Heterogeneous Information and Labor Market Fluctuations*

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Abstract

I introduce dispersed information in a search and matching model of the labor market, where firms are hit by aggregate and idiosyncratic productivity shocks. The latter induce larger responses in recruiting activity than the former - because aggregate shocks have general equilibrium effects which partially offset the change in fundamentals. Informational frictions prevent firms from disentangling aggregate and idiosyncratic shocks. With Bayesian updating, firms attribute aggregate shocks largely to idiosyncratic factors, because the latter have significantly larger variances. This misattribution translates into an increased responsiveness of employment to aggregate shocks, relative to an economy with full information. I show that in a calibrated model, this channel has quantitatively significant effects and offers a potential solution to a well-known puzzle - the inability of standard search and matching models to generate sufficient volatility in labor market variables. In particular, the model with dispersed information brings the relative volatilities of employment and market tightness very close to those observed in the data.

JEL Classification: D82, E24, E32, J64

Keywords: Real business cycles, unemployment volatility puzzle, search and matching, incomplete information

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1 Introduction

The ability of models with search and matching frictions to match the observed cyclical properties of labor market activity has received a lot of attention in recent years. Shimer (2005) showed that, in a calibrated version of the standard Mortensen and Pissarides (1994) model, productivity shocks cannot generate the large movements in unemployment or labor market tightness observed in the data. Several authors have proposed modifications to the standard model with a view to resolving this puzzle[1]. In this paper, I propose a new explanation - informational frictions - and show that they can significantly resolve the discrepancy between the data and model-predicted movements.

At the heart of this result is a key feature of the firm-level data - evidence on firm-level decisions point to idiosyncratic shocks that induce large adjustments in firm-level employment. I interpret these shocks as innovations to firm-level productivity and embed them in a stochastic general equilibrium model with aggregate productivity shocks and search frictions in the labor market. These idiosyncratic factors lead to bigger changes in hiring activity at the firm level than aggregate shocks of comparable magnitude. This occurs for two reasons. Firstly, with search frictions, hiring decisions are based on expectations of future conditions and since idiosyncratic shocks are more persistent than aggregate shocks, they call for bigger changes in recruiting effort. Secondly, and more importantly, aggregate shocks have general equilibrium effects - on tightness of the labor market and on wages. These effects work to dampen the firm's incentives to change hiring in response to the change in the fundamental. For example, a positive aggregate shock increases labor market tightness and thereby makes it harder for firms to find workers. It also increases wages by affecting the households' marginal rate of substitution between consumption and leisure. A positive shock to firm-specific productivity, on the other hand, does not affect overall labor market conditions or wages (at least, not to the same degree as an aggregate shock). As a result, firms respond more aggressively to a idiosyncratic shock than to a aggregate shock of the same size.

In Figure 1, I plot monthly data on the total number of job separations and new hires in the US economy. As the graph shows, every month, about four to five million matches are destroyed every month and about the same number of new ones created. It seems natural to assume that firms, faced with this massive turbulence, find it difficult to disentangle shocks hitting the whole

[1]The literature on the empirical performance of labor seach models is too large to cite every worthy paper. Mortensen and Nagypal (2007) is a recent survey. Any reading list will certainly include Andolfatto (1996), Cole and Rogerson (1999), den Haan, Ramey and Watson (2000), Hall (2005), Farmer and Hollenhorst (2006), Gertler and Trigari (2006), Guerrieri (2007), Krause and Lubik (2007), Hall and Milgrom (2008), Costain and Reiter (2008), Hagedorn and Manovskii (2008) and Brugemann and Moscarini (2010).

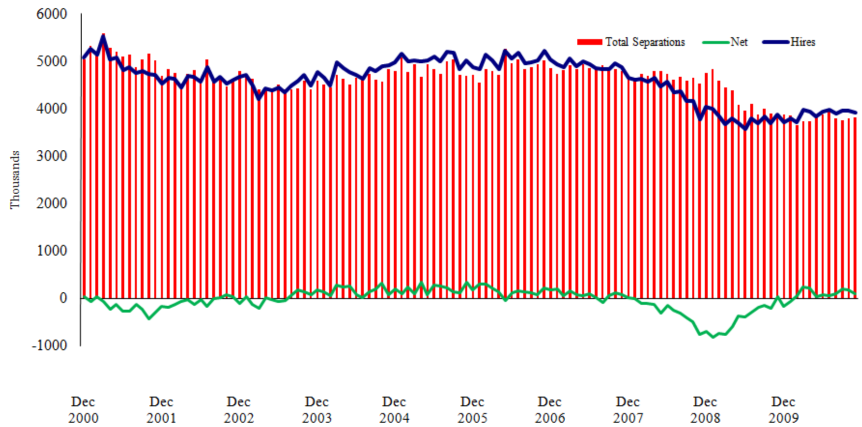

Figure 1: Job Destruction and Creation in the US

economy from the shocks affecting their own situation. This difficulty is compounded by the fact that fluctuations caused by economy-wide shocks are much more modest in comparison. This is illustrated by the green line in the figure, which plots the net hires, a proxy for overall economic conditions.

Now, suppose firms observe a combination of aggregate and idiosyncratic productivity shocks and use Bayesian updating to form forecasts of the individual components. In such a scenario, optimal filtering will lead firms to attribute aggregate shocks largely to idiosyncratic factors (because the latter tend to be an order of magnitude larger) and therefore, respond more aggressively (for the reasons discussed earlier) than they would under full information. This increased responsiveness at the firm level in turn translates into an amplified response of overall labor market activity to the aggregate shock. To evaluate the quantitative importance of this mechanism, I calibrate the model to both aggregate and micro data and show that informational frictions generates significant amplification in the volatilities of employment and hiring activity (relative to output volatility). In particular, in my benchmark calibration, informational frictions make market tightness slightly more volatile than in the data and close most of the gap between model-implied and observed relative volatilities of employment.

Quantitative evaluation of the implications of heterogeneous information has been hampered by two challenges. First, these models are not easy to solve, particularly in the kind of rich settings essential to answering quantitative questions. Second, since agents' information is seldom directly observable, it is not obvious how to calibrate these models. This paper makes a broader contribution to the literature by developing a flexible and tractable method, that has applicability beyond the specific context studied here. To address the first issue, this paper combines standard techniques for solving dynamic optimization problems using linear approximations with the approach for handling dynamic learning developed in Hellwig (2002). The result is a solution algorithm which can handle a large number of state variables - both fundamental and informational - without losing tractability. A key assumption that makes this possible is that aggregate shocks are revealed with a finite lag. In addition to being an intuitive way of capturing the diffusion of information about aggregates, this assumption also converts the learning problem into a finite-dimensional filtering problem. This allows a recursive characterization of the state space and considerably simplifies solving for equilibrium.

The second element of the paper's methodological contribution is a calibration strategy, which uses firm-level data to impose discipline on the information structure. This strategy is based on the intuitive idea that, before reliable information about aggregates arrives, firms use information generated in the normal course of their market activities to form forecasts about market conditions[2]. Examples of such signals include own productivity realizations, wages and labor market outcomes. Idiosyncratic variation in these signals is an important (and in this paper, the only) source of informational heterogeneity. Micro-level data (e.g. on firm-level employment or wages) provide direct evidence on the properties of these shocks and thus, provide a useful way to calibrate the degree of informational heterogeneity.

The basic idea that agents mistake aggregate shocks for idiosyncratic ones and therefore, react differently is not a new one in macroeconomics. A well-known application is Lucas (1972) where agents try to distinguish aggregate nominal shocks from 'island-specific' demand shocks before making a static labor input decision. Lucas (1972) shows that this confusion can generate real effects from purely nominal shocks, since agents respond partly to perceived movements in idiosyncratic demand shocks. More recently, Hellwig and Venkateswaran (2009) investigate the implications of this mechanism in an environment where firms attempt to disentangle aggregate nominal shocks and idiosyncratic demand/cost shocks from market-generated signals before setting prices[3] Apart

[2]Another justification for limiting the amount of information to just these signals comes from theories of costly information. For example, in Mackowiak and Wiederholt (2009), firms face a constraint on the amount of attention (defined as in Sims, 2003) and choose to allocate almost all of it to signals containing information about idiosyncratic factors. While that finding was in a price-setting context, the underlying economic rationale is a general one - firms will pay most attention to factors that have the greatest effect on their payoffs. Since idiosyncratic shocks are large and relatively persistent, they matter a great deal to firm profits.

[3]For other recent applications where heterogeneity in information is induced by payoff-relevant shocks (as opposed

from obvious differences in the context and the nature of the shocks, there are 2 major differences between this paper and the analysis in Hellwig and Venkateswaran (2009). The first difference relates to the dynamic nature of the choice variables in this paper compared to the static choices in Hellwig and Venkateswaran (2009). This introduces a new source of persistence and presents additional methodological challenges. The second difference lies in the nature of the conclusions reached in the two papers. In Hellwig and Venkateswaran (2009), informational frictions turn out to be largely irrelevant for the aggregate economy. Even though agents remain confused about the true nature of the shock, the economy behaves very much like a full information economy. The intuition behind this surprising result is that, despite being very uninformative about idiosyncratic vs. aggregate shocks, the firms' demand and wage signals provide a parsimonious, yet reasonably accurate indicator of their optimal full information pricing decisions. Consequently, even though firms are confused about the true source of the shocks, they manage to set prices quite close to their optimal levels. This implies that aggregate nominal shocks are reflected in prices very quickly, significantly dampening any effects on quantities (e.g. output, labor). Here, the economy behaves very differently under heterogeneous information. Confusion about the true nature of the shocks affects firms' incentive to recruit more workers and leads to 'excess' volatility in the aggregate.

Finally, for the calibration, I draw on a large literature studying idiosyncratic variability in employment and hiring activity. The literature on firm dynamics has documented two robust features of firm-level growth rates in the US - one, they show significant variability, considerably more than the volatility of aggregate employment, and two, they are independent of firm size. The first fact is suggestive of large idiosyncratic productivity shocks which an order of magnitude larger than shocks to aggregate productivity. The latter feature, often referred to as Gibrat's Law, implies that these shocks induce almost permanent changes in the level of employment, indicating that the effect on productivity must also be permanent (or at least, highly persistent). Together, these observations help calibrate the stochastic process for idiosyncratic productivity shocks. In addition, the environment of this paper also assumes idiosyncratic shocks to the efficiency of recruiters. These shocks serve to slow down the speed of learning about aggregate conditions from labor market outcomes. Recent work by Davis, Faberman and Haltiwanger (2010) studies the cross-sectional distribution of vacancies and hiring patterns and reports significant variation in the rate at which vacancies are converted into hires. Their estimates for the relationship between firm growth rates and vacancy yields provide a useful way to impose discipline on the size of these recruiting efficiency shocks.

to observational noise), see Amador and Weill (2010) or Graham and Wright (2010).

This paper is related to several branches of literature. Obviously, there is a direct relationship with the large body of work using search and matching frictions in models of business cycles. One of the key findings of this literature is that standard calibrations (like the one in Shimer (2005)) lead to relatively small fluctuations in the value of hiring activity for a firm in response to aggregate productivity shocks. As a result, resources devoted to recruiting activity - and consequently, employment - also do not fluctuate much with the cycle, leading to much more subdued movements in labor market activity than observed in the data. In this paper, informational frictions cause the *perceived* value of recruiting to be more volatile and thus induce bigger responses in firm decisions. This is in contrast to a number of papers which have introduced modifications to make the *actual* value more responsive to aggregate shocks. For example, Hall (2005) assumes that wages do not adjust to productivity changes, leading to bigger fluctuations in the value appropriated by the firm out of a match. However, Pissarides (2009) argues that the relevant wage for determining the value of hiring is the wage of the the *marginal* worker, which moves one-for-one with productivity in the data. As an alternative, he introduces, as does Cheremukhin (2010), fixed costs associated with hiring and shows that they can generate the required volatilities in the value of posting a vacancy. Hagedorn and Manovskii (2008) pursue an alternative calibration strategy which reduces the level of the match surplus, but leads to larger *percentage* fluctuations. Costain and Reiter (2008) argue that such a calibration solves the volatility puzzle but creates another one - it implies counterfactually high elasticities of unemployment to labor market policies. In the context of this debate, the main contribution of this paper is to introduce a novel channel of amplification, distinct from the mechanisms discussed above. To highlight the contribution of informational frictions, I abstract from the other modifications and stick as close as possible to the standard model[4].

