

# Offshoring and Unemployment: The Role of Search Frictions and Labor Mobility\*

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## Abstract

In a two-sector, general-equilibrium model with labor-market search frictions, we find that wage increases and sectoral unemployment decreases upon offshoring in the presence of perfect intersectoral labor mobility. If, as a result, labor moves to the sector with the lower (or equal) vacancy costs, there is an unambiguous decrease in economywide unemployment. With imperfect intersectoral labor mobility, unemployment in the offshoring sector can rise, with an unambiguous unemployment reduction in the non-offshoring sector. Imperfect labor mobility can result in a mixed equilibrium in which only some firms in the industry offshore, with unemployment in this sector rising.

Keywords: Trade, Offshoring, Search, Unemployment

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# 1 Introduction

"Offshoring" is the sourcing of inputs (goods and services) from foreign countries. When production of these inputs moves to foreign countries, the fear at home is that jobs will be lost and unemployment will rise. In the recent past, this has become an important political issue. The remarks by Greg Mankiw, when he was Head of the President's Council of Economic Advisers, that "outsourcing is just a new way of doing international trade" and is "a good thing" came under sharp attack from prominent politicians from both sides of the aisle. Recent estimates by Forrester Research of job losses due to offshoring equaling a total of 3.3 million white collar jobs by 2015 and the prediction by Deloitte Research of the outsourcing of 2 million financial sector jobs by the year 2009 have drawn a lot of attention from politicians and journalists (Drezner, 2004), even though these job losses are only a small fraction of the total number unemployed, especially when we take into account the fact that these losses will be spread over many years.<sup>1</sup> Furthermore, statements by IT executives have added fuel to this fire. One such statement was made by an IBM executive who said "[Globalization] means shifting a lot of jobs, opening a lot of locations in places we had never dreamt of before, going where there is low-cost labor, low-cost competition, shifting jobs offshore", while another statement was made by Hewlett-Packard CEO Carly Fiorina in her testimony before Congress that "there is no job that is America's God-given right anymore" (Drezner, 2004). The alarming estimates by Bardhan and Kroll (2003) and McKinsey (2005) that 11 percent of our jobs are potentially at risk of being offshored have provided anti-offshoring politicians with more ammunition for their position on this issue.

While the relation between offshoring and unemployment has been an important issue for politicians, the media and the public, there has hardly been any careful theoretical analysis of this relationship by economists. In this paper, in order to study the impact of offshoring on sectoral and economywide rates of unemployment, we construct a two-sector, general-equilibrium model in which unemployment is caused by search frictions a la Pissarides (2000).<sup>2</sup> There is a single factor of production, labor. Firms in one sector, called sector  $Z$ , use labor to produce two inputs which are then assembled into output. The production of one of these inputs (production input) can be offshored, but the other input (headquarter services) must be produced using domestic labor only. There is another sector,  $X$ , that uses only domestic labor to produce its output. Goods  $Z$  and  $X$  are combined to produce the consumption good  $C$ .

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<sup>1</sup>The average number of gross job losses per week in the US is about 500,000 (Blinder, 2006). Also see Bhagwati, Panagariya and Srinivasan (2004) on the plausibility and magnitudes of available estimates of the unemployment effects of offshoring.

<sup>2</sup>For a comprehensive survey of the search-theoretic literature on unemployment, see Rogerson, Shimer and Wright (2005).

An important result of this paper is that in the presence of perfect intersectoral labor mobility, offshoring leads to wage increases and unemployment reductions in both sectors. The very basic intuition is that there will be gains from international trade which in this case takes the form of offshoring. In a truly single-factor model, this would mean that this factor of production gains from trade, and that explains why, when labor is intersectorally perfectly mobile, real wage increases and unemployment declines. When there are impediments to intersectoral labor mobility, it is possible for unemployment to increase in the  $Z$  sector (offshoring sector), however, unemployment in the  $X$  sector must decrease. The very basic intuition is that with impediments to labor mobility, we are effectively moving away from a one-sector model. Thus, even with overall gains from trade, we can have winners and losers. When labor is totally immobile across sectors, in our set up we truly have a two-factor model, and both factors need not necessarily be winners from offshoring (trade). Since offshoring is similar to a technological improvement in in the  $Z$  sector, the relative supply of  $Z$  increases and its relative price falls as a result (the relative price of  $X$  rises). Given that  $X$ -sector labor has to win from trade due to the positive relative price effect in its favor, the only possible loser, if at all there is one, is  $Z$ -sector labor.

Moving from the very basic to more detailed intuition, offshoring reduces the cost of production and hence the relative price of good  $Z$ , since one of the inputs is offshored and is cheaper. The resulting increase in the relative price of the non-offshoring sector  $X$  leads to greater job creation and hence reduced unemployment there. The impact of offshoring on  $Z$ -sector unemployment depends on the relative strengths of two mutually opposing forces, namely the decrease in the relative price of  $Z$ , and the increase in the marginal product of workers engaged in headquarter activities there (each such worker now working with more production input, since it is cheaper). In the presence of perfect labor mobility, the no arbitrage condition ensures that the second effect dominates and that increases job creation and wages in sector  $Z$ . Otherwise, labor would keep flowing out of this sector. Even though offshoring of the production input destroys the jobs of workers engaged in the production of this input in the  $Z$  sector, additional  $Z$ -sector headquarter jobs and  $X$ -sector jobs, in excess of the production jobs offshored, are created.

In the imperfect labor mobility case, it is possible in the  $Z$  sector for the negative relative price effect to dominate the positive productivity effect. Among many factors, the net effect will also depend on the per-unit cost post offshoring of the input that has been offshored. If this cost is low enough, we get complete offshoring of the production of the offshorable input. The reason is that in this case the domestic demand for good  $Z$  is very high. Therefore, all domestic employment in that sector has to be used in headquarter activity to be combined with a large amount of the cheap imported input (whose production has been offshored). For

low enough cost of offshoring, we get an increase in wage and a reduction in unemployment in the  $Z$  sector, as a very large amount of the imported input per unit of headquarter labor yields a very high marginal product of domestic headquarter labor in that sector.

