Media competition: a two-sided duopoly with costly differentiation

Guillaume Roger [*]
The University of Southern California

This version: September 2007

Abstract

We model a duopoly in which media compete in both the consumer and the advertising markets. The advertisers' payoffs depend on the coverage in the consumer market, hence there are cross-market externalities but no direct transfers are possible. At a (sunk) cost, platforms select the quality they offer consumers, and the prices they charge consumers and advertisers. Under well-defined conditions, the pure strategy equilibrium of this game is unique and can be computed. Generically, a mixed-strategy equilibrium is shown to always exist and the distributions are characterised. Compared to an established benchmark (Shaked and Sutton, (1982)), consumer prices are distorted downward and so is the quality offered to consumers. The introduction of advertising revenue enhances price competition for consumers, and the necessary consumer discount relaxes the need to provide quality. Hence quality (on the consumer side) and advertising revenue are substitutes. Competition is shown to promote the investment in quality, as contrasted to a monopoly. The market may be preempted not as a result of an exogenous contraction, but as a consequence of an *expansion*, of (advertising) demand.

1 Introduction

> *"The only thing advertisers care about is circulation, circulation, circulation."*
>
> Edward J. Atorino, analyst
> Fulcrum Global Partners, New York
> June 17, 2004 (The Boston Globe).

[*]Department of Economics, USC. Email: roger@usc.edu

The standard *modus operandi* for the media requires of them to satisfy two constituencies: consumers on one side and advertisers on the other. Typically advertisers also prefer reaching as large an audience as possible, but direct transfers to consumers are effectively impossible to implement. Unlike a more classical multiproduct firm problem where consumer internalise the benefits of each product in their consumption decision of the other one(s), here each side fails to so. That is, there are complementarities that parties cannot take advantage of. From this simple observation they can be construed as platforms competing in a two-sided market. But unlike e-Bay, say, whose only purpose is to facilitate transactions between buyers and sellers, a medium also provides an information (or entertainment) good to attract consumers. In this paper we develop a model of media competition in which *a*) platforms select the quality of the consumer good, where quality is costly; and *b*) competition cannot be reduced to the sole problem of attracting consumers. That is, media compete non-trivially in *both* the consumer and the advertising market.

We compute the unique pure-strategy equilibrium of this game for a restricted set of parameters. Quite naturally it is asymmetric – like Shaked and Suton (1982). More generally a mixed-strategy equilibrium always exists and the distributions over the (subset of) actions are characterised and are symmetric. Beyond the positive analysis, consumer prices are distorted downward – as in any two-sided market problem – in order to take advantage of the complementarity they exert on advertising demand. The main result is that, when a pure-strategy equilibrium exists, the optimal quality level of the top firm is lower than in the corresponding Shaked and Sutton (1982) benchmark. Profit maximisation alone is sufficient for this phenomenon to arise; that is, interference is not necessary. Quality and advertising become *substitutes* in the platforms' problem. Indeed, without advertising revenue, a high quality is a means of extracting consumer surplus, at the cost of giving away market share to the competition. With ancillary revenue, every consumer becomes more valuable because the platform can extract surplus from advertisers, hence the substitution phenomenon. It is further established that quality is declining in the magnitude of the advertising revenue. As the value of advertising increases, the discount offered to consumers deepens. Therefore the quality level required to induce the marginal consumer to purchase from the high-quality platform decreases. Beyond a well-defined threshold, the quality-adjusted price of the top firm is so low that the consumer market is preempted. However this fails to be an equilibrium as the excluded firm simply has to offer a marginally higher quality level to monopolise the market. Then platforms play in mixed strategies. *Ex post* either both media operate or both markets are monopolised (by the same firm), depending on their exact play. This latter phenomenon arises not because there is too little surplus to extract but thanks to a large advertising market, which induces more competition.

More formally, the model calls on Gabszewicz and Thisse's (1979) (also Shaked and Sutton's (1982)) vertical differentiation construct on the consumer side, however adding a convex, sunk cost of quality. Advertisers are not strategic: their payoffs do no depend on what other advertisers do, but they do derive some idiosyncratic benefit from advertising. This fits a framework of informative advertising, or one where commodity producers do not compete in this dimension[1], as in Anderson and Coate (2005). Because advertisers care only about the market coverage of each medium – and not to *whom* they advertise, vertical differentiation emerges endogenously on that side. The game is played in three stages: first media platforms simultaneously select their quality level. In the second stage, having observed each other's quality, they set consumer prices. Last, knowing the consumer market's configuration (their marketshare) they choose advertising prices. Consumers and advertisers purchase at most one unit of the good of interest – this is called the single-homing assumption. This important detail will be discussed at greater length later; in particular it implies that price competition is still vivid in the advertising market, unlike in Gabszewicz, Laussel and Sonnac (2001).

The media sector carries a significant weight in industrialised countries, with worldwide advertising expenditure estimated at US$ 600 billion in 2006 – approximately half of which were incurred in the United States. Print media is still a healthy subset of it wit $60Bn in advertising revenue in the US, a figure that is comparable to that of broadcast television. The industry tends to be concentrated with few large companies such as Time Warner (with $44.2 Bn in 2006 revenue) Disney (34.3 Bn, 2006) or Tribune company ($5.5 Bn 2006 revenue) and Google ($12 Bn, 2007). The significance of media extends beyond easily measurable gains from trade: agents also have to invest leisure time in its consumption. On average an American consumers spends 4 hours a day watching TV, 40 minutes a day reading print media and, for those participating, 60 minutes a day on the internet. Yet unlike other large sectors of the economy (such as automobile) or industries fraught with externalities (airlines or telecommunications), it has not been the object of much academic research. According to Simon Wilkie, former chief economist of the the FCC, regulators suffer from this lack of interest

[1] In fact competition between producers is entirely side-stepped

on the part of economists. And in spite of this lack of information this sector of the economy is one of the most regulated one: ownership, access (cable, internet interconnection), reach (broadcasting) and even content are the object of edicts of the FCC.

While this paper does not claim to be a policy prescription, it may inform policy makers and actors in the industry. In considering applications, the results seem to match the observation that in most US cities either a single newspaper survives, or a very large one dominates a (or a fringe of) small outlet(s)[2]. This lone (or dominant) newspaper is nonetheless quite inexpensive. Although the static model we present cannot capture the dynamic evolution of print media competition, its predictions correspond to these observations. Two scenarii may be drawn. In the first one, a single newspaper remains at the end of a competitive process that eventually drives out other players. Alternatively, an exogenous shock affecting advertising profits disrupts the pure strategy equilibrium and leads to exit, as observed in the recent past across the US. Thus some players may be driven out not because of an exogenous market contraction, but because of a(n) (advertising) market *expansion*.

The model analysed herein is too stark to claim being a faithful description of the media industry, however it does contribute to its study by departing from much of the literature in three important ways. First quality is costly, which fits much of the industry: better, more accurate information requires more investment to retrieve and verify it, and better shows do cost more to produce. Second, it ignores whatever disutility consumers may suffer from advertising. This choice can be debated[3], however what should not be is that a (commonly used) convex disutility function ($\delta(q^A)$) reduces the value of the marginal advertiser from the perspective of the platform. In other words, it modifies the rate of substitution between surplus extraction from consumers and from advertisers. Introducing such disutility would extend the range of parameters on which the pure-strategy equilibrium can be sustained, as it reduces the value of advertising to the platform. It otherwise does not modify the results qualitatively. Last, the model presumes of unit demand, and therefore of single-homing on both sides, which can be rationalised through a budget constraint imposed on consumers and a liquidity constraint on advertisers. This constraint has non-trivial implications. It defines a proper subgame in the last stage, which would not arise in its ab-

[2]Poynter Online, circulation rankings as of November 6, 2002. Only New York City has more than one significant daily paper.

[3]In particular, such a disutility function may not be consistent with the framework of informative advertising, in which the latter is necessary for consumer to discover goods.

sence; that is, there is meaningful price competition between platforms in the advertising market. In turn this hardens competition for consumers, in that a marginal increase in consumer coverage yields different marginal benefits in the advertising market, depending on the ranking of the platform in the consumer market. It also renders the profit function bi-modal (hence not quasi-concave) at the consumer pricing stage, with associated equilibrium existence concerns. These features are generated *not* by the single-homing assumption but by the fact that platforms compete in *both* markets. They would also arise if advertisers could multihome but made subject to a liquidity constraint. In that case platforms would compete for the marginal unit, hence the model can be directly cast in terms of unit demand.

2 Literature

Rochet and Tirole (2002, 2004), Armstrong (2004) and Caillaud and Jullien (2003) are the seminal references when it comes to studying two-sided markets. In this paper, consumers' utility is *not* directly dependent on the number of advertisers. It may be affected indirectly by the price system. That is, the externality is one-sided only. The works closest to ours are those of Gabszewicz, Laussel and Sonnac (2001), hereafter GLS, Ferrando, Gabszewicz, Laussel and Sonnac (FGLS 2003), Gabszewicz and Wauthy (2006) and Anderson and Coate (2005). GLS (2001) characterise pure-strategy equilibria in a Hotelling model with multi-homing on the advertising side and endogenous locations. In contrast, FGLS (2003) take the locations as fixed. Gabszewicz and Wauthy (2006) do consider endogenous *costless* quality, however in a rational expectation model with simultaneous price-setting in the consumer and advertising market and with the option of multi-homing. Anderson and Coate (2005) conduct a welfare analysis of the broadcasting market; advertising may be underprovided, depending on its nuisance cost and its expected benefit to advertisers. There is no direct competition between broadcasters for the advertisers business. Our results also contrast the simple model developed by Thorson (2003), which she summarises as

$$Newsroom\ investment \rightarrow Quality \rightarrow Circulation \rightarrow Revenue$$

and according to which a quality investment uniformly improves revenue. Ellman and Germano (2005) disregard editorial independence and show that competing newspapers may optimally select what to report in order to alleviate advertisers' discomfort with the news content[4]. Along the same vein, Strömberg (2004) shows that profit-maximising media report information relevant to larger audiences, thereby providing incentives to political competitors to distort their messages to please this subset of the population. Crampes, Harichabalet and Jullien (2005) analyse the problem of entry in the media market using a model where platforms derive revenues from both consumers and advertising.

This work is also related to an older strand of the industrial organisation literature. Building on the work of Gabszewicz and Thisse (1979), Shaked and Sutton (1982) show that when firms compete in a vertical differentiation model, their profits, prices and market shares are ranked according to their quality choices. The equilibrium is unique and duopolists exhibit maximum differentiation to soften price competition. This model is slightly modified to reflect ours, and used as a benchmark when comparing quality levels.

Throughout, obvious or longer proofs are collected in the Appendix, which also contains a supplementary section on symmetric equilibria.

3 Introducing the model

There are two platforms, identified with the subscripts 1 and 2, and a continuum of consumer of mass 1 with private valuation b for information (in the common-language understanding of the word). The benefit b is distributed on an interval $\left[\underline{\beta}, \overline{\beta}\right]$ following the function $F(\beta)$ and everywhere positive density $f(\beta)$. All consumers value quality in the sense of vertical differentiation – there is no ambiguity for consumers as to what quality is. Let $\theta \in \Theta = \left[\underline{\theta}, \overline{\theta}\right]$ denote the quality parameter of each good. For simplicity, it is assumed that one must purchase the medium to consume it: there is no free viewing. A consumer's net utility function is expressed as

$$u(b, \theta_i, p_i^R) = \theta_i b - p_i^R; \quad i = 1, 2$$

when facing a price p_i^R, where the superscript R stands for 'reader'. Let $\mathbf{p}^R = \left(p_1^R, p_2^R\right), \theta = (\theta_1, \theta_2)$, and consumers buy at most one medium. Suppose further $\theta_1 > \theta_2$ without loss of generality. This induces the measure

$$D_1^R\left(\mathbf{p}^R, \theta\right) \equiv Pr\left(\theta_1\beta - p_1^R \geq \max\left\{0, \theta_2\beta - p_2^R\right\}\right) \tag{1}$$

on the part of readers; $D_1^R\left(p^R, \theta\right)$ is simply the number of subscribers. Without loss of generality, consumers will purchase from provider 1 over provider

[4]The recent feud between the LA Times and Chrysler does lend some credence to this thesis.

2 as long as $\beta \geq \max\left\{\hat{\beta} \equiv \frac{p_1^R - p_2^R}{\theta_1 - \theta_2}, \tilde{\beta} \equiv \frac{p_1^R}{\theta_1}\right\}$ and have demand

$$D_1^R\left(\mathbf{p}^R, \theta\right) = \min\left\{1 - F\left(\frac{p_1^R}{\theta_1}\right), 1 - F\left(\frac{p_1^R - p_2^R}{\theta_1 - \theta_2}\right)\right\}$$

while the demand for information good 2 is determined for values of the parameter $\beta \in \left[\frac{p_2^R}{\theta_2}, \hat{\beta}\right]$, or zero, and expressed as

$$D_2^R\left(\mathbf{p}^R, \theta\right) = \max\left\{0, F\left(\frac{p_1^R - p_2^R}{\theta_1 - \theta_2}\right) - F\left(\frac{p_2^R}{\theta_2}\right)\right\}$$

Thus demand functions exhibit a kink at $\hat{\beta} = \tilde{\beta}$.

Advertisers have a profit function $\mathcal{A}(y, x)$ separable in x, y; $x \in \{0, 1\}$ denotes advertising consumption and y is a (vector of) variable(s). These include any other action a platform may undertake, such as hiring sales personnel, R&D and so forth. Let e_i denote the quality of platform i as perceived by the advertisers. This plays the same role as θ_i on the consumer side, but pertains to the advertisers decision. For any $\hat{y}$, they may choose to purchase one unit of space at most at price p_i^A if

$$e_i\left(A(\hat{y}, 1) - A(\hat{y}, 0)\right) - p_i^A = e_i a - p_i^A \geq 0; \quad i = 1, 2$$

that is, they derive an increase in (expected) profit a. In practice, while producers routinely place their messages on different media, it is also true that they are cash-constrained [5]. The one unit limit can be interpreted as a tight liquidity constraint. Announcers may value the benefit from advertising differently according to the parameter a, which is considered private and distributed following $G(\alpha)$ on $[\underline{\alpha}, \overline{\alpha}]$ with mass 1. It is quite natural to let $e_i \equiv e\left(D_i^R\right)$, with $\frac{\partial e\left(D_i^R\right)}{\partial D_i^R} > 0$ and $\frac{\partial^2 e\left(D_i^R\right)}{\left(\partial D_i^R\right)^2} \leq 0$, and of course, $e\left(D_i^R\right) = 0$ for $D_i^R = 0$: the more consumers the advertiser can reach, the more they value an ad, but this benefit is (weakly) concave. For example, following Shaked and Sutton (1982), if one thinks of the parameter taste b as disposable income, the value of the marginal consumer to advertisers is clearly decreasing. The distribution $G(\alpha)$ is restricted to having a monotonically increasing hazard rate. Like consumers, potential advertisers act as price takers and there is no strategic interaction between them, nor between advertisers and platforms. The ranking of the platforms' market shares in the consumer market defines their relative quality in the advertising market. Given prices $\mathbf{p}^A = \left(p_1^A, p_2^A\right)$, a producer purchases from channel 1 over

[5] say because of risk aversion or credit markets imperfections

channel 2 (without loss of generality), only if $e_1\alpha - p_1^A \geq \max\left\{0, e_2\alpha - p_2^A\right\}$. This decision rule generates the measure

$$Pr\left(e_1\alpha - p_1^A \geq \max\left\{0, e_2\alpha - p_2^A\right\}\right) \equiv q_1^A\left(p^A, e\right)$$

where $e = (e_1, e_2)$, whence we derive demands

$$q_1^A\left(\mathbf{p}^A, \mathbf{e}\right) = \min\left\{1 - G\left(\frac{p_1^A}{e_1}\right), 1 - G\left(\frac{p_1^A - p_2^A}{e_1 - e_2}\right)\right\}$$

and

$$q_2^A\left(\mathbf{p}^A, \mathbf{e}\right) = \max\left\{0, G\left(\frac{p_1^A - p_2^A}{e_1 - e_2}\right) - G\left(\frac{p_2^A}{e_2}\right)\right\}$$

We assume neither constraint on advertising space (the medium can always print one more page, for example), nor that advertising affects readership[6]. The cost of running adverts is set at zero. Quality however is costly to provide and is modeled as an investment according to $k\theta_i^2$.

Platforms (say, magazine or television channels) first choose a quality level. Given this quality, set prices first to consumers and, in a third stage, to advertisers. Upon observing these prices, they choose whether to purchase. The three-stage game is denoted Γ.

