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A model of Civil war with the risk of death

Introduction

The world has seen numerous civil wars both historically and in recent years. Civil wars differ in their nature and causes but they all involve severe economic loss and loss of human life. English civil war (1642-1651) was a series of armed conflicts and political machinations between Parliamentarians and Royalists. The first (1642-1646) and second (1648-1649) civil wars pitted the supporters of King Charles I against the supporters of the Long Parliament, while the third war (1649-1651) saw fighting between supporters of King Charles ii and supporters of the Rump Parliament. The Civil war ended with the Parliamentary victory at the battle of Worcester. The American Civil War (1861-1865) was a civil war between the United States of America (the “Union”) and eleven Southern slave states that declared their secession from the U.S. and formed the Confederate States of America (the “Confederacy”). An Shi rebellion (755-763) and Taiping rebellion (1850-1865) in China results in huge death toll comparable to the death toll in the two World Wars. In the recent years, many African countries after their independence witnessed civil wars fought mainly over the control of natural resources. Congo civil war, Rwanda civil war, Angolan civil war, Burundi Civil war, Chad civil war, Liberian civil war, Somali civil war, Ugandan civil war, Sierra- Leone civil war etc. are some of the examples of contemporary civil wars.

Civil wars are usually fought between two groups and in most of the cases these groups are formed along the ethnic lines. Without institutions to protect the right to life or property rights, the act of killing others to capture their property will not be punished sufficiently, if at all.

Failure to protect such rights can make the benefit of capturing the resources of others exceeds the costs. One of the important factors that can prevent the decision to fight the war is the risk of death for someone who is involved in actual combat. When one party is attacked, it has no choice, but to fight, that is, to defend his life and property. Defense requires participation in combat, but that brings with it the risk of death. Parties can this risk by let others fight the war on their behalf. In this case, those actively participating must be compensated for their opportunity cost and for the risk of death.

Participation by all members of a group involved in a war is not likely. In any group, those who fight the war are typically those who do not own significant amount of resources. These individuals do not get any share of the captured resources and the contested resources are not owned by them either. The rich individuals within the group hire the poor to fight on their behalf and thereby avoid the risk of death. The poor own some amount of resources which they can use for productive purpose. In order to induce the poor to fight, rich must compensate them for the loss of output as well as for the risk of death. Rich choose the number of poor they are going to hire for fighting the war. The objective of this paper is to analyze how compensation for the possibility of death affects the decisions made by rich. I am interested in comparing my results under different situations like when rich fight for themselves and when the risk of death itself depends upon the number of people fighting for both sides. I am also looking at the situation when there is no risk of death and when the rich fight for themselves for one group but in other group the rich hire the poor to fight on their behalf.

Literature Review

There is a huge literature on different civil wars and authors look at the civil wars from various points of view by concentrating on the important factors which are behind these civil wars. Among the empirical works, Fearon and Laitin (2002) and Collier and Hoeffler (2004) are the important ones. Fearon and Laitin (2002) use the data on the civil wars after 1945. Collier and Hoeffler (2004) use the data on the civil wars during 1960-99. According to Fearon and Laitin (2002), the conditions that favor civil wars are poverty, weak state, large population and instability. One of the surprising results of their paper is that these civil wars can't be explained by variables such as ethnic or religious diversity, lack of democracy or civil liberties, economic inequalities, state discrimination against minority religions or languages. Intense grievances are not a cause of civil wars but it is one of the results of civil wars. In my paper, I am considering the hiring of the poor by the rich and no punishment for the violation of property rights. Fearon and Laitin (2002) finding that the large population of poor is supporting my assumption that poor could be hired to fight the war. Collier and Hoeffler (2004) also find that poorer countries are more likely to experience civil wars. The opportunity cost for potential rebels are low and thus makes recruitment easier. Countries with abundant natural resources have a higher risk of conflict, they hypothesize that natural resources could be used as a source of financing the rebel.

Collier and Hoeffler (2004) use two competing models to analyze the civil wars, one is a greed model and the other is a grievance model. The grievance model examines inequality, political oppression, and ethnic and religious divisions as causes of conflict, while the greed model focuses on the source of financing. The authors consider three grievances, inter-group hatred, political exclusion and vengeance. Inter-group hatred can't be quantified, instead they use polarization as a proxy variable. In spite of relatively low explanatory power of the grievance model they can't reject it in favor of the greed model and thus they combine the two models.

Fearon and Laitin (2002) find that civil wars can't be explained by ethnic or religious diversity. Although, civil wars can't be explained by ethnic or religious diversity, the groups which are engaged in civil wars could be formed along these lines. Herbst (2000) explains the ways through which the leaders motivate their soldiers. These forms of motivation could be economic incentives, ethnic mobilization, political indoctrination and coercion. In my model, I assume that there are two groups which could be formed along ethnic or religious lines and the individuals can't switch from one group to other group.