At relatively high costs of the offshored input (even though lower than the autarky cost of producing that input at home), we get incomplete offshoring (mixed equilibrium) where some firms offshore and others do not. Wages in the  $Z$  sector are lower and unemployment there higher as compared to autarky. The intuition here is as follows: At these levels of offshoring costs, the price of  $Z$  is not low enough for the quantity demanded to call for complete offshoring. Firms in the incomplete offshoring case are indifferent in the offshoring equilibrium between offshoring and not offshoring. This means that the domestic cost of producing the offshorable input gets equalized to the cost of the imported offshored input, which brings the domestic wage down and the unemployment up relative to autarky. Incomplete offshoring makes domestic labor and the imported input perfect substitutes at the margin. This channel of competitive pressure on the domestic price of labor goes away when there is complete offshoring. Thus, the relationship between offshoring costs and sectoral unemployment (and wage) in sector  $Z$  is non-monotonic in the imperfect mobility case.

The impact of offshoring on overall economywide unemployment (even though sectoral unemployment rates fall in both sectors) also depends on how the structure or the composition of the economy changes. The exception is the case where, in addition to perfect intersectoral labor mobility, the search costs and hence unemployment rates (and the reduction in them upon offshoring) in the two sectors are identical and equal to the overall economywide unemployment rate (and its reduction). Now maintaining perfect labor mobility, when search costs (and therefore equilibrium unemployment rates) are made unequal across the two sectors, whether the overall unemployment rate goes up or down will also depend on which sector's share in the economy's labor force expands. This is true even though we have unambiguously falling sectoral unemployment rates. Since the employment share of the offshoring sector (sector  $Z$ ) falls upon offshoring for most of the parameter space, aggregate unemployment for those parameter values will fall if this sector has the higher search cost (higher unemployment rate). Obviously, there is ambiguity in the opposite case, where the search cost is smaller in the  $Z$  sector.

Our theoretical results are consistent with the empirical results of Amiti and Wei (2005a, b) for the US and the UK. They find no support for the "anxiety" of "massive job losses" associated with offshore

outsourcing from developed to developing countries.<sup>3</sup> Using data on 78 sectors in the UK for the period 1992-2001, they find no evidence in support of a negative relationship between employment and outsourcing. In fact, in many of their specifications the relationship is positive. In the US case, they find a very small, negative effect of offshoring on employment if the economy is decomposed into 450 narrowly defined sectors which disappears when one looks at more broadly defined 96 sectors. Alongside this result, they also find a positive relationship between offshoring and productivity. These results are consistent with opposing effects on employment (and unemployment) created by offshoring. In this context, Amiti and Wei (2005a) write: “On the one hand, every job lost is a job lost. On the other hand, firms that have outsourced may become more efficient and expand employment in other lines of work. If firms relocate their relatively inefficient parts of the production process to another country, where they can be produced more cheaply, they can expand their output in production for which they have comparative advantage. These productivity benefits can translate into lower prices generating further demand and hence create more jobs. This job creation effect could in principle offset job losses due to outsourcing.” This intuition is consistent with the channels in our model and the reason for obtaining a result that shows a reduction in sectoral and overall unemployment as a result of offshoring.

A discussion of the related theoretical literature is useful here, as it puts in perspective the need for our analysis. While the relationship between offshoring and unemployment has not been analytically studied before by economists, there is now a vast literature on offshoring and outsourcing.<sup>4</sup> All the models in that literature, following the tradition in standard trade theory, assume full employment. In spite of this assumption in the existing literature, it is important to note that our results are similar in spirit to those in an important recent contribution by Grossman and Rossi-Hansberg (2008) where they model offshoring as "trading in tasks" and show that even factors of production whose tasks are offshored can benefit from offshoring due to its productivity enhancing effect. Our paper is also closely related to the fragmentation literature which analyzes the economic effects of breaking down the production process into different components, some of which can be moved abroad.<sup>5</sup> In this literature, the possibility of fragmentation leading to the equivalent of technological improvement in an industry has been shown.<sup>6</sup>

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<sup>3</sup>The offshoring variable they use, which they call offshoring intensity, is defined as the share of imported inputs (material or service) as a proportion of total nonenergy inputs used by the industry.

<sup>4</sup>See Helpman (2006) for a review of this literature.

<sup>5</sup>See for instance Arndt (1997), Jones and Kierzkowski (1990 and 2001) and Deardorff (2001a and b).

<sup>6</sup>See for instance Jones and Kierzkowski (2001).

Also closely related to our work is a very recent working paper by Davidson, Matusz and Shevchenko (2006) that uses a model of job search to study the impact of offshoring of high-tech jobs on low and high-skilled workers' wages, and on overall welfare. Another paper looking at the impact of offshoring on the labor market is Karabay and McLaren (2006) who study the effects of free trade and offshore outsourcing on wage volatility and worker welfare in a model where risk sharing takes place through employment relationships. Bhagwati, Panagariya and Srinivasan (2004) also analyze in detail the welfare and wage effects of offshoring.

It is also important to note that there does exist a literature on the relationship between trade and search induced unemployment (e.g. Davidson and Matusz (2004), Moore and Ranjan (2005), Helpman and Itskhoki (2007)). The main focus of this literature, as discussed in Davidson and Matusz, has been the role of efficiency in job search, the rate of job destruction and the rate of job turnover in the determination of comparative advantage.<sup>7</sup> Using an imperfectly competitive set up, Helpman and Itskhoki look at how gains from trade and comparative advantage depend on labor market rigidities as captured by search and firing costs and unemployment benefits, and how labor-market policies in a country affect its trading partner. Moore and Ranjan, whose focus is quite different from the rest of the literature on trade and search unemployment, show that the impact of skill-biased technological change on unemployment can be quite different from that of globalization. None of these models deals with offshoring.

## 2 The Model

### 2.1 Preferences

All agents share the identical lifetime utility function from consumption given by

$$\int_t^\infty \exp^{-r(s-t)} C(s) ds, \tag{1}$$

where  $C$  is consumption,  $r$  is the discount rate, and  $s$  is a time index. Asset markets are complete. The form of the utility function implies that the risk-free interest rate, in terms of consumption, equals  $r$ .