A platform collect revenues from both readers and advertisers, with monies from either side perfectly substitutable. *Ceteris paribus*, a provider simply cares about total revenue. For any medium $i = 1, 2$, the objective function takes the form

$$\Pi_i = D_i^R\left(\mathbf{p}^R, \theta\right) p_i^R - k\theta_i^2 + q_i^A\left(\mathbf{p}^A, \mathbf{e}\right) p_i^A \tag{2}$$

4 Equilibrium characterisation

Section 7.9 of the Appendix presents an analysis of symmetric equilibria in general form, which arise when platforms select symmetric quality (either because they are constrained by design, or as an equilibrium outcome), or when consumer prices are fixed. While the latter situation occurs quite naturally, say by mandate or simply because consumption cannot be monitored[7], there is no reason for platforms to limit themselves to symmetric quality. Moving away from symmetry softens price competition and leads

[6] Although we duly note restrictions on advertising time for free-to-air television, as well as an obvious distaste for advertising on TV by viewers, for example

[7] as in the case of broadcasting

to positive profits (Shaked and Sutton (1982, 1983), Tirole (1988)). In line with the literature, we impose some structure by assuming a uniform distribution on the bounded supports of the private parameters β and α in the consumer market and the advertising market, respectively. To prevent exogenous market preemption (in the consumer market) we impose

Assumption 1 $\overline{\beta} - 2\underline{\beta} > 0$

This rules out the trivial case in which the low-quality platform necessarily faces zero demand in the price game. Let $e\left(D_i^R\right) = e \times D_i^R$ for (non-trivial) simplicity, which lets us interpret e as a scaling parameter. For the analysis to remain tractable we restrict the range of equilibrium outcomes to those were full (advertising) market coverage arises, that is

Assumption 2 *$e \leq 1$ and $\frac{\overline{\alpha}}{\underline{\alpha}} \in \left(2, \frac{2D_i^R - D_j^R}{e(D_i^R - D_j^R)}\right)$*

where $D_i^R > D_j^R$ and the demands are evaluated at equilibrium. This is not without loss of generality, as shown by Wauthy (1996): coverage is an equilibrium outcome. Wauthy (1996) shows it is optimal only in this range of parameter values. The condition on e guarantees the interval $\left(2, \frac{2D_i^R - D_j^R}{e(D_i^R - D_j^R)}\right)$ to be non-trivial and well defined. As Assumption 1, it also prevents exogenous preemption. Further discussion is postponed until after the analysis. Also, the parameter k needs to be sufficiently large for the cost function to have some bite; specifically,

Assumption 3 $k > \frac{(2\overline{\beta} - \underline{\beta})^2}{18\overline{\theta}}$

to guarantee an interior solution in the benchmark case[8]. Taken together, Assumptions 1 and 3 guarantee that the consumer market is covered in equilibrium, which greatly simplifies the analysis. To see why, suppose there is no externality and denote the equilibrium quality levels are given by $\theta_1^0 = \frac{1}{2k}\left(\frac{2\overline{\beta} - \underline{\beta}}{3}\right)^2 > \theta_2^0 = \underline{\theta}$ [9]. The condition for a covered market is $\frac{\overline{\beta} - 2\underline{\beta}}{3}(\theta_1^0 - \theta_2^0) \leq \underline{\beta}\theta_2^0$ (see for example, Tirole, (1988)). Substituting for the values of θ_1^0, θ_2^0 and re-arranging, the market is covered for $k \geq \frac{1}{2\underline{\theta}}\left(\frac{2\overline{\beta} - \underline{\beta}}{3}\right)^2\left(\frac{\overline{\beta} - 2\underline{\beta}}{\overline{\beta} - \underline{\beta}}\right)$,

[8] to be developed in Section 5. In the absence of a sufficiently large parameter k the Shaked and Sutton boundary result prevails, which renders across-model comparisons meaningless.

[9] Refer section 5 for details of this equilibrium

which is necessarily satisfied by Assumption 3. It follows that both firms operate and the relevant demand functions in the consumer market are the competitive ones. It will be obvious that it is satisfied in an equilibrium of this game. We proceed in two steps, first focusing on agents' behaviour in the advertising market, which is not *directly* affected by quality choices. Let $\hat{\alpha} = \frac{\hat{p}_1^A - \hat{p}_2^A}{\Delta e}$ and $e\Delta D^R = e(D_1^R - D_2^R)$ denote the difference in the platforms' quality. The following lemma reflects these specifications.

Lemma 1 *Suppose $D_1^R \geq D_2^R$ w.l.o.g. There may be three pure strategy equilibria in the advertising market. When $D_1^R > D_2^R > 0$, the profit functions write:*

$$\overline{\Pi}_1^A = e\Delta D^R \left(\frac{2\overline{\alpha} - \underline{\alpha}}{3}\right)^2 ; \ \underline{\Pi}_2^A = e\Delta D_2^R \left(\frac{\overline{\alpha} - 2\underline{\alpha}}{3}\right)^2$$

When $D_1^R > D_2^R = 0$, platform 1 is a monopolist its profits are:

$$\Pi_1^{AM} = eD_1^R \left(\frac{\overline{\alpha}}{2}\right)^2$$

For $D_1^R = D_2^R$, the Bertrand outcome prevails and platforms have zero advertising profits.

Up to the endogenous quality in the advertising market, this is exactly the result of the classical analysis of vertical differentiation. Note that an equilibrium resting on $D_i^R > D_j^R = 0$ may arise as a non-trivial equilibrium in the advertising market: take for example $\theta_i = \theta_j$ and $p_i^R < p_j^R$, which may, or may not, be an out-of-equilibrium outcome in the consumer market.

Proof. The proof is standard and therefore omitted (it follows the proof of Lemma 10, Section 7.9). The low-quality firm's survival requires sufficient heterogeneity among buyers, which is imposed by Assumption 2. ■

Thanks to Assumption 1, consumer demand for the information good simply writes

$$D_i^R = \overline{\beta} - \frac{p_i^R - p_j^R}{\Delta\theta} \quad ; \quad D_j^R = \frac{p_i^R - p_j^R}{\Delta\theta} - \underline{\beta}$$

for $\theta_i > \theta_j$ and with $\Delta\theta = \theta_i - \theta_j$. When choosing its strategy in the consumer market, each firm knows what to expect in the announcers' market: the platform that holds the larger market share in the consumer market will enjoy high advertising profits and conversely. Following Lemma 1 the profit function (2) rewrites as

$$\Pi_i = p_i^R D_i^R(\mathbf{p}^R, \theta) - k(\theta_i) + \Pi_i^A \left(e\Delta D^R(\mathbf{p}^R, \theta)\right) \tag{3}$$

on the equilibrium path. It is useful to bear in mind that when firm 1 is dominant in the consumer market, $\Delta D^R(\mathbf{p}^R,\theta) = D_1^R(\mathbf{p}^R,\theta) - D_2^R(\mathbf{p}^R,\theta)$, and conversely if firm 2 dominates.

4.1 Consumer price subgame

From Lemma 1 three distinct configurations may arise on the equilibrium path. In the first case, platform i dominates the consumer market, in the second one both share the consumer market equally and in the last one it is dominated by firm j. The profit function (3) of each firm $i = 1,2$ writes

$$\Pi_i = p_i^R D_i^R(\mathbf{p}^R,\theta) - k\theta_i^2 + \begin{cases} \overline{\Pi}_i^A, & \text{if } D_i^R > D_j^R; \\ 0, & \text{if } D_i^R = D_j^R; \\ \underline{\Pi}_i^A, & \text{if } D_i^R < D_j^R. \end{cases} \tag{4}$$

This function is continuous, but not quasi-concave and has a kink at the profile $\tilde{\mathbf{p}}^R$ of consumer prices such that $D_i^R = D_{-i}^R$. Thus the conditions of Theorem 2 of Dasgupta and Maskin's paper (1986a) are not met. Their Theorem 5 is of limited use as it pertains to mixed strategies. In characterising the equilibrium of this subgame we face discontinuous best-response correspondences owing the lack of quasi-concavity, which follows from the externality generated by advertising revenue. Proceeding by construction it is nonetheless possible to show that at least one equilibrium in pure strategies always exists. Denote $\Delta\theta = \theta_1 - \theta_2$, $\overline{A} = \left(\frac{2\overline{\alpha}-\underline{\alpha}}{3}\right)^2$ and $\underline{A} = \left(\frac{\overline{\alpha}-2\underline{\alpha}}{3}\right)^2$. The space $P_i^R \times P_{-i}^R$ of action profiles (prices) can be divided into three regions: region I such that $D_i^R > D_{-i}^R$, region II such that $D_i^R < D_{-i}^R$ and region III such that $D_i^R = D_i^R$. We begin with

Definition 1 *For $i = 1,2$, the platforms' 'quasi-best responses' are defined as the solution to the problem $\max_{p_i^R} \Pi_i\left(p_i^R, D_i^R(\mathbf{p}^R,\theta); \Pi_i^A(D_i^R, D_j^R)\right)$, where the profit function is defined by (4). Therefore, letting $\theta_1 > \theta_2$ w.l.o.g,*

$$p_1^R\left(p_2^R\right) = \begin{cases} \underline{p}_1^R\left(p_2^R\right) = \frac{1}{2}\left(p_2^R + \Delta\theta\overline{\beta} - 2e\overline{A}\right), & \textit{if } D_1^R > D_2^R; \\ \frac{1}{2}\left(p_2^R + \Delta\theta\overline{\beta}\right), & \textit{if } D_1^R = D_2^R; \\ \overline{p}_1^R\left(p_2^R\right) = \frac{1}{2}\left(p_2^R + \Delta\theta\overline{\beta} + 2e\underline{A}\right), & \textit{if } D_1^R < D_2^R; \end{cases}$$

and

$$p_2^R\left(p_1^R\right) = \begin{cases} \underline{p}_2^R\left(p_1^R\right) = \frac{1}{2}\left(p_2^R - \Delta\theta\underline{\beta} - 2e\overline{A}\right), & \textit{if } D_1^R < D_2^R; \\ \frac{1}{2}\left(p_1^R - \Delta\theta\underline{\beta}\right), & \textit{if } D_1^R = D_2^R; \\ \overline{p}_2^R\left(p_1^R\right) = \frac{1}{2}\left(p_2^R - \Delta\theta\underline{\beta} + 2e\underline{A}\right), & \textit{if } D_1^R > D_2^R; \end{cases}$$

While it is always possible to find some point where 'quasi-best responses' intersect (e.g. such that both play as if $D_1^R < D_2^R$), it by no means necessarily defines an equilibrium. Doing so assumes that in some sense platforms coordinate on a particular market configuration, say, such that $D_1^R < D_2^R$, which may not been immune from unilateral deviation. After all this is a non-cooperative game. To find the equilibrium, if it exists, we first need to pin down the firms' true best replies.

Lemma 2 *Let $\theta_1 > \theta_2$ w.l.o.g. There exists a pair of actions $(\hat{p}_1, \hat{p}_2)$ such that the best response correspondences are defined as*

$$p_1^R\left(p_2^R\right) = \begin{cases} \underline{p}_1^R\left(p_2^R\right), & \text{for } p_2 \geq \hat{p}_2; \\ \overline{p}_1^R\left(p_2^R\right), & \text{for } p_2 < \hat{p}_2; \end{cases} \tag{5}$$

and

$$p_2^R\left(p_1^R\right) = \begin{cases} \overline{p}_2^R\left(p_1^R\right), & \text{for } p_1 < \hat{p}_1; \\ \underline{p}_2^R\left(p_1^R\right), & \text{for } p_1 \geq \hat{p}_1; \end{cases} \tag{6}$$

Lemma 2 thus defines the 'true' best-response of each player. It says that platform 1, for example, prefers responding with $\underline{p}_1^R\left(p_2^R\right)$ for any prices $p_2 \geq \hat{p}_2$ and switches to $\overline{p}_1^R\left(p_2^R\right)$ otherwise. Recall that consumer profits and advertising profits are substitutes for the platforms. The best reply correspondence is discontinuous at that point where platforms are indifferent between being the dominant platform and not, that is, between the combination of prices $\left(\underline{p}_i^R(p_j^R), p_i^A(\underline{p}_i^R)\right)$ and $\left(\overline{p}_i^R(p_j^R), p_i^A(\overline{p}_i^R)\right)$.

Proof. First notice that playing a profile $\tilde{\mathbf{p}}^R$ such that $D_1^R = D_2^R$ can never be a best reply. When $D_1^R = D_2^R$ advertising profits Π_i^A are nil for both platforms. So both players have a deviation strategy $p_i^R + \varepsilon$ in either direction since $\overline{\Pi}_i^A > \underline{\Pi}_i^A > 0,\ i = 1,2$ as soon as $D_i^R \neq D_{-i}^R$. Maximising the profit function (4) taking p_{-i} as fixed leaves us with two 'quasi-reaction correspondences', for each competitor, depending on whether $D_1^R > D_2^R$ or the converse. Player i's profit function can be rewritten $\Pi_i\left(p_1^R(p_2^R), p_2^R; \Pi_i^A\right)$. Depending on firm 2's decision, platform 1's profit is either

$$\Pi_1 = \begin{cases} \Pi_1\left(\underline{p}_1^R(p_2^R), p_2^R; \Pi_i^A\right) = \Pi_1\left(\frac{1}{2}\left(p_2^R + \Delta\theta\overline{\beta} - 2e\overline{A}\right), p_2^R; \Pi_i^A\right), & \text{or;} \\ \Pi_1\left(\overline{p}_1^R(p_2^R), p_2^R; \Pi_i^A\right) = \Pi_1\left(\frac{1}{2}\left(p_2^R + \Delta\theta\overline{\beta} + 2e\underline{A}\right), p_2^R; \Pi_i^A\right). \end{cases}$$

Define $g_1(p_2^R) \equiv \Pi_1\left(\overline{p}_1^R(p_2^R), p_2^R; \Pi_i^A\right) - \Pi_1\left(\underline{p}_1^R(p_2^R), p_2^R; \Pi_i^A\right)$. This quantity is the difference in profits generated by firm 1 when it chooses one 'quasi-best response' over the other, as a function of the consumer price set by firm 2. For p_2^R sufficiently low, $g_1(.) > 0$. This function is continuous and a.e differentiable, for it is the sum of two continuous, differentiable functions. Using the definitions of equilibrium advertising profits (in Lemma 1), it is immediate to compute $\frac{dg_1}{dp_2^R} = \frac{d\Pi_1^A(\overline{p}_1^R, p_2^R)}{dp_2^R} - \frac{d\Pi_1^A(\underline{p}_1^R, p_2^R)}{dp_2^R} < 0$, and $\frac{d^2 g_1}{d(p_2^R)^2} = 0$, whence there exists a point $\hat{p}_2^R$ such that $g_1(\hat{p}_2^R) = 0$. At $\hat{p}_2^R$, $\Pi_i\left(\underline{p}_1^R(\hat{p}_2^R), \hat{p}_2^R\right) = \Pi_i\left(\overline{p}_1^R(\hat{p}_2^R), \hat{p}_2^R\right)$ and platform 1 is indifferent between these two profit functions, that is between either best response $\underline{p}_1^R(\hat{p}_2^R)$ or $\overline{p}_1^R(\hat{p}_2^R)$. The same follows for platform 2, which defines $\hat{p}_1^R$. Computing the profit functions, it is immediate that

$$\Pi_1\left(\underline{p}_1^R(p_2^R), p_2^R; \Pi_i^A\right) \geq \Pi_1\left(\overline{p}_1^R(p_2^R), p_2^R; \Pi_i^A\right) \Leftrightarrow p_2^R \geq \hat{p}_2^R \equiv -\left(\Delta\theta\underline{\beta} + e(\overline{A} - \underline{A})\right)$$

and

$$\Pi_2\left(p_1^R, \underline{p}_2^R(p_1^R); \Pi_i^A\right) \geq \Pi_2\left(p_1^R, \overline{p}_2^R(p_1^R); \Pi_i^A\right) \Leftrightarrow p_1^R \geq \hat{p}_1^R \equiv \Delta\theta\overline{\beta} - e(\overline{A} - \underline{A})$$

■

An equilibrium has a flavour of rational expectations, in that platforms must select actions that are consistent with each other (for example, *both* must play as if $D_1^R > D_2^R$), as well as compatible with an equilibrium. Such a rationality requirement however is not necessary, as will soon be obvious; instead we let media play a standard Nash equilibrium. Hence by definition, for each firm, its action must be an element of the best reply correspondence and these correspondences must intersect. We define a condition that captures both these features, and will show next that it is both necessary and sufficient for an equilibrium to exist. From the 'quasi-best responses', an equilibrium candidate is a pair of prices such that

$$\left(p_1^{*R}, p_2^{*R}\right) = \begin{cases} \underline{p}_1^R\left(p_2^R\right) \cap \overline{p}_2^R\left(p_1^R\right), & \text{if } D_1^R > D_2^R \text{ or;} \\ \overline{p}_1^R\left(p_2^R\right) \cap \underline{p}_2^R\left(p_1^R\right), & \text{if } D_1^R < D_2^R; \end{cases}$$

These actions may form an equilibrium only if their intersections are non-empty. Together, the definitions of a best-response profile (relations (5) and (6)) and of an equilibrium candidate sum to

Condition 1 *Either*

$$\hat{p}_1^R \geq p_1^{*R} \textit{ and } \hat{p}_2^R \leq p_2^{*R}$$

or

$$\hat{p}_1^R \leq p_1^{*R} \textit{ and } \hat{p}_2^R \geq p_2^{*R}$$

or both.

