R&D and productivity: The knowledge capital model revisited*

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Abstract

Investment in knowledge at the firm level is a primary source of productivity growth. To account for the fact that such investments create long-lived assets for firms, Griliches (1979) developed the knowledge capital model that has remained a cornerstone of the productivity literature for more than 25 years. The link between R&D and productivity, however, is much more complex. In this paper, we relax the assumptions on the R&D process and examine the impact of the investment in knowledge on the productivity of firms.

We develop a simple estimator for production functions in the presence of endogenous productivity change that allows us to retrieve productivity and its relationship with R&D at the firm level. We illustrate our approach on an unbalanced panel of more than 1800 Spanish manufacturing firms in nine industries during the 1990s. Our findings indicate that the link between R&D and productivity is subject to a high degree of uncertainty, nonlinearity, and heterogeneity across firms. Abstracting from uncertainty and nonlinearity, as is done in the knowledge capital model, or assuming an exogenous process for productivity, as is done in the recent literature on structural estimation of production functions, overlooks some of its most interesting features.

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1 Introduction

Investment in knowledge at the firm level is a primary source of productivity growth. Firms invest in R&D and related activities to develop and introduce process and product innovations that enhance their productivity. To account for the fact that investment in knowledge creates long-lived assets for firms, Griliches (1979) augmented the production function with the stock of knowledge. This knowledge capital model has remained a cornerstone of the productivity literature for more than 25 years and has been applied in hundreds of empirical studies on firm-level productivity and also extended to macroeconomic growth models (see Griliches (1995) for a comprehensive survey). While useful as a practical tool, the knowledge capital model has a long list of known drawbacks as explained, for example, in Griliches (2000). The critical (but implicit) assumptions of the basic model include the linear and certain accumulation of knowledge from period to period in proportion to R&D expenditures as well as the linear and certain depreciation.

The link between R&D and productivity, however, is much more complex. The outcome of the R&D process is likely to be subject to a high degree of uncertainty. Discovery is, by its very nature, uncertain. Once discovered an idea has to be developed and applied, and there are the technical and commercial uncertainties linked to its practical implementation. In addition, current and past investments in knowledge are likely to interact with each other in many ways. For example, there is evidence of complementarities in the accumulation of knowledge (Klette 1996). In general, there is little reason to believe that this and other features such as economies of scale can be adequately captured by simple functional forms.

The first goal of this paper is thus to relax the assumptions on the R&D process that are at the center of the knowledge capital model. In particular, we recognize the uncertainties in the R&D process in the form of shocks to productivity. We model the interactions between current and past investments in knowledge in a flexible fashion. Furthermore, we relax the assumption that the obsolescence of previously acquired knowledge can be described by a constant rate of depreciation. This allows us to better assess the impact of the investment in knowledge on the productivity of firms.

Capturing the uncertainties in the R&D process also paves the way for heterogeneity across firms. Whereas firms with the same time path of R&D expenditures have necessarily the same productivity in the knowledge capital model, in our setting this is no longer the case because we allow the shocks to productivity to accumulate over time. The second goal of this paper is thus to assess the role of R&D in determining the differences in productivity across firms and the evolution of firm-level productivity over time.

To achieve these goals we develop a dynamic model that allows for investment in knowl-

¹See Hall & Mairesse (1996) for a classic application. The knowledge capital model has evolved in many directions. Pakes & Schankerman (1984) modeled the creation of knowledge by specifying a production function in terms of R&D capital and R&D labor. Jaffe (1986) initiated ways of accounting for the appropriability of the external flows of knowledge or spillovers. For recent examples see Griffith, Redding & Van Reenen (2004) or Griffith, Harrison & Van Reenen (2006).

edge, thereby endogenizing productivity change, and derive a simple estimator for production functions in this setting. We use our approach to study the relationship between R&D and productivity in Spanish manufacturing firms during the 1990s.

We start by modeling a firm that can invest in R&D in order to improve its productivity over time in addition to carrying out a series of investments in physical capital. Both investment decisions depend on the current productivity and capital stock of the firm. The evolution of productivity is subject to random shocks. We interpret these innovations to productivity as representing the resolution over time of all uncertainties. They capture the factors that have a persistent influence on productivity such as absorption of techniques, modification of processes, and gains and losses due to changes in labor composition and management abilities. R&D governs the evolution of productivity up to an unpredictable component. Hence, for firms that engage in R&D, the productivity innovations additionally capture the uncertainties inherent in the R&D process such as chance in discovery and success in implementation. Productivity thus follows a first-order Markov process that can be shifted by R&D expenditures. Subsequently decisions on variable (or "static") inputs such as labor and materials are taken according to the current productivity and capital stock of the firm.

To estimate the parameters of the production function and retrieve productivity at the level of the firm, we build on recent advances in the structural estimation of production functions. Starting with Olley & Pakes (1996) (hereafter OP) this literature has stressed that the decisions that a firm makes depend on its productivity. Because the productivity of the firm is unobserved by the econometrician, this gives rise to an endogeneity problem. The insight of OP is that if (observed) investment is a monotone function of (unobserved) productivity, then this function can be inverted to back out productivity. Controlling for productivity resolves the endogeneity problem as well as, eventually, the selection problem that may arise if a firm's decision to exit the industry depends on its productivity.² In addition to OP, this line of research includes contributions by Levinsohn & Petrin (2003) (hereafter LP), Wooldridge (2004), Ackerberg, Caves & Frazer (2005) (hereafter ACF) as well as a long list of applications. Common to the existing estimators is the assumption that the productivity process is exogenous, thereby making them ill-suited to study the link between R&D and productivity. Endogenizing the productivity process by incorporating R&D expenditures into the dynamic investment model of OP is difficult as Buettner (2005) has shown (see Section 3 for details).

We develop a simple estimator for production functions that can accommodate the controlled Markov process that results from the impact of R&D on the evolution of productivity. Similar to LP and ACF, we use the fact that decisions on variable inputs are based on current productivity. These inputs are chosen with current productivity known and therefore

²See Griliches & Mairesse (1998) and Ackerberg, Benkard, Berry & Pakes (2005) for reviews of the problems involved in the estimation of production functions.

contain information about it. The resulting input demands are invertible functions of unobserved productivity (as first shown by LP). This enables us to control for productivity and obtain consistent estimates of the parameters of the production function. We differ from the previous literature in that we recognize that, given a parametric specification of the production function, the functional form of these inverse input demand functions is known. Because we make full use of the structural assumptions, we do not have to rely on nonparametric methods to estimate the inverse input demand function. This renders identification and estimation more tractable.

We apply our estimator to an unbalanced panel of more than 1800 Spanish manufacturing firms in nine industries during the 1990s. A number of interesting findings emerge. To begin with, the R&D process must be treated as inherently uncertain. We estimate that, depending on the industry, between 20% and 50% of the variance in actual productivity is explained by productivity innovations that cannot be predicted when decisions on R&D expenditures are made. Despite this, the expected productivity of firms that perform R&D is systematically more favorable in the sense that their distribution of expected productivity tends to stochastically dominate the distribution of firms that do not perform R&D. The mean of the distribution of expected productivity is higher by around 5% in most cases and up to 9% in some cases. However, R&D may not inject additional uncertainty into the evolution of productivity over time.

Expected productivity is increasing in attained productivity and R&D expenditures. While this relationship takes a simple separable form in some cases, in most cases the impact of current R&D on future productivity depends crucially on current productivity. Our setting nests the knowledge capital model, but the data very clearly reject the functional form restrictions implied by it, thus suggesting that the linearity assumption in the accumulation and depreciation of knowledge may have to be relaxed in order to better assess the impact of the investment in knowledge on the productivity of firms.

We estimate that the contribution of firms that perform R&D explains between 50% and 85% of productivity growth in the industries with intermediate or high innovative activity. R&D expenditures are thus indeed a primary source of productivity growth. Finally, we find large average rates of return to R&D in most industries, although our estimates are within the range of the previous literature. Hidden behind these averages, however, is a substantial amount of heterogeneity across firms.

Overall, the link between R&D and productivity is subject to a high degree of uncertainty, nonlinearity, and heterogeneity across firms. Abstracting from uncertainty and nonlinearity, as is done in the knowledge capital model, or assuming an exogenous process for productivity, as is done in the literature following OP and LP, overlooks some of its most interesting features.

2 A model for investment in knowledge

A firm carries out two types of investments, one in physical capital and another in knowledge through R&D expenditures. The investment decisions are made in a discrete time setting with the goal of maximizing the expected net present value of future cash flows. The firm has the Cobb-Douglas production function

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_k k_{jt} + \omega_{jt} + e_{jt},$$

where y_{jt} is the log of output of firm j in period t, l_{jt} the log of labor, and k_{jt} the log of capital. We follow the convention that lower case letters denote logs and upper case letters levels and focus on a value-added specification to simplify the exposition. Capital is the only fixed (or "dynamic") input among the conventional factors of production, and accumulates according to $K_{jt} = (1 - \delta)K_{jt-1} + I_{jt-1}$. This law of motion implies that investment I_{jt-1} chosen in period t-1 becomes productive in period t. The productivity of firm j in period t is ω_{jt} . We follow OP and often refer to ω_{jt} as "unobserved productivity" since it is unobserved from the point of view of the econometrician (but known to the firm). Productivity is presumably highly correlated over time and perhaps also across firms. In contrast, e_{jt} is a mean zero random shock that is uncorrelated over time and across firms. The firm does not know the value of e_{jt} at the time it makes its decisions for period t.

The assumption usually made about productivity (see OP, LP, and the subsequent literature) is that it follows an exogenous first-order Markov process with transition probabilities $P(\omega_{jt}|\omega_{jt-1})$. This rules out that the firm spends on R&D and related activities. However, investment in knowledge has always been thought of as aimed at modifying productivity for given conventional factors of production (see, e.g., the tradition started by Griliches (1979)). Our goal is thus to assess the role of R&D in determining the differences in productivity across firms and the evolution of firm-level productivity over time.

We therefore consider productivity to be governed by a controlled first-order Markov process with transition probabilities $P(\omega_{jt}|\omega_{jt-1},r_{jt-1})$, where r_{jt-1} is the log of R&D expenditures. The Bellman equation for the firm's dynamic programming problem is

$$V(k_{jt}, \omega_{jt}) = \max_{i_{jt}, r_{jt}} \pi(k_{jt}, \omega_{jt}) - c_i(i_{jt}) - c_r(r_{jt}) + \frac{1}{1+\rho} E\left[V(k_{jt+1}, \omega_{jt+1}) | k_{jt}, \omega_{jt}, i_{jt}, r_{jt}\right],$$

where $\pi(\cdot)$ denotes per-period profits and ρ is the discount rate. In the simplest case the cost functions $c_i(\cdot)$ and $c_r(\cdot)$ just transform logs into levels, but their exact forms are irrelevant for our purposes.

The dynamic problem gives rise to two policy functions, $i(k_{jt}, \omega_{jt})$ and $r(k_{jt}, \omega_{jt})$ for the investments in physical capital and knowledge, respectively. The main difference between the two types of investments is that they affect the evolution of different state variables, i.e., either the capital stock k_{jt} or the productivity ω_{jt} of the firm.

When the decision about investment in knowledge is made in period t-1, the firm is only able to anticipate the expected effect of R&D on productivity in period t. The Markovian assumption implies

$$\omega_{jt} = E\left[\omega_{jt} | \omega_{jt-1}, r_{jt-1}\right] + \xi_{jt} = g(\omega_{jt-1}, r_{jt-1}) + \xi_{jt}.$$

That is, actual productivity ω_{jt} in period t can be decomposed into expected productivity $g(\omega_{jt-1}, r_{jt-1})$ and a random shock ξ_{jt} . Our key assumption is that the impact of R&D on productivity can be expressed through the dependence of the conditional expectation function $g(\cdot)$ on R&D expenditures. In contrast, ξ_{jt} does not depend on R&D expenditures. This productivity innovation may be thought of as the realization of the uncertainties that are naturally linked to productivity plus the uncertainties inherent in the R&D process (e.g., chance in discovery, degree of applicability, success in implementation). It is important to stress the timing of decisions in this context: When the decision about investment in knowledge is made in period t-1, the firm is only able to anticipate the expected effect of R&D on productivity in period t as given by $g(\omega_{jt-1}, r_{jt-1})$ while its actual effect also depends on the realization of the productivity innovation ξ_{jt} that occurs after the investment has been completely carried out. Of course, the conditional expectation function $g(\cdot)$ is unobserved from the point of view of the econometrician (but known to the firm) and must be estimated nonparametrically.

If we consider a ceteris paribus increase in R&D expenditures that changes ω_{jt} to $\tilde{\omega}_{jt}$, then $\tilde{\omega}_{jt} - \omega_{jt}$ approximates the effect of this change in productivity on output in percentage terms, i.e., $(\tilde{Y}_{jt} - Y_{jt})/Y_{jt} = \exp(\tilde{\omega}_{jt} - \omega_{jt}) - 1 \simeq \tilde{\omega}_{jt} - \omega_{jt}$. That is, the change in ω_{jt} shifts the production function and hence measures the change in total factor productivity. Also $g(\cdot)$ and ξ_{jt} can be interpreted in percentage terms and decompose the change in total factor productivity. Finally, $\frac{\partial \omega_{jt}}{\partial r_{jt-1}} = \frac{\partial g(\omega_{jt-1}, r_{jt-1})}{\partial r_{jt-1}}$ is the elasticity of output with respect to R&D expenditures.

