Monopoly Regulation Under Incomplete Information: Prices vs. Quantities

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Abstract

In this paper we compare two mechanisms to regulate a monopoly: fixing a price for the good or service, or fixing the quantity to be produced. We consider an environment of asymmetric information in which the regulated firm has private information about the demand it faces or its costs. We completely characterize the optimal mechanism in both case and compare their performance. The problem is solved considering a (sophisticated) mechanism design approach, and also an environment where (simple) bunching mechanism must be used. When demand is known imperfectly by the regulator, and he is allowed to choose any mechanism, price regulation dominates quantity regulation for increasing marginal costs, while simple price bunching regulation dominates for decreasing marginal costs. In the remaining cases, price and quantity regulation can both be preferable, depending on parameter values. In particular, for sophisticated regulation and decreasing marginal cost, the importance the regulator gives to monopoly profits play a major role. Keywords: *Price regulation, quantity regulation, monopoly regulation, mechanism design, asymmetric information. JEL: D42, D82, L51.*

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1. Introduction

The design of regulatory mechanisms in order to control monopoly market power, when the firm has better information than the regulator is a problem whose practical relevance is undoubtable. It is then no surprise that many articles have looked into the issue, taking advantage of the development of the Bayesian mechanism design literature. David Baron and David Besanko (1984), Baron and Roger Myerson (1982), Jean-Jacques Lafont and Jean Tirole (1986), David Sappington (1983) and Sappington and David Sibley (1988) among others, focus on cases where the firm has private information about its costs. Michael Riordan (1984) and Trace Lewis and Sappington (1988), on the other hand, focus on the case where the firm's private information is about its demand. The difference matters: it is now almost a textbook example of Bayesian mechanism design to regulate a monopoly with private information about its (constant) marginal cost, it is optimal to delegate the choice of price to the firm, who must choose from a menu of pairs of prices and transfers; the price will be above marginal cost for all type of firms but the most efficient. It seems to be less well-known though that if private information is on demand and marginal costs are increasing, the first best (price equal marginal cost and zero profit for the firm) is incentive compatible and therefore optimal, while when marginal costs are decreasing, the optimal mechanism is to name one price and one transfer, regardless of demand.

In all the above cases, the form of controlling the firm is through the price mechanism. Yet, in many other real cases and settings, particularly in the case of externalities, economists and authorities have considered implementing quantity-based mechanisms in order to induce some desired outcome. This is the case, for example, of choosing the optimal number of pollution permits, or fixing to some level the number of flights (departures and arrivals) per unit of time at an airport (as in the four slot-controlled airports Chicago O'Hare, New York La Guardia, New York Kennedy, and Washington National). A natural question then emerges: given the Principal's objective function, are quantity-based mechanisms preferable to price mechanisms? The first formalization of the 'prices vs quantities' comparison is the article by Martin Weitzman (1974), who acutely pointed out that if there is any advantage to employing price or quantity control modes it must be due to inadequate information or uncertainty, since in an environment of perfect information there will be formal identities between the two and hence –leaving implementation issues aside– both should deliver the same results. Weitzman found that if the objective function is to maximize social welfare, and there is uncertainty in costs and demand, then the relative advantage of one control mechanism over the other would depend on the slopes of demand and cost functions; later, Laffont (1977) complemented these results. This seminal work has since then been used and expanded in the environmental economics literature (e.g. Adar y Griffin, 1976; Montero, 2002; Kelly, 2005) and more recently in the airport management literature (Achim Czerny, 2009; see also Jan Brueckner, 2009, and Leonardo Basso and Anming Zhang, 2009, or more on price versus quantity based mechanisms for airport control). Yet, somewhat surprisingly, quantity based mechanisms have not been studied as an option for the regulation of monopolies and therefore, whether price is comparatively better than quantity for regulation of market power has not been addressed. This article attempts to fill this gap, providing regulators and authorities with insights about the consequences of choosing different instruments for regulation. Our main research question is then the following: in an environment of asymmetric information in which the firm has superior knowledge about its demand or costs, which instrument is preferable, prices or quantities? We assume, from the outset, that the decision to regulate the firm has been taken and therefore will not consider options other than regulation.

Now, if the results from Weitzman (1974) and the paper that followed were directly applicable to the regulation case through simple relabeling of variables, and thus the same qualitative insights would be obtained, the contribution would be relatively little. But this is not the case because of two reasons. First, the setup proposed by Weitzman applies to firms that are price takers and do not face the demand directly as is the case with monopoly; we discuss this with more detail in the next section but the differences are sizeable. Second, these papers do not make use of mechanism design in the sense that is currently understood, that is, of carefully designing mechanisms that delegate the pricing or quantity decision to agents but inducing them to choose what is best for the regulator. Instead, they consider a case where the decisions are not delegated, that is, the principal uses its imperfect information to set one price or one quantity and does not attempt to induce revelation of information. In mechanism design jargon, the regulator is restricted to choosing bunching mechanisms from the outset. What we do in this paper is to consider for both sophisticated Bayesian mechanisms and simpler bunching mechanisms, both for price and quantity modes, and then compare. The idea is to obtain insights on when is one instrument better than the other, what drives the relative advantage, and whether the degree of sophistication of the regulator affects the choice of instrument. We believe this last point is important because, while Bayesian mechanims are undoubtedly elegant and solve the principal's problem optimally, there have been some discussion regarding their applicability in practice and, in fact, have not really been implemented. While we do not take position in this debate, we do realize that simple mechanisms that directly name a price or quantity, while obviously being less beneficial for the principal, may be in fact something more implementable in reality.

Our results show that when the firm has private information about its costs, the two instruments are equivalent for all cost structures and independently of the sophistication of the regulation. However, when demand is known imperfectly by the regulator, sophisticated price regulation will dominate quantity regulation for increasing marginal cost, while simple price regulation dominates for decreasing marginal costs. In the remaining cases, price and quantity regulation can both be preferable, depending on parameter values. In particular, for sophisticated regulation and decreasing marginal cost, the importance the regulator gives to monopoly profits play a major role. All this, in a context in which we assume that under quantity regulation, the firm chooses (or is asked) to produce a certain level of production, and then a price is set such that these units are actually sold. In other words, we rule out the possibility that the monopoly decides to ration its production and keep some units unsold. We discuss this assumption more in the next section. [NOTE TO DRAFT: *not sure about leaving the last sentence here*]

The structure of the paper is as follows: in Section 2 we lay out our general setup and link our model to the literature, providing a comprehensive view of the results that are already available and what is needed for a full comparison of regulatory mechanisms. In this section we also provide our first result, for the case of unknown costs. Section 3 is devoted to obtaining sophisticated (Bayesian) price and quantity mechanisms when demand is unknown, and comparing their performance. Section 4 does the same but for the case of simpler regulatory mechanisms. In Section 5 we discuss our results and compare simple to sophisticated mechanisms (what we call, the benefits of sophistication). Section 6 concludes.

2. The Setup and the Case of Unknown Costs

The setup we consider is a classical principal-agent framework of adverse selection. We consider that there is asymmetric information i.e. the firm has private information regarding its costs *or* its demand but not both.[1]The information is indexed by a parameter θ, known to the firm (its *type*) but unobservable to the regulator. It is common knowledge though, that the firm's parameter belongs to a compact interval $\Theta = \left[\underline{\theta}, \overline{\theta}\right] \subset \mathbb{R}$, while the regulator has a subjective belief that the distribution of the parameter is $G : \Theta \to [0,1]$ with continuous density $g(\cdot)$. Before going ahead with the problem the regulator faces it is important to discuss why it could be the case that either costs or demand are unobservable to the regulator. [NOTE TO DRAFT: *put here baron-myerson & weitzman insights for unknown costs....put lewis and sappington insights for unknown demand..... add the vertical structure insight using as examples airports and electric generation]*.

The regulator has to make three choices. First, it has to choose a regulatory instrument, namely price p or quantity q. Then it has to choose the level for that instrument, which we denote in general by $x \in \mathbb{R}_+$, and the size of a lump-sum transfer $T \in \mathbb{R}$ between the firm and the consumers, which corresponds to a tax if T is positive and a subsidy if it is negative. In Bayesian mechanism design, the regulator chooses (x, T) contingent on message delivered by the firm about its type. Following the *revelation principle*, the regulator can focus without loss of generality on direct mechanisms that are incentive compatible, offering to the firm different values of x and T depending on θ. Note that, in practice, this is equivalent to offer a schedule or menu of (x, T) pairs, delegating to the firm the choice of price or production and the associated transfer. Clearly, this way of proceeding requires quite an amount of sophistication both from the part of the regulator and the firm. Hence, we define:

Definition 1 *Let $\theta \in \Theta$ be the monopoly's private information about its costs or demand. A* ***sophisticated regulatory mechanism*** *consist of an assignment rule $x : \Theta \longrightarrow \mathbb{R}_+$ (where x is either p or q), and a lump-*

[1]It is not that the case where both are private information is not interesting, but mechanism design for multidimensional asymmetric information is less tractable and the insights much more dependent on assumptions. See Jean-Charles Rochet and Lars Stole (2003) and Mark Armstrong and Sappington (2007) for surveys of multidimensional screening.

