

Trade, Prices, and the Exchange Rate with Heterogeneous Producers and Endogenous Markups

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Abstract

We develop a model of trade with monopolistic competition, heterogeneous firms, and endogenous markups, to study the implications of firm reallocations and average productivity changes on exchange-rate pass-through to import prices and trade flows. We obtain that pass-through elasticities are strictly decreasing with firm productivity. For an individual exporting firm, the exchange rate pass-through to the import price (in the importer's currency) reflects two opposite forces: an industry-wide competition effect, driven by the response of the productivity cutoff rule; and a firm-specific effect, driven by the relative position of a firm with respect to the cutoff point. Both effects produce asymmetric responses of import prices (at the firm level) to appreciations and depreciations. With respect to aggregate prices, we find that due to firm reallocations, average import prices and trade flows are disconnected. That is, average import prices, which are directly affected by changes in the extensive margin of trade, are poorly suited to derive conclusions about the expenditure-switching effects of exchange rate changes. We then extend the model to a general equilibrium New Open Economy Macroeconomics setup with sticky wages. The general equilibrium model replicates the results of the partial equilibrium model for an endogenous exchange rate change coming from an unexpected monetary shock. However, the GE model is solved using a first-order linearization, so it loses the second order effects (reflected in asymmetric responses) of the partial equilibrium model.

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1 Introduction

Recent trade and open economy models have been enriched with the incorporation of firm heterogeneity. In particular, the model by Melitz (2003) provided a very tractable approach to deal with this issue. In this line, many papers have focused on the impacts of firm heterogeneity on trade flows, the so called intensive and extensive margins of trade.¹ Nevertheless, papers remain to be written about the implications of firm heterogeneity and firm reallocations for the impact of exchange rate changes on prices. This topic was indirectly suggested by Rudiger Dornbusch in his 1987 AER paper, which reads: “In particular, it will be interesting to see how pricing decisions are affected by entry and relocation possibilities at an international level...”.

There are abundant exchange rate pass-through studies. An empirical regularity of these papers is a low level of exchange-rate pass-through to final consumer import prices. Goldberg and Knetter (1997) present a good survey of the evolution of empirical studies on exchange rates and prices up to the mid-1990s and present evidence in favor of pricing-to-market models. They conclude that destination-specific changes in markups are a very significant factor for the lack of response of prices to exchange rate changes.

More recently, a new wave of works on exchange rates and prices surged with the development of New Open Economy Macroeconomics (NOEM) models. These are general equilibrium models that incorporate microeconomic foundations, imperfect competition, nominal rigidities, and allow for welfare analysis.² Depending on the assumption on how a good’s price is preset for its sale in another country, producer currency pricing (PCP) or local currency pricing (LCP), NOEM models yield two different conclusions. Under PCP, the optimal monetary policy involves flexible exchange rates (see Obstfeld and Rogoff (2000)). On the other hand, under LCP, the conclusion is that the exchange rate must be fixed (see Devereux and Engel (2003)).³

As in the Mundell-Fleming model, the PCP assumption implies that changes in the exchange rate are reflected fully and immediately in import prices (a pass-through rate of 1), so immediate changes in aggregate demand are expected. This is known as the traditional expenditure-switching effect of changes in the exchange rate. To the contrary, under LCP the good’s price is preset in the currency of the destination country, which implies a zero response

¹See, for example, Chaney (2006) and Helpman, Melitz, and Rubinstein (2006).

²See Lane (2001) for an excellent survey of NOEM models.

³Local currency pricing is closely related to the concept of pricing-to-market, with the extra assumption that the producer is invoicing in the currency of the importer.

of import prices in terms of the importer's currency (a pass-through rate of 0). LCP implies that the expenditure-switching role of changes in the exchange rate is zero.

As mentioned above, empirical evidence based on consumer prices does not support the PCP assumption. Engel (2002) presents evidence of a low and slow pass-through of nominal exchange rate changes to consumer prices of imports in rich countries. These results led Devereux and Engel (2003) to conclude that LCP was a more realistic assumption, so that with no expenditure-switching effects of the exchange rate, there was no case for exchange rate flexibility.

Obstfeld (2002) mentions two problems from concluding about expenditure-switching effects from looking into consumer prices. The first is that there are differences between the final consumer price and the entry import price. The second, and most important for our case, is that firms' decisions are the relevant for the expenditure-switching effect of exchange rates. Also, Obstfeld and Rogoff (2000) argue in favor of high pass-through rates to border import prices and important expenditure-switching effects, but mention that this is hardly reflected in the consumer price due to factors like distribution services, advertising, other nontradable costs, and pricing policies of importing firms.

From there, recent studies have explored the importance of distribution services in deviations from the law of one price (LOOP) and in the degree of exchange rate pass-through. Supporting the importance of local distribution costs in the failure of PPP for consumer prices, Burstein, Neves, and Rebelo (2003) obtain that they account for 40% of retail prices in the US and for 60% in Argentina.⁴ Campa and Goldberg (2005) find average pass-through rates to import prices — before distribution costs — of 46 and 64 percent in the short and long-run, respectively, for OECD countries.⁵ They suggest that microeconomic phenomena have played a major role in pass-through elasticities and that the missing link — for low pass-through rates at the consumer level — can be found in the middle of the distribution chain.

The final question is: Should we really trust in prices, even at disaggregated levels or at the border price, to make conclusions about the expenditure-switching effects of exchange rate changes? A simple model of trade, heterogeneous firms, and exchange rates would say that the answer is no. Firm reallocations coming from a heterogeneous-firm model can generate a disconnection between import prices and expenditure-switching effects of exchange rate changes.

⁴See also Campa and Goldberg (2004), who get distribution margins going from 20 to 30 percent over producer costs for 13 OECD countries.

⁵For the US the estimates are 23 and 42 percent.

Consider exclusively import price indexes, in which most pass-through studies base their estimates. The simplest trade model with heterogeneous firms and exchange rates can show how firm reallocations can nullify the impact of exchange rates on aggregate prices. Consider the case of two countries, A and B, that trade varieties of a differentiated good with a constant elasticity of substitution between varieties. Suppose that initially country A imports four varieties from B. Suppose now that a depreciation of country A's currency drives two of country B's firms out of A. For simplicity purposes, assume that the initial prices for country B's varieties were 3 for two of them, and 5 for the other. If they have the same weight, the aggregate import price is given by 4. After a 30% depreciation, our CES assumption implies a full exchange rate pass-through for the surviving firms from B. Therefore, the new aggregate import price will be 3.9. That is, there is a decrease in the aggregate import price in A, even though each surviving firm is fully passing through to consumer prices the exchange rate change. Note that there are important expenditure switching effects. Country B reduced its presence in country A by half, but this was not reflected in the aggregate import price. In other words, the aggregate import price is directly affected by changes in the extensive margin of trade (the number of varieties traded), which is the main dynamic force driving trade flows in heterogeneous-firm models.

The previous example assumes full pass-through rates for exchange rate changes at the firm level. This is the natural result from the use of Dixit-Stiglitz preferences (CES utility function), that imply exogenous markups given by the elasticity of substitution. From an individual firm's perspective, markups need not be exogenous. Dornbusch (1987) identifies this issue and develops a partial equilibrium model with sticky wages, in which individual firms can adjust their markups according to market competition conditions.

In the spirit of that paper, we develop a partial equilibrium model with heterogeneous firms and endogenous markups in which exogenous exchange rate shocks will have an impact on the competition environment, and hence in the pass-through to prices. The model builds on Melitz and Ottaviano (2005) and keeps Dornbusch's assumption of sticky wages.

The core of our monopolistic competition model is a quadratic utility function that will generate a linear demand system with endogenous markups. Each firm sets its own markup, which depends on each firm's productivity and on the industry competition environment, which is captured by the average price and the number of competitors.

Due to our endogenous markups, we obtain that more productive firms have always lower

exchange-rate pass-through elasticities. Moreover, the partial equilibrium model identifies two opposite forces for the pass-through rate to the price of an individual exporting firm: a firm-specific effect, related to the firm's own productivity level; and an industry-wide effect, which will mirror the competition environment. Furthermore, the model derives a disconnection between the average import price and the expenditure-switching effect of exchange rate changes. Due to cross-country firm reallocations, the pass-through rate to the average import price is negative, even when individual firms' pass-through rates are greater than zero. Trade flows present substantial changes.

The partial model also derives asymmetric responses of import prices (at the firm level) for appreciations and depreciations of a currency. These second order effects are also a consequence of the asymmetric impacts of exchange rate changes on both: the competition environment, and the relative productivity of a firm with respect to its competitors. If asymmetries exist, the link between aggregate pass-through rates and expenditure-switching effects become even weaker.⁶

We then show how to incorporate this model into a general equilibrium NOEM model with sticky wages. The structure of the model follows the *Redux* model of Obstfeld and Rogoff (1995) and will be closely related to the model of Ghironi and Melitz (2005). However, our model differs from the Redux model in that we incorporate firm heterogeneity, a non-conventional quadratic consumption index, and we do not assume PPP, as heterogeneous firms can effectively discriminate between consumers at home and abroad. On the other hand, our basic departures from the model of Ghironi and Melitz (2005), which is the first one to incorporate heterogeneous producers in a dynamic general equilibrium setting, are that our model assumes perfect-foresight, with nominal rigidities, and a quadratic consumption index, while theirs is stochastic, with flexible prices (real), and a CES consumption index.

In our NOEM model, exchange rate changes are derived endogenously from an unexpected and permanent monetary shock. Wages are set a period in advance and the economy reaches its new steady state one period after the shock. The short-run results for the general equilibrium model are the same as in the partial equilibrium model. However, due to the first-order

⁶Asymmetric responses of import prices have been explored before in the literature. Marston (1990) suggests an asymmetric behavior of pass-through rates driven by market share considerations. When exporters face a depreciation of the importer's currency, they are willing to absorb an important amount of the shock in order to avoid a decrease in their market share (by increasing their prices in the importer's currency). If an appreciation occurs, they decrease their prices (in the importer's currency) faster to capture a bigger part of the market. He finds this kind of asymmetry for 5 out of 17 Japanese products. Froot and Klemperer (1989) also highlight the importance of market share for exchange rate pass-through.

linearization realized to solve the model, the general equilibrium model loses the second order effects. That is, it exhibits symmetric pass-through rates for appreciations and depreciations.

The paper is organized as follows. Section 2 presents the partial equilibrium model with heterogeneous firms and endogenous markups. Section 3 shows the implications of the partial model for exchange-rate pass-through at the firm and aggregate levels. It also analyzes the connection between trade flows and aggregate prices. In Section 4, we embody the heterogeneous-firm model in Section 2 into a full blown general equilibrium NOEM model with sticky wages. Section 5 explores the implications of the extended model for pass-through and trade flows. Finally, Section 6 presents our conclusions.

2 A Partial Equilibrium Model with Heterogeneous Firms and Endogenous Markups

In this Section we present a partial equilibrium model of heterogenous firms and endogenous markups. We develop a simple extension of the model by Melitz and Ottaviano (2005) to allow for exchange-rate pass-through analysis.

There are two countries, Home and Foreign, each of which is inhabited by atomistic households on the interval $[0, 1]$. Each household provides labor to each of the two producing sectors in a country: a homogeneous-good sector and a differentiated-good sector. The homogeneous good is nontradable and is produced under perfect competition. On the other hand, each variety of the differentiated good is tradable and produced under monopolistic competition. Moreover, firms in the latter sector are heterogenous with respect to their relative productivity levels.

We begin by specifying preferences and obtaining the demand, then we describe the production sector, and finally we solve the model. We use a star (*) to denote Foreign variables. However, in some parts of this Section we only refer to the Home country, as analogous expressions will hold for the Foreign country.

2.1 Preferences and Demand

The utility function for a representative Home household j , C^j , is given by

$$C^j = c_N^j + \alpha \int_{\omega \in \Omega} c_T^j(\omega) d\omega - \frac{\gamma}{2} \int_{\omega \in \Omega} c_T^j(\omega)^2 d\omega - \frac{\eta}{2} \left(\int_{\omega \in \Omega} c_T^j(\omega) d\omega \right)^2 \quad (1)$$

where c_N^j is the household j 's consumption of the nontraded good and $c_T^j(\omega)$ is its consumption of the differentiated variety $\omega \in \Omega$, where Ω is the continuum set of differentiated varieties sold at Home. The varieties can be Home or Foreign-produced. Parameters α and η stand for the level of substitution between the nontraded good and the different varieties, while γ represents the degree of product differentiation between the varieties.

This quadratic quasi-linear utility function was introduced by Ottaviano, Tabuchi, and Thisse (2002) and later incorporated by Melitz and Ottaviano (2005) in a heterogenous-firm model. It differs from the conventional constant-elasticity-of-substitution (CES) index in that, while exhibiting love-for-variety, it also allows us to derive endogenous firms' markups, which will depend on the market's level of competition. The endogeneity of markups is a consequence of the linear demand system associated with the quadratic function.

From C^j , we get that household j 's demand for each variety ω of the differentiated good is

$$c_T^j(\omega) = \frac{\alpha}{\gamma} - \frac{1}{\gamma} \frac{p(\omega)}{\rho} - \frac{\eta}{\gamma} \frac{N}{\gamma + \eta N} \left(\alpha - \frac{\bar{p}}{\rho} \right)$$

where $p(\omega)$ is the Home currency price of variety ω , ρ is the Home price of the nontraded good, N is the mass of varieties sold at Home, and $\bar{p} = \frac{1}{N} \int_{\omega \in \Omega} p(\omega) d\omega$ is the unweighted average price for the differentiated good.