Our setting encompasses as a particular case the knowledge capital model (see Griliches (1979, 2000)). In this model, a conventional Cobb-Douglas production function is augmented by including the log of knowledge capital c_{jt} as an extra input yielding

$$y_{it} = \beta_0 + \beta_l l_{it} + \beta_k k_{it} + \varepsilon c_{it} + e_{it}, \tag{1}$$

where ε is the elasticity of output with respect to knowledge capital. Knowledge capital is assumed to accumulate with R&D expenditures and to depreciate from period to period at a rate δ . Hence, its law of motion can be written as

$$C_{jt} = (1 - \delta)C_{jt-1} + R_{jt-1} = C_{jt-1} \left(1 - \delta + \frac{R_{jt-1}}{C_{it-1}} \right).$$

Taking logs we have

$$c_{jt} \simeq c_{jt-1} + \left(\frac{R_{jt-1}}{C_{it-1}} - \delta\right),$$

where $\frac{R_{jt-1}}{C_{jt-1}}$ is the rate of investment in knowledge. Letting $\omega_{jt} = \varepsilon c_{jt}$ it is easy to see that

$$\omega_{jt} = \omega_{jt-1} + \varepsilon \left(\frac{\exp(r_{jt-1})}{\exp(\omega_{jt-1}/\varepsilon)} - \delta \right)$$

and hence $\omega_{jt} = g(\omega_{jt-1}, r_{jt-1})$. That is, the "classical" accumulation of knowledge capital induces a particular expression for the conditional expectation function $g(\cdot)$ that depends on both productivity and R&D expenditures in the previous period.

The knowledge capital model ignores that the accumulation of improvements to productivity is likely to be subjected to shocks. To capture this assume that the effect of the rate of investment in knowledge has an unpredictable component ξ_{jt} . The law of motion becomes $C_{jt} = C_{jt-1} \left(1 - \delta + \frac{R_{jt-1}}{C_{jt-1}} + \frac{1}{\varepsilon} \xi_{jt}\right)$. This simple extension causes the law of motion of productivity to be $\omega_{jt} = g(\omega_{jt-1}, r_{jt-1}) + \xi_{jt}$, which turns out to be our controlled first-order Markov process. Therefore, a useful way to think of our setting is as a generalization of the knowledge capital model to the more realistic situation of uncertainty in the R&D process.³

In addition, our setting overcomes other problems of the knowledge capital model, in particular the linear accumulation of knowledge from period to period in proportion to R&D expenditures and the linear depreciation. The absence of functional form restrictions on the combined impact of R&D and already attained productivity on future productivity is an important step in the direction of relaxing all these assumptions. Of course, there is a basic difference between the two models. In the case of the knowledge capital model, given data on R&D and a guess for the initial condition, one must be able to construct the stock of knowledge capital at all times and with it control for the impact of R&D on productivity. In our setting, in contrast, the random nature of accumulation and the unspecified form of the law of motion prevents the construction of the "stock of productivity," which remains unobserved. Consequently, no guess for the initial condition is required. Moreover, our empirical strategy takes into account that the endogeneity problem in production function estimation may not be completely resolved by adding the stock of knowledge capital to the conventional factors of production.

³We note that there are ways of introducing uncertainty into the knowledge capital model, although we are not aware of any such attempts in the literature. Let the law of motion for the log of knowledge capital be $c_{jt} = (1 - \delta)c_{jt-1} + R_{jt-1} + \xi_{jt}$. Then $c_{jt} = (1 - \delta)^t c_0 + \sum_{\tau=1}^t (1 - \delta)^{t-\tau} R_{j\tau-1} + \sum_{\tau=1}^t (1 - \delta)^{t-\tau} \xi_{j\tau}$ can be split into a deterministic and a stochastic part that is incorporated into the error term of the estimation equation. In this case, however, using R&D expenditures as a proxy for the stock of knowledge gives rise to an endogeneity problem that invalidates the traditional estimation strategies such as running OLS on first-differences of logs. A further problem is that the ability to split the log of knowledge capital into a deterministic and a stochastic part relies heavily on functional form. In particular, it is no longer possible if, as is customary in the literature, the law of motion for the level of knowledge capital is assumed to be linear.

3 Empirical strategy

Our model relaxes the assumption of an exogenous Markov process for ω_{jt} . As emphasized in Ackerberg, Benkard, Berry & Pakes (2005), making this process endogenous is problematic for the standard estimation procedures. First, it tends to invalidate the usual instrumental variables approaches. Given an exogenous Markov process, input prices are natural instruments for input quantities. Since all quantities depend on all prices, this is, however, no longer the case if the transitions from current to future productivity are affected by the choice of an additional unobserved "input" such as R&D. Second, the absence of data on R&D implies that a critical determinant of the distribution of ω_{jt} given ω_{jt-1} is missing. Recovering ω_{jt} from k_{jt} , i_{jt} , and their lags, the key step in OP, may thus be difficult.

Buettner (2005) extends the OP approach by studying a model similar to ours while assuming transition probabilities for unobserved productivity of the form $P(\omega_{jt}|\psi_t)$, where $\psi_t = \psi(\omega_{jt-1}, r_{jt-1})$ is an index that orders the probability distributions for ω_{jt} . The restriction to an index excludes the possibility that current productivity and R&D expenditures affect future productivity in qualitatively different ways. Under certain assumptions it ensures that the policy function for investment in physical capital is still invertible and that unobserved productivity can hence still be written as an unknown function of the capital stock and the investment as $\omega_{jt} = h(k_{jt}, i_{jt})$. Buettner (2005) further notes, however, that there are problems with identification even when data on R&D is available.

Our estimation procedure solves entirely the identification problem when there is data on R&D by using a known function $h(\cdot)$ that is derived from the demand for variable inputs such as labor and materials in order to recover unobserved productivity. These variable inputs are chosen with current productivity known, and therefore contain information about it. This allows us to back out productivity without making assumptions on the firm's dynamic investment problem. In particular, our approach does not rely on an index and frees up the relationship between current productivity, R&D expenditures, and future productivity. It can also solve potentially the identification problem when there is no data on R&D but this point needs further research.⁴

While our approach pertains to production functions that are written in terms of either gross output or value added, in what follows we focus on the value added case for the sake of simplicity. The extension to the gross output case is straightforward.

Given the Cobb-Douglas production function $y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_k k_{jt} + \omega_{jt} + e_{jt}$, the assumption that the firm chooses labor based on the expectation $E(e_{jt}) = 0$ gives the

⁴Muendler (2005) suggests to use investment in physical capital interacted with industry-specific competition variables to proxy for endogenously evolving productivity. His rationale is that firms make R&D decisions in light of their expectations about future market prospects. Hence, in the absence of data on R&D, these competition variables should to some extent capture the drivers of R&D decisions.

demand for labor as

$$l_{jt} = \frac{1}{1 - \beta_l} \left(\beta_0 + \ln \beta_l + \beta_k k_{jt} + \omega_{jt} - (w_{jt} - p_{jt}) \right).$$
 (2)

Solving for ω_{jt} we obtain the inverse labor demand function

$$h(l_{it}, k_{it}, w_{it} - p_{it}) = \lambda_0 + (1 - \beta_l)l_{it} - \beta_k k_{it} + (w_{it} - p_{it}),$$

where λ_0 combines the constant terms $-\beta_0$ and $-\ln \beta_l$ and $(w_{jt} - p_{jt})$ is the relative wage (homogeneity of degree zero in prices). From hereon we call $h(\cdot)$ the inverse labor demand function and use h_{jt} as shorthand for its value $h(l_{jt}, k_{jt}, w_{jt} - p_{jt})$.

Substituting the inverse labor demand function $h(\cdot)$ for ω_{jt} in the production function cancels out parameters of interest and leaves us with the marginal productivity condition for profit maximization, i.e., $\ln \beta_l + (y_{jt} - l_{jt}) = w_{jt} - p_{jt} + e_{jt}$. Using its value in period t-1 in the controlled Markov process, however, we have

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_k k_{jt} + g(h(l_{jt-1}, k_{jt-1}, w_{jt-1} - p_{jt-1}), r_{jt-1}) + \xi_{jt} + e_{jt}.$$
 (3)

Both k_{jt} , whose value is determined in period t-1 by i_{t-1} , and r_{jt-1} are uncorrelated with ξ_{jt} by virtue of our timing assumptions. Only l_{jt} is correlated with ξ_{jt} (since ξ_{jt} is part of ω_{jt} and l_{jt} is a function of ω_{jt}). Nonlinear functions of the other variables can be used as instruments for l_{jt} , as can be lagged values of l_{jt} and the other variables. If firms can be assumed to be perfectly competitive, then current wages and prices are exogenous and constitute the most adequate instruments (since demand for labor is directly a function of current wages and prices).

As noted by LP and ACF, backing out unobserved productivity from the demand for either labor or materials is a convenient alternative to backing out unobserved productivity from investment as in OP. In the tradition of OP, however, LP and ACF use nonparametric methods to estimate the inverse input demand function. This forces them either to rely on a two-stage procedure or to jointly estimate a system of equations as suggested by Wooldridge (2004). The drawback of the two-stage approach is a loss of efficiency whereas the joint estimation of a system of equations is numerically more demanding (see Ackerberg, Benkard, Berry & Pakes (2005) for a discussion of the relative merits of the two approaches).

We differ from the previous literature in that we recognize that the parametric specification of the production function implies a known form for the inverse labor demand function $h(\cdot)$ that can be used to control for unobserved productivity. As a consequence, only the conditional expectation function $g(\cdot)$ is unknown and must be estimated nonparametrically. This yields efficiency gains. In addition, because we make full use of the structural assumptions, we have but a single equation to estimate, thus easing the computational burden. A drawback of our approach is that, in principle, it requires firm-level wage and price data to estimate the model, although the model remains identified, however, if the log of relative wage is replaced by a set of dummies.⁵

Apart from the presence of R&D expenditures, our estimation equation (3) is similar in structure to the second equation of OP and LP when viewed through the lens of Wooldridge's (2004) GMM framework. In our setting the first equation of OP and LP is the marginal productivity condition for profit maximization. We note that combining it with our estimating equation (3) may help to estimate the labor coefficient, but this point needs further research.

Our model nests, as a particular case, the dynamic panel model proposed by Blundell & Bond (2000). Suppose the Markov process is simply an autoregressive process that does not depend on R&D expenditures so that we have $g(\omega_{jt-1}) = \rho \omega_{jt-1}$. Using the marginal productivity condition for profit maximization to substitute ρy_{jt-1} for $\rho(-\ln \beta_l + (w_{jt-1} - p_{jt-1}) + l_{jt-1})$, we are in the Blundell & Bond (2000) specification. Hence, the differences between their and our approach lie in the generality of the assumption on the Markov process and the strategy of estimation. In the tradition of OP and LP our method basically proposes the replacement of unobservable autocorrelated productivity by an expression in terms of observed variables and an unpredictable component, whereas their method models the same term through the use of lags of the dependent variable (see ACF for a detailed description of these two literatures).

Below we discuss how imperfect competition can be taken into account and the likelihood of sample selection. Then we turn to identification, estimation, and testing.

Imperfect competition. Until now we have assumed a perfectly competitive environment. But when firms have some market power, say because products are differentiated, then output demand enters the specification of the inverse input demand functions (see, e.g., Jaumandreu & Mairesse 2005). Consider firms facing a downward sloping demand function that depends on the price of the output P_{jt} and the demand shifters Z_{jt} . Profit maximization requires that firms set the price that equates marginal cost to marginal revenue $P_{jt}\left(1-\frac{1}{\eta(p_{jt},z_{jt})}\right)$, where $\eta(\cdot)$ is the absolute value of the elasticity of demand evaluated at the equilibrium price and the particular value of the demand shifter and, for convenience, is written as a function of $p_{jt} = \ln P_{jt}$ and $z_{jt} = \ln Z_{jt}$. With firms minimizing costs, marginal cost and conditional labor demand can be determined from the cost function and combined with marginal revenue to give the inverse labor demand function

$$h^{IC}(l_{jt}, k_{jt}, w_{jt} - p_{jt}, p_{jt}, z_{jt}) = \lambda_0 + (1 - \beta_l)l_{jt} - \beta_k k_{jt} + (w_{jt} - p_{jt}) - \ln\left(1 - \frac{1}{\eta(p_{jt}, z_{jt})}\right).$$

⁵This may be an appropriate solution in the absence of wage and price data if the industry can be considered perfectly competitive.

Thus, the estimation equations is

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_k k_{jt} + g \left(h_{jt-1} - \ln \left(1 - \frac{1}{\eta(p_{jt-1}, z_{jt-1})} \right), r_{jt-1} \right) + \xi_{jt} + e_{jt}.$$
 (4)

As both p_{jt} and z_{jt} enter the equations lagged they are expected to be uncorrelated with the productivity innovation ξ_{jt} .⁶

Sample selection. A potential problem in the estimation of production functions is sample selection. If a firm's dynamic programming problem generates an optimal exit decision, based on the comparison between the sell-off value of the firm and its expected profitability in the future, then this decision is a function of current productivity. The simplest model, based on an exogenous Markov process, predicts that if an adversely enough shock to productivity is followed immediately by exit, then there will be a negative correlation between the shocks and the capital stocks of the firms that remain in the industry. Hence, sample selection will lead to biased estimates.

