

# Downstream mergers and producer's capacity choice: why bake a larger pie when getting a smaller slice?

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*We study the effect of downstream horizontal mergers on the upstream producer's capacity choice. Contrary to conventional wisdom, we find a non-monotonic relationship: horizontal mergers induce a higher upstream capacity if the cost of capacity is low, and a lower upstream capacity if this cost is high. We explain this result by decomposing the total effect into two competing effects: a change in hold-up and a change in bargaining erosion.*

## 1 Introduction

A growing debate in the antitrust arena concerns the negative long term effects of downstream horizontal integration on upstream producers' investment incentives. For example, the position held by the European Commission in two recent decisions on mergers of leading retailers, *Kesko/Tuko* and *Rewe/Meinl*, created jurisprudence in merger control: competitive assessment should no longer be restricted to the downstream market but should also look for adverse effects on input markets.<sup>1</sup>

The present paper studies how downstream horizontal mergers affect the capacity choice of an upstream producer. It seems intuitive that following a merger the producer's bargaining power vis-a-vis the downstream firms is weakened and, since the producer then gets a lower share of the surplus, his incentives to invest in capacity are also reduced. Perhaps surprisingly then, working from fundamentals, we find a non-monotonic relationship between downstream horizontal integration and the upstream equilibrium capacity.

We study a two-stage game. In the first stage the upstream producer chooses capacity and pays for its cost. In the second stage the producer bargains with downstream firms over input supply. Since the solution concept determines the allocation of the bargaining surplus, and therefore investment incentives, this choice is a crucial step. Like other authors studying the effects of integration, we use the Shapley value (e.g. Hart and Moore, 1990; Inderst and Wey, 2003; Segal, 2003).

Our main result is that the cost of capacity provides a simple criterion for evaluating claims about the effect of downstream horizontal mergers on the producer's capacity choice: a downstream merger induces a higher equilibrium capacity upstream if the cost of capacity is low, and the converse is true if the cost of capacity is high. We explain this result by decomposing the total effect into two effects. On the one hand, as expected, the merger decreases the share of the surplus accruing to the producer, i.e. the extent to which downstream firms are able to *hold-up*

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<sup>1</sup>*Kesko/Tuko*, Case IV/M.784, Commission decision of November 20 1996. *Rewe/Meinl*, Case COM/M.1121, Commission decision of February 3 1999. This concern is also present in the European Commission's guidelines on the applicability of the Article 81 (2001) with regard to purchasing agreements.

the producer *is increased*. On the other hand, for any given market configuration, increasing capacity erodes the bargaining power of the producer since competition for an input gets weaker when this input becomes more abundant. A downstream merger *reduces the rate* at which this *bargaining erosion* takes place. This latter effect counteracts the former, and will dominate when the cost of capacity is low.

Surprisingly, while the hold-up literature has studied the effect of vertical integration, there exists little formal analysis of the *investment* effects of downstream horizontal integration—the growing literature on buyer power has focused on mergers’ *distributive* effect. The paper closest to ours is Hart and Moore (1990) which, assuming that marginal contributions are independent of the level of investment, finds that “...as one might expect, if two competing traders merge, this will worsen the incentives of the owner-manager of a firm that trades with them” (Hart and Moore, 1990, p. 1148). Other papers studying *continuous* investment choices find a similar negative relationship between downstream mergers and upstream investment (e.g. Chae and Heidhues, 1999; Inderst and Shaffer, 2005). The present paper shows precisely that this result may change significantly when the above mentioned assumption is not satisfied.

A few other papers look at *discrete* technology choices. Stole and Zwiebel (1996a, 1996b) find that a firm dealing with independent workers has a preference for technologies which give rise to more concave surplus functions since this leverages its bargaining power; this bias is absent when the workforce is unionized. Inderst and Wey (2003, 2006) show that downstream horizontal mergers reduce a similar bias, inducing the producer to focus on value creation itself. Our work extends this analysis in two ways. First, and similar to the property rights literature, we focus on a continuous investment choice and show that downstream horizontal mergers may result in *more* upstream investment (a meaningless statement for a discrete technological choice). Second, we account for the problem of producer’s opportunism in related output markets studied in the vertical contracting literature (e.g. McAfee and Schwartz, 1994; Segal and Whinston, 2003).

Most of the above-mentioned literature uses linear bargaining solutions—e.g. the Shapley value. Recent work has shown that results derived using a linear bargaining solution may not be robust in settings with nonlinear bargaining solutions (e.g. Chiu, 1998; de Meza and Lockwood, 1998; Inderst and Wey, 2005). While our results remain valid in examples with nonlinear bargaining solutions, the present paper is, to our knowledge, the first using a linear bargaining solution which finds that the integration of competing players may actually increase the level of investment made by a complementary player.

The remainder of the paper is organized as follows. In section 2 we illustrate the main ideas of this paper with a simple example. We present the model in section 3 and the analysis in section 4. In section 5 we extend the model to related output markets. We conclude in section 6. All proofs can be found in the appendix.

## 2 A simple example

Suppose a producer  $p$  chooses, at date 0, a capacity  $Q \in \{0, 1, 2\}$ . The cost of each unit of capacity is  $c$ . There are two outlets, each in a distinct market. At date 1, in *each* outlet one unit can be sold for a value of 1, but there is no demand for a second unit. Denote the maximal revenue obtained with  $m$  outlets and a capacity of  $Q$  by  $\phi(Q, m)$  (e.g.  $\phi(1, 1) = \phi(1, 2) = 1$ ).

No contracts are signed at date 0 and at date 1 gains from trade are split according to each player’s Shapley value. Let  $Z$  denote the industry configuration and  $S_Z(Q)$  denote the producer’s Shapley value in industry  $Z$  when the producer’s capacity is  $Q$ . (Below we compare two situations,  $A$  and  $B$ , so  $Z \in \{A, B\}$ .)

The producer's problem is to maximize its date 1 revenue minus date 0 cost of capacity:

$$\max_Q \{S_Z(Q) - cQ\} \text{ with } Q \in \{0, 1, 2\}.$$

He will choose an additional unit of capacity at stage 0 if the incremental benefit of capacity is larger than the cost  $c$ , i.e. if

$$\Delta S_Z(Q) \equiv S_Z(Q) - S_Z(Q-1) > c.$$

We first focus on the producer's date 1 revenue. A well-known interpretation of the Shapley value has all players in the game being randomly set in an ordered sequence, with each sequence being equally likely. Suppose each player gets his marginal contribution to the coalition formed by those players who precede him in the sequence. The Shapley value is the expectation taken over all possible sequences.

In situation *A* each of the two downstream firms,  $i$  and  $j$ , owns one outlet. There are six possible sequences:  $pij$ ,  $pji$ ,  $ipj$ ,  $jpi$ ,  $ijp$  and,  $jip$ . In two of them the producer comes first, so his marginal contribution is 0. In two other he comes second, so his marginal contribution is equal to  $\phi(Q, 1)$ —the value of allocating all capacity to a single outlet. Finally, there are two sequences in which the producer comes last, so he gets  $\phi(Q, 2)$ —the total industry surplus. Taking expectations we get  $S_A(0) = 0$ ,

$$S_A(1) = 2\frac{1}{6}(0) + 2\frac{1}{6}(1) + 2\frac{1}{6}(1) = \frac{2}{3}$$

and

$$S_A(2) = 2\frac{1}{6}(0) + 2\frac{1}{6}(1) + 2\frac{1}{6}(2) = 1.$$

In situation *B* the downstream firm  $i$  owns both outlets. In this case there are only two possible sequences:  $pi$  and  $ip$ . So  $S_B(0) = 0$ ,

$$S_B(1) = \frac{1}{2}(0) + \frac{1}{2}(1) = \frac{1}{2},$$

and

$$S_B(2) = \frac{1}{2}(0) + \frac{1}{2}(2) = 1.$$

To study the extent to which downstream firms are able to hold-up the producer we also define the share of the industry surplus accruing to the producer as

$$\alpha_Z(Q) \equiv \frac{S_Z(Q)}{\phi(Q, 2)}.$$

We have

$$\alpha_A(1) = \frac{2}{3}, \alpha_A(2) = \frac{1}{2} \text{ and } \alpha_B(1) = \alpha_B(2) = \frac{1}{2}.$$

The share of date 1 surplus accruing to the producer is (weakly) higher in situation *A*, so the hold-up is more severe in situation *B*. We could therefore expect the incremental benefit of capacity to be larger in *A* than in *B*. But things are in fact not so simple. While this is true from  $Q = 0$  to  $Q = 1$ , since

$$\Delta S_A(1) = \frac{2}{3} \text{ and } \Delta S_B(1) = \frac{1}{2},$$

it is not true from  $Q = 1$  to  $Q = 2$ , since

$$\Delta S_A(2) = \frac{1}{3} \text{ and } \Delta S_B(2) = \frac{1}{2}.$$

The table below represents the equilibrium choice of capacity,  $Q_Z^*$ , as a function of  $c$ : upstream equilibrium capacity is larger in *B* when the cost of capacity is low and lower when this cost is high (it is zero in both cases if the cost is too high i.e.  $c \geq 2/3$ ):