This paper also relates to a growing literature on the role of heterogeneous information in business fluctuations[5]. In their seminal work, Phelps (1970) and Lucas (1972) showed that an economy with agents that are imperfectly informed about aggregate fundamentals can exhibit fluctuations in real variables from purely nominal shocks. Several papers have extended this basic hypothesis, by adding strategic interactions, as in Woodford (2003) and Mankiw and Reis (2002),

[4]Moreover, since my focus here is the job creation margin, I also abstract from other margins of labor adjustment explored by the literature, e.g. endogenous job destruction (as in Cheremukhin, 2010, den Haan and Ramey, 2000, Mortensen and Nagypal, 2007 or Eyigungor, 2010) or on-the-job search (as in Menzio and Shi, 2009).

[5]In addition to the papers mentioned in the text, an inexhaustive list from this literature includes Moscarini (2004), Bacchetta and Wincoop (2006), Reis (2006), Adam (2007), Veldkamp and Wolfers (2007), Angeletos and Pavan (2007), Gorodnichenko (2008), Angeletos and La'O (2008, 2010b), Hellwig (2008b, 2008a) and Lorenzoni (2009, 2010).

or limited capacity for processing information, as in Mackowiak and Wiederholt (2009), or market-generated information, as in Hellwig and Venkateswaran (2009) or Graham and Wright (2010). While many of these papers, especially the earlier ones, focus on the effects of monetary shocks on real variables, more recent work, notably Mackowiak and Wiederholt (2010), Lorenzoni (2009) and Angeletos and La'o (2010a), consider implications of heterogeneous information for 'real' shocks, which is the focus of this paper as well. In these papers, informational frictions dampen (and delay) the response of the economy to aggregate productivity shocks. In contrast to these findings, I show that informational frictions can amplify the responsiveness to productivity shocks. This difference in findings stems from differences in the signal structure. In the benchmark environments of Mackowiak and Wiederholt (2010) and Angeletos and La'o (2010a), informational frictions takes the form of pure observational noise[6]. Therefore, an aggregate shock enters the signals of all agents but is partly attributed to noise and to that extent, does not affect agents' responses at all. In this paper, on the other hand, signals are combinations of aggregate and idiosyncratic factors, causing the former to be mistaken for the latter and thereby inducing a stronger response[7].

The rest of this paper proceeds as follows. Section 2 presents a simple 2-period example which highlights the basic economic force at work and highlights the potential for amplification in this mechanism. Section 3 lays out the full model and defines the relevant equilibrium concepts. Sections 4 and 5 describe the solution algorithm and the calibration strategy respectively. Section 6 discusses the numerical results, along with some robustness exercises. Sections 7 and 8 present extensions with alternative wage determination rules and matching functions, and section 9 concludes.

2 A simple example

In this section, I lay out a simple example, which will highlight the economic forces at work in the more general model and illustrate the potential for amplification. A 2-period economy is populated by a unit measure of both firms and workers. Let $N_{it},\ \ t = 1, 2$ denote the mass of employed workers at firm i in period t. At the beginning of period 1, workers and firms are uniformly matched i.e. $N_{i1} = 1\ \forall i$. Period 1 output is produced according to:

[6]Lorenzoni (2009) has agent-specific productivity shocks as in this paper, but assumes that these shocks are iid. As a result, they have no effect on forward looking pricing decisions and therefore, act very much like observational noise.

[7]Li and Weinberg (2003) employ a similar confusion to generate differences in the cyclical response of investments at small and large firms.

$$Y_{i1} = e^{a_1 + a_{i1}} (N_{i1})^{\alpha} = e^{a_1 + a_{i1}} \ ,$$

where $e^{a_1+a_{i1}}$ represents the firm's productivity in period 1. Productivity has an aggregate and an idiosyncratic component, denoted a_1 and a_{i1} respectively. Both are mean-zero, normally distributed random variables

$$a_1 \sim N(0, \sigma^2)$$
$$a_{i1} \sim N(0, \hat{\sigma}^2).$$

I assume that a law of large numbers applies[8] to the cross-sectional distribution of a_{i1} i.e.

$$\int a_{i1} \ di = 0.$$

After period 1 production, the firm's matches with its current labor force are exogenously destroyed. In other words, to be able to produce in period 2, the firm is required to go through a hiring process. This involves posting vacancies in period 1 at a constant marginal cost of 1 unit of output. Employment in period 2 is related to vacancies through an aggregate matching function[9], which depends on the mass of unemployed in the economy (denoted U) and the the total number of vacancies posted V:

$$M = \zeta f(U, V) = \zeta U^{\eta} \ V^{1-\eta} = \zeta V^{1-\eta} \ , \qquad \eta \in (0, 1) \ ,$$

where η is the elasticity of the matching function with respect to the measure of unemployed. The last equality follows from the assumption that all matches are destroyed in first period, leading to $U = 1$.

The parameter η is also the elasticity of the job-filling rate, $\frac{M}{V} = \zeta V^{-\eta}$, with respect to the total mass of vacancies. Firm i's employment in period 2 is given by

$$N_{i2} = V_i \ \ \zeta V^{-\eta}.$$

Output in period 2 is produced according to the same decreasing returns-to-scale technology

[8] I maintain this assumption about idiosyncratic shocks throughout the paper. See Sun (2006) for the precise construction of a probability space where the exact law of large numbers holds for a continuum of pairwise independent random variables.

[9] I assume that ζ is sufficiently small so that the condition $M \leq \min(U, V)$ holds with a very high probability.

$$Y_{i2} = e^{a_2 + a_{i2}} (N_{i2})^{\alpha} \ ,$$

where a_2 and a_{i2} are the aggregate and idiosyncratic components of period 2 productivity. They are linked to their period 1 counterparts as follows:

$$\begin{aligned} a_2 &= \rho\ a_1 & \rho \in [0,1] \\ a_{i2} &= \hat{\rho}\ a_{i1}\ , & \hat{\rho} \in [0,1]. \end{aligned}$$

The parameters ρ and $\hat{\rho}$ index the persistence of the aggregate and idiosyncratic components respectively[10].

In both periods, wages are assumed to be proportional to output - in particular, firms pay one-half of their output to workers as wages. Dividend in the 2 periods are therefore,

$$\begin{aligned} D_{i1} &= Y_{i1} - W_{i1} - V_i = \frac{e^{a_1 + a_{i1}}}{2} - V_i \\ D_{i2} &= Y_{i2} - W_{i2} - V_i = \frac{e^{a_2 + a_{i2}} (N_{i2})^{\alpha}}{2} \end{aligned}$$

Firms maximize the expected value of dividends. Formally, firm i solves

$$\max_{V_i} \ \ D_{i1} \ + \ \mathbb{E}_i \ D_{i2}$$

where

$$\begin{aligned} D_{i1} &= \frac{e^{a_1 + a_{i1}}}{2} - V_i \\ N_{i2} &= \zeta V^{-\eta} \ \ V_i \\ D_{i1} &= \frac{e^{a_2 + a_{i2}} \ \ (N_{i2})^{\alpha}}{2} \ , \end{aligned}$$

and the expectation operator, $\mathbb{E}_i$, is with respect to firm i's information.

2.1 Optimality

The first-order condition of the above problem is

$$1 = \underbrace{\zeta V^{-\eta}}_{\text{Probability}} \quad \underbrace{\frac{1}{2}}_{\text{Firm's share}} \quad \underbrace{\alpha\ \mathbb{E}\left(e^{a_2 + a_2^i}\right) \ \ (N_{i2})^{\alpha - 1}}_{\text{Match surplus}}. \tag{1}$$

[10]Introducing additional shocks to period 2 productivities - whether aggregate or idiosyncratic - will not change any of the results in this section. Therefore, I abstract from any additional uncertainty.

The left-hand side of (1) is the cost of a vacancy and the right side is the expected value from posting a vacancy. Assuming conditional log-normality (which will be shown to hold), substituting for second period employment and taking logs on both sides, this can be written as

$$(1-\alpha)v_i = \mathbb{E}_i(a_2 + a_{i2}) - \alpha\eta\mathbb{E}_i(v) + \text{Const.} \tag{2}$$

where $v \equiv \int v_i di$ and the constant term is a function of parameters and the (commonly known) second moments of random variables on the right hand side. Equation (2) is intuitive - the firm's hiring decision depends positively on its expected productivity and negatively on the aggregate number of vacancies posted in the economy. The latter effect arises because the vacancy yield of the firm (i.e. the number of new hires each vacancy generates) decreases with economy-wide hiring activity. As a result, for a given level of its expectation of fundamentals (i.e. future productivity), the number of vacancies posted by a firm is decreasing in its beliefs about aggregate hiring activity. The greater the aggregate number of vacancies posted in the economy, the lower is the incentive of an individual firm to post vacancies. In the language of the heterogeneous information literature, this feature makes hiring decisions strategic substitutes. The effect of this interaction is greater (i.e. more negative), the larger the elasticity of the vacancy yield with respect to aggregate vacancies (i.e. η).

The firm's belief about v takes the form of a conjecture about its relationship with a_1, the only source of aggregate uncertainty in the economy. In a rational expectations equilibrium, this conjecture must be the same as the true relationship. I start with a guess that this relationship is linear (upto a constant):

$$v = \phi \;\; a_1 \; , \tag{3}$$

where ϕ is an endogenous coefficient to be determined. Substituting this conjecture and the laws of motion for productivity in (2),

$$(1-\alpha)v_i = \rho\,\mathbb{E}_i(a_1) + \hat{\rho}\,\mathbb{E}_i(a_{i1}) - \alpha\eta\phi\,\mathbb{E}_i(a_1) + \text{Const.} \tag{4}$$

$$= (\rho - \alpha\eta\phi)\,\mathbb{E}_i(a_1) + \hat{\rho}\,\mathbb{E}_i(a_{i1})\; + \text{Const..} \tag{5}$$

As (5) shows, the firm's optimal response to a perceived aggregate shock, a_1, is dampened by the effects of labor market congestion (captured by the $-\alpha\eta\phi$ in the first term). Obviously, the firm-specific component of productivity does not affect this congestion and therefore, the response to perceived idiosyncratic shocks, a_{i1} depends only on its persistence.

2.2 Full Information

First, suppose that firms have full information about the nature of shocks i.e. they observe both components of their productivity shock separately. Therefore,

$$\mathbb{E}_i(a_1) = a_1 \qquad \mathbb{E}_i(a_{i1}) = a_{i1}.$$

Substituting in (5) and integrating over all i, ignoring the constant terms

$$\begin{aligned}(1-\alpha)\int v_i\, di &= (\rho - \alpha\eta\phi)\int \mathbb{E}_i(a_1)\, di \;+\; \hat{\rho}\int \mathbb{E}_i(a_{i1})\, di \\ \Rightarrow (1-\alpha)\, v \; &= (\rho-\alpha\eta\phi)a_1 \;+\; \hat{\rho}\int a_{i1}\, di = (\rho-\alpha\eta\phi)\, a_1.\end{aligned}$$

Thus, under full information, the firm's conjecture (3) is verified when

$$(1-\alpha)\,\phi^{\text{Full}} = (\rho - \alpha\eta\,\phi^{\text{Full}}) \quad \Rightarrow \quad \phi^{\text{Full}} = \frac{\rho}{1-\alpha+\alpha\eta}\;. \tag{6}$$

2.3 Dispersed Information

Now, assume that, at the time of making its hiring decision, each firm only observes its composite productivity $a_1 + a_{i1}$ and nothing else. The firm uses Bayes rule to form expectations of the aggregate and idiosyncratic components. This leads to

$$\mathbb{E}_i(a_1) = \pi(a_1 + a_{i1}) \qquad \mathbb{E}_i(a_{i1}) = (1-\pi)(a_1 + a_{i1}), \text{ where } \pi = \frac{\sigma^2}{\sigma^2 + \hat{\sigma}^2}.$$

In other words, Bayes rule implies that the signal is allocated to the 2 components according to the ratio of their variances. Again, substituting in (5) and integrating over all i,

$$\begin{aligned}(1-\alpha)\int v_i\, di &= (\rho - \alpha\eta\phi)\int \mathbb{E}_i(a_1)\, di \;+\; \hat{\rho}\int \mathbb{E}_i(a_{i1})\, di \\ (1-\alpha)\, v \; &= (\rho-\alpha\eta\phi)\pi\int (a_1 + a_{i1})\, di \;+\; \hat{\rho}(1-\pi)\int (a_1 + a_{i1})\, di \\ \Rightarrow (1-\alpha)\, v \; &= (\rho-\alpha\eta\phi)\pi\; a_1 + \hat{\rho}(1-\pi)\; a_1\;,\end{aligned}$$

where the last equality makes uses of the fact that $\int a_{i1}\, di = 0$. Under heterogeneous information, the firms' conjecture (3) is verified when

$$(1-\alpha)\,\phi^{\text{Het}} = (\rho - \alpha\eta\,\phi^{\text{Het}})\pi + \hat{\rho}(1-\pi) \tag{7}$$

$$\Rightarrow \quad \phi^{\text{Het}} = \frac{\pi\,\rho + (1-\pi)\,\hat{\rho}}{1-\alpha+\alpha\eta\pi}\,. \tag{8}$$

A comparison of (6) and (8) reveals that heterogeneous information changes the equilibrium response of hiring activity to aggregate shocks in 2 ways. The first comes from changes in the forecast of future productivity in response to an aggregate shock. Under full information, firms observe a_1 perfectly and correctly forecast its implications for their productivity next period. Under heterogeneous information, firms attribute a fraction $1-\pi$ of all changes in their signals (including those coming from aggregate shocks) to an idiosyncratic shock. Therefore, the firm's forecast for period 2 productivity is computed using a weighted average of the persistence of the 2 components, with weights determined by π. This is reflected in the numerator in (8).