Accounting for R&D expenditures in the Markov process complicates matters. On the one hand, a firm now has an instrument to try to rectify an adverse shock and the optimal exit decision is likely to become more complicated. To begin with, there are many more relevant decisions such as beginning, continuing, or stopping innovative activities whilst remaining in the industry, and exiting in any of the different positions. On the other hand, a firm now is more likely to remain in the industry despite an adverse shock. Innovative activities often imply large sunk cost which will make the firm more reluctant to exit the industry or at least to exit it immediately. This will tend to alleviate the selection problem. At this stage we do not model any of these decisions. Instead, we simply explore whether there is a link between exit decisions and estimated productivity.

3.1 Identification

Our estimation equation (3) is a semiparametric, so-called partially-linear, model with the additional restriction that the inverse labor demand function $h(\cdot)$ is of known form. To see how this restriction aids identification, suppose to the contrary that $h(\cdot)$ were of unknown form. In this case, the composition of $h(\cdot)$ and $g(\cdot)$ is another function of unknown form. The fundamental condition for identification is that the variables in the parametric part of the model are not perfectly predictable (in the least squares sense) by the variables in the nonparametric part (Robinson 1988). In other words, there cannot be a functional relationship between the variables in the parametric and nonparametric parts (see Newey, Powell & Vella (1999) and also ACF for an application to the OP/LP framework). To see that this condition is violated, recall that $K_{jt} = (1 - \delta)K_{jt-1} + \exp(i(k_{jt-1}, \omega_{jt-1}))$ by the

⁶Note that this setting yields an estimate of the average elasticity of demand. The reason by which this is possible is the same by which correcting the Solow residual for imperfect competition allows for estimating margins and elasticities (see, e.g., Hall 1990).

law of motion and the policy function for investment in physical capital. But k_{jt-1} is one of the arguments of $h(\cdot)$ and ω_{jt-1} is by construction a function of all arguments of $h(\cdot)$, thereby making k_{jt} perfectly predictable from the variables in the nonparametric part.

Of course, in our setting the inverse labor demand function $h(\cdot)$ is of known form. The central question thus becomes whether k_{jt} is perfectly predictable from the value of $h(\cdot)$ (as opposed to its arguments) and r_{jt-1} . Since h_{jt-1} is identical to ω_{jt-1} , we have to ask if k_{jt-1} and hence k_{jt} (via $i(k_{jt-1}, \omega_{jt-1})$) can be inferred from r_{jt-1} . This may indeed be possible. Recall that $r_{jt-1} = r(k_{jt-1}, \omega_{jt-1})$ by the policy function for investment in knowledge. Hence, if its R&D expenditures happen to be increasing in the capital stock of the firm, then $r(\cdot)$ can be inverted to back out k_{jt-1} .

Fortunately, there is little reason to believe that this is the case. In fact, even under the fairly stringent assumptions in Buettner (2005), it is not clear that $r(\cdot)$ is invertible. Moreover, there is empirical evidence that invertibility may fail even for investment in physical capital (Greenstreet 2005) and it seems clear that R&D expenditures are even more fickle.

Even if $r(\cdot)$ happens to be an invertible function of ω_t , anything that shifts the costs of the investments in physical capital and knowledge over time guarantees identification. The price of equipment goods is likely to vary, for example, and the marginal cost of investment in knowledge depends greatly on the nature of the undertaken project. Using x_{jt} to denote these shifters, the policy functions become $i(k_{jt}, \omega_{jt}, x_{jt})$ and $r(k_{jt}, \omega_{jt}, x_{jt})$. Obviously, x_{jt} cannot be perfectly predicted from h_{jt-1} and r_{jt-1} . This breaks the functional relationship between $K_{jt} = (1 - \delta)K_{jt-1} + \exp(i(k_{jt-1}, \omega_{jt-1}, x_{jt}))$ and h_{jt-1} and r_{jt-1} .

3.2 Estimation

The problem can be cast in the nonlinear GMM framework

$$E\left[z_{jt}'(\xi_{jt}+e_{jt})\right] = E\left[z_{jt}'v_{jt}(\theta)\right] = 0,$$

where z_{jt} is a vector of instruments and we write the error term $v_{jt}(\cdot)$ as a function of the parameters θ to be estimated. The objective function is

$$\min_{\theta} \left[\frac{1}{N} \sum_{j} z'_{j} v_{j}(\theta) \right]' A_{N} \left[\frac{1}{N} \sum_{j} z'_{j} v_{j}(\theta) \right],$$

where z'_j and $v_j(\cdot)$ are $L \times T_j$ and $T_j \times 1$ vectors, respectively, with L being the number of instruments, T_j being the number of observations of firm j, and N the number

⁷Depending on the construction of the capital stock in the data, we may also be able to account for uncertainty in the impact of investment in physical capital. But once an error term is added to the law of motion for physical capital, k_{jt} can no longer be written as a function of h_{jt-1} and r_{jt-1} , and identification is restored.

of firms. We first use the weighting matrix $A_N = \left(\frac{1}{N}\sum_j z_j'z_j\right)^{-1}$ to obtain a consistent estimator of θ and then we compute the optimal estimator which uses weighting matrix $A_N = \left(\frac{1}{N}\sum_j z_j'v_j(\hat{\theta})v_j(\hat{\theta})'z_j\right)^{-1}$.

Time trend. Our preliminary estimates indicate that in some industries it is useful to add a time trend to the production function. In these cases the inverse input demand function $h(\cdot)$ contains, in addition to the constant, a term equal to $-\beta_t t$. One can say that there is an "observable" trend in the evolution of productivity that is treated separately from ω_{jt} but of course taken into account when substituting h_{jt} for ω_{jt} . Our goal is thus to estimate

$$y_{jt} = \beta_0 + \beta_t t + \beta_l l_{jt} + \beta_k k_{jt} + \beta_m m_{jt} + g(h_{jt-1}, r_{jt-1}) + \xi_{jt} + e_{jt}.$$

Series estimator. As suggested by Wooldridge (2004) when modeling an unknown function q(v, u) of two variables v and u we use a series estimator made of a "complete set" of polynomials of degree Q (see Judd 1998), i.e., all polynomials of the form $v^j u^k$, where j and k are nonnegative integers such that $j+k \leq Q$. When the unknown function $q(\cdot)$ has a single argument, we use a polynomial of degree Q to model it, i.e., $q(v) = \rho_0 + \rho_1 v + \ldots + \rho_Q v^Q$.

Taking into account that there are firms that do not perform R&D, the most general formulation is

$$y_{jt} = \beta_0 + \beta_t t + \beta_l l_{jt} + \beta_k k_{jt} + \beta_m m_{jt} + 1(R_{jt-1} = 0)g_0(h_{jt-1}) + 1(R_{jt-1} > 0)g_1(h_{jt-1}, r_{jt-1}) + \xi_{jt} + e_{jt}.$$
 (5)

This allows for a different unknown function when the firm adopts the corner solution of zero R&D expenditures and when it chooses positive R&D expenditures.

It is important to note that any constant that its arguments may have will be subsumed in the constant of the unknown function. Our specification is therefore

$$g_0(h_{jt-1}) = g_{00} + g_{01}(h_{jt-1} - \lambda_0),$$

$$g_1(h_{jt-1}, r_{jt-1}) = g_{10} + g_{11}(h_{jt-1} - \lambda_0, r_{jt-1}),$$

where in g_{00} and g_{10} we collapse the constants of the unknown functions $g_0(\cdot)$ and $g_1(\cdot)$ and the constant of h_{jt-1} . The constants g_{00} , g_{10} , and β_0 cannot be estimated separately. We thus estimate the constant for nonperformers g_{00} together with the constant of the production function β_0 and include a dummy for performers to measure the difference between constants $\beta_0 + g_{10} - (\beta_0 + g_{00}) = g_{10} - g_{00}$.

In the case of imperfect competition, where we have to nonparametrically estimate the absolute value of the elasticity of demand, we impose the theoretical restriction that $\eta(\cdot) > 1$ by using the specification $\eta(p_{jt-1}, z_{jt-1}) = 1 + \exp(q(p_{jt-1}, z_{jt-1}))$, where $q(\cdot)$ is modeled as described above.

Instrumental variables. As discussed before, k_{jt} is always a valid instrument because it is not correlated with ξ_{jt} because the latter is unpredictable when i_{t-1} is chosen. Labor and materials, however, are contemporaneously correlated with the innovation to productivity. The lags of these variables are valid instruments but when the demand for one of these inputs is being used to substitute for ω_{jt} it appears itself in h_{jt-1} . We can use the lag of the other input. Constant and trend are valid instruments. Therefore, we have four instruments to estimate the constant and the coefficients for the trend, capital, labor, and materials. This leaves us with the need for at least one more instrument. We use as instruments for the whole equation the complete set of polynomials of degree Q in the variables which enter h_{jt-1} , the powers up to degree Q of r_{jt-1} , and the interactions up to degree Q of the variables which enter h_{jt-1} and r_{jt-1} . The nonlinear functions of all exogenous variables included in these polynomials provide enough instruments.

We set Q=3 and use polynomials of order three. Hence, when there are four variables in the inverse input demand function $h(\cdot)$, say l_{jt-1} , k_{jt-1} , $w_{jt-1}-p_{jt-1}$, and $p_{Mjt-1}-p_{jt-1}$, we use as instruments the polynomials which result from the complete set of polynomials of degree 3 corresponding to the third power of h_{jt-1} (34 instruments), plus 3 terms which correspond to the powers of r_{jt-1} (3 instruments) and 12 interactions formed from the products $h_{jt-1}r_{jt-1}$, $h_{jt-1}^2r_{jt-1}$, and $h_{jt-1}r_{jt-1}^2$ (12 instruments). In fact when we enter p_{jt-1} linearly we use it detached from the other prices and we also need a dummy for the firms that perform R&D (2 more instruments). In addition, when there are enough degrees of freedom we instrument separately h_{jt-1} for nonperformers and h_{jt-1} and r_{jt-1} for performers by interacting the instruments with the dummy for performers. In addition, we have the exogenous variables included in the equation: constant, trend, current capital and lagged materials (4 instruments). This gives a total of 34+34+3+12+1+2+4=90 instruments. When we combine the demands for labor and materials, both equations have the same number of instruments (recall that not all are equal) and hence we have a total of 190 instruments.

Given these instruments, our estimator has exactly the form of the GMM version of Ai & Chen's (2003) sieve minimum distance estimator, a nonparametric least squares technique (see Newey & Powell 2003). This means that, if the conditional expectation function $g(\cdot)$ is specified in terms of variables which are correlated with the error term of the estimation equation, we still obtain a consistent and asymptotically normal estimator of the parameters by specifying the instrumenting polynomials in terms of exogenous conditioning variables.

3.3 Testing

The value of the GMM objective function for the optimal estimator, multiplied by N, has a limiting χ^2 distribution with L-P degrees of freedom, where L is the number of instruments

and P the number of parameters to be estimated.⁸ We use it as a test for overidentifying restrictions or validity of the moment conditions based on the instruments.

We test whether the model satisfies certain restrictions by computing the restricted estimator using the weighting matrix for the optimal estimator and then comparing the values of the properly scaled objective functions. The difference has a limiting χ^2 distribution with degrees of freedom equal to the number of restrictions.

We use this to test for misspecification. Recall that the production function parameters appear both in the production function and in the inverse labor demand function. If the inverse labor demand function is misspecified (e.g., because our implicit assumption of perfect competition is violated or because labor is not a variable input), then this causes β_l and β_k in the inverse labor demand function to diverge from their counterparts in the production function. By testing the null hypothesis that the structural parameters in the two parts of the model are equal, we may thus rule out that our model is misspecified.

We test for imperfect competition by adding an unknown function in the equilibrium price p_{jt-1} and the demand shifter z_{jt-1} to h_{jt-1} inside the conditional expectation function $g(\cdot)$ in equation (4). Under the null hypothesis of perfect competition p_{jt-1} and z_{jt-1} play no role.

In addition, we test whether R&D plays a role in the conditional expectation function and whether this function is separable. In the first case, we test whether all terms in r_{jt-1} can be excluded from the conditional expectation function $g_{11}(h_{jt-1} - \lambda_0, r_{jt-1})$ for performers plus the equality of the common part of the conditional expectation functions for performers and nonperformers, i.e., $g_{11}(h_{jt-1} - \lambda_0, r_{jt-1}) = g_{01}(h_{jt-1} - \lambda_0)$ for all r_{jt-1} . In the second case, we test whether $g_{11}(h_{jt-1} - \lambda_0, r_{jt-1})$ can be broken up into two additively separable functions $g_{11}(h_{jt-1} - \lambda_0)$ and $g_{12}(r_{jt-1})$.

We test whether the conditional expectation function is consistent with the knowledge capital model. Recall from Section 2 that the knowledge capital model implies that $g_1(h_{jt-1}, r_{jt-1}) = g_{10} + g_{11}(h_{jt-1} - \lambda_0, r_{jt-1})$ has a particular functional form:

$$\omega_{jt} = \omega_{jt-1} + \varepsilon \left(\frac{\exp(r_{jt-1})}{\exp(\omega_{jt-1}/\varepsilon)} - \delta \right) = h_{jt-1} + \varepsilon \left(\frac{\exp(r_{jt-1})}{\exp(h_{jt-1}/\varepsilon)} - \delta \right)$$

$$= \lambda_0 + (h_{jt-1} - \lambda_0) + \varepsilon \exp(-\lambda_0/\varepsilon) \frac{\exp(r_{jt-1})}{\exp((h_{jt-1} - \lambda_0)/\varepsilon)} - \varepsilon \delta$$

$$= (\lambda_0 - \varepsilon \delta) + (h_{jt-1} - \lambda_0) + \gamma \frac{\exp(r_{jt-1})}{\exp((h_{jt-1} - \lambda_0)/\varepsilon)}$$

$$= g_{10} + g_{11}(h_{jt-1} - \lambda_0, r_{jt-1}),$$

where γ is a parameter to be estimated.