*sum transfer rule $T : \Theta \longrightarrow \mathbb{R}$, such that $\{x(\theta), T(\theta)\}_{\theta\in\Theta}$ is a menu that induces truthful self-selection for all $\theta \in \Theta$. We will say that a regulator is **sophisticated** if she regulates using a sophisticated mechanism.*

These type of mechanism, while very popular by now in the literature, have been criticized because they would not really be applicable in practice (see Michael Crew and Paul Kleindorfer, 2002; Ingo Vogelsang, 2002). In fact, there is no record that regulatory mechanisms as sophisticad as the Bayesian have ever been implemented.[2]While we do not want to wage in that debate, we do agree that sophisticated mechanisms may be hard to implement and thus we address the choice of regulatory instrument in a context where the regulator restricts herself to simpler mechanisms. By simple mechanisms we mean that the regulator do not delegate the choice of price or production level to the firm, but instead uses its imperfect information to set one price or one quantity without attempting to induce revelation of information. This simpler mechanisms is what in Bayesian mechanism design are called bunching mechanisms.

Definition 2 *Let $\theta \in \Theta$ be the monopoly's private information about its costs or demand. A **simple regulatory mechanism** consists of a unique assignment and transfer $\{x^*, T^*\}$ (where x is either p or q), for all state $\theta \in \Theta$. We will say that a regulator is **simple** if she regulates using a simple mechanism.*

The only missing ingredient in our setup is the regulator's objective function. First, we write the monopolist profit function as $\pi(x, T, \theta) = R(x, \theta) + T$ where $R : \mathbb{R}_+ \times \Theta \to \mathbb{R}_+$ are revenues and therefore, θ appears either in the demand or in the cost function depending on where is private information present. As is usual in settings where transfers are feasible (see Armstrong and Sappington, 2007 for a survey), we consider that the regulator seeks to maximize a social welfare function SW which is a weighted average of consumer benefits and firm's profits, namely,

$$SW(x, T, \theta) = CS(x, \theta) - T + \alpha\pi(x, T, \theta) \quad \alpha \in [0, 1]$$

where $CS : \mathbb{R}_+ \times \Theta \to \mathbb{R}_+$ is consumer surplus. For a mechanism to be feasible, it has to ensure the monopoly's participation in the regulation game (positive profits), and the incentive compatibility constraint to induce truth-telling. Thus, the problem the regulator faces, once it has chosen the regulatory instrument and if it uses a sophisticated mechanism is:

$$\begin{aligned}
\max_{x,T} \quad & \int_{\Theta} SW(x(\theta), T(\theta), \theta) g(\theta) d\theta \\
s.a \quad & \pi(x(\theta), T(\theta), \theta) \geq \pi(x(\hat{\theta}), T(\hat{\theta}), \theta) \quad \forall(\theta, \hat{\theta}) \in \Theta^2 \\
& \pi(x(\theta), T(\theta), \theta) \geq 0 \quad \forall\theta \in \Theta
\end{aligned}$$

The problem for simple mechanisms is identical, but with$\{x(\theta), T(\theta)\}$ replaced by $\{x, T\}$. Note that, while what we call sophisticated mechanism are indeed Bayesian mechanisms (despite the fact that quantity

[2]According to Vogelsang (2004) the FCC has tried out price regulation with menus for the Bell operating companies in the early 1990s, but the practice was abandoned after a few years.

has not been considered as an instrument before), our simple mechanisms are more general than what was consider by Weitzman (1974) and Laffont (1977) because they considered as objective function the unweighted sum of consumer benefits and firms' profits, and did not allow for transfers. This is equivalent, in our setup, to a simple mechanism restricted to $\alpha = 1$ and $T = 0$.

Then, in order to choose a regulatory instrument, the regulator needs to solve the above problem for both instruments, and then compare them to see which leads to a larger value of the social welfare function. If the problem is solved in general, the what is expected is that conditions for one instrument to be superior to the other will emerge.

The case of unknown costs

We focus now in the case of unknown costs, as is quite trivial to compare regulatory instruments here. The seminal work on optimal (sophisticated) price regulation –i.e. $x(\cdot) = p(\cdot)$– when private information is on cost –i.e. $C(q, \theta)$– as in Baron and Myerson (1982). They considered a setup like ours, where θ induces parallel shifts of the constant marginal cost function of the firm. The main result is that the regulator designs a mechanism $(p^*(\theta), T^*(\theta))$ such that if a firm is of the highest cost, it ends up with zero profits, while any other type of firm is left with informational rents. Moreover, all type of firms but the most efficient self-select into prices that are above marginal cost, a result known as *no distortion at-the-top.* It is easy to prove that these results extend to any cost function, as long as θ induces only parallel shifts. As mentioned before, there is no literature on quantity regulation for unknown costs yet, when the demand function $Q(p)$ is common knowledge, the optimal quantity regulation can be trivially obtained from the optimal price mechanism: if the sophisticated price mechanism is $(p^*(\theta), T^*(\theta))$, then the sophisticated quantity mechanism is $(q^*(\theta) = Q(p^*(\theta)), T^*(\theta))$ and they will both obviously lead to the same level of social welfare, price, production and so on. Note further that the same happens when comparing the two instruments in simple mechanisms: if (p^*, T^*) is the best simple price mechanism, then $(q^* = Q(p^*), T^*)$ will be the best simple quantity mechanism and again social welfare levels will be the same. Therefore, we can conclude that:

Proposition 3 *When there is private information on costs, independently of whether the regulator chooses sophisticated or simple mechanisms, both regulatory instruments –price and quantity– lead to the same results in terms of final price, production, transfers, consumer benefit, firm profits and objective function.*

Readers familiar with results from Weitzman (1974) may found this proposition puzzling; his main results is that the relative advantage of one control mechanism over the other depends on the slopes of demand and cost functions and the variance of the cost function, while any demand uncertainty is completely irrelevant. Hence, private information on costs would matter. The explanation of the difference is very important conceptually: Weitzman results do not apply to regulation as we now explain. Recall that in Weitzman models $\alpha = 1$ and $T = 0$; assuming for expositional simplicity that there is only information asymmetry in costs, Weitzman's quantity mechanism, in our notation, would be given by $q^* \in \arg\max_q \mathbb{E}_\theta[CS(q) - C(q,\theta)]$; the price mechanism, on the other hand would be given by $p^* \in \arg\max_p \mathbb{E}_\theta[CS(h(p,\theta),\theta) - C(h(p,\theta),\theta)]$, where $h(p,\theta) = \tilde{q} \in \arg\max_q pq - C(q,\theta)$. It is this last fact which separates this case from monopoly regulation: what Weitzman assumes is that, once a regulated price is given, the firm (or firms) will act as price takers and will choose a quantity of production according to its (their) cost function, and not the demand. In other words, the firm assumes that it will be able to sell all the units it wants at the regulated price. This may be the case if there are many firms –such that each sells a small portion of the total– and thus it is reasonable to think that they act as price takers. Or, perhaps in a procurement case, where the principal tells the firm that it will buy all units that the agent want to produce at the regulated price. But in the case of monopoly regulation, the firm does face the whole demand, and thus the quantity produced for a regulated price will not be given by profit maximization but simply by $Q(p^*)$, as assumed by Baron and Myerson. In fact, for the monopoly case, the problem of the regulator is $p^* \in \arg\max_p \mathbb{E}_\theta[CS(Q(p)) - C(Q(p),\theta)]$, which indeed leads to the result in Proposition 3.