The second change comes from the fact that informational frictions also affect the firm's expectation of aggregate labor market conditions (through the $\mathbb{E}_i(v)$ term). For example, a positive aggregate shock increases overall hiring activity and therefore, leads to lower vacancy yields. However, to the extent aggregate shocks are mistakenly attributed to idiosyncratic factors, firms underestimate this effect. Since their hiring decisions depend negatively on expectations of overall market tightness, this underestimation works to increase the number of vacancies posted. This effect shows up through the π multiplying the $\alpha\eta$ term in the denominator of (8).

The following proposition shows that if the idiosyncratic shock is sufficiently persistent, the confusion caused by informational frictions leads to an amplified response in hiring activity.

Proposition 1 *Suppose $\hat{\rho} \geq \rho\left(\frac{1-\alpha}{1-\alpha+\alpha\eta}\right)$. Then, the economy under heterogeneous information is more responsive to aggregate shocks i.e. $\phi^{Het} \geq \phi^{Full}$.*

Note that $\left(\frac{1-\alpha}{1-\alpha+\alpha\eta}\right) < 1$, so informational frictions can lead to amplification even when idiosyncratic shocks are less persistent than aggregate shocks. This occurs due to the second effect discussed above (i.e. through firms' expectations about aggregate labor market conditions). To see this, consider the case as the strategic interactions disappear i.e. $\eta \to 0$. Then,

$$\phi^{\text{Full}} \;\to\; \frac{\rho}{1-\alpha} \qquad\qquad \phi^{\text{Het}} \;\to\; \frac{\pi\,\rho + (1-\pi)\,\hat{\rho}}{1-\alpha}.$$

In other words, without the strategic links introduced by the aggregate matching function, the differences in responses under full and heterogeneous information stem purely from the relative persistence of the two shocks. In this limiting case, heterogeneous information leads to amplification in aggregate responsiveness if and only if idiosyncratic shocks are more persistent than aggregate ones. More generally, the greater the link between vacancy yield and aggregate hiring, i.e. the higher the value of η, the lower is the threshold level of idiosyncratic persistence necessary to generate amplification.

While this example is too stylized for a full-fledged calibration and quantitative evaluation, I use a simple numerical calculation to illustrate that this mechanism can generate economically significant amplification. I set[11] the returns to scale parameter $\alpha = 0.9$, the persistence of the aggregate shock, $\rho = 0.98$ and make the idiosyncratic shocks perfectly persistent i.e. $\hat{\rho} = 1$. Figure 2 plots the equilibrium response of vacancies in the economy under heterogeneous information relative to the full information case i.e. $\phi^{\text{Het}}/\phi^{\text{Full}}$, for various values of η, the elasticity parameters as well as for the relative variance of the idiosyncratic shocks $\hat{\sigma}/\sigma$. Recall that higher the relative volatility of firm-specific shocks, the greater the fraction of the signal attributed to them.

As the graph shows, the degree of amplification generated by informational frictions can be quite large. For the empirically relevant range of the relative variance,(i.e. towards the high end of the X-axis in Figure 2), vacancies are anywhere from 3 to 9 times as volatile under heterogeneous information compared to the full information economy. This is approximately the factor by which standard search and models underpredict labor market volatility, indicating that informational frictions can go a long way in closing the gap between these models and the data. The primary purpose of the general model in the sections that follow is to verify the robustness of this finding.

2.4 Relation to the Literature

This simple example also offers a convenient way to relate the mechanism to others used by the literature to amplify cyclical fluctuations in the labor market. Recall, from the FOC (1), that firms equate costs of a vacancy to its expected benefits.

$$\text{Cost of a vacancy} = \mathbb{E}_i \left[\text{Probability of a match} \times (\text{Firm's share} \times \text{Match surplus})\right] \qquad (9)$$

[11] These parameter values are similar to the ones used in the calibration of the general model in Section 5.

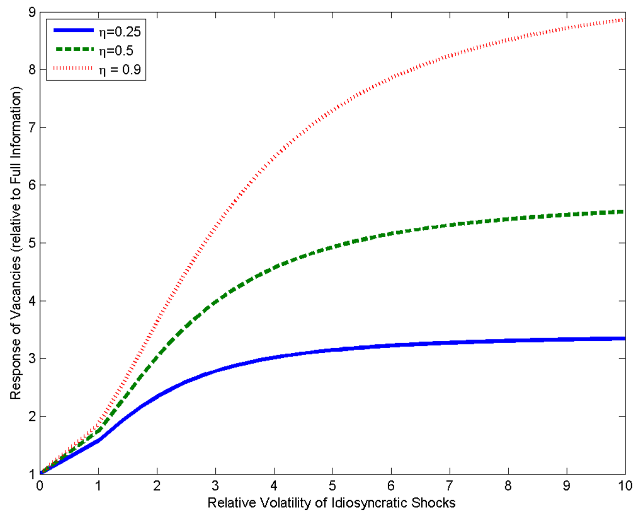

Figure 2: Effect of Confusion and Congestion

The volatility puzzle in Shimer (2005) can be represented in terms of this expression. Assume that the left hand side does not vary with the cycle. Given a matching function and data on employment/vacancies, cyclical movements in the probability of a match can be directly observed in the data. Under standard functional form assumptions for a matching function, that probability turns out to be quite volatile i.e. declines (rises) sharply during booms(recessions). Therefore, if the model is to match the data, it must generate significant volatility in the term within parentheses i.e. the benefit to the firm from a successful match must rise (decline) sharply during booms (recessions). This is the source of the puzzle - the standard search model does not generate sufficient movement in this object in response to productivity shocks. Various modifications to the standard model have been proposed to make the firm's value from a match more responsive to shocks. For example, when wages are sticky, as in Hall (2005), the firm's share becomes more cyclical. Training costs, as in Pissarides (2009), or higher outside options of workers, as in Hagedorn and Mannovskii (2008), serve to generate bigger fluctuations in the net surplus from a match. In contrast, I argue that, in the presence of information frictions, firms equate the cost of a vacancy to the *perceived* benefit from posting one. In such a scenario, when firms mistake an aggregate shock for an idiosyncratic one, they underestimate the movements in the probability of a match and overestimate the change in the other terms. This mechanism acts through the $\mathbb{E}_i$ operator and is present even when the *actual* benefit from a vacancy behaves as in the standard model. In other words, informational frictions can generate amplification, without the need to rely on the other modifications used by the literature.

3 The Full Model

In this section, I lay out a micro-founded dynamic stochastic general equilibrium model with search frictions. The model setup is very similar to those used in the search and matching literature and closely follows the approach in Shimer (2010), augmented to allow for heterogeneity and dispersed information. The full model will have a number of features - infinite horizon, a more standard specification of preferences and technology, a richer set of signals - that were not present in the simple example discussed above. These features will preclude an analytical characterization of the solution but will allow a more robust quantitative evaluation of the role of informational frictions. They will also present a few challenges in solving the model - a dynamic learning problem, both physical and informational state variables, to cite a couple. Addressing these challenges is part of the methodological contribution of this paper and I discuss them in greater detail in Section 4.

3.1 Model description

I denote the history of the economy upto time t by s^t, with an associated probability $\Pi(s^t)$. The state space for s^t includes all aggregate and idiosyncratic shocks (to be specified later).

Households: There is a representative household with a measure 1 of members, who work in one of a continuum of markets i. The household maximizes the expected discounted sum of utilities of all its members

$$\sum_t \int_{s^t} \beta^t \, \Pi(s^t) \left(\ln \, C_t(s^t) - \gamma \int N_{it}(s^t) Z_{it}(s^t) di \right) ,$$

where $C_t(s^t)$ is aggregate household consumption, $N_{it}(s^t)$ is employment in i and Z_{it} is an idiosyncratic preference shock which affects the disutility of working in i. The measure of employed members of the household, $N_t(s^t)$, is :

$$N_t(s^t) \equiv \int N_{it}(s^t) \, di \, .$$

The household is assumed to have access to complete contingent claims markets and faces a lifetime budget constraint:

$$\sum_t \int_{s^t} Q_t(s^t) C_t(s^t) = \sum_t \int_{s^t} Q_t(s^t) \int (W_{it}(s^t) N_{it}(s^t) + D_{it}(s^t)) di \ .$$

where $Q(s^t)$ is price of an Arrow-Debreu security which pays off 1 unit in state s^t, W_{it} and D_{it} denote the wage rate and dividends paid out by the representative firm in i. Standard optimization arguments imply

$$\lambda Q_t(s^t) = \frac{1}{C_t(s^t)} \ ,$$

where λ is the multiplier on the lifetime budget constraint.

Firms: The representative firm in market i maximizes the expected discounted sum of dividends:

$$\mathbb{E}_i \qquad \sum_{t=0}^{\infty} \beta^t Q_t D_{it} \ ,$$

subject to

$$D_{it} + K_{i,t+1} = Y_{it} - W_{it} N_{it} + K_{i,t}(1-\delta) + \frac{\psi}{2}\left(\frac{I_{it}}{K_{it}} - \delta\right)^2 \ ,$$

where I have suppressed the explicit dependence on s^t for brevity. The expectation $\mathbb{E}_i$ is taken with respect to firm i's information set (to be defined later). As in the simple example, informational frictions will act through this operator.

The firm operates a Cobb-Douglas technology:

$$Y_{it} = A_t A_{it} K_{it}^{\alpha_1} (N_{it}(1 - V_{it}))^{\alpha_2}$$

where V_{it} is the fraction of the firm's labor force allocated to recruiting efforts. Note that this is slightly different from the recruiting technology used in the simple example. Here, I assume that recruiting is a labor-intensive activity and so, instead of posting vacancies, the firm devotes a fraction of its labor force to recruiting activity. Aggregate and idiosyncratic productivity shock processes are denoted by A_t and A_{it} respectively. I assume decreasing returns at the firm level i.e. $\alpha_1 + \alpha_2 < 1$.

Labor Markets: The law of motion for employment in i is given by:

$$N_{i,t} = (1 - \delta_n) N_{it-1} + V_{i,t-1} N_{it-1} F_{i,t-1} \ . \tag{10}$$

where δ_n is the (exogenous) separation rate and $F_{i,t-1}$ is the (endogenous) number of hires each recruiter deployed in i in period $t-1$ is able to attract. $L_{i,t}$ is an (exogenous) idiosyncratic shock to recruiter efficiency[12]. The firm takes as given $F_{i,t-1}$ while choosing $V_{i,t-1}$. The household, on the other hand, takes this law of motion as completely exogenous.

Finally, I specify the relationship between matches created in sector i and labor market conditions. I adopt a flexible parameterization, which allows the effectiveness of each recruiter in a sector to depend on both i-specific variables as well as overall conditions in the labor market.

$$F_{it} = \bar{\mu}\left(\Theta_t\right)^{-\eta} \; L_{it}, \qquad \eta \in (0,1) \tag{11}$$

$$\text{where} \qquad \Theta_t \equiv \frac{V_t N_t}{1-N_t}. \tag{12}$$

Thus, the expected efficiency of a firm's recruiting efforts are declining in Θ_t, which captures the degree of tightness in the overall labor market.