We apply the Rivers & Vuong (2002) test for model selection among nonnested models. After multiplying the difference between the GMM objective functions by \sqrt{N} , the test

⁸Our baseline specification has 18 parameters: constant, trend, three production function coefficients, and thirteen coefficients in the series approximations.

statistic has an asymptotic normal distributed with variance

$$\begin{split} \sigma^2 &= 4 \Big[(\sum_j z_j' v_j(\hat{\theta}))' A_N (\sum_j z_j' v_j(\hat{\theta}) v_j(\hat{\theta})' z_j) A_N (\sum_j z_j' v_j(\hat{\theta})) \\ &+ (\sum_j z_j' v_j(\hat{\theta}^{KCM}))' A_N (\sum_j z_j' v_j(\hat{\theta}^{KCM}) v_j(\hat{\theta}^{KCM})' z_j) A_N (\sum_j z_j' v_j(\hat{\theta}^{KCM})) \\ &- 2 (\sum_j z_j' v_j(\hat{\theta}))' A_N (\sum_j z_j' v_j(\hat{\theta}) v_j(\hat{\theta}^{KCM})' z_j) A_N (\sum_j z_j' v_j(\hat{\theta}^{KCM})) \Big], \end{split}$$

where $\hat{\theta}$ and $\hat{\theta}^{KCM}$ are the unrestricted and restricted parameter estimates, respectively, the instruments in z_j are kept the same, and A_N is a common first-step weighting matrix.

4 Data

We use an unbalanced panel of Spanish manufacturing firms in nine industries during the 1990s. This broad coverage of industries is unusual, and it allows us to examine the link between R&D and productivity in a variety of settings that potentially differ in the importance of R&D.

Our data come from the ESEE (Encuesta Sobre Estrategias Empresariales) survey, a firm-level survey of Spanish manufacturing sponsored by the Ministry of Industry. The unit surveyed is the firm, not the plant or the establishment. At the beginning of this survey in 1990, 5% of firms with up to 200 workers were sampled randomly by industry and size strata. All firms with more than 200 workers were asked to participate, and 70% of all firms of this size chose to respond. Some firms vanish from the sample, due to both exit and attrition. The two reasons can be distinguished, and attrition remained within acceptable limits. In what follows we reserve the word exit to characterize shutdown by death or abandonment of activity. To preserve representativeness, samples of newly created firms were added to the initial sample every year.

We account for the survey design as follows. First, to compare the productivities of firms that perform R&D to those of firms that do not perform R&D we conduct separate tests on the subsamples of small and large firms. Second, to be able to interpret some of our descriptive statistics as aggregates that are representative for an industry as a whole, we replicate the subsample of small firms $\frac{70}{5} = 14$ times before merging it with the subsample of large firms. Details on industry and variable definitions can be found in Appendix A.

Given that our estimation procedure requires a lag of one year, we restrict the sample to firms with at least two years of data. The resulting sample covers a total of 1879 firms (before replication). Table 1 shows the number of observations and firms by industry. The samples are of moderate size. Firms tend to remain in the sample for short periods, ranging

⁹This data has been used elsewhere, e.g., in Gonzalez, Jaumandreu & Pazo (2005) to study the effect of subsidies to R&D and in Delgado, Farinas & Ruano (2002) to study the productivity of exporting firms.

from a minimum of two years to a maximum of 10 years between 1990 and 1999. The descriptive statistics in Table 1 are computed for the period from 1991 to 1999 and exclude the first observation for each firm.¹⁰ The small size of the samples is compensated for by the quality of the data, which seems to keep noise coming from errors in variables at relatively low levels.

Entry and exit reported in Table 1 refer to the incorporation of newly created firms and to exit. Newly created firms are a large share of the total number of firms, ranging from 15% to one third in the different industries. In each industry there is a significant proportion of exiting firms (from 5% to above 10% in a few cases).

Table 1 shows that the 1990s were a period of rapid output growth, coupled with stagnant or at best slightly increasing employment and intense investment in physical capital. The growth of prices, averaged from the growth of prices as reported individually by each firm, is moderate.

The R&D intensity of Spanish manufacturing firms is low by European standards, but R&D became increasingly important during the 1990s (see, e.g., European Commission 2001).¹¹ The manufacturing sector consists partly of transnational companies with production facilities in Spain and huge R&D expenditures and partly of small and medium-sized companies that invested heavily in R&D in a struggle to increase their competitiveness in a growing and already very open economy.

Government funded R&D in the form of subsidies and other forms of support amounts to 7.7% of firms' total R&D expenditures in the EU-15, 9.3% in the US, and 0.9% in Japan (European Commission 2004a). In Spain at most a small fraction of the firms that engaged in R&D received subsidies. The typical subsidy covers between 20% and 50% of R&D expenditures and its magnitude is inversely related to the size of the firm. Subsidies are used efficiently without crowding out private funds and even stimulate some projects. Their effect is mostly limited to the amount that they add to the project (see Gonzalez et al. 2005). This suggests that R&D expenditures irrespective of their origin are the relevant variable for explaining productivity.

Table 1 reveals that the nine industries are rather different when it comes to innovative activities of firms. This can be seen along three dimensions: the share of firms that perform R&D, the degree of persistence in performing R&D over time, and R&D intensity among performers defined as the ratio of R&D expenditures to output.

Three industries are highly active: Chemical products (3), agricultural and industrial machinery (4), and transport equipment (6). The share of firms that perform R&D during at least one year in the sample period is two thirds, with slightly more than 40% of stable performers that engage in R&D in all years and slightly more than 20% of occasional

¹⁰Since R&D expenditures appear lagged in our estimation equation (3), we report them for the period 1990 to 1998.

 $^{^{11}\}text{R\&D}$ intensities for manufacturing firms are 2.1% in France, 2.6% in Germany, and 2.2% in the UK as compared to 0.6% in Spain (European Commission 2004b).

performers that engage in R&D in some (but not all) years. Dividing the share of stable performers by the combined share of stable and occasional performers yields the conditional share of stable performers and gives an indication of the persistence in performing R&D over time. With about 65% the degree of persistence is is very high. Finally, the average R&D intensity among performers ranges from 2.2% to 2.7%.

Four industries are in an intermediate position: Metals and metal products (1), non-metallic minerals (2), food, drink and tobacco (7), and textile, leather and shoes (8). The share of performers is lower than one half, but it is near one half in the first two industries. With a conditional share of stable performers of about 40% the degree of persistence tends to be lower. The average R&D intensity among performers is between 1.1% and 1.5% with a much lower value of 0.7% in industry 7.

Two industries, timber and furniture (9) and paper and printing products (10), exhibit low innovative activity. The first industry is weak in the share of performers (below 20%) and degree of persistence. In the second industry the degree of persistence is somewhat higher with a conditional share of stable performers of 46% but the share of performers remains below 30%. The average R&D intensity is 1.4% in both industries.

This heterogeneity in the three dimensions of innovative activities makes it difficult to fit a single model to explain the impact of R&D on productivity. In addition, the standard deviation of R&D intensity is of substantial magnitude in the nine industries. This suggests that that heterogeneity across firms within industries is important, partly because firms engage in R&D to various degrees and partly because the level of aggregation used in defining these industries encompasses many different specific innovative activities.

5 Estimation results

We first present our estimates of the production function and the Markov process that governs the evolution of productivity and test the linearity and certainty assumptions of the knowledge capital model. Next we turn to the link between R&D and productivity. In order to assess the role of R&D in determining the differences in productivity across firms and the evolution of firm-level productivity over time, we examine three aspects of this link in more detail: productivity levels and growth and the rate of return to R&D.

5.1 Production function and Markov process

Table 2 summarizes different production function estimates. The first three columns report the coefficients estimated from OLS regressions of the log of output on the logs of inputs. The coefficients are reasonable as usual when running OLS on logs (but not when running OLS on first-differences of logs), and returns to scale are remarkably close to constancy. The share of capital in value added, as given by the capital coefficient scaled by the sum of the labor and capital coefficients, is between the values of 0.15 and 0.35 as expected.

The next six columns of Table 2 report the coefficients estimated when we use the demand for labor to back out unobserved productivity. Specifying the law of motion of productivity to be an exogenous Markov process that does not depend on R&D expenditures yields the coefficients reported in columns four to six. Compared to the OLS regressions, the changes go in the direction that is expected from theory. The labor coefficients decrease considerably in all industries while the capital coefficients increase somewhat in 7 industries. The materials coefficients show no particular pattern. Changes are as expected not huge because we are comparing estimates in logs (as opposed to first-differences of logs) for both the exogenous and the controlled Markov process. All this matches the results in OP and LP.

Columns seven to nine show the coefficients obtained when specifying a controlled Markov process. Again, compared to the OLS regressions, the changes go in the expected direction. The labor coefficients decrease in 8 cases, the capital coefficients increase in 5 cases and are virtually the same in 2 more cases. In fact, changes from the exogenous to the controlled Markov process do not exhibit a distinct pattern. This leaves open the question whether it is possible to obtain consistent estimates of the parameters of the production function in the absence of data on R&D, although it is clear that omitting R&D expenditures from the Markov process substantially distorts the retrieved productivities (see Section 5.2 for details).

*** ADD COMPARISON TO ALTERNATIVE SPECIFICATIONS USING THE DEMAND FOR MATERIALS OR THE DEMANDS FOR BOTH LABOR AND MATERIALS TO BACK OUT UNOBSERVED PRODUCTIVITY (PUT TABLES IN APPENDIX). ***

*** ADD COMPARISON TO OP METHOD (PUT TABLES IN APPENDIX). ***

To check the validity of our estimates we have conducted a series of tests as reported in Table 3. We first test for overidentifying restrictions or validity of the moment conditions based on the instruments as described in Section 3.3. The test statistic is too high for the usual significance levels in only the case of industry 1. The other values indicate the validity of the moment conditions by a wide margin.

The data very clearly reject the assumption of a perfectly competitive environment. Our estimates of the average elasticity of demand are around 2. *** ALL THE ESTIMATES REPORTED IN THIS DRAFT OF THE PAPER ARE DONE ASSUMING PERFECT COMPETITION. PRELIMINARY ESTIMATES SHOW THAT BASIC RESULTS DO NOT CHANGE WITH IMPERFECT COMPETITION. ***

Recall that misspecification causes the parameters in the inverse labor demand function to diverge from their counterparts in the production function. Fortunately, while we must reject the null hypothesis of equality in industries 1, 7, and 10, in the remaining industries the test suggests by a wide margin that we may rule out that our model is misspecified.

Nonlinearity. We next turn to the conditional expectation function $g(\cdot)$ that describes the Markov process of unobserved productivity. We test for the role of R&D by comparing the controlled with the exogenous Markov process. The result is overwhelming: In all cases the constraints imposed by the model with the exogenous Markov process are clearly rejected. We also test whether the conditional expectation function $g_{11}(\cdot)$ for firms that perform R&D is separable in its arguments ω_{jt-1} and r_{jt-1} . The test statistics indicate that this is only the case in industries 1, 4, and perhaps 9. From hereon we impose separability on industry 4, where it slightly improves the estimates, but we keep nonseparability in industry 1, where separability does not seem to change anything. Given the limited number of firms that perform R&D in industries 9 and 10, we also impose separability in the interest of parsimony. The main result, however, is that the R&D process can hardly be considered separable. From the economic point of view this stresses that the impact of current R&D on future productivity depends crucially on current productivity, and that current and past investments in knowledge interact in a complex fashion.

Finally, we test whether the conditional expectation function is consistent with the knowledge capital model. Our estimates of the elasticity of output with respect to knowledge capital are between the perfectly reasonable values of 0.12 and 0.50 for the different industries. Nevertheless, the data very clearly reject the functional form restrictions implied by the knowledge capital model. This suggests that the linearity assumption in the accumulation and depreciation of knowledge that underlies the knowledge capital model may have to be relaxed in order to better assess the impact of the investment in knowledge on the productivity of firms.

Uncertainty. Once the model is estimated we can compute ω_{jt} , h_{jt} , and $g(\cdot)$ up to a constant. We can also obtain an estimate of ξ_{jt} up to a constant as the difference between the estimates of ω_{jt} and $g(\cdot)$. Recall that the productivity of firm j in period t is given by $\beta_t t + \omega_{jt} = \beta_t t + g(\omega_{jt-1}, r_{jt-1}) + \xi_{jt}$ with $\omega_{jt} = h_{jt}$. Using the notational convention that $\widehat{\omega}_{jt}$, \widehat{h}_{jt} , and $\widehat{g}(\cdot)$ represent the estimates up to a constant, we have

$$\widehat{\omega}_{jt} = \widehat{h}_{jt} = -\widehat{\beta}_t t + (1 - \widehat{\beta}_l - \widehat{\beta}_m)l_{jt} - \widehat{\beta}_k k_{jt} + (1 - \widehat{\beta}_m)(w_{jt} - p_{jt}) + \widehat{\beta}_m(p_{Mjt} - p_{jt})$$

and

$$\begin{split} \widehat{g}(\widehat{h}_{jt-1}, r_{jt-1}) &= 1(R_{jt-1} = 0)\widehat{g}_{01}(\widehat{h}_{jt-1}) \\ &+ 1(R_{jt-1} > 0)[\widehat{g}_{10} - \widehat{g}_{00}) + \widehat{g}_{11}(\widehat{h}_{jt-1}, r_{t-1})]. \end{split}$$

This implies that we can estimate $Var(\omega_{jt}), Var(g(\cdot))$ and $Var(\xi_{jt})$ as well as $Cov(g(\cdot), \xi_{jt})$ and the correlation coefficient $Corr(g(\cdot), \xi_{jt}) = Cov(g(\cdot), \xi_{jt}) / \sqrt{Var(g(\cdot))Var(\xi_{jt})}$. We can also estimate the random shocks e_{jt} and their variance $Var(e_{jt})$. When we combine multiple input demands, we compute the variances and covariances of ω_{jt} , $g(\cdot)$, and ξ_{jt} from

an average of the input-specific estimates.

