain the NBER recession dates present in our estimation sample, each represented by a pink area. The NBER recessions present in our sample are 1990Q3-1991Q1, 2001Q1-2001Q4.

As the figures make clear, 90 percent of the MCMC draws set the left-hand side of inequality (42) and A_t^e well above zero, implying that the listed conditions are indeed satisfied.

¹⁶We also have to check that given the history of aggregate shocks and the uncertainty about the estimated parameters, workers, firm owners and labor intermediaries all extract a positive surplus from the match in every period in the sample. This is done in the technical appendix to the paper.

Figure 2: Equilibrium Reduction Condition



Note: The thick red line is the posterior mean path, the grey area delineated by thin, black lines is the 90 percent HPD interval, and the pink bars indicate the NBER recession dates present in our sample.

Interestingly, figure 2 offers a clear illustration of the working of our model. It reveals notably that in the run up to and at the very beginning of an NBER recession, workers precautionary savings A_t^e has a tendency to build up, thus ultimately precipitating and aggravating the recession. In contrast, workers deplete their assets after the trough of the recession. The figure also illustrates that rebuilding precautionary savings can take some time. This is particularly apparent in the expansion phase following the 1991Q1 trough.

5.5.2 Variance decomposition

Finally, Table 2 reports the conditional variance decomposition for output, consumption, investment, all taken in logarithm, and employment. For each of these variables, the table reports the portion of the variance explained by each of the seven structural shocks, ϵ_h , $h \in \{R, c, i, w, p, z, s\}$, at quarters 0, 4, 8, and 12. This decomposition is obtained from the infinite moving average representation of the

linearized model, computed at the posterior mean. The infinite moving average representation derives from the transition equation (40).

Table 2 shows that when it comes to output, employment, and investment, the main driver at business cycle frequencies is the investment shock ϵ_i . Indeed, this shock explains about or more than 50 percent of the variance on impact and more than 20 percent at the fourth quarter. This is reminiscent of results reported in Justiniano et al. (2010).

By contrast, the main driver of consumption is the preference shock ϵ_c . It accounts for about 60 percent of the variance on impact, and still more than 40 percent 8 quarters later. This shock also explains a non-negligible part of the variance of output and employment.

The monetary policy shock ϵ_R explains about 10 percent of the variance of the variables up to the fourth quarter, and still about 5 percent at the eighth quarter. Combined with the preference shock, this implies that demand shocks explain about 20 percent of the variance of output and employment, and substantially more for consumption.

On impact, technology shocks ϵ_z account for about 4 percent of the variance of output, 14 percent for consumption, and close to 20 percent for employment. This contribution is substantially smaller for investment. By construction, ϵ_z accounts for a greater and greater portion of the variance of output, investment, and consumption at higher horizons. Asymptotically, this share converges to 100 percent.

The mark-up shock ϵ_p explains more than 10 percent of the variance of output, consumption, investment, and employment. The contribution is the smaller for consumption and the higher for employment. Finally, wage and separation shocks, ϵ_w and ϵ_s respectively, account only for a small portion of the business cycle.