$c$	$Q_A^*$	$Q_B^*$
$\in [0, \frac{1}{3}]$	2	2
$\in [\frac{1}{3}, \frac{1}{2}]$	1	2
$\in [\frac{1}{2}, \frac{2}{3}]$	1	0

To understand *why* the producer's incremental benefit is lower in  $A$  than in  $B$  from  $Q = 1$  to  $Q = 2$  we start by writing the change in the producer's share as  $\Delta\alpha_Z(2) = \alpha_Z(2) - \alpha_Z(1)$ . We have

$$\Delta\alpha_A(2) = -\frac{1}{6} < 0 \text{ and } \Delta\alpha_B(2) = 0,$$

that is, the producer's share of the industry surplus changes with capacity in situation  $A$ , while it is constant in  $B$ . Thus bargaining power depends on the relative scarcity of an input. We name *bargaining erosion* this negative effect of increasing capacity on the producer's share. Writing the producer's benefit as a share of the total surplus, we have:

$$\Delta S_Z(2) = \alpha_Z(2) \cdot \phi(2, 2) - \alpha_Z(1) \cdot \phi(1, 2).$$

Denote the difference in the industry surplus by  $\Delta\phi(2, 2) = \phi(2, 2) - \phi(1, 2)$ . We can now decompose the incremental benefit of capacity as follows:

$$\Delta S_Z(2) = \Delta\phi(2, 2) \cdot \alpha_Z(1) + \Delta\alpha_Z(2) \cdot \phi(2, 2).$$

Recognizing the latter, we look at how the incremental benefit changes from situation  $A$  to situation  $B$ :

$$\begin{aligned} \Delta S_B(2) - \Delta S_A(2) &= \Delta\phi(2, 2) \underbrace{[\alpha_B(1) - \alpha_A(1)]}_{\Delta \text{ Hold-up}} + \underbrace{[\Delta\alpha_B(2) - \Delta\alpha_A(2)]}_{\Delta \text{ Bargaining erosion}} \phi(2, 2) \\ \Leftrightarrow \Delta S_B(2) - \Delta S_A(2) &= 1 \cdot \left[ \frac{1}{2} - \frac{2}{3} \right] + \left[ 0 - \left( -\frac{1}{6} \right) \right] \cdot 2 = \frac{1}{6} > 0. \end{aligned}$$

We find that while downstream horizontal integration increases hold-up (the first negative term), it also induces the producer to focus more on increasing the industry surplus and less on the effect of the capacity choice on its bargaining position (the second positive term). From  $Q = 1$  to  $Q = 2$  the second effect dominates the first. Below, we establish these results in a much more general setting.

### 3 The Model

We consider an industry with one upstream producer  $p$ , a set  $N = \{1, \dots, n\}$  of downstream firms and  $m_N$  identical outlets. Let  $Z$  denote a generic allocation of outlets across firms. Each downstream firm  $i$  is the single owner of  $m_i(Z)$  outlets, and

$$\sum_{i \in N} m_i(Z) = m_N.$$

The net revenue at each outlet is described by  $R(q_x)$ , and  $q_x$  is the quantity sold in outlet  $x$ —*net* meaning it accounts for any costs of delivery, transforming the intermediate good into a final good, or both.  $R$  is twice continuously differentiable with  $R(0) = 0$ ,  $R'' < 0$ .  $R'(0) > 0$

but  $R'(q_x) < 0$  for all large  $q_x$ , so there is an unique revenue-maximizing quantity at each outlet, which we denote by

$$\bar{q} \equiv \arg \max_{q_x} R(q_x).$$

Here each single market revenue is independent of the quantities allocated to the remaining outlets, as when downstream firms compete only on the input market—e.g. downstream firms produce different final goods or sell in distinct geographical markets. (We relax this assumption in section 5.)

The timing of the game is the following. At stage 0 the producer chooses a capacity  $Q$  (or stock) and pays its cost  $C(Q) = cQ$  with  $c > 0$  (we also discuss affine and convex cost functions in the text). This investment allows the producer, at date 1, to produce up to  $Q$  at some non-negative constant marginal cost which, without loss of generality, is assumed to be zero. In common with the property rights literature, no supply contracts are signed at date 0 so date 1 bargaining determines quantities and transfers between the producer and each downstream firm. Our model therefore concerns situations in which the producer's revenue is mainly determined in post-investment bargaining. This assumption captures in a stark way that, in general, contracts do not cover the overall economic life of upstream investment.<sup>2</sup>

**Bargaining.** For simplicity, we take a cooperative approach to bargaining and use the Shapley value, a well known benchmark, as our solution concept.<sup>3</sup> A cooperative game consists of two elements: a set of players  $\underline{Y}$  (with a typical subset  $Y$ ) and a function  $v(Y)$  specifying the *value* created by different subsets of players. In our game  $\underline{Y} = N \cup p$  and  $v(Y)$  is the highest revenue a subset of players can achieve on its own.<sup>4</sup> Player  $i$ 's Shapley value is the following linear combination of his marginal contributions (the weights add up to unity):<sup>5</sup>

$$s^i = \sum_{Y \subseteq \underline{Y} | i \in Y} \frac{(|Y| - 1)! (|\underline{Y}| - |Y|)!}{|\underline{Y}|!} [v(Y \cup i) - v(Y \setminus i)].$$

To characterize the value of the present game we start by defining the highest revenue achieved with  $m$  outlets and a capacity of  $Q$  as

$$\phi(Q, m) \equiv \max \sum_{x=1}^m R(q_x) \text{ subject to } \sum_{x=1}^m q_x \leq Q.$$

This objective is globally concave in the vector of quantities—the Hessian is a diagonal with negative elements since  $R$  is strictly concave. At the optimum each outlet sells the same fraction of  $Q$ . We thus have

$$\phi(Q, m) = \begin{cases} mR(\frac{Q}{m}) & \text{if } Q < \bar{q}m, \\ mR(\bar{q}) & \text{if } Q \geq \bar{q}m. \end{cases} \quad (1)$$

<sup>2</sup>Contracts covering the full economic life of capital, while used in some energy markets (e.g. Joskow, 1987), are the exception rather than the norm. Supply contracts longer than twelve months duration are rare; but most investments in capacity done by intermediate goods producers have a longer life. We are unaware of empirical work tying the two aspects together.

<sup>3</sup>For a general survey see e.g. Winter (2002); extensive-form bargaining games between downstream firms and a producer over supply contracts with equilibrium payoffs coinciding with the Shapley value can be found e.g. in Inderst and Wey (2003), de Fontenay and Gans (2004, 2005) and Montez and Ungern-Sternberg (2006).

<sup>4</sup>We write  $Y \cup i$  for  $Y \cup \{i\}$ ,  $Y \setminus i$  for  $Y \setminus \{i\}$  and  $|Y|$  for the cardinality of  $Y$ .

<sup>5</sup> $Y \subseteq \underline{Y} | i \in Y$  represents a set  $Y \subseteq \underline{Y}$  such that  $i$  belongs to  $Y$ .

which is strictly concave in  $Q$  if  $Q/m \leq \bar{q}$ , otherwise linear in  $m$  and constant in  $Q$ . Denote by  $L \subseteq N$  a generic subset of downstream firms and, with abuse of notation, let

$$m_L(Z) = \sum_{i \in L} m_i(Z)$$

denote both the number and the set of outlets owned by  $L$  (with  $m_N(Z) = m_N$  for every  $Z$ ). For a given  $Z$  and capacity  $Q$ , the *value* is:

$$v(L \cup p | Q, Z) = \phi(Q, m_L(Z)); v(L | Q, Z) = \phi(0, m_L(Z)) = 0. \quad (2)$$

For brevity we omit the superscript  $p$  and, since the value of our game is conditional on  $Q$  and  $Z$ , we denote the producer's Shapley value by  $S_Z(Q)$ . With (2), the producer's Shapley value simplifies to:

$$S_Z(Q) = \sum_{L \subseteq N} \omega_L v(L \cup p | Q, Z) \text{ with } \omega_L = \frac{|L|!(n - |L|)!}{(n + 1)!}. \quad (3)$$

It can be checked, with (1), that  $S_Z(Q)$  is strictly concave for all  $Q \in [0, \bar{q}m_N)$  and constant for all  $Q$  larger than  $\bar{q}m_N$ .