Wages: For now, I assume a general wage determination rule:

$$W_{it} = G(\{Y_{it-s}, N_{it-s}, V_{it-s}, C_{t-s}, N_{t-s}, V_{t-s}, Z_{it-s}\}_{s=0}^{\infty}) \,. \tag{13}$$

This specification is flexible enough to nest a large class of wage determination processes. In Section 3.6, I discuss my choices of functional forms for $G(\cdot)$ and relate them to some of the standard wage determination protocols used in the literature.

3.2 Optimality

The first-order condition for capital takes the standard form:

$$\left[1+\frac{\psi}{K_{it}}\left(\frac{K_{it+1}}{K_{it}}-1\right)\right]\mathbb{E}_{it}Q_t = \beta\mathbb{E}_{it}Q_{t+1}\left[\alpha_1\frac{Y_{it+1}}{K_{it+1}}+1-\delta+(1-\delta)\frac{\psi}{K_{it+1}}\left(\frac{K_{it+2}}{K_{it+1}}-1\right)\frac{K_{it+2}}{K_{it+1}}\right] \,.$$

Under the assumption that the firm chooses an interior solution for the fraction of recruiters[13], the optimality condition is,

[12] Cheremukhin and Restrepo-Echavarria (2010) find a significant role for *aggregate* shocks to matching efficiency in explaining the US post-war data. Here, I employ only idiosyncratic ones.

[13] With unbounded shocks, this will in general not be true for all i. Unfortunately, explicitly modeling this situation makes the problem almost intractable numerically. However, if shocks are sufficiently small, the probability that a firm will want to choose a corner value will be small as well.

$$\alpha_2 \frac{Y_{it}}{N_{it}(1-V_{it})}\mathbb{E}_{it}Q_t = \beta \mathbb{E}_{it}\ \xi_{it+1}F_{it}\ .$$

where ξ_{it} is the shadow price of employment (i.e. the multiplier on (10), the law of motion for employment) and follows:

$$\xi_{it} = \left(\frac{\alpha_2 Y_{it}}{N_{it}(1-V_{it})} - W_{it}\right)\mathbb{E}_{it}Q_t + \beta(1-\delta_n)\mathbb{E}_{it}\ \xi_{it+1}\ .$$

3.3 Aggregate and Idiosyncratic Shock Processes

In this subsection, I specify the nature of the stochastic process followed by the aggregate and idiosyncratic shocks. All the shocks are assumed to follow AR(1) processes in logs

$$\ln A_t = \rho \ln A_{t-1} + u_t \tag{14}$$

$$\ln A_{it} = \rho_a \ln A_{it-1} + u^a_{it} \tag{15}$$

$$\ln Z_{it} = \rho_z \ln Z_{it-1} + u^z_{it} \tag{16}$$

$$\ln L_{it} = \rho_m \ln L_{it-1} + u^l_{it}. \tag{17}$$

where u_t, u^a_{it}, u^z_{it} and u^l_{it} are normally distributed with mean zero and variances $\sigma^2, \sigma^2_a, \sigma^2_z$ and σ^2_m respectively.

I also make the standard assumption that a law of large numbers applies to the cross-sectional distribution of idiosyncratic shocks i.e. $\forall\ t$,

$$\int u^j_{it} di = 0\ , \qquad j = a, z, l.$$

3.4 Information Structure

All shocks become common knowledge after a lag of T^* periods. This is a natural, though somewhat stark, way of modeling the delay in the collection of direct information about aggregates from asset prices, published aggregate data. This will also allow me to examine the robustness of my results to the arrival of additional information in a direct way.

Apart from these, firms do not observe any aggregates directly. They only have access to variables which arise in the natural course of their business - wages, productivities, outcomes of their labor market activities. Formally, I assume that firm i's information set at time t, denoted s^t_i

- Productivities: $\{a_{t-\tau} + a_{i,t-\tau}\}^{\infty}_{\tau=0}$

- Wages: $\{W_{i,t-\tau}\}_{\tau=0}^{\infty}$
- All firm-specific variables: $\{V_{i,t-\tau-1}, K_{i,t-\tau}, N_{i,t-\tau}\}_{\tau=0}^{\infty}$
- $\{U_{t-T^*-\tau},\ \ U_{it-T^*-\tau},\ \ Z_{it-T^*-\tau},\ \ L_{it-T^*-\tau}\}_{\tau=1}^{\infty}$

3.5 An Approximate Equilibrium

A rational expectations equilibrium for this economy is defined in the usual manner (see, for example, Townsend, 1983). Appendix A presents the formal equilibrium definition. Solving for the exact equilibrium allocations in this economy, however, is quite challenging - with both fundamental and informational heterogeneity, the distribution of types is a high dimensional state variable. Therefore, I construct and solve for an equilibrium in the neighborhood of a deterministic steady state(i.e. one without aggregate or idiosyncratic shocks). I assume that the equations characterizing individual decisions are well-approximated by a first order log-approximation. Further, log-deviations of aggregate variables are assumed to be well approximated by the cross-sectional averages of the log-deviations of individual state variables. Formally, an approximate equilibrium is a set of log-deviations of

i. Aggregate variables q_t, c_t, n_t, v_t as linear functions of $(u_t, u_{t-1}, ..)$

ii. Wages w_{it} as a linear function of $\left(u_t, u_{it}^a, u_{it}^z, u_{it}^l, u_{t-1}, u_{it-1}^a ...\right)$

iii. Firm-level employment n_{it}

iv. Firm decisions k_{it+1}, v_{it}, d_{it} as linear functions of the variables that make up the individual histories

such that, to a log-linear approximation,

- (i)-(iv) are consistent with the wage determination equation (13) and the law of motion for employment (10)
- The choices in (iv) solve the firm's problem, taking as given (ii), the law of motion (10) and forecasts of the aggregate variables in (i) made using the firm's information set
- The aggregate allocations are consistent with choices of individual firms and market clearing for state-contingent claims i.e.

$$n_t = \int n_{it} \, di$$
$$\bar{C} c_t = \bar{W} \bar{N} \int (w_{it} + n_{it}) \, di + \bar{D} \int d_{it} \, di \quad ,$$

where $\bar{X}$ denotes the steady state value of X.

3.6 Wages

Next, I specify the functional form of $G(\cdot)$, the wage determination equation. The standard assumption for wages in the search and matching literature is through a Nash bargaining rule. Applying that protocol to the environment of this paper raises a few issues. The first is the need to specify - and impose discipline on - what firms and workers know about fundamentals as well as each others' information. Unlike the natural assumption that firms use the information generated by their market activities to form forecasts, there is no obvious way to model the interconnections between information sets of firms and the workers they hire. The second issue is one of tractability. Existing work on bargaining under informational frictions (Brugemann and Moscarini 2010, Kennan 2010) work in environments that are much simpler and more stylized than the one studied in this paper[14]. The final issue is a pedagogical one. Menzio (2005) and Kennan (2010) show informational asymmetry can lead to a form of wage stickiness. This is good news from the perspective of the volatility puzzle - we know, from Hall (2005), that dampening the response of wages to productivity shocks increases the response of employment. However, the main contribution of this paper is to highlight the implications of informational frictions acting through the confusion about the true nature of shocks. Using a wage determination process that induces wage stickiness confounds this mechanism with the well-known effects of wage rigidity.

For these reasons, I take a different approach. I present results under two alternative assumptions for wage determination. In the first, wages are determined through period-by-period Nash bargaining and the tractability problems caused by informational asymmetries are avoided by assuming that workers and firms are symmetrically informed[15]. Note that under this assumption,

[14] Menzio (2005) and Guerrieri (2007, 2008) also study search models with informational asymmetries but under the assumption that firms post wages.

[15] One way to interpret this is that the firm bargains with an in-house agent of the representative household. The agent does not communicate with the household and has access to only the information that the firm has. In this interpretation, workers who move across firms (or between the the unemployment pool and firms) from period to period are assumed to carry no information with them.

wages bring no additional information to the firm's information set. In the second specification, I directly make assumptions about the relationship of wages to aggregate and idiosyncratic variables. This will allow for additional learning about aggregate conditions through wages. Both these specifications will have identical implications for aggregate behavior under full information but will differ when information is heterogeneous.

I start by presenting results under the first approach[16]. In Appendix D, I show that the Nash bargaining leads to the following expression for the after-tax wage:

$$(1-\tau)W_{it} = (1-\phi)\,\frac{\mathbb{E}_{it}(\gamma Z_{it} + \Upsilon_t)}{\mathbb{E}_{it}Q_t} + \phi\,\frac{\alpha_2 Y_{it}(1-\tau)}{N_{it}(1-V_{it})}\ , \tag{18}$$

where Υ_t is the future value to the household of an unemployed worker and ϕ indexes the bargaining power of the worker. Thus, the wage is a weighted average of the expected value of an additional worker to the firm and the household, with the weights determined by the bargaining power parameter, ϕ.

3.7 Solution under Full Information

Now, consider the approximate equilibrium under full information i.e. assuming all shocks are common knowledge. The log-linear approximations of the equations characterizing the solution to firm's problem depend only on parameters and the relationship between aggregate variables and aggregate shocks. Importantly, they do not directly depend on variances of the idiosyncratic (or the aggregate) shock processes. Therefore, given a conjecture about aggregates, the response of agents to either aggregate or idiosyncratic shocks are unaffected by variances. Next, note that due to certainty equivalence, only the expected values of future variables are relevant to the firm's problem. Under full information, the actual realizations of shocks are assumed to be commonly known and so, expectations are common knowledge and unaffected by the variances as well. Finally, by construction, variances play no role in a linear aggregation. These features point to an equilibrium in which the relative magnitude of idiosyncratic shocks plays no role in aggregate dynamics. The next proposition formalizes this idea and delivers a key insight: fundamental heterogeneity by itself does have any implications for aggregate behavior in this economy, at least to a first-order approximation.

Proposition 2 *Under full information, the laws of motion for aggregate variables are independent of σ_a^2. In particular, they are identical to the case with $\sigma_a^2 = 0$ i.e. a representative agent economy*

[16]In Section 7, I discuss the results under the alternative approach.

with the same preferences, technology and Nash bargaining over wages.

This benchmark result stems from the fact that a log-linear approximation splits the state and optimal policies of each firm into two separate components - one arising from the history of aggregate shocks and the other from the realizations of idiosyncratic shocks. These coefficients are independent of second (or higher) moments. Upon aggregation, the component due to idiosyncratic conditions averages out to zero (because of the law of large numbers assumption). Therefore, aggregate laws of motion depend only on aggregate states and expectations of future aggregate conditions. Without informational frictions, these expectations depend only on (the commonly known) realizations of current and past aggregate productivity shocks. A representative agent version of the model using the same preferences and technology is subject to the same optimality and market clearing conditions and so generates the same relationship between aggregates and the history of aggregate shocks. With informational frictions, this logic no longer holds. Relative variances now matter for the firm's forecasting problem, which changes the response of aggregate variables to aggregate shocks, which in turn affects an individual firm's optimal response to aggregate conditions - in other words, a different equilibrium emerges. In the following section, I describe the solution algorithm to numerically solve for such an equilibrium.

4 Solution Strategy

This section describes the algorithm for computing the approximate equilibrium described above. For the full information case, Proposition 2 directly points to a solution strategy - simply solve the representative agent model i.e an economy without idiosyncratic shocks. Solving the heterogeneous information case presents two challenges. The first stems from the fact that firms care not only about the realizations of the shocks but also about the reactions of other firms. This strategic linkage arises because (a) the stochastic discount factor Q_t used by the firm depends on aggregate household consumption (and therefore, on the recruiting and investment decisions of all the firms in the economy) (b) the effectiveness of recruiters in sector i depends partly on overall labor market conditions as well, through (11) and (c) future wages depend on aggregate conditions in the economy. Therefore, firms have to form forecasts not just of fundamentals but also of actions of other firms. Since they in turn depend on the others actions, the entire structure of higher order beliefs (what firms believe about other firms' beliefs about others actions, what firms believe about others beliefs about others beliefs and so on) about becomes relevant for determining equilibrium actions. In a one-shot game (like the example in Section 2), all these higher-order beliefs are functions of a single random variable. This allowed the use of a simple method of undetermined coefficients to solve the problem. With more periods, higher-order beliefs depend in an arbitrary way on the history of signals. As a result, the set of relevant state variables can become quite large as the number of periods increases. Consider the case where past realizations are never revealed i.e. $T^* = \infty$. In this case, strategic linkages lead to the well-known "infinite regress" problem (Townsend, 1983). The evolution of the economy depends on the realizations of an infinite history of signals, making the problem generally intractable. The heterogeneous information literature has dealt with this problem either by restricting attention to special cases where the relevant history can be summarized in a finite dimensional state variable (e.g. Woodford, 2003) or by truncating the dependence of equilibrium actions on higher order beliefs (e.g. Graham and Wright, 2010 or Nimark, 2008) or by modeling the history dependence using finite-order ARMA processes(e.g. Sargent, 1991 or Mackowiak and Wiederholt, 2010).