The first column of Table 4 tells us the ratio of the variance of the random shock e_{jt} to the variance of unobserved productivity ω_{jt} . Despite differences among industries, the variances are quite similar in magnitude. This suggests that unobserved productivity is at least as important in explaining the data as the host of other factors that are embedded in the random shock. The next column gives the ratio of the variance of the productivity innovation ξ_{jt} to the variance of actual productivity ω_{jt} . The ratio shows that the unpredictable component accounts for a large part of attained productivity, between 20% and 50%, thereby casting doubt on the certainty assumption of the knowledge capital model. Interestingly enough, a high degree of uncertainty in the R&D process seems to be characteristic for both some of the most and some of the least R&D intensive industries.

The last column of Table 4 gives the correlation between expected productivity and the innovation to productivity. Since the correlation is low, this further validates the model. Only in industries 1 and 8 the correlation tends to be a bit higher, as could have already been guessed from the specification test. *** ALSO CHECK WHETHER l_t AND m_t ARE UNCORRELATED WITH ξ_{t+1} , SEE OP P. 1284 AND LP P. 326. ***

5.2 Productivity levels

To describe differences in expected productivity between firms that perform R&D and firms that do not perform R&D, we employ kernels to estimate the density and the distribution functions associated with the subsamples of observations with R&D and without R&D. To be able to interpret these descriptive measures as representative aggregates, we proceed as described in Section 4. Figure 1 shows the density and distribution functions for performers (solid line) and nonperformers (dashed line) for each industry. In all industries but 4, 9, and 10, the distribution for performers is to the right of the distribution for nonperformers. This strongly suggests stochastic dominance. In contrast, in industries 4 and 10 the distribution functions openly cross: Attaining the highest levels seems more likely for the nonperformers than for the performers. In industry 9 the distribution for nonperformers dominates the one for performers.

Before formally comparing the means and variances of the distributions and the distributions themselves, we illustrate the impact of omitting R&D expenditures from the Markov process of unobserved productivity. We have added the so-obtained density and distribution functions to Figure 1 (dotted line). Comparing them to the density and distribution functions for a controlled Markov process reveals that the exogenous process takes a sort of average over firms with heterogeneous innovative activities and hence blurs remarkable differences in the impact of the investment in knowledge on the productivity of firms.

Mean and variance. Turning to the moments of the distributions, the difference in means is computed as

$$\widehat{\overline{g}}_{0} - \widehat{\overline{g}}_{1} = \frac{1}{NT_{0}} \sum_{j} \sum_{t} 1(r_{jt-1} = 0) \widehat{g}_{01}(\widehat{h}_{jt-1}) - \frac{1}{NT_{1}} \sum_{j} \sum_{t} 1(r_{jt-1} > 0) [(g_{10} - g_{00}) + g_{11}(\widehat{h}_{jt-1}, r_{t-1})],$$

where NT_0 and NT_1 are the size of the subsamples of observations without and with R&D, respectively. We compare the means using the test statistic

$$t = \frac{\widehat{\overline{g}}_0 - \widehat{\overline{g}}_1}{\sqrt{Var(g_{01})/(NT_0 - 1) + Var(g_{11})/(NT_1 - 1)}}$$

which follows a t distribution with $\min(NT_0, NT_1) - 1$ degrees of freedom and the variances using

$$F = \frac{Var(g_{01})}{Var(g_{11})}$$

which follows an F distribution with $NT_0 - 1$ and $NT_1 - 1$ degrees of freedom.

Column four of Table 5 reports the difference in means $\hat{g}_1 - \hat{g}_0$ (with the opposite sign of the test statistic for the sake of intuition) and the next four columns report the standard deviations and the test statistics along with their probability values separately for the subsamples of small and large firms. The difference in means is positive for firms of all sizes in all industries that exhibit medium or high innovative activity, with the striking exception of industry 4. The differences are sizable, with many values between 4% and 5% and up to 9%. They are often larger for the smaller firms. In the two industries that exhibit low innovative activity, however, one size group shows a lower mean of expected productivity than the other: The small firms in industry 9 and the large firms in industry 10. The formal statistical test duly rejects, at the usual significance levels, the hypothesis of a higher mean of expected productivity among performers than among nonperformers in these two cases and in both size groups in industry 4.

The hypothesis of greater variability for performers than for nonperformers is rejected in many cases, although there does not seem to be a recognizable pattern. As can be seen in columns 9 and 10 of Table 5, it is rejected for both size groups in industries 4, 6, 7, and 10, for small firms in industries 2, 3 and 9, and for large firms in industries 1 and 8.

Distribution. The above results suggest to compare the distributions themselves. We use a Kolmogorov-Smirnov test to compare the empirical distributions of two independent samples (see Barret & Donald (2003) and Delgado et al. (2002) for similar applications). Since this test requires that the observations in each sample are independent, we consider as the variable of interest the average of expected productivity for each firm, where for occasional performers we average only over the years with R&D (and discard the years

without R&D). This avoids dependent observations and sets the sample sizes equal to the number of nonperformers and performers, N_0 and N_1 , respectively.

Let $F_{N_0}(\cdot)$ and $G_{N_1}(\cdot)$ be the empirical cumulative distribution functions of nonperformers and performers, respectively. We apply the two-sided test of the hypothesis $F_{N_0}(\overline{g}) - G_{N_1}(\overline{g}) = 0$ for all \overline{g} , i.e., the distributions of expected productivity are equal, and the one-sided test of the hypothesis $F_{N_0}(\overline{g}) - G_{N_1}(\overline{g}) \leq 0$ for all \overline{g} , i.e., the distribution $G_{N_1}(\cdot)$ of expected productivity of performers stochastically dominates the distribution $F_{N_0}(\cdot)$ of expected productivity of nonperformers. The test statistics are

$$S^{1} = \sqrt{\frac{N_{0}N_{1}}{N_{0} + N_{1}}} \max_{g} \left\{ |F_{N_{0}}(\overline{g}) - G_{N_{1}}(\overline{g})| \right\}, \quad S^{2} = \sqrt{\frac{N_{0}N_{1}}{N_{0} + N_{1}}} \max_{g} \left\{ F_{N_{0}}(\overline{g}) - G_{N_{1}}(\overline{g}) \right\},$$

respectively, and the probability values can be computed using the limiting distributions $P(S^1 > c) = -2 \sum_{k=1}^{\infty} (-1)^k \exp(-2k^2c^2)$ and $P(S^2 > c) = \exp(-2c^2)$.

Because the test tends to be inconclusive when the number of firms is small, we limit it to cases in which we have at least 20 performers and 20 nonperformers. This allows us to carry out the tests for the small firms in 8 industries and for the large firms in industries 7 and 8. The results are reported in the last four columns of Table 5. Equality of distributions is rejected in six out of ten cases. Stochastic dominance can hardly be rejected anywhere with the exception of industry 4.

To further illustrate the consequences of omitting R&D expenditures from the Markov process of unobserved productivity, we have redone the above tests for the case of an exogenous Markov process. The results are striking: We can no longer reject the equality of the productivity distributions of performers and nonperformers in eight out of ten cases. This once more makes apparent that omitting R&D expenditures substantially distorts the retrieved productivities.

Conditional expectation function. To obtain a more complete picture of the conditional expectation function $g(\cdot)$ and its dependence on R&D expenditures, we average this function conditional on three sets of values for r_{jt-1} : When R&D expenditures are zero, when they are above zero but below the median value for performers, and when they are above the median value. That is, for all three sets of values for R&D expenditures we compute

$$E[g(\omega_{jt-1}, r_{jt-1}) | \omega_{jt-1}] = \int g(\omega_{jt-1}, r_{jt-1}) f_{r|\omega}(r_{jt-1} | \omega_{jt-1}) dr_{t-1},$$

where $f_{r|\omega}(r_{jt-1}|\omega_{jt-1})$ is the conditional density of r_{jt-1} given ω_{jt-1} in the sample. The graphs on the left of Figure 2 show in line with intuition that expected productivity is increasing in attained productivity for all three sets of values for R&D expenditures. In addition, with a few exceptions, this relationship shifts upwards with higher R&D expenditures. Sometimes, however, some or part of the dashed lines that represent the expected transitions from current to future productivity for performers lie below the solid line that

pertains to nonperformers. This is clearly the case in industries 4, 9, and 10 and once again mirrors the lack of stochastic dominance.

The graphs on the right of Figure 2 depict

$$E[g(\omega_{jt-1}, r_{jt-1})|r_{jt-1}] = \int g(\omega_{jt-1}, r_{jt-1}) f_{\omega|r}(\omega_{jt-1}|r_{jt-1}) d\omega_{t-1}.$$

They may be read as the marginal effect of R&D expenditures on expected productivity, once we average out over the values of attained productivity in the sample. Note that the horizontal axis is R&D expenditures in thousands of euros. The dotted line represents the density of R&D expenditure in the sample. The graphs show that higher R&D expenditures tend to be associated with higher expected productivity, even in the case of industry 4 for the bulk of the R&D expenditures. The exceptions here are just industries 9 and 10 with low innovative activity.

*** ADD COMPARISON TO $E[g(\omega_{jt-1},r_{jt-1})|\omega_{jt-1}]$ AND $E[g(\omega_{jt-1},r_{jt-1})|r_{jt-1}]$ IN KNOWLEDGE CAPITAL MODEL. ***

In sum, comparing expected productivity across firms that perform R&D and firms that do not perform R&D we find strong evidence of stochastic dominance in most industries. It remains to be explained why expected productivity appears eventually lower in some industries. One possible explanation is heterogeneity across firms within industries, i.e., stochastic dominance may hold if we were able to split these industries into more homogeneous innovative activities.

5.3 Productivity growth

We compute the expectation of productivity growth as

$$\beta_t + E(\omega_{jt} - \omega_{jt-1} | \omega_{jt-2}, r_{jt-1}, r_{jt-2})$$

$$= \beta_t + E[g(\omega_{jt-1}, r_{jt-1}) | \omega_{jt-2}, r_{jt-1}, r_{jt-2}] - g(\omega_{jt-2}, r_{jt-2})$$
(6)

because $E(\xi_{jt}|\omega_{jt-2},r_{jt-1},r_{jt-2})=E(\xi_{jt-1}|\omega_{jt-2},r_{jt-1},r_{jt-2})=0$. For firm j in period t this productivity growth is, given attained productivity and R&D expenditures in period t-2, a function of R&D expenditures r_{jt-1} and the innovation to productivity ξ_{jt-1} . By taking the expectation of $g(\omega_{jt-1},r_{jt-1})$ we average out over the realizations of these innovations. We estimate the expectation of productivity growth $\frac{1}{N}\sum_{j}\sum_{t}\frac{1}{T_{j}}[\widehat{g}(\widehat{h}_{jt-1},r_{jt-1})-\widehat{g}(\widehat{h}_{jt-2},r_{jt-2})]$. The first three columns of Table 6 report the results for the entire sample and for the subsamples of observations with and without R&D. In what follows we drop 2.5% of observations at each tail of the distribution to guard against outliers.

We also compute a weighted version to be able to interpret the expectation of productivity growth as representative for an industry as a whole. The weights $\mu_{jt} = Y_{jt} / \sum_j Y_{jt}$ are given by the share of output of each firm in each year. Total averages are then computed

as $\sum_{j} \sum_{t} \frac{1}{T_{j}} \mu_{jt} [\widehat{g}(\widehat{h}_{jt-1}, r_{jt-1}) - \widehat{g}(\widehat{h}_{jt-2}, r_{jt-2})]$. Columns four to six of Table 6 report the results. The last two columns show a decomposition into the contributions of observations with and without R&D.

Productivity growth, both unweighted and weighted, is higher on average for performers than for nonperformers in 5 industries, sometimes considerably so. The industries in which this relationship is reversed coincide again with industries 4, 9, and 10 to which we must now add industry 8. Given that the standard deviations are an order of magnitude larger than the averages, it is clear that there are considerable differences in productivity growth within firms that engage in R&D as well as within those that do not.

A comparison of unweighted and weighted productivity growth shows that there is no definite pattern in productivity growth by size group: The productivity of small firms grows more rapidly in some industries and less in others.

The last two columns are particularly important. The contribution to the expectation of weighted productivity growth of firms that perform R&D is estimated to explain between 70% and 85% of productivity growth in the industries with high innovative activity and between 50% and 65% in the industries with intermediate innovative activity (with the exception of industry 8). This is all the more remarkable since in these industries between 55% and 60% and between 30% and 35% of firms engage in R&D. While these firms manufacture between 70% and 75% of output in the industries with high innovative activity and between 45% and 55% in the industries with intermediate innovative activity, their contribution to productivity growth exceeds their share of output by between 5% and 15%. That is, firms that engage in R&D tend not only to be larger than those that do not but also to grow even larger over time. R&D expenditures are thus indeed a primary source of productivity growth.