What we are left with, and what is the focus of the rest of the paper is the case when private information is on demand. Here, the comparison between regulatory instruments is far for simple and requires quite an amount of work. There are two papers on regulation under unknown demand that are of central importance: Laffont (1977) and Lewis and Sappington (1988). Let us start by the latter; Lewis and Sappington (1988) consider price regulation when the direct demand function is given by $Q(p,\theta)$ –with θ inducing parallel shifts– while the cost function is common knowledge. The results are very different to the case of unknown costs. When marginal costs are non-decreasing, the first-best mechanism, which involves price equal marginal cost and transfers that induce zero profits for all demand types, is incentive compatible. This implies that the regulator can delegate the pricing decision to the firm who will not create social costs as it cannot take advantage of its superior information. On the contrary, when marginal costs are decreasing, it is never optimal for the regulator to use a menu; Lewis and Sappington show that there always exist a bunching mechanism that does better: hence, the regulator will set a single price and transfer independently of the firm's message about its demand. Using our definitions, the optimal sophisticated price mechanism is, in fact, a simple mechanism. Hence, in what respects to sophisticated mechanisms with unknown demand, there already exists results for price regulation; but in order to undertake our comparison of the two instruments we need to (i) obtain the sophisticated quantity mechanism, and (ii) explicitly obtain the price mechanism when marginal costs are decreasing (bunching), since Lewis and Sappington's proof is not constructive. All this is undertaken in Section 3.

The other paper that interest us is Laffont (1977). This author pointed out that Weitzman had in reality, different assumptions regarding information in his model. Laffont shows convincingly that Weitzman considers *uncertainty* about demand, –in that demand is unknown to both the regulator and the firm–, but private information on costs –the firm knows but the regulator only knows a distribution.

Laffont points out that private information is a more interesting case that uncertainty since in the latter, the firm cannot do better than a regulator and vice-versa. Thus, he analyzed what he called *Weitzman's dual problem*, that is, private information on demand but uncertainty on costs. As important for us as this fact, is that Laffont crafted his model in a way that is indeed applicable to the monopoly case (although he did not make the distinction): contrary to Weitzman, he did considered that once a regulated price is set, the quantity to be produced is given by the demand, $Q(p,\theta)$, just as in Lewis and Sappington (1988). Therefore, Laffont provides the first comparison between the two regulatory instruments when demand is unknown; in our terms, Laffont compares simple mechanisms when transfers are zero and profits have the same weight as the consumer surplus ($\alpha = 1$). His result indeed show that in this case, the choice of instrument matters; he finds that the relative advantage of the price instrument over the quantity instrument is equal to $\frac{(-b+C'')\sigma_\theta}{2b^2}$, where $b > 0$ is the slope of the inverse demand function, $C'' > 0$ is the slope of the marginal cost function and σ_θ is the variance of the demand. Hence, in what respects to the comparison between regulatory instruments within the context of simple mechanisms, the challenge is to derive results for the case where transfers are feasible and profits may weigh less than consumer surplus; this is analyzed in Section 4.

Now, before closing this section and moving on to detailed analysis of specific regulation mechanisms it is important to stop a moment to discuss what quantity regulation means. Because, indeed, price regulation seem to be quite natural to understand: either the regulator sets a price or the firm chooses one from the menu it was offered, but, as soon as the price is fixed, the firm has to sell to every consumer that demands a unit at that price. In fact, Lewis and Sappington show that it is not in the interest of the firm to do otherwise. With quantity regulation, however, the firm is either asked to produce a certain amount in the simple mechanism, or chooses the quantity to produce from a menu. But, how are those units rationed between consumers? The assumption we make, is that the price at which the produced units go is given by the inverse demand $P(q,\theta)$, that is, that a price that clears the market *emerge*. It may not be in the firm interest, however, to choose such a price; after all, once the units are produced, the firm has now a marginal cost function which is zero up to the point given by the produced units and, therefore, if the monopoly is left alone, it would choose to sell less units than those produced, according to the point where the marginal revenue function intersects the x axis. What we are thinking of when we speak of quantity regulation here is that the regulator will take care of the rationing, for example, through an auction. Indeed, a simple generalized Vickrey auction or a Uniform price auction would result in a value for each unit equal to the price that clears the market (with the revenues of the sale going to the firm). Two things are important to note regarding this assumption: on one hand, in the airport market the FAA has been indeed considering auctions to ration slots (see e.g. Frank Bernardino, 2009; Brueckner, 2009); on the other hand, although never mentioned explicitly, both Weitzman (1978) and Laffont (1977) did assume that such market-clearing rationing was going on when considering quantity mechanisms. While we consider the analysis of the quantity mechanism when the monopoly is allowed to ration freely –and the comparison of this to the price instrument– to be indeed very interesting and relevant, we see this as a second step in the research agenda.

3. Sophisticated Regulatory Mechanisms with Unknown Demand

There is private information about the market demand, indexed by a parameter θ, which is observable by the firm but not by the regulator. This parameter belongs to some compact interval $\Theta = \left[\underline{\theta}, \overline{\theta}\right] \subset \mathbb{R}$. As usual, the regulator has some prior probability distribution, $G : \Theta \to [0,1]$ with continuous and strictly positive density $G'(\cdot) = g(\cdot) > 0$. The next assumption is standard in mechanism design and is satisfied, among others, by any log-concave distribution.

Assumption 4 *$G(\theta)$ satisfies increasing hazard rate, i.e, $\frac{g(\theta)}{1-G(\theta)}$ is increasing in θ*

We assume that quantity and price are linked by the equation, $q = Q(p,\theta)$, or equivalently $p = P(q,\theta)$, with the following properties: $Q_p(\cdot,\theta) \leq 0 \quad \forall \theta \in \Theta, Q_\theta(p,\cdot) \geq 0 \quad \forall p \in \mathbb{R}_+, P_q(\cdot,\theta) \leq 0 \quad \forall \theta \in \Theta, P_\theta(q,\cdot) \geq 0 \quad \forall q \in \mathbb{R}_+$. In other words, in the $p-q$ space the demand function has negative slope, and higher realizations of θ means higher demand. To ensure that the single crossing property holds, we make the following assumption (which implies that a change in θ implies parallel shifts).

Assumption 5 *We consider $Q : \mathbb{R}_+ \times \Theta \to \mathbb{R}_+$ and $P : \mathbb{R}_+ \times \Theta \to \mathbb{R}_+$ such that $Q_{\theta p}(p,\theta) = 0 \; \forall \theta \in \Theta \;, p \in \mathbb{R}_+$ and $P_{\theta q}(q,\theta) = 0 \; \forall \theta \in \Theta \;, q \in \mathbb{R}_+$*

There is a production cost $C(\cdot)$, with $C'(\cdot) \geq 0$. If the marginal costs are decreasing, for the regulator's problem to be concave, we assume that the demand function is more steeply sloped than marginal costs.

Assumption 6 *If the marginal costs are decreasing, we assume $\mid C''(Q(p,\theta)) \mid \cdot \mid Q_p(p,\theta) \mid < 1$. Equivalently, $\mid C''(q) \mid < \mid P_q(q,\theta) \mid$*

Now, using The Revelation Principle, we define formally a quantity and price regulatory mechanisms.

Definition 7 *A **price regulatory mechanism** is a pair of functions (p,T) such that, $p : \Theta \longrightarrow \mathbb{R}_+$ and $T : \Theta \longrightarrow \mathbb{R}$, where $p(\theta)$ represents the regulated price when the firm reports θ and $T(\theta)$ specifies the transfers (subsidies if positive, taxes otherwise).*
*Analogously, a **quantity regulatory mechanism** is a pair of functions (q,T) such that, $q : \Theta \longrightarrow \mathbb{R}_+$ and $T : \Theta \longrightarrow \mathbb{R}$.*

So, in a price regulatory mechanism, the profit of a firm of type θ that declares a type $\hat{\theta}$ is given by

$$\pi(\hat{\theta},\theta) = p(\hat{\theta})Q(p(\hat{\theta}),\theta) - C(Q(p(\hat{\theta}),\theta)) + T(\hat{\theta})$$

Analogously, in a quantity regulatory, mechanism, profits are given by

$$\pi(\hat{\theta}, \theta) = P(q(\hat{\theta}), \theta)q(\hat{\theta}) - C(q(\hat{\theta})) + T(\hat{\theta})$$

In both cases, we can consider, without loss of generality, regulatory mechanisms that ensure truthful revelation. So we impose that,

$$\Pi(\theta) := \max_{\hat{\theta}\in\Theta} \pi(\hat{\theta}, \theta) = \pi(\theta, \theta) \quad \textbf{(IC)}$$

Also we assume that the firm cannot be forced to accept the contract, hence a mechanism must satisfy the voluntary participation constraint. [3]

$$\Pi(\theta) \geq 0 \quad \forall\theta \in \Theta \quad \textbf{(VP)}$$

Definition 8 *We say that a regulatory mechanism is **feasible** if and only if it satisfies **(IC)** and **(VP)**.*

The regulator's objective function is a weighted social welfare, putting more weight on the consumers than the firm's profits. We now write the objective function in terms of prices or quantities.

$$\begin{aligned}
\mathbb{E}_\theta SW(p, T, \theta) &= \mathbb{E}_\theta \left\{ \int_p^\infty Q(x, \theta)dx - T + \alpha\left[pQ(p, \theta) - C(Q(p, \theta)) + T\right] \right\} \\
\mathbb{E}_\theta SW(q, T, \theta) &= \mathbb{E}_\theta \left\{ \int_0^q [P(x, \theta) - P(q, \theta)]dx - T + \alpha\left[P(q, \theta)q - C(q) + T\right] \right\}
\end{aligned}$$

With $\alpha \in [0, 1]$. Note that when $\alpha < 1$, a dollar out of the consumers' pockets hurts society more than the benefit of a dollar received by the firm and, therefore, the regulator may be willing to accept some output contraction (i.e. *allocative inefficiency*), in order to diminish the size of the subsidy. Intuitively, the regulator will need to raise fund through taxes to subsidize the firm, which is costly unless $\alpha = 1$.