In this paper, this issue is resolved by the assumption that information is fully revealed after a finite number of periods. Now, only the history over the last T^* periods is relevant for determining the structure of higher-order beliefs. This allows me to summarize the effects of informational frictions on current equilibrium objects in the form of a solution to a finite-dimensional filtering problem. This approach for dealing with the infinite-regress follows Hellwig (2002) and Hellwig (2008a).

The second challenge arises because of the presence of two 'physical' state variables, capital and employment. Now, the history of signals received by a firm affects current decisions in two ways. One, since the learning problem is a dynamic one, they directly affect current forecasts of fundamentals. Two, they also determine the level of its employment and capital at the beginning of the period [17]. Both these effects could be significant in determining the response of equilibrium objects and therefore, it becomes important to keep track of them separately. One of the contributions of this paper is to develop a method to do this in a tractable manner. The methodology combines standard techniques for solving dynamic optimization problems using linear approximations with the approach for dealing with heterogeneous information from Hellwig (2002). The result is a flexible (and tractable) methodology that can handle a rich set of individual state variables as well as complicated signal structures (with both endogenous and exogenous as well as public or private components).

Note that all the shocks - aggregate and idiosyncratic - are assumed to be ultimately transitory,

[17] This channel is missing, for example, in Hellwig and Venkateswaran (2009) and Mackowiak and Wiederholt (2010).

though they may be persistent to an arbitrarily high degree. Also, both capital and employment are subject to exogenous depreciation rates. Therefore, the effects of past shocks on current allocations will become arbitrarily small over time. I exploit this feature in my numerical analysis and assume that there exists some large lag T, such that shocks more than T periods old do not have any effect on current variables. In the discussion that follows, any reference to the 'entire' history of shocks in period t means the history of shocks upto $t - T$.

A summary of the iterative procedure that is used to solve for allocations in an approximate equilibrium is as follows:

- **Step 1:** Conjecture a (linear) relationship of aggregates to aggregate shocks
- **Step 2:** Derive full information equilibrium policy functions using a log-linear approximation
- **Step 3:** Invoke certainty equivalence to replace unknowns by their conditional expectations
- **Step 4:** Aggregate individual policy rules to express aggregate variables as (linear) functions of aggregate state variables and 'average' conditional expectations
- **Step 5:** Exploit normality to write 'average' conditional expectations as (linear) functions of aggregate shock realizations
- **Step 6:** Combine to express aggregates in terms of the aggregate shocks
- **Step 7:** Verify conjecture and iterate until convergence

Appendix C provides a more detailed exposition of the solution algorithm.

5 Calibration

Calibration of most of the aggregate parameters is fairly standard and closely follows the strategy in Shimer (2010). Table 1 presents the calibrated values. Appendix E discusses the moments used to pin down most of these parameters. The adjustment cost parameter, ψ, is set to match the volatility of investment in the data.

Next, I turn to my choice of parameters for the 3 idiosyncratic shock processes - productivity, disutility of working and recruiter efficiency. Given the AR(1) assumption, these processes can be summarized by two parameters - autocorrelation and variance of the innovation (expressed relative to the variance of the innovation to aggregate productivity). The general strategy is to use cross-sectional and time-series properties of firm-level employment and vacancies to pin down these

	Description	Value
	Preferences and Production	
	Time period	1 month
β	Discount factor	0.996
α_1	Share of capital	0.23
α_2	Share of labor	0.67
δ	Dep. of capital	0.0028
γ	Disutility of leisure	0.43
ρ	Persistence of agg. TFP	0.98
	Labor Markets	
δ_n	Rate of exogenous destruction	0.034
τ	Labor tax	0.4
ϕ	Workers' share of surplus	0.5
η	Elasticity of job filling rate	0.5
$\bar{\mu}$	Scale parameter of job filling rate	2.32

Table 1: Calibrated values of aggregate parameters

parameters[18]. In picking targets, to the extent possible, I will attempt to be as conservative - i.e. geared towards more learning and smaller effects. The shocks to disutility of working play no role in this version of the model (because wages are of no informational value to the firm). Therefore, for now, I set its relative variance and persistence to zero[19].

First, I make use of moments from the large body of work documenting the empirical properties of firm growth rates in the US. Two robust findings emerge from this literature. One, there is a large cross sectional dispersion in firm level growth rates. Davis et al. (2007) report that from 1970 to 1980, the cross-sectional variance in annual growth rates ranged from 20% to 25%. As a conservative starting point, I use an estimate of 10 % in my calibration. Second, firm growth rates are independent of firm size. This observation, commonly referred to as Gibrat's law, implies that firm-level shocks induce permanent changes in the level of the firm's employment. In the context of my model, this implies that innovations to idiosyncratic productivity have to be permanent[20].

[18]Interpreting i, the informational unit in my model, as a firm is a natural starting point for my analysis. It seems reasonable to assume that the sources of information in my model - productivity, labor market outcomes - are most likely to be aggregated at the firm level.

[19]The properties of disutility shock will play a key role in Section 7, where I present results under the alternative wage determination mechanism.

[20]This also finds support from the work of Franco and Philippon (2007), who document the presence of large and permanent firm-specific shocks along with significantly smaller, transitory aggregate shocks using data on large US

For my baseline calibration, I use a value of 0.995 for the persistence of the idiosyncratic process ρ_a. In Section 6.3, I discuss the sensitivity of the results with respect to this choice.

Next, I calibrate the idiosyncratic shocks to recruiter efficiency. The primary role played by these shocks is to slow down learning about aggregates from past labor market outcomes. Recall that firms' information set includes the history of its past recruiting efforts. Without additional sources of variation, the success of last period's recruiting efforts will perfectly reveal the market tightness (and therefore, the aggregate state) last period. In other words, without shocks to recruiter efficiency, the confusion can last at most one period. On the other hand, the presence of idiosyncratic shocks to recruiter efficiency slows down learning. In order to focus on the informational effects of these shocks, I abstract from any direct effects on intertemporal incentives and assume that these shocks are completely transitory i.e.

$$l_{it} = u^l_{it} \ .$$

To calibrate the variance of u^l_{it}, I use recent work on vacancies and hiring by Davis, Faberman and Haltiwanger (2010). They find evidence of a strong positive relationship between a firm's employment growth rate and its vacancy yield i.e. the fraction of vacancies that are converted into new hires. Their estimate for the elasticity of vacancy yield to hiring of 0.72. To see how this is related to the efficiency shock in my model, consider the case without these idiosyncratic shocks. In that case, the analogue of the vacancy yield in my model, the expected number of new hires brought in by each recruiter, depends only on overall market tightness, and is therefore the same for every firm in the economy. In other words, absent shocks to recruiter efficiency, firm growth and vacancy yields are uncorrelated. Efficiency shocks affect both growth and vacancy yield in the same direction (by construction) and therefore, are a source of positive comovement. The greater the variance of these efficiency shocks, the greater the degree of positive correlation. Now, there could be other mechanisms, potentially non-random, at work behind this positive comovement[21]. Therefore, I conservatively choose a lower elasticity target than than the observed value. In particular, I set the relative variance of shocks to recruiting efficiency to deliver an elasticity of 0.5.

The other crucial parameter is the full information revelation lag T^*. One option is to use actual delays in the release of data. But that would also require taking a stand on measurement errors or other sources of aggregate uncertainty. I pursue a different approach in this paper. In my baseline calibration, I set T^* to a very large number (in effect, shocks become common knowledge

firms.

[21] For example, Davis, Faberman and Haltiwanger (2010) conjecture that growing firms might be varying an unobserved recruiting intensity per vacancy or that there are increasing returns to vacancies at the micro-level.

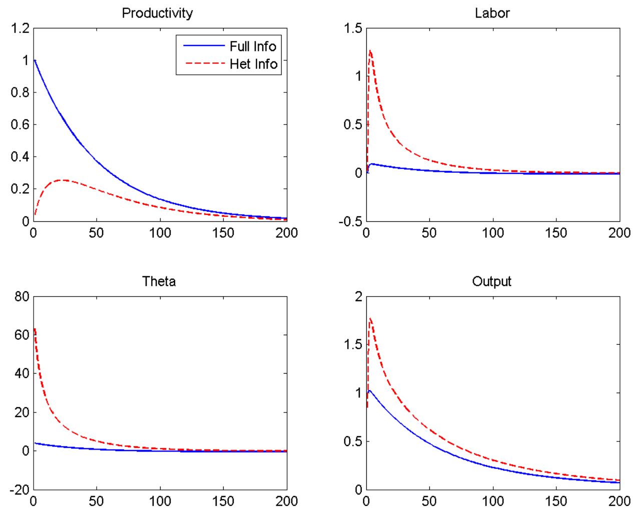

Figure 3: Response of aggregate variables to a positive productivity shock

only after they have ceased to have any meaningful effect on equilibrium actions). In Section 6.3, I present some robustness exercises which will show that the quantitative results are still significant even for reasonably low values of T^*.

6 Results

Figure 3 presents impulse responses of the aggregate variables in response to a positive aggregate productivity shock in the economy under full information as well as under heterogeneous information. The first panel plots the evolution of the aggregate productivity along with the average expectations about aggregate productivity. As the graph shows, under the baseline calibration, learning occurs relatively slowly. The remaining panels reveal that this slow learning amplifies the response of other variables to the aggregate shock. In particular, the response of employment and market tightness is an order of magnitude larger under heterogeneous information.

So why does confusion about the nature of the shock amplify the firms' responses ? Or to put it differently, why do firms respond more aggressively to perceived changes in idiosyncratic productivity shocks, relative to aggregate ones ? The basic intuition is the same as in the simple example, though there are more channels for the general equilibrium and learning effects in the full model. Amplification happens for four reasons, only two of which were present in the simple example in Section 2. First, in the baseline calibration, firm-specific shocks are slightly more persistent than

	y	n	θ	c	c-y
Data	1.00	0.64	14.70	0.61	0.66
Full Info	1.00	0.09	3.80	0.57	0.43
Het. Info	1.00	0.79	37.27	0.58	0.92
Sticky Wages	1.00	0.51	21.88	0.26	0.81

Table 2: Relative Standard Deviation (Annual growth rates)

aggregate shocks. Second, a firm-specific productivity shock, unlike an aggregate shock, has no effect on aggregate labor market conditions. Therefore, when hit by a positive idiosyncratic shock, the firm can expect to be more successful in its hiring efforts (as measured by the expected efficiency of its recruiters) and so, it responds more aggressively when it mistakes an an aggregate shock for an idiosyncratic one. Thirdly, wages respond differently to the two types of shocks[22]. Recall from (18) that wages are a weighted average of the value of a worker to the firm and the representative household. Both aggregate and idiosyncratic shocks affect the surplus (through their effect on the marginal product of labor) but the former has additional effects because of general equilibrium considerations. For one, an aggregate shock affects the household's marginal rate of substitution (because it affects aggregate consumption and therefore, Q_t). It also changes overall labor market conditions, affecting the household's outside option (as captured by Ω_t). Both these additional effects make wages more responsive to aggregate shocks than idiosyncratic ones. A firm which perceives a positive shock specific to itself expects a greater surplus from new hires and is therefore more inclined to expand its hiring activity. Lastly, to the extent that firms attribute aggregate shocks to idiosyncratic factors, they also do not make any adjustments to their expectations of the stochastic discount factor.

6.1 Moments

Table 2 presents the key second moments of aggregate variables in the model and compares them to their counterparts in the post-war US data. The first panel reports the standard deviation of the annual growth rates of aggregates. For the relevant moments in data, I use the numbers reported by Shimer (2010). Recall that, by Proposition 2, the response of aggregates under full information case is the same as in an economy with a representative agent. Not surprisingly, the row labeled 'Full Info' is almost identical to the results reported by Shimer (2010) using a very similar model as

[22]This channel was absent in the example because wages were assumed to be proportional to firm output.

the one in this paper, but without heterogeneity. It highlights the inability of the standard model to generate sufficient volatility in labor market activity - the standard deviations of employment and market tightness are off by an order of magnitude.