5.5.3 Impulse response to a monetary policy shock

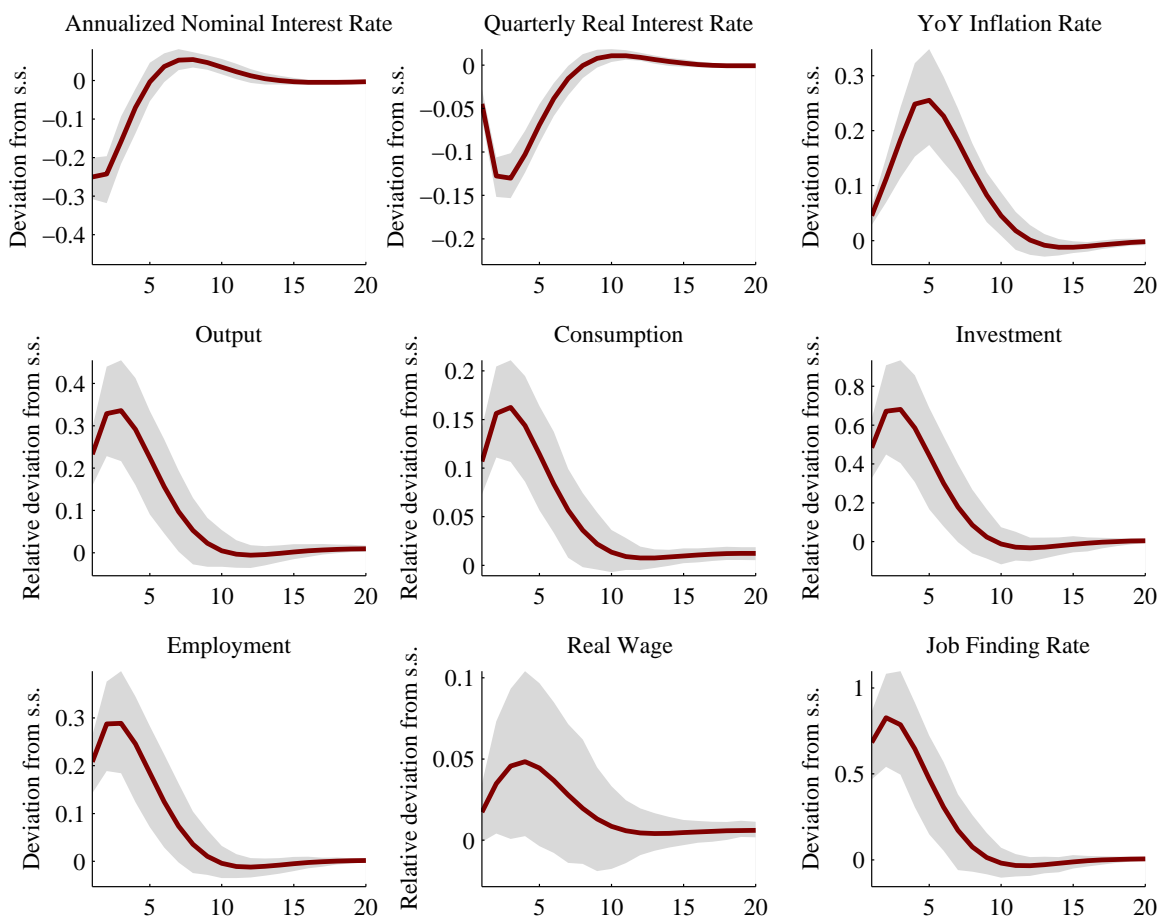
Next, as an illustration of the model’s dynamic behavior, figure 3 reports the impulse response functions (IRF) to a monetary policy shock.¹⁷ The shock is normalized so that the posterior mean impact response of the annualized nominal interest rate corresponds to a drop by 25 basis points. The red line is the posterior mean of IRF while the grey area delineates the 90 percent HPD interval. In addition to the the annualized nominal interest rate, we consider the real ex ante quarterly interest rate, the year-on-year inflation rate (all three in deviation from their steady-state values), output, consumption, investment (all three in percent deviation from their steady-state values), the employment rate (in deviation from its steady-state value), the real wage (in percent deviation from its steady-state value), and the job separation rate (in deviation from its steady-state value). The IRFs have the standard hump shape widely discussed in the empirical literature and resemble the typical outcome of a New Keynesian DSGE model, e.g. Christiano et al. (2005).

¹⁷The IRFs to the other shocks appear in the Appendix

Table 2: Conditional Variance Decomposition

Output							
Quarter	ϵ_R	ϵ_c	ϵ_i	ϵ_w	ϵ_p	ϵ_z	ϵ_s
0	14.28	7.96	54.93	1.00	16.46	4.32	1.04
4	9.11	2.65	23.28	3.43	25.16	36.18	0.18
8	6.25	1.89	15.35	4.00	20.28	52.04	0.20
12	5.25	1.80	12.89	3.53	17.08	59.27	0.18
Consumption							
Quarter	ϵ_R	ϵ_c	ϵ_i	ϵ_w	ϵ_p	ϵ_z	ϵ_s
0	8.80	62.80	2.83	0.43	10.47	14.63	0.04
4	5.07	41.22	0.81	1.00	15.54	36.09	0.28
8	3.16	30.87	0.50	1.06	12.77	51.33	0.32
12	2.44	24.86	0.45	0.88	10.11	60.97	0.29
Investment							
Quarter	ϵ_R	ϵ_c	ϵ_i	ϵ_w	ϵ_p	ϵ_z	ϵ_s
0	10.85	0.20	73.30	0.88	12.32	0.49	1.96
4	8.11	4.65	39.36	3.96	21.26	22.37	0.28
8	6.10	9.29	28.98	5.06	17.87	32.50	0.22
12	5.55	10.53	26.28	4.78	16.55	36.11	0.20
Employment							
Quarter	ϵ_R	ϵ_c	ϵ_i	ϵ_w	ϵ_p	ϵ_z	ϵ_s
0	12.81	6.84	47.31	2.28	11.30	19.05	0.41
4	15.22	4.43	35.49	8.47	28.87	7.24	0.27
8	13.94	4.12	31.40	11.61	29.23	9.29	0.40
12	13.75	4.28	31.15	11.72	29.47	9.22	0.41

Figure 3: Impulse Response Functions to a Monetary Policy Shock



Note: The thick red line is the posterior mean IRF, the grey area is the associated 90 percent HPD interval.