In the same way, the demand for the nontraded homogeneous good is given by

$$c_N^j = \frac{1}{\rho} \left[I^j - N \left(\frac{\alpha - \frac{\bar{p}}{\rho}}{\gamma + \eta N} \bar{p} - \frac{\sigma^2}{\gamma \rho} \right) \right]$$

where I^j is the total expenditure in consumption by household j , and $\sigma^2 = \frac{1}{N} \int_{\omega \in \Omega} (p(\omega) - \bar{p})^2 d\omega$ is the variance of the prices in the differentiated-good sector.

Plugging the previous two expressions into (1), we can write the indirect utility function as

$$C^j = \frac{I^j}{\rho} + \frac{N}{2} \left[\frac{\left(\alpha - \frac{\bar{p}}{\rho} \right)^2}{\gamma + \eta N} + \frac{\sigma^2}{\gamma \rho^2} \right]. \quad (2)$$

2.2 Production and Pricing

The nontraded good is competitively supplied and employs only labor for production. The production function for the nontraded good is

$$y_N = L,$$

where L is a labor index whose nominal price is given by W . Given that the good is supplied under perfect competition, it follows that $\rho = W$.

As in the nontraded good sector, the differentiated-good industry uses labor as the only factor of production. Producers are heterogeneous and they will know their relative productivity, φ , only after entry. The production function for a Home firm with relative productivity φ is given by

$$y(\varphi) = Z\varphi L$$

where Z is aggregate labor productivity and L is defined as above.

There is an entry cost in the differentiated-good sector. This is a sunk cost that accounts for the research and investment necessary to start producing a variety. Let f_E denote the entry cost in units of effective labor. In nominal terms, this cost is given by $\frac{f_E W}{Z}$. After paying the entry cost and realizing its relative productivity, φ , a firm will produce as long as it can set a price no less than its constant marginal cost, $\frac{W}{Z\varphi}$.

Firms draw their productivity from a common distribution $G(\varphi)$. As usual in these models, we assume that productivity is Pareto distributed with support $[\varphi_{\min}, \infty)$ and shape parameter k . That is, $G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^k$. Higher k means lower heterogeneity, with firms' productivities clustered near the lower bound.

Home and Foreign markets are segmented. Given constant marginal costs, a producer will independently decide whether or not to sell in each country. The only cost of exporting is an iceberg cost. In particular, for each unit of the good that reaches the Foreign market, a Home exporter must ship $\tau > 1$ units.

Let $p_D(\varphi)$ and $p_X(\varphi)$ denote, respectively, the nominal domestic and export prices of a Home firm with relative productivity φ . These prices are set in the currency of the destination country. Also, let \mathcal{E} be the nominal exchange rate, measured as the Home-currency price of the Foreign currency. Given the demand for each variety of the differentiated good obtained in Section 2.1,

and making use of the fact that $\rho = W$, the profit maximizing prices for Home and Foreign firms are

$$\begin{aligned} p_D(\varphi) &= \frac{1}{2} \left[\hat{p} + \frac{W}{Z\varphi} \right] & p_X(\varphi) &= \frac{1}{2} \left[\hat{p}^* + \frac{\tau W}{\mathcal{E}Z\varphi} \right] \\ p_D^*(\varphi) &= \frac{1}{2} \left[\hat{p}^* + \frac{W^*}{Z^*\varphi} \right] & p_X^*(\varphi) &= \frac{1}{2} \left[\hat{p} + \frac{\tau^* \mathcal{E}W^*}{Z^*\varphi} \right] \end{aligned}$$

where

$$\hat{p} = \frac{\alpha\gamma W + \eta N \bar{p}}{\gamma + \eta N} \quad \hat{p}^* = \frac{\alpha\gamma W^* + \eta N^* \bar{p}^*}{\gamma + \eta N^*}$$

are the maximum prices a firm can set in the Home and Foreign markets, respectively, to have a non-negative demand for its variety. This can be clearly seen from the following equations for the equilibrium quantities demanded

$$\begin{aligned} y_D(\varphi) &= \frac{1}{2\gamma W} \left[\hat{p} - \frac{W}{Z\varphi} \right] & y_X(\varphi) &= \frac{1}{2\gamma W^*} \left[\hat{p}^* - \frac{\tau W}{\mathcal{E}Z\varphi} \right] \\ y_D^*(\varphi) &= \frac{1}{2\gamma W^*} \left[\hat{p}^* - \frac{W^*}{Z^*\varphi} \right] & y_X^*(\varphi) &= \frac{1}{2\gamma W} \left[\hat{p} - \frac{\tau^* \mathcal{E}W^*}{Z^*\varphi} \right]. \end{aligned}$$

Finally, we have that total profits in the seller's currency from selling in each market are given by

$$\begin{aligned} \pi_D(\varphi) &= \frac{1}{4\gamma W} \left[\hat{p} - \frac{W}{Z\varphi} \right]^2 & \pi_X(\varphi) &= \mathcal{E} \frac{1}{4\gamma W^*} \left[\hat{p}^* - \frac{\tau W}{\mathcal{E}Z\varphi} \right]^2 \\ \pi_D^*(\varphi) &= \frac{1}{4\gamma W^*} \left[\hat{p}^* - \frac{W^*}{Z^*\varphi} \right]^2 & \pi_X^*(\varphi) &= \frac{1}{\mathcal{E}} \frac{1}{4\gamma W} \left[\hat{p} - \frac{\tau^* \mathcal{E}W^*}{Z^*\varphi} \right]^2. \end{aligned}$$

2.3 Cutoff Productivity Rules

From the equilibrium quantity and profit functions above, we can now obtain the productivity cutoff rules. These rules are determined by the minimum productivity a producer can have and still have non-negative production and profits, without taking into account the sunk cost. Let φ_D and φ_X denote the productivity cutoff rules for Home firms selling at Home and Foreign,

respectively. These rules are then defined as

$$\begin{aligned}\varphi_D &= \inf\{\varphi : \pi_D(\varphi) > 0 \text{ and } y_D(\varphi) > 0\} = \frac{W}{Z\hat{p}} \\ \varphi_X &= \inf\{\varphi : \pi_X(\varphi) > 0 \text{ and } y_X(\varphi) > 0\} = \frac{\tau W}{\mathcal{E}Z\hat{p}^*}.\end{aligned}$$

Analogously, $\varphi_D^* = \frac{W^*}{Z^*\hat{p}^*}$ and $\varphi_X^* = \frac{\tau^*\mathcal{E}W^*}{Z^*\hat{p}^*}$.

Note that we can derive the following relations from the cutoff rules

$$\varphi_D = \frac{1}{\tau^*} \left[\frac{Z^*}{Z} \right] \left[\frac{W}{\mathcal{E}W^*} \right] \varphi_X^* \quad (3)$$

$$\varphi_D^* = \frac{1}{\tau} \left[\frac{Z}{Z^*} \right] \left[\frac{\mathcal{E}W^*}{W} \right] \varphi_X. \quad (4)$$

Now, we can rewrite the price, quantity and profit equations in terms of the cutoff rules and the firm's own productivity level. Rewriting the price equations:

$$\begin{aligned}p_D(\varphi) &= \frac{W}{Z\varphi} + \frac{1}{2} \frac{W}{Z} \left[\frac{1}{\varphi_D} - \frac{1}{\varphi} \right] & p_X(\varphi) &= \frac{\tau W}{\mathcal{E}Z\varphi} + \frac{1}{2} \frac{\tau W}{\mathcal{E}Z} \left[\frac{1}{\varphi_X} - \frac{1}{\varphi} \right] \\ p_D^*(\varphi) &= \frac{W^*}{Z^*\varphi} + \frac{1}{2} \frac{W^*}{Z^*} \left[\frac{1}{\varphi_D^*} - \frac{1}{\varphi} \right] & p_X^*(\varphi) &= \frac{\tau^*\mathcal{E}W^*}{Z^*\varphi} + \frac{1}{2} \frac{\tau^*\mathcal{E}W^*}{Z^*} \left[\frac{1}{\varphi_X^*} - \frac{1}{\varphi} \right]\end{aligned}$$

where the second term in each expression is the absolute markup over the marginal cost, in terms of the destination country's currency.

In the same way, we rewrite the equilibrium quantities of production and the profit functions as

$$\begin{aligned}y_D(\varphi) &= \frac{1}{2\gamma W} \left[\frac{W}{Z} \right] \left[\frac{1}{\varphi_D} - \frac{1}{\varphi} \right] & y_X(\varphi) &= \frac{1}{2\gamma W^*} \left[\frac{\tau W}{\mathcal{E}Z} \right] \left[\frac{1}{\varphi_X} - \frac{1}{\varphi} \right] \\ y_D^*(\varphi) &= \frac{1}{2\gamma W^*} \left[\frac{W^*}{Z^*} \right] \left[\frac{1}{\varphi_D^*} - \frac{1}{\varphi} \right] & y_X^*(\varphi) &= \frac{1}{2\gamma W} \left[\frac{\tau^*\mathcal{E}W^*}{Z^*} \right] \left[\frac{1}{\varphi_X^*} - \frac{1}{\varphi} \right]\end{aligned}$$

and

$$\begin{aligned}\pi_D(\varphi) &= \frac{1}{4\gamma W} \left[\frac{W}{Z} \right]^2 \left[\frac{1}{\varphi_D} - \frac{1}{\varphi} \right]^2 & \pi_X(\varphi) &= \mathcal{E} \frac{1}{4\gamma W^*} \left[\frac{\tau W}{\mathcal{E}Z} \right]^2 \left[\frac{1}{\varphi_X} - \frac{1}{\varphi} \right]^2 \\ \pi_D^*(\varphi) &= \frac{1}{4\gamma W^*} \left[\frac{W^*}{Z^*} \right]^2 \left[\frac{1}{\varphi_D^*} - \frac{1}{\varphi} \right]^2 & \pi_X^*(\varphi) &= \frac{1}{\mathcal{E}} \frac{1}{4\gamma W} \left[\frac{\tau^*\mathcal{E}W^*}{Z^*} \right]^2 \left[\frac{1}{\varphi_X^*} - \frac{1}{\varphi} \right]^2.\end{aligned}$$

2.4 Averages and Number of Firms

The key feature of this type of heterogeneous-firm model is that it gathers all the information of the differentiated-good sector in simple cutoff productivity rules. In this section we derive some useful results for averages and firm composition in the differentiated-good industry in terms of these rules.

All the following results make use of the Pareto parametrization mentioned above. Besides, we only compute unweighted averages as, given our utility function, any weighted version will be proportional to the unweighted one. Then, for example, average productivities of Home and Foreign firms producing for their own markets are respectively given by

$$\begin{aligned}\bar{\varphi}_D &= \int_{\varphi_D}^{\infty} \varphi dG(\varphi \mid \varphi > \varphi_D) = \frac{k}{k-1} \varphi_D \\ \bar{\varphi}_D^* &= \int_{\varphi_D^*}^{\infty} \varphi dG(\varphi \mid \varphi > \varphi_D^*) = \frac{k}{k-1} \varphi_D^*.\end{aligned}$$

Let N_D and N_X be the mass of Home firms producing for the Home and Foreign market, respectively. With similar expressions for the Foreign firms, we have that the total mass of varieties sold at Home is $N = N_D + N_X^*$. Analogously, the mass of varieties sold at Foreign is $N^* = N_D^* + N_X$.

Using the composition of sellers, the average price for the differentiated good (at Home) can be written as

$$\bar{p} = \frac{1}{N} (N_D \bar{p}_D + N_X^* \bar{p}_X^*).$$

where we have that

$$\bar{p}_D = \frac{2k+1}{2(k+1)} \frac{W}{Z\varphi_D} \quad \bar{p}_X^* = \frac{2k+1}{2(k+1)} \frac{\tau^* \mathcal{E} W^*}{Z^* \varphi_X^*}.$$

From the cutoff rules, we can see that the relation between φ_D and φ_X^* in (3) implies that $\bar{p}_D = \bar{p}_X^*$. Hence, the average price of the differentiated good is simplified to

$$\bar{p} = \frac{2k+1}{2(k+1)} \frac{W}{Z\varphi_D}, \tag{5}$$

from which we can see that an increase in the domestic cutoff rule drives down not only the average price of domestic varieties, but also the average price of the imported ones.

As we mentioned above, a producer will sell in a market if and only if its relative productivity is no less than the cutoff rule. Let N_E and N_E^* denote the mass of entrants at Home and Foreign. Therefore, our parametrization implies that the composition of sellers is given by

$$\begin{aligned} N_D &= \left[\frac{\varphi_{\min}}{\varphi_D} \right]^k N_E & N_X &= \left[\frac{\varphi_{\min}}{\varphi_X} \right]^k N_E \\ N_D^* &= \left[\frac{\varphi_{\min}}{\varphi_D^*} \right]^k N_E^* & N_X^* &= \left[\frac{\varphi_{\min}}{\varphi_X^*} \right]^k N_E^*. \end{aligned}$$

We can also derive expressions for the number of entrants, N_E and N_E^* , in terms of the cutoff rules. We first solve for N and N^* from the threshold price expressions, \hat{p} and \hat{p}^* , in Section 2.2. We then use the cutoff rules to write the threshold prices in terms of the cutoff levels φ_D and φ_D^* . Using (5) and an analogous expression for the average price at Foreign, we find that $N = \frac{2\gamma(k+1)}{\eta}(\alpha Z \varphi_D - 1)$ and $N^* = \frac{2\gamma(k+1)}{\eta}(\alpha Z^* \varphi_D^* - 1)$. Finally, plugging in the previous two equations and the expressions for the composition of sellers into $N = N_D + N_X^*$ and $N^* = N_D^* + N_X$, we get a system of two equations with two unknowns, N_E and N_E^* . Solving, we get

$$N_E = \frac{2\gamma(k+1)}{\varphi_{\min}^k \eta [(\tau\tau^*)^k - 1]} \left[(\tau\tau^*)^k \varphi_D^k (\alpha Z \varphi_D - 1) - \left[\frac{\tau Z^* W}{Z \mathcal{E} W^*} \right]^k \varphi_D^{*k} (\alpha Z^* \varphi_D^* - 1) \right] \quad (6)$$

$$N_E^* = \frac{2\gamma(k+1)}{\varphi_{\min}^k \eta [(\tau\tau^*)^k - 1]} \left[(\tau\tau^*)^k \varphi_D^{*k} (\alpha Z^* \varphi_D^* - 1) - \left[\frac{\tau^* Z \mathcal{E} W^*}{Z^* W} \right]^k \varphi_D^k (\alpha Z \varphi_D - 1) \right]. \quad (7)$$

2.5 Free-Entry Conditions

Producers in both countries will enter the differentiated-good industry as long as their expected value of entry is no less than the sunk entry cost. Since the potential number of entrants is unbounded, the expected value of entry and the sunk entry cost are equal in equilibrium. Therefore, the free-entry conditions can be written as

$$\begin{aligned} \int_{\varphi_D}^{\infty} \pi_D(\varphi) dG(\varphi) + \int_{\varphi_X}^{\infty} \pi_X(\varphi) dG(\varphi) &= \frac{f_E W}{Z} \\ \int_{\varphi_D^*}^{\infty} \pi_D^*(\varphi) dG(\varphi) + \int_{\varphi_X^*}^{\infty} \pi_X^*(\varphi) dG(\varphi) &= \frac{f_E^* W^*}{Z^*} \end{aligned}$$

where the profit functions are as defined in Section 2.3.