Adding informational frictions improves the picture significantly. The relative volatility of both employment and market tightness are increased significantly, with the latter now in excess of observed levels. For purposes of comparison, Table 2 also shows the results from a model with wage stickiness. Many authors (notably Hall, 2005) have argued that the standard model generates too little cyclical movement in the incentives to create new jobs because it assumes that wages respond 'too much' to productivity shocks. In other words, wage movements - both current and expected - largely offset the changes in the value of a vacancy from the firm's perspective. If, on the other hand, wages were rigid (or sufficiently sticky), then the response of vacancies is considerably magnified. The last row of the table shows results (as reported in Shimer (2010)) for a model similar to the one used here but with a representative firm and a sticky wage assumption of the form:

$$W_t = 0.95 \ \ W_{t-1} + 0.05 \ \ W_t^{\mathrm{Nash}} \ ,$$

where W_t^{Nash} is the wage under Nash bargaining. As the table shows, heterogeneous information leads to a greater degree of amplification in labor market volatility compared to the sticky wage model[23]. However, informational frictions in this paper operate through a very different channel and therefore lead to different implications for other moments in the data. Later in the paper, I elaborate on this point using two sets of moments. In the next subsection, I show that the model with heterogeneous information leads to a negative comovement between the aggregate labor wedge and employment. This is an important finding, because generating such countercyclical movements in the labor wedge has proved to be a big challenge to the business cycle literature. The second set of moments pertains to the volatility of wages - or more precisely, the elasticity of average wages to aggregate shocks. The assumption of Nash bargaining under symmetric information dampens the response of wages to aggregate shocks. This occurs due to the fact that in equilibrium, aggregate shocks are attributed partly (in fact, largely) to idiosyncratic factors. Since firm-specific shocks have a smaller effect on wages (because, by construction, they do not affect labor market conditions - in particular, the job-finding rate and through that, the outside option of the worker), this misattribution leads to a smaller adjustment of the wage. The wage stickiness model also implies

[23]To be fair, if the degree of rigidity is sufficiently high, then the sticky wage model also overshoots on the volatility front.

	y	n	θ	c	c-y
Data	1.00	0.65	15.30	0.59	0.64
Full Info	1.00	0.07	2.80	0.76	0.33
Het. Info	1.00	0.46	18.77	0.71	0.53
Sticky Wages	1.00	0.26	11.21	0.68	0.55

Table 3: Relative Standard Deviation (Logs)

	y	n	θ	$\hat{\tau}$	c-y
Data	0.14	-0.46	-0.11	1.00	-0.49
Full Info	0.98	0.85	0.99	1.00	-0.99
Het. Info	0.04	-0.34	0.65	1.00	-0.59
Wage Rig.	0.92	0.67	0.75	1.00	-0.88

Table 4: Correlation with Labor Wedge (Growth Rates)

a dampened response of wages. This might seem to suggest that the heterogeneous information mechanism works essentially through this channel i.e. by making wages sticky. However, this intuition is not quite correct. Firms in the heterogeneously informed economy learn relatively slowly about the aggregate state. As a result, the response of the economy to an aggregate shock comes largely from the responses to perceived movements in idiosyncratic factors. And since the responsiveness of wages to *aggregate* shocks does not affect firms' desired response to *idiosyncratic* shocks, the dampening of the wage elasticity contributes little by way of amplification. I return to this point in Section 7. There, I study an alternative assumption for wages, under which heterogeneous information will make wages more responsive to aggregate shocks, but will still generate amplified responses in hiring and employment, similar to Table 2.

Table 3 repeats the exercise in Table 2 with the logs of the variables (as opposed to growth rates) and tells a very similar story - informational frictions significantly close the gap between volatilities predicted by the model and those observed in the data. Given that the model abstracts from a number of other modifications to the standard model emphasized by other papers, this improvement in performance is quite remarkable.

6.2 The Labor Wedge

The *aggregate labor wedge*, denoted $\hat{\tau}_t$, is defined as the deviation between the marginal product of labor and marginal rate of substitution, both computed using aggregate data:

$$\alpha_2 \frac{Y_t}{N_t}(1-\hat{\tau}_t) = \frac{u'(N_t)}{u'(C_t)} \; .$$

The wedge is thus the implicit tax on labor income that emerges when an econometrician looks at the data through the eyes of a frictionless representative agent model[24] Many authors have documented two key properties of this object in the post-war US data - one, it shows significant amounts of volatility and two, it comoves negatively with the cycle, in particular with aggregate employment. In other words, relative to the frictionless model, the data behave as if the implied labor tax rises significantly during recessions, exacerbating the decline in employment. The opposite happens in a boom.

A good theory of the business cycle, therefore, must be able to match these cyclical patterns. The frictionless real business cycle framework with only productivity shocks implies a constant or *acyclical* wedge. Shimer (2009) surveys alternative explanations and argues that search and matching frictions are a natural framework for analyzing the wedge between marginal product of labor is not equated to the marginal rate of substitution. However, as he shows in Shimer (2010), models with search frictions tend to induce *procyclical* movements in the wedge. This occurs because search frictions, at some level, act like labor adjustment costs and therefore, serve to dampen the response of labor to shocks.

Table 4 shows the correlation of various aggregate variables with the labor wedge $\hat{\tau}_t$. As with relative volatilities, the full information case replicates the earlier findings of the literature i.e. search frictions lead to a strongly procyclical labor wedge. Heterogeneous information acts in the opposite direction. In a boom, for example, firms attribute a positive aggregate shock to a favorable idiosyncratic disturbance and therefore, increase hiring aggressively. From the perspective of the frictionless model, they behave as if they are being subsidized (relative to the full information case). In other words, the misattribution shows up as a reduction in the implicit tax on labor. The net effect of these two opposing forces is an aggregate labor wedge that comoves negatively with employment[25].

[24] Note that, with search frictions, the wedge $\hat{\tau}_t$ is not the same as the actual labor tax rate τ, though the two objects are closely related.

[25] Angeletos and La'o (2010a) also generate a source of countercyclical movement in the labor wedge in an economy with heterogeneous information. But, in their setup, this comes from the economy's response to 'noise' in the

			Full Revelation Lag, in months			
	Data	Full Info	T	12	6	3
Employment	0.65	0.07	0.46	0.47	0.42	0.33
Tightness	15.30	2.80	18.77	19.55	17.98	14.91

Table 5: Effect of the Full Revelation Lag, T^*

Table 4 also highlights an important dimension in which heterogeneous information outperforms the model with wage stickiness. The latter achieves only a modest reduction in the procyclicality of the labor wedge. For example, the correlation with employment drops to 0.67 under sticky wages[26], whereas in the data, it is -0.46. Informational frictions, on the other hand, generate the right sign for this comovement.

6.3 Robustness

In this section, I present some robustness exercises to examine the sensitivity of the results to key parameter assumptions. I start with T^*, the full revelation lag. As mentioned earlier, the baseline calibration sets this parameter to a very large number, implying that the only sources of learning are productivity and labor market outcomes. In Table 5, I report the change in results[27] when all shocks are revealed with much shorter lags. Obviously, the shorter this lag, the less the amplification generated by the confusion between aggregate and idiosyncratic shocks. But, as the table shows, even when the confusion lasts only for a quarter or two, there is a quantitatively significant amount of amplification.

Finally, in Table 6, I show the sensitivity of my results to the persistence of idiosyncratic productivity shocks. As in the simple example, strategic interactions lead to amplification even when the persistence of the idiosyncratic shocks is less than that of aggregate shocks (recall that

public signal about productivity. False good news about the economy causes an increase in output and employment, without a corresponding increase in technology. This lowers the MPL and raises the MRS, leading to a drop in the observed wedge. However, in their model, informational frictions lead to *procyclical* movements in the labor wedge in response to aggregate productivity shocks - because output and employment respond less than they would under full information. Here, on the other hand, informational frictions induce a negative comovement between the labor wedge and employment.

[26]Of course, increasing the degree of rigidity will help matters. If wages are completely rigid, as in Hall (2005), then the wedge is indeed countercyclical. See Table 4.3 in Shimer (2010).

[27]I report the volatilities of the log-deviations, as in Table 3, instead of growth rates. This is because the discrete nature of information causes additional variability in growth rates. Adding measurement errors would help smoothen the arrival of new information and therefore, the response of growth rates, but I do not pursue this approach here.

	Pers. of idio. prod., ρ_a					
	Data	Full Info	0.995	0.98	0.95	0.5
Employment	0.65	0.07	0.46	0.46	0.43	0.13
Tightness	15.30	2.80	18.77	18.77	18.05	5.32

Table 6: Sensitivity to Persistence of Idiosyncratic Shocks

aggregate shocks have a persistence of 0.98). Table 6 shows that these effects are quite strong - even for relatively transitory processes, heterogeneous information increases the volatility of employment and tightness by a factor of two.

7 Extension I: An Alternative Assumption for Wages

In this section, I present results under an alternative assumption for wage determination. Specifically, instead of specifying a bargaining protocol (Nash) and an information structure (symmetric), I directly make assumptions about the relationship between wages and the relevant state variables. Using this reduced form specification for wages will serve 2 purposes. First, it will allow me to examine the robustness of the results in the previous sections to the introduction of additional learning through wages. In order to facilitate this comparison, I adopt a form of the wage equation which, under full information, leads to the same aggregate behavior as the case analyzed in the previous sections. Under heterogeneous information, however, the two specifications will have very different implications, precisely because of the differences in the amount of learning. Second, it will also help distinguish the mechanism at work in this paper from the sticky wage hypothesis. Under the wage specification I adopt, the average wage rate in the economy with heterogeneous information will be *more* responsive to aggregate shocks than in the full information benchmark, revealing that the amplification generated by heterogeneous information is not coming from a dampened response of wages.

My starting point is a version of my model without heterogeneity. Appendix F shows that, for sufficiently small aggregate shocks, Nash bargaining leads to the following expression for wages:

$$(1-\tau)W_t = \phi(1-\tau)\frac{\alpha_2 Y_t}{N_t(1-V_t)}(1+\Theta_t) + (1-\phi)C_t\gamma \ , \tag{19}$$

where, as before, ϕ is a parameter indexing the bargaining power of the worker and τ is the constant labor tax rate. This expression has an intuitive interpretation - the wage is then a weighted average of the marginal product of labor (adjusted for the dynamic nature of the hiring decision) and the marginal rate of substitution between consumption and leisure, with the relative weights determined by ϕ.

Under heterogeneity, I assume that the wage determination equation takes the same form as (19), with some modifications:

$$(1-\tau)W_{it} = \phi(1-\tau)\frac{\alpha_2 Y_{it}}{N_{it}(1-V_{it})}(1+\Theta_{it}) + (1-\phi)Z_{it}C_t\gamma \; . \tag{20}$$

Note that the interpretation of this expression as the outcome of a bargaining protocol is no longer true when we have heterogeneity. Nevertheless, under full information, the log-linear approximation of the average wage rate will be the same as in the representative agent case. Along with Proposition 2, this assumption about wages implies that the approximate equilibrium under full information will be identical to that of an economy with a representative agent with wages determined through Nash bargaining.

Equation (20) has another striking implication. If there is no uncertainty about Z_{it} (as is the case under the calibration in Section 5), it can be shown that observing the (log-linear approximation of the wage) is informationally equivalent to observing a linear combination of C_t and Θ_t, aggregate consumption and market tightness respectively. Given that both these variables depend only on the realization of the aggregate shock, the history of wages would then perfectly reveal the aggregate state. However, when Z_{it} is random, it introduces idiosyncratic noise into the wage signal, slowing down learning.

To discipline the size of this idiosyncratic shock, I interpret it as a cause of person-specific effects in wage distribution[28]. This interpretation follows from my specification of wages, where z_i shows up an disturbance orthogonal to both the firm-specific and aggregate components of the wage. A number of papers have found a significant role for person-specific effects in the cross-sectional distribution of wages. Abowd, Kramarz and Margolis (1999) find that they account for 50-60% of wage dispersion in the French data. Postel-Vinay and Robin (2002) estimate a structural model using French labor market data and find that person-effects can account for as much as 40% of wage dispersion at high skill levels though this share declines quite sharply for low skilled jobs. Davis et al. (1991) report that within-plant heterogeneity explains as much as 35-40% of the variance in wages.