Esteban and Ray (2008) consider two types of division in the society, one is by income and the other is by ethnicity. Esteban and Ray provide a theory for why ethnic conflict is more likely to occur relative to economic/class conflict assuming that ethnicity is uncorrelated with the economic inequality. In their analysis, the rich within the group supply the resources for the conflict whereas poor can supply cheap labor, together, they form an alliance. James Robinson (2001) analyzes how the aggregate amount of conflict depends on which group form and oppose each other. Ethnic groups and social classes are considered and author shows that the class conflict is not necessarily worse than ethnic conflict.

Esteban and Ray (2008) show that in the presence of economic inequality there is a systematic bias towards ethnic conflict. In the setting of their model, three outcomes are possible namely peace, ethnic conflict and class conflict. Both the rich and the poor have their preferences over these three outcomes. Rich prefer peace but in order to avoid the class conflict, they might end up with ethnic conflict. They extended the basic model by considering factors such as flexible transfers, costs of conflict and polarization. I am using the assumption of their model that each rich has same endowment and each poor has same endowment.

In most of the civil wars, people who actually fight in the war are the poor and the resources with which they fight the war are usually not owned by them. Since death toll is huge, it should be included in the model of civil war. The important questions are (i) who provides the resources used in the war (ii) who own the resources over which we are having the war (iii) who is going to get the resources gained through the war (iii) if the resources are lost in the war who is losing these resources. The income distribution is not uniform among the individuals in the same ethnic group: instead we have both rich and poor. Ignoring income asymmetries within groups might not be proper way of analysis. When we have two types of people, rich and poor, they are likely to participate in the civil war in quite different way. The rich are likely to avoid getting themselves engaged in physical combat so as not to risk death particularly when they can find someone else who can fight on their behalf. The poor own some amount of resources which they can use for production. The rich can hire poor to fight the war on their behalf. The rich can compensate the poor for fighting the war and the compensation would include not only loss in production but also risk of death involved for those who engage in the combat. However, the gained go to the rich, since the war is fought over the resources of the rich only. World Bank policy research report (2003) says that the failure of development is the main cause of civil wars and a private military organization is an army and a business.

In many civil wars, we have seen warlords who provide resources for the civil wars and they have people who fight in the war on their behalf. In other circumstances, when we have some group or organization involved in the war, the war is usually financed by people who do not fight the war themselves. We do not have any model of civil war which incorporates this difference in the stakes for rich and poor. The question arises why do not the rich fight the war for themselves? Could we explain the hiring of poor by some other reason apart from risk of death?

In this paper, I am trying to answer these questions by using a model of civil war which includes the risk of death. I am comparing the outcomes under different situations when poor are hired by both groups, when rich fight for themselves in both groups and when rich in one group fight for themselves but in the other group poor are hired to fight the war. I am showing that hiring of poor by rich in both groups will be the Nash equilibrium. Extending the basic model by making the risk of death dependent on the numbers of people fighting for both groups will not change the results. I am looking at the utilities of rich, number of people engaged in the combat, amount of resources used for fighting, death toll for both groups.

I assume that there are two groups A and B. These groups are formed along the ethnic lines and individuals can't switch from one group to another group. Within each group, we have both rich and poor. This is a one period model and the population of rich in group A is N_w^A and the population of rich in group B is N_w^B. Each rich in both groups own R_w amount of resources. The population of poor in group A and group B are N_p^A and N_p^B respectively. Each poor in both groups own R_p amount of resources. I assume that there is no saving in the model and whatever is there to consume is consumed in every period by both rich as well as poor. Both rich and poor are risk neutral. I assume that there is no problem of commons or free rider problem in the sense that all the rich individuals behave in order to maximize the utility of the group as a whole. This seems to be a reasonable assumption since in the case of war, it is likely that we have a central decision making body which is going to decide the amount of resources to be used for fighting. Risk of death is exogenous in the model as it does not depend on the number of people fighting on both sides. Risk of death is only for people who are engaged in the combat.

Case 1: When poor are hired by rich in both groups

Each poor own R_p amount of resource which he can use for the production. The production function is $F(R_p) = R_p$, which he can consume if he is not going to get hired for fighting the war. Utility from consuming R_p amount of resource is assumed to be equal to R_p as I assume that individuals are risk neutral. When a poor is hired for fighting the war, he faces the risk of death and he can't use his resources for the productive purpose. He should be compensated for both the risk of death and the loss in the production, otherwise he will not fight the war. Rich will give just enough resources to the poor such that he is indifferent between fighting the war and not fighting it. Let us assume that θ is the risk of death which is exogenous and same for both groups. Amount of resources received by each poor as compensation if he is hired to fight the war must be such that the survival probability times the compensation must be equal to R_p. The survival probability is equal to (1-θ).