6 Quantitative results

In this section, we seek to identify the effects of the interaction between the lack of insurance markets and nominal frictions. We proceed sequentially. We first define a set of alternative economies in which we tilt the market arrangement to affect the precautionary savings motive. We first consider an economy in which impatient households always hit their borrowing constraint. Then we consider a representative-agent version of our baseline economy. For each specification, we then perturb the amount of nominal (price and wage) rigidities.

6.1 Alternative model specifications

To identify the effect of time-varying precautionary savings, we now compare the outcome of the baseline model to two alternative specifications of the model, in which there is no time-varying precautionary saving.

HM economy. In the first alternative economy, impatient households face complete insurance markets. As a consequence, they do not form precautionary savings. It is as if impatient households belonged to a family which ensures risk-sharing within the family. As these households have the lowest discount factor and there is no idiosyncratic risk, this framework is similar to that of Kiyotaki and Moore (1998). Impatient households always want to borrow and they all have the same saving decision:

$$a^{eI} = a^{uI} = \underline{a}e^z = 0,$$

The Euler equation for impatient households no longer determines the equilibrium allocation. All the other equations are the same. Using the budget constraint (6) and the balance of the UI scheme condition (1), one finds that the per capita consumption of impatient households obeys:

$$c^I = \frac{n(1 - \tau)w + (1 - n)b^ue^z}{n} = w$$

In the previous equality, we used the fact that $\underline{a} = 0$ for the baseline specification. This economy can thus be thought of as a hand-to-mouth (HM) model where impatient households consume all their income in all periods. As a consequence, we label this economy the HM economy. Finally, all parameter values are the same as in the baseline economy. It is worth noting that the HM economy and the baseline economy have the same steady-state values for aggregate consumption, output, investment, and unemployment. This is so because in each economy, the steady-state values of these variables are determined by the steady-state real interest rate which is pinned down by the discount factor of patient households.

RA economy. The second economy is an economy which has the same production structure, but which is populated by a representative agent (RA) which collects all resources, and faces complete markets. It is assumed that the representative agent has the same utility function as patient households (in particular the same discount factor), and that the unit of labor services provided by the representative household ψ^{RA} is such that the supply of labor services is the same in the baseline economy and in the RA-economy. This economy can be represented by the same equations as in the baseline case with a share of impatient households equal to $\Omega = 0$ and a productivity parameter $\psi^{RA} = \Omega + (1 - \Omega)\psi$. This correction to the productivity parameter ensures that the average productivity is the same in the RA economy as in our baseline model with precautionary saving. Since the steady state interest rate is also unchanged at $1/\beta^F - 1$, this in turn implies that the two economies have the same levels of net aggregate wealth.

6.2 Comparing impulse responses across model specifications

To investigate the difference between these three economies, we focus on two demand and two supply shocks. The demand shocks are the preference shock φ_c and the interest rate shock φ_R . The supply

Table 3: Simulation Results

	Consumption		Investment		Output	
	HM	RA	HM	RA	HM	RA
Monetary Policy Shocks	-20.21	-35.21	-0.21	-0.05	-5.94	-10.27
Preference Shocks	-11.86	17.71	-7.58	27.38	-17.32	6.08
Technology Shocks	10.68	-4.95	1.68	1.95	1.38	-1.97
Separation Shock	-5.63	-35.54	1.80	-5.37	10.63	-8.67

shocks are the permanent technology shock z and the shock job separation rate φ_s . For each of these shocks, we draw the associated Impulse Response Functions (IRF) for output, consumption, and investment. These are computed at the posterior mean.