The regulator problem is, in each case, to choose a feasible regulatory mechanism which will maximize a expected weighted social welfare subject to the incentive compatibility and individual rationality constraints.

Now, following standard methods in mechanism design, it easy to show that the incentive compatibility constraint translates in two conditions.

Lemma 9 *Suppose that assumption 5 holds. A price regulatory mechanism is incentive compatible if and only if:*

i. $p(\cdot)$ *is non-decreasing in* θ.

ii. $\Pi'(\theta) = [p(\theta) - C'(Q(p(\theta), \theta))]\, Q_\theta(p(\theta), \theta)$

[3] We normalize the outside option to zero.

A quantity regulatory mechanism is incentive compatible if and only if:

i. $q(\cdot)$ *is non-decreasing in* θ.

ii. $\Pi'(\theta) = P_\theta(q(\theta), \theta)q(\theta)$

As a benchmark we define the first-best regulatory mechanism which is the mechanism that the regulator would choose under complete information.

Definition 10 *The* ***first-best regulatory mechanism*** *consists of prices* $p(\theta)$*, quantities* $q(\theta)$ *and transfers* $T(\theta)$ *such that for all* $\theta \in \Theta$*:*

1. $p(\theta) = P(q(\theta), \theta)$ *or* $q(\theta) = Q(p(\theta), \theta)$

2. $p(\theta) = C'(q(\theta))$

3. $T(\theta)$ *such that* $\Pi(\theta) = 0$.

The first thing to note is that if marginal costs are non-decreasing, the first-best price mechanism is feasible and therefore optimal, in contrast to the first-best quantity mechanism. Since higher θ means higher demand, and therefore, higher marginal costs, the first best allocation implies both an increasing quantity and price schedule, satisfying condition (i) in lemma 9. Moreover, for a price mechanism a price that equals marginal cost implies condition (ii) as long as $\Pi'(\theta) = 0$, i.e., the firm's profits are independent of the demand type. So, if we adjust the transfers to leave with zero profits to any type, the price regulatory mechanism that achieves the first best is feasible. However, for a quantity regulatory mechanism, profits must by increasing in type, so it is impossible to satisfy both VP and zero profits for the firm.

It is important to remark that it is always possible to achieve an efficient production level (quantity that equals price with marginal cost) with an adequate transfer. Yet, optimally, for $\alpha < 1$, in a quantity mechanism the regulator chooses not to do so since such a schedule requires transfers that are costly. This is very different to the result obtained when price regulation is considered and marginal cost are increasing, where an efficient production level can be achieved while leaving the firm with no profits, therefore attaining the first best.

Proposition 11 *Let* $z_\alpha(\theta) = (1-\alpha)\frac{1-G(\theta)}{g(\theta)}$*. Suppose that assumption 4 holds, and that* $P_\theta(q, \theta)$ *is decreasing in* θ *for all* q*, then the optimal quantity regulatory mechanism* (q^o, T^o) *satisfies:*

$$P(q^o(\theta), \theta) = C'(q^o(\theta)) + z_\alpha(\theta)P_\theta(q^o(\theta), \theta) \quad (1)$$

$$T^o(\theta) = C(q^o(\theta)) - P(q^o(\theta), \theta)q^o(\theta) + \int_{\underline{\theta}}^{\theta} P_2(q^o(x), x)q^o(x)dx \quad (2)$$

The previous characterization of optimal quantity mechanism holds for any costs structure and, moreover, only achieves the allocative first best when $\alpha = 1$ (and that, leaving positive profits to firms). As we discussed before, price mechanisms achieve the first best with increasing marginal costs, and to complete the analysis we discuss below what happens with decreasing marginal costs.

Lewis and Sappington (1988) find that in such a case the optimal price regulatory mechanism involves complete bunching. In other words, the regulator fixes a price (and a transfer) independent of the firm's report. However, their proof is not constructive, and in what follows we characterize the optimal pair (p^*, T^*) in order to perform comparisons with the optimal quantity mechanism.

From lemma 7, we see that choosing a price p^* implies that $\Pi'(\theta) = [p^* - C'(Q(p^*,\theta))]\, Q_\theta(p^*,\theta)$. Since marginal costs are decreasing, we have that there exists $\theta_c(p^*)$ such that $\Pi'(\theta) \leq 0$ if and only if $\theta \leq \theta_c(p^*)$ (in particular, θ_c could be either $\underline{\theta}$ or $\overline{\theta}$ if the price is too big or too low). Because of this, the type which is most reluctant to participate is precisely $\theta_c(p^*)$, which will then be left with exactly zero profits. Therefore, we immediately have that the transfers can also be written as a function of the chosen p^* as $T^*(p^*) = C'(Q(p^*, \theta_c(p^*))) - p^* Q(p^*, \theta_c(p^*))$. It is easy to see that any price below $\underline{p}$, defined by $\underline{p} = C'(Q(\underline{p}, \overline{\theta}))$ is dominated by $\underline{p}$, and analogously for $\overline{p} = C'(Q(\overline{p}, \underline{\theta}))$. Therefore, the problem of the regulator can be written as

$$\max_{p^* \in [\underline{p}, \overline{p}]} \mathbb{E}_\theta SW(p^*,\theta) = \mathbb{E}_\theta \left\{ \int_{p^*}^{\infty} Q(x,\theta)dx - T(p^*) + \alpha \left[p^* Q(p^*,\theta) - C(Q(p^*,\theta)) + T(p^*) \right] \right\}$$

Moreover, from the above discussion, we can characterize the allocative inefficiencies that a bunching mechanism introduces:

Lemma 12 *If marginal costs are decreasing, a bunching mechanism (p^*, T^*) satisfies that $p^* \leq C'(Q(p^*,\theta)$ if and only if $\theta \leq \theta_c(p^*)$*

As we can see, these allocative inefficiencies can be different for low and high realizations of the demand. If low, production is higher than efficient, if high, production is lower. In the next example, we explicitly compute the optimal price mechanism in the case of linear demand and quadratic costs.

Example 13 *Consider a direct demand function $Q(p,\theta) = \frac{A+\theta-p}{b}$ and quadratic costs $C(q) = c_0 q - k\frac{q^2}{2}$, with $b > k$. In this case, the best bunching price mechanism is represented by:*

$$\begin{aligned} p^* &= \frac{bc_0 - k(A + \mathbb{E}(\theta))}{b-k} && (3) \\ T^* &= \frac{k(A - c_0 + \mathbb{E}(\theta))^2}{2(b-k)^2} && (4) \\ \theta_c(p^*) &= \mathbb{E}(\theta) && (5) \end{aligned}$$

Note that this optimal mechanism does not depend on α, but this is not general. What is always true is that, given a price, the transfers do not depend on α. However, α may influence the selection of the optimal p, and with it the allocative inefficiencies induced by the bunching mechanism.

Up to now, we have characterized the optimal regulatory mechanism for the cases where the regulator con monitor prices or quantities. Now, we turn our attention to the performance comparison between the optimal quantity mechanism and the optimal price mechanism. Remember that the optimal regulation through quantities when demand is unknown never involves bunching, independent of the cost structure of the monopolist. Moreover, when $\alpha = 1$, the first-best allocation is attained: the provision of incentives is exclusively done through transfers (without distorting the optimal allocation), and since transfers are irrelevant in this case, the quantity mechanism weakly dominates any other one.[4] When $\alpha < 1$ and marginal costs are increasing, the price mechanism attains the first-best, yet the quantity mechanism induces allocative inefficiencies and leaves the firms with informational rents (which are undesirable when $\alpha < 1$), therefore the price mechanism dominates. Note that the first-best allocation could be attained with a quantity mechanism, but it would involve large profits for the firm. Since this is this prohibitively costly for the regulator (unless $\alpha = 1$), a mechanism which involves allocative distortions is chosen.