$$(1-\theta)\ \text{Compensation} = R_p \tag{1}$$

$$\text{Compensation} = \frac{Rp}{(1-\theta)} \tag{2}$$

This compensation includes two parts one is due to loss in production which is equal to R_p, other part is due to the risk of death which is equal to

$$\frac{Rp}{(1-\theta)} - R_p = \frac{\theta\, Rp}{(1-\theta)} \tag{3}$$

When the number of poor hired by both group A and B is equal to F_A and F_B respectively, we must have $F_A \le N_p^A$ and $F_B \le N_p^B$. Total amount resources owned by the rich in both groups is equal to $N_w^A R_w + N_w^B R_w$, out of which the transfer to poor is equal to

$$\text{Total transfer} = F_A \frac{Rp}{(1-\theta)} + F_B \frac{Rp}{(1-\theta)} \tag{4}$$

The remaining amount over which rich in both groups are fighting is equal to

$$\text{Contested resources} = N_w^A R_w + N_w^B R_w - F_A \frac{Rp}{(1-\theta)} - F_B \frac{Rp}{(1-\theta)} \quad (5)$$

The sequence of important events in this game is, (i) Fighters are hired (ii) they receive the compensation (iii) war takes place (iv) Gain/losses are realized and consumed by the rich.

Both groups are going to capture the contested resources with some probabilities which depend on the number of people fighting for both groups according to the Tullock's (1980) contest success functions. For group A it is equal to $\frac{F_A}{F_A+F_B}$ and for group B it is equal to $\frac{F_B}{F_A+F_B}$.

The utility function for the rich in group A is U_A, and then we must have U_A equals to

$$U_A = \frac{F_A}{F_A+F_B} \{ N_w^A R_w + N_w^B R_w - F_A \frac{Rp}{(1-\theta)} - F_B \frac{Rp}{(1-\theta)} \} \quad (6)$$

Similarly, the utility function for group B is U_B, where

$$U_B = \frac{F_B}{F_A+F_B} \{ N_w^A R_w + N_w^B R_w - F_A \frac{Rp}{(1-\theta)} - F_B \frac{Rp}{(1-\theta)} \} \quad (7)$$

Since the rich in group A choose the number of poor to hire (F_A), they will choose it such that their utility is maximized. Suppose that an interior solution exists for the model, and then we will have the first order condition as

$$\frac{dU_A}{dF_A} = \frac{F_B}{(F_A+F_B)^2} \{ N_w^A R_w + N_w^B R_w - F_A \frac{Rp}{(1-\theta)} - F_B \frac{Rp}{(1-\theta)} \} - \frac{F_A}{F_A+F_B} \frac{Rp}{(1-\theta)} = 0 \quad (8)$$

Equation (8) shows that the marginal effect of hiring one more poor for fighting. The first part is the change in the added proportion of resources captured times the total amount of contested resources. This is the marginal benefit of hiring one more poor for fighting, and it is positive. The second part represents the utility cost due to the increase in cost of hiring one more poor at the margin. In equilibrium, the marginal cost balances against the marginal benefit. From equation (8) we get,

$$\{ N_w^A R_w + N_w^B R_w - F_A \frac{Rp}{(1-\theta)} - F_B \frac{Rp}{(1-\theta)} \} = \frac{F_A(F_A+F_B)}{F_B} \frac{Rp}{(1-\theta)} \quad (9)$$

For group B, the rich will try to maximize their utility U_B by choosing F_B, such that

$$\frac{dU_B}{dF_B} = \frac{F_A}{(F_A+F_B)^2} \{ N_w^A R_w + N_w^B R_w - F_A \frac{Rp}{(1-\theta)} - F_B \frac{Rp}{(1-\theta)} \} - \frac{F_B}{F_A+F_B} \frac{Rp}{(1-\theta)} = 0 \quad (10)$$

By rearranging equation (10), we will get

$$\{ N_w^A R_w + N_w^B R_w - F_A \frac{Rp}{(1-\theta)} - F_B \frac{Rp}{(1-\theta)} \} = \frac{F_B(F_A+F_B)}{F_A} \frac{Rp}{(1-\theta)} \quad (11)$$

Since the left hand side of equations (9) and (11) are equal and it is equal to the total contested resources. By equating the right hand sides of equation (9) and (11), we get

$$\frac{F_A(F_A+F_B)}{F_B} \frac{Rp}{(1-\theta)} = \frac{F_B(F_A+F_B)}{F_A} \frac{Rp}{(1-\theta)} \quad (12)$$

From equation (12), we get $F_A = F_B$ (13)

In the equilibrium both group will hire same number of poor for fighting. Putting $F_A = F_B$ in the equation (11), we can solve for the number of poor hired by each group in terms of risk of death (θ), population of rich in both groups (N_w^A, N_w^B) and the resources owned by rich and poor (R_w , R_p).