Consider an action profile $\mathbf{p}^{*R}$ satisfying this condition; from Lemma 2 each p_i^{*R} is an element of i's best response. Now, for it to be an equilibrium, players must choose reaction functions that intersect. This is exactly what Condition 1 requires. For example, the first pair of inequalities tells us that player 1's optimal action has to be low enough and simultaneously that of 2 must be high enough. When they hold, player 2's reaction correspondence is necessarily continuous until 1 reaches the maximiser p_1^{*R}, and similarly for 1's best reply. Then

Lemma 3 *Condition 1 is necessary and sufficient for at least one equilibrium* $\mathbf{p}^{*R} = \left(p_1^{*R}, p_2^{*R}\right)$ *to exist. When both inequalities are satisfied, the game admits two equilibria.*

When Condition 1 holds, the best-reply correspondences intersect in at least one subset of the action profile space $P_1^R \times P_2^R$. In this case, the Nash correspondence $p_1^R(p_2^R) \times p_2^R(p_1^R)$ has a closed graph and standard theorems apply. The potential multiplicity of equilibria owes to the discontinuity of the best-reply correspondences.

Proof. The profit function Π_i $i = 1, 2$ is strictly concave in p_i^R and therefore quasi-concave. Since player i's action set is $P_i^R \subseteq \mathbb{R}$, it is compact and convex. For each platform this can be partitioned into two subsets $\underline{P}_i^R = \left[p_i^{R,min}, \hat{p}_i^R\right]$ and $\overline{P}_i^R = \left[\hat{p}_i^R, p_i^{R,max}\right]$, on which the best-response correspondences defined by (5) and (6) are continuous for each platform i. Consider any equilibrium candidate $\left(p_1^{*R}, p_2^{*R}\right)$. By construction it is defined as the intersection of the 'quasi-best responses', which is not necessarily an equilibrium. But when Condition 1 holds, following the definitions given by equations (5) and (6), either $p_1^{*R} \in \underline{p}_1^R(p_2^R)$ and $p_2^{*R} \in \overline{p}_2^R(p_1^R)$, or $p_1^{*R} \in \overline{p}_1^R(p_2^R)$ and $p_2^{*R} \in \underline{p}_2^R(p_1^R)$ (or both, if two equilibria exist). Thus at the point $\left(p_1^{*R}, p_2^{*R}\right)$ the reaction correspondences necessarily intersect at least once, whence the Nash correspondence has a closed graph and the Kakutani fixed-point theorem applies. To show necessity, suppose a pair $\left(p_1^{*R}, p_2^{*R}\right)$ is a Nash equilibrium. By definition, $p_2^R\left(p_1^R\right) \cap p_1^R\left(p_2^R\right) \neq \varnothing$, and by Lemma 2, either $\left(p_1^{*R}, p_2^{*R}\right) = \underline{p}_1^R\left(p_2^R\right) \cap \overline{p}_2^R\left(p_1^R\right)$ or $\left(p_1^{*R}, p_2^{*R}\right) = \overline{p}_1^R\left(p_2^R\right) \cap \underline{p}_2^R\left(p_1^R\right)$, or both if two equilibria exist. For the first equality to hold, the first line of Condition 1 must hold, and for the second one, the second line of Condition 1 must be satisfied. ■

Let $C = \left[2e\left(\overline{A} + \underline{A}\right)\right]^2 = \left[2e\left(\left(\frac{2\overline{\alpha}-\underline{\alpha}}{3}\right)^2 + \left(\frac{\overline{\alpha}-2\underline{\alpha}}{3}\right)^2\right)\right]^2$. Condition 1 provides us with a pair of easy-to-verify conditions in terms of prices. Thus we can establish

Lemma 4 ***Existence***. *An equilibrium in pure strategies of the consumer price subgame always exists. It is unique and located in region I.*

There cannot exist a pair of alternative consumer prices $\left(p_1^{**R}, p_2^{**R}\right)$ compatible with a pair of quality choices (θ_1^*, θ_2^*) solving the platforms' problem: they fail the necessary condition laid out in Lemma 3.

Proof. The proof is somewhat lengthy – and therefore collated in the Appendix, Section 7.1 – but simple. Candidate equilibria can be constructed from the 'quasi-reaction correspondences' of Definition 1. By Lemma 3, it is enough to verify that these candidates satisfy Condition 1 for them to form a Nash equilibrium. One of them always does, while the other one never can. ■

The lack of quasi-concavity of the payoff functions induces discontinuity of the best-reply correspondences. To paraphrase Dasgupta and Maskin (1986a) however, this discontinuity *is* essential. Given sunk quality the configuration $\langle \theta_1 > \theta_2, D_1 < D_2 \rangle$ corresponding to the second line of Condition 1 can be interpreted as a need for advertising profits to be large enough for a second equilibrium to exist. However it entails playing a weakly dominated strategy for player 2: if she finds it attractive to reduce her price so much, so must player 1. Thus the discontinuity set is not a trivial one – it has certainly not measure zero. It follows that mixed strategies cannot restore this second candidate equilibrium (Dasgupta and Maskin (1986a), Theorem 5). Furthermore it implies that we need not call on the rational expectation framework. Elimination of weakly dominated strategies is sufficient to rule it out and play a less strenuous Nash equilibrium. This is depicted in Figure 1. Now we are ready to collect these results and to compute equilibrium consumer prices in this subgame.

Proposition 1 ***Consumer prices.*** *Let $\theta_1 > \theta_2$ w.l.o.g. There may be two possible configurations arising in the consumer price subgame. For each, there exists a unique Nash equilibrium in pure strategies characterised as*

- *For* $\Delta\theta > \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}$

$$\begin{aligned} p_1^{R*} &= \tfrac{1}{3}\left[\Delta\theta\left(2\overline{\beta} - \underline{\beta}\right) + 2e\left(\underline{A} - 2\overline{A}\right)\right] \\ p_2^{R*} &= \tfrac{1}{3}\left[\Delta\theta\left(\overline{\beta} - 2\underline{\beta}\right) + 2e\left(2\underline{A} - \overline{A}\right)\right] \end{aligned}$$

- *If* $\Delta\theta \leq \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}$

$$\begin{aligned} p_1^{R*} &= \frac{\Delta\theta\overline{\beta}}{2} - 2e\overline{A} \\ p_2^{R*} &= 0 \end{aligned}$$

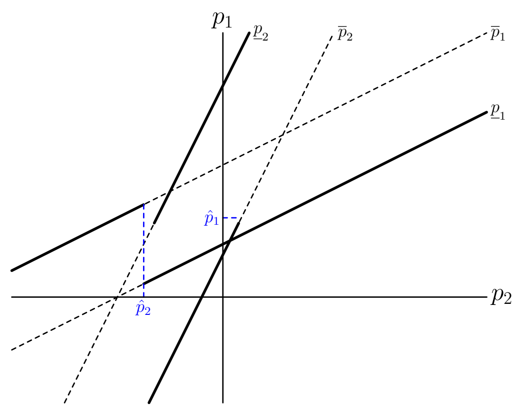

Figure 1: Best reply functions and unique equilibrium

Proof. Directly from Lemma 4, which establishes existence and uniqueness of this equilibrium. In particular no such alternative equilibrium can exist when $\Delta\theta < \frac{\sqrt{C}}{2\overline{\beta}-\underline{\beta}}$. Consider such a situation, then the prices

$$p_1^R = \frac{\Delta\theta\overline{\beta}}{2} - 2e\overline{A}$$
$$p_2^R = 0$$

do form an equilibrium for they satisfies Condition 1. But the pair

$$p_1^R = 0$$
$$p_2^R = -\frac{\Delta\theta\underline{\beta}}{2} - 2e\overline{A}$$

cannot be best responses to each other. At the price-setting stage, the cost of quality is sunk. So with $\theta_1 > \theta_2$, there always exists some price $p_1^R \geq p_2^R$ such that consumers prefer purchasing from platform 1. ■

Consumer prices thus include a 'discount' as platforms engage in cross-subsidisation. The lure of advertising revenue intensifies the competition for consumers because they become more valuable than just for their willingness to pay for the information good. But the intuitive reasoning whereby the low-quality firm may find it profitable to behave very aggressively in order to access large advertising revenue does not hold true (Lemma 4). Moreover, unlike in the Shaked and Sutton (1982) model, $\overline{\beta} - 2\underline{\beta} > 0$ is not sufficient to afford the low-quality firm some positive demand: $\Delta\theta$, defined in the first stage, may be too narrow to sustain two firms. That is, the high-quality platform may choose to act so as to exclude firm 2.

4.2 First-stage actions

In the first stage, platforms face the profit function (4), which they each maximise by choice of their quality variable θ_i. That is, each of them solves

Problem 1

$$\max_{\theta_i \in [\underline{\theta}, \overline{\theta}]} p_i^{R*} D_i^R \left(\theta, \mathbf{p}^{R*}\right) + \Pi_i^A \left(e, \Delta D^R(\mathbf{p}^{R*}, \theta)\right) - k\theta_i^2$$

subject to

$$\theta_{-i} \in \Theta_{-i}^N$$

and

$$\hat{\beta} = \frac{p_i^R - p_j^R}{\theta_i - \theta_j} \in \left[\underline{\beta}, \overline{\beta}\right] \tag{7}$$

where $\mathbf{p}^{R*}$ is the relevant equilibrium price profile characterised in Proposition 1, and p_i^{R*} the corresponding price chosen by platform i. Θ_{-i}^N denotes the set of best responses of player i's opponents. The second constraint does not limit quality choices *per se* but is a natural restriction guaranteeing that platforms' demands remains bounded by the market size[10]. It can be rearranged as a pair of inequalities: $\Delta\theta\left(2\overline{\beta} - \underline{\beta}\right) + \sqrt{C} \geq 0$ and $\Delta\theta\left(\overline{\beta} - 2\underline{\beta}\right) - \sqrt{C} \geq 0$. Only the second one is constraining. On the equilibrium path the objective function of Problem 1 reads

$$\Pi_1 = \begin{cases} \frac{1}{9}\left(\Delta\theta(2\overline{\beta} - \underline{\beta})^2 + B_1 + \frac{C}{\Delta\theta}\right) - k\theta_1^2, & \text{if } \Delta\theta > \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}; \\ \frac{1}{9}\left(\Delta\theta(2\overline{\beta} - \underline{\beta})^2 + B_1 + \sqrt{C}(\overline{\beta} - 2\underline{\beta})\right) - k\theta_1^2, & \text{if } \Delta\theta \leq \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}} \end{cases} \tag{8}$$

where $B_1 = (2\overline{\beta} - \underline{\beta})2e\left(2\underline{A} - \overline{A}\right) + 3e\left(\overline{\beta} + \underline{\beta}\right)\overline{A}$ (is constant in θ). The second line of the definition of Π_1 rules out the artificial case of firm 1 facing a demand larger than the whole market (profits are bounded). It is derived by taking $\frac{C}{\Delta\theta}$ as fixed at its lowest value, that is, where $\Delta\theta = \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}$. For platform 2, profits are

$$\Pi_2 = \begin{cases} \frac{1}{9}\left(\Delta\theta(\overline{\beta} - 2\underline{\beta})^2 + B_2 + \frac{C}{\Delta\theta}\right) - k\theta_2^2, & \text{if } \Delta\theta(\overline{\beta} - 2\underline{\beta}) > \sqrt{C}; \\ 0, & \Delta\theta(\overline{\beta} - 2\underline{\beta}) \leq \sqrt{C} \text{ and } \theta_2 = 0; \\ -k\theta_2^2, & \Delta\theta(\overline{\beta} - 2\underline{\beta}) \leq \sqrt{C} \text{ and } \theta_2 > 0; \end{cases} \tag{9}$$

[10] observe that $\theta_i \to \theta_j \Rightarrow \hat{\beta} \to \infty$

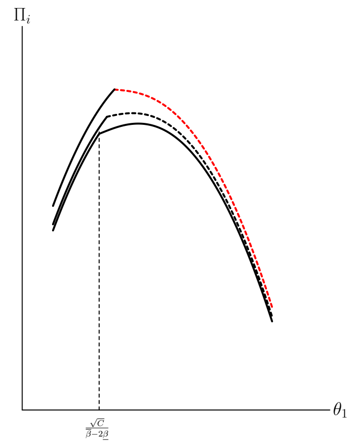

Figure 2: Profit functions for different values of advertising

with $B_2 = (\overline{\beta} - 2\underline{\beta})2e\left(\underline{A} - 2\overline{A}\right) + 3e\left(\overline{\beta} + \underline{\beta}\right)\underline{A}$. A major difficulty may arise in solving this problem. Both platforms' profit functions are the difference of two convex functions, which may be concave or convex. Section 7.2 of the Appendix studies the profit function $\Pi_1(\theta_1, \theta_2)$ in the details necessary to support our results. In particular it identifies a threshold $C^f \equiv \left[\frac{(2\overline{\beta}-\underline{\beta})^2}{27k} - \underline{\theta}\right]^2 \left(\frac{(2\overline{\beta}-\underline{\beta})^2}{3}\right)$ such that the function remains well behaved if C does not exceed C^f. Otherwise, the necessary first-order condition fails to hold entirely. For $C \geq C^f$, the high-quality medium would like to pick $\tilde{\theta}(e) \equiv \underline{\theta} + \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$, where $\Pi_1(.,.)$ reaches is maximum. At that point its rival is excluded ($\Delta\theta$ is low enough), and it still extracts as much surplus from consumers as it can without losing its status as monopolist. But then firm 2 can 'leap' over it and become the monopolist at a negligible incremental cost. Intuitively, when advertising returns are large enough every consumer becomes very valuable to both platforms. Consequently the profit function $\Pi_1(.,.)$ is linearly increasing up to $\theta_1 = \tilde{\theta}(e)$. For $C < C^f$ the function $\Pi_1(.,.)$ remains increasing, but now concave, on the portion *beyond* $\theta_1 = \tilde{\theta}(e)$ as well, where it admits a maximiser. This is illustrated in Figure 2, where the higher curve corresponds to the case of $C > C^f$. The solid lines represent the first line, and the dashed ones the second line, of (8). To overcome this problem, Assumption 2 is strengthened and turned into

Assumption 4 $e < \bar{e} \equiv \min\left\{1, \left(\frac{(2\overline{\beta}-\underline{\beta})^2}{27k} - \underline{\theta}\right)\frac{\overline{\beta}-2\underline{\beta}}{2(\overline{A}+\underline{A})}\right\}$ [11].

in order to study the optimal action profile in pure strategies. Assumption 4 guarantees that when $\hat{\theta}_1$ solves the first-order condition, $(\hat{\theta}_1 - \underline{\theta})(\overline{\beta} - 2\underline{\beta}) > \sqrt{C}$, so that both media operate. If this restriction is met we can claim

Lemma 5 *Let $\theta_1 > \theta_2$ w.l.o.g. and Assumption 4 hold. Optimal actions consist of $\theta_2^* = \underline{\theta}$ and $\theta_1^* = \hat{\theta}_1$, where $\hat{\theta}_1$ uniquely solves*

$$(2\overline{\beta} - \underline{\beta})^2 = 18k\theta_1 + \frac{C}{(\Delta\theta)^2} \tag{10}$$

Both platforms operate.

We label the term $\frac{C}{(\Delta\theta)^2}$ the 'market share effect': it acts as an incentive to reduce quality and is similar to that arising in standard Bertrand competition. In condition (10), firm 1 trades off the marginal benefit of quality (the left-hand side) not only with its marginal investment cost but also with the marginal advertising revenue that it must forego because of higher consumer prices induced by higher quality (the RHS). For a positive k the low-quality firm cannot deviate by 'leaping' over its rival and offering a slightly higher-quality good. Given that it markets a lesser good, platform 2 selects $\underline{\theta}$ to mitigate the price war. This is the Differentiation Principle at work, but here it is subsumed by the 'market share effect'.