Integrating, we rewrite the free-entry conditions as

$$\frac{\varphi_{\min}^k}{2\gamma(k+1)(k+2)} \frac{W}{Z} \left[\frac{1}{W\varphi_D^{k+2}} + \frac{\tau^2}{\mathcal{E}W^*\varphi_X^{k+2}} \right] = f_E \quad (8)$$

$$\frac{\varphi_{\min}^k}{2\gamma(k+1)(k+2)} \frac{W^*}{Z^*} \left[\frac{1}{W^*\varphi_D^{*k+2}} + \frac{\tau_t^{*2}\mathcal{E}}{W\varphi_X^{*k+2}} \right] = f_E^*. \quad (9)$$

This concludes the setup of the partial equilibrium model. We are now ready to solve for the cutoff productivity rules when the rest of the variables are exogenously given.

2.6 Solution of the Model: Equilibrium Cutoff Rules

In this partial equilibrium model, the equilibrium cutoff rules are determined by solving the system of equations given by (3), (4), (8) and (9). Note that this model analyzes only competition effects of entry and exit of firms, as wages are kept fixed.⁷

Given that we want to focus on the impact of exogenous exchange rate shocks, without loss of generality we can simplify our analysis by assuming that $f_E = f_E^*$, $\tau = \tau^*$, $Z = Z^*$, and $W = W^*$.

The equilibrium productivity rules are then given by

$$\begin{aligned} \varphi_D &= \left[\frac{A(\tau^{2k} - 1)}{\tau^k(\tau^k - \mathcal{E}^{k+1})} \right]^{\frac{1}{k+2}} \\ \varphi_X &= \left[\frac{A(\tau^{2k} - 1)}{\tau^k(\tau^k - \frac{1}{\mathcal{E}^{k+1}})} \right]^{\frac{1}{k+2}} \frac{\tau}{\mathcal{E}} \\ \varphi_D^* &= \left[\frac{A(\tau^{2k} - 1)}{\tau^k(\tau^k - \frac{1}{\mathcal{E}^{k+1}})} \right]^{\frac{1}{k+2}} \\ \varphi_X^* &= \left[\frac{A(\tau^{2k} - 1)}{\tau^k(\tau^k - \mathcal{E}^{k+1})} \right]^{\frac{1}{k+2}} \tau \mathcal{E} \end{aligned}$$

where $A = \frac{\varphi_{\min}^k}{2\gamma(k+1)(k+2)Zf_E}$ is an aggregate index reflecting differentiation, productivity dispersion and aggregate technology. Note, for example, that more product differentiation (higher γ), lower heterogeneity (higher k), and higher aggregate productivity and entry costs, all imply lower relative productivity cutoff rules.

From the previous equations, we can see that in order to have an interior solution under the

⁷In Melitz (2003), changes in wages are the driving force of firm reallocations.

symmetry assumptions above, the exchange rate must range between $\frac{1}{\tau^{k/(k+1)}}$ and $\tau^{\frac{k}{k+1}}$. These bounds are a natural consequence of the use of a linear demand system.

In the next Section we move to the study of the effects of exchange rate changes in prices and trade flows in partial equilibrium.

3 The Impact of Exchange Rate Changes in Partial Equilibrium

In this Section we analyze the transmission of exchange rate shocks to firm reallocation decisions, prices and trade flows, when individual firms adjust their profit markups according to market competition conditions.

An important driving force in this model involves cross-country firm reallocations, that is, changes in the pattern of entry in each country. Melitz and Ottaviano (2005) distinguish between two scenarios, one with cross-country reallocations (the long-run steady state) and another without them (the short-run). In our case, we focus on the scenario that allows cross-country firm reallocations and leave an analysis of the second scenario for the Appendix.

We start by looking into the relation between the exchange rate and the cutoff productivity rules. Then we see how this is reflected in disaggregated and aggregated prices. Finally, we look into trade flows and their relation with prices.

3.1 The Cutoff Rules and the Exchange Rate

This model captures all the information on entry and changes in the competition environment in the cutoff rules. Therefore, to understand how an exchange rate change is reflected in prices, we analyze first its impact on the productivity cutoff rules.

Let us focus on the the impact of exchange rate changes on the productivity cutoff rule for Home firms selling in their own country, φ_D . The elasticity of φ_D with respect to the exchange rate, $\zeta_{\varphi_D, \mathcal{E}}$, is

$$\zeta_{\varphi_D, \mathcal{E}} = \frac{k+1}{k+2} \left(\frac{\mathcal{E}^{k+1}}{\tau^k - \mathcal{E}^{k+1}} \right) > 0. \quad (10)$$

Moreover, we also have that

$$\frac{\partial^2 \varphi_D}{\partial \mathcal{E}^2} = \frac{\varphi_D \zeta_{\varphi_D, \mathcal{E}}}{\mathcal{E}^2} [(k+3)\zeta_{\varphi_D, \mathcal{E}} + k] > 0,$$

that is, φ_D is strictly increasing and strictly convex in the exchange rate.

In Section 2.4 we saw that the average productivity is directly proportional to the productivity cutoff rule. Therefore, the positive relation between φ and \mathcal{E} implies that a depreciation of the Home currency drives up the average productivity of Home firms selling at Home. Moreover, there are second order effects, as the magnitude of the impact is increasing in the exchange rate. In other words, a depreciation of the Home currency has a higher impact on φ_D than a proportional appreciation.

With a depreciation of the Home currency, Home firms become more competitive in the Foreign market. The new profit opportunities abroad will increase the entrance of Home producers. At the same time, Foreign firms will exit the Home and Foreign markets. The key issue is that even though entrance is motivated by export opportunities, the new entrants will also want to produce for the domestic market. At the end, competition increases at Home, driving up the productivity cutoff rule φ_D .

The opposite occurs in case of an appreciation. The value of entry at Home goes down as Home firms become less competitive at Home and abroad. Less entry of Home producers decreases competition, driving down the cutoff rule φ_D .

The convexity of φ_D in \mathcal{E} implies that a Home currency depreciation has a larger impact on the cutoff rule than a proportional appreciation. We have seen that there is an increase in competition from new Home entrants when there is a depreciation of the exchange rate. At the same time, competition decreases from Foreign firms at Home, which leave because they are less competitive. However, the rate at which Foreign firms leave the Home market is decreasing with the exchange, as most productive firms can absorb the exchange rate change in their markups. The net effect is an important increase in competition, which is translated to higher demand elasticities, and then to the productivity cutoff rule. In the case of a proportional appreciation, the decrease in competition caused by Home firms leaving their own market, is compensated by a faster entry of Foreign firms. Foreign exporters compete against each other to enter the Home market. Hence, the increase in the markups they get from Home after an appreciation is modest. Overall, the impact on the price elasticity and therefore in the cutoff rule is smaller than with a depreciation.

Even though the average productivity of Home firms selling at Home increases with a depreciation, this does not mean that the average productivity of Home exporters increases. From

the equilibrium cutoff rules, it is easy to see that φ_X goes down with a Home currency depreciation. Firms that were not exporting become internationally competitive due exclusively to the exchange rate change. Therefore, the average productivity of exporters goes down. This result has important policy implications, as it shows how less productive firms from a country with a permanently undervalued currency can displace more productive firms from other countries.

3.2 Exchange Rate Pass-Through to Disaggregated Prices

In Section 2.3 we got that the Home currency price of an imported variety from a firm with relative productivity φ is given by

$$p_X^*(\varphi) = \frac{\tau^* \mathcal{E} W^*}{Z^* \varphi} + \frac{1}{2} \frac{\tau^* \mathcal{E} W^*}{Z^*} \left[\frac{1}{\varphi_X^*} - \frac{1}{\varphi} \right]. \quad (11)$$

Hence, under the symmetry assumptions of Section 2.6, we get that the pass-through rate of an exchange rate shock to the previous price, $\mu_X^*(\varphi) = \frac{\partial p_X^*(\varphi)}{\partial \mathcal{E}} \frac{\mathcal{E}}{p_X^*(\varphi)}$, is

$$\mu_X^*(\varphi) = 1 - \frac{\zeta_{\varphi_X^*, \mathcal{E}}}{1 + \frac{\varphi_X^*}{\varphi}} \quad (12)$$

where $\zeta_{\varphi_X^*, \mathcal{E}}$ is the elasticity of the productivity cutoff rule for Foreign exporters, φ_X^* , with respect to the exchange rate. From equation (3) it is not difficult to see that $\zeta_{\varphi_X^*, \mathcal{E}} = 1 + \zeta_{\varphi_D, \mathcal{E}}$, so that φ_X^* follows the same behavior as φ_D .

From the previous equation, we can observe that there are two opposite forces when the exchange rate changes. Note, for example, that a depreciation of the Home currency will increase the denominator in the second term of the equation (through the increase in φ_X^*), pushing for a higher pass-through, while the numerator also increases (through the increase in $\zeta_{\varphi_X^*, \mathcal{E}}$), pushing down the pass-through rate. The effect in the denominator, driven by $\frac{\varphi_X^*}{\varphi}$, is firm-specific and establishes the relative position of the firm with respect to the cutoff rule. On the other hand, the effect in the numerator, driven by $\zeta_{\varphi_X^*, \mathcal{E}}$, is industry-wide, and reflects the industry's competition environment.⁸

Independently of the exchange rate, the firm-specific effect implies pass-through rates that are decreasing in φ . That is, more productive exporting firms have always lower pass-through rates.⁹

⁸Given the close relation between $\zeta_{\varphi_X^*, \mathcal{E}}$ and $\zeta_{\varphi_D, \mathcal{E}}$, we use them indistinctly to refer to the industry-wide competition effect.

⁹Note that if $\varphi \rightarrow \infty$, the firm-specific effect vanishes and the pass-through rate is given by $1 - \zeta_{\varphi_X^*, \mathcal{E}}$.

The firm-specific effect is related to the magnitude of the markup a Foreign firm can charge, which decreases monotonically with the exchange rate. For example, with a Home currency depreciation, the price the exporting firm charges at Home gets closer to its marginal cost because of the increase in φ_X^* , which then pushes for a higher pass-through rate. If the industry-wide effect is constant, the firm-specific effect generates pass-through rates for individual imported varieties that are always higher for depreciations than for appreciations.

The industry-wide competition effect mimics exactly the behavior of φ_D with respect to \mathcal{E} described in the previous Section. If the Home currency depreciates, firms exporting to Home face a tougher competition environment, which is reflected in higher demand elasticities. Then, as firms adjust down their markups to reflect the higher elasticities, the pass-through rate goes down. On the other hand, with a proportional appreciation there is a decrease in competition but not as important as with the depreciation. Then, the smaller elasticity change is reflected in an smaller increase in the producer's markup. This is finally reflected in a higher rate of pass-through.

We know from the previous section that φ_D , and then φ_X^* , is strictly increasing and strictly convex in the exchange rate. Therefore, we can expect to see the industry-wide effect dominating for high levels of the exchange rate, while the firm-specific effect should be dominating for low levels. That is, the exchange rate pass-through function to the Home price of each imported variety has an inverted U-shape. Obviously, this relation need not be symmetric. For very productive firms, the industry-wide effect will dominate along most of the way.

3.3 Aggregate Prices and Trade Flows

In Section 2.4 we got that the average prices of Home and Foreign varieties sold at Home were equivalent. Equation (5) presents this average price, which implies an exchange-rate pass-through of

$$-\zeta_{\varphi_D, \mathcal{E}} < 0.$$

Even though we have positive pass-through rates for prices of individual varieties, this is not reflected in the aggregate import price. Moreover, the pass-through rate is always negative, implying decreases in the average import price from Home currency depreciations, and increases from appreciations.¹⁰

¹⁰As mentioned before, any weighted version of the average price will be proportional to the unweighted version

This striking result is due to the fact that the aggregate import price is being affected by the changes in the pattern of entry and in average productivity. For example, in case of a Home currency depreciation, Foreign firms leave the Home market as they become less competitive. The new average import price is computed taking into account only the surviving firms, who are also the most productive. But this is not the end, surviving Foreign firms have to adjust their markups down because of the increase in competition coming from Home entrants. At the end, the average price will only reflect the industry-wide competition effect, showing a lower average price when competition increases and a higher price when it decreases.

Even though the model only considers the competition channel for the transmission of exchange rate changes, it highlights the importance of firm reallocations for long-run effects of permanent exchange rate changes on aggregate prices. As we mentioned in the Introduction, this pervasive effect can be obtained even in models with firm-level pass-through rates of 1.