**The producer's problem.** The producer solves

$$\max_Q \{S_Z(Q) - C(Q)\}.$$

Since  $S_Z$  is concave and  $c > 0$ , first order conditions are necessary and sufficient to determine the optimal capacity. Let the superscript  $(')$  denote the partial derivative of a function with respect to  $Q$ . The optimal capacity level, denoted by  $Q_Z^*$ , satisfies

$$S'_Z(Q_Z^*) = c.$$

The producer's marginal return to capacity is lower than the industry marginal return, i.e.

$$\begin{aligned} 0 &< S'_Z(Q) < \phi'(Q, m_N) \text{ for } Q \in [0, \bar{q}m_N) \\ 0 &= S'_Z(Q) = \phi'(Q, m_N) \text{ for } Q \geq \bar{q}m_N, \end{aligned}$$

so there is always underinvestment. We therefore focus on the relevant range  $Q \in [0, \bar{q}m_N)$ .

We write  $S_Z(Q)$  as the share  $\alpha_Z(Q)$  of the industry surplus  $\phi(Q, m_N)$ , i.e.

$$S_Z(Q) = \alpha_Z(Q) \cdot \phi(Q, m_N),$$

and the marginal return to capacity can be written as

$$S'_Z(Q) = \phi'(Q, m_N) \cdot \alpha_Z(Q) + \alpha'_Z(Q) \cdot \phi(Q, m_N). \quad (4)$$

This equation shows two effects from increasing capacity: it increases the total date 1 industry revenue and affects the bargaining power (share) of the producer. We show in Proposition 1 below that  $\alpha'_Z(Q) \leq 0$ , so increasing capacity erodes the producer's bargaining power.

We now consider a pre-merger structure  $A$  and a post-merger structure  $B$ . Using the specification in (4), we decompose the total effect of a downstream horizontal merger on the producer's marginal return to capacity into two effects: the *change in hold-up* and the *change in bargaining erosion*, i.e.

$$S'_B(Q) - S'_A(Q) = \phi'(Q, m_N) \underbrace{[\alpha_B(Q) - \alpha_A(Q)]}_{\Delta \text{ Hold-up}} + \underbrace{[\alpha'_B(Q) - \alpha'_A(Q)]}_{\Delta \text{ Bargaining erosion}} \phi(Q, m_N). \quad (5)$$

In the next section we show that a merger increases the hold-up, so the first right-hand term is negative (Proposition 2). However, a merger also reduces the rate at which the bargaining erosion takes place, so the second right-hand term is positive (Proposition 3). The overall effect on the producer's marginal return and equilibrium capacity depends on the relative magnitude of these two effects. We find that the producer's marginal return to capacity is lower after a merger if  $Q$  is low, and higher if  $Q$  is high (Proposition 4). It follows that a downstream merger induces a higher equilibrium capacity if the marginal cost of capacity is low, and the converse is true if this cost is high (Proposition 5).

Importantly, note that consumers may benefit from downstream integration when the cost of capacity is low since higher output levels are associated with lower prices and a higher consumer surplus.

**Ownership structures.** To study the effect of an exogenous change in the outlets' ownership structure we use the concept of downstream horizontal *integration*. A typical *integration contract* gives one player control over the assets owned by another player.

*Definition 1.* An *integration contract* allocates those outlets owned by any two firms  $i$  and  $j$ , with  $m_i > 0$  and  $m_j > 0$ , to a single firm  $i$ .

It can be understood as a merger, an acquisition, or a purchase agreement between two firms. An integration contract changes  $m_L(Z)$ , thereby changing the value in the following way:

$$v(L \cup p | Q, B) = \begin{cases} v(L \cup j \cup p | Q, A) & \text{if } i \in L \wedge j \notin L, \\ v(L \setminus j \cup p | Q, A) & \text{if } i \notin L \wedge j \in L, \\ v(L \cup p | Q, A) & \text{otherwise.} \end{cases}$$

More generally we have:

*Definition 2.*  $B$  is an *integrated* ownership structure of  $A$  if  $B$  can be obtained from  $A$  by successive *integration contracts*.

Note that *integration* is transitive, i.e. if  $C$  is an integrated structure of  $B$ , and if  $B$  is an integrated structure of  $A$ , then  $C$  is also an integrated structure of  $A$ . Finally, we present a definition that will prove useful below.

*Definition 3.* An ownership structure is *symmetric* if each downstream firm owns the same fraction of outlets, i.e. if  $m_i(Z) = \frac{m_N}{n}$  for each  $i \in N$ .

## 4 Analysis

In this section we derive the results mentioned in subsection 3.3. We also illustrate our analysis with an example of linear pricing to final consumers in an industry with three outlets ( $m_N = 3$ ). In situation  $A$ , each outlet is owned by a distinct downstream firm. In situation  $B$ , two outlets

are owned by a single firm while another firm owns the remaining one. In particular, when the inverse demand at each outlet is given by  $p_x(q_x) = 1 - (m_N \cdot q_x)$  the total industry revenue is

$$\phi(Q, m_N) = \begin{cases} (1 - Q)Q & \text{if } Q \leq \frac{1}{2}, \\ \frac{1}{4} & \text{if } Q > \frac{1}{2}. \end{cases} \quad (6)$$

This example is used in figure 1 below (attached at bottom).

**The bargaining erosion effect.** We first look at the effect of increasing capacity on the producer's bargaining position, i.e. the second right-hand term of equation (4). The negative relation we find, which we name *bargaining erosion*, reduces the producer's incentive to invest in capacity. As we will see, this effect is ultimately driven by the concavity of  $R$ .

*Proposition 1 (Bargaining erosion).* If  $m_i(Z) \neq m_N$  for all  $i$  we have

$$\alpha'_Z(Q) < 0$$

- i) for all  $Q \in (0, \bar{q}m_N)$  if  $\phi$  is log-supermodular.<sup>6</sup>
- ii) for all  $Q$  sufficiently close to  $\bar{q}m_N$ .

If  $m_i(Z) = m_N$  for some  $i$  (bilateral monopoly) we have  $\alpha'_Z(Q) = 0$ .

This effect is illustrated in figure 1 (attached at bottom). The lines representing the share accruing to the producer are downward sloping in both situation  $A$  and  $B$ , so the share accruing to the producer decreases as  $Q$  increases.

The accompanying intuition builds on the following sequence of arguments. In a bargaining situation power is determined by the players' relative marginal contributions. We measure the marginal contribution of a subset of firms  $N \setminus L$  to the industry  $N \cup p$  with a first-order difference operator defined as

$$\Delta_{N \setminus L}(Q, Z) \equiv v(N \cup p | Q, Z) - v(L \cup p | Q, Z).$$

Recall that  $v(N \cup p | Q, Z) = \phi(Q, m_N)$ . We add and subtract  $\phi(Q, m_N)$  to  $S_Z(Q)$  (see (3)) and normalize the result with  $\phi(Q, m_N)$ . Since the weights of the Shapley value add up to unity, rearranging the expression we get:

$$\alpha_Z(Q) = 1 - \sum_{L \subseteq N} \omega_L \frac{\Delta_{N \setminus L}(Q, Z)}{\phi(Q, m_N)}. \quad (7)$$

Increasing capacity reduces the bargaining power (share) of the producer when this decreases the normalized marginal contributions of downstream firms. This is satisfied when the growth rate of the industry revenue with respect to  $Q$  is increasing in the number of outlets, i.e.  $\phi(Q, m)$  is log-supermodular at  $Q$ .

Moreover,  $\phi$  is log-supermodular when the cross derivative of the log of  $\phi$  with respect to  $Q$  and  $m$  is positive, which is the case if the elasticity of  $R$  is decreasing in  $q_x$

$$\frac{\partial}{\partial q_x} \left( \frac{R'(q_x)}{R(q_x)} q_x \right) < 0 \text{ for all } q_x \in (0, \bar{q}). \quad (8)$$

That is, when the percentage increase in  $R(q_x)$  achieved with a one percent increase of  $q_x$  decreases as  $q_x$  increases. Developing (8) shows this reasonable condition is verified if  $R$  is sufficiently

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<sup>6</sup>  $\phi(Q, m)$  is *log-supermodular* at  $Q$  when for  $m_1 < m_2 \leq m_N$  and  $\phi'(Q, m_2) \neq 0$  we have  $\frac{\phi'(Q, m_1)}{\phi(Q, m_1)} < \frac{\phi'(Q, m_2)}{\phi(Q, m_2)}$ .



concave. For example, with linear pricing to consumers it is verified for all concave, all linear, and many commonly used convex inverse demand functions.