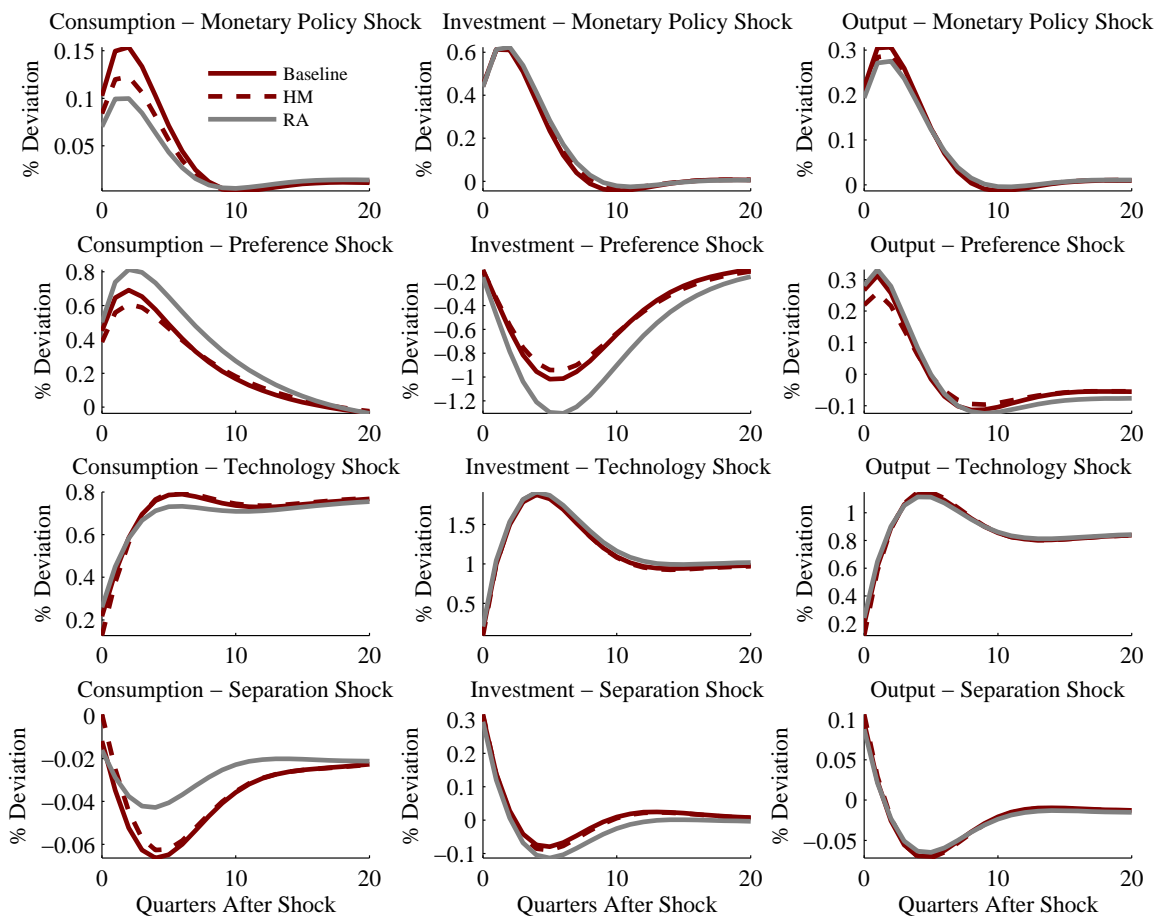
Although the model has seven shocks, we focus on these four shocks because they best help to understand the quantitative properties of the model. In particular, Ravn and Sterk (2014) investigate the effect of a shock on the job separation rate to assess the macroeconomic effects of precautionary saving in a calibrated model without capital. We can thus compare our results with theirs. The IRFs for other shocks can be found in the Technical Appendix.

To complement those impulse-responses, Table 3 shows the proportional differences in the responses of the main aggregates to the shocks under consideration. In each case, the difference is measured at the peak of the response generated by the baseline precautionary saving model.

Monetary policy shock. As shown in the first row of Figure 4, consumption increases much more after an expansionary policy shock than in the HM or RA economy. It increases 35 percent more than in the RA economy and roughly 20 percent more than in the HM economy. Investment behaves similarly in the three economies. The result is an additional amplification mechanism of 10 percent on output compared to the RA economy and 7 percent compared to the HM economy (even though investment is *smoother* under time-varying precautionary savings, due to the aggregate supply effects discussed above). The amplification effect captures the interaction between precautionary saving and the fall in aggregate demand, as will be shown in the next Section. The amplification effect disappears when prices are flexible.

Preference shock. The second row of Figure 4 reports the IRFs for the preference shock. One can first observe that the RA-economy exhibits the highest amplification mechanism, essentially because this shock is directly applied to the representative agents which is a patient agent. In the other two economies, it also affects impatient agents who react less to this shock. Comparing the baseline with the HM-economy, one finds that consumption increases is 16 percent higher after a preference shock.