Proof. The proof is collected in the Appendix, Section 7.3. For C not too large the FOC binds at zero and the maximiser of Π_1 exists. Because its profit function is locally decreasing from $\theta_2 = \underline{\theta}$ on (by Claim 4), the only deviation for platform 2 would consist in 'leaping' over firm 1 and become the high-quality firm. But this cannot be profit-maximising when the FOC holds. ■

Collecting the results from Lemmata 1 and 5 and Proposition 1, and letting platform 1 be the high-quality medium w.l.o.g., we can finally state

Proposition 2 ***Equilibrium characterisation.*** *Suppose Assumption 4 holds. The game Γ admits exactly one equilibrium in pure strategies in which both platforms operate and choose different qualities. It is characterised by the triplet of profiles $\left(\theta^*, \mathbf{p}^{*R}, \mathbf{p}^{*A}\right)$ defined by Lemma 5, Proposition 1 and Lemma 1, respectively.*

[11] this arises from the condition $(\hat{\theta}_1 - \underline{\theta})(\overline{\beta} - 2\underline{\beta}) > (\theta_1^f - \underline{\theta})(\overline{\beta} - 2\underline{\beta}) \geq \sqrt{C}$, where $\theta_1^f = \frac{(2\overline{\beta}-\underline{\beta})^2}{27k}$ is defined in Section 7.2

Proof. Suppose $\theta_1 > \theta_2$. Since each proper subgame admits a unique Nash equilibrium by Lemma 5, Proposition 1 and Lemma 1, the equilibrium of the game Γ must be unique. ■

Proposition 2 can be appended with the obvious

Corollary 1 *If a Nash equilibrium of Γ exists, platforms may also play in non-trivial mixed strategies.*

for which the intuition is that of the Battle of the Sexes. Further we note

Remark 1 *Unlike the situtation where $C \geq C^f$, in which platforms should select symmetric qualities with probability zero*[12]*, the case $\theta_1 = \theta_2$ may arise with positive probability following Corollary 1. Recall that when a pure-strategy equilibrium exists, firms randomise over the set $\{\underline{\theta}_i, \theta_i^*\}$ instead of a continuous support. Each event carries a mass-point. Thus Proposition 10 (in the Appendix) is relevant to the case of endogenous quality, however only when a pure-strategy equilibrium exists.*

That aside, when $C \geq C^f$ (Lemma 6 in the Appendix, Section 7.3), it is not immediate that the game admits a mixed strategy equilibrium, for the payoff correspondences are not upper-hemicontinuous and their sum is not necessarily so either. Nonetheless it is possible to show that

Proposition 3 *When $\frac{\overline{\alpha}}{\underline{\alpha}} \in \left(2, \frac{2D_i^R - D_j^R}{e(D_i^R - D_j^R)}\right)$ a mixed-strategy equilibrium of the game Γ always exists.*

The proof and discussion of this statement can be found in the Appendix, Section 7.4. The conditions of the Proposition guarantee that the market is covered. Let $H_i(\theta_i)$ be the probability distribution over i's play, Θ_i^N the relevant support and θ_i^c the upper bound of the support. Let also $R_i(\theta_i, \theta_j)$ denote the revenue accruing to i. We can claim

Proposition 4 *There exists are pair of symmetric distributions H_1, H_2 satisfying*

$$H_i(\underline{\theta}_i)R_j(\underline{\theta}_i, \theta_j) + \int_{\tilde{\theta}_i}^{\theta_i^c} R_j(s, \theta_j) dH_i(s) = k(\theta_j^c)^2$$

with

$$H_i(s) \begin{cases} \in (0,1), & s = \underline{\theta}_i; \\ = 1, & s = \theta_i^c. \end{cases}$$

[12] refer Section 7.4 in the Appendix.

and

$$h(s) = 0, \qquad s \in \left(\underline{\theta}_i, \tilde{\theta}_i\right)$$

and θ_i^c *defined in Lemma 9.*

Noticeably $\Theta_j^N \subset \Theta_j$. Indeed it is obvious from the profit function (9) that playing any $\theta_j \in \left(\underline{\theta}, \tilde{\theta}(e)\right)$ is strictly dominated by selecting the lower bound. Furthermore, platforms place some mass at the lower bound $\underline{\theta}$. This is because $\Pi_i(\underline{\theta}_i, \theta_j) > 0$ for $\theta_j > \tilde{\theta}(e)$: if j plays anything in the support Θ_j^N but $\underline{\theta}_j$, i necessarily derives positive profits.

Proof. Please refer to the Appendix, Section 7.5. ■

Having characterised the equilibria of the game, we want to understand the impact of the externality $e(.)$ on players' behaviour and on the breakdown of the equilibrium. This discussion is the object of the next section.

5 The role of externalities

The results of the preceding analysis are first contrasted with earlier ones established in the literature. We then investigate the impact of advertising profits on the behaviour of quality by computing some comparative statics. Last we compare the level of quality provided in our duopoly to that arising in a monopoly, with and without externality.

5.1 Quality distortion and advertising revenue

An important goal of this paper is to understand the behaviour of quality when information providers have access to a substitute source of revenue. To study this problem it is useful to first establish a benchmark. The major reference in the vertical differentiation literature is Shaked and Sutton's 1982 paper, where however costs are completely ignored. It is immediate to adapt their model, and easy to show below that this second source of revenue acts as a *substitute* for quality for the platforms. Indeed

Proposition 5 ***Quality distortion****. In any pure-strategy equilibrium of the game* Γ*, quality is lower than it would be absent advertising.*

Advertising income puts emphasis back on market share – while the introduction of differentiation had the opposite effect. This leads to more intense price competition for consumers, and the discount extended by the platforms increases in the advertising profits (as shown in Propositions 1 and 6). Lower consumer prices uniformly relax the need to provide costly quality: at lower prices, the marginal consumer demands a lesser product to make a purchase. An intuition for the exact trade-off can be sketched as follows: given any quality, in the second stage firms *must* offer a discount to consumers – quality is sunk by then. The extent of that discount, given fixed quality, is determined by profits to be collected on the advertising market. In the quality-setting stage, the high-quality firm can *further* increase this discount by lowering quality: its consumer price is $p_1^* = \frac{1}{3}\left[\Delta\theta(2\overline{\beta} - \underline{\beta}) + 2e(\underline{A} - 2\overline{A})\right]$, while that of its rival is $p_2^* = \frac{1}{3}\left[\Delta\theta(\overline{\beta} - 2\underline{\beta}) + 2e(2\underline{A} - \overline{A})\right]$. Taking θ_2 fixed, for each $d\theta_1$ firm 1 can decrease its consumer price by more than firm 2 can.

Proof. In the Shaked and Sutton (1982) model the game admits a unique equilibrium, as shown in the Appendix, Section 7.6. In the first stage players select $\theta_2^0 = \underline{\theta}$ and $\theta_1^0 = \frac{1}{2k}\left(\frac{2\overline{\beta}-\underline{\beta}}{3}\right)^2 < \overline{\theta}$. Firm 1's first-order condition in this problem reads $\left(\frac{2\overline{\beta}-\underline{\beta}}{3}\right)^2 - 2k\theta_1^0 = 0$ while that of Problem 1 is $\left(\frac{2\overline{\beta}-\underline{\beta}}{3}\right)^2 - 2k\hat{\theta}_1 = \frac{C}{(3\Delta\theta)^2} > 0$. Therefore $\hat{\theta}_1 < \theta_1^0$. ■

If quality were interpreted as accuracy of reporting, this model implies that it is optimal for newsmedia to under-invest in said accuracy. Thus a contraction of the quality spread need not result from exogenous constraints, nor from outsiders' intervention (interference), but from players' profit-maximising behaviour. This phenomenon resembles that observed in other industries such as software or game development: a widely used operating system need not provide the same intrinsic quality as a more marginal one because it supports so many applications.

Proposition 5 suggests that firms' behaviour depends very essentially on the externality $e(.)$. Things break down when $C(e)$, the sum of marginal advertising profits per unit of quality, becomes too large[13]. Recalling the definitions of θ_1^0, θ_1^f and C^f, we can rearrange $\theta_1^f = \frac{2}{3}\theta_1^0$. Following Assumption 4, there exists some threshold $e < \bar{e}$ and $\theta_1^* \in \left(\frac{2}{3}\theta_1^0, \theta_1^0\right)$. Thus with advertising externalities a pure strategy equilibrium requires more consumer heterogeneity[14] to exist than in the classical Shaked and Sutton (1982) case. In summary,

Corollary 2 ***Taxonomy*** *Let $\theta_1 > \theta_2$ w.l.o.g. and $\frac{\overline{\alpha}}{\underline{\alpha}} \in \left(2, \frac{2D_i^R - D_j^R}{e(D_i^R - D_j^R)}\right)$,*

For $e = 0,\ k > 0$ *The equilibrium is that of Problem 2 (adapted from Shaked*

[13] recall the condition $(\theta_1^* - \underline{\theta})(\overline{\beta} - 2\underline{\beta}) > \sqrt{C}$

[14] a larger difference $\overline{\beta} - \underline{\beta}$

and Sutton (1982)) with both firms operating;

For $e > 0,\ k = 0$ *Maximum differentiation obtains with both firms having positive demand;*

For $\bar{e} > e > 0,\ k > 0$ *The equilibrium is characterised by Proposition 2;*

For $1 > e > \bar{e},\ k > 0$ *Proposition 4 applies.*

Note that the case $e > 0, k = 0$ yields the same differentiation result as Shaked and Sutton (1982) and Gabszewicz, Laussel and Sonnac (2001 and 2002). A positive externality, single-homing and costly quality are necessary to Proposition 5 and its Corollary 2. Intuitively, with free quality, why choose anything but the one that allows the largest surplus extraction from consumers? When the externality is powerful ($e > \bar{e}$)no equilibrium (in pure strategies) exists.

Proof. For lines 1 and 3 the proof follows directly from Propositions 2 and 5, as well as the analysis of $\Pi_1(.,.)$ in Section 7.2. When $k = 0$, because quality is a sunk cost in the original model, nothing is altered until platforms' have to choose their quality variable. That is, the analysis of the third and second stages remains valid. In the first stage, they now face profit functions

$$\Pi_1 = \begin{cases} \frac{1}{9}\left[\Delta\theta(2\overline{\beta} - \underline{\beta})^2 + B_1 + \frac{C}{\Delta\theta}\right], & \text{if } \Delta\theta > \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}} \text{ and ;} \\ \frac{1}{9}\left[\Delta\theta(2\overline{\beta} - \underline{\beta})^2 + B_1 + \frac{C}{\frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}}\right], & \text{if } \Delta\theta \leq \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}. \end{cases}$$

and

$$\Pi_2 = \begin{cases} \frac{1}{9}\left[\Delta\theta(\overline{\beta} - 2\underline{\beta})^2 + B_2 + \frac{C}{\Delta\theta}\right], & \text{if } \Delta\theta > \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}} \text{ and ;} \\ \frac{1}{9}\left[\Delta\theta(\overline{\beta} - 2\underline{\beta})^2 + B_2 + \frac{C}{\frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}}\right], & \text{if } \Delta\theta \leq \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}. \end{cases}$$

The FOC of the first line of Π_1 identifies a minimiser of Π_1. That is, firms will necessarily play the second line. Then we are back to Shaked and Sutton's model of maximum differentiation. ■

5.2 The behaviour of quality

Since $e(D_i^R) = e \times D_i^R$ we continue to parametrise the value of the advertising market for platforms by e, as in Assumption 4. Thus the magnitude of the term $C \equiv (2e(\overline{A} + \underline{A}))^2$ is governed by e only, which can be construed as a measure of market size for the advertisers, for example. When a pure strategy equilibrium exists, we have

Proposition 6 ***Comparative statics***. *Let $\theta_1 > \theta_2$ w.l.o.g. At an equilibrium $\left(\theta^*, \mathbf{p}^{*R}, \mathbf{p}^{*A}\right)$*

a. $\frac{d\theta_2}{de} = 0$, *but* $\frac{d\theta_1}{de} < 0$ *and* $\frac{d^2\theta_1}{de^2} < 0$

b. $\frac{dp_1^A}{de} > \frac{dp_2^A}{de} > 0$ *and* $\frac{dp_1^R}{de} < \frac{dp_2^R}{de} < 0$

c. $\frac{dD_1^R}{de} = -\frac{dD_2^R}{de} > 0$

d. $\frac{d\Pi_1}{de} > 0$ *and* $\frac{d^2\Pi_1}{de^2} < 0$

The effect of e on Π_2 is ambiguous.

Proof. The proof is somewhat lengthy, but straightforward and relegated to the Appendix, Section 7.7. It is necessary to first characterise the behaviour of θ_1 with respect to the parameter e, on which all other results depends. ■

The presence of a second source of revenue not only depresses the quality of the consumer good, it does increasingly so as the advertising market becomes more valuable. Price competition is correspondingly more intense in the consumer market, but less in the advertising market – where differentiation is endogenous. As advertising revenues' weight increases, every consumer becomes more valuable to both platforms, so the consumer discount deepens – and does so faster for the high-quality firm. It finds it easier to enlarge its market share and therefore to become an increasingly better platform for advertisers.

5.3 Properties of the mixed-strategy equilibrium

Although the distributions H_1, H_2 do not lend themselves to easy interpretation, more can be said about the nature of the equilibrium. Here we claim

Proposition 7 *Suppose $e > e^*$. When no platform plays at the lower bound $\underline{\theta}$, the market is necessarily monopolised* ex post.

Thus the dominated firm loses its investment $k\theta_i^2$. The reason is that the length of the interval $\theta^c - \tilde{\theta}$ is not sufficient to accommodate two firms (because $\Delta\theta < \frac{2e(\overline{A}+\underline{A})}{\overline{\beta}-2\underline{\beta}}$). That is, if not trying to be the high-quality medium, the platform should seek maximal differentiation. Of course, whether it obtains depends on its opponent's exact play.

Proof. Please refer to the Appendix, Section 7.8. ■

Interestingly either of monopolisation or the competitive situation may be an *ex post* outcome, which fits industry patterns[15]. In addition *ex post* profits in mixed-strategy case are not monotonically ranked: consider the play $\langle \underline{\theta}_1, \theta_2^c \rangle$, which implies $\Pi_1 > \Pi_2 = 0$ although $\theta_2 > \theta_1$. This result also compares favourably to the industry's idiosyncrasies, where the higher-quality publications or shows do not necessarily yield higher profits.

5.4 Gabszewicz *et al.* (2001 and 2002)

Our results stand in contrast to those obtained by these authors. In their horizontal differentiation setup, Gabszewicz *et al.* (2001) find (mutually exclusive) pure strategy equilibria in which platforms either play at the extrema or converge to the centre. Identical qualities in the present model leads to a mixed-strategy equilibrium, with one firm exiting *ex post* (by Proposition 10 in the Appendix). A pure strategy equilibrium always exists in their setup, thanks to a well-behaved profit function. This rests on both costless quality and 'multi-homing'. The latter results in no price competition in the advertising market, hence each platform acts as a monopolist on its audience – it is a bottleneck to advertisers. Technically no proper subgame is defined at the advertising pricing stage, so the payoff function remains quasi-concave (advertising profits are simply increasing in consumer market share). Therefore the best-response correspondence in the consumer pricing subgame remains continuous. In contrast, 'single-homing' enhances competition for consumers: there is a higher premium to being the dominant platform. Not only does one additional consumer bring more revenue from both sides, it does so at a higher rate for all infra-marginal consumers as well. Had we allowed for 'multi-homing' in this game, the unique equilibrium would also be symmetric. Costly quality necessarily leads to an interior solution and introduces a smooth trade-off between surplus extraction from consumers and extraction from advertisers. For high values of e, this trade-off is extreme, and combined with the asymmetric nature of the equilibrium, leads to the low-quality firm being excluded. In their 2002 paper the authors impose single-homing in a similar model, however keeping differentiation fixed. Unlike in our construct, multiple (asymmetric) equilibria may arise in theirs. When platforms are symmetric at the price-setting stage, playing either of $\overline{p}_2^R\left(p_1^R\right)$ or $\underline{p}_2^R\left(p_1^R\right)$ is perfectly reasonable and does not involve dominated strategies, unlike in the present paper.

[15] Only New York City and Los Angeles have more than one significant newspaper, for example.