$$F_A = F_B = \frac{(1-\theta)(N_W^A\, Rw + N_W^B\, Rw)}{4Rp} \quad (14)$$

The number of fighters hired depends negatively on the risk of death (θ) and the amount of resources owned by each poor (R_p), but it depends positively on the population of rich in both groups (N_w^A, N_w^B) and the resources owned by each rich (R_w). The cost of hiring one poor for fighting is equal to $\frac{\mathrm{Rp}}{(1-\theta)}$, this cost is increasing in both risk of death (θ) and the amount of resources owned by each poor (R_p). If the marginal cost of hiring one more poor increases, it makes sense to hire less poor for fighting. Poor are hired to capture the resources, as the amount of contested resources increases, more people will be hired. The contested resources depend positively on the population of rich in both groups and the resources owned by each rich. As the population of rich in either group increases or the resources owned by them increases, the marginal benefit of hiring one more poor for fighting increases so more people will be hired for fighting. Total amount spent on hiring poor for fighting by group A will be same as that of group B, and it will be equal to

$$F_A \frac{\mathrm{Rp}}{(1-\theta)} = F_B \frac{\mathrm{Rp}}{(1-\theta)} = \frac{(N_W^A \mathrm{Rw} + N_W^B \mathrm{Rw})}{4} \quad (15)$$

Total amount of resources used by both group for hiring the poor will be equal to

$$F_A \frac{\mathrm{Rp}}{(1-\theta)} + F_B \frac{\mathrm{Rp}}{(1-\theta)} = \frac{(N_W^A \mathrm{Rw} + N_W^B \mathrm{Rw})}{2} \quad (16)$$

The remaining amount of resources is also equal to $\frac{(N_W^A \mathrm{Rw} + N_W^B \mathrm{Rw})}{2}$, both group A and group B will capture this with probability ½, as they are hiring the same number of people for fighting, so the utility of rich in both groups will be

$$U_A = U_B = \frac{(N_W^A \mathrm{Rw} + N_W^B \mathrm{Rw})}{4} \quad (17)$$

The utilities of the rich in both groups are independent of the risk of death and the amount of resources owned by each poor. It only depends on the population of rich in both groups and the resources owned by each rich.

The number of combined death toll will be equal to $\theta(F_A + F_B)$ which can be written as

$$\text{Total death toll} = \theta\,(F_A + F_B) = \frac{\theta(1-\theta)(\,N_W^A\,Rw + N_W^B\,Rw\,)}{2Rp} \tag{18}$$

Total death toll increases as the risk of death (θ) increases from 0 to 0.5, when θ = 0.5, the death toll is maximum, after that as θ increases from 0.5 to 1, death toll decreases. When risk of death is either 0 or 1, the death toll will be 0. When the risk of death is 1, it means that death is certain for those who fight then in order to induce someone to fight the war, the required compensation will be infinity, so nobody can be hired. As the risk of death increase from 0 to 0.5, the cost of hiring increases so although less number of people will be hired for fighting, the increase in the risk death of death outweigh it such that the death toll be more. When the risk of death increases from 0.5 to 1, the effect of increase in cost of hiring outweigh the high risk of death such that although risk of death is high, death toll decreases because less number of people are hired for fighting the war.

Case 2: When rich in both groups hire someone among themselves to fight the war

I want to compare the different situations, when poor are fighting for both groups, when rich are fighting for both groups, when rich is fighting for one group and poor are fighting for the other group. When rich are going to hire someone among themselves to fight the war, the required amount of compensation will be

$$\text{Compensation} \;=\; \frac{R_W}{(1-\theta)} \tag{19}$$

When group A hires F_A number of rich and group B hire F_B number of rich the total compensation will be equal to

$$\text{Total compensation} = F_A\frac{Rw}{(1-\theta)} + F_B\frac{Rw}{(1-\theta)} \tag{20}$$

Remaining amount of resources over which both groups will contest is equal to

$$\text{Contested resources} = N_w^A R_w + N_w^B R_w - F_A \frac{Rw}{(1-\theta)} - F_B \frac{Rw}{(1-\theta)} \qquad (21)$$

In this case the utility function of both groups A and B will be equal to

$$U_A = \frac{F_A}{F_A+F_B} \left\{ N_w^A R_w + N_w^B R_w - F_A \frac{Rw}{(1-\theta)} - F_B \frac{Rw}{(1-\theta)} \right\} \qquad (22)$$

and

$$U_B = \frac{F_B}{F_A+F_B} \left\{ N_w^A R_w + N_w^B R_w - F_A \frac{Rw}{(1-\theta)} - F_B \frac{Rw}{(1-\theta)} \right\} \qquad (23)$$