We can now look into the true impact of exchange rate changes in each economy: trade flows. Although pass-through rates can be small at the firm-level, and are even negative at the aggregate level, this model predicts important expenditure switching effects. Our trade flow equations are given by

$$VE = N_X \frac{\mathcal{E}}{2\gamma(k+2)W^*} \left[\frac{\tau W}{\mathcal{E}Z} \right]^2 \frac{1}{\varphi_X^2}$$

$$VE^* = N_X^* \frac{1}{2\gamma(k+2)W} \left[\frac{\tau^* \mathcal{E} W^*}{Z^*} \right]^2 \frac{1}{\varphi_X^{*2}}$$

where VE and VE^* are the values of Home and Foreign exports, respectively, in Home currency and final prices.

Just as we know that φ_D is strictly increasing and strictly convex in \mathcal{E} , we have that φ_D^* is strictly decreasing and strictly convex in the exchange rate. Using equations (3) and (4), we can rewrite the trade flow equations in terms of φ_D and φ_D^* as

$$VE = N_X \frac{\mathcal{E}}{2\gamma(k+2)W^*} \left[\frac{W^*}{Z^* \varphi_D^*} \right]^2$$

$$VE^* = N_X^* \frac{1}{2\gamma(k+2)W} \left[\frac{W}{Z \varphi_D} \right]^2.$$

above. For example, if we weight by the value of exports, the average price is given by $\frac{k+1}{k+2} \frac{W}{Z \varphi_D}$; therefore, the pass-through rate does not change.

From these equations we can clearly see unambiguous expenditure switching effects. The value of Home exports, VE , is strictly increasing in the exchange rate, as $N_X \mathcal{E}$ increases and φ_D^* decreases with \mathcal{E} . On the other hand, the value of Home imports (or Foreign exports), VE^* , is strictly decreasing with the exchange rate, as N_X^* decreases and φ_D increases with \mathcal{E} . It can also be shown that VE and VE^* are strictly convex, highlighting important second order effects.

Summarizing, in the presence of firm heterogeneity and reallocation effects, exchange rate pass-through studies using aggregate prices can lead to erroneous conclusions about the true expenditure switching effects. Moreover, the use of disaggregated prices for pass-through studies can also be of limited use for making conclusions about expenditure switching effects, as individual firms could be adjusting their markups, not their quantities, in response to changes in the elasticity of demand.

3.4 A Numerical Example

For illustrative purposes, Figure 1 shows the graphical representation of our results. For simplicity, we normalize W , W^* , Z , Z^* , f_E , and f_E^* to 1. For the parameters in the demand for each variety of the differentiated good, we set $\alpha = 10$, $\gamma = 0.5$, and $\eta = 1$. These numbers are chosen in an arbitrary manner. They only have to satisfy the conditions for an interior solution and a positive consumption of the nontraded good. We also set $\varphi_{\min} = 0.1$, so that it agrees with the value of α for a positive number of sellers in each market. The model works best for high values for τ and k , as the range for \mathcal{E} is increasing in both parameters. Therefore, for the iceberg trade costs we fix $\tau = \tau^* = 1.4$, which is higher than the value of 1.3 mostly used in the literature, and we set k at 4, which is close to the value of 3.4 used by Ghironi and Melitz (2005).

Panel 1a presents the elasticity of φ_D with respect to the exchange rate, $\zeta_{\varphi_D, \mathcal{E}}$. This elasticity is positive, increasing, and strictly convex in the exchange rate. This behavior is explained by the cross-country reallocation of firms, as shown in Panels 1b and 1c.

A depreciation of the Home currency decreases the number of Foreign entrants, N_E^* , and increases the number of Home entrants, N_E , through two channels: first, Foreign firms become less competitive at Home, thus, the least productive Foreign exporters leave the Home market and are replaced by new Home firms; and second, Home firms become more competitive in the Foreign market, driving in existing and new Home firms and displacing the least productive

Foreign firms. At the end, the increase in the number of Home entrants, who enter motivated by the profit opportunities at Foreign, will increase the competition at Home itself. This is finally reflected in a higher N and a lower domestic cutoff rule.

Panel 1d presents the exchange rate pass-through elasticity for the price at Home of a Foreign variety produced with the average productivity of Foreign exporting firms when $\mathcal{E}=1$. Note that the pass-through function follows very strongly the behavior of $-\zeta_{\varphi_D, \mathcal{E}}$, which is driving the industry-wide competition effect. For a firm with this productivity, the industry-wide effect dominates the firm-specific effect along all the way. However, the most unproductive exporting firms will show pass-through rates in which the firm-specific effect dominates for low levels of the exchange rate, implying inverted U-shaped pass-through functions.

Let $c_D = \frac{W}{Z\varphi_D}$ and $c_X^* = \frac{W^*}{Z^*\varphi_X^*}$ represent the cutoff marginal costs (before trade costs), in each country's currency, for firms selling at Home. Panel 1f presents a plot of \bar{p} , c_D , and c_X^* , against the exchange rate.¹¹ This Panel shows how the average import price bears no relation with individual firms' pass-through rates, as shown in Panel 1d. It even shows a negative pass-through rate: lower average import price for depreciations and higher for appreciations. The aggregation is missing the entire expenditure switching effects. For example, it is not that import prices are going down with a depreciation of the Home currency, but that the Foreign firms that survived are the most productive and have lower but positive pass-through rates. Expenditure switching effects are, actually, very important. The last panel in Figure 1 shows the relation between trade flows and the exchange rate. The gap between the value of exports (VE) and the value of imports (VE^*) at Home is strictly increasing with the exchange rate.

4 A Sticky-Wage General Equilibrium Model with Heterogeneous Firms and Endogenous Markups

In this Section, we embody the previous heterogeneous-firm model with endogenous markups into a perfect-foresight general equilibrium NOEM framework à la Obstfeld and Rogoff (1995). Short-run nominal rigidities are incorporated in the form of sticky wages, which are determined a period in advance.

As mentioned in the Introduction, our model has the same building blocks as the model

¹¹Remember from Section 2.4 that the average import price is equal to the average price of domestic firms.

by Ghironi and Melitz (2005), who embody Melitz's (2003) model into a dynamic stochastic general equilibrium framework with flexible prices. Our model, however, is monetary and with perfect-foresight.

This model provides an important edge over the partial equilibrium model analyzed in the previous two Sections, as it will give microeconomic foundations to the sticky-wage assumption and will allow us to derive exchange rate changes endogenously.

Compared to the NOEM literature, this model provides an alternative to PCP and LCP models. While assuming pricing-to-market, our model moves away from complete pass-through rates at the firm level by the use of endogenous markups. Moreover, the sticky-wage heterogeneous-firm framework very nicely derives a disconnection between aggregate price indexes and expenditure switching effects. This has important implications for the PCP-LCP debate, as LCP models make their case for their crucial assumption of no expenditure switching effects (and the implication of fixed exchange rates optimality), in empirical evidence of low pass-through rates to consumer prices.¹²

The general equilibrium model keeps the basic structure of the partial equilibrium version. There are two countries, Home and Foreign, each of them populated by atomistic households in the interval $[0,1]$. Households provide labor to the two sectors in each economy: a nontraded-good sector, produced under perfect competition, and a differentiated-good sector, produced under monopolistic competition by heterogeneous producers. For this extension, we add the assumption that households are monopolistic suppliers of labor.

In the rest of the Section, we first describe the preferences of the representative household, then we describe production, labor demand, and the household's budget constraint. Then we compute the Euler equations and present the final equations that close the model. Finally, we solve the model following the next steps: description of the symmetric steady-state, linearization of the model, and solution for the short and long-run of an unexpected monetary shock.

As usual, Foreign variables are denoted with a star (*). Again, in most of the Section we refer only to the Home country, as analogous expressions apply for the Foreign country.

¹²See Engel (2002) for a discussion about the PCP-LCP debate and the low exchange rate pass-through to consumer prices.

4.1 Model Setup

4.1.1 Preferences

Households in both countries have the same preferences over consumption, money and labor. The intertemporal utility function for the representative Home household j is given by

$$U_t^j = \sum_{s=0}^{\infty} \beta^s \left[\log C_{t+s}^j + \chi \log \frac{M_{t+s}^j}{\rho_{t+s}} - \frac{\kappa}{2} \ell_{t+s}(j)^2 \right] \quad (13)$$

where C^j is the consumption index, $\frac{M^j}{\rho}$ are money balances in terms of the nontraded good, and $\ell(j)$ is the amount of labor devoted to production.

C_j is as defined in equation (1). The corresponding demands for the nontraded good and each variety of the differentiated good still apply. The indirect utility function, expressed in equation (2), becomes our indirect consumption index and will be crucial in the first-order conditions.

Contrary to the CES case, C^j is non-homogeneous, and as such, it is not possible to obtain a price index. Therefore, we use the nontraded good price, ρ , as the deflator for money balances. This is a straightforward step, as ρ is the deflator of the expenditure in consumption, I^j , in the indirect consumption index and will be linked to the wage at Home.

The last term in the utility function corresponds to the disutility of labor, $\frac{\kappa}{2} \ell_{t+s}(j)^2$. Household j 's labor is differentiated and will be monopolistically supplied to both, the homogeneous and the differentiated-good sectors. The household incorporates this information when maximizing equation (13) with respect to ℓ_t . This characteristic of our model implies a household labor supply below the efficient level, which provides the underpinnings for short-run wage stickiness.

4.1.2 Production and Labor Demand

Each household is a monopolistic supplier of a differentiated type of labor. As before, the nontraded good is competitively supplied and employs only labor for production. However, it demands every type of labor in the interval $[0, 1]$.¹³ The production function for the nontraded good at time t is just $y_{N,t} = L_t$, where

$$L_t = \left[\int_0^1 \ell_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}. \quad (14)$$

¹³See Section 10.4 in Obstfeld and Rogoff (1996) for a brief description on how to introduce sticky wages in a NOEM framework.

That is, the labor index follows a Dixit-Stiglitz technology, where $\theta > 1$ is the elasticity of substitution of between the different types of labor.

It follows that the demand for type j 's labor in the nontraded sector is given by $\ell_{N,t}(j) = \left[\frac{w_t(j)}{W_t} \right]^{-\theta} y_{N,t}$, where $w_t(j)$ is the nominal wage for type j 's labor and $W_t = \left[\int_0^1 w_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}$ is the price for labor index L_t . Moreover, perfect competition implies that $\rho_t = W_t$. As we saw in Section 2, this result simplifies our calculations for the differentiated good sector, as the representative consumer demand for each variety of the differentiated good depends on ρ .

The description for the differentiated good sector is the same as in Sections 2.2-2.5. We only need to add time subscripts and make the following assumption: firms only live for one period. A firm pays the entry cost at the beginning of the period, realizes its productivity immediately after entrance, produces if productivity is higher than any of the cutoff rules, and dies at the end of the period. This assumption magnifies the overall impact of entry, to the point of generating an inverse relation between the average import price and the exchange rate as in Section 3.3, but it makes our model more tractable and helps us gain insights on the keys impacts of firm reallocation and competition on prices.

Let us now derive an expression for the labor demand. As we have seen above, demand for composite units of labor, L_t , in the nontraded good sector is given simply by $y_{N,t}$. On the other hand, demand for labor in the differentiated-good sector at time t has two components: the demand to cover the entry costs, and the demand coming from the firms that are actually producing at t . The former is given by $N_{E,t} \frac{f_{E,t}}{Z_t}$, while the latter is

$$N_{D,t} \int_{\varphi_{D,t}}^{\infty} \frac{y_{D,t}(\varphi)}{Z_t \varphi} dG(\varphi \mid \varphi > \varphi_{D,t}) + N_{X,t} \int_{\varphi_{D,t}}^{\infty} \tau_t \frac{y_{X,t}(\varphi)}{Z_t \varphi} dG(\varphi \mid \varphi > \varphi_{X,t}).$$

Solving for this expression, we get that total labor demand at Home at time t , L_t^D , is given by

$$L_t^D = N_{E,t} \frac{f_{E,t}}{Z_t} + N_{E,t} \frac{k \varphi_{\min}^k}{2\gamma(k+1)(k+2)Z_t^2} \left[\frac{1}{\varphi_{D,t}^{k+2}} + \tau_t^2 \frac{W_t}{\mathcal{E}_t W_t^*} \frac{1}{\varphi_{X,t}^{k+2}} \right] + c_{N,t}, \quad (15)$$

where we substitute $c_{N,t}$ for $y_{N,t}$ because the nontraded-good sector clears.

Given L_t^D , it is not difficult to show that the total demand for a particular type of labor j is

$$\ell_t^D(j) = \left[\frac{w_t(j)}{W_t} \right]^{-\theta} L_t^D. \quad (16)$$

To conclude the model setup, we now describe the representative household's budget constraint.

4.1.3 Household Budget Constraint

The representative Home household j can hold Home and Foreign riskless bonds denominated in units of each country's labor. Following the notation by Ghironi and Melitz (2005), the Home household begins time t with B_t^j and B_{*t}^j holdings of the Home and Foreign bonds, respectively. Bond holdings at the end of t are given by B_{t+1}^j and B_{*t+1}^j . Interest rates from t to $t+1$, in terms of each country's labor, are given by r_{t+1} and r_{t+1}^* .

The other components of the budget constraint are labor income, tax payments and money holdings. Dividends do not enter the budget constraint, as profits are zero in the nontraded-good sector and the positive profits for firms in the differentiated-good sector cancel out with the entry costs for successful and unsuccessful entrants.¹⁴

For labor income, household j receives a nominal wage $w_t(j)$ for each unit of labor provided. On the other hand, it pays lump-sum taxes in the amount of T_t^j and maintains M_t^j in nominal money balances. With these elements, the Home household's budget constraint can be written as

$$I_t^j + W_t B_{t+1}^j + \mathcal{E}_t W_t^* B_{*t+1}^j + M_t^j + T_t^j = w_t(j) \ell_t(j) + W_t(1 + r_t) B_t^j + \mathcal{E}_t W_t^*(1 + r_t^*) B_{*t}^j + M_{t-1}^j$$

where I_t^j is the nominal expenditure in consumption, as defined in Section 2.1.