Obviously, assuming that this heterogeneity in its entirety serves to confound the signalling value of wages is too extreme an assumption. One could argue that firms might be able to observe

[28] Nimark (2008) also makes use of a similar interpretation to find empirical counterparts for idiosyncratic marginal cost shocks.

	y	n	θ	c	c-y
Data	1.00	0.64	14.70	0.61	0.66
Full Info	1.00	0.09	3.66	0.58	0.42
Sym. Bargaining	1.00	0.79	37.27	0.58	0.92
Wage Equation	1.00	0.36	18.24	0.46	0.84
Wage Equation *	1.00	0.76	35.52	0.54	0.95

Table 7: Relative Standard Deviation of Growth Rates

* : Without learning from wages

certain characteristics of the individuals they hire and so may have some information about the realization of z_{it}. In the limit, if they could observe the sources of heterogeneity perfectly, wages will fully reveal the aggregate state, undoing the informational frictions entirely. As with the other shocks, I take a conservative approach and calibrate the relative variance of z_{it} so that it accounts for only 25% of the cross-sectional variance in wages.

7.1 Results

Table 7 compares the relative standard deviations of growth rates under the alternative wage specification to the symmetric case studied earlier. The key feature that emerges from these results is the smaller amplification under the reduced form wage specification compared to the symmetric bargaining case studied earlier. The intuition behind this result is that when bargaining happens with symmetric information, wages contain no additional information about aggregates. Firms learn only from productivity and past labor market outcomes. With the wage equation, firms also learn from their wage bill every period. As a result, the misattribution of shocks is much less severe. The last row of the table provides support for this intuition - it shows results using the same wage equation (20), but under the assumption that firms do not learn from wages. Now, with the additional learning from wages turned off, the reduced form wage equation and symmetric bargaining assumption yield similar results.

Table 7 also illustrates an important distinction between heterogeneous information and wage stickiness as sources of amplification in labor market volatility. As mentioned earlier (in Section 6), the assumption of Nash bargaining under symmetric information for wage determination implies a dampened response of average wages to aggregate shocks (compared to the full information case). However, the reduced form specification considered here has the opposite implication, i.e. average wages in the heterogeneously informed economy display an increased sensitivity to aggregate shocks (again compared to the fully informed economy). This is true irrespective of whether firms learn from wages or not. In other words, the third and last rows in Table 7 show very different elasticities of wages to aggregate shocks, yet lead to very similar volatilities of labor market activity. This is an important finding, particularly in light of the critique of the sticky wage assumption by Pissarides (2009). Using micro data, he finds that wages of marginal workers are much more responsive to aggregate shocks than those of average workers. He shows that firms' incentives to create new jobs are affected only by wages of marginal workers and therefore, argues that any explanation that relies on unresponsive wages is unlikely to be a big driver of the cyclical behavior of hiring activity. The results presented in Table 7 essentially show that a dampened wage elasticity is not central to the amplification generated by informational frictions, making them immune to this critique.

8 Extension II: A More General Matching Function

The analysis in the previous sections assumes that all firms in the economy hire from a centralized market and face the same average job-filling rate. Since the general equilibrium effects arising from centralized labor markets is key to the amplification result, it becomes important to examine the robustness of the results to a less extreme specification of the labor market. This section achieves that objective by using a generalized relationship between a firm's recruiter efficiency and labor market conditions. Specifically, I assume

$$F_{it} = \bar{\mu} \left(\Theta_{it}\right)^{-\eta} \; L_{it}, \qquad \eta \in (0,1) \tag{21}$$

$$\text{where} \qquad \Theta_{it} = \frac{\left(V_{it} N_{it}\right)^{1-\kappa} \left(\overline{VN}_t\right)^{\kappa}}{1 - N_t} \qquad \kappa \in [0,1]. \tag{22}$$

The parameter κ controls how Θ_{it}, the relevant measure of tightness for i, depends on the overall level of recruiting in the economy. Higher the value of κ, the greater the degree to which recruiter effectiveness of a particular firm is affected by hiring efforts of other firms. This specification is a slightly generalized version of functional forms commonly used in the literature. In particular, the analysis in the previous sections sets $\kappa = 1$. At the other extreme, $\kappa = 0$ implies segmented labor markets i.e. where each i has its own labor market. However, as the next result shows, aggregate matching elasticities are invariant to κ, at least in a log-linear approximation around a steady state.

Lemma 1 *In the economy with (21) as the matching function, upto a first order log-approximation, the elasticities of the total measure of matches with respect to total recruiting effort and the measure of unemployed are given by* $(1-\eta)$ *and* η *respectively.*

Relative Volatility of Growth Rates

			Elas. to aggr. conditions, κ			
	Data	Full Info	1	0.75	0.5	0
Employment	0.64	0.09	0.79	0.67	0.64	0.65
Tightness	14.70	3.66	37.27	30.91	29.07	29.14

Table 8: Sensitivity to κ, the elasticity of tightness to aggregate conditions

Next, I examine the effect of κ on the amplification generated by informational frictions. Under full information, κ has not effect on the behavior of aggregate variables. Under heterogeneous information, however, varying κ has 2 opposing effects. On the one hand, a higher κ increases the degree of strategic interaction. Since informational frictions cause firms to underestimate the change in market conditions, a higher κ i.e. a higher degree of strategic interation tends to generate greater amplification. However, a higher κ also increases the flow of information from past labor market outcomes. The results in Table 8 show that these two opposing forces work to make the degree of amplification only mildly sensitive to κ.

9 Conclusion

A well-known puzzle in modern macroeconomics is the finding that models with search frictions fail to generate quantitatively significant aggregate variability in employment and other labor market variables, compared to what is observed in the data. The main point of this paper is to argue that this anomaly can be significantly resolved by relaxing the assumption that aggregate conditions are perfectly observed by firms while making hiring decisions. In particular, in an environment where firms are hit by both aggregate and idiosyncratic shocks, the firms' inference problem causes them to be attribute aggregate shocks largely to idiosyncratic factors. Since idiosyncratic shocks do not have offsetting general equilibrium effects, perceived movements in them induce larger adjustments in hiring. As a result, misattribution of aggregate shocks to idiosyncratic factors amplifies the response of the economy to aggregate shocks. In a calibrated model, this mechanism closes most of the gap between a full information benchmark model and the data.

Much work remains ahead. Due to considerations of tractability, the analysis has abstracted from several interesting amplification mechanisms highlighted by the large literature on search and matching models. Endogenous job destruction and on-the-job search are two examples. An investigation of the interactions between these frictions and the information friction that is the focus of this paper holds a lot of promise, both for explaining aggregate behavior but also in understanding features of the cross-sectional distribution. Similarly, the analysis in this paper uses simple wage determination protocols which serve the pedagogical motive of isolating the direct effects of the confusion on firms' incentives to create jobs. Exploring the implications of informational heterogeneity for environments with alternative mechanisms for wage determination is another important and challenging direction for future research. On the informational front, endogenizing the information structure - either with explicit costs or under the rational inattention paradigm - is a natural next step.

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Appendix A Equilibrium Definition

An equilibrium is a set of

i. State-contingent prices Q_t

ii. Aggregate allocations C_t, N_t, V_t

iii. Wages W_{it}

iv. Firm-level employment N_{it}

v. Firm decisions K_{it+1}, V_{it}, D_{it}

such that

- (ii)-(v) satisfy the wage determination equation (13) and the law of motion for employment (10)

- The choices in (v) above solve the firm's problem, taking as given the wages in (iii), the law of motion for employment (10) and beliefs about the aggregate variables in (i)-(ii) based on the firm's information set

- The aggregate allocations are consistent with individual choices and market-clearing conditions for the state-contingent securities i.e.

$$
\begin{aligned}
N_t &= \int N_{it}\, di \\
C_t &= \int (W_{it} N_{it} + D_{it})\, di
\end{aligned}
$$

Appendix B Steady State

Here, I characterize the deterministic steady state of this economy i.e. assuming no aggregate or idiosyncratic shocks. Note that since firms are identical, informational frictions are irrelevant.

$$
\begin{aligned}
\bar{Y} &= \bar{K}^{\alpha_1} \bar{N}^{\alpha_2} (1 - \bar{V})^{\alpha_2} \\
\bar{C} &= \bar{Y} - \delta \bar{K} \\
\frac{1}{\beta} &= \alpha_1 \frac{\bar{Y}}{\bar{K}} + (1 - \delta) \\
\bar{\Theta} &= \frac{\bar{V}\bar{N}}{1 - \bar{N}} \\
\delta_n &= \bar{V}\bar{F} \\
\bar{W} &= \frac{\alpha_2 \bar{Y}}{\bar{N}(1 - \bar{V})\bar{F}} \left(\frac{1 - \beta(\bar{F} + 1 - \delta_n)}{\beta} \right) \\
\bar{W} &= G(\bar{Y}, \bar{N}, \bar{V}, \bar{C}, \bar{N})
\end{aligned}
$$

Appendix C Numerical Solution Methodology

First, I introduce some notation. Define

$$\Omega_{it} \equiv \begin{pmatrix} U_t \\ U_{it}^a \\ U_{it}^z \\ U_{it}^l \end{pmatrix} \quad \text{where} \quad U_t \equiv (u_t, u_{t-1}, ..u_{t-T})' \quad .$$

and $U_{it}^j, \;\; j = a, z, l$ are defined analogously.

Step 1: The effect of aggregates on an individual firms problem can be summarized in 2 variables - the (log of the) stochastic discount factor q_t and overall market tightness θ_t. The starting point of the algorithm is a conjecture for the (linear) relationship between these two variables and the entire history of aggregate shocks:

$$\begin{bmatrix} q_t \\ \theta_t \end{bmatrix} = P \; U_t \; , \tag{23}$$

where P is a 2 x T matrix. Note that this specification allows aggregate variables, upto a log-linear approximation, to depend on the history of aggregate shocks in an arbitrary way.

Step 2: Next, I solve the problem of firm i, assuming it is perfectly informed i.e. assuming it knows the entire history of aggregate and idiosyncratic shocks affecting it. The firm enters period t with its current capital stock k_{it} as well as the level of employment and choice of recruiting effort in $t-1$, n_{it-1} and v_{it-1} respectively. The last two are state variables because they affect the level of employment in the current period. In addition, the entire history of innovations to all the shock processes affecting the firm's payoffs (Ω_{it}) will affect the firm's decisions. These shocks are relevant not only because they directly affect the firm's current productivity, wages etc., but also because they form the basis for the firm's forecasts of future values of aggregate and firm-specific factors. The solution to this problem can be expressed in the form of a law of motion for the firm's state and policy variables:

$$X_{it} = BX_{i,t-1} + D\Omega_{it} \qquad \text{where } X_{it} \equiv \begin{pmatrix} k_{it+1} \\ n_{it} \\ v_{it} \\ w_{it} \\ d_{it} \end{pmatrix} \quad . \tag{24}$$

Step 3: The next step makes use of certainty equivalence implicit in the linear approximation of the policy rules and replaces the actual realizations in (24) with conditional expectations.

$$X_{it} = BX_{i,t-1} + D\mathbb{E}_{it}\Omega_{it} \ . \tag{25}$$

Step 4: To derive the laws of motion for the aggregate state variables, I add (25) over all i. The result is a law of motion which depends only on aggregate state variables and 'average' expectations about aggregate and firm-specific shocks in the economy.

$$X_t = BX_{t-1} + D\bar{\mathbb{E}}_t\Omega_{it} \qquad \text{where} \quad \bar{\mathbb{E}}_t(\cdot) = \int \mathbb{E}_{it}(\cdot) \ di \ . \tag{26}$$

Step 5: Next, I characterize the average expectations in this economy. To achieve this, note that the signals received firm i are linear combinations of Ω_{it} i.e., for a suitably defined Γ,

$$s_i^t = \Gamma \ \ \Omega_{it} \ .$$

Then, standard filtering results using normally distributed variables imply that

$$\mathbb{E}_{it}\Omega_{it} \equiv \mathbb{E}(\Omega_{it}|s_{it}) = \Sigma \ \Gamma' \ (\Gamma \ \Sigma \ \Gamma')^{-1} s_{it} \tag{27}$$

$$= \Sigma \ \Gamma' \ (\Gamma \ \Sigma \ \Gamma')^{-1} \Gamma \ \Omega_{it} \ , \tag{28}$$

where Σ is the variance-covariance matrix of Ω_{it}. Average expectations are given by

$$\bar{\mathbb{E}}_t(\Omega_{it}) = \int \mathbb{E}_{it}\Omega_{it} \ di = \int \mathbb{E}(\Omega_{it}|s_{it}) \ \ di = \Sigma \ \Gamma' \ (\Gamma \ \Sigma \ \Gamma')^{-1} \ \Gamma \ \int \Omega_{it} \ \ di$$
$$= \Sigma \ \Gamma' \ (\Gamma \ \Sigma \ \Gamma')^{-1} \ \Gamma\bar{\Omega}_t \ ,$$

where $\bar{\Omega}_{it}$ is the cross-sectional average of the shocks. Since we assume a law of large numbers to be in effect for the idiosyncratic shock realizations,

$$\bar{\Omega}_t = \begin{pmatrix} U_t \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \ .$$