Each worker has one unit of labor to devote to market activities at every instant of time. The total size of the workforce is  $L$ . The final consumption good  $C$  is produced under CRS using two goods  $Z$  and  $X$  as

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<sup>7</sup>See also the influential and well cited paper by Davidson, Martin and Matusz (1999) for a careful analysis of these relationships under very general conditions.

inputs (or equivalently can be considered to be a composite basket of these two goods) as follows:

$$C = F(Z, X) \tag{2}$$

We choose the final consumption good  $C$  as numeraire. Let  $P_z$  and  $P_x$  be the prices of  $Z$  and  $X$ , respectively. Since the price of  $C = 1$ , we get

$$1 = g(P_z, P_x) \tag{3}$$

where  $g$  is increasing in both  $P_z$  and  $P_x$ . Therefore, an increase in  $P_z$  implies a decrease in  $P_x$ . Also, (2) implies that the relative demand for  $Z$  is given by

$$\left(\frac{Z}{X}\right)^d = f\left(\frac{P_z}{P_x}\right); f' < 0 \tag{4}$$

In addition to the utility from consumption, workers also have idiosyncratic preferences for working in a particular sector which is captured by a per-period non-pecuniary utility (or disutility) to individual- $j$  of  $\varepsilon_i^j$  from being part of the labor force in sector- $i$ .<sup>8</sup> This can arise from individual-specific preference for the region in which this industry is located or from the individual specific costs of updating one's human capital that may differ across sectors.<sup>9</sup> Define  $\varphi^j \equiv \varepsilon_z^j - \varepsilon_x^j$ . The distribution function of  $\varphi$  is denoted by  $G(\varphi)$ . This is our way of introducing mobility cost in the model. If  $\varphi^j > 0$ , then  $\varphi^j$  is the cost to worker  $j$  of moving from sector  $Z$  to sector  $X$ . Similarly, if  $\varphi^j < 0$ , it is costly for worker  $j$  to move from sector  $X$  to sector  $Z$ .  $\varphi^j = 0, \forall j$ , will capture perfect mobility.

## 2.2 Goods and labor markets

Production of good  $X$  is undertaken by perfectly competitive firms. To produce one unit of  $X$  a firm needs to hire one unit of labor.

$Z$  is also produced by competitive firms, but using a slightly more sophisticated technology involving two separate stages which are then combined. The production function for  $Z$  is given as follows.

$$Z = (\tau m_h^\rho + (1 - \tau)m_p^\rho)^{\frac{1}{\rho}} \tag{5}$$

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<sup>8</sup>In the case of an extra utility,  $\varepsilon_i^j > 0$ , while in the case of an extra disutility,  $\varepsilon_i^j < 0$ .

<sup>9</sup>As a simplifying assumption, one can assume full obsolescence or depreciation of one's human capital or skills each period. In order to work or search each period in a particular sector, an individual has to incur costs each period to acquire the updated sector-specific human capital. These costs can be assumed to be individual- and sector-specific.

where  $m_h$  is the labor input into certain core activities (say headquarter services) which have to remain within the home country and  $m_p$  is the labor input for production activities which can potentially be offshored.  $\sigma = \frac{1}{1-\rho}$  is the elasticity of substitution between headquarter services and production services.

If we denote the total amount of labor employed by a firm by  $N$ , then we have

$$N = m_h + m_p \tag{6}$$

To produce either  $X$  or  $Z$ , a firm needs to open job vacancies and hire workers. The cost of vacancy in terms of the numeraire good is  $c_i$  in sector  $i = X, Z$ .<sup>10</sup> Let  $L_i$  be the total number of workers who look for a job in sector  $i$ . Any job in either sector can be hit with an idiosyncratic shock with probability  $\delta$  and be destroyed. Define  $\theta_i = \frac{v_i}{u_i}$  as the measure of market tightness in sector  $i$ , where  $v_i L_i$  is the total number of vacancies in sector  $i$  and  $u_i L_i$  is the number of unemployed workers searching for jobs in sector  $i$ . The probability of a vacancy filled is  $q(\theta_i) = \frac{m(v_i, u_i)}{v_i}$  where  $m(v_i, u_i)$  is a constant returns to scale matching function. Since  $m(v_i, u_i)$  is constant returns to scale,  $q'(\theta_i) < 0$ . The probability of an unemployed worker finding a job is  $\frac{m(v_i, u_i)}{u_i} = \theta_i q(\theta_i)$  which is increasing in  $\theta_i$ .

### 2.3 Profit maximization by firms

Denote the number of vacancies posted by a firm in the  $Z$  sector by  $V$ . Assuming that each firm is large enough to employ and hire enough workers to resolve the uncertainty of job inflows and outflows, the dynamics of employment for a firm is

$$\dot{N}(t) = q(\theta_z(t))V(t) - \delta N(t) \tag{7}$$

The wage for each worker is determined by a process of Nash bargaining with the firm separately which (along with alternative modes of bargaining, including multilateral bargaining) is discussed later. While deciding on how many vacancies to open up the firm correctly anticipates this wage. Effectively, the firm solves a two stage problem where in stage 1 it chooses vacancies and in stage 2 it enters into bargaining with workers to determine wages.<sup>11</sup> Therefore, the profit maximization problem for an individual firm can

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<sup>10</sup>The robustness of our results to alternatively defining and fixing vacancy costs in terms of good  $Z$  or in terms of labor is discussed in the penultimate section of this paper.

<sup>11</sup>As shown by Stole and Zwiebel (1996), the subgame perfect equilibrium of this type of set up can possibly involve a choice of employment greater than what a wage taking firm would do. This is because by choosing higher employment in stage 1 a firm can lower the marginal product of a worker and thus reduce the wage it has to pay in the second stage. As we will see shortly for the autarky case (and later for the offshoring case), the value of marginal product of labor in our set up will be constant for



be written as

$$\underset{V(s), m_h(s), m_p(s)}{Max} \int_t^\infty e^{-r(s-t)} \{P_z(s)Z(s) - w_z(s)N(s) - c_z V(s)\} ds \quad (8)$$

The firm maximizes (8) subject to (5), (6), and (7). We provide details of the firm's maximization exercise in the appendix. Since we are going to study only the steady state in this paper, we suppress the time index hereafter. The steady-state is characterized by  $\dot{N}(t) = 0$ . From the first-order conditions of the firm's maximization problem, the optimal mix of headquarter and production labor is given by

$$\frac{m_h}{m_p} = \left( \frac{\tau}{1 - \tau} \right)^\sigma \quad (9)$$

which in turn makes the output effectively linear in the total employment of the firm as follows:

$$Z = \tau' N \quad (10)$$

where  $\tau' \equiv [\tau^\sigma + (1 - \tau)^\sigma]^{\frac{1}{\sigma-1}}$ .