From the indirect consumption index in equation (2), we substitute W_t for ρ_t , express the second term in the right hand side in terms of cutoff rules, and then solve for I_t^j . Plugging in the result in the budget constraint above, we can rewrite it as

$$\begin{aligned} W_t C_t^j - W_t \frac{(\alpha Z_t \varphi_{D,t} - 1) [(k+2)\alpha Z_t \varphi_{D,t} - k - 1]}{2\eta(k+2)(Z_t \varphi_{D,t})^2} + W_t B_{t+1}^j + \mathcal{E}_t W_t^* B_{*t+1}^j \\ + M_t^j + T_t^j = w_t(j) \ell_t(j) + W_t(1 + r_t) B_t^j + \mathcal{E}_t W_t^*(1 + r_t^*) B_{*t}^j + M_{t-1}^j. \end{aligned} \quad (17)$$

The expression for I_t^j in terms of the consumption index and the cutoff rules, can also be

¹⁴We might include, as in Ghironi and Melitz (2005), a mutual fund composed by Home entrants in the differentiated good sector from which household j buys a share x_t at the beginning of the period. Then, at the end of the period it receives the share x_t of total profits generated by the successful entrants. This will simply generate an Euler equation identical to the free-entry condition at Home, given in equation (8).

used in the demand for the nontraded good, so that we can write it as

$$c_{N,t}^j = C_t^j - \frac{(\alpha Z_t \varphi_{D,t} - 1) [(k+2)\alpha Z_t \varphi_{D,t} + k+1]}{2\eta(k+2)(Z_t \varphi_{D,t})^2}. \quad (18)$$

4.2 Closing the Model: First-Order Conditions and Net Foreign Assets

We now solve the household maximization problem. The representative household maximizes the intertemporal utility function (13) subject to the budget constraint (17), and the demand for its labor (16). The best strategy is to solve for $w_t(j)$ in equation (16), where the household takes L_t as given, and plug the resulting equation into equation (17). We then solve for C_t^j in the modified budget constraint and substitute the result in the intertemporal utility function.

The Euler equations with respect to B_{t+1}^j , B_{*t+1}^j , M_t^j , and $\ell_t(j)$ are respectively given by

$$C_{t+1} = \beta(1 + r_{t+1})C_t \quad (19)$$

$$C_{t+1} = \beta(1 + r_{t+1}^*) \frac{Q_{t+1}}{Q_t} C_t \quad (20)$$

$$\frac{M_t}{W_t} = \frac{1 + i_{t+1}}{i_{t+1}} \chi C_t \quad (21)$$

$$\ell_t^{\frac{\theta+1}{\theta}} = \frac{\theta-1}{\theta\kappa} \frac{L_t^{\frac{1}{\theta}}}{C_t} \quad (22)$$

where $Q_t = \frac{\mathcal{E}_t W_t^*}{W_t}$ and $1 + i_{t+1} = (1 + r_{t+1}) \frac{W_{t+1}}{W_t}$. We also drop the superscript j from the Euler equations as households are symmetric.

In the symmetric equilibrium where every household receives the same wage w_t and offers the same amount of labor ℓ_t , we get that $W_t = w_t$ and $L_t = \ell_t$. Therefore, we can rewrite the labor-leisure Euler expression (22) as

$$L_t = \frac{\theta-1}{\theta\kappa} \frac{1}{C_t}. \quad (23)$$

As L_t is below its efficient value, labor and hence output will be demand determined in the presence of small shocks and wage stickiness, as the wage will still be above the value of the marginal disutility of labor. In other words, the labor-leisure Euler equation (23) does not bind in the short-run, when wages are sticky.

To close the model, we need to establish two more conditions. First, we must satisfy that

the internationally traded bonds are in zero net supply. That is,

$$B_{t+1} + B_{t+1}^* = 0$$

$$B_{*t+1} + B_{*t+1}^* = 0$$

The second is the net foreign assets condition that comes from the aggregate budget constraints. We assume that the government's unique role is to use transfers to distribute the seignorage revenue to households, so that $T_t + (M_t - M_{t-1}) = 0$. Therefore, the Home and Foreign aggregate budget constraints (in terms of labor) are respectively given by

$$\begin{aligned} B_{t+1} + Q_t B_{*t+1} &= (1 + r_t)B_t + Q_t(1 + r_t^*)B_{*t} - C_t + \frac{(\alpha Z_t \varphi_{D,t} - 1) [(k+2)\alpha Z_t \varphi_{D,t} - k - 1]}{2\eta(k+2)(Z_t \varphi_{D,t})^2} + L_t \\ \frac{B_{t+1}^*}{Q_t} + B_{*t+1}^* &= \frac{(1 + r_t)}{Q_t}B_t^* + (1 + r_t^*)B_{*t}^* - C_t^* + \frac{(\alpha Z_t^* \varphi_{D,t}^* - 1) [(k+2)\alpha Z_t^* \varphi_{D,t}^* - k - 1]}{2\eta(k+2)(Z_t^* \varphi_{D,t}^*)^2} + L_t^*. \end{aligned}$$

Following the Appendix in Ghironi and Melitz (2005), we multiply the second expression by Q_t , and then we substitute the zero net supply conditions for bonds. Subtracting the resulting expression from the Home aggregate budget constraint, we get that the law of motion of Home net foreign assets, NFA , in terms of Home labor is given by

$$\begin{aligned} NFA_{t+1} &= (1 + r_t)NFA_t + [Q_t(1 + r_t^*) - Q_{t-1}(1 + r_t)] B_{*t} - \frac{1}{2} (C_t - Q_t C_t^*) \\ &+ \frac{1}{2} \left(\frac{(\alpha Z_t \varphi_{D,t} - 1) [(k+2)\alpha Z_t \varphi_{D,t} - k - 1]}{2\eta(k+2)(Z_t \varphi_{D,t})^2} - Q_t \frac{(\alpha Z_t^* \varphi_{D,t}^* - 1) [(k+2)\alpha Z_t^* \varphi_{D,t}^* - k - 1]}{2\eta(k+2)(Z_t^* \varphi_{D,t}^*)^2} \right) \\ &+ \frac{1}{2} (L_t - Q_t L_t^*), \end{aligned} \tag{24}$$

where $NFA_{t+1} = B_{t+1} + Q_t B_{*t+1}$.

Table 1 presents the summary of our model. Note that there are not pricing or quantity equations for the differentiated good sector, as they will depend on the productivity cutoff rules.

4.3 Solving the Model

The main purpose of this paper is to analyze the impact of exchange rate changes on prices and trade flows in a framework of heterogeneous firms and endogenous markups. The NOEM structure allows us to derive exchange rate changes endogenously and enrich our understanding of the dynamics of adjustment.

In this Section we log-linearize the model around a symmetric steady state. Then we analyze

the impact of a permanent unexpected monetary shock at two different points in time, the time of the shock and the new steady state. The new steady state is reached after a period, as the only rigidity comes from wages set a period in advance.

For the purposes of this paper, it is enough to consider exogenous monetary shocks. Therefore, we abstract from any other type of shocks by assuming that $Z_t = Z_t^* = 1$, $f_{E,t} = f_{E,t}^* = f_E$, and $\tau_t = \tau_t^* = \tau$ for every t .

4.3.1 Log-linearization around the Symmetric Steady-State

The symmetric steady state, which will be represented by a subscript 0, is characterized by a net foreign asset position of zero, that is $NFA_0 = B_0 = B_{*0} = 0$. As in the Obstfeld and Rogoff (1995) Redux model, it is natural to start from a position of financial autarky, as the long-run effects of a permanent unexpected shock are a consequence of the short-run impact of the shock on the net foreign asset position.

Table 2 presents the solution of the model for the symmetric steady state. Obviously, the solution for the cutoff rules when $M_0 = M_0^*$ is identical to the solution in the partial equilibrium model (Section 2.6) when $\mathcal{E} = 1$ and $Z = 1$. As usual in the NOEM literature, we get that the symmetric steady state exchange rate is given by the ratio of Home and Foreign money balances.

We now proceed with the linearization of the model at Table 1 around the symmetric steady state. Table 3 presents the linearized system. We use sanserif and bold fonts (for cutoff rules), to represent log-deviations.

With an exception, all variables are log-linearized, which means that they represent percentage deviations from the initial steady state. The exception is the net foreign asset position, NFA , which cannot be log-linearized because it has an initial value of zero. Moreover, even if initial NFA were different from zero, it is prevented from being log-linearized because it represents a balance (that is, it can take negative values). For terms involving NFA , we just make a direct first order Taylor approximation. Note also that the coefficient on B_{*t} in the NFA equation in Table 1 has a value of zero in the symmetric steady state. Therefore, this term vanishes in the linearization.

4.3.2 A Permanent Unexpected Monetary Shock

We now proceed with the classic experiment of international macroeconomics of a permanent unexpected monetary shock. As we know, monetary shocks under flexible prices are immediately transmitted to nominal variables and do not have a real impact in the economy. In this case, however, we consider one period nominal rigidities in the form of sticky wages.

The dynamics of adjustment are as follows. At time 0 the countries are in the symmetric steady state described above. Home (and hence Foreign) households enter period 1 with zero bond holdings. At time 1 there is an unexpected permanent monetary shock. The percentage deviations for the money supplies at Home and Foreign from their initial steady state levels, M_0 and M_0^* , are given by m and m^* , respectively. Wages are set a period in advance. As mentioned above, at the time of the shock the labor Euler condition does not bind, as the wage continues to be higher than the value of the disutility of labor (for a small enough shock). Therefore, at period 1, labor and output are demand determined. The rest of the equations at Table 3 still apply at period 1. Period 1 variables will be denoted without time subscripts.

The new steady state is reached in period 2, which differs from the initial steady state in that the net foreign asset position is different from zero. In this new equilibrium, the current account goes back to zero and the monetary transmission is completed. We denote the new steady state variables with a bar.

At the end we have a system of 33 equations with 33 unknowns. For the first period we have 16 endogenous variables: $\varphi_D, \varphi_D^*, \varphi_X, \varphi_X^*, n_E, n_E^*, c, c^*, l, l^*, c_N, c_N^*, r, r^*, e$, and \overline{NFA} .¹⁵ On the other hand, the 17 endogenous variables for the second period are: $\bar{\varphi}_D, \bar{\varphi}_D^*, \bar{\varphi}_X, \bar{\varphi}_X^*, \bar{n}_E, \bar{n}_E^*, \bar{c}, \bar{c}^*, \bar{l}, \bar{l}^*, \bar{c}_N, \bar{c}_N^*, \bar{r}, \bar{r}^*, \bar{w}, \bar{w}^*$, and \bar{e} . Remember that all of them, except \overline{NFA} , represent log-deviations from the symmetric steady state.

Given that the solution of this system of equations is rather complicated, we just describe some important results. In the next Section we explore further the solution to analyze the impact of exchange rate changes on prices and trade flows.

An easy result concerns the new steady state interest rates, \bar{r} and \bar{r}^* . From the bonds Euler equations, it is easy to see that in the steady state they are always equal to $\frac{1-\beta}{\beta}$. Therefore, \bar{r} and \bar{r}^* are zero.

¹⁵ r and r^* represent the interest rate, in terms of labor, from period 1 to period 2. NFA is barred because the net foreign asset position at the end of period 1 is the new steady state NFA .

Another important result is that the exchange jumps immediately at the time of the shock to its new steady state value, that is, $e = \bar{e}$. There is no overshooting, though it could be easily generated through the incorporation in equation (13) of an intertemporal elasticity of substitution for real money balances greater than one.

The consumption indexes log-deviations in period 1, c and c^* , change in the same proportion as the money supplies, m and m^* , respectively. On the other hand, in the new steady state we have that $\bar{c} = -\bar{c}^*$, so that there is only a redistribution of wealth, but no changes in world's income. For changes in the nontraded good consumption in the new steady state, we also find that $\bar{c}_N = -\bar{c}_N^*$.

For the productivity cutoff rules, we get that

$$\frac{\varphi_D}{\varphi_D^*} = \frac{\varphi_X}{\varphi_X^*} = \frac{\bar{\varphi}_D}{\bar{\varphi}_D^*} = \frac{\bar{\varphi}_X}{\bar{\varphi}_X^*} = -1,$$

so that a change in average productivity in one country implies an exact and opposite change in the other country's productivity. We also get that

$$\frac{\varphi_D}{\varphi_X} = \frac{\varphi_D^*}{\varphi_X^*} = \frac{\bar{\varphi}_D}{\bar{\varphi}_X} = \frac{\bar{\varphi}_D^*}{\bar{\varphi}_X^*} = -\frac{k+1}{\tau^k(k+2)-1} < 0,$$

which indicates that the average productivities for producers selling in the domestic market and exporters change in opposite direction, though the changes in the cutoff rules for the domestic market are always smaller in absolute value, as $0 < \frac{k+1}{\tau^k(k+2)-1} < 1$. Combining the previous two relations between the cutoff rules, we can see that

$$\frac{\varphi_D}{\varphi_X^*} = \frac{\varphi_D^*}{\varphi_X} = \frac{\bar{\varphi}_D}{\bar{\varphi}_X^*} = \frac{\bar{\varphi}_D^*}{\bar{\varphi}_X} = \frac{k+1}{\tau^k(k+2)-1},$$

implying that changes in the productivity of sellers in the same market always move in the same direction, but the changes for the domestic sellers are always smaller.