**Integration and hold-up.** Following a downstream merger the scope the producer has for playing one downstream firm off against another will be reduced, so the first right-hand side element of (5) is expected to be negative. This intuition is proven correct. (In figure 1 the line associated with the producer's share is lower in situation  $B$  than in situation  $A$ .)

However, when the industry is not capacity-constrained ( $Q = \bar{q}m_N$ ) the producer's pay-off is independent of the outlet ownership structure since downstream firms do not actually have to compete with each other for the producer's input.

*Proposition 2 (Integration aggravates the hold-up).* If  $B$  is an integrated structure of  $A$  then

$$\alpha_B(Q) < \alpha_A(Q) \quad \forall Q \in (0, \bar{q}m_N).$$

and

$$\alpha_B(Q) = \alpha_A(Q) = \frac{1}{2} \quad \text{for } Q = \bar{q}m_N.$$

This result is driven by the substitutability of downstream firms. We capture the notion of substitution with the following second-order difference operator:

$$\Delta_{ij}^2 v(L \cup p | Q, Z) \equiv \tag{9}$$

$$v(L \cup i \cup j \cup p | Q, Z) - v(L \setminus i \cup j \cup p | Q, Z) - v(L \setminus j \cup i \cup p | Q, Z) + v(L \setminus i \setminus j \cup p | Q, Z).$$

Firms  $i$  and  $j$  are substitutes at  $Q$  if  $\Delta_{ij}^2 v$  is negative, complements otherwise (trivially we also have  $\Delta_{ij}^2 v = 0$  if  $m_i(Z) = 0$ ). As expected, downstream firms are substitutes:

*Lemma 1.* For any two downstream firms  $i$  and  $j$ , with  $m_i(Z) \neq 0$  and  $m_j(Z) \neq 0$ , we have

$$\Delta_{ij}^2 v(L \cup p | Q, Z) \begin{cases} < 0 \text{ if } Q \in (0, \bar{q}m_{L \cup i \cup j}), \\ = 0 \text{ if } Q \geq \bar{q}m_{L \cup i \cup j}. \end{cases} \tag{10}$$

Segal (2003) finds that the integration of two substitutable players hurts players who are indispensable—here the producer is indispensable, since  $v(L | Q, Z) = 0$ . Formally:

*Lemma 2.* If  $B$  can be obtained from  $A$  by an integration contract we have

$$S_B(Q) - S_A(Q) = \sum_{L \subseteq N | j \in L \wedge i \notin L} \omega_L \cdot \Delta_{ij}^2 v(L \cup p | Q, A). \tag{11}$$

These two lemmas imply that an integration contract therefore reduces the producer's bargaining surplus, and hence his share. Since integration is transitive we obtain Proposition 2.

**Integration and bargaining erosion.** We now turn to the question of how the magnitude of the bargaining erosion changes with integration. We find that the rate at which this effect takes place decreases with integration, so the second right-hand side element of (5) is positive.

*Proposition 3 (Integration softens the bargaining erosion).* If  $B$  is an integrated structure of  $A$ , there exists a critical value  $\tilde{Q}_{AB} < \bar{q}m_N$  such that

$$-\alpha'_B(Q) < -\alpha'_A(Q) \text{ for all } Q \in \left( \tilde{Q}_{AB}, \bar{q}m_N \right).$$

$\tilde{Q}_{AB} = 0$  when  $B$  can be obtained by an integration contract from a symmetric structure  $A$  and  $\phi$  is log-supermodular.

Note that in figure 1 (attached at bottom) the rate at which the share of the producer decreases with additional capacity is always higher in  $A$  than in  $B$ .

**Integration and capacity choice.** Propositions 2 and 3 show that the two effects we identified in (5) counteract each other. Here we look at the total effect of integration on the producer's investment incentives and equilibrium capacity choice.

We find that marginal returns to capacity are lower in an integrated ownership structure if  $Q$  is low, and higher if  $Q$  is high. Therefore no single ownership structure provides unambiguously higher investment incentives.

*Proposition 4.* If  $B$  is an integrated structure of  $A$  there exists a critical value  $\underline{Q}_{AB}$  such that

$$S'_B(Q) - S'_A(Q) < 0 \text{ for } Q \in \left[ 0, \underline{Q}_{AB} \right)$$

and critical value  $\bar{Q}_{AB} \geq \underline{Q}_{AB}$  such that

$$S'_B(Q) - S'_A(Q) > 0 \text{ for } Q \in \left( \bar{Q}_{AB}, \bar{q}m_N \right).$$

In figure 1 the relationship between  $Q$  and the producer's marginal return to capacity illustrates this result. To determine the optimal level of capacity we intercept the marginal return line with a marginal cost line. It is clear that  $A$  induces a higher capacity if  $c$  is high, and the opposite is true if  $c$  is low. Since with linear pricing to consumers higher output levels are associated with lower prices, consumers will benefit from downstream integration when the cost of capacity is low.

More generally, we find that the marginal cost of capacity provides a simple criterion to whether a positive or negative effect of integration is expected in the equilibrium capacity. Let

$$\bar{c} = S'_A(0), \tilde{c} = S'_A(\underline{Q}_{AB}) \text{ and } \underline{c} = S'_A(\bar{Q}_{AB}).$$

We have:

*Proposition 5 (Integration induces a higher (lower) equilibrium capacity if the cost of capacity is low (high)).* We have:

$$\begin{aligned} Q_A^* &= Q_B^* = 0 \text{ if } c \geq \bar{c}, \\ Q_A^* &> Q_B^* \text{ if } c \in (\tilde{c}, \bar{c}), \\ Q_A^* &< Q_B^* \text{ if } c \in (0, \underline{c}). \end{aligned}$$

With affine and convex capacity cost functions we still find a positive effect of integration on the equilibrium capacity if fixed costs are low and marginal costs of capacity remain low for all  $Q \in [0, \bar{q}m_N)$ . Only when the cost of capacity is high—but not too high, profits must be positive—does the hold-up effect dominate, and the conventional wisdom goes through.

## 5 Related output markets

Here we study the case in which, in addition, each outlet's revenue depends negatively on the quantities sold in the remaining outlets, as when downstream firms compete on both the input and output market. We consider two cases.

In the first case the value of the game is again the maximal revenue an industry subset can obtain on its own. We call this the *committed value*. This case is very similar to the previous sections and similar results apply. In the context of an example, we study the role of downstream differentiation. We show that integration is more likely to increase upstream capacity when both downstream differentiation is high and the marginal cost of capacity is low, and reduce upstream capacity when either this cost is high or downstream differentiation is low.

We then identify the potential for producer's opportunism, a well documented problem in vertical contracting, which results in oversupply and a lower aggregate revenue (e.g. McAfee and Schwartz, 1994; Segal and Whinston, 2003). Taking this problem into account we obtain a second value, which we call *opportunistic value*. The use of the former or the latter value should depend whether producer's opportunism can be avoided, e.g. by using public contracting or exclusive territories (see McAfee and Schwartz, 1994; we adopt the terms *committed* and *opportunistic* from their work).

In this second case we find several interesting results. i) Equilibrium capacity may be lower in the *opportunistic* case than in the *committed* case—low capacity works here as a commitment that the producer will not oversupply the industry. ii) While downstream integration always hurts the producer in the *committed* case, it may help in the *opportunistic* case since downstream integration helps the producer overcome its opportunism problem. iii) Importantly we show that, also in this case, a downstream merger induces a higher equilibrium capacity upstream if the marginal cost of capacity is low, and vice versa.

**Setup.** The setup is similar to the one presented in section 3, except that outlets' revenue is now described by a function  $R(q_x, Q_{-x})$ ,  $q_x$  is the quantity sold in outlet  $x$  and

$$Q_{-x} = \sum_{y \in m_N \setminus x} q_y$$

the aggregate quantity sold in the remaining outlets.<sup>7</sup>  $R$  is twice continuously differentiable and decreasing in the quantities sold in the remaining outlets

$$R_2 \begin{cases} = 0 & \text{if } q_x = 0, \\ < 0 & \text{if } q_x > 0. \end{cases}$$

$R(0, Q_{-x}) = 0$  for all  $Q_{-x}$ ,  $R_1(0, 0) > 0$  but  $R_1 < 0$  for sufficiently large  $q_x$ ,  $Q_{-x}$  or both.  $R_{11}$ ,  $R_{12}$  and  $R_{22}$  are such that, for all non-negative  $q_x$  and  $Q_{-x}$ , the Hessian matrix of

$$\sum_{x \in m_L(Z)} R(q_x, Q_{-x})$$

is negative definite for all  $L \subseteq N$ .