The key equation from the firm's optimal choice of vacancy, derived in the appendix, is given by

$$\frac{\tau' P_z - w_z}{(r + \delta)} = \frac{c_z}{q(\theta_z)} \quad (11)$$

The expression on the left-hand side is the marginal benefit from creating a job which equals the present value of the stream of the value of marginal product net of wage of an extra worker after factoring in the probability of job separation each period. The expression on the right-hand side is the cost of creating a job which equals the cost of posting a vacancy,  $c_z$ , multiplied by the average duration of a vacancy,  $\frac{1}{q(\theta_z)}$ . The left hand side of (11) is also the asset value of an extra job for a firm which will be useful in the wage determination below. An alternative way to write (11) is

$$\tau' P_z = w_z + \frac{(r + \delta)c_z}{q(\theta_z)} \quad (12)$$

That is, the value of marginal product of a worker is equal to the marginal cost of hiring a worker. This is the modified pricing equation in the presence of search frictions where in addition to the standard wage cost, expected search cost is added to compute the marginal cost of hiring a worker. This is also known as the job creation condition in the literature.

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a given  $P_z$ , and therefore, a firm has no such strategic motive. Hence, the second stage wage is effectively independent of the first stage employment choice (see Cahuc and Wasmer (2001) for a formal proof).

Since the  $X$  sector uses one unit of labor to produce one unit of output, the marginal revenue product of labor in the  $X$  sector simply equals  $P_x$ , and therefore, the profit maximization by firms in the  $X$  sector yields the following analogue of (12)

$$P_x = w_x + \frac{(r + \delta)c_x}{q(\theta_x)} \quad (13)$$

## 2.4 Determination of Unemployment

Denoting the rate of unemployment in sector- $i$  by  $u_i$ , in steady-state the flow into unemployment must equal the flow out of unemployment:

$$\delta(1 - u_i) = \theta_i q(\theta_i) u_i; \quad i = x, z$$

The above implies

$$u_i = \frac{\delta}{\delta + \theta_i q(\theta_i)}; \quad i = x, z \quad (14)$$

The above is the standard Beveridge curve in Pissarides type search models where the rate of unemployment is positively related to the probability of job destruction,  $\delta$ , and negatively related to the degree market tightness  $\theta_i$ .

## 2.5 Wage Determination

Wage is determined for each worker through a process of Nash bargaining with his/her employer. Workers bargain individually and simultaneously with the firm.<sup>12</sup> Rotemberg (2006) justifies this assumption by viewing it as a situation where each worker bargains with a separate representative of the firm. Thus each worker and the representative that he bargains with assume at the time of bargaining that the firm will reach a set of agreements with the other workers that leads these to remain employed.

Denoting the unemployment benefit in terms of the final good by  $b$ , it is shown in the appendix that the expression for wage is the same as in a standard Pissarides model and is given by

$$w_i = b + \frac{\beta c_i}{1 - \beta} \left[ \theta_i + \frac{r + \delta}{q(\theta_i)} \right]; \quad i = x, z \quad (15)$$

where  $\beta$  represents the bargaining power (weight) of the worker relative to the employer (See appendix). The above wage equation along with the (12) and (14) derived earlier are the three key equations determining  $w_z$ ,  $\theta_z$ , and  $u_z$  for a given  $P_z$ . For the  $X$  sector the three key equations are (13), (14), and (15).

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<sup>12</sup>As explained in the previous footnote, under CRS to labor, the bargaining outcome is the same as in Stole and Zweibel (1996), the outcome of which in turn is similar to the Shapley value of a worker obtained under multilateral bargaining.

For each of the two sectors, for a given price we can determine the wage,  $w_i$  and the market tightness,  $\theta_i$  as follows. Equation (15) represents the wage curve,  $WC$  which is clearly upward sloping in the  $(w, \theta)$  space in Figure 1. The greater is the labor market tightness, the higher is the wage that emerges out of the bargaining process (as the greater is going to be the value of each occupied job). Note that the position of this curve is independent of the price,  $P_i$ . The job creation curve,  $JC$ , depicting (12) for sector  $Z$  and (13) for sector  $X$ , is downward sloping in the  $(w, \theta)$  space. The capitalized value of the hiring cost is increasing in market tightness,  $\theta_i$ . The tighter the market the longer it takes to fill up a vacancy. Therefore, for a given value of the marginal product of labor, there is a tradeoff between the wage and the market tightness. The intersection of  $WC$  and  $JC$  gives the partial equilibrium levels of  $w_i$  and  $\theta_i$  for a given  $P_i$ . As the price,  $P_i$ , increases,  $JC$  shifts up, leading to an increase in  $w_i$  and  $\theta_i$ , and thus from the Beveridge curve a reduction in unemployment.

## 2.6 Sectoral choice of workers

Since unemployed workers can search in either sector, they search in the sector where their expected utility is higher. As shown in equation (36) in the appendix, the asset value of unemployed worker- $j$  searching in sector- $i$  is given by  $rU_i^j = \varepsilon_i^j + b + \frac{\beta}{1-\beta}c_i\theta_i$ . Recall that  $\varepsilon_i^j$  is the non-pecuniary benefit of worker- $j$  from being affiliated with sector- $i$ , while the market tightness variable  $\theta_i$  positively affects the wage and job finding rate in sector- $i$ .