There is also the following intertemporal relation between the cutoff rules

$$\frac{\bar{\varphi}_D}{\varphi_D} = \frac{\bar{\varphi}_D^*}{\varphi_D^*} = \frac{\bar{\varphi}_X}{\varphi_X} = \frac{\bar{\varphi}_X^*}{\varphi_X^*} = -\frac{1-\beta}{\beta},$$

that is, there is a negative intertemporal relation between the cutoff rules given by the negative of the steady state interest rate, $\frac{1-\beta}{\beta}$. This also implies that the changes in the new steady state

cutoff rules are much smaller. This negative relation is a consequence of the current account going back to zero in the new steady state. After a change in the *NFA* position in the short-run, the debtor country will need to run trade surpluses every period from time 2 to cover the interest payments to the creditor country. The amount of interest payment is given by $\frac{1-\beta}{\beta}$ times the *NFA* position resulting at the time of the shock. Hence, the model implies that if a country has a trade surplus at the time of the shock, it must run trade deficits thereafter, adjusting the cutoff rules in the opposite direction.

The log-deviations for the number of entrants are straightforward. As we can expect from the relations in the cutoff rules, we have that $\frac{n_E}{n_E^*} = \frac{\bar{n}_E}{\bar{n}_E^*} = -1$ and $\frac{\bar{n}_E}{n_E} = \frac{\bar{n}_E^*}{n_E^*} = -\frac{1-\beta}{\beta}$.

In the following Section we go deeper into the solution, and analyze the model's implications for exchange-rate pass-through and trade flows.

5 The Impact of Exchange Rate Changes in General Equilibrium

Deviations from monetary neutrality in the long-run are given exclusively by the interest income on the *NFA* position resulting at the time of the shock. The creditor country will enjoy more leisure, while the opposite happens for the debtor country.

Given that the relative importance of interest income should be rather small, we expect a much smaller impact of a monetary shock in real variables in the long-run. We already saw in the previous section that this is the case for the productivity cutoff rules, whose long-run percent changes have the opposite sign than in the short-run but with a much smaller magnitude, given simply by the steady state interest rate times the short-run change.

This implies that wages and prices are almost neutral in the long-run, and therefore, import prices for individual varieties should have pass-through rates of exchange rate changes close to 1. Thus, in this Section we focus on the impact of exchange rate changes at the time of the shock, and we only refer to the long run impact in the final numerical exercise.

5.1 Exchange-Rate Pass-Through and Trade Flows in the Short-Run

Log-linearizing equation (11), the Home currency price of an imported variety from a firm with productivity φ , around the symmetric steady state and expressing log-deviations with sanserif

font, we get that

$$p_X^*(\varphi) = e - \frac{\varphi}{\varphi + \tau\varphi_{D,0}} \varphi_X^*,$$

where we make use of the fact that wages are sticky at the time of the shock, that is, $w^* = 0$.

Therefore, the exchange rate pass-through elasticity is simply given by

$$\frac{p_X^*(\varphi)}{e} = 1 - \frac{\frac{\varphi_X^*}{e}}{1 + \frac{\tau\varphi_{D,0}}{\varphi}}, \quad (25)$$

which is almost identical to equation (12). Note that we are preserving both, the firm-specific effect, driven by $\frac{\tau\varphi_{D,0}}{\varphi}$, and the industry-wide competition effect, driven by $\frac{\varphi_X^*}{e}$. The GE model also preserves the inverse relation between a firm's productivity and the pass-through rate to the export price (in the importer's currency) of its variety. For the most productive firms, those with $\varphi \rightarrow \infty$, only the industry-wide effect matters.

There is an important difference, however, from the partial equilibrium model. To solve the general equilibrium model, we made use of a first-order linearization. Therefore, in the solution of the GE model we lose the second order effects which implied asymmetric responses of import prices to appreciations and depreciations. In the solution for the linearized model, we get that the industry-wide effect is always constant and given by

$$\frac{\varphi_X^*}{e} = \frac{\tau^k(k+2) - 1}{(k+2)(\tau^k - 1)} > 0.$$

On the other hand, the firm-specific component for an individual exporting firm is also constant, as it depends on $\tau\varphi_{D,0}$.

As before, the exchange-rate pass-through elasticity for the average import price is given by $-\frac{\varphi_D}{e}$. In the GE solution, this is equivalent to

$$-\frac{k+1}{(k+2)(\tau^k - 1)} < 0.$$

As in the partial equilibrium model, the average import price reflects exclusively the industry-wide competition effect.¹⁶

¹⁶Remember from the partial equilibrium model that we can also use $\frac{\varphi_D}{e}$ to refer to the industry-wide competition effect, as $\frac{\varphi_D}{e} > 0$ and $\frac{\varphi_X^*}{e} = 1 + \frac{\varphi_D}{e}$.

The trade flows elasticities with respect to the exchange rate are given by

$$\frac{ve}{e} = k + \frac{n_E}{e} - (k+2)\frac{\varphi_D^*}{e} > 0$$

$$\frac{ve^*}{e} = -k + \frac{n_E^*}{e} - (k+2)\frac{\varphi_D}{e} < 0,$$

where ve and ve^* are the log deviations of Home exports (Foreign imports) and Home imports (Foreign exports), respectively, in terms of the Home currency. Each component in the previous equations reinforce each other. Therefore, important changes in trade flows are expected.

5.2 A Numerical Example

We now simulate the model for an unexpected and permanent 5% increase in the money supply at Home. We set $\beta = 0.9$ as the length of the period should be long enough to allow for firm reallocations. $\frac{\theta-1}{\theta\kappa}$ is set at 1000, which ensures a positive consumption of the nontraded good. The rest of the parameters are set as in Section 3.4.

The solution is reported in Table 4. As we can observe, changes in real variables in the new steady state are very small compared to the changes at the time of the shock. The exchange rate, which jumps immediately to its new steady state value, changes by less than the money supply shock. This happens because the country receiving more income will also demand more leisure, and hence a smaller change in the exchange rate is needed. Note also the important reallocation effects in the short-run.

Under these assumptions, the exchange rate pass-through elasticity for the Home price of an imported variety produced with relative productivity φ is given by

$$\frac{p_X^*(\varphi)}{e} = 1 - \frac{1.293}{1 + \frac{0.178}{\varphi}}.$$

Note that we have not imposed any lower bound condition for the pass-through rate, so that in theory, it can be negative for extremely productive firms. In this case, the pass-through rate becomes negative for firms with φ greater than 0.606. However, according to our distribution and our chosen parameter values, only the 0.07% of initial Foreign entrants and the 0.74% of the Foreign firms that were exporting in the initial steady state are above this level.

The pass-through rate to the average import price is -0.293, which is striking when it is

compared with the 99.25% of initial Foreign varieties with positive pass-through rates. As mentioned above, this price goes down because we are averaging over more productive firms, which have lower pass-through rates, and who also adjust their markups down because of the increase in competition generated by Home entrants.

The changes in trade flows this model generates are extremely big for any combination of parameter values. It is enough to see from the previous Section, that their elasticities with respect to the exchange rate are bounded below by k , in absolute value.

6 Conclusion

This paper has presented partial and general equilibrium versions of a model of trade with monopolistic competition, heterogeneous firms and endogenous markups.

As a consequence of endogenous markups, more productive firms have always lower pass-through rates. Moreover, for an individual exporting firm, we were able to distinguish between two opposite effects coming from exchange rate changes: an industry-wide competition effect, reflecting changes in average prices and in the extensive margin of trade; and a firm-specific effect, related to the firm's position with respect to the cutoff rule. Besides, we found that these effects are asymmetric for appreciations and depreciations of a currency.

An interesting result from the model is that the average import price does not reflect the real pass-through rates of surviving firms. This is the case because aggregate price indexes are affected directly by the impact of exchange rate changes on the extensive margin of trade, which is the main dynamic force driving trade flows in heterogeneous-firm models. In case of a Home currency depreciation, that pushes Foreign firms out of the Home market, the new average import price is computed over the most productive surviving Foreign firms, who have lower pass-through rates. Besides, there is an increase in competition from Home producers that shifts down the markups of the surviving firms. Our model derives negative pass-through rates for the average import price. This is certainly overstating the impact of exchange rates on aggregate price indexes, as it considers strong cross-country reallocation effects, but it sends a clear signal of the possible misleading results that pass-through studies might have when making conclusions about expenditure-switching effects. In the Appendix, we show that even in the absence of changes in the pattern of entry, firm heterogeneity and endogenous markups tend to deliver very low (but positive) pass-through rates at the aggregate level and important

expenditure-switching effects.

Future lines of research include the incorporation of second order effects in the NOEM model solution. Empirical work is also needed to look for evidence of asymmetric exchange-rate pass-through elasticities at the firm-level. Finally, to validate the conclusions of this paper about the disconnection between final consumer import prices and expenditure-switching effects, it is also necessary to empirically estimate, first, the impact of exchange rates on the extensive margin of trade, and second, the impact of the latter on aggregate import price indexes.

Appendix

The Partial Equilibrium Model without Cross-Country Firm Reallocations

Most of the conclusions of the paper, including the asymmetric pass-through elasticities at the firm level and the negative pass-through elasticity of the average import price, rely on the exchange rate impact on cross-country firm reallocations. As in Melitz and Ottaviano (2005), we now study the implications of the partial equilibrium model when changes in the pattern of entry are not allowed. This analysis can be regarded as the immediate effects of exchange rate changes, when cross-country firm reallocations are not possible.

Each country has a fixed number of producers, or incumbents, denoted by \bar{N}_E and \bar{N}_E^* for Home and Foreign, respectively. As before, the productivity of this incumbents is Pareto distributed on the interval $[\varphi_{\min}, \infty)$. Depending on the productivity cutoff rules, incumbents can produce or shut down. If an incumbent is not producing and the cutoff rules change so that it can produce now, it will not have to pay the sunk cost again.

As long as the productivity cutoff rules are above φ_{\min} , so that not all incumbent firms produce, our equilibrium must satisfy

$$\left[\frac{\varphi_{\min}}{\varphi_{D,NR}} \right]^k \bar{N}_E + \left[\frac{\varphi_{\min}}{\varphi_{X,NR}^*} \right]^k \bar{N}_E^* = N_{NR} \quad (\text{A-1})$$

$$\left[\frac{\varphi_{\min}}{\varphi_{X,NR}} \right]^k \bar{N}_E + \left[\frac{\varphi_{\min}}{\varphi_{D,NR}^*} \right]^k \bar{N}_E^* = N_{NR}^* \quad (\text{A-2})$$

where $N_{NR} = \frac{2\gamma(k+1)}{\eta}(\alpha Z \varphi_{D,NR} - 1)$ and $N_{NR}^* = \frac{2\gamma(k+1)}{\eta}(\alpha Z^* \varphi_{D,NR}^* - 1)$. The subscript NR indicates “no cross-country reallocations”.

Substituting equations (3) and (4) into (A-1) and (A-2), respectively, we get that

$$\frac{\eta\varphi_{\min}^k}{2\gamma(k+1)} \left[\bar{N}_E + \left(\frac{Z^*W}{\tau^*Z\mathcal{E}W^*} \right)^k \bar{N}_E^* \right] = \varphi_D^k(\alpha Z\varphi_D - 1) \quad (\text{A-3})$$

$$\frac{\eta\varphi_{\min}^k}{2\gamma(k+1)} \left[\left(\frac{Z\mathcal{E}W^*}{\tau Z^*W} \right)^k \bar{N}_E + \bar{N}_E^* \right] = \varphi_D^{*k}(\alpha Z^*\varphi_D^* - 1). \quad (\text{A-4})$$

Even though we cannot solve directly for φ_D and φ_D^* , it is easy to see from the previous equation that there is a negative relation between φ_D and \mathcal{E} , and a positive relation between φ_D^* and \mathcal{E} . These results are contrary to those found when we allow for cross-country firm reallocations.

The elasticity of the cutoff rule $\varphi_{D,NR}$ with respect to the exchange rate is given by

$$\zeta_{\varphi_{D,NR},\mathcal{E}} = -\frac{k}{k+1} \left[\frac{\varphi_{D,NR}^k(\alpha Z\varphi_{D,NR} - 1) - \frac{\eta\varphi_{\min}^k}{2\gamma(k+1)}\bar{N}_E}{\varphi_{D,NR}^k(\alpha Z\varphi_{D,NR} - 1) + \frac{\varphi_{D,NR}^k}{k+1}} \right], \quad (\text{A-5})$$

which is always negative and smaller than $\frac{k}{k+1}$ in absolute value, as the numerator in the second term is always positive (by equation (A-3)) and smaller than the denominator.

The intuition for this result is clear. If the Home currency depreciates, Foreign firms lose competitiveness at Home and some of them will have to exit. These Foreign firms are then replaced by Home firms that were already in the industry, but that were not producing because their productivity was lower than the cutoff rule. Therefore, $\varphi_{D,NR}$ decreases, shifting down the average productivity of Home producers selling at Home. On the other hand, an appreciation will displace some Home firms, which are replaced by Foreign firms that are now competitive. The displaced Home firms are the most unproductive ones of the original producers. Therefore, $\varphi_{D,NR}$ increases.

We can also show that $\varphi_{D,NR}$ is strictly convex in the exchange rate, that is, $\frac{\partial^2 \varphi_{D,NR}}{\partial \mathcal{E}^2} > 0$. Contrary to the cross-country reallocations case, in which the impact of a Home currency depreciation on φ_D was larger than the impact of a proportional appreciation, in this case the opposite is true.

The convexity of $\varphi_{D,NR}$ also implies that $\zeta_{\varphi_{D,NR},\mathcal{E}}$ is strictly increasing and approaching to zero as \mathcal{E} increases. Intuitively, for high levels of the exchange rate (a highly undervalued Home currency) and no entry allowed, import competition will be very small; therefore, effects of the exchange rate on the domestic cutoff rule should be small. On the other hand, for low levels

of the exchange rate (Home currency highly overvalued), import competition is high and bigger changes in the cutoff rule are expected.