A well-known example satisfying the above conditions is the case of linear pricing to consumers with an inverse demand at each outlet given by

$$1 - b(q_x + \gamma Q_{-x})$$

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<sup>7</sup>For the remainder of the section, for a given function  $f(u_1, u_2)$  we denote by  $f_h = \partial f / \partial u_h$ ; similarly  $f_{hg} = \partial^2 f / \partial u_h \partial u_g$ .

with  $b > 0$  and  $\gamma \in (0, 1)$  measures the degree of differentiation—outlets are less differentiated for larger values of  $\gamma$ . We call this the linear model.

**The committed case.** Similar to section 3, the value is here the maximal revenue an industry subset can obtain on its own. We have

$$\phi(Q, m) = \max \sum_{x=1}^m R(q_x, Q_{-x}) \text{ subject to } \sum_{x=1}^m q_x \leq Q,$$

with  $q_y = 0$  for all  $y \notin m$ . Let  $Q^m \equiv \arg \max_Q \phi(Q, m)$ . With  $v(L \cup p | Q, Z) = \phi(Q, m_L(Z))$  we have

$$v(L \cup p | Q, Z) = \begin{cases} m_L(Z)R\left(\frac{Q}{m_L(Z)}, \frac{m_L(Z)-1}{m_L(Z)}Q\right) & \text{if } Q < Q^m, \\ m_L(Z)R\left(\frac{Q^m}{m_L(Z)}, \frac{m_L(Z)-1}{m_L(Z)}Q^m\right) & \text{if } Q \geq Q^m. \end{cases}$$

This value is increasing in  $m_L(Z)$  and, for a given  $m_L(Z)$ , it is increasing and strictly concave in  $Q$  for  $Q < Q^m$ , it is constant otherwise. The properties of this value are similar to those of section 3. Not surprisingly, if the producer's date 1 revenue corresponds to its Shapley value with this *efficient value*,  $S_Z(Q)$ , all section 4 results apply as well to the present case.<sup>8</sup>

To study how differentiation affects the producer's capacity investment we study the linear model defined above. To keep the comparative statics unbiased, we require the total market size to be unaffected by differentiation. With

$$b = \frac{m_N}{1 + \gamma(m_N - 1)}$$

the first-best capacity level is invariant in  $\gamma$  and, for all  $\gamma \in (0, 1)$  and  $m_N$ , the total industry revenue is given by (6).

We focus on the particular example of section 4. The industry has three outlets ( $m_N = 3$ ). In situation *A* each outlet is owned by a distinct downstream firm, while in situation *B* two outlets are owned by a firm and another firm owns the remaining one. (When  $\gamma \rightarrow 0$  we have the independent market case we studied before.) In this example marginal returns to capacity cross once and the optimal quantities satisfy

$$Q_B^* \begin{cases} > Q_A^* & \text{if } c \in (0, \underline{c}(\gamma)) \\ \leq Q_A^* & \text{if } c \geq \underline{c}(\gamma) \end{cases} \quad \text{with } \underline{c}(\gamma) = \frac{5}{2} \left( \frac{1 - \gamma}{7 - \gamma} \right).$$

This critical value  $\underline{c}(\gamma)$  is decreasing in  $\gamma$  (and converges to 0 as  $\gamma \rightarrow 1$ ), suggesting that integration is more likely to increase upstream capacity when downstream differentiation is high and the marginal cost of capacity is low, and reduce upstream capacity when either this cost is high or downstream differentiation is low.

**The opportunistic case.** As in the contracting with externalities literature, if traded quantities maximize the industry's revenue, the producer and each single firm have an incentive to secretly trade a larger quantity at the expense of the remaining firms' revenue. To see this, let

$$Q^L = (Q_1^L, \dots, Q_n^L) \text{ with } Q_i^L = 0 \text{ if } i \notin L.$$

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<sup>8</sup>Montez and Ungern-Sternberg (2006) study a non-cooperative bargaining game over supply contracts which implements this payoff when bargained quantities are observed by the firms. When quantities are unobserved the same game implements the payoff used in the next subsection.

denote a vector of quantities available to  $L$  downstream firms. Like Segal and Whinston (2003), we assume for simplicity that downstream firms can not withhold part of their quantities from the market. Due to concavity, each downstream firm  $i$  sells  $Q_i^L/m_i(Z)$  in each of its outlets and its revenue is

$$\pi_i(Q^L) = m_i(Z)R\left(\frac{Q_i^L}{m_i(Z)}, \sum_{i \in N} Q_j^L - \frac{Q_i^L}{m_i(Z)}\right).$$

Note that the derivative of firm  $i$ 's revenue with respect to  $Q_i^L$  evaluated at the vector inducing the subsets highest revenue is strictly positive—it is zero only when  $i$  is a retail monopolist, i.e.  $m_i(Z) = m_L(Z)$ . Each single downstream firm could therefore increase its own revenue by negotiating a higher supply. However, this additional revenue comes in part at the expense of a lower revenue to the remaining firms: the derivative of each firm's  $j \neq i$  revenue with respect to  $Q_i^L$  is negative.

The *committed value* is therefore unstable. In an extensive-form bargaining game, deFontenay and Gans (2004) show that the producer's pay-off corresponds to its Shapley value over a stable value  $\hat{v}$ , with the producer and each firm being unable to increase their bilateral surplus. This value satisfies

$$\hat{v}(L \cup p | Z) = \max \left\{ \sum_{i \in L} \max_{Q_i^L} \pi_i(Q^L) \right\}.$$

Each term in the sum maximizes firm  $i$ 's revenue taking the quantities of the remaining firms in  $L$  as given, thus it implicitly defines the best response correspondence of a firm  $i$  in a Cournot setting.  $\hat{v}(L \cup p | Z)$  is therefore the highest aggregate Cournot revenue of  $L$ .

Aggregate Cournot quantities may however exceed the producer's capacity  $Q$ . We extend  $\hat{v}$  to take this constraint into account (for more details see Montez and Ungern-Sternberg, 2006). This new value depends also on  $Q$ , i.e.

$$\hat{v}(L \cup p | Q, Z) = \max \left\{ \sum_{i \in L} \max_{Q_i^L} \pi_i(Q^L) \text{ subject to } \sum_{i \in L} Q_i^L \leq Q \right\},$$

and we call it *opportunistic value* since it is immune to secret bilateral renegotiations between any firm  $i \in L$  and the producer, given the capacity constraint.

We denote the producer's Shapley value with the *opportunistic value* by  $\hat{S}_Z(Q)$ . To solve the producer's problem using first order conditions we assume  $Q^{m_N}$ , the capacity maximizing the industry aggregate revenue, is lower than aggregate Cournot quantities of any subset  $L$  when  $m_L(Z) \neq m_i(Z)$  for every  $i \in L$ . In the invariant linear model of subsection 5.2 this condition is satisfied if product differentiation is not too high ( $\gamma$  close to 1), precisely the case which contrasts with the independent output market setup we studied in the previous sections (which corresponds to  $\gamma = 0$ ). For all  $Q < Q^{m_N}$  and  $L$  such that  $m_L(Z) \neq m_i(Z)$  for every  $i \in L$  we have

$$\hat{v}(L \cup p | Q, Z) = m_L(Z)R\left(\frac{Q}{m_L(Z)}, \frac{m_L(Z) - 1}{m_L(Z)}Q\right), \quad (12)$$

which is strictly concave and increasing (decreasing) in  $Q$  for  $Q$  lower (larger) than  $Q^{m_L(Z)}$ —the producer oversupplies if his capacity is high and aggregate revenue is therefore lower than it could otherwise be.

$\hat{S}_Z(Q)$  is then non-increasing for  $Q > Q^{m_N}$  and strictly concave for  $Q \leq Q^{m_N}$ . Since  $c > 0$ , the producer's optimal capacity  $\hat{Q}_Z^*$  is unique and satisfies

$$\hat{S}'_Z(\hat{Q}_Z^*) = c$$

In addition, there exists a critical value  $Q^o$  such that

$$\begin{aligned}\phi'(Q, m_N) &> S'_Z(Q) = \widehat{S}'_Z(Q) > 0 \quad \forall Q \in [0, Q^o], \\ \phi'(Q, m_N) &> S'_Z(Q) > \widehat{S}'_Z(Q) \quad \forall Q \in (Q^o, Q^{m_N}), \\ \phi'(Q, m_N) &= S'_Z(Q) = 0 > \widehat{S}'_Z(Q) \quad \text{for } Q = Q^{m_N}.\end{aligned}$$

if  $m_j(Z) + m_i(Z) \neq m_N$  for all  $i$  and  $j$ —otherwise  $S'_Z(Q) = \widehat{S}'_Z(Q)$ . First, we will have again underinvestment in upstream capacity. We therefore focus on the relevant range  $Q \in [0, Q^{m_N})$ . Second, the producer's marginal return to capacity is (weakly) lower in the *opportunistic* case than in the *committed* case, thus equilibrium capacity may be lower in the *opportunistic* case. A low capacity level works here as a commitment that the producer will not oversupply the industry—a related point is made by Baake, Kamecke and Norman (2004).