Figure 4: Comparison of IRFs Across Specifications



Note: Impulse response functions of Output, Consumption and Investment, after a monetary shock (first row), a preference shock (second row), a technology shock (third row), and a shock on the job separation rate (fourth row). The size of each shock is set to the respective standard deviation obtained at the estimation stage, except for the monetary shock, for which it is set to induce a 25-basis-point drop in the annualized nominal interest rate. The black solid line is the baseline, the dashed line is the HM-economy, the grey line is the RA economy.

This is due to the decrease in precautionary saving. Investment falls more in the baseline economy than in the HM-economy (but less so than in the RA-economy). As a result, output increases in the baseline economy by 20 percent more than in the HM-economy. It increases less by 6 percent at the peak compared to RA-economy.

Technology shock. The three economies appear very similar concerning technology shock (which is a permanent shock in the three models). Although the RA economy exhibits a little bit less amplification concerning consumption, the baseline and the HM-economies are almost indistinguishable for the three variables. We conclude that the precautionary motive does not significantly alter the impact of TFP shocks.

Job separation shock. After a shock on the job separation rate, the baseline economy exhibits much more amplification for aggregate consumption than the RA economy. The fall in consumption is 10 percent higher at the trough in the baseline economy than in the HM economy. The IRFs for investment are very similar for the HM and the baseline economy. Investment falls a little bit more in the RA economy. Overall, the three economies have very similar behavior of output after such a shock. We conclude that precautionary saving induces a smaller amount of amplification for supply shock than for demand shock.

6.3 Effect of nominal frictions

We now exhibit the role of nominal frictions in the amplification mechanism found in the previous Section. Whereas the interaction of incomplete insurance market for the unemployment risk and labor market frictions has been studied under flexible price in Krusell Mukoyama and Sahin (2010), the quantitative effects of nominal frictions has not been investigated yet in such a framework.

We now compare the previous three economies, imposing ex post a set of parameter restrictions such that price are flexible.

There are two types of nominal frictions in the baseline economy. First, wholesale good firms adjust their price only with a probability $1 - \alpha$ in each period. The higher α , the “stickier” prices on the goods market. Second, there are rigidities in nominal wage adjustments, because of the inertia in nominal wage dynamics. This inertia is captured by the parameter γ^w in the wage equation (26). When $\gamma^w = 0$ there are no nominal frictions on the wage determination (but only real frictions). For the estimated parameters, we found $\alpha = 0.73$ and $\gamma^w = 0.89$.

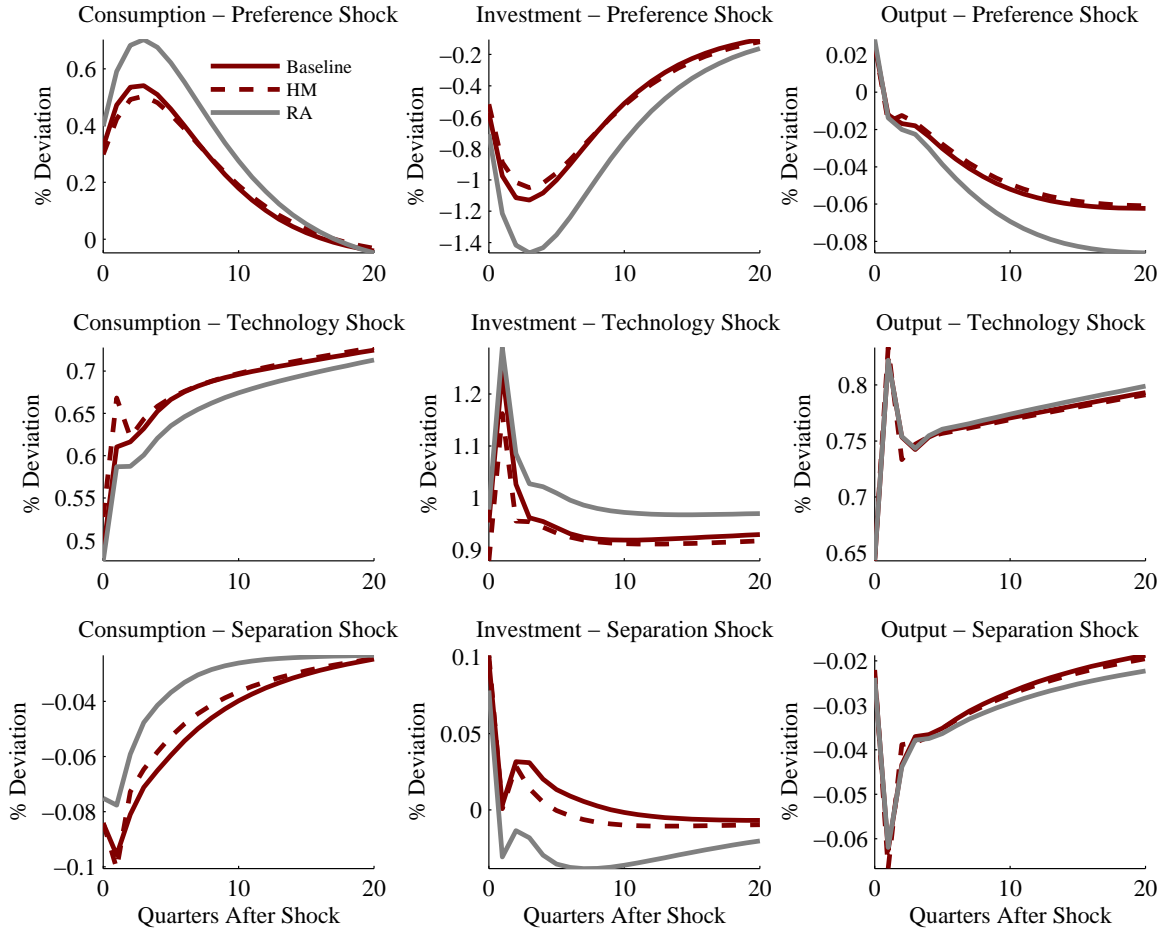
The flex-baseline economy. In the first economy, we consider the baseline economy, where we set $\alpha = 0.05$ and $\gamma^w = 0$ instead of $\alpha = 0.83$ and $\gamma^w = 0.89$. This economy is thus baseline economy with flexible price. We keep a low price rigidity on the goods market for the nominal inflation rate to be well defined with our Taylor rule.