5.5 Comparison to the monopoly case

A question of interest is whether competition fosters the provision of quality in this economy. In the externality-free environment, the answer to this question is unambiguously positive: with a uniform distribution it is straightforward to show that the monopolist selects price $p^M = \frac{\theta\overline{\beta}}{2}$ and quality $\theta^M = \frac{1}{2k}\left(\frac{\overline{\beta}}{2}\right)^2$, and it is immediate to verify that $\theta^M < \theta_1^0$ when Assumption 1 holds. Given that quality θ_1^* decreases in equilibrium when the externality e is introduced, it is not *a priori* obvious that this statement remains true in our duopoly. When the medium is a monopolist the profit function for the advertising market writes $\Pi^A = eD\left(\frac{\overline{\alpha}}{2}\right)^2$ (directly from Lemma 1). Total profits are $\Pi^M = p^R\left(\overline{\beta} - \frac{p^R}{\theta}\right) + e\left(\overline{\beta} - \frac{p^R}{\theta}\right)\left(\frac{\overline{\alpha}}{2}\right)^2 - k\theta^2$, whence $p^{R*} = \frac{1}{2}\left(\theta\beta - e(\frac{\overline{\alpha}}{2})^2\right) < p^M$. Substituting we can finally write the monopolist's profit function as $\Pi^M = \frac{1}{4\theta}\left[\theta\overline{\beta} + e\left(\frac{\overline{\alpha}}{2}\right)\right]^2 - k\theta^2$, with first-order condition

$$\left(\frac{\overline{\beta}}{2}\right)^2 - \left(\frac{e}{2\theta}\right)^2\left(\frac{\overline{\alpha}}{2}\right)^4 = 2k\theta \tag{11}$$

which may not necessarily hold for all values of e. Suppose it does, we can define the function $e(\theta) \equiv 2\theta\left\{\left(\frac{2}{\overline{\alpha}}\right)^4\left[\left(\frac{\overline{\beta}}{2}\right)^2 - 2k\theta\right]\right\}^{\frac{1}{2}}$. The maximiser θ^e of this function must satisfy $\frac{\partial e(.)}{\partial\theta} = 0$ (the SOC can easily be verified), and takes value $\theta^e = \frac{\left(\frac{\overline{\beta}}{2}\right)^2\left(\frac{2}{\overline{\alpha}}\right)^4}{k\left[1+2\left(\frac{2}{\overline{\alpha}}\right)^4\right]}$. Thus the largest value of e such that the first-order condition holds is $e^M = 2\left(\frac{2}{\overline{\alpha}}\right)^2\frac{\left(\frac{\overline{\beta}}{2}\right)^2\left(\frac{2}{\overline{\alpha}}\right)^4}{k\left[1+2\left(\frac{2}{\overline{\alpha}}\right)^4\right]}\left\{\left(\frac{\overline{\beta}}{2}\right)^2 - 2\frac{\left(\frac{\overline{\beta}}{2}\right)^2\left(\frac{2}{\overline{\alpha}}\right)^4}{\left[1+2\left(\frac{2}{\overline{\alpha}}\right)^4\right]}\right\}^{\frac{1}{2}}$, which is always positive. From now on we suppose that $e \leq e^M$. The optimality condition (11) admits two solutions; rewrite it as $\left(\frac{\overline{\beta}}{2}\right)^2 = \varphi(\theta)$ with $\varphi'(\theta) = 2k - \left(\frac{\overline{\alpha}}{2}\right)^4\left(\frac{e^2}{2\theta^3}\right)$, which may be positive or negative. Since $\varphi'(\theta) \geq 0$ for the SOC to be satisfied, the maximiser is unique and identified by the larger of the two roots of the first-order condition. Let $\theta^{**} = \arg\max \Pi^M(\theta)$, $\theta^{**} < \theta^M$ immediately from (11). That is, (exogenously) reducing advertising opportunities would increase quality. This claim departs from Anderson (2004), whose (monopoly) broadcasting model shows that a binding cap on advertising quantity leads to *lower* (costly) quality. That is, increasing advertising volumes helps improve quality. In that model, advertising is the only source of revenue: no surplus is extracted from consumers.

In the present paper, consumer revenue and advertising revenue are substitutes. If advertising were capped, the platform would offer consumers a smaller discount and would therefore have to *increase* its quality.

To evaluate quality across all market structures, we have to call on numerical estimations of the optimality conditions (10) and (11). With the parameter values $\overline{\alpha} = \overline{\beta} = 3/2, \underline{\alpha} = \underline{\beta} = 1/2,\ k = 1/4$ they yield the following results

Parameters	θ^{**}	θ^M	θ_1^*	θ_1^0
$e = 1/4$	1.12	$\frac{9}{8}$	1.37	$2\left(\frac{5}{3}\right)^2$
$e = 1/2$	1.09		1.3	
$e = 3/4$	1.04		1.15	
$e = 8/10$	1.03		1.08	

On the proviso that an equilibrium in pure strategies exists, competition uniformly improves quality for a small enough externality. More precisely,

Proposition 8 *Suppose both (10) and (11) hold. There exists some* e^* *such that for* $e < e^*,\ \theta^{**} < \theta^M < \theta_1^* < \theta_1^0$ *and for* $e > e^*,\ \theta^{**} < \theta_1^* < \theta^M < \theta_1^0$.

When e becomes large enough the lure of advertising profits leads the high-quality platform in the competitive environment to drop its quality below the monopoly level θ^M. Thus the introduction of advertising revenue can reduce quality below the level provided even in the most distortionary environment.

5.6 Market coverage

Under Assumption 4, both firms operate and the market is fully covered. More precisely, $e < \bar{e}$ not only ensures that both platforms are viable, but also that full market coverage is the unique equilibrium outcome in the advertising market. Given market coverage in advertising, Assumption 3 guarantees market coverage in the consumer market. Under these restrictions, the formulation of advertising profits (Lemma 1) and of the demand functions D_1^R, D_2^R are valid and represent the only equilibrium configuration that can arise in this game. This is no longer true absent those constraints, as established by Wauthy (1996). He extends the now standard Gabszewicz and Thisse (1979) model (or Shaked and Sutton (1982)), and shows market coverage to be an *endogenous* choice on the part of the firm. With a broader parameter space, the low-quality firm may optimally choose to *not* cover the market, that is, to *not* select the lowest quality possible[16]. The difficulty

[16] obviously some consumers do not purchase in this case

we face in relaxing Assumption 4 is twofold. First, it implies that the profit functions defined by Lemma 1 are no longer the correct ones to use; others (well defined by Wauthy (1996)) have to be called for. In other words, the solution (θ_1^*, θ_2^*) computed in this paper likely bears no resemblance to the new one. Second, the reaction functions $p_i^R(p_j^R)$ become fourth-order polynomials and it is impossible to compute the cut-offs $\hat{p}_i^R(p_j^R)$.

6 Conclusion

This paper has developed an analysis of platform competition when the production of a good is necessary to entice one of the parties onto the platform, and where firms compete on *both* sides of the market. Specifically, players compete for consumers by choice of quality (in the sense of vertical differentiation) and prices. Using the language of the media industry, given an audience that determines their perceived quality by advertisers, they compete in prices in the advertising market. That is, vertical differentiation arises endogenously in the advertising market. Because of the assumption of inelastic demand, audience size not only induces a ranking in the advertising market, but also a *premium* to being the better platform for advertisers (as in the standard vertical differentiation models). This exacerbates the competition for consumers.

In equilibrium maximum differentiation does not obtain, in departure from the standard literature. It is hampered because too costly in terms of market share. Indeed, the opportunity for additional revenue from each consumer renders them more valuable. Platforms therefore engage in cross-subsidisation, and lower prices relax the need for better quality to induce a consumer to purchase the more expensive good. Qualities can come so close to each other that the low-quality platform becomes strictly dominated, at which point the equilibrium breaks down. It would face zero demand, not because of exogenous parameters, but because of firm 1's actions.

Three additional ingredients are necessary to these results: costly quality, here modeled as an investment, an externality from one side of the platform to the other and 'single-homing'. In particular, the externality itself is not sufficient and would lead to maximal differentiation, however with a different distribution of profits. The model lends support to popular claims of reduction in newsroom investment following the takeover of media by professional management. No ingerence is required for this phenomenon to arise: profit-maximising behaviour is sufficient, for it turns quality and advertising first into substitutes.

The results we report owe in part to the simple structure chosen, and in particular to the assumption of complete market coverage. When it is no longer assumed that markets are covered, lower prices resulting from the cross-subsidisation contribute to expanding market size; that is, they improve trade. In this case welfare analysis becomes more sensible. Importantly, media operate in conglomerates and strive to segment consumer markets (using real or perceived correlation between media and commodity consumption) to better serve their advertisers. These important characteristics are so far left out.

7 Appendix

The Appendix contains the lengthier or obvious proofs of the propositions developed in the main text, as well as two propositions that rest on some exogenous restrictions on the players' behaviour.

7.1 Proof of Lemma 4

First construct a candidate equilibrium as follows. Suppose that platform maximise $\Pi_1^H = p_1^R D_1^R(\mathbf{p^R}, \theta) - k\theta_1^2 + \overline{\Pi}_1^A$ and $\Pi_2^H = p_2^R D_2^R(\mathbf{p^R}, \theta) - k\theta_2^2 + \underline{\Pi}_2^A$, respectively. Solving for the first-order conditions laid out in Definition 1 yields

$$\begin{aligned} p_1^{*R} &= \tfrac{1}{3}\left[\Delta\theta\left(2\overline{\beta} - \underline{\beta}\right) + 2e\left(\underline{A} - 2\overline{A}\right)\right] \\ p_2^{*R} &= \tfrac{1}{3}\left[\Delta\theta\left(\overline{\beta} - 2\underline{\beta}\right) + 2e\left(2\underline{A} - \overline{A}\right)\right] \end{aligned}$$

From equilibrium prices it is straightforward to compute consumer demand: $D_1^R = \frac{1}{3}\left[(2\overline{\beta} - \underline{\beta})^2 + \sqrt{C}\right]$ and $D_2^R = \frac{1}{3}\left[(\overline{\beta} - 2\underline{\beta})^2 - \sqrt{C}\right]$, hence the restriction $D_2^R > 0$ provided $\Delta\theta > \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}$ and

$$\begin{aligned} p_1^{*R} &= \tfrac{\Delta\theta\overline{\beta}}{2} - e2\overline{A} \\ p_2^{*R} &= 0 \end{aligned}$$

otherwise. When $D_i^R = 0 \; i = 1, 2$, p_j^R is determined by platform j's reaction correspondence only. Thus it easy to verify that the first line of Condition 1 is satisfied and that $\left(p_1^{*R}, p_2^{*R}\right)$ indeed constitutes an equilibrium by Lemma 3. This equilibrium *always* exists because $\hat{p}_1^R \geq p_1^{*R}$ and $\hat{p}_2^R \leq p_2^{*R}$ are always satisfied. Indeed, either both hold when both platforms are active, for $\Delta\theta\left(\overline{\beta} + \underline{\beta}\right) + e\left(\overline{A} + \underline{A}\right) \geq 0$ is always true, or $p_2^{*R} = 0 > \hat{p}_2^R$ and $\hat{p}_1^R > p_1^{*R}$ can be immediately verified when only firm 1 is active.

Another candidate equilibrium $\left(p_1^{**R}, p_2^{**R}\right)$ can be constructed by letting platform 1 play as if $\Pi_1^L = p_1^R D_1^R(\mathbf{p^R}, \theta) - k\theta_1^2 + \underline{\Pi}_1^A$ and platform 2 as if $\Pi_2^L = p_2^R D_2^R(\mathbf{p^R}, \theta) - k\theta_2^2 + \overline{\Pi}_2^A$, whence

$$\begin{aligned} p_1^{**R} &= \tfrac{1}{3}\left[\Delta\theta\left(2\overline{\beta} - \underline{\beta}\right) + 2e\left(2\underline{A} - \overline{A}\right)\right] \\ p_2^{**R} &= \tfrac{1}{3}\left[\Delta\theta\left(\overline{\beta} - 2\underline{\beta}\right) + 2e\left(\underline{A} - 2\overline{A}\right)\right] \end{aligned}$$

with $D_1^R = \frac{1}{3}\left[(2\overline{\beta} - \underline{\beta})^2 - \sqrt{C}\right]$ and $D_2^R = \frac{1}{3}\left[(\overline{\beta} - 2\underline{\beta})^2 + \sqrt{C}\right]$, therefore $D_1^R > 0$ if $\Delta\theta > \frac{\sqrt{C}}{2\overline{\beta} - \underline{\beta}}$. Notice that an equilibrium such that

$$\begin{aligned} p_1^{*R} &= 0 \\ p_2^{*R} &= -\tfrac{\Delta\theta\underline{\beta}}{2} - e2\overline{A} \end{aligned}$$

cannot exist, for these prices are not best response to each other. At the price setting stage the cost of quality is sunk, so for $\theta_1 > \theta_2$ there always exists some price $p_1^R \geq p_2^R$ such that consumers prefer purchasing from platform 1. Then when both firms are active Condition 1 holds as long as $\Delta\theta\left(\overline{\beta} + \underline{\beta}\right) - e\left(\overline{A} + \underline{A}\right) \leq 0$. Given that $\Delta\theta \geq \frac{\sqrt{C}}{2\overline{\beta} - \underline{\beta}}$, take the lower bound and substitute into the second line of Condition 1. Recalling $\sqrt{C} = 2e(\overline{A} + \underline{A})$,

$$e(\overline{A} + \underline{A})\left(\frac{2(\overline{\beta} + \underline{\beta})}{\overline{\beta} - 2\underline{\beta}} - 1\right) > 0, \ \forall \underline{\beta} \geq 0$$

which violates the second pair of inequalities of the necessary Condition 1. So the second candidate can never be an equilibrium. For completeness, Condition 1 is also sufficient to rule out deviations from the pairs $\left(p_1^{*R}, p_2^{*R}\right)$ and $\left(p_1^{**R}, p_2^{**R}\right)$. The SOC of the profit function (4) is satisfied at prices p_i^{*R} and p_i^{**R} $\forall i, \forall p_{-i}^R$, there cannot be any local deviation. Consider now deviations involving inconsistent actions, that is, such that both platforms maximise either $p_i^R D_i^R(p^R, \theta) - k\theta_i^2 + \overline{\Pi}_i^A$ or $p_i^R D_i^R(p^R, \theta) - k\theta_i^2 + \underline{\Pi}_i^A$. Since $\left(p_1^{*R}, p_2^{*R}\right)$ always exists, the first line of Condition 1 always holds. It immediately follows from (5) and (6) that $\overline{p}_1^R\left(p_2^R\right) \cap \overline{p}_2^R\left(p_1^R\right) = \varnothing$ and $\underline{p}_1^R\left(p_2^R\right) \cap \underline{p}_2^R\left(p_1^R\right) = \varnothing$ as well. ■

7.2 Analysis of the high-quality firm's profit function

In the sequel $\theta_1 > \theta_2$ without loss of generality. The profit function $\Pi_1(.,.)$ is obviously continuous for $\theta_1 < \underline{\theta} + \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}$ or the converse. Furthermore, assume $e < \infty$, then

Claim 1 *The function* Π_1 *is continuous for* $\Delta\theta = \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$

Proof. For ease of notation, let $\Pi_1 = \Pi_1^L$ for all $\Delta\theta \geq \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$ and $\Pi_1 = \Pi_1^R$ otherwise. These are the definitions of $\Pi_1(\theta_1, \underline{\theta})$ to the left and the right of the point such $\Delta\theta = \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$ for any pair (θ_1, θ_2). To the left platform 1 is a monopolist whose profits Π_1^L are necessarily bounded. The function is defined as $\Pi_1^L : \Theta_1 \times \Theta_2 \subseteq \mathbb{R}^2 \mapsto \mathbb{R}$, therefore Theorem 4.5 in Haaser and Sullivan (page 66) applies: a mapping from a metric space into another metric space is continuous if and only if the domain is closed when the range is closed. So $\Pi_1^L(\theta_1, \theta_2)$ is continuous at $\Delta\theta = \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$, and is necessary the left-hand limit of the same function Π_1^L. Now consider a sequence θ_1^n such that $\Delta\theta > \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$ converging to $\frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$ from above for some fixed θ_2 . This sequence exists and always converges for $\Theta_1 \subseteq \mathbb{R}$ is complete. As $e < \infty$ and $\overline{A}$ and $\underline{A}$ are necessarily bounded, C is finite so there is some n and some arbitrarily small δ such that $\Pi_1^R(\theta_1^n, \theta_2) - \Pi_1^L(\theta_2 + \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}, \theta_2) < \delta$. That is, $\lim_{\theta_1^n \to \theta_2 + \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}} \Pi_1^R(\theta_1^n) = \Pi_1^L(\theta_2 + \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}, \theta_2)$. Hence Π_1 is continuous for $\Delta\theta = \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$. ■

$\Pi_1(.,.)$ being the difference of two convex functions its exact shape is affected by that of these two primitives. Indeed, when C becomes large enough, it is no longer well behaved.