Since the rich in both groups will maximize their utilities by choosing the number of rich hired for fighting, suppose that the interior solution exists then we must have the first order conditions as

$$\frac{dU_A}{dF_A} = \frac{F_B}{(F_A+F_B)^2} \left\{ N_w^A R_w + N_w^B R_w - F_A \frac{Rw}{(1-\theta)} - F_B \frac{Rw}{(1-\theta)} \right\} - \frac{F_A}{F_A+F_B} \frac{Rw}{(1-\theta)} = 0 \qquad (24)$$

and

$$\frac{dU_B}{dF_B} = \frac{F_A}{(F_A+F_B)^2} \left\{ N_w^A R_w + N_w^B R_w - F_A \frac{Rw}{(1-\theta)} - F_B \frac{Rw}{(1-\theta)} \right\} - \frac{F_B}{F_A+F_B} \frac{Rw}{(1-\theta)} = 0 \qquad (25)$$

From equation (24) and equation (25) we get

$$F_A = F_B = \frac{(1-\theta)(N_W^A + N_W^B)}{4} \qquad (26)$$

We can also get this result from equation (14), by replacing R_p with R_w, since in this case instead of poor we have rich fighting for both groups. The number of rich hired by both group depend only on the population of both group ($N_w^A + N_w^B$) and the risk of death (θ). The numbers of fighters hired will be independent of the amount of resources owned by each rich (R_w). The number of rich hired for fighting will depend positively on the population of rich in both group and it will depend negatively on the risk of death (θ). Since the amount of resources owned by rich is greater than the amount owned by poor, we have ($\frac{R_w}{R_p} > 1$), comparing the number of fighters hired in case (1) with the number in case (2), we compare the right hand side of equations (14) and (26), we can say that when rich are fighting less number of fighters will be hired. The reason behind this is that when rich are hired the marginal cost of hiring is more but there is no change in marginal benefit. As a result, less number of people will be hired.

Total amount of resources used by both groups for hiring the rich will be equal to

$$F_A \frac{Rw}{(1-\theta)} + F_B \frac{Rw}{(1-\theta)} = \frac{(N_W^A\,Rw + N_W^B\,Rw)}{2} \qquad (27)$$

Comparing equation (27) with equation (16), we see that the right hand sides of both these equations are same, so the total amount of resources used for fighting is same when rich are hired as it was when poor were hired. The increase in the marginal cost of hiring is neutralized by less number of people hired. Since both groups are hiring the same number of people so the utility of both groups will be same and it is equal to

$$U_A = U_B = \frac{(N_W^A\,Rw + N_W^B\,Rw)}{4} \qquad (28)$$

The utilities of the rich in both groups are independent of the risk of death and the amount of resources owned by each poor. It only depends on the population of rich in both groups and the resources owned by each rich. Equation (28) is same as equation (17), which means that rich are equally well off in this case as they where earlier when poor where hired to fight the war.

The number of combined death toll will be equal to $\theta\ (F_A + F_B)$ which can be written as

$$\text{Total death toll} = \theta\ (F_A + F_B) = \frac{\theta(1-\theta)(N_W^A + N_W^B)}{2} \qquad (29)$$

Total death toll in this case is lower than it was earlier when poor were hired to fight the war. Rich are equally well off in this case as they were earlier except that in this case they can't transfer the risk of death. Rich can choose between a small death toll of rich and large death toll of poor. Apart from the avoiding the risk of death, could there be any other reason for choosing to hire poor for fighting? The answer is yes, but then we have to look at the third possibility in which for one group rich are fighting but in the other group poor are hired to fight the war.

Case 3: When rich fight for group A but poor are hired to fight for group B

When rich are fighting for group A, the compensation received by each rich fighting must be enough to cover the loss of production as well as the risk of death, thus it is equal to

$$\text{Compensation received by each rich hired in group A} = \frac{R_w}{(1-\theta)} \tag{30}$$

Since the poor are hired to fight for group B, each poor fighting will receive a compensation which is equal to

$$\text{Compensation received by each poor hired in group B} = \frac{Rp}{(1-\theta)} \tag{31}$$

When group A hires F_A number of rich and group B hire F_B number of rich the total compensation will be equal to

$$\text{Total compensation} = F_A \frac{Rw}{(1-\theta)} + F_B \frac{Rp}{(1-\theta)} \tag{32}$$

The remaining amount of resources over which both groups will contest is equal to

$$\text{Contested resources} = N_w^A R_w + N_w^B R_w - F_A \frac{Rw}{(1-\theta)} - F_B \frac{Rp}{(1-\theta)} \tag{33}$$