5.4 Return to R&D

The growth in expected productivity in equation (6) can be decomposed (excluding the trend) as

$$[g(\omega_{jt-2}, r_{jt-1}) - g(\omega_{jt-2}, r_{jt-2})] + [E[g(\omega_{jt-1}, r_{jt-1}) | \omega_{jt-2}, r_{jt-1}, r_{jt-2}] - g(\omega_{jt-2}, r_{jt-1})].$$
(7)

The first term reflects the change in expected productivity that is attributable to R&D expenditures r_{jt-1} , the second the change that is attributable to the "inertia" derived from the previously attained productivity ω_{jt-2} and the productivity innovation ξ_{jt-1} . Table 7 reports unweighted averages of the growth in expected productivity (third column) as well as unweighted averages of the contributions of R&D expenditures (fourth column) and of inertia (fifth column). Column four shows that the contribution of R&D expenditures is positive except for industries 4 and 8. Moreover, the contribution of R&D expenditures represents either a large part of the growth in expected productivity or it may even exceed

it and only become smaller because of the negative contribution of inertia.

The term $[g(\omega_{jt-2}, r_{jt-1}) - g(\omega_{jt-2}, r_{jt-2})]$ in equation (7) is the increment to expected productivity in period t-1. Multiplying it by a measure of expected output, say Y_{jt-1} , gives the rent that the firm can expect from this increment. Dividing it further by R&D expenditures R_{jt-1} gives an estimate of the rate of return to R&D, or dollars obtained by spending one dollar on R&D.

Columns six and seven of Table 7 report weighted averages of the rates of return to R&D computed in this way, where the weights $\mu_{jt} = R_{jt}/\sum_j R_{jt}$ are given by the share of R&D expenditures of each firm in each year. Our rates of return to R&D are well within the range of estimates in the literature (see, e.g., Nadiri (1993) or Griliches (2000)). In a few cases, however, they become close to zero and in one case decidedly negative (industry 4). Exploring the reason for these abnormal values, it became clear that there are important differences in the rates of return to R&D belonging to observations that switch from zero R&D to positive R&D or vice versa. When there is a switch we have an extra benefit in some (but not all) industries or an extra cost in industries 4 and 8. Our model thus makes apparent the particular properties of the corner solution of zero R&D expenditures. Extra positive returns could be linked to the sunk costs implied by starting or re-starting innovative activities, but the interpretation of the extra negative returns is more challenging.

*** ADD COMPARISON OF RATES OF RETURN TO R&D AND TO INVEST-MENT IN PHYSICAL CAPITAL. THE DIFFERENCE IS AN ECONOMIC MEASURE OF MAGNITUDE OF THE UNCERTAINTY IN THE R&D PROCESS. ***

*** DISCUSS PERSISTENCE OF PRODUCTIVITY. ***

To facilitate the comparison with the existing literature, we have estimated the simplest version of the knowledge capital model as given in equation (1) by regressing the first-difference of the log of output on the first-differences of the logs of inputs and the ratio R_{jt-1}/Y_{jt-1} of R&D expenditures to output. The estimated coefficient of this ratio can be interpreted as the rate of return to knowledge capital.¹²

As usual when running OLS on first-differences of logs, returns to scale and capital coefficients tend to be low. The coefficients on the ratio of R&D expenditures to output are, however, sensible for most of the industries. The last column of Table 7 presents the rates of return estimated from the knowledge capital model. When these rates are compared with the rates estimated from our model the differences are not overwhelming with the exception, perhaps, of industries 4 and 8. The question is then whether and why our rates of return to R&D should be considered more reliable and whether this justifies the extra effort of pursuing the more structural approach.

One difference is that the knowledge capital model yields an average rate of return to

¹²Recall that ε is the elasticity of output with respect to knowledge capital. Since $\varepsilon \Delta c_{jt} = \frac{\partial Y}{\partial C} \frac{C_{jt-1}}{Y_{jt-1}} \Delta c_{jt} \simeq \frac{\partial Y}{\partial C} \frac{\Delta C_{jt}}{Y_{jt-1}}$ and R_{jt-1} approximates ΔC_{jt} (by the law of motion for knowledge capital), the estimated coefficient is $\frac{\partial Y}{\partial C}$. Since spending one dollar on R&D adds one unit of knowledge capital $\frac{\partial Y}{\partial C}$ is, in turn, equal to $\frac{\partial Y}{\partial R}$ or the rate of return to R&D.

R&D as the estimated coefficient on the firm- and year-specific ratio of R&D expenditures to output. The equivalent measure in our model is a weighted average of individual rates that include all the information contained in the firm- and year-specific values of the conditional expectation function $g(\cdot)$. This reveals a more fundamental difference. Our specification is able to handle and describe heterogeneity. We model at the same time performers and nonperformers, and different amounts of R&D expenditures. We obtain the distribution of productivities in its entirety. This allows us to compute different rates of return to R&D for different firms or different groups of firms.

In addition, our individual rates are computed from more reliable coefficient estimates than what the simplest knowledge capital model provides because our estimator takes into account the possibility of endogeneity bias in assessing the role of R&D. Because our model is structural we are more confident in the causality of the estimated relationship between expected productivity, current productivity, and R&D expenditures. The drawback of our approach is that it depends on the informational and timing assumptions that we make. These assumptions, however, appear to be broadly accepted in the literature following OP.

In sum, our approach does not contradict the knowledge capital model. On the contrary, as we have shown in Section 2, a simple extension of the knowledge capital model that allows the accumulation of improvements to productivity to be subjected to shocks is a special case of our model (albeit one that is rejected by the data). Our model is richer, in particular with regard to the absence of functional form restrictions and the treatment of heterogeneity. Yet, it is simple enough to apply.

6 Concluding remarks

In this paper we develop a simple estimator for production functions. The basic idea is to exploit the fact that decisions on variable inputs such as labor and materials are based on current productivity. This results in input demands that are invertible functions and thus can be used to control for unobserved productivity in the estimation. Moreover, the parametric specification of the production function implies a known form for these functions. This renders identification and estimation more tractable. As a result, we are able to accommodate a controlled Markov process, thereby capturing the impact of R&D on the evolution of productivity.

We illustrate our approach to production function estimation on an unbalanced panel of more than 1800 Spanish manufacturing firms in nine industries during the 1990s. We obtain sensible parameters estimates. Our estimator thus appears to work well. Yet, it is simple to use.

A number of interesting findings emerge. To begin with, the R&D process must be treated as inherently uncertain. We estimate that, depending on the industry, between 20% and 50% of the variance in actual productivity is explained by productivity innovations that

cannot be predicted when R&D expenditures are carried out.

Despite this uncertainty, the distribution of expected productivity of firms that perform R&D tends to stochastically dominate the distribution of expected productivity of firms that do not perform R&D. That is, while the distributions of productivities are overlapping, performers have a higher probability of achieving any particular value of productivity than nonperformers. There is an apparent tendency for the mean of the distribution of expected productivity to be higher for performers than for nonperformers. We estimate this difference in expected productivity to be around 5% in most cases and up to 9% in some cases. There is no clear rule when it comes to the variance. Thus, R&D may not inject additional uncertainty into the evolution of productivity over time.

Expected productivity is increasing in attained productivity and R&D expenditures. Specifically, we find that the transitions from current to future productivity tend to become in expectation more favorable with R&D expenditures. While the conditional expectation function takes a simple separable form in some cases, in most cases the impact of current R&D on future productivity depends crucially on current productivity. Hence, current and past investments in knowledge interact in a complex fashion.

We further test whether the relationship between current and future productivity and R&D expenditures is consistent with the knowledge capital model. While our estimates of the elasticity of output with respect to knowledge capital are between 0.12 and 0.50, the data very clearly reject the functional form restrictions implied by the knowledge capital model. This suggests that the linearity assumption in the accumulation and depreciation of knowledge that underlies the knowledge capital model may have to be relaxed in order to better assess the impact of the investment in knowledge on the productivity of firms.

R&D expenditures stimulate productivity growth. The growth in expected productivity corresponding to observations with R&D expenditures is often higher than the growth corresponding to observations without R&D. In addition, we estimate that the contribution of firms that perform R&D explains between 50% and 85% of productivity growth in the industries with intermediate or high innovative activity. Decomposing the expected productivity changes, we are able to separate the impact of carrying out investments in knowledge from that of inertia in the evolution of productivity. On average the contribution of R&D expenditures to productivity growth is positive and large in almost all industries.

Consequently, we find large average rates of return to R&D in most industries, although our estimates are within the range of the previous literature. Hidden behind these averages, however, is a substantial amount of heterogeneity across firms. Our approach allows us to obtain the distribution of productivities in its entirety. Hence, we are able to compute different rates of return for different firms in different years. In particular, we find that in many industries there is a large gap between the rates of return for starting and for continuing R&D. The rates of return for starting R&D are higher, possibly due to sunk costs.

Our method can be applied to other contexts, for example, to model and test for two types of technological progress in production functions: Hicks-neutral technological progress that shifts the production function in its entirety and labor-saving technological progress that shifts the ratio of labor to capital. Economists have been for a long time interested in disentangling these effects. In ongoing work we have begun to explore how this can be done by further exploiting the known form of the inverse input demand functions for labor and materials to recover two unobservables, one for Hicks-neutral and one for labor-saving technological progress.

Appendix A

Table 8 gives the equivalence between our grouping of industries and the manufacturing breakdown of ESEE. We exclude industry 5 because of data problems.

In what follows we define the variables.

- Capital. Capital at current replacement values is computed recursively from an initial estimate and the data on current investments in equipment goods (but not buildings or financial assets), updating the value of past stocks by means of a price index of investment in equipment goods, and using industry-specific estimates of the rates of depreciation. That is, $K_{jt} = (1 \delta) \frac{p_{It}}{p_{It-1}} K_{jt-1} + I_{jt-1}$, where capital and investment are in current values and p_{It} is the price index of investment. Real capital is obtained by deflating capital at current replacement values by the price index of investment.
- *Investment*. Value of current investments in equipment goods deflated by the price index of investment as needed.
- Labor. Number of workers multiplied by hours per worker (normal hours of work plus overtime minus lost hours per worker).
- *Materials*. Value of intermediate consumption deflated by the firm's price index of materials.
- Output. Value of produced goods and services computed as sales plus the variation of inventories deflated by the firm's price index of output.
- Price of investment. Equipment goods component of the index of industry prices computed and published by the Ministry of Industry.
- Wage. Hourly wage rate (total labor cost divided by effective total hours of work).
- Price of materials. Paasche-type price index computed starting from the percentage variations in the prices of purchased materials, energy, and services as reported by the firm.
- Price of output. Paasche-type price index computed starting from the percentage price changes that the firm reports to have made in the markets in which it operates.
- R&D expenditures. Total R&D expenditures including the cost of intramural R&D activities, payments for outside R&D contracts, and expenditures on imported technology (patent licenses and technical assistance).
- Market dynamism. Weighted index of the dynamism of the markets (slump, stability, and expansion) as reported by the firm for the markets in which it operates.

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Table 1: Descriptive statistics by industries, 1991-1999 (sample numbers and averages).

						Ŗ	Rates of growth	wth			Wit	With R&D	
Industry	Obs.	Firms	Entry (%)	Exit (%)	Output	Labor	Capital	Materials	Price	Obs. (%)	Stable (%)	Occas. (%)	R&D intens.
1. Metals and metal products	1235	289	88 (30.4)	17 (5.9)	0.050 (0.238)	0.010 (0.183)	0.086 (0.278)	0.038 (0.346)	0.012 (0.055)	420 (34.0)	63 (21.8)	72 (24.9)	0.0126 (0.0144)
2. Non-metallic minerals	029	140	20 (14.3)	$\frac{15}{(10.7)}$	0.039 (0.209)	0.002 (0.152)	0.065 (0.259)	0.040 (0.304)	0.011 (0.057)	226 (33.7)	$\frac{22}{(15.7)}$	44 (31.4)	0.0112 (0.0206)
3. Chemical products	1218	275	64 (23.3)	$15 \tag{5.5}$	0.068 (0.196)	0.007 (0.146)	0.093 (0.238)	0.054 (0.254)	0.007 (0.061)	672 (55.2)	$124 \tag{45.1}$	55 (20.0)	0.0268 (0.0353)
4. Agric. and ind. machinery	576	132	36 (27.3)	$6 \tag{4.5}$	0.059 (0.275)	0.010 (0.170)	0.078 (0.247)	0.046 (0.371)	0.013 (0.032)	322 (55.9)	52 (39.4)	35 (26.5)	0.0219 (0.0275)
6. Transport equipment	637	148	$\frac{39}{(26.4)}$	$10 \\ (6.8)$	0.087 (0.0354)	0.011 (0.207)	0.114 (0.255)	0.087 (0.431)	0.007 (0.037)	$361 \\ (56.7)$	62 (41.9)	$\frac{35}{(23.6)}$	0.0224 (0.0345)
7. Food, drink and tobacco	1408	304	47 (15.5)	22 (7.2)	0.025 (0.224)	-0.003 (0.186)	0.094 (0.271)	0.019 (0.305)	0.022 (0.065)	386 (27.4)	$\frac{56}{(18.4)}$	64 (21.1)	0.0071 (0.0281)
8. Textile, leather and shoes	1278	293	77 (26.3)	49 (16.7)	0.020 (0.233)	-0.007 (0.192)	0.059 (0.235)	0.012 (0.356)	0.016 (0.040)	378 (29.6)	$\frac{39}{(13.3)}$	66 (22.5)	0.0152 (0.0219)
9. Timber and furniture	569	138	52 (37.7)	$\frac{18}{(13.0)}$	0.038 (0.278)	0.014 (0.210)	0.077 (0.257)	0.029 (0.379)	0.020 (0.035)	66 (12.6)	7 (5.1)	$\frac{18}{(13.8)}$	0.0138 (0.0326)
10. Paper and printing products	s 665	160	42 (26.3)	10 (6.3)	0.035 (0.183)	-0.000 (0.140)	0.099 (0.303)	0.026 (0.265)	0.019 (0.089)	113 (17.0)	21 (13.1)	25 (13.8)	0.0143 (0.0250)

Table 2: Industry production function estimates (output, 1991-1999). Using the demand for labor to control for unobserved productivity.