Since  $\varphi^j \equiv \varepsilon_z^j - \varepsilon_x^j$ , the sectoral choice of workers is given as follows.

$$\begin{aligned} \text{If } \varphi^j &\geq \frac{\beta}{1-\beta}(c_x\theta_x - c_z\theta_z) \text{ then search in sector-}Z \\ \text{If } \varphi^j &< \frac{\beta}{1-\beta}(c_x\theta_x - c_z\theta_z) \text{ then search in sector-}X \end{aligned}$$

Given the above relationship, the equilibrium sectoral choice is determined by a cutoff value of  $\varphi$  denoted by  $\widehat{\varphi}$  where

$$\widehat{\varphi}(\theta_x, \theta_z) = \frac{\beta}{1-\beta}(c_x\theta_x - c_z\theta_z) \quad (16)$$

such that a fraction  $1 - G(\widehat{\varphi})$  of workers are affiliated with sector  $Z$ , while the remaining fraction  $G(\widehat{\varphi})$  are affiliated with sector  $X$ . That is,

$$L_z = (1 - G(\widehat{\varphi}))L; L_x = G(\widehat{\varphi})L$$

The case of perfect mobility can be captured by setting  $\varphi^j = 0$  for all  $j$ , which would imply the following no

arbitrage condition

$$c_x\theta_x = c_z\theta_z \quad (17)$$

### 3 Autarky Equilibrium

The autarky equilibrium can be solved by deriving the relative supply curve for  $Z$  since the relative demand is given by (4). To derive the relative supply corresponding to each relative price  $p$  obtain the values of  $P_z$  and  $P_x$  from (3) which is the zero profit condition ( $ZPC$ ) for the numeraire good,  $C$ . Next, for these values of  $P_i$  determine  $w_i$  and  $\theta_i$  from the intersection of  $WC$  and  $JC$  for sector  $i$  as shown in Figure 1. Having determined  $\theta_i$ , find the corresponding  $\hat{\varphi}$  from (16). Denote  $\hat{\varphi}$  as a function of  $p$  in the case of autarky by  $\hat{\varphi}^A$ . The relative supply of  $Z$  is given by

$$\left(\frac{Z}{X}\right)^s = \frac{\tau'(1-u_z)L_z}{(1-u_x)L_x} = \frac{\tau'(1-u_z)(1-G(\hat{\varphi}^A))}{(1-u_x)G(\hat{\varphi}^A)} \quad (18)$$

where  $u_i$  which is a decreasing function of  $\theta_i$  is given by (14). To see what happens to the relative supply when  $p$ , which is the relative price of  $Z$ , increases note from (3) that an increase in  $p$  must imply an increase in  $P_z$  and a decrease in  $P_x$  to satisfy the zero profit condition for the numeraire. This also implies an increase in  $\theta_z$  and a decrease in  $\theta_x$ , which in turn implies a decrease in  $\hat{\varphi}$  from (16). Therefore,  $\hat{\varphi}^{A'}(p) < 0$ , which is shown in figure 2a. What it implies is that  $L_z$  increases and  $L_x$  decreases with  $p$ , that is, some workers move from sector  $X$  to sector  $Z$ . As well, an increase in  $\theta_z$  implies a decrease in  $u_z$ , while a decrease in  $\theta_x$  implies an increase in  $u_x$ . Thus, the relative supply of  $Z$  is increasing in the relative price,  $p$ .

In order to analyze the implications of varying degrees of intersectoral labor mobility, let us assume that  $\varepsilon_z$  and  $\varepsilon_x$  are independent of each other and each follows the same extreme value distribution as in Artuc, Chaudhuri and McLaren (2008), which is represented by the following special case of the Gumbel cumulative distribution function:

$$F(\varepsilon_i, i = x, z) = \exp\left(-\exp\left(-\frac{\varepsilon_i}{\alpha} - \gamma\right)\right), \varepsilon_i \in (-\infty, \infty)$$

where  $\gamma = 0.5772$  is Euler's constant and  $\alpha$  is the scale parameter. The mean of  $\varepsilon_i$  is zero and variance is  $\pi^2\alpha^2/6$  (where the constant,  $\pi \approx 3.14$ ). In this case,  $\varphi = \varepsilon_z - \varepsilon_x$  follows a symmetric distribution with mean zero and a variance equal to  $\pi^2\alpha^2/3$ , and this distribution is given by

$$G(\varphi) = \frac{\exp(\varphi/\alpha)}{1 + \exp(\varphi/\alpha)}, \varphi \in (-\infty, \infty)$$

Based on the above distribution, we have

$$L_z = (1 - G(\widehat{\varphi}^A))L = \frac{L}{1 + \exp(\widehat{\varphi}^A/\alpha)} \quad (19)$$

$$L_x = L - L_z \quad (20)$$

It should be clear from (18), (19), and (20) that the relative supply can be written as

$$\left(\frac{Z}{X}\right)^s = \frac{\tau'(1 - u_z)}{(1 - u_x) \exp(\widehat{\varphi}^A/\alpha)} \quad (21)$$

We depict this relative supply curve in Figure 2b.

Recall that  $\widehat{\varphi}^A$  depicted in Figure 2a is solely a function of  $p$  and independent of  $\alpha$ . Therefore, relative supply is increasing in  $\alpha$  when  $\widehat{\varphi}^A > 0$  and decreasing in  $\alpha$  when  $\widehat{\varphi}^A < 0$ . At  $\widehat{\varphi}^A = 0$ , relative supply becomes independent of  $\alpha$ . Denote the solution to  $\widehat{\varphi}^A(p) = 0$  by  $p^A$ . It is easy to see that the relative supply curves given by (21) for different values of  $\alpha$  all pass through the same point at  $p = p^A$ . This is shown in Figure 2b (in which and in all subsequent figures, we normalize the unemployment benefit,  $b$  to zero for simplicity). For  $p < p^A$ ,  $\widehat{\varphi}^A > 0$ , and hence the relative supply curve for higher  $\alpha$  lies to the right of the one for lower  $\alpha$ , and for  $p > p^A$ , it is the opposite. Thus, as  $\alpha$  goes down, the relative supply curve rotates clockwise around  $p = p^A$  (Figure 2b). Clearly around that point, labor mobility goes up with a decrease in  $\alpha$ , i.e., at that point any given price shock leads to a bigger movement in labor from one sector to another, the smaller is  $\alpha$ . In the limit, when  $\alpha \rightarrow 0$ ,  $\varphi^j \rightarrow 0 \forall j$ . In this case the relative supply is zero for any  $p < p^A$  because no one wants to work in the  $Z$  sector, and it becomes horizontal at  $p = p^A$  since all workers are indifferent between working in the two sectors. This is the case of perfect labor mobility.