For tractability, we now restrict our attention to the case in which downstream firms are symmetric before the integration contract. The producer's payoff changes in the following way:

*Lemma 3.* If  $B$  can be obtained with an integration contract from a symmetric structure  $A$  then, for all  $Q \in [0, Q^{m_N})$ ,

$$\begin{aligned}\widehat{S}_B(Q) - \widehat{S}_A(Q) = \\ \frac{1}{N(N-1)} \left[ (\phi(Q, m_N) - 2\widehat{S}_A(Q)) + \frac{2}{N+1} (v(\{i, j\} \cup p | Q, A) - \widehat{v}(\{i, j\} \cup p | Q, A)) \right].\end{aligned}$$

A similar result holds for the difference  $S_B(Q) - S_A(Q)$ , except  $\widehat{S}_A(Q)$  is replaced by  $S_A(Q)$  and the second right-hand term is absent. This second right-hand term captures the fact that for  $L = \{i, j\}$  the oversupply problem is present in  $A$  but not in  $B$ , since with a retail monopoly the traded quantity is efficient. This term is positive if  $Q > Q^{m_{\{i, j\}}(A)}$ , it is zero otherwise.

We know from section 4 that  $\alpha_Z(Q) \geq 1/2$  for all  $Z$ . Therefore downstream integration was detrimental to the producer in the *committed* case ( $\alpha_B(Q) < \alpha_A(Q)$ ). Here however, we may have  $\widehat{\alpha}_A(Q) < 1/2$  for sufficiently large  $Q$ . So both right hand-terms can be positive for large  $Q$ . In this case downstream integration may help the producer in the *opportunistic* case by helping the producer overcome the oversupply problem.

We now look at the effect of integration on the producer's equilibrium capacity. We find that:

*Proposition 6.* When  $B$  can be obtained with an integration contract from a symmetric structure  $A$ , there exist positive values  $\bar{c}^o, \underline{c}^o$  and  $\underline{c}^o$  such that

$$\begin{aligned}\widehat{Q}_A^* &= \widehat{Q}_B^* = 0 \quad \text{if } c \geq \bar{c}^o, \\ \widehat{Q}_A^* &> \widehat{Q}_B^* \quad \text{if } c \in (\underline{c}^o, \bar{c}^o), \\ \widehat{Q}_A^* &< \widehat{Q}_B^* \quad \text{if } c \in (0, \underline{c}^o).\end{aligned}$$

Again the cost of capacity provides a simple criterion for evaluating claims about the effect of a downstream merger on the producer's capacity choice.

## 6 Conclusion

This paper studied how integration of players who compete for an input affects the capacity choice (or stock) of a monopolist supplying that input. We found that the cost of capacity provides a simple criterion for evaluating claims about the effect of competing players' integration on the monopolist's capacity choice: if the cost of capacity is low integration induces a higher equilibrium capacity, and the converse is true if this cost is high.

At a practical level, these results are useful to merger analysis since they help sign the effect of downstream integration on upstream capacity and, indirectly, welfare. Since the difference in the producer’s payoff can be measured with a second-order difference operator, our work also suggests that further research on the effect of different investments on this measure of substitutability should lead to additional results on the investment effects of mergers.

At a more theoretical level, these results provide support to Galbraith’s idea of Countervailing Power. Our results support the idea that when competition fails on both sides of the market, allocative efficiency may be nurtured not by increased competition but by a process of concentration on the most competitive side. Galbraith’s informal argument is that “..the existence of market power creates an incentive to the organization of another position of power that neutralizes it” (Galbraith, 1952, p. 119). In our model the mechanism at play is different. Bargaining power arises from control over a scarce resource. This gives the producer a strategic incentive to keep his input *relatively scarce* to leverage his bargaining power. This effect, which distorts supply downwards, is particularly important when the retail sector exhibits a low level of concentration, since in this case limiting capacity encourages downstream competition the most. Downstream integration, by relaxing this effect, may result in a higher industry capacity which can in turn translate into lower consumer prices.

These results are of interest for other settings. For example, they cast doubt on the usual claim that unionization reduces a firm’s incentive to invest in labor-complementary capital—see e.g. Grout’s (1984) seminal work on the hold-up problem. Another application concerns public procurement. As an example, a reason for Roche’s recent reluctance to increase its stock of the Tamiflu antiviral may be that a low stock leverages Roche’s bargaining power. Our results suggest that it could have been easier to reach an agreement to increase the production of the antiviral had governments (the buyers) been negotiating through a single entity.

## Appendix

All proofs follow.

*Proof of Proposition 1.* From (7) the share of the surplus accruing to the producer can be written as follows

$$\alpha_Z(Q) = 1 - \sum_{L \subseteq N} \omega_L \frac{\Delta_{N \setminus L}(Q, m_L(Z))}{\phi(Q, m_N)}.$$

Thus

$$\alpha'_Z(Q) = - \sum_{L \subseteq N} \omega_L \frac{\partial}{\partial Q} \left( \frac{\Delta_{N \setminus L}(Q, m_L(Z))}{\phi(Q, m_N)} \right) \quad (\text{A1})$$

where

$$\frac{\partial}{\partial Q} \left( \frac{\Delta_{N \setminus L}(Q, m_L(Z))}{\phi(Q, m_N)} \right) = \frac{\phi'(Q, m_N) \cdot \phi(Q, m_L(Z)) - \phi'(Q, m_L(Z)) \cdot \phi(Q, m_N)}{(\phi(Q, m_N))^2}, \quad (\text{A2})$$

since  $v(L \cup p | Q, Z) = \phi(Q, m_L(Z))$ . We consider first those ownership structures where  $m_i(Z) \neq m_N$  for all  $i$  (Step 1) and then those where  $m_i(Z) = m_N$  for some  $i$  (Step 2).

*Step 1.*  $m_i(Z) \neq m_N$  for all  $i$ : it follows from (A2) together with (A1) that  $\alpha'_Z(Q) < 0$  if for all  $L \subset N$

$$\frac{\phi'(Q, m_L(Z))}{\phi(Q, m_L(Z))} \leq \frac{\phi'(Q, m_N)}{\phi(Q, m_N)}$$

and the inequality is strict for at least one  $L$ .

Since  $m_i(Z) < m_N$  and  $m_L(Z) \leq m_N$  for all  $L \subseteq N$  it follows that  $\alpha'_Z(Q) < 0$  if  $\phi(Q, m)$  is log-supermodular at  $Q$ . Moreover, for all  $Q$  close to  $\bar{q}m_N$  we have that

$$\phi'(Q, m_N) > \phi'(Q, m_L(Z)) = 0 \text{ if } m_L(Z) < m_N.$$

So  $\phi(Q, m)$  is log-supermodular at  $Q$  for all  $Q$  close to  $\bar{q}m_N$ .

*Step 2.*  $m_i(Z) = m_N$  for some  $i$ : for all  $L \subseteq N$  we have  $m_L \in \{0, m_N\}$ . The normalized difference operator is constant and equal to either 1 if  $m_L(Z) = 0$  or 0 if  $m_L(Z) = m_N$ . We have  $\alpha'_Z(Q) = 0$ . Q.E.D.

*Proof of Proposition 2.* Since  $m_i(A) \neq m_N$  and  $m_j(A) \neq m_N$ , Lemma 1 (see proof below) together with lemma 2 (adapted from Segal, 2003) imply that the share of the surplus accruing to the producer is lower after an integration contract since all the elements of (11) are negative, and strictly negative for  $L$  such that  $m_{L \cup i \cup j}(A) = m_N$ . (It remains unchanged at  $Q = \bar{q}m_N$ .)

We now use transitivity of integration to complete the proof. An integrated structure results from successive integration contracts. If the share is reduced, for all  $Q \in (0, \bar{q}m_N)$ , from  $B$  to  $C$  and from  $A$  to  $B$ , then it is also reduced from  $A$  to  $C$ . Thus:

- i) The share accruing to the producer is reduced after integration for all  $Q \in (0, \bar{q}m_N)$ .
- ii) It remains unchanged for all  $Z$  when  $Q = \bar{q}m_N$ .

When  $m_i(Z) = m_N$  (a bilateral monopoly situation) the producer gets one half of the surplus. It follows that  $\alpha_Z(\bar{q}m_N) = \frac{1}{2}$  for all  $Z$ . Q.E.D.