Claim 2 *There exists some* C^f *such that* $\Pi_1(.,.)$ *admits a binding first-order condition for* $C \leq C^f$ *only. When* $C > C^f$*, its maximum is reached at the kink:* $\theta_1 = \underline{\theta} + \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$.

Proof. Seeking first-order conditions of $\Pi_1(.,.)$ with respect to θ_1 yields

$$\frac{\partial \Pi_1}{\partial \theta_1} = \begin{cases} \left(\frac{2\overline{\beta}-\underline{\beta}}{3}\right)^2 - 2k\theta_1 = 0, & \text{for } \Delta\theta \leq \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}; \\ \left(\frac{2\overline{\beta}-\underline{\beta}}{3}\right)^2 - \frac{C}{(3\Delta\theta)^2} - 2k\theta_1 = 0, & \text{for } \Delta\theta > \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}} \text{ and } C \leq C^f; \\ \left(\frac{2\overline{\beta}-\underline{\beta}}{3}\right)^2 - \frac{C}{(3\Delta\theta)^2} - 2k\theta_1 < 0, & \text{for } \Delta\theta > \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}} \text{ and } C > C^f; \end{cases} \tag{12}$$

When binding, the second line of system (12) can be rearranged as $\left(2\overline{\beta} - \underline{\beta}\right)^2 = \phi(\theta_1)$, with slope $\phi'(\theta_1) = 18k - \frac{2C}{(\Delta\theta)^3}$. Since $\Delta\theta > 0$, this FOC has at most two solutions: one where $\phi'(\theta_1) < 0$ and the other with $\phi'(\theta_1) > 0$. The SOC requires $\phi'(\theta_1) \geq 0$ for the FOC to identify a maximiser, so there exists a unique local maximiser of Π_1, denoted $\hat{\theta}_1$. Let θ_1^0 be the (unique) maximiser of the first line of system (12). It is immediate that $\hat{\theta}_1 < \theta_1^0$ and consequently $\theta_1^0 - \theta_2 \leq \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$, $\theta_1 \in BR_1(\theta_2)$ can never be true. That is, the two statements of the first line of (12) cannot be simultaneously satisfied: firm 1 would not play the first line of (8), but the second one. We rewrite:

$$\frac{\partial \Pi_1}{\partial \theta_1} = \left(\frac{2\overline{\beta} - \underline{\beta}}{3}\right)^2 - 2k\theta_1 > 0; \quad for\ \Delta\theta \leq \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}$$

Recall that the profit function is continuous, so it does not jump anywhere. Because Π_1 is monotonically increasing below $\hat{\theta}_1$ and the SOC is monotonic beyond $\hat{\theta}_1$, it is concave for $C \leq C^f$ and $\hat{\theta}_1$ is a global maximiser. The binding first-order condition defines a function $C(\theta_1, \theta_2) \equiv (\Delta\theta)^2 \left[(2\overline{\beta} - \underline{\beta})^2 - 18k\theta_1\right]$, whence $\frac{dC(.)}{d\theta_1} = 0 \Leftrightarrow \theta_1^f = \frac{(2\overline{\beta}-\underline{\beta})^2}{27k}$. Substituting back into $C(\theta_1, \theta_2)$ gives the cut-off value $C^f \equiv \left[\frac{(2\overline{\beta}-\underline{\beta})^2}{27k} - \theta_2\right]^2 \left(\frac{(2\overline{\beta}-\underline{\beta})^2}{3}\right)$. When $C > C^f$, the first-order condition (12) is everywhere negative, hence

$$\begin{aligned} &\tfrac{d\Pi_1}{d\theta_1}\big|_{\theta_1 < \underline{\theta} + \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}} > 0 \\ &\tfrac{d\Pi_1}{d\theta_1}\big|_{\theta_1 > \underline{\theta} + \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}} < 0 \end{aligned}$$

While this profit function is not differentiable for $\Delta\theta = \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$, it has been established that it is nonetheless continuous for any such pair (θ_1, θ_2). It is monotonic on either side of $\Delta\theta = \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$, so that $\hat{\theta}_1$ such that $\Delta\theta = \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}$, is the unique maximum of $\Pi_1(\theta_1, \theta_2)$ given some fixed θ_2. ■

Last in this section we examine the behaviour of the quality variable θ_1 when the first-order condition (12) does bind.

Claim 3 *Let $\hat{\theta}_1$ solve $\left(2\overline{\beta} - \underline{\beta}\right)^2 - \frac{C}{(\Delta\theta)^2} - 18k\theta_1 = 0$, then $\frac{d\hat{\theta}_1}{de} < 0$ and $\frac{d\hat{\theta}_1}{dk} < 0$.*

Proof. Differentiate the first-order condition (12); after some manipulations we can write

$$\frac{d\theta_1^*}{de} = \frac{8\Delta\theta e(\overline{A} + \underline{A})}{2\left(2e(\overline{A} + \underline{A})\right)^2 - 18k(\Delta\theta)^3}$$

$\frac{d\theta_1^*}{de} \geq (\leq) 0 \Leftrightarrow 2C - 18k(\Delta\theta)^3 = -(\Delta\theta)^3\phi'(\theta_1)|_{\theta_1=\theta_1^*} \geq (\leq) 0$ so that $\frac{d\theta_1^*}{de} < 0$ (assuming the SOC holding strictly at θ_1^*, otherwise $\frac{d\theta_1^*}{de}$ is not defined and we need to consider the left derivative). The second statement is similar: differentiate the first-order condition of (8) to find $2C(\Delta\theta)^{-3}\frac{d\theta_1}{dk} - 18\theta_1 - 18k\frac{d\theta_1}{dk} = 0$, which is rearranged as $\frac{d\theta_1}{dk} = \frac{18\theta_1(\Delta\theta)^3}{2C-18k(\Delta\theta)^3}$. The denominator is exactly the SOC of (8), which we know to hold, multiplied by $(\Delta\theta)^3$. ■

7.3 Proof of Lemma 5

First off the following simplifies the analysis and lets us focus on platform 1's problem.

Claim 4 *In any pure-strategy Nash equilibrium (θ_1^*, θ_2^*) such that $\theta_1^* > \theta_2^*$, $\theta_2^* = \underline{\theta}$ necessarily.*

Proof. Assume the FOC (12) binds so that $\theta_1^* = \hat{\theta}_1$. Computing the slope of the profit function Π_2 yields

$$\frac{d\Pi_2}{d\theta_2} = \begin{cases} -(\overline{\beta} - 2\underline{\beta})^2 + \frac{C}{(\Delta\theta)^2} - 2k\theta_2 < -2k\theta_2, & \text{if } \Delta\theta(\overline{\beta} - 2\underline{\beta}) > \sqrt{C}; \\ -2k\theta_2, & \text{if } \Delta\theta(\overline{\beta} - 2\underline{\beta}) \leq \sqrt{C}. \end{cases}$$

whence it is immediate that $\frac{d\Pi_2}{d\theta_2}|_{\theta_2 > \underline{\theta}} < \frac{d\Pi_2}{d\theta_2}|_{\underline{\theta}} < 0$. ■

Next delineate an impossibility. When C is said to be 'large' the profit function $\Pi_1(.,.)$ is no longer well behaved, as shown in Section 7.2. This leads to

Lemma 6 *Let $\theta_1 > \theta_2$ w.l.o.g. and $C \geq C^f \equiv \left[\frac{(2\overline{\beta}-\underline{\beta})^2}{27k} - \theta_2\right]^2 \left(\frac{(2\overline{\beta}-\underline{\beta})^2}{3}\right)$, a Nash equilibrium in pure strategies cannot exist.*

Proof. Follows directly from Claims 4 and 2 in Section 7.2. Any pair $\left(\theta_2 + \frac{\sqrt{C}}{\overline{\beta}-2\underline{\beta}}, \theta_2\right)$ cannot be an equilibrium because firm 2 can 'jump' and assume the monopolist's role at incremental cost $k\varepsilon^2$. ■

In line with the previous section of the Appendix, firm 1's first-order condition reads $\left(2\overline{\beta} - \underline{\beta}\right)^2 - \frac{C}{(\Delta\theta)^2} - 18k\theta_1 = 0$ and admits a unique maximiser $\hat{\theta}_1$. This analysis does not yet identify an equilibrium of this game but only platform 1's behaviour, taking that of firm 2 fixed. Suppose firm 1 plays $\hat{\theta}_1$; by Claim 4, platform 2 cannot increase its quality to any $\theta_2 \in \left(\underline{\theta}, \hat{\theta}_1\right)$. So the pair $\left(\hat{\theta}_1, \underline{\theta}\right)$ is an equilibrium as long as firm 2 cannot 'jump' over firm 1 and become the high-quality firm. It will necessarily do so if platform 1 turns out to be a monopolist. To guarantee firm 2 operates we need $(\hat{\theta}_1 - \underline{\theta})(\overline{\beta} - 2\underline{\beta}) > \sqrt{C}$ – Assumption 4 must holds. When firm 2 does operate, the smallest 'leap' it can undertake is such that $\tilde{\theta}_2 \geq \hat{\theta}_1 + \varepsilon$. Hence the no-deviation condition is $\Pi_2\left(\hat{\theta}_1, \underline{\theta}\right) \geq \Pi_2\left(\hat{\theta}_1, \hat{\theta}_1 + \varepsilon\right)$, or

$$\begin{aligned}
(\hat{\theta}_1 - \underline{\theta})(\overline{\beta} - 2\underline{\beta})^2 + B_2 + \tfrac{C}{(\hat{\theta}_1 - \underline{\theta})} \geq &\ B_1 + \sqrt{C}(\overline{\beta} - 2\underline{\beta}) - 9k(\hat{\theta}_1 + \varepsilon)^2 \\
(\hat{\theta}_1 - \underline{\theta})\left[(\overline{\beta} - 2\underline{\beta})^2 + (2\overline{\beta} - \underline{\beta})^2\right] - 18k\hat{\theta}_1^2 + B_2 \geq &\ B_1 + \sqrt{C}(\overline{\beta} - 2\underline{\beta}) - 9k(\hat{\theta}_1 + \varepsilon)^2 \\
(\hat{\theta}_1 - \underline{\theta})\left[(\overline{\beta} - 2\underline{\beta})^2 + (2\overline{\beta} - \underline{\beta})^2\right] - 9k\hat{\theta}_1^2 + B_2 \geq &\ B_1 + \sqrt{C}(\overline{\beta} - 2\underline{\beta})
\end{aligned}$$

using the FOC $(2\overline{\beta} - \underline{\beta})^2 - 18k\hat{\theta}_1 - \frac{C}{(\hat{\theta}_1 - \underline{\theta})^2} = 0$ and the fact that $k\hat{\theta}_1\underline{\theta} = k\underline{\theta}^2 = 0$ (by assumption). Noting $\hat{\theta}_1 - \underline{\theta} > \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}$, this condition is generically satisfied. ■

7.4 Discussion and Proof of Proposition 3

The assertion of Proposition 3 holds trivially by Corollary 1 when Assumption 4 holds. The balance focuses on the case where it fails.

Denote $\tilde{\theta} = \underline{\theta} + \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}$ from now on. As briefly alluded to in the main text, it is not immediate that the game Γ admits a mixed strategy equilibrium, for the payoffs are not everywhere continuous. To see why, first define by θ_1^c the threshold such that $\Pi_1(\theta_1^c, \underline{\theta}) = 0$ when $\theta_1 > \theta_2$. This point exists and exceeds $\tilde{\theta}_1$ because $\frac{d\Pi_1}{d\theta_1}|_{\theta_1 > \tilde{\theta}_1} < 0$ and the cost function is convex. Obviously neither platform will want to exceed that threshold, so we may as well restrict the set of pure actions over which firms randomise to be $[\underline{\theta}, \theta_i^c] \subseteq \Theta_i,\ i = 1, 2$. Next observe that any distribution over this set must assign zero mass to any $\theta_i \in (\underline{\theta}, \tilde{\theta})$ by Claim 4: any action in this interval is dominated by either $\underline{\theta}$ or $\tilde{\theta}$. For $[\tilde{\theta}, \theta_i^c]$ large enough (and $\theta_2 \geq \tilde{\theta}$) there may be outcomes such that $\Delta\theta > \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}$, in which case both platforms are active, or $\Delta\theta \leq \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}$, in which case only the high-quality firm operates. Take $\theta_1 > \theta_2 > \underline{\theta}$ and suppose $\Delta\theta > \frac{\sqrt{C}}{\overline{\beta} - 2\underline{\beta}}$ and $\Pi_1 > \Pi_2 > 0$. Let θ_2 increase, both Π_1 and Π_2 vary smoothly. But while $\lim_{\theta_2^n \uparrow \theta_1} \Pi_1 = \Pi_1 > 0$, $\lim_{\theta_2^n \downarrow \theta_1} \Pi_1 = -k\theta_1^2$, and similarly for firmm 2. Both payoff functions are discontinuous at the point $\theta_1 = \theta_2$. In this case neither the payoffs nor their sum are even upper-hemicontinous. Following Dasgupta and Maskin's (1986a) Theorem 5, it is first necessary to characterise the discontinuity set. If it has Lebesgue measure zero, a mixed strategy equilibrium does exist. Consider the case where $\theta_1 \geq \theta_2$ w.l.o.g. and define $\Upsilon_0 = \left\{(\theta_1, \theta_2) | \theta_1 = \theta_2, \theta_i \in [\tilde{\theta}_i, \theta_i^c] \; \forall i\right\}$, the set on which the payoffs are discontinuous. Further define the probability measure $\mu(\theta_1, \theta_2)$ over the set $\Theta^N = \{\underline{\theta}_1\} \cup [\tilde{\theta}_1, \theta_1^c] \times \{\underline{\theta}_2\} \cup [\tilde{\theta}_2, \theta_2^c]$. It is immediate that Υ_0 has Lebesgue measure zero, so that $\Pr\left((\theta_1, \theta_2) \in \Upsilon_0\right) = 0$. Note that excluding the set Υ_0 is remarkably convenient, for we do not know whether an equilibrium of the price subgame even exists (refer Section 7.9). Through this construct we can side-step this problem entirely. Next we claim

Lemma 7 *Suppose $\theta_1 = \theta_2 = \underline{\theta}$, an equilibrium in mixed strategies exists in the consumer price subgame.*

The reader may recall that existence of a pure strategy Nash equilibrium is ruled out by Proposition 10 when $\theta_1 = \theta_2$. In addition, a mixed strategy equilibrium in prices is not guaranteed to exist for all values of $\theta_1 = \theta_2$. Here it holds because $\theta_1 = \theta_2 = \underline{\theta}$ implies no *ex post* loss for either party. As each platform's payoffs are bounded below at zero and only one of them can operate (except at $p_1^R = p_2^R$), their sum is almost everywhere continuous, except for the set of pairs $(p_1^R = p_2^R)$, which has measure zero.

Proof. Let $\theta_1 = \theta_2 = \underline{\theta}$. The sum of profits $\Pi = \Pi_1 + \Pi_2$ is almost everywhere continuous. Either $\Pi = \Pi_1 > 0 \; \forall p_1^R < p_2^R$, or $\Pi = \Pi_2 > 0 \; \forall p_1^R > p_2^R$, both of which are continuous except at $p_1^R = p_2^R$, where $\Pi = \Pi_1 + \Pi_2 = 0$. But the set $\Psi = \left\{(p_1^R, p_2^R) | p_1^R = p_2^R, (p_1^R, p_2^R) \in \mathbb{R}^2\right\}$ has Lebesgue measure zero. Theorem 5 of Dasgupta and Maskin (1986a) directly applies and guarantees existence of an equilibrium in mixed strategies. ■

Therefore the pair $\theta_1 = \theta_2 = \underline{\theta}$ may be part of an equilibrium of the overall game. Then Proposition 3 asserts that a mixed strategy equilibrium of the game Γ exists.