	4	No $control^2$	7	Exogeno	${\rm Exogenous~Markov~process}^3$	$^{\circ}$ process ³	proces	$\begin{array}{c} {\rm Markov} \\ {\rm process~including~R\&D^{3,4}} \end{array}$	$ m R\&D^{3,4}$
${\rm Industry}^1$	1	k	m	1	k	m	1	k	m
1. Metals and metal products	0.251 (0.022)	0.109 (0.013)	0.643 (0.020)	0.168 (0.031)	0.111 (0.019)	0.672 (0.014)	0.118 (0.019)	0.107 (0.012)	0.693 (0.009)
2. Non-metallic minerals	0.277 (0.032)	0.091 (0.020)	0.662 (0.028)	0.182 (0.039)	0.152 (0.024)	0.651 (0.016)	0.146 (0.014)	0.154 (0.009)	0.646 (0.009)
3. Chemical products	0.239 (0.021)	0.060 (0.010)	0.730 (0.020)	0.161 (0.029)	0.116 (0.020)	0.725 (0.013)	0.129 (0.020)	0.104 (0.014)	0.730 (0.009)
4. Agric. and ind. machinery	0.284 (0.038)	0.051 (0.017)	0.671 (0.027)	0.227 (0.026)	0.088 (0.014)	0.647 (0.014)	0.322 (0.015)	0.050 (0.009)	0.648 (0.009)
6. Transport equipment	0.289 (0.033)	0.080 (0.023)	0.636 (0.046)	0.223 (0.030)	0.096 (0.018)	0.668 (0.016)	0.186 (0.016)	0.118 (0.009)	0.647
7. Food, drink and tobacco	0.173 (0.016)	0.095 (0.014)	0.740 (0.016)	0.154 (0.028)	0.077 (0.020)	0.757 (0.010)	0.155 (0.015)	0.082 (0.014)	0.746 (0.008)
8. Textile, leather and shoes	0.327 (0.024)	0.063 (0.010)	0.607 (0.019)	0.285 (0.031)	0.038 (0.021)	0.625 (0.014)	0.262 (0.020)	0.055 (0.015)	0.611 (0.010)
9. Timber and furniture	0.283 (0.029)	0.079 (0.019)	0.670 (0.029)	0.184 (0.019)	0.103 (0.012)	0.719 (0.012)	0.207 (0.025)	0.101 (0.014)	0.712 (0.014)
10. Paper and printing products	0.321 (0.029)	0.092 (0.016)	0.621 (0.025)	0.263 (0.023)	0.129 (0.011)	0.606 (0.017)	0.287 (0.024)	0.118 (0.015)	0.602 (0.018)

¹Estimates for industries 1,2,3,6 and 10 include a time trend.

²OLS of log of output on a constant, the log of the variables and (for the indicated industries) a time trend ³Reported coefficients are optimal nonlinear GMM estimates and standard errors are robust to heteroskedasticity and autocorrelation. ⁴Results for industries 4,9 and 10 are from the separable model.

Table 3: Industry production function estimates (output, 1991-1999). Markov process including R&D: testing the specification.

Industry	Overidentifying restrictions $\chi^2(df)$ p val.	ntifying tions p val.	Misspec te $\chi^2(3)$	Misspecification test $\chi^2(3)$ p val.	R&D test $\chi^2(10)$ p v	test p val.	Separability test $\chi^2(3)$ p va	bility t p val.	N(0,1)	Knowledge capital test p. val. ε^1	$ \begin{array}{c} \text{lge} \\ \text{est} \\ \varepsilon^{1}(std.dev) \end{array} $	$\chi^2(9)$	Imperfect competition test p. val. η^2 (st	$ \begin{array}{c} \operatorname{sct} \\ \eta^2 \left(std.dev. \right) \\ \end{array} $
1. Metals and metal products	106.19 (72)	0.005	7.04	0.071	38.21	0.001	3.71	0.295	-12.6	0.000	0.386 (0.122)	174.71	0.000	2.09 (0.12)
2. Non-metallic minerals	76.29 (72)	0.342	1.41	0.703	204.51	0.000	24.52	0.000	-12.6	0.000	0.115 (0.067)	938.36	0.000	$\frac{1.85}{(0.20)}$
3. Chemical products	77.13 (72)	0.318	2.93	0.403	81.06	0.000	9.88	0.020	-27.2	0.000	0.331 (0.113)	23.91	0.004	1.99 (0.03)
4. Agric. and ind. machinery	79.81 (64)	0.088	6.18	0.103	110.81	0.000	0.65	0.885	-10.3	0.000	0.329 (0.478)	821.59	0.000	1.95 (0.08)
6. Transport equipment	81.75 (72)	0.203	3.38	0.337	485.44	0.000	211.57	0.000	-10.8	0.000	0.286 (0.078)	767.79	0.000	1.93 (0.10)
7. Food, drink and tobacco	87.72 (73)	0.115	10.61	0.014	49.08	0.000	12.30	0.006	-19.5	0.000	0.297 (0.114)	32.66	0.000	2.15 (0.09)
8. Textile, leather and shoes	90.86 (73)	0.077	2.64	0.450	69.62	0.000	21.90	0.000	-13.2	0.000	0.495 (0.193)	131.84	0.000	$\frac{1.90}{(0.08)}$
9. Timber and furniture	31.78 (29)	0.330	2.73	0.436	230.39	0.000	7.54	0.057	-10.0	0.000	0.365 (0.402)	102.80	0.000	1.81 (0.10)
10. Paper and printing products 28.76 0.425 15.18 0.002 43.97 0.000 26.5 (28)	28.76 (28)	0.425	15.18	0.002	43.97	0.000	26.57	0.000	-13.5	0.000	0.123 (0.043)	61.59	0.000	1.95 (0.15)

¹ Estimate of the implicit elasticity with respect to knowledge capital and its standard deviation.
² Estimate of the average implicit elasticity and its standard deviation.

Table 4: Industry production function estimates (output, 1991-1999). Markov process including R&D: testing the specification.

Industry	$\frac{Var(e)}{Var(\omega)}$	$\frac{Var(\xi)}{Var(\omega)}$	$Corr(g, \xi)$
1. Metals and metal products	1.080	0.240	0.325
2. Non-metallic minerals	1.059	0.324	0.150
3. Chemical products	0.781	0.219	0.227
4. Agric. and ind. machinery	2.498	0.393	-0.096
6. Transport equipment	1.810	0.546	-0.027
7. Food, drink and tobacco	1.825	0.293	0.136
8. Textile, leather and shoes	1.228	0.226	0.457
9. Timber and furniture	2.046	0.475	0.109
10. Paper and printing products	0.836	0.419	0.163

Table 5: Comparing productivity levels without and with R&D, 1991-1999.

		${ m Firms~with}$	vith	Diff. of	Standard dev. ¹	d dev. ¹	Mean w is gr	Mean with R&D is greater	Var. wi	Var. with R&D is greater	Distrib	Distributions are equal	Nonnogorov-Simurnov tests- stributions Distribution w are equal R&D dominat	Distribution with R&D dominates
Industry	Size	No R&D	R&D	means ¹	No R&D	R&D	t	Prob.	F	Prob.	S_1	Prob.	S_2	Prob.
1. Metals and metal products	<pre> 200 > 200 > 200</pre>	143 11	71 64	0.045 0.038	$0.083 \\ 0.107$	$0.085 \\ 0.079$	-6.050 -3.455	1.000	0.943 1.835	0.691	1.817	0.003	0.388	0.740
2. Non-metallic minerals	<pre> 200 > 200 > 200</pre>	65	27 39	0.090 0.045	$0.119 \\ 0.071$	$0.105 \\ 0.070$	-6.771 -4.964	1.000	1.279 1.023	0.089	1.705	0.006	0.324	0.811
3. Chemical products	<pre> 200 > 200 > 200</pre>	91	81 98	0.047 0.033	0.087	$0.076 \\ 0.095$	-7.555 -2.276	1.000 0.987	1.305 1.056	0.010 0.374	1.673	0.007	0.144	0.959
4. Agric. and ind. machinery	<pre></pre>	39	56 31	-0.027	0.104 0.083	0.091 0.067	2.746 5.095	0.003	1.312 1.561	0.031	1.085	0.190	1.085	0.095
6. Transport equipment	<pre></pre>	37 14	32 65	$0.081 \\ 0.052$	0.090	$0.070 \\ 0.061$	-7.915 -6.135	1.000	1.648 1.578	0.005	2.236	0.000	0.000	1.000
7. Food, drink and tobacco	<pre></pre>	155 29	49	$0.013 \\ 0.024$	$0.092 \\ 0.082$	$0.074 \\ 0.060$	-1.631	0.947 1.000	1.558	0.003	0.960 1.012	0.315 0.258	0.308	$0.827 \\ 0.361$
8. Textile, leather and shoes	<pre></pre>	165 23	59 46	0.046 0.004	$0.094 \\ 0.122$	0.119 0.104	-4.929 -0.394	1.000 0.653	0.620 1.373	1.000	1.762 0.681	0.004	$0.127 \\ 0.596$	$0.968 \\ 0.492$
9. Timber and furniture	<pre></pre>	112	18	-0.036 0.004	0.082	$0.045 \\ 0.064$	4.128	0.000 0.581	3.347 0.371	0.000				
10. Paper and printing products	<pre> < 200</pre>	98	24 22	0.007	$0.104 \\ 0.126$	0.037 0.046	-0.962 1.345	0.830 0.092	8.121	0.000	1.251	0.088	1.023	0.123
1 Computed with all observations. 2 Computed with the distribution of firm's time means when the	s. n of firm's	s time mean	s when		compared samples have more than 20 firms each.	es have n	ore than	$20 \text{ firms } \epsilon$	ach.					

Table 6: Assessing the role of R&D in expected productivity growth, 1991-1999.

	Prod. grc	Prod. growth $(unweighted)^1$	$hted)^1$	P_1	Prod. growth (weighted) ²	$eighted)^2$
I	Total	R&D	No R&D	Total	R&D	No R&D
${\rm Industry}$	(Std.dev.)	(Std.dev.)	(Std.dev)		obs. (%contrib.)	obs. (%contrib.)
1. Metals and metal products	0.0126 (0.1340)	0.0183 (0.1435)	0.0112 (0.1315)	0.0181	0.0209 (64.9)	0.0145 (35.1)
2. Non-metallic minerals	0.0185 (0.1780)	0.0213 (0.1670)	0.0177 (0.1811)	0.0174	0.0171 (48.6)	0.0168 (51.4)
3. Chemical products	0.0183 (0.1488)	0.0232 (0.1517)	0.0157 (0.1471)	0.0212	0.0267 (85.0)	0.0109 (15.0)
4. Agric. and ind. machinery	0.0123 (0.1840)	0.0074 (0.1592)	$0.0164 \\ (0.2021)$	0.0135	0.0126 (70.5)	0.0181 (29.5)
6. Transport equipment	0.0233 (0.1475)	0.0347 (0.1688)	0.0155 (0.1303)	0.0257	0.0286 (85.6)	0.0160 (14.4)
7. Food, drink and tobacco	0.0081 (0.1425)	0.0102 (0.1376)	0.0078 (0.1432)	0.0081	0.0098 (52.2)	0.0062 (47.8)
8. Textile, leather and shoes	0.0120 (0.1434)	0.0030 (0.1528)	0.0145 (0.1406)	0.0120	0.0069 (27.6)	0.0151 (72.4)
9. Timber and furniture	0.0088 (0.1287)	0.0052 (0.1724)	0.0090 (0.1250)	0.0068	0.0008 (7.8)	0.0075 (92.2)
10. Paper and printing products	0.0141 (0.1829)	0.0136 (0.1515)	0.0142 (0.1862)	0.0127	0.0128 (28.1)	0.0131 (71.9)

¹ Unweighted means and standard deviations computed with 95% of the data.

² Yearly weighted means of 95% of the data. The reported numbers are simple averages of the means over the years. Notice that the total average can then lie slightly outside of the interval set by the partial averages.

Table 7: Decomposing productivity growth and computing rates of return to R&D, 1991-1999.