*Proof of Lemma 1.* (For brevity we here drop the notational dependence on  $Z$ .) For all  $L, i$  and  $j$

$$m_{L \setminus i \setminus j} + m_{L \cup i \cup j} = m_{L \cup i \setminus j} + m_{L \cup j \setminus i}.$$

If  $m_i \neq 0$  and  $m_j \neq 0$  there exists a  $\hat{\lambda} \in (0, 1)$  such that (adding up vertically the expression below we obtain the expression above):

$$\begin{aligned} \hat{\lambda}m_{L \setminus i \setminus j} + (1 - \hat{\lambda})m_{L \cup i \cup j} &= m_{L \cup j \setminus i} \\ (1 - \hat{\lambda})m_{L \setminus i \setminus j} + \hat{\lambda}m_{L \cup i \cup j} &= m_{L \cup i \setminus j}. \end{aligned} \tag{A3}$$

Recall from (1) that  $\phi(Q, m)$  is increasing and concave in  $m$  if  $Q < mq$ . It is linear if  $Q \geq mq$ . So for all  $\lambda \in (0, 1)$  we have

$$\begin{aligned} \lambda\phi(Q, m_{L \setminus i \setminus j}) + (1 - \lambda)\phi(Q, m_{L \cup i \cup j}) &< \phi(Q, \lambda m_{L \setminus i \setminus j} + (1 - \lambda)m_{L \cup i \cup j}) \text{ if } Q < m_{L \cup i \cup j}\bar{q} \\ \lambda\phi(Q, m_{L \setminus i \setminus j}) + (1 - \lambda)\phi(Q, m_{L \cup i \cup j}) &= \phi(Q, \lambda m_{L \setminus i \setminus j} + (1 - \lambda)m_{L \cup i \cup j}) \text{ if } Q \geq m_{L \cup i \cup j}\bar{q} \end{aligned}$$

From (A3), the expression above and (2) we have

$$v(L \cup i \cup j \cup p | Q) + v(L \setminus i \setminus j \cup p | Q) \leq v(L \cup L \cup j \setminus i \cup p | Q) + v(L \cup L \cup i \setminus j \cup p | Q).$$

The inequality is strict if  $Q < m_{L \cup i \cup j}\bar{q}$ . It is verified with equality if  $Q \geq m_{L \cup i \cup j}\bar{q}$ . We apply the definition of  $\Delta_{ij}^2 v$  and obtain lemma 1. Q.E.D.

*Proof of Proposition 3.* We proceed in 3 steps. In step 1 we show that the rate at which the bargaining erosion takes place decreases if the second order difference operator increases with  $Q$ . This is verified if  $Q$  is high—step 2. In step 3 we look at the particular case where the ownership structure is symmetric before the integration contract.

*Step 1.* Dividing (11) from lemma 2 by  $\phi(Q, m_N)$  we write

$$\alpha'_B(Q) - \alpha'_A(Q) = \sum_{L \subseteq N | j \in L \wedge i \notin L} \omega_L \cdot \frac{\partial}{\partial Q} \left( \frac{\Delta_{ij}^2 v(L \cup p | Q, A)}{\phi(Q, m_N)} \right)$$



with

$$\frac{\partial}{\partial Q} \left( \frac{\Delta_{ij}^2 v(L \cup p | Q, A)}{\phi(Q, m_N)} \right) = \frac{\frac{\partial}{\partial Q} \Delta_{ij}^2 v(L \cup p | Q, A) \cdot \phi(Q, m_N) - \phi'(Q, m_N) \cdot \Delta_{ij}^2 v(L \cup p | Q, A)}{(\phi(Q, m_N))^2}.$$

From Lemma 1, for  $Q \in [0, \bar{q}m_N]$  we have  $\Delta_{ij}^2 v(L \cup p | Q, A) \leq 0$  for all  $L$ . This inequality is strict for  $L$  such that  $m_{L \cup i}(A) = m_N$  and there exists at least one such  $L$ . Because  $\phi'(Q, m_N) > 0$ , we have

$$\sum_{L \subseteq N | j \in L \wedge i \notin L} \omega_L \phi'(Q, m_N) \cdot \Delta_{ij}^2 v(L \cup p | Q, A) > 0.$$

Thus

$$-\alpha'_B(Q) < -\alpha'_A(Q) \text{ if } \sum_{L \subseteq N | j \in L \wedge i \notin L} \omega_L \frac{\partial}{\partial Q} \Delta_{ij}^2 v(L \cup p | Q, A) \geq 0$$

$$\text{or } \frac{\partial}{\partial Q} \Delta_{ij}^2 v(L \cup p | Q, A) \geq 0 \text{ for all } L \subseteq N | j \in L \wedge i \notin L,$$

i.e. if the the substitutability of  $i$  and  $j$  in  $L$  is decreasing in  $Q$ .

*Step 2.* The derivative of the second-order difference operator is

$$\frac{\partial}{\partial Q} \Delta_{ij}^2 v(L \cup p | Q, A) =$$

$$v'(L \cup i \cup j \cup p | Q, A) - v'(L \setminus i \cup j \cup p | Q, A) - v'(L \setminus j \cup i \cup p | Q, A) + v'(L \setminus i \setminus j \cup p | Q, A). \quad (\text{A4})$$

Since  $m_i(A) \neq 0$  and  $m_j(A) \neq 0$ , from (1) and (2) we have that for all  $Q \geq \bar{q}m_{L \setminus j \cup i}(A)$

$$v'(L \cup i \cup j \cup p | Q, A) \geq v'(L \setminus i \cup j \cup p | Q, A)$$

$$v'(L \setminus j \cup i \cup p | Q, A) = v'(L \setminus i \setminus j \cup p | Q, A) = 0.$$

Therefore

$$\frac{\partial}{\partial Q} \Delta_{ij}^2 v(L \cup p | Q, A) \geq 0 \text{ for all } Q \geq \bar{q}m_{L \setminus j \cup i}(A). \quad (\text{A5})$$

We conclude that

$$-\alpha'_B(Q) < -\alpha'_A(Q) \text{ for all } Q \in (\tilde{Q}_{AB}, \bar{q}m_N),$$

where  $\bar{q} \min \{m_{N \setminus i}(A), m_{N \setminus j}(A)\}$  is an upper bound of  $\tilde{Q}_{AB}$ . (Again, by transitivity of integration, for all  $Q$  sufficiently close to  $Q$  we have that if  $\alpha'_C(Q) - \alpha'_B(Q) > 0$  and  $\alpha'_B(Q) - \alpha'_A(Q) > 0$  then  $\alpha'_C(Q) - \alpha'_A(Q) > 0$ .)

*Step 3.* Assume  $A$  is symmetric. We show in step 1 of the proof of Lemma 3 (see below) that the difference of the producer's payoff when  $B$  can be obtained from  $A$  by an integration contract can be written as:

$$S_B(Q) - S_A(Q) = \frac{1}{N(N-1)} (\phi(Q, N) - 2S_A(Q)).$$

Therefore

$$\alpha'_B(Q) - \alpha'_A(Q) = -\frac{2}{N(N-1)} \alpha'_A(Q).$$

If  $\phi$  is log-supermodular,  $\alpha'_Z(Q) < 0$  for all  $Q \in (0, \bar{q}m_N)$  (Proposition 1). We thus have  $-\alpha'_B(Q) < -\alpha'_A(Q)$  for all  $Q \in (0, \bar{q}m_N)$ . Q.E.D.

*Proof of Proposition 4.* With Lemma 2, we write

$$S'_B(Q) - S'_A(Q) = \sum_{L \subseteq N | j \in L \wedge i \notin L} \omega_L \cdot \frac{\partial}{\partial Q} \Delta_{ij}^2 v(L \cup p | Q, A).$$

We first show that the producer's marginal return to capacity is always higher before integration for  $Q$  is close to 0 (step 1). We then show the opposite for  $Q$  close to  $\bar{q}m_N$  (step 2).