With cross-country firm reallocations φ_D and φ_X^* move in the same direction with exchange rate changes. This is not the case when cross-country reallocations are shut down. From equation (A-5), we have that $\zeta_{\varphi_{D,NR},\mathcal{E}}$ is in the interval $(-k/(k+1), 0)$. Therefore, as $\zeta_{\varphi_{X,NR}^*,\mathcal{E}} = \zeta_{\varphi_{D,NR},\mathcal{E}} + 1$, we have that $\zeta_{\varphi_{X,NR}^*,\mathcal{E}}$ is in the interval $(1/(k+1), 1)$. Analogously, we have that $\zeta_{\varphi_{D,NR}^*,\mathcal{E}} \in (0, k/(k+1))$ and $\zeta_{\varphi_{X,NR},\mathcal{E}} \in (-1, -1/(k+1))$. In words, this implies that a Home currency depreciation creates a more favorable competition environment for Home producers at Home and abroad. Home firms are substituting imports at Home and displacing Foreign firms in their own market. On the other hand, Foreign producers see tougher competition environments in both countries.

Under the symmetry assumptions of Section 2.6, the exchange-rate pass-through elasticity for import prices at the firm level is as in equation (12), but now the industry-wide competition effect is much weaker, which is translated into higher pass-through rates. It is important to mention that the conclusion about more productive firms having lower pass-through rates is sustained. As before, the industry-wide effect drives down the pass-through rate with increases in \mathcal{E} , while the opposite occurs with the firm-specific effect. However, in this case the industry-wide effect dominates at low-levels of the exchange rate, while the firm-specific effect dominates for high levels. Therefore, the exchange rate pass-through for import prices at the firm level is U-shaped.

As before, the pass-through rate for the average import price is given by $-\zeta_{\varphi_{D,NR},\mathcal{E}}$. Although this rate is now positive, it will not be a reliable indicator of the expenditure-switching effects. Note that the average import price will reflect only the industry-wide competition effect, showing a lower pass-through rate for a depreciation than for a proportional appreciation. This rate is totally different to the one we got for an individual exporting firm. Even though we do not allow for cross-country reallocation effects, there are still changes in the extensive margin of trade as there are selection effects in the pool of incumbent firms in both countries.

The trade flow equations in Home currency are as given in Section 3.3. With no cross-country reallocation effects, the value of Home exports increases with the exchange rate if and only if $\zeta_{\varphi_{X,NR},\mathcal{E}} < -\frac{1}{k+2}$, which is always satisfied. On the other hand, the value of Foreign exports (or Home imports) in Home currency decreases with the exchange rate if and only if

$\zeta_{\varphi_{D,NR},\mathcal{E}} > -\frac{k}{k+2}$. Therefore, there is a range for $\zeta_{\varphi_{D,NR},\mathcal{E}}$, between $-\frac{k}{k+1}$ and $-\frac{k}{k+2}$, in which the value of Home imports increases with a depreciation. Note, however, that the value of Foreign exports measured in Foreign currency always decreases with a Home currency depreciation.¹⁷

For illustrative purposes, Figure 2 shows the effects of exchange rate changes when cross-country reallocation effects are absent. For this case, we assume that \bar{N}_E and \bar{N}_E^* are given by the number of entrants when $\mathcal{E} = 1$.

Panel 2a presents the elasticity of $\varphi_{D,NR}$ with respect to the exchange rate. This elasticity is negative and converging to zero. As mentioned above, a Home currency depreciation substitutes Foreign producers selling at Home with inefficient Home producers, whose increase in competitiveness comes exclusively from the exchange rate change. Panel 2b shows the changes in the composition and total number of sellers at Home.

The exchange-rate pass-through for the imported variety coming from the mean Foreign exporter is showed in Panel 2c. The pass-through function is U-shaped, implying that the industry-wide effect dominates for low levels of the exchange rate, while the firm-specific effect dominates for high levels. Note, however, that the range for the pass-through rate is very limited in this case.

Panel 2d presents the evolution of the cutoff marginal cost rules (before trade costs) for firms selling at Home and the average price. Though positive, as given by $-\zeta_{\varphi_{D,NR},\mathcal{E}}$, the pass-through rate to the average price is small and very asymmetric compared to individual pass-through rates. This result implies that the use of average prices in pass-through studies underestimates actual pass-through rates. Moreover, even if we were able to estimate correct pass-through rates at the disaggregated level, they would be of little help to conclude about expenditure-switching effects, as firms may be adjusting their markups, not their quantities, and important changes in the extensive margin of trade might be present. Finally, Panel 2e shows that, although not as important as with cross-country reallocations, exchange rate changes have an important impact on trade flows.

¹⁷The value of Foreign exports in Foreign currency decreases with the exchange rate if and only if $\zeta_{\varphi_{D,NR},\mathcal{E}} > -\frac{k+1}{k+2}$, which is always true.

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Table 1: Model Summary

Cutoff productivity rules	$\varphi_{D,t} = \frac{1}{\tau_t^*} \left[\frac{Z_t^*}{Z_t} \right] \left[\frac{W_t}{\varepsilon_t W_t^*} \right] \varphi_{X,t}^*$ $\varphi_{D,t}^* = \frac{1}{\tau_t} \left[\frac{Z_t}{Z_t^*} \right] \left[\frac{\varepsilon_t W_t^*}{W_t} \right] \varphi_{X,t}$
Free-entry conditions	$\frac{\varphi_{\min}^k}{2\gamma(k+1)(k+2)} \frac{W_t}{Z_t} \left[\frac{1}{W_t \varphi_{D,t}} + \frac{\tau_t^2}{\varepsilon_t W_t^* \varphi_{X,t}} \right] = f_{E,t}$ $\frac{\varphi_{\min}^k}{2\gamma(k+1)(k+2)} \frac{W_t^*}{Z_t^*} \left[\frac{1}{W_t^* \varphi_{D,t}} + \frac{\tau_t^{*2} \varepsilon_t}{W_t \varphi_{X,t}} \right] = f_{E,t}^*$
Number of entrants	$N_{E,t} = \frac{2\gamma(k+1)}{\varphi_{\min}^k \eta(\tau_t \tau_t^*)^{k-1}} \left[(\tau_t \tau_t^*)^k \varphi_{D,t}^k (\alpha Z_t \varphi_{D,t} - 1) - \left[\frac{\tau_t Z_t^* W_t}{Z_t \varepsilon_t W_t^*} \right]^k \varphi_{D,t}^k (\alpha Z_t^* \varphi_{D,t}^* - 1) \right]$ $N_{E,t}^* = \frac{2\gamma(k+1)}{\varphi_{\min}^k \eta(\tau_t \tau_t^*)^{k-1}} \left[(\tau_t \tau_t^*)^k \varphi_{D,t}^k (\alpha Z_t^* \varphi_{D,t}^* - 1) - \left[\frac{\tau_t^* Z_t \varepsilon_t W_t^*}{Z_t^* W_t} \right]^k \varphi_{D,t}^k (\alpha Z_t \varphi_{D,t} - 1) \right]$
Labor demands	$L_t^D = N_{E,t} \frac{f_{E,t}}{Z_t} + N_{E,t} \frac{\varphi_{\min}^k}{2\gamma(k+1)(k+2)Z_t^2} \left[\frac{1}{\varphi_{D,t}} + \tau_t^2 \frac{W_t}{\varepsilon_t W_t^*} \frac{1}{\varphi_{X,t}} \right] + c_{N,t}$ $L_t^{*D} = N_{E,t}^* \frac{f_{E,t}^*}{Z_t^*} + N_{E,t}^* \frac{\varphi_{\min}^k}{2\gamma(k+1)(k+2)Z_t^{*2}} \left[\frac{1}{\varphi_{D,t}} + \tau_t^{*2} \frac{\varepsilon_t W_t^*}{W_t} \frac{1}{\varphi_{X,t}} \right] + c_{N,t}^*$
Nontraded good consumption	$c_{N,t} = C_t - \frac{(\alpha Z_t \varphi_{D,t} - 1) \varphi_{D,t}^k}{2\gamma(k+2)(Z_t \varphi_{D,t})^2}$ $c_{N,t}^* = C_t^* - \frac{(\alpha Z_t^* \varphi_{D,t}^* - 1) \varphi_{D,t}^{*k}}{2\gamma(k+2)(Z_t^* \varphi_{D,t}^*)^2}$ $C_{t+1} = \beta(1 + r_{t+1})C_t$ $C_{t+1} = \beta(1 + r_{t+1}^*) \frac{Q_{t+1}}{Q_t} C_t, \text{ where } Q_t = \frac{\varepsilon_t W_t^*}{W_t}$ $C_{t+1}^* = \beta(1 + r_{t+1}^*) C_t^*$ $C_{t+1}^{*} = \beta(1 + r_{t+1}) \frac{Q_t}{Q_{t+1}} C_t^*$
Euler equations (money)	$\frac{M_t}{W_t} = \frac{1+i_{t+1}}{i_{t+1}} \chi C_t \text{ where } 1 + i_{t+1} = (1 + r_{t+1}) \frac{W_{t+1}}{W_t}$ $\frac{M_t^*}{W_t^*} = \frac{1+i_{t+1}^*}{i_{t+1}^*} \chi C_t^* \text{ where } 1 + i_{t+1}^* = (1 + r_{t+1}^*) \frac{W_{t+1}^*}{W_t^*}$
Euler equations (labor)	$L_t = \frac{\theta-1}{\theta \kappa} \frac{1}{C_t}$ $L_t^* = \frac{\theta-1}{\theta \kappa} \frac{1}{C_t^*}$
Net foreign assets	$NFA_{t+1} = (1 + r_t) NFA_t + [Q_t(1 + r_t^*) - Q_{t-1}(1 + r_t)] B_{*t} - \frac{1}{2} (C_t - Q_t C_t^*) + \frac{1}{2} (L_t - Q_t L_t^*)$ $+ \frac{1}{2} \left[\frac{(\alpha Z_t \varphi_{D,t} - 1)(k+2) \alpha Z_t \varphi_{D,t}^{k-1}}{2\gamma(k+2)(Z_t \varphi_{D,t})^2} - Q_t \frac{(\alpha Z_t^* \varphi_{D,t}^* - 1)[(k+2) \alpha Z_t^* \varphi_{D,t}^{*k-1}]}{2\gamma(k+2)(Z_t^* \varphi_{D,t}^*)^2} \right]$

Table 2: Symmetric Steady State

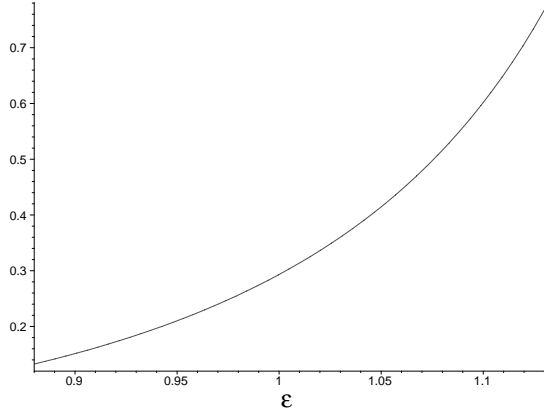
$\varphi_{D,0} = \varphi_{D,0}^* = \frac{1}{\tau} \varphi_{X,0} = \frac{1}{\tau} \varphi_{X,0}^* = \left[\frac{\varphi_{\min}^k}{2\gamma(k+1)(k+2)f_E} \frac{\tau^{k+1}}{\tau^k} \right]^{\frac{1}{k+2}}$
$N_{E,0} = N_{E,0}^* = \frac{\alpha\varphi_{D,0}-1}{\eta(k+2)f_E\varphi_{D,0}^2}$
$C_0 = C_0^* = \frac{(\alpha\varphi_{D,0}-1)[(k+2)\alpha\varphi_{D,0}-k-1]}{4\eta(k+2)\varphi_{D,0}^2} + \frac{1}{2} \sqrt{\left(\frac{(\alpha\varphi_{D,0}-1)[(k+2)\alpha\varphi_{D,0}-k-1]}{2\eta(k+2)\varphi_{D,0}^2} \right)^2 + 4 \left(\frac{\theta-1}{\theta\kappa} \right)}$
$L_0 = L_0^* = \frac{\theta-1}{\theta\kappa} \frac{1}{C_0}$
$c_{N,0} = c_{N,0}^* = \left[\frac{\theta-1}{\theta\kappa} \right] \frac{1}{C_0} - \frac{(k+1)(\alpha\varphi_{D,0}-1)}{\eta(k+2)\varphi_{D,0}^2}$
$r_0 = r_0^* = i_0 = i_0^* = \frac{1-\beta}{\beta}$
$\mathcal{E}_0 = \frac{M_0}{M_0^*} = \frac{W_0}{W_0^*}, \text{ then } Q_0 = 1$
$W_0 = \mathcal{E}_0 W_0^* = \left[\frac{1-\beta}{\chi C_0} \right]^{\frac{1}{\lambda}} M_0$
$NFA_0 = B_{*0} = 0$