Proof. We only need showing that the payoff functions $\Pi_i \; i = 1, 2$ are lower-hemicontinuous in their own argument θ_i. Without loss of generality, fix $\theta_1 > \theta_2$. We know that Π_1 is continuous for any $\theta_1 > \theta_2$ (refer Section 7.2). From Claim 4 it is immediate that Π_2 is continuous for $\theta_1 > \theta_2$. Last, for $i = 1, 2$

$$\Pi_i = \begin{cases} 0, & \text{if } \theta_1 = \theta_2 = \underline{\theta}; \\ -k\theta_i^2, & \text{if } \theta_1 = \theta_2 > \underline{\theta}. \end{cases}$$

that is, $\Pi_i, \; i = 1, 2$ is l.h.c. Since $(\theta_2, \theta_1) \, s.t \; \theta_2 = \theta_1 \in \Upsilon_0$, Theorem 5 in Dasgupta and Maskin (1986a) can be applied, whence an equilibrium in mixed strategies must exist. ■

7.5 Proof of Proposition 4

Let θ_i^c denote the upper bound of the support of the distributions, a precise definition of which we will provide later. While Sharkey and Sibley (1993) provide an appealing approach to characterise mixed strategies in a problem of entry with sunk cost, it does not quite apply here. Indeed there is no proper entry stage and the payoffs depend not just on the ranking of the firms' decisions (θ_1, θ_2), but on the difference $\theta_1 - \theta_2$. This implies, in particular, that playing $\theta_i = \underline{\theta}$ cannot be interpreted as a decision to not enter the market because $\Pi_i(\underline{\theta}_i, \theta_j) > 0$ for θ_j such that $\Delta\theta > \frac{2e(\overline{A}+\underline{A})}{\overline{\beta}-2\underline{\beta}}$. Let $H_i(\theta_i)$ be the distribution over $-i$'s payoff realisations for $\theta_i \in \left[\tilde{\theta}, \theta^c\right] \cup \{\underline{\theta}\}$. For any play θ_2, write the expected profit of firm 2 as

$$\begin{aligned} \mathbb{E}_{\theta_1}\left[\Pi_2(\theta_1, \theta_2)\right] = \quad & H(\underline{\theta}_1)\Pi_2(\underline{\theta}_1, \theta_2) + \\ & \int_{\tilde{\theta}_1}^{\theta_1'=\theta_2} \Pi_2(s, \theta_2) dH_1(s) + \int_{\theta_1'=\theta_2}^{\theta_1^c} \Pi_2(s, \theta_2) dH_1(s) \end{aligned} \tag{13}$$

with an atom at $\underline{\theta}_1$. With probability $\int_{\tilde{\theta}_1}^{\theta_1'=\theta_2} dH_1(s)$ it plays θ_1 such that medium 2 is the dominant firm ($\theta_2 \geq \theta_1$). With probability $\int_{\theta_1'=\theta_2}^{\theta_1^c} dH_1(s)$ it is the dominant firm (the second integral). In this latter case, $\Pi_2(\theta_1, \theta_2) = -k\theta_2^2 < 0$. We first claim

Lemma 8 *There is a mass point at $\underline{\theta}_i$. More precisely,*

$$\forall\ i,\ H_i(\underline{\theta}_i) \in (0, 1)$$

Proof. Suppose $H_1(\underline{\theta}_1) = 1$, then $\arg\max \mathbb{E}_{\theta_1}\left[\Pi_2(\underline{\theta}_1, \theta_2)\right] = \tilde{\theta}_2$, so $H_2(\underline{\theta}_2) = 0$ and $H_2(\theta_2)$ assigns full mass at $\tilde{\theta}_2$. But then firm 1 should play some $\theta_1 > \tilde{\theta}_2$ and become the monopolist. If $H_1(\underline{\theta}_1) = 0$ it necessarily plays on $\left[\tilde{\theta}_1, \theta_1^c\right]$ and playing $\tilde{\theta}_2$ is a dominated strategy for firm 2. It therefore assigns no mass at this point. But then $\forall\ \theta_2 \in \left(\tilde{\theta}_2, \theta_2^c\right]$, $\Pi_1(\underline{\theta}_1, \theta_2) > 0$ and platform 1 should shift some mass to $\underline{\theta}_1$. ■

The equilibrium conditions write $\forall \theta_i \in \Theta_i^N$,

$$\begin{aligned} & \mathbb{E}_{\theta_j}\left[\Pi_i(\theta_i, \theta_j)\right] = \Pi_i(\underline{\theta}_i, \tilde{\theta}_j) \\ & \Pi_i(\underline{\theta}_i, \tilde{\theta}_j) = 0 \end{aligned} \tag{14}$$

The first line asserts that the expected payoff cannot be worse than if not investing for sure and the second one that if not investing for sure, a platform can only expect zero profits. Thus expected profits in the mixed-strategy equilibrium must be zero. We next need to determine the upper bound θ_i^c of the support of $H_i(\theta_i)$ for each platform $i = 1, 2$. As a consequence of Lemma 8 it solves either

$$\Pi_i(\underline{\theta}_j, \theta_i^c) = 0$$

or

$$\Pi_i(\tilde{\theta}_j, \theta_i^c) = 0$$

hence

Lemma 9 $\quad \theta_i^c = \max\left\{\theta_i' | \Pi_i(\underline{\theta}_j, \theta_i') = 0, \Pi_i(\tilde{\theta}_j, \theta_i') = 0\right\}$

Proof. Let θ_i' solve $\Pi_i(\underline{\theta}_j, \theta_i') = 0$ and θ_i'' solve $\Pi_i(\tilde{\theta}_j, \theta_i'') = 0$. Suppose $\theta_i' < \theta_i''$ and $\theta_i^c = \theta_i'$: there is a measure $\theta_i'' - \theta_i'$ on which i places zero weight. Then j should shift at least some weight to $\theta_i' + \epsilon$, $\epsilon > 0$ and small, to obtain $\mathbb{E}_{\hat{H}(\theta_i)}[\Pi_j] > 0 = \mathbb{E}_{\theta_i}[\Pi_j(\theta_i, \theta_j)]$ (where $\hat{H}(.)$ is an alternative distribution). Clearly this extends to any $\theta_i \in [\theta_i', \theta_i'')$. ■

Rewriting the equilibrium condition (14)

$$\forall\ \theta_j \in \Theta_j^N,\ H_i(\underline{\theta}_i) R_j(\underline{\theta}_i, \theta_j) + \int_{\tilde{\theta}_i}^{\theta_i' = \theta_j} R_j(s, \theta_j) dH_i(s) = k\theta_j^2$$

where $R_i(\theta_i, \theta_j)$ stands for platform i's revenue (gross of costs). Hence Proposition 4, the proof of which we complete below. **Proof.** Existence is established by Proposition (3). For any play θ_i, total revenue $R_j(\theta_i, \theta_j$ is decreasing in $\theta_j \in \Theta_j^M$ – refer Conditions (8) and (9). Thus for any distribution $H_i(\theta_i)$ the LHS is decreasing while the RHS is strictly increasing. ■

7.6 Elements of Proof of Proposition 5 – unique subgame perfect equilibrium of the Shaked and Sutton model

In the Shaked and Sutton (1982) model there exists a unique equilibrium in the price subgame. In the first stage of the game, firms solve

Problem 2

$$\max_{\theta_i \in \Theta_i}\ p_i^* D_i - k\theta_i^2$$

subject to

$$\theta_{-i} \in \Theta_{-i}^N$$

for $i = 1,2$ and with demand $D_1 = \frac{1}{3}\left(2\overline{\beta} - \underline{\beta}\right)$, $D_2 = \frac{1}{3}\left(\overline{\beta} - 2\underline{\beta}\right)$ and prices $p_1 = \frac{\Delta\theta}{3}\left(2\overline{\beta} - \underline{\beta}\right)$, $p_2 = \frac{\Delta\theta}{3}\left(\overline{\beta} - 2\underline{\beta}\right)$, respectively. This problem is concave $\forall i$, and, given equilibrium prices $p_i^* \ \forall i$, has obvious maximisers $\theta_2^0 = \underline{\theta}$ and $\theta_1^0 = \frac{1}{2k}\left(\frac{2\overline{\beta}-\underline{\beta}}{3}\right)^2$ with $\theta_1^0 < \overline{\theta}$ thanks to $k > \frac{(2\overline{\beta}-\underline{\beta})^2}{18\overline{\theta}}$. These individually optimal maximisers also form a Nash equilibrium, for although $\Pi_1\left(\theta_1^0, \theta_2^0\right) > \Pi_2\left(\theta_1^0, \theta_2^0\right) \ \forall k > 0$ [17], it is also true that

Claim 5 $\nexists\ \tilde{\theta}_2 > \theta_1^0$ *such that* $\Pi_2\left(\theta_1^0, \tilde{\theta}_2\right) \geq \Pi_2\left(\theta_1^0, \theta_2^0\right)$.

Proof. Consider a deviation $\tilde{\theta}_2 = \theta_1^0 + \epsilon$, ϵ arbitrarily small. We can compute firm 2 profit from this deviation as $\Pi_2\left(\theta_1^0, \tilde{\theta}_2\right) = \epsilon\left(\frac{2\overline{\beta}-\underline{\beta}}{3}\right)^2 - k\tilde{\theta}_2^2 < 0$ and the marginal profit $\left(\frac{2\overline{\beta}-\underline{\beta}}{3}\right)^2 - 2k(\theta_1^0 + \epsilon) < 0$. ■

This completes the equilibrium characterisation of the benchmark model.

■

7.7 Proof of Proposition 6

Uniqueness of the subgame-perfect equilibrium renders the comparative statics exercise valid. For the first line, recall that $\theta_2^* = \underline{\theta}$ is a strictly dominant strategy when an equilibrium exists, whence θ_2^* is independent of e. Item (**b**) is stated and proven in Section 7.2. To show concavity, differentiate $\frac{d\theta_1^*}{de}$ once more and rearrange to find

$$\frac{d^2\theta_1^*}{de^2} = \frac{8\left(\overline{A} + \underline{A}\right)}{[-(\Delta\theta)^3\phi']^2}\left[\frac{d\theta_1^*}{de}e\left((\Delta\theta)^3 18k + 2C\right) - \Delta\theta\left(\phi' + 4C\right)\right]$$

Since $\phi' \geq 0$ it is immediate that $\frac{d^2\theta_1^*}{de^2} < 0$. Next (**c**) obtains from the definitions of equilibrium advertising prices together with items (**b**): $\frac{dp_1^A}{de} = \left((\overline{\beta} + \underline{\beta}) + 4e\frac{\Delta\theta - e\frac{d\theta_1^*}{de}}{(\Delta\theta)^2}\right)\sqrt{\overline{A}} > \left((\overline{\beta} + \underline{\beta}) + 4e\frac{\Delta\theta - e\frac{d\theta_1^*}{de}}{(\Delta\theta)^2}\right)\sqrt{\underline{A}} = \frac{dp_2^A}{de} > 0$. Ascertaining the behaviour of consumer prices (**d**) is equally simple: $\frac{dp_1^R}{de} =$

[17] We can readily compute these profits with closed form solutions: $\Pi_1 > \Pi_2 \Leftrightarrow \left(\frac{2\overline{\beta}-\underline{\beta}}{3}\right)^2\left[\frac{1}{2k}\left(\frac{2\overline{\beta}-\underline{\beta}}{3}\right)^2 - \underline{\theta}\right] > \left(\frac{\overline{\beta}-2\underline{\beta}}{3}\right)^2\left[\frac{1}{2k}\left(\frac{2\overline{\beta}-\underline{\beta}}{3}\right)^2 - \underline{\theta}\right]$, which can be re-arranged as $\theta_1^0\left[\frac{1}{2} + \frac{2}{3}\frac{\overline{\beta}\underline{\beta}}{\overline{\beta}^2 - \underline{\beta}^2}\right] > \underline{\theta}$, and always holds

$\frac{1}{3}\left[\frac{d\theta_1^*}{de}(2\overline{\beta}-\underline{\beta})+2(\underline{A}-2\overline{A})\right] < \frac{1}{3}\left[\frac{d\theta_1^*}{de}(\overline{\beta}-2\underline{\beta})+2(2\underline{A}-\overline{A})\right] = \frac{dp_1^R}{de} < 0$. For (**e**), call on the definitions of consumer demands; in particular, we can compute $\frac{dD_1^R}{de} = \frac{2}{3\Delta\theta}(\overline{A}+\underline{A})\left(1-\frac{e\frac{d\theta_1^*}{de}}{\Delta\theta}\right) > 0$, and the rest of the statement is obvious. To show (**f**), differentiate the profit function, which yields

$$\frac{d\Pi_1}{de} = \frac{1}{9}\left[\frac{d\theta_1^*}{de}(2\overline{\beta}-\underline{\beta})^2+\frac{dB_1}{de}+\frac{8e(\overline{A}+\underline{A})^2\Delta\theta - C\frac{d\theta_1^*}{de}}{(\Delta\theta)^2}\right]-2k\theta_1^*\frac{d\theta_1^*}{de}$$

Collecting the terms and recalling the definition of the FOC of (8), this rewrites $\frac{d\Pi_1}{de} = \frac{dB_1}{de}+\frac{8e(\overline{A}+\underline{A})^2}{\Delta\theta} > 0$. That Π_1 is concave is obvious, too. ■

7.8 Proof of Proposition 7

When e is large enough platform 1 (the high-quality firm) prefers playing such that $\Delta\theta = \frac{2e(\overline{A}+\underline{A})}{\overline{\beta}-2\underline{\beta}} \equiv z(e)$ for any θ_2 (and θ_1 not so large as to induce negative profits). Its payoffs when $\Delta\theta \leq z(e)$ are given by the second line of (8), where $B_1(e) = 2e(2\overline{\beta}-\underline{\beta})(2\underline{A}-\overline{A})$. This can re-arranged as

$$\pi_1(e,\theta) = \frac{1}{9}\left[\Delta\theta\left(2\overline{\beta}-\underline{\beta}\right)^2+2e[\underline{A}(5\overline{\beta}-4\underline{\beta})-\overline{A}(\overline{\beta}+\underline{\beta})]\right]-k\theta_1^2$$

for $\Delta\theta \leq z(e)$ and

$$\pi_1(e,\theta) = \frac{1}{9}\left[\Delta\theta\left(2\overline{\beta}-\underline{\beta}\right)^2+B_1(e)+\frac{[2e(\overline{A}+\underline{A})]^2}{\Delta\theta}\right]-k\theta_1^2$$

if $\Delta\theta > z(e)$. Let $\overline{\pi}_1(e,\theta) = \max \pi_1(e,\theta)$ for any pair $\theta_1 > \theta_2$ such that $\Delta\theta = z(e)$. This is an upper bound on firm 1's profits for any play by firm 2. Clearly $\overline{\pi}_1(e,\theta)$ is maximised for $\theta_2 = \underline{\theta}$. Recall that we denote the corresponding value of θ_1 by $\tilde{\theta}_1$. For any e and θ_2, $\frac{\partial\pi_1(e,\theta)}{\partial\theta_1} > 0$ when $\Delta\theta < z(e)$ and $\frac{\partial\overline{\pi}_1(e,\theta)}{\partial\theta_1} < 0$ when $\Delta\theta = z(e)$ and $\theta_2 > \underline{\theta}$. Therefore $\overline{\pi}_1(e,\theta)$ reaches zero for some value $\theta_1' \leq \theta_1^c$. Thus no firm will play out of these bounds. More precisely,

$$\begin{aligned}\frac{\partial\pi_1(e,\theta)}{\partial\theta_1} &= \frac{2\overline{\beta}-\underline{\beta}}{9}-2k\theta_1 > 0, \quad \text{when } \Delta\theta < z(e) \text{ and}\\ \frac{\partial\overline{\pi}_1(e,\theta)}{\partial\theta_1} &= \frac{2\overline{\beta}-\underline{\beta}}{9}-2k\theta_1 < 0, \quad \text{for } \Delta\theta = z(e),\ \theta_2 > \underline{\theta}.\end{aligned}$$

with $\max\frac{\partial\pi_1(e,\theta)}{\partial\theta_1}$ reached for $\theta_2 = \underline{\theta}$. Since $\arg\max\ \Pi_1(\theta_1,\theta_2) > \tilde{\theta}_1$ when $\theta_2 > \underline{\theta}$, it follows that

$$\frac{\partial\pi_1(e,\theta)}{\partial\theta_1} < \left|\frac{\partial\overline{\pi}_1(e,\theta)}{\partial\theta_1}\right|$$

and therefore $\mid \tilde{\theta}_1 - \theta_1^c \mid < z(e)$.

7.9 Analysis and presentation of symmetric equilibria

In this section we study a constrained version of the problem, that of symmetric equilibria. Given the profit function (2), the following result will be useful throughout. Suppose without loss of generality that firm 1 is the high-quality platform for advertisers, that is $e_1 > e_2$.