Having derived the relative supply curve, the autarky equilibrium can be determined by bringing in the relative demand curve given in (4) which is downward sloping. The intersection of the relative demand curve with the relative supply curve determines the autarky equilibrium. Note that in the case of perfect labor mobility, since the relative supply curve is horizontal at  $p = p^A$  where  $p^A$  solves  $\widehat{\varphi}^A(p) = 0$ , the autarky equilibrium price is necessarily  $p^A$ . At  $p^A$  the no arbitrage condition (17) is satisfied, and therefore, all workers are indifferent between being in the two sectors (since  $\varphi^j = 0$  for all workers).

Autarky equilibrium price with imperfect mobility can be higher or lower than  $p^A$  depending on the position of the relative demand curve. To facilitate comparison of autarky equilibrium with offshoring equilibrium in the presence of various degrees of labor mobility, we will assume that the technology that yields  $C$  in terms of  $Z$  and  $X$  is such that the relative demand curve,  $RD$  passes through the common point of intersection of the autarky relative supply curves with varying degree of intersectoral labor mobility

(Figure 2b). That is, the relative demand is such that the autarky equilibrium for various degrees of labor mobility is  $p^A$ .

## 4 Equilibrium with the possibility of offshoring

Now, suppose firms in the  $Z$  sector have the option of procuring input  $m_p$  from abroad (which we call offshoring in this paper) instead of producing them domestically.<sup>13</sup> The per unit cost of imported input is  $w_s$  in terms of the numeraire good  $C$ , and this country takes this per unit cost as given.<sup>14</sup> This includes transportation cost, tariffs, foreign wage costs and possible search costs, all of which, for analytical tractability, we assume to be proportional to the amount of the input imported. If and when offshoring takes place, the final good  $C$  will be exported to pay for the imports of  $m_p$ . Starting from an autarky equilibrium with relative price  $p^A$  and associated cost of employing a worker in sector  $Z$  given by  $w_z^A + \frac{(r+\delta)c_z}{q(\theta_z^A)}$ , it must be the case that  $w_s < w_z^A + \frac{(r+\delta)c_z}{q(\theta_z^A)}$  so that offshoring production input is cheaper than producing it domestically.<sup>15</sup>

For a firm offshoring its production input, the production function specified in (5) can be written as  $Z = (\tau N^\rho + (1-\tau)m_p^\rho)^{\frac{1}{\rho}}$ , where  $N$  is the domestic labor used for headquarter services. This firm maximizes  $\int_t^\infty e^{-r(s-t)}\{P_z(s)Z(s) - w_z(s)N(s) - w_s m_p(s) - c_z V(s)\}ds$ . The equation of motion for employment given in (7) remains valid.<sup>16</sup>

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<sup>13</sup>The assumption here is that one unit of home (domestic) labor can produce one unit of the production input. Therefore, we use  $m_p$  to denote both the number of units of the imported input in the offshoring case as well as the number of units of production labor in the autarky case.

<sup>14</sup>The assumption that  $w_s$  is fixed is effectively a small country assumption. However, as argued in an earlier version of this paper, there is no loss of generality resulting from it. Large amounts of labor used in the production of a numeraire consumption good in the South (country to which input production is offshored), which forms a large share in the household budget, fixes wage and the unemployment rate also in input production there. One can here easily work out the implications of offshoring for the South.

<sup>15</sup>It is possible that the value of  $w_s > w_z^A + \frac{(r+\delta)c_z}{q(\theta_z^A)}$  and  $w_s < w_z^O + \frac{(r+\delta)c_z}{q(\theta_z^O)}$  when all firms offshore (where the superscript "O" represents variables in the offshoring equilibrium), resulting in the possibility of multiple equilibria - autarky and offshoring. However, starting from autarky, in such a case firms will be faced with a coordination problem that will prevent them from moving into an offshoring equilibrium. Therefore, for our analysis, for offshoring to take place it will be required that  $w_s < w_z^A + \frac{(r+\delta)c_z}{q(\theta_z^A)}$ .

<sup>16</sup>As in the autarky case, following Cahuc and Wasmer (2001), there is no role for strategic overemployment here as well. The marginal product of headquarter labor in  $Z$  gets fixed for a given  $P_z$  as follows:  $w_s$  is equated to the value of marginal product of production input. Under CRS, this fixes the ratio of headquarter to production input for a given  $P_z$ , which in turn fixes the marginal product of headquarter labor. In other words, what we are doing here is implicitly equivalent to the case

We use the following notational simplification.

**Definition 1:**  $\omega \equiv \frac{w_z + \frac{(r+\delta)c_z}{q(\theta_z)}}{w_s}$

In the above definition  $\omega$  is the cost of domestic labor relative to foreign labor. In an offshoring equilibrium it must be the case that  $\omega \geq 1$ . With the above notation, the ratio in which an offshoring firm uses headquarter and production inputs in steady state is given by

$$\frac{N}{m_p} = \left( \frac{\tau}{(1-\tau)\omega} \right)^\sigma \quad (22)$$

The first order condition for the optimal choice of output is given by

$$P_z = \left( \tau^\sigma \left( w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right)^{1-\sigma} + (1-\tau)^\sigma w_s^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad (23)$$

The expression on the right hand side above is the marginal cost of producing an extra unit of good  $Z$ . The above can be written in an alternative form as

$$(\tau + (1-\tau)^\sigma \omega^{\sigma-1})^{\frac{1}{\sigma-1}} P_z = w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \quad (24)$$

The left hand side is the value of marginal product of domestic labor in headquarter activity, which must equal the cost of hiring domestic labor inclusive of the recruitment cost. This is the job creation condition for headquarter jobs for offshoring firms. Note that at  $\omega = 1$  the expression above reduces to the job creation condition (12) derived for autarky which is also the job creation condition for non-offshoring firms.