In this case the utility function of both groups A and B will be equal to

$$U_A = \frac{F_A}{F_A+F_B} \left\{ N_w^A R_w + N_w^B R_w - F_A \frac{Rw}{(1-\theta)} - F_B \frac{Rp}{(1-\theta)} \right\} \tag{34}$$

and

$$U_B = \frac{F_B}{F_A+F_B} \left\{ N_w^A R_w + N_w^B R_w - F_A \frac{Rw}{(1-\theta)} - F_B \frac{Rp}{(1-\theta)} \right\} \tag{35}$$

Since the rich in both groups will maximize their utilities by choosing the number of rich hired for fighting, suppose that the interior solution exists then we must have the first order conditions as

$$\frac{dU_A}{dF_A} = \frac{F_B}{(F_A+F_B)^2} \ \{ N_w^A R_w + N_w^B R_w - F_A \frac{Rw}{(1-\theta)} - F_B \frac{Rp}{(1-\theta)} \} - \frac{F_A}{F_A+F_B} \ \frac{Rw}{(1-\theta)} = 0 \qquad (36)$$

and $$\frac{dU_B}{dF_B} = \frac{F_A}{(F_A+F_B)^2} \ \{ N_w^A R_w + N_w^B R_w - F_A \frac{Rw}{(1-\theta)} - F_B \frac{Rp}{(1-\theta)} \} - \frac{F_B}{F_A+F_B} \ \frac{Rp}{(1-\theta)} = 0 \qquad (37)$$

In this case the marginal benefit of hiring one more fighter is same for both groups but the marginal cost is more for group A, since they are hiring rich which requires more compensation than hiring a poor. From equation (36) and (37), we get

$$\frac{F_A(F_A+F_B)}{F_B} \ \frac{Rw}{(1-\theta)} = \frac{F_B(F_A+F_B)}{F_A} \ \frac{Rp}{(1-\theta)} \qquad (38)$$

This implies that we can get an equation which shows the relationship between F_A, F_B, R_w and R_p as

$$\frac{F_A}{F_B} = (\frac{R_p}{R_w})^{1/2} \qquad (39)$$

Since $\frac{R_p}{R_w} < 1$, the number of rich hired by group A will be less than the number of poor hired by group B ($F_A < F_B$). Combining equation (36) and equation (39), we can get the number of people hired by both groups as a function of risk of death, the population of rich in both groups, the amount of resources owned by each rich and each poor.

$$F_A = \frac{(1-\theta)}{2} \ \frac{N_W^A Rw + N_W^B Rw}{\left(R_w^{1/2} + R_p^{1/2} \right) \ R_w^{1/2}} \qquad (40)$$

and $$F_B = \frac{(1-\theta)}{2} \ \frac{N_W^A Rw + N_W^B Rw}{\left(R_w^{1/2} + R_p^{1/2} \right) \ R_p^{1/2}} \qquad (41)$$

Both groups are hiring more people than what they were hiring in case2 but less than what they were hiring in case1. The number of people hired by both groups depends positively on the population of rich in both group and the amount of resources owned by each rich, but it is inversely related to the risk of death and the amount of resources owned by each poor.

Compensation paid by group A for hiring F_A amount of rich will be equal to

$$F_A \frac{Rw}{(1-\theta)} = \frac{\left(N_W^A Rw + N_W^B Rw\right) R_w^{1/2}}{2\left(R_w^{1/2} + R_p^{1/2}\right)} \quad (42)$$

Compensation paid by group B for hiring F_B amount of poor will be equal to

$$F_B \frac{Rw}{(1-\theta)} = \frac{\left(N_W^A Rw + N_W^B Rw\right) R_p^{1/2}}{2\left(R_w^{1/2} + R_p^{1/2}\right)} \quad (43)$$

Compensation paid by group B for hiring poor is than the compensation paid by group A for hiring rich, and the group B is hiring more people than group A. Group A is spending more for hiring in this case than what it was spending in case1 and case2. Group B is spending less for hiring poor than what it was doing in case1 and case2. Compensation paid by group A is positively related to the population of rich in both groups and the amount of resources owned by each rich, but it is negatively related to the amount of resources owned by each poor. Compensation paid by group B is positively related to the population of rich in both groups, the amount of resources owned by each rich and the amount of resources owned by each poor.

Total compensation paid by both groups is same as it was in the case1 and case2 and it is equal to

$$F_A \frac{Rw}{(1-\theta)} + F_B \frac{Rw}{(1-\theta)} = \frac{\left(N_W^A Rw + N_W^B Rw\right)}{2} \quad (44)$$

The total number of people hired by both sides in this case is equal to

$$(F_A + F_B) = \frac{(1-\theta)}{2} \quad \frac{(N_W^A + N_W^B)\ R_w^{1/2}}{R_p^{1/2}} \tag{45}$$

Total death toll will be risk of death times the number of people fighting

$$\theta\ (F_A + F_B) = \frac{\theta(1-\theta)}{2} \quad \frac{(N_W^A + N_W^B)\ R_w^{1/2}}{R_p^{1/2}} \tag{46}$$

In this case the death toll is less than what it was in case1 but it is more than what it was in case2. The death toll increase by a factor of $\frac{R_w^{1/2}}{R_p^{1/2}}$ as we move from case2 to case3 to case1. Death toll is lowest when rich are fighting on both sides and it is highest when poor are hired to fight by both sides.