Lemma 10 *For any quality profile (θ_1, θ_2) and any pair of action (p_1^R, p_2^R), there are three pure strategy equilibria in the advertising market. When $e_2 > 0$,*

$$p_1^A(\theta, p^R) = c + \frac{1 - G(\hat{\alpha})}{g(\hat{\alpha})}\Delta e > p_2^A(\theta, p^R) = c + \frac{G(\hat{\alpha}) - G(\frac{p_2^A}{e_2})}{e_2 g(\hat{\alpha}) - g(\frac{p_2^A}{e_2})\Delta e} e_2 \Delta e$$

When $e_1 > e_2 = 0$, platform 1 is a monopolist. The equilibrium is a pair of prices such that

$$\frac{p_1^{AM} - c}{p_1^{AM}} = \frac{1}{\eta^A} > p_2^A = 0$$

The third equilibrium entails $e_1 = e_2$, whence

$$p_1^A(\theta, p^R) = p_2^A(\theta, p^R) = c$$

Equilibrium prices, quantities and profits are functions of the actions chosen in the consumer market, which are summarised by the externality variable $e(D_i^R)$. In the first equilibrium quantities are $q_1^A(\theta, p^R) = 1 - G(\hat{\alpha}) > q_2^A(\theta, p^R) = G(\hat{\alpha}) - G\left(\frac{p_2^A}{e_2}\right)$ and necessarily advertising profits $\overline{\Pi}_1^A(\theta, p^R) > \underline{\Pi}_2^A(\theta, p^R) > 0$. In the monopoly case, they read $q_1^{AM}(\theta, p^R) = 1 - G\left(\frac{p_1^{AM}}{e_1}\right) > q_2^A = 0$, and obviously $\overline{\Pi}_1^{AM}(\theta, p^R) > \Pi_2^A(\theta, p^R) = 0$. In the symmetric equilibrium, $q_1^A = q_2^A = \frac{1}{2}\left(1 - G\left(\frac{c}{e}\right)\right)$ and necessarily $\Pi_1^A(\theta, p^R) = \Pi_2^A(\theta, p^R) = 0$. The number of equilibria in this subgame follows the number of permutations of the leadership role.

Proof. The symmetric equilibrium obtains directly from Bertrand competition when $e_1 = e_2$. Suppose $e_1 > e_2 = 0$ – for whatever reason. Platform 2 has no customers in the advertising market since $v_2 \leq 0$ and therefore $q_2^A = 0$. Consequently firm 1 optimally prices in the advertising market using the monopoly pricing rule

$$\frac{p_1^A - c}{p_1^A} = \frac{1}{\eta^A}$$

where η^A denote the price elasticity of demand. The reverse obtains when $e_2 > e_1 = 0$. In an equilibrium where both firms are active and $e_1 > e_2 > 0$ we necessarily have $p_1^A > p_2^A$. If not, $\alpha e_1 - p_1^A > \alpha e_2 - p_2^A$ for any α and $q_2^A = 0$. We also have

Claim 6 *In equilibrium,* $1 - G(\hat{\alpha}) > G(\hat{\alpha}) - G\left(\frac{p_2^A}{e_2}\right)$.

Proof. By contradiction. Suppose that in equilibrium (at prices $\hat{p}_i^A$, $i = 1, 2$), $1 - G(\hat{\alpha}) \leq G(\hat{\alpha}) - G\left(\frac{\hat{p}_2^A}{e_2}\right)$. The necessary first-order conditions for both firms imply

$$\frac{g(\hat{\alpha})}{\Delta e}(\hat{p}_1^A - c) \leq \left(\frac{g(\hat{\alpha})}{\Delta e} - \frac{g\left(\frac{\hat{p}_2^A}{e_2}\right)}{e_2}\right)(\hat{p}_2^A - c)$$

which is obviously impossible when $\hat{p}_1^A > \hat{p}_2^A$. ■

The fact that $\overline{\Pi}_1^A > \underline{\Pi}_2^A$ follows directly. Furthermore, $\Pi_2^A > 0$ since $\hat{p}_2^A > c$ and firm 2 faces strictly positive demand. This concludes the proof of the Lemma. ■

7.9.1 Symmetric equilibria

This section presents a fairly intuitive result when platforms are constrained in their actions. Such constraints may owe to technological limitations, as in Proposition 9, below or be somewhat more arbitrary (but nonetheless plausible), as in Proposition 10, further. Both are impossibility results, the first one resting on quite a familiar empirical observation.

Proposition 9 *Fix* $p_1^R = p_2^R = 0$. *A pure strategy equilibrium does not exist. If a mixed-strategy equilibrium exists, platforms set their advertising price as if each were a monopolist.*

The conditions of this proposition and the results are a stylised version of the problem faced by free-to-air broadcasters, where access to the good cannot be controlled by the providers. Proposition 9 helps rationalising the giving away of free goods by media platforms that are naturally constrained in their pricing (for example local radio stations).

Proof. Given $p_1^R = p_2^R = 0$, consumer demand is given by

$$D_i^R = \begin{cases} 1, & \text{if } \theta_i > \theta_j; \\ \frac{1}{2}, & \text{if } \theta_i = \theta_j; \text{ and} \\ 0, & \text{if } \theta_i < \theta_j. \end{cases}$$

for $i \neq j,\ i = 1, 2$, whence platform i faces payoffs

$$\Pi_i = \begin{cases} \Pi^{AM} - k(\theta_i), & \text{if } \theta_i > \theta_j; \\ -k(\theta_i), & \text{if } \theta_i = \theta_j; \\ 0, & \text{if } 0 = \theta_i < \theta_j; \text{ or} \\ -k(\theta_i), & \text{if } \underline{\theta} < \theta_i < \theta_j. \end{cases}$$

following directly from Lemma 10, and where Π^{AM} denotes monopoly profits in the advertising market when demand in the consumer market is $D_i^R(0, \theta_i) = 1$. In particular, suppose $\theta_i = \theta_j$ such that $\Pi_i^A(e, \theta_i) \geq 0$ and one of the platforms charges $k(\theta_i)$, j can offer $k(\theta_i) - \epsilon$ and become a monopolist in advertising. Thus

Claim 7 *When consumer prices are identical a pure strategy equilibrium cannot exist.*

Proof. Given $p_i^R = p_j^R = 0$, $\theta_i > \theta_j$ implies that i is a monopolist and j faces negative payoffs for $\theta_j > \underline{\theta}$. Suppose $\theta_i = \theta_j$; then $e_i = e_j$ and any price advertising price $k(\theta_i)$ leading to $\Pi_i^A(e, \theta_i) \geq 0$ is dominated by $k(\theta_i) - \epsilon$, and j is a monopolist. There exists some quality level $\tilde{\theta}$, such that $\Pi_i^{AM}(\tilde{\theta}_i, p_i^A) = 0$. At this point firm j should not enter, but then i deviate slightly to $\tilde{\theta}_i - \epsilon$ and make a positive profit. Hence no pure strategy is deviation proof. ■

A symmetric equilibrium in mixed strategies of this game is a profile $\left(\sigma^A, \sigma^\theta\right)$ of distributions over price and quality parameter $\left(p_i^A, \theta_i\right)_{i=1,2}$. Since no dominated strategy can be part of a mixed strategy equilibrium, the set of pure strategies over which platforms randomise can be restricted to the subsets $\left[\underline{\theta}, \tilde{\theta}\right]$, where $\tilde{\theta} = k^{-1}\left(\Pi^{AM}\right)$, and p_i^A as defined in Lemma 10. It is easier to first deal with the advertising market.

Claim 8 ***Irrelevance.*** *In any symmetric, mixed-strategy equilibrium platforms will behave as strict monopolists in the advertising market.*

Proof. The proof follows directly from the fact that firms necessarily play a mixed strategy equilibrium. Since $\theta_i \in \left[\underline{\theta}, \tilde{\theta}_i\right]\ \forall i$, each event has probability zero and one platform will be a monopolist in the advertising market with certainty. It will therefore not face competition in this market and behave regardless of what its opponent does. ■

Thus σ^A is necessarily degenerate and the only source of uncertainty is the choice of quality. It is not the object of this paper to characterise σ^θ, the existence of which is not certain. ■

Given symmetric consumer prices, platforms can only choose symmetric strategies in the remaining variables: they lose an instrument to compete and must follow the Bertrand logic of competition, however in quality. But this costly quality is a sunk cost by the time the pricing decision must be made in the advertising market. Therefore marginal-cost pricing in advertising cannot include the cost of producing the information good, which they also fail to recover from consumers. Symmetric qualities necessarily lead to the Bertrand result in advertising, whence the quality investment in the consumer market is necessarily too costly. However, if an equilibrium can be sustained in mixed strategies, players can foresee that only one of them will remain. So they select their distributions anticipating the monopoly rent in the advertising market. The behaviour of the opponent becomes irrelevant; it is *as if* there were no competition in the advertising market[18]. More precisely, the restrictions imposed on the game necessarily lead players to a subgame where only one of them will survive.

Next we lay out a proposition analysing the price subgame when quality θ_i is (arbitrarily) restricted to be symmetric. This kills differentiation and leads to an outcome that somewhat mirrors Bertrand competition. Here it takes an extreme form owing to the positive externality afforded by the advertising market: firms' competition is intensified in the consumer market. Duopolists each maximise the profit function (2) by choice of prices $\left\{p_i^R, p_i^A\right\}_{i=1,2}$, in the last two stages, given some fixed quality level. We can think of this situation as a degenerate version of the game without the quality-setting stage, or as some case where quality turns out to be symmetric in the first stage.

Proposition 10 *Fix $\theta_i = \theta_j$. An equilibrium in pure strategies in the price subgame does not exist. If a mixed-strategy equilibrium exists, platforms set prices as if each were a monopolist in the advertising market.*

If platforms are symmetric in the consumer market (identical quality and prices), they necessarily are so in the advertising market as well. But with costly quality, there is a limit to incurring losses in the consumer market: they may not be covered by advertising profits. Thanks to randomisation, it is optimal for firms to price as if they were a monopolist in the advertising market: indeed, either they are a monopolist, or they are absent altogether.

[18] This result is extremely sensitive to the choice of cost function. With marginal cost increasing in quality – as opposed to an investment cost– the sunk-cost problem vanishes. Instead, marginal-cost pricing in advertising can include the marginal cost of quality. Thus it is *as if* platforms where setting prices to marginal cost on *both* sides of the market. The externality is then neutralised and we refer to this phenomenon as *independence*.

Irrelevance strikes again: competition in the advertising market is ignored at the consumer price-setting stage.

Proof. Using the result of Lemma 10, given a profile (e_1, e_2) and assuming a covered market, the payoffs in the advertising market read

$$(\Pi_1^A, \Pi_2^A) = \begin{cases} (\overline{\Pi}_1^A, \underline{\Pi}_2^A), & \text{if } e_1 > e_2; \\ (\underline{\Pi}_1^A, \overline{\Pi}_2^A), & \text{if } e_1 < e_2; \\ (0, 0), & \text{if } e_1 = e_2; \\ (\Pi_1^{AM}, 0), & \text{if } e_1 > e_2 = 0; \\ (0, \Pi_2^{AM}), & \text{if } 0 = e_1 < e_2. \end{cases}$$

where Π_i^{AM} denotes monopoly profits when firm j reaches no consumer. This can arise when $\theta_i = \theta_j$ but $p_i^R < p_j^R$, for then $e_j = 0$. For any firm i, total profits are

$$\Pi_1 = p_i^R D_i^R \left(\theta, p^R\right) - k(\theta_i) + \begin{cases} \overline{\Pi}_i^A; \\ \underline{\Pi}_i^A; \\ 0; \\ \Pi_i^{AM}. \end{cases}$$

Whoever ends up with a larger market share in the consumer market necessarily dominates in the advertising market.

Claim 9 *When $\theta_1 = \theta_2$, a pure strategy equilibrium cannot exist.*

Proof. Note that for any $\theta_1 = \theta_2$ in the consumer market any firm playing $p_j^R > p_i^R$ surrenders a monopoly position in the advertising market. It is obvious that any price $p_i^R \geq \frac{k(\theta_i)}{\frac{1}{2}D_i^R}$, $\forall i$ is dominated by $\frac{k(\theta_i)}{\frac{1}{2}D_i^R} - \epsilon$, for then j becomes a monopolist in the advertising market at the cost ϵ. Suppose $\theta_1 = \theta_2 = \underline{\theta}$. Both engage in this form of Bertrand competition for the monopoly privilege in advertising until reaching some price $\underline{p}_i^R$ such that $\Pi_1 = \underline{p}_i^R D_i^R(\theta, \mathbf{p}^R) + \Pi_i^{AM} = 0 \ \forall i$. At this point, j should play some $p_j^R > \underline{p}_i^R$ and stay out, which is costless. But then $p_i^R = p_j^R - \epsilon > \underline{p}_i^R$ becomes a best response. Thus there is no pure-strategy equilibrium. If $\theta_1 = \theta_2 > \underline{\theta}$, $\exists \ \underline{p}_i^R \ \ni \ \Pi_1 = \underline{p}_i^R D_i^R(\theta, \mathbf{p}^R) - k(\theta) + \Pi_i^{AM} = 0 \ \forall i$ as well. But at that point, either $\underline{p}_i^R < p_j^R$ and j realises $\Pi_j = -k(\theta) < 0$, or $\underline{p}_i^R = p_j^R$ and $\Pi_i = \Pi_j = -k(\theta) < 0$ since Bertrand competition prevails in advertising as well. So j has a strict incentive to plays $p_j^R < \underline{p}_i^R$. Following this logic, $\exists \ \check{p}_i^R \ \ni \ \Pi_1 = \check{p}_i^R D_i^R(\theta, \mathbf{p}^R) - k(\theta) + \Pi_i^{AM} = -k(\theta) \ \forall i$. If j plays $p_j^R = \check{p}_i^R$, $\Pi_i = \Pi_j < -k(\theta)$ while for any $p_j^R > \check{p}_i^R$, $\Pi_i(p_j^R, \check{p}_i^R) = \Pi_j(p_j^R, \check{p}_i^R) = -k(\theta)$.

But then again $p_i^R = p_j^R - \epsilon > \check{p}_i^R$ becomes a best response. Therefore no pure-strategy can exist. ■

Irrelevance also works here, of course. An equilibrium in mixed strategies of this game is a profile $\left(\sigma^R, \sigma^A\right)$ of distributions over prices $\left(p_i^R, p_i^A\right)_{i=1,2}$, where σ^A is degenerate. The only source of uncertainty affecting the payoffs is the choice of consumer price p_i^R. ■

8 References

Anderson, S. (2004) "Regulation of television advertising", *mimeo*, The University of Virginia.

Anderson, S. and John McLaren (2004) "Media mergers and media bias", *mimeo*, The University of Virginia.

Anderson, S. and Stephen Coate (2005) "Market provision of broadcasting: a welfare analysis", *mimeo*, The University of Virginia.

Armstrong, M. (2004) "Competing in two-sided markets", *mimeo*, UCL.

Crampes, C, Carole Harichatbalet and Bruno Jullien (2005) "Competing with advertising revenue", *mimeo* IDEI, Toulouse.

Ellmand M. and Fabrizio Germano (2005) "What do newspapers sell?" *mimeo*, Universitat Pompeu Fabra.

Ferrando, J. Jean-J. Gabszewicz, Didier Laussel and Nathalie Sonnac, (2003) "Intermarket network externalities and competition: an application to the media industry.", *mimeo*

Gabszewicz, J.-J. Didier Laussel and Nathalie Sonnac, (2001) " Press advertising and the ascent of of the pensée unique", *European Economic Review*, 45.

Gabszewicz, J.-J. Didier Laussel and Nathalie Sonnac, (2002) " Concentration in the press industry and the theory of the circulation spiral", *mimeo*.

Gabszewicz, J.-J. and Xavier Wauthy (2006) "Two-sided markets and price competition with multi-homing", *mimeo*, Universite Catholique de Louvain.

Hasser, N. and Joseph Sullivan (1991) "Real Analysis" Dover Publications, New York.

Laffont, J.-J, Scott Marcus, Patrick Rey and Jean Tirole (2004) "The off-net-cost pricing principle", *Rand Journal of Economics*.

Owen, B. and Michael Spence (1977) "Television programming, monopolistic competition and welfare", *The Quarterly Journal of Economics*, Vol 91, No. 1.

Rochet, J.-C. and Jean Tirole (2002) “Platform competition in two-sided markets”, *mimeo*, IDEI, Toulouse.

Rochet, J.-C. and Jean Tirole (2004) “Two-sided markets: an overview”, *mimeo*, IDEI, Toulouse.

Shaked, A. and John Sutton (1982) “Relaxing price competition through product differentiation”, *The Review of Economic Studies.*

Shaked, A. and John Sutton (1983) “Natural Oligopolies”, *Econometrica.*

Shaked, A. and John Sutton (1987) “Product differentiation and industrial structure”, *The Journal of Industrial Economics.*

Strömberg, D (2004) “Mass media competition, political competition and public policy”, *The Review of Economic Studies.*

Tirole, J. (1988) “The Theory of Industrial Organisation”, MIT Press.

Wauthy, X. (1996) “Quality choice in models of vertical differentiation”, *The Journal of Industrial Economics.*