Since in steady-state the value of a headquarter job in the  $Z$  sector must still equal the capitalized value of recruitment cost,  $\frac{c_z}{q(\theta_z)}$ , the Nash bargained wage is still given by

$$w_z = b + \frac{\beta c_z}{1-\beta} \left[ \theta_z + \frac{r+\delta}{q(\theta_z)} \right]$$

To derive the offshoring equilibrium, we first derive the offshoring relative supply curve as follows. For any  $w_s < w_z^A + \frac{(r+\delta)c_z}{q(\theta_z^A)}$ , define  $\underline{P}_z^o = \frac{w_s}{\tau}$ , that is,  $\underline{P}_z^o$  is such that the corresponding autarky domestic labor cost in sector  $Z$  given by  $w_z + \frac{(r+\delta)c_z}{q(\theta_z)}$  equals  $w_s$ . It is the minimum price of  $Z$  required for offshoring to take place if allowed. Denote the  $P_x$  that satisfies the  $ZPC$  for  $P_z = \underline{P}_z^o$  by  $\underline{P}_x^o$ . The relative price  $p$  corresponding to  $\underline{P}_z^o$  and  $\underline{P}_x^o$  is denoted by  $\underline{p}^o$ . For  $p < \underline{p}^o$ , there is no offshoring because the autarky labor

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where the firm first sets employment and then wage, followed by its decision on how much of the input to import.

cost in the  $Z$  sector is below  $w_s$ . Therefore, the offshoring relative supply curve coincides with the autarky relative supply curve for  $p < \underline{p}^o$ .

To find the offshoring relative supply at each  $p > \underline{p}^o$  we need to determine  $\widehat{\varphi}$ , the cutoff for idiosyncratic differences in non-pecuniary utility between the two sectors, denoted by  $\widehat{\varphi}^o$ , where the superscript  $O$  stands for offshoring. For  $p < \underline{p}^o$ , allowing for offshoring leaves  $\widehat{\varphi}(p)$  unchanged. For  $p \geq \underline{p}^o$ ,  $P_z$  and  $P_x$  are still given by (3). Therefore,  $\theta_x$  and  $w_x$  remain unchanged from autarky for a given  $p$ . However,  $\theta_z$  and  $w_z$  are now determined by (23). From (23) it is clear that for each  $P_z > \underline{P}_z^o$ , the corresponding  $\left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)}\right)$  is greater than its value in autarky. Therefore,  $\theta_z$  and  $w_z$  are higher than in autarky<sup>17</sup>. Since  $\theta_z$  is higher while  $\theta_x$  is unchanged for each  $p > \underline{p}^o$ , (16) implies that the  $\widehat{\varphi}^o(p)$  curve lies to the left of the  $\widehat{\varphi}^A(p)$  curve as is shown in Figure 2a.

In the appendix we prove the following lemma on the shift in the offshoring relative supply curve compared to the autarky relative supply curve.

**Lemma 1:** *There is a step shift in the offshoring relative supply curve compared to autarky. For  $p < \underline{p}^o$  the offshoring relative supply curve corresponds to the autarky supply curve. For  $p = \underline{p}^o$ , the offshoring supply curve has a horizontal segment and for  $p > \underline{p}^o$ , the offshoring supply curve lies to the right of the autarky supply curve.*

For the perfect mobility case relative supply curve we prove the following lemma.

**Lemma 2:** *The offshoring relative supply curve in the case of perfect mobility is horizontal at  $p^o$  where  $p^o < p^A$  is the solution to  $\widehat{\varphi}^o(p) = 0$ .*

Proof: The offshoring relative supply curve in the case of perfect mobility can be obtained as the limiting case of offshoring relative supply curve with imperfect mobility with  $\alpha \rightarrow 0$  as shown in the appendix. Alternatively, note that at  $p^o$  the no arbitrage condition (17) is satisfied by definition, therefore, workers are indifferent between working in either sector at a price of  $p^o$ , and hence the offshore relative supply curve in the case of perfect mobility is horizontal at  $p^o$ . As shown in the appendix,  $\widehat{\varphi}^o(p)$  lies to the left of  $\widehat{\varphi}^A(p)$ , and hence  $p^o < p^A$  (as shown in Figure 2a).

Figure 3 depicts the shifts in relative supply curves for  $\alpha_1$  and  $\alpha_2$  such that  $\alpha_2 > \alpha_1$  and those for the perfect mobility case. Note that similar to autarky, offshoring relative supply curves with various degrees of labor mobility all pass through the same point at  $p = p^o$ .

Given the offshoring relative supply curves derived above, there are two possible types of offshoring

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<sup>17</sup>For  $p = \underline{p}$  which implies  $P_z = \underline{P}_z$ , we get  $w_z + \frac{(r+\delta)c_z}{q(\theta_z)} = w_s$ .



equilibria in the imperfect mobility case.

1) *Complete Offshoring Equilibrium.* If the relative demand curve intersects the offshoring relative supply curve on the rising part, then we get a complete offshoring equilibrium.

2) *Mixed Offshoring Equilibrium.* If the relative demand curve intersects the horizontal part of the offshoring relative supply curve, then we get a mixed equilibrium. This is the case where in which only some firms in the industry offshore and others remain fully domestic. This equilibrium is shown in Figure 4. In this case the equilibrium price is necessarily equal to  $\underline{p}^o$ . And if we look at Figure 3 and again provided that the relative demand curve is such that it passes through point  $A$ , it should be clear, given the negative slope of the relative demand curve that a mixed equilibrium is more likely as labor becomes less mobile intersectorally. In the appendix, we also show that such an equilibrium becomes less likely as  $w_s$  goes down.

In the perfect mobility case, there cannot be a mixed equilibrium because  $p^o > \underline{p}^o$ . Therefore, the equilibrium in this case involves complete offshoring.

In all cases there is a decrease in the relative price of  $Z$ . This implies an increase in  $P_x$  and a decrease in  $P_z$  from the zero profit condition for the numeraire good  $C$ . That is, the price of good  $X$  in terms of the numeraire increases while the price of good  $Z$  in terms of the numeraire decreases. An increase in the price of  $X$  increases job creation in the  $X$  sector. In terms of Figure 1, there would be a rightward shift in the  $JC$  curve in the  $X$  sector, while the  $WC$  curve remains unchanged. Therefore,  $w_x$  and  $\theta_x$  increase relative to autarky while  $u_x$  decreases.