Theorem 14 *When demand is private information, a price mechanism weakly dominates if marginal costs are non decreasing. However, if marginal costs are strictly decreasing and α is in a neighborhood of 1, then the quantity mechanism dominates.*

Although the above theorem is a general result for the case of decreasing marginal costs, it does not help us understand the choice of prices or quantities for any value of α. Therefore, to push the comparisons further, we specialize our analysis to the case of linear demand and quadratic costs as in the past example.

Consider the inverse demand function $P(q,\theta) = A + \theta - bq$ with $A, b \geq 0$, the cost function $C(q)) = c_0 q - k\frac{q^2}{2}$ and assume that θ is uniformly distributed in $[0, \overline{\theta}]$. Furthermore, let us assume that $A + (1 - \alpha)(\overline{\theta}) > c_0 > k\frac{A+\overline{\theta}}{b}$ to ensure both the regulated quantity and the ex-post price that clears the market are positives for all realizations of θ.[5]

Using the proposition 11 the optimal quantity mechanism is given by

$$q^o(\theta) = \frac{A + \theta - z_\alpha(\theta) - c_0}{b - k} \tag{6}$$

$$T^o(\theta) = c_0 q^o(\theta) - \frac{1}{2} k q^o(\theta)^2 + \int_0^{\overline{\theta}} q^o(x)dx \tag{7}$$

[4] For increasing marginal costs both mechanisms lead to efficiency.

[5] One can interpret this condition as the monopolist have the enough capacity to satisfy any demand realization leaving no type out of the market.

The optimal price mechanism is as in example 13 using $\mathbb{E}(\theta) = \frac{\overline{\theta}}{2}$. Then, simple calculations lead us to

$$\begin{aligned}\triangle SW &:= SW^{quantity} - SW^{price} \\ &= \frac{\overline{\theta}(-12Ab^2(1-\alpha) + bk(1-\alpha)\overline{\theta} + k^2\alpha\overline{\theta} + 4b^2(1-\alpha)(3c_0 - \alpha\overline{\theta}))}{24b^2(b-k)}\end{aligned}$$

Then, it is possible to prove that $\triangle SW|_{\alpha=0} < 0$, $\triangle SW|_{\alpha=1} > 0$, $\triangle SW(\alpha)$ is convex and $\triangle SW'|_{\alpha=0} \geq 0$, leading us to the next theorem.

Theorem 15 *When marginal costs are decreasing, the difference between a price and a quantity mechanism is increasing in α. Moreover there exists a threshold $\alpha^* \in [0,1]$ such that a price mechanism dominates if and only if $\alpha \leq \alpha^*$.*

`[Figure here]`

It is possible to see that the quantity mechanism does not fare well for small values of α. Intuitively, the less important are producers for the regulator, the worse the quantity mechanism is. Since transfers are more costly as α decreases, the optimal quantity mechanism induces large allocative inefficiencies in order to induce truthful revelation, which impacts negatively the consumer surplus. That is the reason behind the surprising fact that a bunching mechanism dominates the optimal quantity menu. Moreover, the allocative inefficiencies induced by the price mechanism are not affected by α, making social welfare linear in α in that case, while social welfare , grows more than linearly in a quantity mechanism, as both the transfers matter less and the inefficiencies are reduced. Using quantity mechanisms is costly, either the quantity chosen is too small and decreases the consumer surplus, or it is large and involves large transfers to the firms.

A couple of comparative statics are interesting. We have that

$$\frac{\partial \triangle SW}{\partial A} = \frac{-(1-\alpha)\overline{\theta}}{2(b-k)} < 0, \quad \text{and} \quad \frac{\partial \triangle SW}{\partial c_0} = \frac{(1-\alpha)\overline{\theta}}{2(b-k)} > 0$$

An increase in market size, or equivalently a decrease in fixed costs, has a bigger positive effect in the case of price regulation. While the price mechanism leaves the firm's profit invariant, quantity regulation makes them larger in order to induce self selection. While this is good because it allows to reduce the allocative inefficiencies, an important part of the extra surplus goes to firms, which is less desirable from a social point of view if $\alpha < 1$.

It is possible to do other comparative statics, for example with respect the slope of marginal costs (k) and demand (b). However, the comparison between both mechanisms is ambiguous. An increase in k makes both mechanisms better, both for firms and consumers and, for example, $\left.\frac{\partial \triangle SW}{\partial k}\right|_{\alpha=1} > 0$, but $\left.\frac{\partial \triangle SW}{\partial k}\right|_{\alpha=0} < 0$.

4. Simple Regulatory Mechanisms with Unknown Demand

In this section we characterize the optimal bunching mechanisms in both quantities and prices. As we explained before, this kind of mechanisms are easier to implement in practice and it is important to understand how the choice of instrument changes when the regulator regulates with simple mechanisms.

As we noted in the previous section, the choice of a price (or quantity) determines the critical type, and thus the transfers. For a quantity mechanism, given q the critical type will be some $\theta_c(q) \in \Theta$ and hence the transfers $(T(q))$ will be determined by $\pi(q, \theta_c(q), T(q)) = 0$.

Then, the regulator's problem in each case is to find p^* or q^* that maximizes the expected weighted social welfare,

$$\begin{aligned} q^* &\in \arg\max_q \mathbb{E}_\theta SW(q, \theta, T(q)) \\ p^* &\in \arg\max_p \mathbb{E}_\theta SW(p, \theta, T(p)) \end{aligned}$$

To obtain clear cut comparisons, equivalent to the ones in the previous section, we concentrate in the linear-quadratic case.

Quantity Regulation

Note that for any $q \in \mathbb{R}_+$ the firm's profits are $\pi(q, \theta, T) = P(q, \theta)q - C(q) + T$, and as $P_\theta \geq 0$ it follows that the critical type will always be $\underline{\theta}$ with transfers $T = C(q) - P(q, \underline{\theta})q$. The regulator's problem is

$$\max_q \mathbb{E}_\theta[S(P(q, \theta)) - C(q) + P(q, \underline{\theta})q + \alpha(P(q, \theta)q - P(q, \underline{\theta})q)]$$

The first order condition is given by $\alpha\mathbb{E}_\theta P(\hat{q}, \theta) + (1 - \alpha)P(\hat{q}, \underline{\theta}) = C_q(\hat{q})$ and using the linear demand and quadratic costs, the optimal bunching mechanism, and the social welfare associated are given by:

$$\begin{aligned} q^* &= \frac{A + \alpha\mathbb{E}(\theta) - c_0}{b + k} \\ T^* &= -(A - bq^*)q^* + \left(c_0 q^* + \frac{kq^{*2}}{2}\right) \\ SW^{quantity} &= \frac{[A + \alpha\mathbb{E}(\theta) - c_0]^2}{2(b + k)} \end{aligned}$$

We need to put conditions over the parameters to ensure a positive quantity and a positive price for any realization of the demand. For this is enough to have $A > c_0 \geq \frac{k}{b}(A + \overline{\theta}) + \alpha\mathbb{E}(\theta) - \overline{\theta}$.