Table 3: Linearized Model

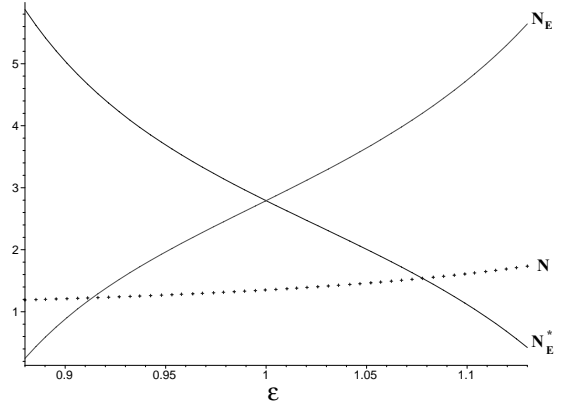
Cutoff productivity rules	$\varphi_{D,t} = w_t - e_t - w_t^* + \varphi_{X,t}^*$ $\varphi_{D,t}^* = -w_t + e_t + w_t^* + \varphi_{X,t}$
Free-entry conditions	$w_t = -k\varphi_{D,t} + \frac{1}{\tau^k+1} [(\tau^k+2)w_t - e_t - w_t^* - (2\tau^k - k)\varphi_{D,t} - (k+2)\varphi_{X,t}]$ $w_t^* = -k\varphi_{D,t}^* + \frac{1}{\tau^k+1} [(\tau^k+2)w_t^* + e_t - w_t - (2\tau^k - k)\varphi_{D,t}^* - (k+2)\varphi_{X,t}^*]$
Number of entrants	$(\tau^k - 1)n_{E,t} = \left[\frac{\alpha\varphi_{D,0}}{\alpha\varphi_{D,0}-1} + k \right] (\tau^k\varphi_{D,t} - \varphi_{D,t}^*) + k(-w_t + e_t + w_t^*)$ $(\tau^k - 1)n_{E,t}^* = \left[\frac{\alpha\varphi_{D,0}}{\alpha\varphi_{D,0}-1} + k \right] (\tau^k\varphi_{D,t}^* - \varphi_{D,t}) + k(w_t - e_t - w_t^*)$
Labor demands	$\left[\frac{\theta-1}{\theta\kappa} \frac{1}{C_0} \right] \left[\frac{\eta(k+2)\varphi_{D,0}^2}{\alpha\varphi_{D,0}-1} \right] (l_t - c_{N,t}) = (k+1)n_{E,t} + k\frac{\tau^k}{\tau^k+1} \left[\frac{1}{\tau^k} (w_t - e_t - w_t^*) - (k+2)(\varphi_{D,t} + \frac{1}{\tau^k}\varphi_{X,t}) \right] - (k+1)c_{N,t}$ $\left[\frac{\theta-1}{\theta\kappa} \frac{1}{C_0} \right] \left[\frac{\eta(k+2)\varphi_{D,0}^2}{\alpha\varphi_{D,0}-1} \right] (l_t^* - c_{N,t}^*) = (k+1)n_{E,t}^* + k\frac{\tau^k}{\tau^k+1} \left[\frac{1}{\tau^k} (-w_t + e_t + w_t^*) - (k+2)(\varphi_{D,t}^* + \frac{1}{\tau^k}\varphi_{X,t}^*) \right] - (k+1)c_{N,t}^*$
Nontraded good consumption	$\left[\frac{\theta-1}{\theta\kappa} \frac{1}{C_0} - \frac{(k+1)(\alpha\varphi_{D,0}-1)}{\eta(k+2)\varphi_{D,0}^2} \right] c_{N,t} = C_0 c_t - \frac{1}{2\eta(k+2)} \left[\frac{\alpha}{\varphi_{D,0}} + \frac{2(k+1)}{\varphi_{D,0}^2} \right] \varphi_{D,t}$ $\left[\frac{\theta-1}{\theta\kappa} \frac{1}{C_0} - \frac{(k+1)(\alpha\varphi_{D,0}-1)}{\eta(k+2)\varphi_{D,0}^2} \right] c_{N,t}^* = C_0 c_t^* - \frac{1}{2\eta(k+2)} \left[\frac{\alpha}{\varphi_{D,0}} + \frac{2(k+1)}{\varphi_{D,0}^2} \right] \varphi_{D,t}^*$ $c_{t+1} = c_t + (1-\beta)r_{t+1}$
Euler equations (bonds)	$c_{t+1} = q_{t+1} - q_t + c_t + (1-\beta)r_{t+1}^*$, where $q_t = e_t + w_t^* - w_t$ $c_{t+1}^* = c_t^* + (1-\beta)r_{t+1}^*$ $c_{t+1}^* = q_t - q_{t+1} + c_t^* + (1-\beta)r_{t+1}$
Euler equations (money)	$m_t - w_t = c_t - \beta r_{t+1} - \frac{\beta}{1-\beta} (w_{t+1} - w_t)$
Euler equations (labor)	$m_t^* - w_t^* = c_t^* - \beta r_{t+1}^* - \frac{\beta}{1-\beta} (w_{t+1}^* - w_t^*)$ $l_t = -c_t$ $l_t^* = -c_t^*$
Net foreign assets	$NFA_{t+1} = \frac{1}{\beta} NFA_t - \frac{1}{2} C_0 (c_t - c_t^* - q_t) + \frac{1}{4\eta(k+2)} \left[-(k+2)\alpha^2 q_t + \frac{(2k+3)\alpha}{\varphi_{D,0}} (\varphi_{D,t} - \varphi_{D,t}^* + q_t) \right]$ $-\frac{k+1}{4\eta(k+2)\varphi_{D,0}^2} (2\varphi_{D,t} - 2\varphi_{D,t}^* + q_t) + \frac{\theta-1}{\theta\kappa} \frac{1}{C_0} (l_t - l_t^* - q_t)$

Table 4: 5% Increase in Home Money Supply

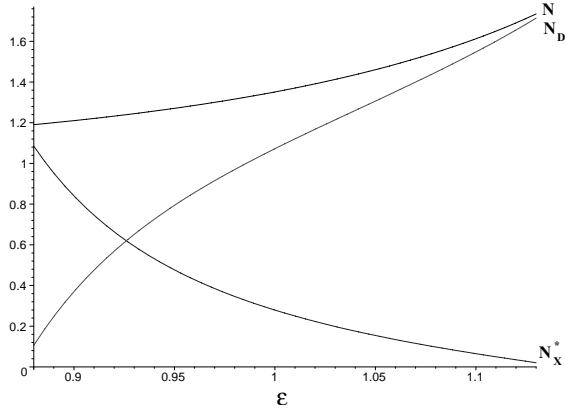
Time of the shock		New steady state	
φ_D	0.011	$\bar{\varphi}_D$	-0.001
φ_D^*	-0.011	$\bar{\varphi}_D^*$	0.001
φ_X	-0.049	$\bar{\varphi}_X$	0.005
φ_X^*	0.049	$\bar{\varphi}_X^*$	-0.005
n_E	0.216	\bar{n}_E	-0.024
n_E^*	-0.216	\bar{n}_E^*	0.024
c	0.050	\bar{c}	0.004
c^*	0.000	\bar{c}^*	-0.004
l	0.136	\bar{l}	-0.004
l^*	-0.079	\bar{l}^*	0.004
c_N	0.065	\bar{c}_N	0.013
c_N^*	0.040	\bar{c}_N^*	-0.013
r	-0.458	\bar{r}	0.000
r^*	-0.042	\bar{r}^*	0.000
e	0.038	\bar{e}	0.038
\overline{NFA}	2.598	\bar{w}	0.046
		\bar{w}^*	0.004



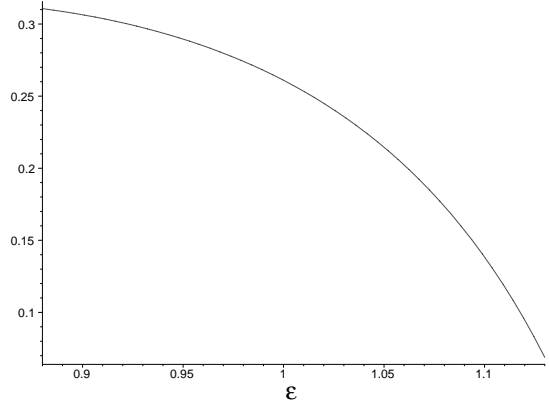
(a) Elasticity of φ_D with respect to ε



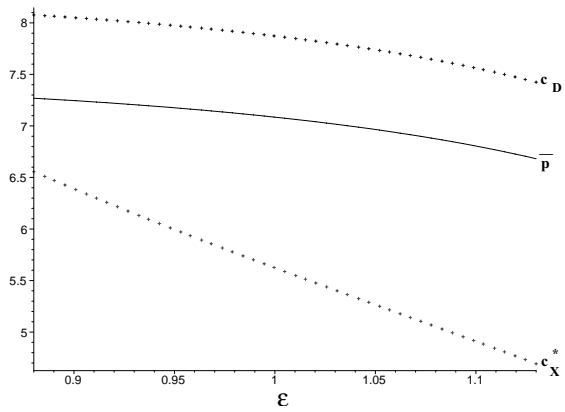
(b) Cross-country firm reallocations



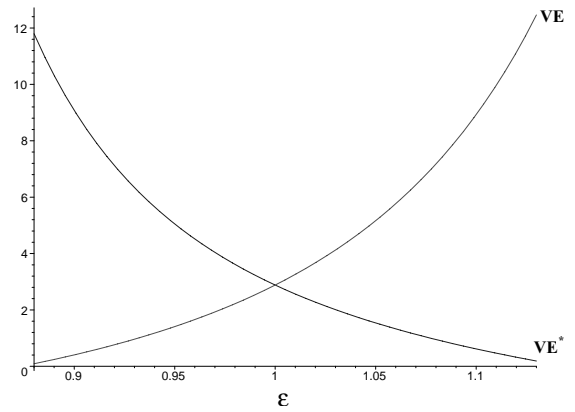
(c) Composition of sellers



(d) Exchange rate pass-through to firm import price

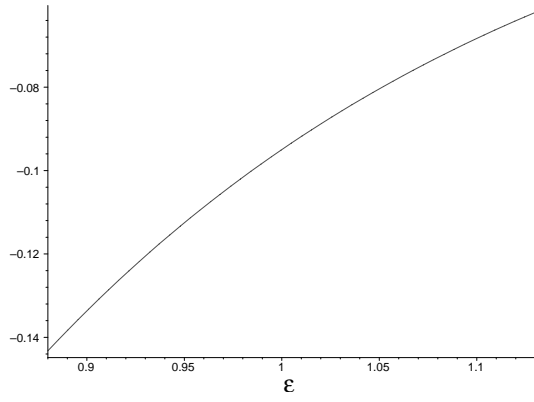


(e) Cutoff costs and average price

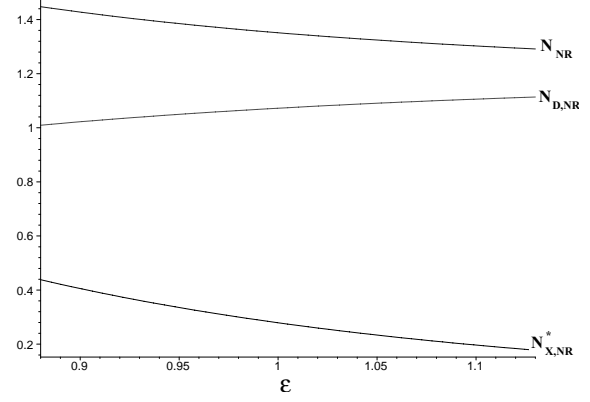


(f) Trade flows

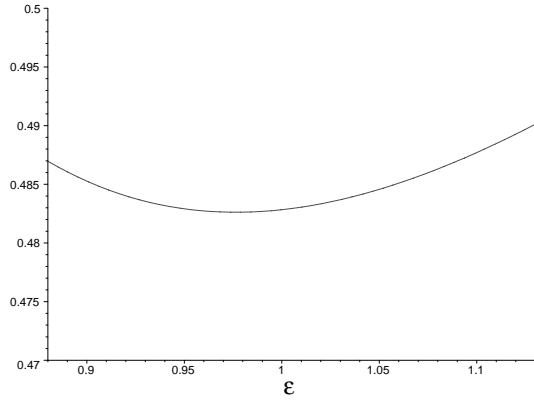
Figure 1: Effects of Exchange Rate Changes in the Partial Equilibrium Model



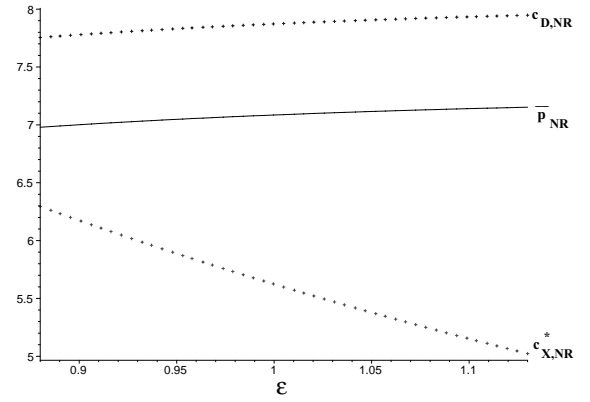
(a) Elasticity of $\varphi_{D,NR}$ with respect to ε



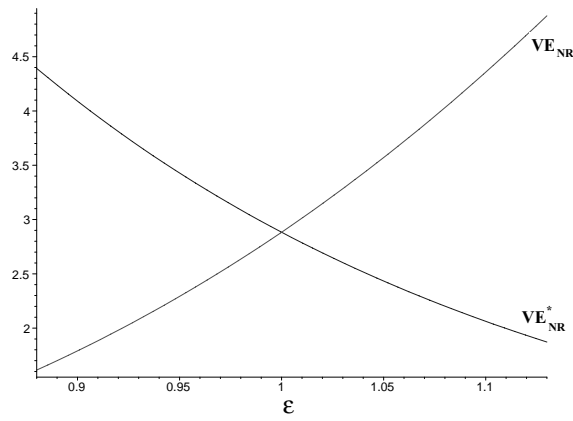
(b) Composition of sellers



(c) Exchange rate pass-through to firm import price



(d) Cutoff costs and average price



(e) Trade flows

Figure 2: Partial Equilibrium Model without Cross-Country Firm Reallocations