The flex-HF economy. In this last economy we consider the HM economy where we set $\alpha = 0.05$ and $\gamma^w = 0$. This economy exhibits no precautionary saving and has flexible prices.

The flex-RA economy. The third economy is the RA economy defined in the previous section, where we also set $\alpha = 0.05$ and $\gamma^w = 0$. This economy is the representative agent economy with flexible prices.

We now focus on three shocks: the preference shock, the technology shock and the shock to job separation, all shocks having the same magnitude as in the previous Section. We no longer consider monetary policy shock, since this shock has a negligible effect in these three economies.

Figure 5: Comparison of IRFs Under Flexible Prices



Note: Impulse response functions of Output, Consumption and Investment after a preference shock (first row), a technology shock (second row) and a shock on the job separation rate (third row) in the three economies with flexible prices. The black solid line is the flex-baseline, the dashed line is the flex-HM economy, The grey line is the flex-RA economy.

Preference shock. The first row of Figure 5 plots the IRFs for the preference shock. One can first observe that the RA-economy exhibits again the highest amplification mechanism. Comparing the baseline with the flex-HM economy, one finds that consumption increases now only 10 percent more after a preference shock, whereas the corresponding number was 16 percent when comparing economies with nominal rigidities. As before, investment exhibits a symmetrical pattern: It decreases more in the flex-baseline economy. The overall effect on output is now very close between the three economies. If anything, the flex-HM economy exhibits more persistence.

Technology shock. After a technology shock, the flex-HM economy exhibits now the highest amplification. Both flex-HM and flex-baseline exhibit greater reaction for both consumption and investment after a technology shock. The overall effect on output is again indistinguishable between the three economies.

Job separation shock. After a shock on the job separation rate, the flex-HM economy generates the biggest effect on consumption at the though with less persistence than the flex-baseline economy. Investment has the shape behavior in the two economies, and is a little bit more persistent in the flex-baseline economy. Concerning output, the flex-HM economy generates again the highest deviation, but the response of the three economies is very similar.

We conclude that nominal frictions amplify the effect of precautionary saving for demand shock. For supply, shock they create a small amplification mechanism which is absent when prices are flexible.

6.4 Investigating the effect of nominal frictions

We now investigate separately the role of nominal frictions on the goods and labor markets. We do so by comparing three alternative economies. The first one is the baseline economy. The second one is an economy with sticky price only, where nominal wages are flexible (i.e. the baseline parameters except for $\gamma_w = 0$). The third one is an economy with sticky nominal wage only, where prices on the goods market are flexible (i.e the baseline parameters except for $\alpha = 0.05$).

Comparing demand and supply shocks, one observes that the sticky price economy generates higher amplification for demand shocks, whereas sticky wage generate higher amplification for supply shocks. We conclude that both frictions are necessary to obtain the effects of demand and supply shocks.