Price Regulation

In section 3, we found the optimal bunching for non-increasing marginal costs. Here we do the same for increasing marginal costs. From lemma 9 it is easy to see that $\Pi(\theta)$ is concave, so the minimum values are attained at the extremes of $[\underline{\theta}, \overline{\theta}]$. It is possible to prove that $\theta_c = \underline{\theta}$ if and only if $p \geq \overline{p}$ (and $\theta_c = \overline{\theta}$ otherwise), and in the linear quadratic case $\overline{p} = \frac{2bc_0 + k(2A + \overline{\theta})}{2(b+k)}$. Straightforward computations shows that the optimal bunching depend on the expected value of θ. For symmetric distributions $p^* = \overline{p}$ for all α, and we refer the reader to the appendix for a complete description of the solution in the non-symmetric case. With this, the optimal mechanism, and the corresponding social welfare are given by:

$$
\begin{aligned}
p^* &= \frac{2bc_0 + k(2A + \overline{\theta})}{2(b+k)} \\
T^* &= -\overline{p}Q(\overline{p}, \overline{\theta}) + C(Q(\overline{p}, \overline{\theta})) \\
SW^{price} &= \frac{\left(4A^2b^2 - (1-\alpha)\overline{\theta}^2 k(b+k) + b^2(-2c_0 + \overline{\theta})(4A - 2c_0 + \overline{\theta})\right)}{8b^2(b+k)} + \frac{(b - k\alpha)Var(\theta)}{2b^2}
\end{aligned}
$$

Comparisson

When marginal costs are non-increasing the comparison is easy to compute, with the optimal bunching as in example 13 and the optimal quantity mechanism being independent of α. Then, [6]

$$
\triangle SW = \frac{(A - c_0 + \alpha\mathbb{E}(\theta))^2}{2(b-k)} - \frac{(A - c_0 + \mathbb{E}(\theta))^2}{2(b-k)} - \frac{(b + k\alpha)Var(\theta)}{2b^2} \tag{8}
$$

It is easy to see that $\triangle SW \leq 0$ always, i.e., price regulation is better than quantity regulation. Intuitively, quantity regulation clears the market adjusting the uniform auction price. Furthermore, as the quantity is fixed the costs of the firm will be fixed too. So, the higher the demand realization the higher will be the price that clears the market, an hence the consumers surplus go down while the firm's profits go up -which is less value than consumer surplus. The price regulation, however, clears the market by adjusting the quantity to be sold. The firm in this case will not face fixed costs, so the the higher the demand realization the greater is the firm's production, leaving better off both the consumers and the firm since marginal cost are decreasing.

When marginal costs are non-decreasing we have

$$
\triangle SW = \frac{(-1+\alpha)\overline{\theta}\left(4Ab^2 - bk\overline{\theta} - k^2\overline{\theta} + b^2(-4c_0 + \overline{\theta} + \alpha\overline{\theta})\right)}{8b^2(b+k)} - \frac{(b - k\alpha)Var(\theta)}{2b^2}
$$

In particular, for $\alpha = 1$ the choice of instrument depends on the slope of both marginal costs and demand. If the slope of demand is greater (less) than the slope of marginal costs, then price regulation is better (worse) than quantity regulation. Moreover, if there were no private information, then both mechanisms would have achieved the same social welfare. This result is consistent with Laffont (1977) and analogous to Weitzman (1974) if the firm does not ration.

[6] We do not restrict ourselves to symmetric distributions in this case.

5. Discussions

Our results can be summarized in the following table:

	Prices vs. Quantities			
	Simple Mechanisms		**Sophisticated Mechanisms**	
	Costs	*Demand*	*Costs*	*Demand*
$C'' > 0$	*Equivalent*	*Depend on parameters*	*Equivalent*	*Price dominates*
$C'' = 0$	*Equivalents*	*Price dominates*	*Equivalent*	*Price dominates*
$C'' < 0$	*Equivalent*	*Price dominates*	*Equivalent*	*Depend on parameters*

In section 3 we see that when marginal costs are non-decreasing and the regulator is sophisticated and $\alpha < 1$, price regulation dominates quantity regulation since the former achieves the first-best with zero profits to the firm, while the latter generates allocative inefficiencies in order to induce self-selection. On the other hand, if $\alpha = 1$, both mechanisms coincide, but if we restrict the regulator to choose a simple mechanism, the optimal selection will depend on parameter values. In particular, when restricted to simple mechanisms, the choice depend on the difference between the slope of the demand and marginal costs. As a corollary, if marginal costs are constant, the price mechanism dominates for with and without the constraint to simple mechanisms.

The phenomenon is totaly inverse when marginal cost are decreasing. First, the best sophisticated price mechanism is a simple mechanism, and thus both simple and sophisticated price mechanisms are equivalent. In other words, price regulation is independent on the regulator's sophistication. Second, which mechanism leaves the society better off will depend on parameter values. In particular, if the regulator cares sufficiently about firm's profits (equivalently if the transfers are costless enough) then sophisticated quantity regulation dominates since allocative inefficiencies are reduced given that transfers are "irrelevant" to the regulator. However, if the regulator is restricted to use simple mechanisms, then price regulation dominates mainly because in this case quantities are adjusted in order to clear the market. So, as simple and sophisticated price regulation are the same, the main difference is given by quantity regulation. Using a scheme or menu the regulator is able to increase the social welfare turning back the domination of price regulation. In fact, straightforward calculations show us that $SW^{quantity}_{sophist} - SW^{quantity}_{simple} = \frac{(-2+\alpha)^2\bar{\theta}^2}{24(b+k)} > 0$. One can do comparative statics to see when the advantage of using sophisticated quantity mechanism is sizeable.[7] All in all, regulating with simple mechanism is in the best case at most as good as using sophisticated mechanisms.

The question that is remaining is, whether or not is more convenient to use sophisticated mechanisms instead of simple ones. In the real world, probably it would not be easy for the regulator to design a

[7]For instance, the advantage of using sophisticated mechanism is increasing with the support of θ, and decreasing with k and b.

sophisticated scheme: either the menu (x, T) could not be carefully designed or the firm could not choose the pair designed for it. To fix ideas, assume that there is a cost $\kappa > 0$ for the society related to the use of a sophisticated mechanism.[8] Therefore, they might be cases where such specialization could be worthy. For instance, when marginal costs are non-increasing and $\alpha \geq \alpha^*$ (see theorem 15) the society is better off if the regulator regulates with a quantity mechanism in the case of sophisticated mechanisms, and with a price mechanism in the case of simple mechanisms. The value of the sophistication (V) is then the difference between the social welfare that generates the best sophisticated mechanism versus the best simple mechanism, therefore, $V = \triangle SW$, where $\triangle SW$ is as in section 3. and the cost is κ, hence the answer about the convenience of regulating with sophisticated schemes will depend on the sign of $\triangle SW - \kappa$. Moreover, when marginal cost are increasing, the regulator does not need any prior belief about the distribution of the uncertainty demand in order to implement the first-best price mechanism, and thus the sophistication's cost should be lower.

`[Rationing? Some Extensions? Not always you can choose p or q?]` An important question is when this sophistication's value is either big or small. Recall the case that we have been discussed, if $\alpha < \alpha^*$ then $V = 0$ since the best sophisticated mechanism is equal to the best simple mechanism, and thus no gain of sophistication are obtained. The same effect happen if marginal costs are constant. In the case of increasing marginal cost, if the slope is sufficiently large then $V = 0$ since with a simple quantity mechanism the regulator is able to achieve the allocative first-best leading the same result that with a sophisticated price mechanism. The same intuition one can get if we consider a demand function with elasticity close to zero.

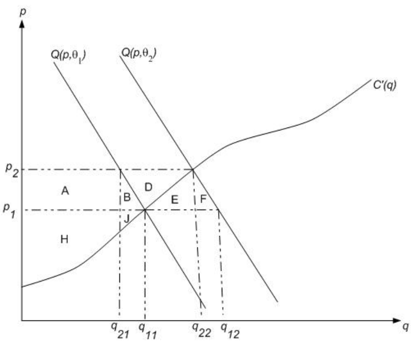

Figure 1: Price regulation

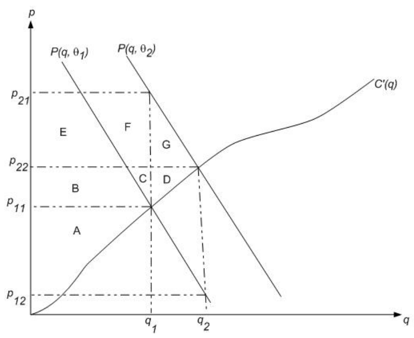

Figure 2: Quantity regulation

Observe that in the first-best case, the incentive compatibility constraint in price regulation goes in the opposite direction in comparison to quantity regulation. Without transfers, in price regulation the

[8]This cost could represent the time, effort, learning, instruction, etc. that both firm and regulator would have to do. Note that if $\kappa \to +\infty$, the communication between the firm and the regulator is unfeasible, and thus simple mechanisms are the only option.

low demand type would like to overstate his true demand in order to get a higher price (p_2), and in quantity regulation, the high demand type would like to understate his true demand in order to reduce his production (q_2) and charge a larger price (p_{21}). Moreover, from lemma 9, in both mechanisms the assignment rule has to be non-decreasing with the type, therefore, the allocative first-best is feasible in both mechanisms (with transfers correctly adjusted): the assignment rule goes in the same direction of that the regulator would want. However, considering the transfers that the regulator would want –i.e., those that leaves any demand type with zero profits– the incentive compatibility constraint in price regulation is not binding since the transfers (or taxes in this case) for a high type are sufficiently large that makes unprofitable to a low type to misreport its demand, making the first-best feasible. On the other hand, in quantity regulation transfers goes in the same direction as in price regulation but in different direction respect to IC constraint, hence, the incentives to misreport demand are increased (IC is even more binding than before), making the first-best unfeasible in quantity regulation.