Since the number of people hired by the groups is different so the value of contest success functions in this case will be

$$\frac{F_A}{(F_A+F_B)} = \frac{R_p^{1/2}}{\left(R_w^{1/2} + R_p^{1/2}\right)} < \frac{1}{2} \tag{45}$$

and

$$\frac{F_B}{(F_A+F_B)} = \frac{R_w^{1/2}}{\left(R_w^{1/2} + R_p^{1/2}\right)} > \frac{1}{2} \tag{46}$$

The utilities for rich in group A and rich in group B will be different ($U_A < U_B$),

$$U_A = \frac{R_p^{1/2}}{\left(R_w^{1/2} + R_p^{1/2}\right)} \frac{(N_W^A\ Rw + N_W^B\ Rw)}{2} \tag{47}$$

$$U_B = \frac{R_w^{1/2}}{\left(R_w^{1/2} + R_p^{1/2}\right)} \frac{(N_W^A\ Rw + N_W^B\ Rw)}{2} \tag{48}$$

The utility of group A depends positively on the population of rich in both groups, the amount of resources owned by each rich and the amount of resources owned by each poor. This is because group A is not hiring poor so as the resources owned by the poor increases, it gained compared to group B which is hiring poor. Rich in group A are worse off than they were in case1 and case2. The utility of group B depends positively on the population of rich in both groups, the amount of resources owned by each rich but it is negatively related to the amount of resources owned by each poor because they are hiring the poor. As the resources owned by each poor increases, the cost of hiring will increase for group B. Rich in group B are better off than they were in case1 and case2.

Now, we can see that if both groups have to choose between hiring rich and hiring poor. They will choose to hire poor, since it is the dominant strategy and we will have a Nash equilibrium in which both groups will hire poor, the death toll will be highest. The rich will also be able to avoid the risk of death.

		Group B	
		Rich are hired	Poor are hired
Group A	Rich are hired	$\frac{(N_W^A Rw + N_W^B Rw)}{4}, \frac{(N_W^A Rw + N_W^B Rw)}{4}$	$\frac{R_p^{1/2}}{\left(R_w^{1/2} + R_p^{1/2}\right)} \frac{(N_W^A Rw + N_W^B Rw)}{2}$ $\frac{R_w^{1/2}}{\left(R_w^{1/2} + R_p^{1/2}\right)} \frac{(N_W^A Rw + N_W^B Rw)}{2},$
	Poor are hired	$\frac{R_w^{1/2}}{\left(R_w^{1/2} + R_p^{1/2}\right)} \frac{(N_W^A Rw + N_W^B Rw)}{2},$ $\frac{R_p^{1/2}}{\left(R_w^{1/2} + R_p^{1/2}\right)} \frac{(N_W^A Rw + N_W^B Rw)}{2}$	$\frac{(N_W^A Rw + N_W^B Rw)}{4}, \frac{(N_W^A Rw + N_W^B Rw)}{4}$

Case 4: When poor are fighting for both sides and the risk of death depends upon the number of people fighting for both sides

When the risk of death depends upon the number of people fighting on both sides, we will have θ_A and θ_B as the risk of death for group A and group B respectively. To make the risk of death endogenous, we have to make θ_A and θ_B a function of F_A and F_B such that the risk of death for group A is negatively related to the number of people fighting for group A but it is positively related to the number of people fighting for group B. The risk of death for group B should be positively related to the number of people fighting for group A, but it should be negatively related to the number of people fighting for group B. As F_A increases, θ_A should decrease and θ_B should increase. Since, the risk of death is a probability so it should take value between 0 and 1.

If we chose $\theta_A = \frac{F_B}{F_A}$ and $\theta_B = \frac{F_A}{F_B}$, then when $F_A = F_B$, the risk of death will be 1 for both sides. When $F_A \neq F_B$, then one of the risk of death will be greater than 1. We should not choose theses functions, the other option is to use the other group's contest success functions as the risk of death for each group. Then $\theta_A = \frac{F_B}{(F_A+F_B)}$ and $\theta_B = \frac{F_A}{(F_A+F_B)}$, the problem with using these functions as the risk of death is that it always sum to 1 and in the case of symmetry, we will have $\theta_A = \theta_B = \frac{1}{2}$. That will give us same result as we will have when the risk of death is exogenous and it is equal to $\frac{1}{2}$. That is why I chose a generalize form such that I can get any value for the risk of death between 0 and 1. These functions are