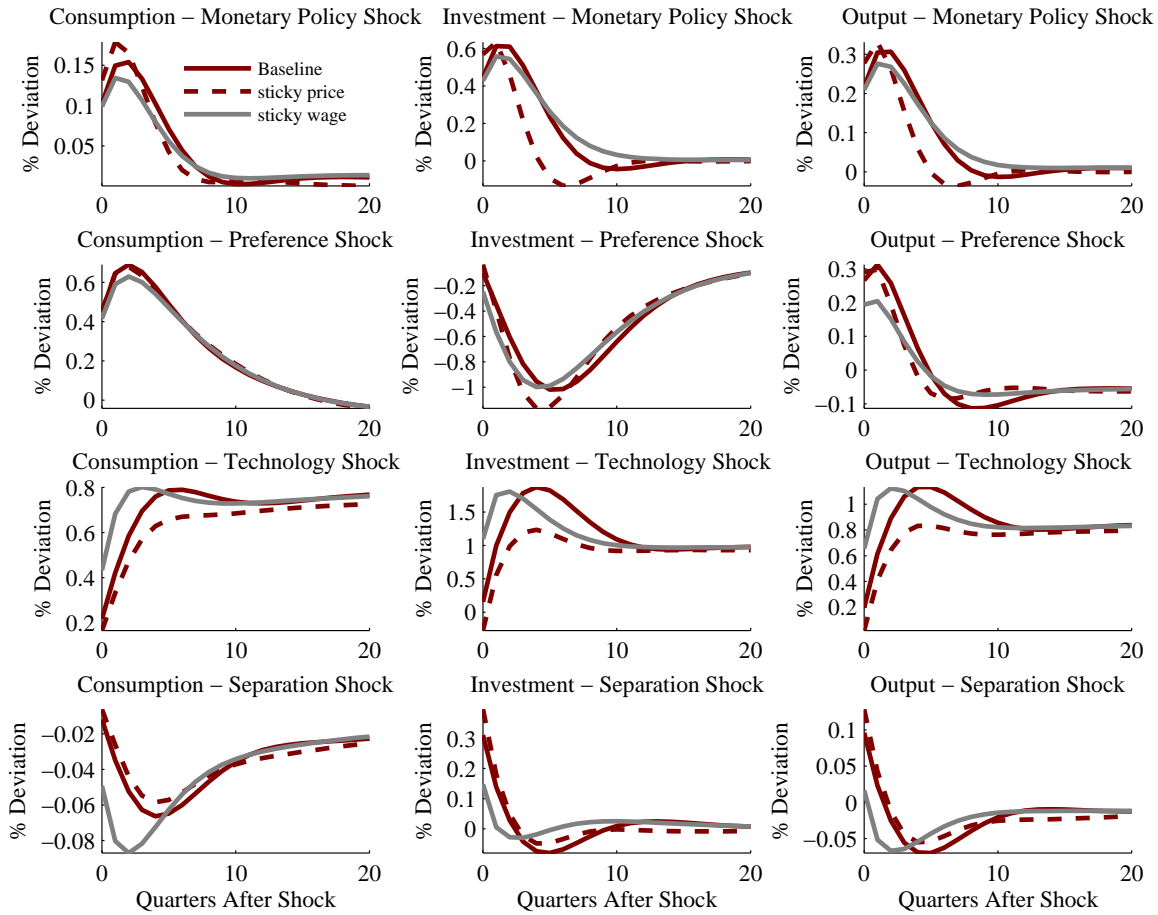
7 Concluding remarks

TBC

References

- [1] Aiyagari, S. R. (1994), ‘Uninsured idiosyncratic risk and aggregate saving’, *Quarterly Journal of Economics*, 109, pp. 659-684.
- [2] Beaudry, P. and Portier, F. (2014), ‘Reconciling Hayek’s and Keynes’ views of recessions’, Working Paper.
- [3] Blanchard, O.J., Gali, J. (2010), ‘Labor markets and monetary policy: A New Keynesian model with unemployment’, *American Economic Journal: Macroeconomics*, 2 (2), 1–3.
- [4] Carroll, C., Slacalek, J. and Sommer, M. (2012), ‘Dissecting saving dynamics: measuring wealth, precautionary, and credit effects’, IMF Working Paper 12/219.
- [5] Challe, E. and Ragot, X. (2014), ‘Precautionary saving over the business cycle’, *Economic Journal*, forthcoming.

Figure 6: Comparison of IRFs in the Baseline Model with or without Flexible Prices and Wages



Note: Impulse response functions of Output, Consumption and Investment, after a monetary shock (first row), a preference shock (second row), a technology shock (third row) and a shock on the job separation rate (fourth row). The black solid line is the baseline, the dashed line is the HM-economy, The grey line is the RA economy.