6. Conclusion

A Appendix: Proofs

Proof of lemma 9:[9] Suppose that incentive compatibility holds. Then the monopolist's problem is to choose $\hat{\theta} \in \arg\max_{\hat{\theta}} \pi(\hat{\theta}, \theta)$. The first order condition implies:

$$P_q(q(\hat{\theta}), \theta)q'(\hat{\theta}) + P(q(\hat{\theta}), \theta)q'(\hat{\theta}) - C'(q(\hat{\theta}))q'(\hat{\theta}) + T'(\hat{\theta}) = 0 \tag{9}$$

And the second order condition requires that:

$$\begin{aligned} q''(\hat{\theta})[P_q(q(\hat{\theta}), \theta)q(\hat{\theta}) + P(q(\hat{\theta}), \theta) - C'(q(\hat{\theta}))] + q'(\hat{\theta})[P_{qq}(q(\hat{\theta}), \theta)q'(\hat{\theta})q(\hat{\theta}) +& \\ P_q(q(\hat{\theta}), \theta)q'(\hat{\theta}) - C''(q(\hat{\theta}))q'(\hat{\theta})] + T''(\hat{\theta}) \leq 0& \end{aligned} \tag{10}$$

Since the mechanism must be incentive compatible, we obtain from (9) that:

$$P_q(q(\theta), \theta)q'(\theta) + P(q(\theta), \theta)q'(\theta) - C'(q(\theta))q'(\theta) + T'(\theta) = 0 \quad \forall \theta \in \Theta \tag{11}$$

Defining $\pi_1(\hat{\theta}, \theta)$ as the partial derivative of π with respect to the first argument, we have that (11) is equivalent to $\pi_1(\theta, \theta) = 0$ for all θ. Differentiating this with respect to θ:

$$\pi_{11}(\theta, \theta) + \pi_{12}(\theta, \theta) = 0 \tag{12}$$

From (10) we know that $\pi_{11}(\theta, \theta) \leq 0$, therefore $\pi_{12}(\theta, \theta) \geq 0$. Noting that $\pi_{12}(\theta, \theta) = q'(\theta)[P_{q\theta}(q(\theta), \theta)q(\theta) + P_\theta(q(\theta), \theta)]$ we obtain that $q'(\theta)$ must be increasing.

[9] We will prove this for the case of quantity regulation. The case of price regulation is analogous.

To prove (ii), consider the total derivative of $\Pi(\theta) = \pi(\theta, \theta)$ with respect to θ, and use the fact that the mechanism is incentive compatible. Then it is easy to see that $\Pi'(\theta) = P_2(q(\theta), \theta)q(\theta)$.

To prove the converse, using i) and ii) it is easy to see that $\Pi(\theta) \geq \Pi(\hat{\theta}) + [P(q(\hat{\theta}), \theta) - P(q(\hat{\theta}), \hat{\theta})]q(\hat{\theta})$, which conclude the proof.

■

Proof of proposition 11: Using lemma 9 and integrating by parts, the regulator's objective function can be written as:

$$\int_{\underline{\theta}}^{\overline{\theta}} \{[V(P(q(\theta), \theta)) - C(q(\theta))] - (1 - \alpha)\frac{1 - G(\theta)}{g(\theta)} P_2(q(\theta), \theta)q(\theta)\}g(\theta)d\theta - (1 - \alpha)\Pi(\underline{\theta})$$

It is clear that in an optimal mechanism $\Pi(\underline{\theta}) = 0$. Then, maximizing pointwise, it's easy to obtain that q^o maximize the regulator's objective function in the relaxed problem. The transfers are determined by the firm's profit function. It remains to check that the solution of the relaxed problem is also solution of the constrained problem. Differentiating $P(q^o(\theta), \theta)$ respect to θ and doing some algebra we obtain:

$$\frac{dq^o(\theta)}{d\theta} = \frac{-P_\theta + z'_\alpha(\theta)P_\theta + z_\alpha(\theta)P_{\theta\theta}}{P_q - C''}$$

Note that both the numerator and denominator of the above fraction are negatives, thus the monotonicity constraint is satisfied, which conclude the proof.

■

Proof of theorem 15: Taking the partial derivative respect to α of $\triangle SW$ and evaluating at $\alpha = 0$, we get

$$\begin{aligned} \frac{\partial \triangle SW}{\partial \alpha}\Big|_{\alpha=0} &= \frac{\overline{\theta}}{24b^2(b-k)}(12b^2(A - c_0) - bk\overline{\theta}) + k^2\overline{\theta} - 4b^2\overline{\theta}) \\ &> \frac{\overline{\theta}^2}{24b^2(b-k)}\left[4b^2 + (2b-k)^2 + bk\right] > 0 \end{aligned}$$

Where the first inequality came by the fact that $A - c_0 > \overline{\theta}$. Thus, as $\triangle SW$ is convex in α for all $\alpha \geq 0$ $\frac{\partial \triangle SW}{\partial \alpha} > 0$ for all α. Furthermore, it is easy to see that $\triangle SW|_{\alpha=0} < 0$, and by theorem 14 we know that $\triangle SW|_{\alpha=1} > 0$, therefore, by continuity there exists α^* such that $\triangle SW|_{\alpha*} = 0$ which concludes the proof.

■

B Appendix: Simple price mechanism with non-decreasing marginal costs

In section 4, we find the optimal bunching when the marginal cost are non-decreasing and the private information distribution was symmetric. Here we complement that case doing it for any distribution.
We know that,

$$\theta_c = \begin{cases} \underline{\theta} & \text{if } p \geq \overline{p} \\ \overline{\theta} & \sim \end{cases}$$

Then, the problem of a regulator insistent in $\theta_c = \underline{\theta}$ will be:

$$\begin{aligned} \max_{p} \quad & \mathbb{E}_\theta[V(Q(p,\theta)) - C(Q(p,\underline{\theta})) + pQ(p,\underline{\theta}) \\ & + \alpha\left(pQ(p,\theta) - C(Q(p,\theta)) + C(Q(p,\underline{\theta})) - pQ(p,\underline{\theta})\right)] \\ s.a \quad & p \geq \overline{p} \end{aligned} \tag{13}$$

The solution to the relaxed problem is:

$$\tilde{p_1} = b\left(\frac{c_0}{b+k}\right) + k\left(\frac{A + \alpha\mathbb{E}(\theta)}{b+k}\right) + (1-\alpha)b\left(\frac{-\mathbb{E}(\theta)}{b+k}\right)$$

The transfers are determined by the profit function of the critical type. Note that whether $\tilde{p_1}$ is greater or less than $\overline{p}$ depend on the expected value of θ. Moreover, if $\mathbb{E}(\theta) < \frac{\overline{\theta}}{2}$ then the constraint is binding and the optimal bunching is equal to $\overline{p}$. On the other hand, if $\mathbb{E}(\theta) \geq \frac{\overline{\theta}}{2}$ the constraint is binding only to a subset of values of α: there is $\underline{\alpha} < 1$ such that the optimal bunching p^* is equal to $\overline{p}$ if $\alpha < \underline{\alpha}$, and p^* is equal to the relaxed solution $\tilde{p_1}$ if $\alpha \in [\underline{\alpha}, 1]$.[10]

The case where the regulator is insistent in $\theta_c = \overline{\theta}$ is analogous. The objective function is the same but the constraint is inverted and the transfers are determined by the profit function of $\overline{\theta}$. The results are quite similar in comparison to the later case. We find that, if $\mathbb{E}(\theta) \geq \frac{\overline{\theta}}{2}$ the constraint is binding and $p^* = \overline{p}$. However, if $\mathbb{E}(\theta) \leq \frac{\overline{\theta}}{2}$ there exist $\overline{\alpha}$ such that $p^* = \overline{p}$ if $\alpha < \overline{\alpha}$, and p^* is equal to the relaxed solution if $\alpha \in [\overline{\alpha}, 1]$.

Observe that if $\mathbb{E}(\theta) \geq \frac{\overline{\theta}}{2}$, the society is better off if the regulator choose $\underline{\theta}$ as critical type, and $\overline{\theta}$ otherwise.

It's important to note that this results are quite different of those founded in the section 3. The growth of marginal costs are extremely important in the characterization of the optimal bunching. As you can see, the calculations done it here are complex rather than simple, even though we are considering linear-quadratic form and simple mechanisms.