*Step 1.* From (1),  $v'(L \cup p | 0, A) = R'(0)$  for all  $m_Z(L) > 0$ , and zero if  $m_Z(L) = 0$ . With (A4) we have

$$\frac{\partial}{\partial Q} \Delta_{ij}^2 v(L \cup p | 0, A) \begin{cases} = 0 & \text{if } m_{L \setminus i \setminus j}(A) \neq 0, \\ -R'(0) & \text{if } m_{L \setminus i \setminus j}(A) = 0. \end{cases}$$

Since  $m_{L \setminus i \setminus j} = 0$  e.g. for  $L = \{j\}$ , we have

$$S'_B(0) - S'_A(0) < 0$$

An integrated structure results from successive integration contracts. If the producer's marginal return to capacity at 0 is higher in  $B$  than in  $C$  and in  $A$  than in  $B$ , then it is also higher in  $A$  than in  $C$ . By transitivity and continuity we conclude that for any two ownership structures  $A$  and  $B$ , such that  $B$  is an integrated structure of  $A$ , there exists  $\underline{Q}_{AB} > 0$  such that

$$S'_B(0) < S'_A(0) \text{ for all } Q \in [0, \underline{Q}_{AB}).$$

*Step 2.* From (A5), for all  $L \subseteq N$  with  $j \in L$  and  $i \notin L$  we have

$$\frac{\partial}{\partial Q} \Delta_{ij}^2 v(L \cup p | Q, A) \geq 0 \text{ for all } Q \geq \bar{q}m_{L \setminus j \cup i}(A).$$

The inequality is strict for  $L$  such that  $m_{L \cup i}(A) = m_N$ . So

$$S'_B(Q) - S'_A(Q) < 0 \text{ for all } Q \in (\bar{q}m_{N \setminus j}, \bar{q}m_N).$$

An integrated structure results from successive integration contracts. If the marginal return to capacity is lower in  $B$  than in  $C$  and in  $A$  than in  $B$ , then it is also lower in  $A$  than in  $C$ . By transitivity and continuity we conclude that for any two ownership structures  $A$  and  $B$ , such that  $B$  is an integrated structure of  $A$ , there exists a  $\bar{Q}_{AB} < \bar{q}(m_N - 1)$  such that

$$S'_B(Q) - S'_A(Q) > 0$$

for all  $Q \in (\bar{Q}_{AB}, \bar{q}m_N)$ . Q.E.D.

*Proof of Proposition 5.* Since  $S$  is strictly concave for  $Q \in [0, \bar{q}m_N]$ , equilibrium capacity is given by

$$S_Z(Q_Z^*) = c.$$

For any  $Z$ , if  $c \geq S'_z(0)$  we have  $Q_Z^* = 0$ . From proposition 4 we know that

$$S'_B(0) < S'_A(0) \text{ for all } Q \in [0, \underline{Q}_{AB}).$$

But then,  $Q_A^* = Q_B^*$  for all  $c \geq S'_A(0)$  and  $Q_A^* > Q_B^*$  for all  $c \in (S'_A(0), S'(\underline{Q}_{AB}))$ .

Since

$$S'_B(Q) - S'_A(Q) > 0 \text{ for all } Q \in (\bar{Q}_{AB}, \bar{q}m_N),$$

a similar argument shows that when  $c \in (S'_A(\bar{Q}_{AB}), \bar{q}m_N)$  we have  $Q_A^* < Q_B^*$ . Q.E.D.

*Proof of Lemma 3.* We first show that (step 1)

$$S_B(Q) - S_A(Q) = \frac{1}{n(n-1)} [\phi(Q, m_N) - 2S_A(Q)],$$

we then prove the remainder of Lemma 3 (step 2).

*Step 1.* When  $A$  is symmetric,  $m_L(A) = hm_N/n$  if  $|L| = h$ . We thus have  $v(L \cup p|Q, Z) = \phi(Q, h\frac{m_N}{n})$  and, the producer's Shapley value simplifies to

$$S_A(Q) = \sum_{h=1}^{h=n} \frac{1}{n+1} \phi\left(Q, h\frac{m_N}{n}\right)$$

since there are  $n!/((n-h)!h!)$  sets  $L$  such that  $|L| = h$  (see Stole and Zwiebel, 1996a).

Recall from Lemma 2 that

$$S_B(Q) - S_A(Q) = \sum_{L \subseteq N | j \in L \wedge i \notin L} \omega_L \cdot \Delta_{ij}^2 v(L \cup p|Q, A) \text{ with } \omega_L = \frac{|L|!(n-|L|)!}{(n+1)!} \quad (\text{A6})$$

and thus, for all  $L$  such that  $|L| = h$ , we have

$$\Delta_{ij}^2 v(L \cup p|Q, A) = \phi\left(Q, (h+1)\frac{m_N}{n}\right) - 2\phi\left(Q, h\frac{m_N}{n}\right) + \phi\left(Q, (h-1)\frac{m_N}{n}\right).$$

In structure  $A$  there are

$$\frac{(n-2)!}{(n-h-1)!(h-1)!}$$

sets  $L \subseteq N$  such that  $j \in L$  and  $i \notin L$  for which  $|L| = h$ . Since  $|L| = h$  we also have

$$\omega_L \cdot \frac{(n-2)!}{(n-h-1)!(h-1)!} = \frac{h(n-h)}{(n+1)n(n-1)}.$$

Recognizing this we can write (A6) as

$$S_B(Q) - S_A(Q) = \frac{1}{(n+1)n(n-1)} \sum_{h=1}^{h=n-1} h(n-h) \left[ \phi\left(Q, (h+1)\frac{m_N}{n}\right) - 2\phi\left(Q, h\frac{m_N}{n}\right) + \phi\left(Q, (h-1)\frac{m_N}{n}\right) \right],$$

and the sum can be rewritten as

$$\begin{aligned} & (n-1)\phi\left(Q, n\frac{m_N}{n}\right) - 2(n-1)\phi\left(Q, (n-1)\frac{m_N}{n}\right) + 2(n-2)\phi\left(Q, (n-1)\frac{m_N}{n}\right) + \\ & \sum_{h=2}^{h=n-2} [(n-(h+1))(h+1) - 2(n-h)h + (n-(h-1))(h-1)] \phi\left(Q, h\frac{m_N}{n}\right) \\ & + 2(n-2)\phi\left(Q, \frac{m_N}{n}\right) - 2(n-1)\phi\left(Q, \frac{m_N}{n}\right) \end{aligned}$$

Since

$$(n-(h+1))(h+1) - 2(n-h)h + (n-(h-1))(h-1) = -2,$$

we can write

$$\begin{aligned} S_B(Q) - S_A(Q) &= \frac{1}{n(n-1)} \left[ \frac{n-1}{n+1} \phi(Q, m_N) - 2 \sum_{h=1}^{h=n-1} \frac{1}{n+1} \phi\left(Q, h\frac{m_N}{n}\right) \right] \\ &= \frac{1}{n(n-1)} \left[ \phi(Q, m_N) - 2 \sum_{h=1}^{h=n} \frac{1}{n+1} \phi\left(Q, h\frac{m_N}{n}\right) \right] = \frac{1}{n(n-1)} [\phi(Q, m_N) - 2S_A(Q)]. \end{aligned}$$

*Step 2.* We now look at  $\widehat{S}$ . For all  $L$  such that  $m_i(Z) \neq m_L(Z)$  we will have that  $\widehat{v}(L \cup p|Q, Z)$  is given by (12), which is similar to  $v(L \cup p|Q, Z)$  except that all capacity is used (this is true for all  $Q \in [0, Q^{m_N})$  since the capacity constraint binds). If  $m_i(Z) = m_L(Z)$  we have  $\widehat{v}(L \cup p|Q, Z) = v(L \cup p|Q, Z)$ . Hence Step 1 applies equally to  $\widehat{S}$ , except for those  $L$  such that  $m_i(A) \neq m_L(A)$  while  $m_i(B) = m_L(B)$  for some  $i$  since the oversupply problem is absent in  $B$  but is present in  $A$ .

In the case of symmetric firms there is only one such  $L = \{i, j\}$ , where  $\widehat{v}(\{i, j\} \cup p|Q, A)$  becomes  $v(\{i, j\} \cup p|Q, A)$ . In the Shapley value this set has a weight

$$\frac{2}{N(N-1)(N+1)}.$$

Adding up the elements we obtain Lemma 3. Q.E.D.

*Proof of Proposition 6.* We proceed in two steps. In the first step we prove the result for the case where  $c$  is high (step 1); in the second step for  $c$  low (step 2).

*Step 1.* For  $Q$  close to 0: we have

$$\widehat{S}_Z(Q) = S_Z(Q)$$

and the proof of Proposition 5 applies.

*Step 2.* For high levels of  $Q$ : If  $m_j(Z) + m_i(Z) \neq m_N$  for some  $i$  and  $j$  we have  $S'_Z(Q) = \widehat{S}'_Z(Q)$  and proof of Proposition 5 applies. If  $m_j(Z) + m_i(Z) \neq m_N$  for all  $i$  and  $j$ , there exists a  $Q_A < Q^{m_N}$  such that:

$$\widehat{S}'_A(Q) \leq 0 \text{ while } \phi'(Q, m_N) > 0 \text{ for all } Q > Q_A.$$

From Lemma 3, and since

$$v'(\{i, j\} \cup p|Q, A) - \widehat{v}'(\{i, j\} \cup p|Q, A) \geq 0,$$

we have that  $\widehat{S}'_B(Q_A) > 0$ . Since  $\widehat{S}'_B(Q)$  is strictly concave, this implies that there exists  $\underline{c}^o$  such that  $\widehat{Q}_A^* < \widehat{Q}_B^*$  if  $c \in (0, \underline{c}^o)$ . Q.E.D.

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