Industry	Total growth ¹ $R\&D$ obs. (from Table 5)	Trend	a^2 $g(\omega_{t-1}, r_{t-1})$ $-g(\omega_{t-2}, r_{t-2})$	b^2 $g(\omega_{t-2}, r_{t-1})$ $-g(\omega_{t-2}, r_{t-2})$	c^2 $g(\omega_{t-1}, r_{t-1})$ $-g(\omega_{t-2}, r_{t-1})$	$\frac{\mathrm{I}}{\left[\frac{g(\omega_{t-2},r_{t-1})-g(\omega_{t-1})}{R_{jt-1}}}\right]}$ All R&D observations ⁴	Rates of return ³ $\frac{g(\omega_{t-2}, r_{t-1}) - g(\omega_{t-2}, r_{t-2})}{R_{j_{t-1}}} \int_{j_{t-1}} X_{j_{t-1}}$ All R&D Continuing bservations ⁴ observations ⁴	Knowledge cap. model ⁵
1. Metals and metal products	0.0183	0.0078	0.0105	0.0036	0.0069	0.3496	0.2166	0.008 (0.330)
2. Non-metallic minerals	0.0213	0.0187	0.0026	0.0054	-0.0027	0.6372	-0.0291	0.515 (0.229)
3. Chemical products	0.0232	0.0164	0.0068	0.0018	0.0049	0.1620	0.1631	0.261 (0.116)
4. Agric. and ind. machinery	0.0074		0.0074	-0.0066	0.0140	-0.1243	0.0431	0.318 (0.284)
6. Transport equipment	0.0347	0.0250	0.0097	0.0137	-0.0040	0.4724	0.2034	0.316 (0.235)
7. Food, drink and tobacco	0.0102		0.0102	0.0020	0.0083	0.4132	0.0644	0.183 (0.238)
8. Textile, leather and shoes	0.0030		0.0030	-0.0069	0.0099	0.0844	0.3223	0.022 (0.213)
9. Timber and furniture	0.0052		0.0052	0.0105	-0.0053	9069.0	0.5614	-0.209 (1.034)
10. Paper and printing products	0.0136	0.0107	0.0029	0.0102	-0.0073	0.2406	-0.0011	0.025 (0.378)

Total productivity growth equals trend +a.

²Columns a to c are unweighted means of the 95% of the data; a = b + c.

⁵OLS using first differences of output, labor, capital and materials. The variable for knowledge capital is $\frac{R_{t-1}}{Y_{t-1}}$. ⁴Continuing observations are the R&D investment observations at t which follow an investment at t-1.

³Rates of return are averages of the individual rates weighted by the individual R&D expenditures over total R&D expenses.

Table 8: Industry definitions and equivalences

	Industry breakdown		ESEE clasiffication
1	Ferrous and non-ferrous metals and metal products	1+4	Ferrous and non-ferrous metals + Metal products
2	Non-metallic minerals	2	Non-metallic minerals
3	Chemical products	3+17	Chemical products + Rubber and plastic products
4	Agricultural and ind. machinery	5	Agricultural and ind. machinery
5	Office and data-processing machines and electrical goods	6+7	Office and data processing machin. + Electrical goods
6	Transport equipment	8+9	Motor vehicles + Other transport equipment
7	Food, drink and tobacco	10+11+12	Meats, meat preparation + Food products and tobacco + Beverages
8	Textile, leather and shoes	13+14	Textiles and clothing + Leather, leather and skin goods
9	Timber and furniture	15	Timber, wooden products
10	Paper and printing products	16	Paper and printing products

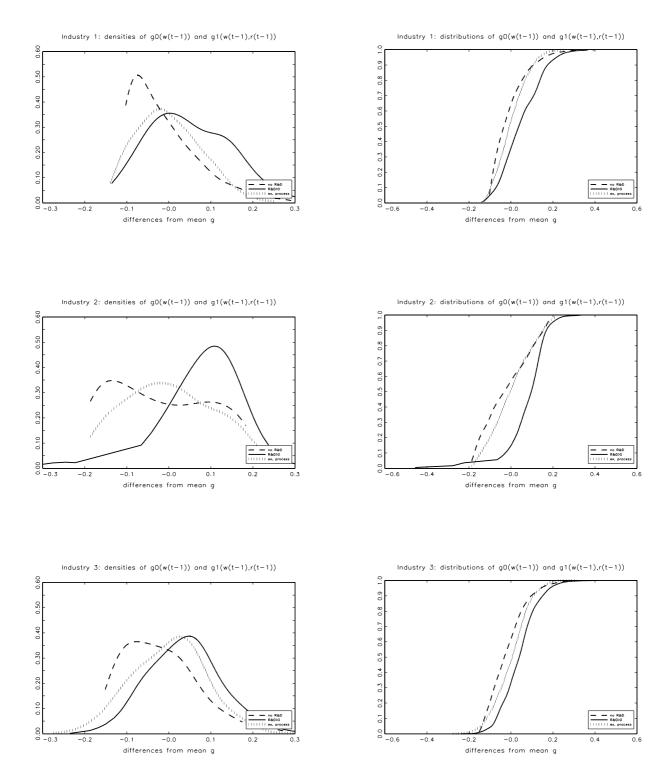
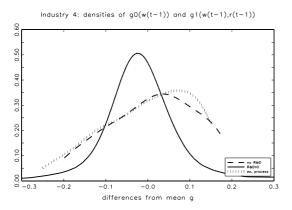
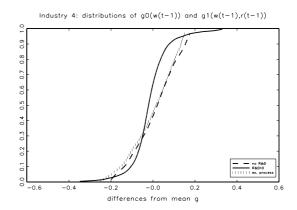
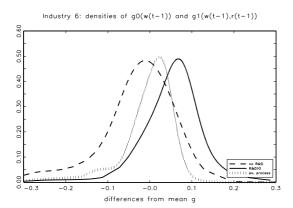
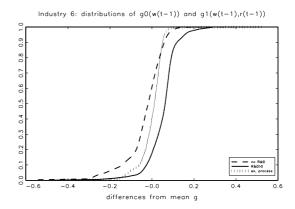


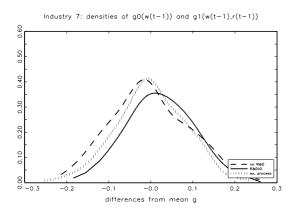
Figure 1: Expected productivities. Density (left panels) and distribution (right panels).











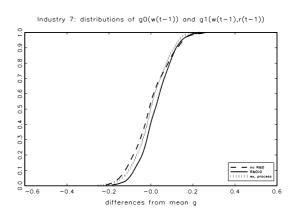
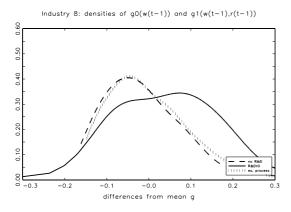
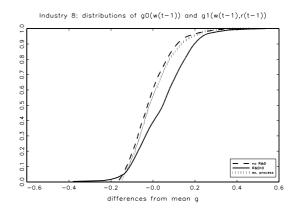
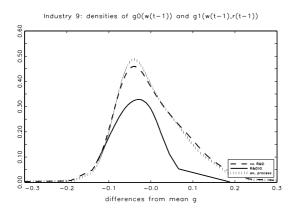
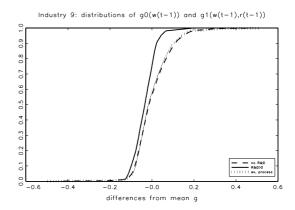


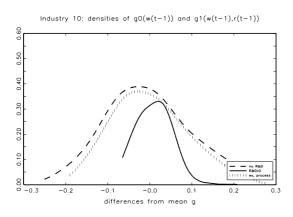
Figure 1: (cont'd) Expected productivities. Density (left panels) and distribution (right panels).











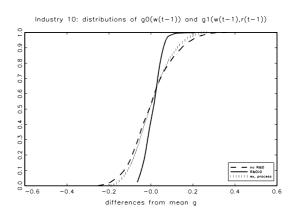


Figure 1: (cont'd) Expected productivities. Density (left panels) and distribution (right panels).

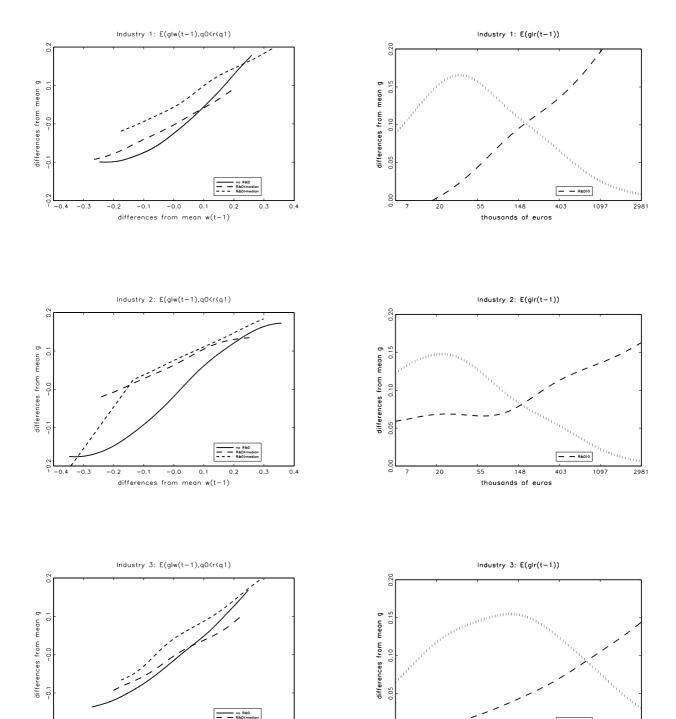


Figure 2: Conditional expectation function. $E\left[g(\omega_{jt-1},r_{jt-1})|\omega_{jt-1}\right]$ (left panels) and $E\left[g(\omega_{jt-1},r_{jt-1})|r_{jt-1}\right]$ (right panels).

differences from mean w(t-1)

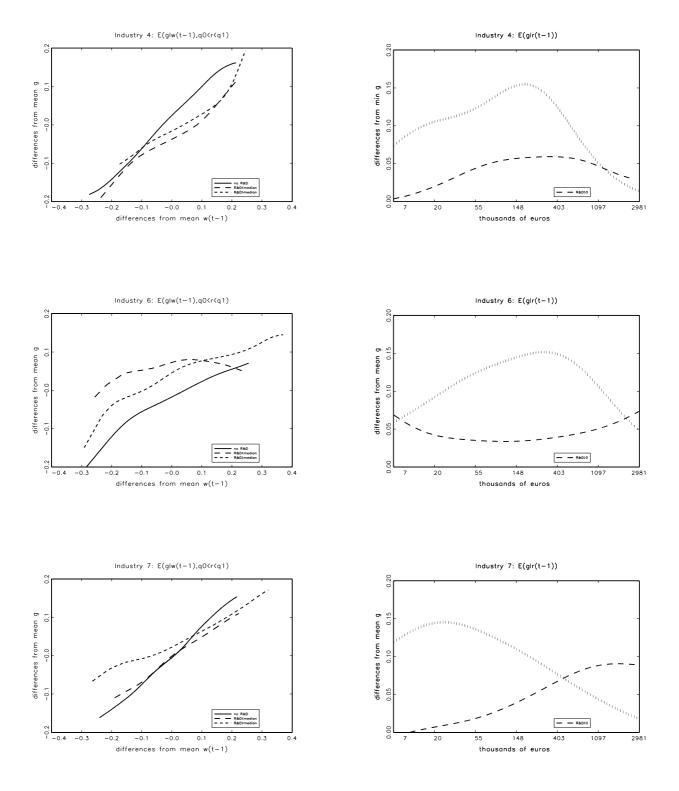


Figure 2: (cont'd) Conditional expectation function. $E\left[g(\omega_{jt-1},r_{jt-1})|\omega_{jt-1}\right]$ (left panels) and $E\left[g(\omega_{jt-1},r_{jt-1})|r_{jt-1}\right]$ (right panels).

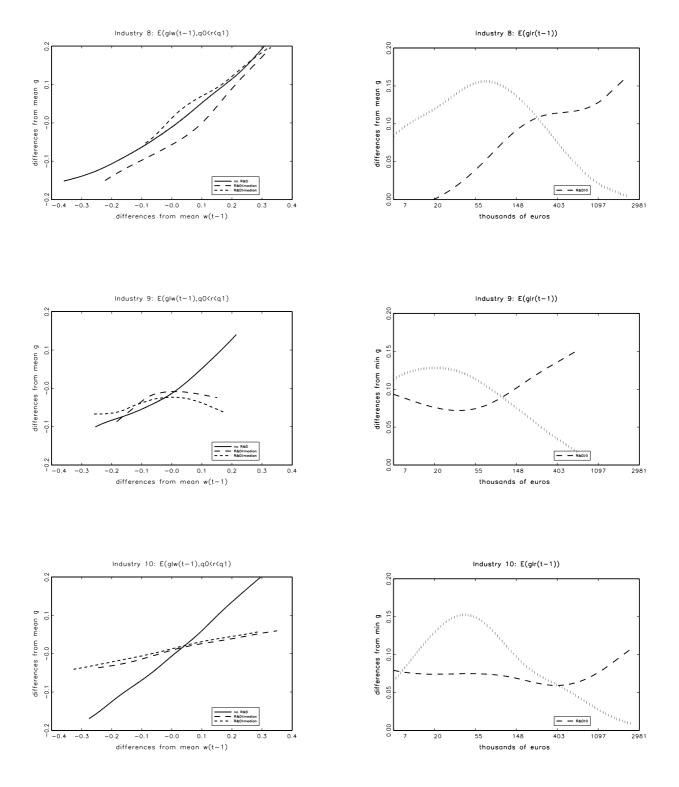


Figure 2: (cont'd) Conditional expectation function. $E\left[g(\omega_{jt-1},r_{jt-1})|\omega_{jt-1}\right]$ (left panels) and $E\left[g(\omega_{jt-1},r_{jt-1})|r_{jt-1}\right]$ (right panels).