[10] To obtain $\underline{\alpha}$ it is enough to find the solution of $\tilde{p_1} - \overline{p} = 0$. Here, $\underline{\alpha} = \frac{k\overline{\theta} + 2b\mathbb{E}(\theta)}{2\mathbb{E}(\theta)(b+k)}$

C Appendix: The General Case with Non-Increasing Marginal Costs

From section 3 we find that when α is in a neighborhood of 1, quantity regulation is better than price regulation (see theorem 14). However, for small values of α it is not clear which mechanism is better. In the linear-quadratic case with private information distributed uniformly, price regulation dominates (theorem 15). In what follows, we give sufficient conditions that hold theorem 15 in the general case.

Theorem 16 *If $G(\theta^*) \leq \frac{1}{2}$, $|P_q(q,\theta)| > 1$ for all q, θ and $|C''(\cdot)| < 1$ for all q, then there exists $\alpha^* \in (0,1)$ where price and quantity mechanisms are equivalent.*

Proof: Using the usual methods in mechanism design, we rewrite the objective function in terms of the assignment rule. In price regulation, the social welfare is:

$$\begin{aligned}
&\int_{\underline{\theta}}^{\theta^*} \Big\{ [V(Q(p^*,\theta)) - C(Q(p^*,\theta)) + p^*Q(p^*,\theta)] \\
&+(1-\alpha)\frac{G(\theta)}{g(\theta)} \left(p^* - C'(Q(p^*,\theta))\right) Q_\theta(p^*,\theta)\Big\} g(\theta)d\theta \\
&+\int_{\theta^*}^{\overline{\theta}} \Big\{ [V(Q(p^*,\theta)) - C(Q(p^*,\theta)) + p^*Q(p^*,\theta)] \\
&-(1-\alpha)\frac{1-G(\theta)}{g(\theta)} \left(p^* - C'(Q(p^*,\theta))\right) Q_\theta(p^*,\theta)\Big\} g(\theta)d\theta
\end{aligned}$$

Where p^* is the marginal cost of the critical type θ^*.

On the other hand, the social welfare with quantity regulation is:

$$\int_{\underline{\theta}}^{\overline{\theta}} \{[V(P(q^o(\theta),\theta)) - C(q^o(\theta))] - (1-\alpha)\frac{1-G(\theta)}{g(\theta)} P_2(q^o(\theta),\theta)q^o(\theta)\} g(\theta)d\theta \tag{14}$$

Where q^o is the optimal quantity mechanism.

To prove the result it is enough to restrict to the case where $\alpha = 0$. We will divide the analysis in two cases: where θ is in either $[\underline{\theta}, \theta^*]$ or $\left[\theta^*, \overline{\theta}\right]$.

Case 1: $\theta \in [\underline{\theta}, \theta^*]$

It is easy to see that for any $\theta \in [\underline{\theta}, \theta^*]$ $p^* < C'(Q(p^*,\theta))$ and $P(q^o(\theta),\theta) > C'(q^o(\theta))$, which implies that

$$V(Q(p^*,\theta)) - C(Q(p^*,\theta)) + p^*Q(p^*,\theta) > V(P(q^o(\theta),\theta)) - C(q^o(\theta))$$

Next, observe that $\frac{G(\theta)}{g(\theta)}\left(p^* - C'(Q(p^*,\theta))\right)Q_\theta(p^*,\theta) < 0$ and $\frac{1-G(\theta)}{g(\theta)}P_2(q^o(\theta),\theta)q^o(\theta) > 0$. We will prove that $|\frac{G(\theta)}{g(\theta)}\left(p^* - C'(Q(p^*,\theta))\right)Q_\theta(p^*,\theta)| < \frac{1-G(\theta)}{g(\theta)}P_2(q^o(\theta),\theta)q^o(\theta)$ for any $\theta \in [\underline{\theta},\theta^*]$. First, it is easy to see that $h(\theta) \equiv |p^* - C'(Q(p^*,\cdot))|$ is decreasing and recall from lemma 9, q^o is increasing. Therefore, we have 3 cases:

(i) $h(\theta) > q(\theta)$ for all $\theta \in [\underline{\theta},\theta^*]$

(ii) $h(\theta) = q(\theta)$ for some $\theta \in [\underline{\theta},\theta^*]$

(iii) $h(\theta) < q(\theta)$ for all $\theta \in [\underline{\theta},\theta^*]$

Case (i) never happen because if it happened then $0 = h(\theta^*) > q^o(\theta^*)$ which is false. Case (ii) never happen too (unless $\theta = \underline{\theta}$) since $|C''| \leq 1$. Hence, the only case that survive is (iii).

Second, $Q_\theta(p^*,\theta) < P_\theta(q,\theta)$ for all θ since $|P_q(q,\theta)| > 1$, and finally $G(\theta) < 1-G(\theta)$ because $G(\theta^*) \leq \frac{1}{2}$, which concludes the first part of the proof.

Case 2: $\theta \in \left[\theta^*,\overline{\theta}\right]$

For the same reasons before it is straightforward to see that:

$$\frac{1-G(\theta)}{g(\theta)}\left(p^* - C'(Q(p^*,\theta))\right)Q_\theta(p^*,\theta) \leq \frac{1-G(\theta)}{g(\theta)}P_2(q^o(\theta),\theta)q^o(\theta)$$

And by continuity $V(Q(p^*,\theta)) - C(Q(p^*,\theta)) + p^*Q(p^*,\theta) > V(P(q^o(\theta),\theta)) - C(q^o(\theta))$ for any θ in a neighborhood of θ^*. However, as $P(q^o(\theta),\theta)$ is decreasing and $P(q^o(\overline{\theta}),\overline{\theta}) = C'(q^o(\overline{\theta}))$, then there exist $\hat{\theta}$ such that $P(q^o(\hat{\theta}),\hat{\theta}) = p^*$. Therefore price dominates for $\theta \in [\underline{\theta},\theta^* + \varepsilon]$...

D Appendix: Price Regulation and Critical Type with Non-Increasing Marginal Costs

Lemma 17 *The critical type θ^* is an interior realization of the demand.*

Proof: It is enough to take the partial derivative of SW respect p^*, where p^* is the optimal price bunching and SW is the social welfare defined as:

$$
\begin{aligned}
SW = & \int_{\underline{\theta}}^{\theta^*} \Big\{ [V(Q(p^*,\theta)) - C(Q(p^*,\theta)) + p^*Q(p^*,\theta)] \\
& +(1-\alpha)\frac{G(\theta)}{g(\theta)}\left(p^* - C'(Q(p^*,\theta))\right) Q_\theta(p^*,\theta)\Big\} g(\theta)d\theta \\
& + \int_{\theta^*}^{\overline{\theta}} \Big\{ [V(Q(p^*,\theta)) - C(Q(p^*,\theta)) + p^*Q(p^*,\theta)] \\
& -(1-\alpha)\frac{1-G(\theta)}{g(\theta)}\left(p^* - C'(Q(p^*,\theta))\right) Q_\theta(p^*,\theta)\Big\} g(\theta)d\theta
\end{aligned}
$$

Using the fact that $p^* = C'(Q(p^*,\theta^*))$,

$$
\begin{aligned}
\frac{\partial SW}{\partial p^*} &= \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial \widetilde{SW}(p^*,\theta)}{\partial p^*} g(\theta)d\theta \\
&+ \int_{\underline{\theta}}^{\theta^*} (1-\alpha)\frac{G(\theta)}{g(\theta)}[1 - C''(Q(p^*,\theta))Q_p(p^*,\theta)]Q_\theta(p^*,\theta)g(\theta)d\theta \\
&- \int_{\theta^*}^{\overline{\theta}} (1-\alpha)\frac{1-G(\theta)}{g(\theta)}[1 - C''(Q(p^*,\theta))Q_p(p^*,\theta)]Q_\theta(p^*,\theta)g(\theta)d\theta \\
&= 0
\end{aligned}
$$

Where $\widetilde{SW}(p^*,\theta) = V(Q(p^*,\theta)) - C(Q(p^*,\theta)) + p^*Q(p^*,\theta)$.

Note that if the regulator is willing to regulate with $\theta^* = \overline{\theta}$, then the last term will disappear and the first term will be positive, hence $\frac{\partial SW}{\partial p^*} > 0$. On the other hand, if the regulator is willing to regulate with $\theta^* = \underline{\theta}$, then the second term will disappear and the first term will be negative, hence $\frac{\partial SW}{\partial p^*} < 0$. Therefore, θ^* must be in $\left(\underline{\theta}, \overline{\theta}\right)$.

■

`[Trade off, indifference ]`

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