$$\theta_A = \frac{kF_B}{(F_A+kF_B)} \tag{49}$$

and

$$\theta_B = \frac{kF_A}{(kF_A+F_B)} \tag{50}$$

In this case k > 0 and by changing the value of k, we can get any value for the risk of death in the equilibrium. When poor are hired then cost of hiring a poor for group A will be equal to $\frac{R_p}{(1-\theta_A)}$ and for group B it will be $\frac{R_p}{(1-\theta_B)}$. When group A and group B are hiring F_A and F_B number of poor respectively, the total compensation paid by rich to the poor will be

$$\text{Total compensation} = F_A \frac{R_p}{(1-\theta_A)} + F_B \frac{R_p}{(1-\theta_B)} \quad (51)$$

By using the equation (49) and equation (50) for the values of θ_A and θ_B in equation (51), we get

$$\text{Total compensation} = (1+k)\,(F_A + F_B)\,R_p \quad (52)$$

In this case, the total amount of contested resources will be equal to

$$\text{Contested resources} = N_w^A R_w + N_w^B R_w - (1+k)\,(F_A + F_B)\,R_p \quad (53)$$

The utility functions for group A and group B will be

$$U_A = \frac{F_A}{F_A+F_B} \{ N_w^A R_w + N_w^B R_w - (1+k)\,(F_A + F_B)\,R_p \} \quad (54)$$

and

$$U_B = \frac{F_B}{F_A+F_B} \{ N_w^A R_w + N_w^B R_w - (1+k)\,(F_A + F_B)\,R_p \} \quad (55)$$

Both groups will try to maximize their utilities by hiring the optimum number of poor, supposing that an interior solution exists, the first order conditions will be

$$\frac{dU_A}{dF_A} = \frac{F_B}{(F_A+F_B)^2} \{ N_w^A R_w + N_w^B R_w - (1+k)\,(F_A + F_B)\,R_p \} - \frac{F_A}{F_A+F_B} (1+k)\,R_p = 0 \quad (56)$$

and

$$\frac{dU_B}{dF_B} = \frac{F_A}{(F_A+F_B)^2} \{ N_w^A R_w + N_w^B R_w - (1+k)\,(F_A + F_B)\,R_p \} - \frac{F_B}{F_A+F_B} (1+k)\,R_p = 0 \quad (57)$$

By solving equations (56) and (57), we will get

$$F_A = F_B = \frac{(N_W^A Rw + N_W^B Rw)}{4(1+k)Rp} \quad (58)$$

Since $F_A = F_B$, we can write risk of death for both groups as

$$\theta_A = \theta_B = \frac{k}{(1+k)} \tag{59}$$

or
$$(1-\theta_A) = (1-\theta_B) = \frac{1}{(1+k)} \tag{60}$$

Let us denote $\theta_A = \theta_B = \theta$, then by using equations (58) and (60), we can write

$$F_A = F_B = \frac{(1-\theta)(N_W^A Rw + N_W^B Rw)}{4Rp} \tag{61}$$

Equation (61) is same as the equation (14), it means that the same amount of people will be hired by both groups as they were hiring when the risk of death was exogenous, so making the risk of death endogenous is not going to change anything. All other results of case 1 will also hold in this case.

Conclusion

Civil wars are usually fought by the poor who do not own the resources with which they are fighting and the contested resources in these wars are also not owned by them. Poor usually do not get any share in the resources gained in the civil wars. They are hired by rich to fight the war on their behalf. One of the possible reasons behind this hiring of poor by the rich is that the rich can avoid the risk of death associated with those who are engaged in the combat. In this paper, I allow for the risk of death and poor are compensated for the risk of death as well as the loss of production when they are hired to fight the war. Utility of rich in both groups involved in the war is same in both cases when they hire poor and when they fight the war for themselves. When poor are hired the marginal cost of hiring is less but the marginal benefit is same so more people are hired. When rich fight the war, the marginal cost of hiring is more as a result of which less people are hired but the total amount of spending for fighting is same in both the cases. Death toll is high when poor are hired because more people are fighting the war.

The rich has to decide between small number of death for rich and large death toll of poor. They chose to hire the poor, so that they can avoid the risk of death.

Apart from the risk of death, hiring poor is the dominant strategy for both groups. When one group is hiring rich and the other group is hiring poor, the group which is hiring poor will be able to hiring more people by spending less resource and they will be able to capture more resources. In the Nash equilibrium, both groups will hire poor. The death toll will be maximum, since it depends upon the risk of death and the number of people fighting. Number of people fighting itself depends on the risk of death. When the risk of death is 0.5, the death toll will be maximum. Making the risk of death endogenous and dependent on the number of people fighting for both sides is not going to change any of the result.

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