Average expectations thus can be written as a function only of the aggregate shock vector U_t i.e.

$$\bar{\mathbb{E}}_t(\Omega_{it}) = H \ \ U_t \ .$$

Step 6: Substituting the expression for average expectations back into (26),

$$X_t = BX_{t-1} + DH \ \ U_t \ . \tag{29}$$

Step 7: Equation (29) implies a relationship between the stochastic discount factor q_t and market tightness θ_t and the realizations of the aggregate shock U_t:

$$\begin{bmatrix} q_t \\ \theta_t \end{bmatrix} = \tilde{P} \ U_t \ . \tag{30}$$

If $\tilde{P} = P$, my initial guess, then the algorithm has converged and the allocations implied by (25), (28) and (29) constitute an approximate equilibrium as defined in Section 3.5. If not, i.e. $\tilde{P} \neq P$, then we repeat the above steps using a new conjecture:

$$P_1 = g \ \ P + (1-g) \ \ \tilde{P} \ , \qquad g \in (0,1).$$

Appendix D Nash Bargaining Under Symmetric Information

Here, I derive the Nash bargaining wage under the assumption that the firm i and its workers have the same information about aggregate and firm-specific histories. Let χ_{it} denote the representative household's value of having a worker employed in i in period t.

$$\chi_{it} = \beta(1-\delta_n)\mathbb{E}\chi_{i,t+1} + W_{it}(1-\tau)Q_t - \gamma Z_{it} - \Upsilon_t \ , \tag{31}$$

where Υ_t is the continuation of value of being unemployed i.e.

$$\Upsilon_t = \beta\mathbb{E}\int \frac{V_{it}N_{it}F_{it}}{1-N_t}di \ .$$

Conditional on firm i's information set, the expected value to a household of the marginal worker in i at an arbitrary wage W is given by

$$H_{it} = (W - W_{it}) \ \mathbb{E}_{it}Q_t + \mathbb{E}_{it}\chi_{it} \ .$$

Similarly, the expected value to the firm

$$J_{it} = (W_{it} - W) \ \mathbb{E}_{it}Q_t + \mathbb{E}_{it}\xi_{it} \ ,$$

where ξ_{it} is firm i's shadow value of a worker, given by

$$\xi_{it} = Q_t\left(\frac{\alpha_2 Y_{it}}{N_{it}(1-V_{it})} - W_{it}\right) + \beta(1-\delta_n)\mathbb{E}\xi_{it+1} \ .$$

The wage under Nash bargaining is the solution to

$$\max_{W} \quad H_{it}^{\phi}\, J_{it}^{1-\phi} \, .$$

The FOC of the above problem evaluated at the equilibrium wage, W_{it}, imply

$$\chi_{it} = \frac{\phi}{1-\phi}(1-\tau)\xi_{it} \, .$$

This relationship can be used to eliminate χ_{it} and χ_{it+1} from (31). This yields

$$\xi_{it} = \frac{1-\phi}{\phi(1-\tau)}\left[W_{it}(1-\tau)\mathbb{E}_{it}Q_t - \gamma\mathbb{E}_{it}Z_{it} - \mathbb{E}_{it}\Upsilon_t\right] + \beta(1-\delta_n)\mathbb{E}_{it}\xi_{it+1}$$

Subtracting the two expressions for ξ_{it} leads to an equation in one unknown, the equilibrium wage W_{it}. The solution is the expression in (18).

Appendix E Calibration

The strategy for picking preference and technology parameters is based largely on Shimer (2010). The values for the discount rate β, the share of labor α_2 and the capital depreciation rate δ are borrowed directly from the real business cycle literature. The share of capital, α_1 is set to target a total share paid to factors of 90%. The persistence of the aggregate shock ρ also corresponds to the values used in the RBC literature, adjusted for the fact that a time period in this paper is a month.

The rate of exogenous separation δ_n is taken from Shimer (2005). Hagedorn and Manovskii (2008) estimate that hiring a worker costs about 4% of a worker's quarterly wage, which implies that each recruiter attracts 25 workers on average per quarter (i.e. the monthly counterpart $\bar{F} = 8.33$). The following steady-state relationship then pins down the fraction of workers engaged in recruiting in the steady state:

$$\delta_n \bar{N} = \bar{V}\bar{N}\bar{F} \qquad \Rightarrow \quad \bar{V} = \frac{\delta_n}{\bar{F}} \, .$$

Given a target for steady state employment ($\bar{N} = 0.95$), the estimate for $\bar{V}$ determines the steady state tightness, $\bar{\Theta}$. The matching function in this paper takes as input recruiting effort, instead of the usual measure of vacancies posted, so there are no direct estimates of the elasticity available. I follow Shimer (2010) and consider the symmetric case i.e. $\eta = 0.5$. Given this choice, equation (11) pins down the scale parameter $\bar{\mu}$. The worker's 'share' of the surplus ϕ is set to 0.5.

The disutility of leisure, γ, is chosen so that steady state unemployment is 5%. Finally, the labor tax τ is set to match the average marginal tax rate.

Appendix F Nash Bargaining without heterogeneity

For this analysis in this section, I assume that there is no heterogeneity. The value to the representative firm of hiring an additional worker at an arbitrary wage W at time t (as before, I suppress the s^t notation for brevity) is given by

$$J_t(W) = Q_t(W_t - W) + \xi_t \ ,$$

where, in a slight abuse of notation, ξ_t is the shadow value of an additional worker (at the equilibrium wage W_t). The law of motion for this value is given by

$$\xi_t = \left(\frac{\alpha_2 Y_t}{N_t(1 - V_t)} - W_{it} \right) Q_t + \beta(1 - \delta_n)\mathbb{E}_t \ \xi_{t+1} \ . \tag{32}$$

To derive an analogous expression for the household, I first specify the household's perceived law of motion for aggregate employment.

$$N_t = N_{t-1}(1 - \delta_n) + (1 - N_{t-1})\Theta_{t-1}F_{t-1} \ .$$

Let χ_t be the multiplier associated with this constraint. As with the firm's multiplier, we can show that χ_t satisfies

$$\chi_t = \frac{1}{C_t}(1 - \tau)W_t - \gamma + (1 - \delta_n - \Theta_t F_t)\beta\mathbb{E}_t \ \chi_{t+1} \ . \tag{33}$$

The household's value from having an additional worker employed at an arbitrary wage W is

$$H_t(W) = \frac{1}{C_t}(1 - \tau)(W - W_t) + \chi_t \ .$$

Nash bargaining implies that the equilibrium wage W_t solves

$$\max_{W} \quad H_t(W)^{\phi} \ J_t(W)^{1-\phi}$$

The FOC of this problem

$$
\begin{aligned}
\phi\,\frac{1}{C_t}(1-\tau)\left(\frac{1}{H_t}\right)_{W=W_t} &= (1-\phi)Q_t\left(\frac{1}{J_t}\right)_{W=W_t}\\
\phi\,\frac{1}{C_t}(1-\tau)\frac{1}{\chi_t} &= (1-\phi)Q_t\frac{1}{\xi_t}\\
\phi\,(1-\tau)\frac{1}{\chi_t} &= (1-\phi)\frac{1}{\xi_t}\\
\chi_t &= \frac{\phi(1-\tau)}{1-\phi}\,\xi_t\ ,
\end{aligned}
$$

where the second step makes use of the fact that $H_t = \chi_t$ and $J_t = \xi_t$ at the equilibrium wage and the third equality simply notes that $Q_t = \frac{1}{C_t}$. Substituting the last relationship into (33) and rearranging,

$$
\xi_t = \frac{(1-\phi)}{\phi(1-\tau)}\left(\frac{1}{C_t}(1-\tau)W_t - \gamma\right) + (1-\delta_n - \Theta_t F_t)\,\beta\mathbb{E}_t\,\xi_{t+1}\ . \tag{34}
$$

Next, note that the FOC for the firm's choice of V_t implies[29]

$$
\beta\mathbb{E}_t\,\xi_{t+1} = \frac{\alpha_2 Y_t}{N_t(1-V_t)F_t}Q_t\ .
$$

Using this to eliminate $\mathbb{E}_t\xi_{t+1}$ from the right sides of (32) and (34), we get

$$
\begin{aligned}
\xi_t &= \left(\frac{\alpha_2 Y_t}{N_t(1-V_t)} - W_t\right)Q_t + (1-\delta_n)\frac{\alpha_2 Y_t}{N_t(1-V_t)F_t}Q_t\\
\xi_t &= \frac{(1-\phi)}{\phi(1-\tau)}\left(\frac{1}{C_t}(1-\tau)W_t - \gamma\right) + (1-\delta_n - \Theta_t F_t)\frac{\alpha_2 Y_t}{N_t(1-V_t)F_t}Q_t\ .
\end{aligned}
$$

Equating the right-hand sides and noting that $Q_t = \frac{1}{C_t}$, I solve for the after tax wage:

$$
(1-\tau)W_t = \phi(1-\tau)\frac{\alpha_2 Y_t}{N_t(1-V_t)}(1+\Theta_t) + (1-\phi)C_t\gamma\ .
$$

Appendix G Proofs of Results

G.1 Proof of Proposition 1

Follows from a direct comparison of the expressions for ϕ^{Full} and ϕ^{Het}.

[29] Note that this assumes an interior solution for V_t. This is true at the steady state and will hold in a neighborhood around the steady state for sufficiently small shocks.

G.2 Proof of Proposition 2

Under full information, the log-linearized optimality and equilibrium conditions can be represented as the solution to a dynamic system of the form

$$R_1 \; X_{it} = R_2 \; X_{it-1} + R_3 \; \Omega_{it}$$

where X_{it} is a vector of all state and endogenous variables of interest, both aggregate and firm-specific. Integrating over all i yields

$$R_1 \; X_t = R_2 \; X_{t-1} + R_3 \; \bar{\Omega}_{it} \tag{35}$$

where $\bar{\Omega}_{it}$ is the cross sectional average of shocks and, by the law of large numbers assumption, is simply $[U_t \;\; \mathbf{0} \;\; \mathbf{0} \;\; \mathbf{0}]'$. Subtracting one equation from the other,

$$R_1 \; [X_{it} - X_t] = R_2 \; [X_{it-1} - X_{t-1}] + R_3 \; [\bar{\Omega}_{it} - \Omega_{it}] \tag{36}$$

Now, a solution to the original dynamic system can be found by simply adding the solutions to the two decoupled systems (35) and (36) separately. To see that they are indeed decoupled, simply note that the driving error processes for the two systems are completely different. Equation (35) is driven solely by the aggregate shocks while (36) is affected only by realizations of the idiosyncratic shocks.

Next, it is straightforward to show log-linearization of the wage equation (18) yields an expression for the *average* wage rate (in logs) that is identical to the log-linearized version of the wage rate in a model with Nash Bargaining and no heterogeneity. This observation, along with direct comparison of the linearized equilibrium conditions of a representative agent model, implies that the coefficient matrices R_1, R_2 and R_3 in equation (35) are exactly the same as those of the log-linearized representative agent model (because the first-order conditions take the same form). Therefore, both systems have identical solutions, establishing the result of the proposition.

G.3 Proof of Lemma 1

The total number of matches, $M_t \equiv \int V_{it} N_{it} F_{it} \; di$, upto a first-order approximation (in log deviations from steady state values) can be written as

$$\begin{aligned} m_t &= v_t + n_t + f_t = v_t + n_t - \eta\theta_t \\ &= v_t + n_t - \eta \left((1-\kappa) \int (v_{it} + n_{it}) di + \kappa(v_t + n_t) - \tilde{n}_t \right) \\ &= (1-\eta)(v_t + n_t) + \eta \tilde{n}_t \end{aligned}$$

where $\tilde{n}_t$ is the log-deviation of the unemployment rate $1 - N_t$. The result follows immediately.