- [6] Chamley, C. (2014), ‘When demand creates its own supply: saving traps’, *Review of Economic Studies*, forthcoming.
- [7] Christiano, L.J., Eichenbaum, M., Evans, C.L. (2005), ‘Nominal rigidities and the dynamic effects of a shock to monetary policy’, *Journal of Political Economy*, 113 (1), 1–45.
- [8] Christiano, L.J., Motto, R., Rostagno, M. (2014), ‘Risk Shocks’, *American Economic Review*, 104 (1), 27–65.
- [9] Faia, E., 2008. ‘Optimal monetary policy rules in a model with labor market frictions’, *Journal of Economic Dynamics and Control*, 32 (5), 1600–1621.
- [10] Gertler, M., Sala, L., Trigari, A. (2008), ‘An estimated monetary DSGE model with unemployment and staggered nominal wage setting’, *Journal of Money, Credit and Banking*, 40 (8), pp. 1713–1764.

- [11] Gali, J. (2011), ‘Monetary policy and unemployment’, in *Handbook of Monetary Economics*, Vol. 3.A., pp. 487-546.
- [12] Gornemann, N., Kuester, K. and Nakajima, M. (2012), ‘Monetary policy with heterogenous agents’, Working Paper.
- [13] Guerrieri, V. and Lorenzoni, G. (2011), ‘Credit crises, precautionary savings, and the liquidity trap’, NBER Working Papers 17583.
- [14] Hall, R. (2005), ‘Employment fluctuations with equilibrium wage stickiness’, *American Economic Review*, 95 (1), pp. 50–64.
- [15] Heathcote, J. and Perri, F. (2013), ‘Wealth and volatility’, Working Paper.
- [16] Iacoviello, M. (2005), ‘House prices, borrowing constraints, and monetary policy in the business cycle’, *American Economic Review*, 95(3), pp. 739-764.
- [17] Justiniano, A., Primiceri, G.E. and Tambalotti, A. (2010), ‘Investment shocks and business cycles’, *Journal of Monetary Economics*, 57, pp. 132-145.
- [18] Kehoe, P., Midrigan, V. and Pastorino, E. (2014), ‘Debt constraints and unemployment’, Federal Reserve Bank of Minneapolis Working Paper.
- [19] Krusell, P. and Smith, A.A. (1998), ‘Income and wealth heterogeneity in the macroeconomy’, *Journal of Political Economy*, 106(5), pp. 867-896.
- [20] Krusell, P., Mukoyama, T and Sahin, A. (2011), ‘Labor-market matching with precautionary savings and aggregate fluctuations’, *Review of Economic Studies*, 77 (4), pp. 1477-1507.
- [21] Lucas, R. (1990), ‘Liquidity and interest rates’, *Journal of Economic Theory*, 50, pp. 237-264.
- [22] McKay, A. (2013), ‘Idiosyncratic risk and the business cycle: A likelihood perspective’, Boston University Working Paper
- [23] McKay, A. and Reis, R. (2013), ‘The role of automatic stabilizers in the U.S.business cycle’, Working Paper.
- [24] Michaillat, P. and Saez, E. (2014a), ‘A model of aggregate demand, idleness, and unemployment’, Working Paper.
- [25] Michaillat, P. and Saez, E. (2014a), ‘An economical business cycle model’, NBER Working Paper 18826.

- [26] Mortensen, D.T. and Pissarides, C. (1994), ‘Job creation and job destruction in the theory of unemployment’, *Review of Economic Studies*, 61, pp. 397-415.
- [27] Oh, H. and Reis, R. (2012), ‘Targeted transfers and the fiscal response to the Great Recession’, *Journal of Monetary Economics*, 59, pp. S50-S64.
- [28] Ravn, M.O. and Sterk, V. (2013), ‘Job uncertainty and deep recessions’, Working Paper
- [29] Rendhal, P. (2014), ‘Fiscal policy in an unemployment crisis’, Working Paper.
- [30] Shimer, R. (2005), ‘The cyclical behavior of equilibrium unemployment and vacancies’, *American Economic Review*, 95(1), pp. 25-49.
- [31] Trigari, A. (2009), ‘Equilibrium unemployment, job flows, and inflation dynamics’, *Journal of Money, Credit and Banking*, 41 (1), pp. 1-33.
- [32] Walsh, C. (2005), ‘Labor market search, sticky prices, and interest rate rules’, *Review of Economic Dynamics*, 8, pp. 829–849.