The impact on wage and unemployment in the  $Z$  sector is less straightforward. In the case of a mixed equilibrium, since the equilibrium price is  $\underline{p}^o < p^A$ , the offshoring equilibrium labor cost in the  $Z$  sector, which equals  $w_s$ , is less than the autarky equilibrium labor cost in the  $Z$  sector given by  $w_z^A + \frac{(r+\delta)c_z}{q(\theta_z^A)}$ . Therefore, both  $w_z$  and  $\theta_z$  decrease relative to autarky, and hence the unemployment rate is higher in the  $Z$  sector. In an offshoring equilibrium with perfect mobility, the no arbitrage condition implies that  $\theta_z$  must increase because  $\theta_x$  increases. Therefore, offshoring must lead to a decrease in unemployment in the  $Z$  sector if labor is perfectly mobile. Finally, in the case of complete offshoring equilibrium with imperfectly mobile labor, the impact of offshoring on unemployment and wage in the  $Z$  sector is ambiguous.

Intuitively, offshoring has two effects on the job creation in the  $Z$  sector. This can be seen by comparing the job creation condition (24) with (12) for autarky. Note that for  $\omega = 1$  (24) reduces to (12). The value of marginal product of labor in the  $Z$  sector in the case of offshoring differs from autarky if either  $\omega \neq 1$  and/or  $P_z$  is not equal to its autarky value. We have seen that the offshoring equilibrium price of  $Z$  is less than the autarky price of  $Z$ . This reduces the value of marginal product of labor in the offshoring equilibrium leading

to less job creation. If  $\omega = 1$ , as is the case in a mixed equilibrium, the reduction in  $P_z$  leads to a definite decrease in the value of marginal product of labor in the  $Z$  sector and consequently a decline in  $Z$ -sector wage and an increase in  $Z$ -sector unemployment. If  $\omega > 1$ , as is the case with complete offshoring equilibrium, then the value of marginal product of labor increases due to this effect. This is the productivity enhancing effect of offshoring on the labor used in headquarter activities in the  $Z$  sector. Since  $P_z$  is lower compared to autarky, while  $\omega > 1$ , the impact on the value of marginal product of labor relative to autarky is ambiguous, in general. In Figure 1, the positive productivity effect shifts the  $JC$  curve for sector  $Z$  to the right and the negative price effect shifts it to the left. The former is a partial-equilibrium effect and the latter a general-equilibrium effect. The net direction of shift of the  $JC$  curve is thus ambiguous. However, if labor is perfectly mobile across sectors, then the no-arbitrage condition ensures that the productivity effect must dominate the negative price effect, and hence there must be an increase in the wage and a decrease in unemployment in the  $Z$  sector. Thus, the  $JC$  curve shifts in net terms to the right. Just as in Figure 1, the productivity effect takes the  $JC$  curve to the right to  $JC'$  and the price effect shifts it back in the other direction to  $JC''$  but not all the way back up to  $JC$ .

The result on the impact of offshoring on sectoral wages and unemployment is summarized in a proposition below.

**Proposition 1** *In the case of imperfect labor mobility, in an offshoring equilibrium, the unemployment rate in the non-offshoring sector goes down and the wage rate goes up, relative what we obtain in the autarky equilibrium. In the offshoring sector, the unemployment rate goes up and the wage rate goes down in a mixed offshoring equilibrium, but the impact is ambiguous in a complete offshoring equilibrium. The likelihood of a mixed equilibrium goes down with the extent of labor mobility and with a decrease in the cost of the imported input. In the case of perfect labor mobility, however, a mixed equilibrium is not possible (only a complete offshoring equilibrium is possible under offshoring) and sectoral wages are unambiguously higher and sectoral unemployments unambiguously lower in an offshoring equilibrium compared to the autarky equilibrium.*

Using a continuity argument, we can derive the following corollary.

**Corollary 1:** *For any  $w_s < w_z^A + \frac{(r+\delta)c_z}{q(\theta_z^A)}$ , there exists an  $\alpha^*$  such that for  $\alpha < \alpha^*$ ,  $w_z^o > w_z^A$  and  $\theta_z^o > \theta_z^A$ .*

The Corollary above implies that with sufficient degree of labor mobility, the sectoral unemployment rates decrease in both sectors.

In the appendix (see the section on “comparing equilibria with differing degrees of labor mobility”),

we also show that the equilibrium relative price of  $Z$  in an offshoring equilibrium under imperfect labor mobility is lower than in the case of perfect mobility, and the equilibrium price keeps decreasing as the labor mobility keeps decreasing. So the adverse relative price effect is stronger as labor becomes more and more intersectorally immobile, and also as a result the likelihood that the  $Z$  sector wage falls and unemployment in that sector rises goes up.<sup>18</sup>

While we have derived results on the impact of offshoring on sectoral unemployment rates, the economywide unemployment rate is a weighted average the sectoral unemployment rates with the weights being the share of each sector in the total labor force. Now, even if the sectoral unemployment rates go down, economywide unemployment rate may increase if workers move from low unemployment sector to high unemployment sector. Alternatively, even if the unemployment rate in the  $Z$  sector increases upon offshoring (as happens in a mixed equilibrium), economywide unemployment rate may go down if workers move to the low unemployment sector upon offshoring. Providing analytical results on the movement of labor consequent upon offshoring is not possible in the case of imperfect mobility, however, in the case of perfect mobility no arbitrage condition allows us to derive analytical results which we provide below. We discuss several cases depending on the search costs in the two sectors.

Case I: In the special case of  $c_x = c_z$ , no arbitrage condition (17) implies  $\theta_x = \theta_z$  and hence  $u_x = u_z$ . Since offshoring reduces sectoral unemployment rates, the aggregate unemployment rate must fall as well.

When  $c_x \neq c_z$ , we have  $\theta_x \neq \theta_z$ , and therefore, the two sectors have different unemployment rates. Now, the impact of offshoring on economywide unemployment depends on the direction of labor movement, that is whether labor moves to the high unemployment sector or low unemployment sector. The direction of labor movement depends on the parameters of the model, particularly the elasticities of substitution in consumption and production. Assume a constant elasticity of substitution production function for  $C$  where the elasticity of substitution is  $\phi$ . Recall that the elasticity of substitution in  $Z$  production is  $\sigma$ . We prove the following lemma in the appendix.

**Lemma 3:** *When  $c_x = c_z$ , except when  $\phi > 1$  and  $\sigma < 1$ , the size of the labor force in the  $Z$  sector post-offshoring is less than in the autarky equilibrium. When  $\phi > 1$  and  $\sigma < 1$ , it is possible for the post-offshoring labor force in the  $Z$  sector to exceed its autarky level.*

Intuitively, since production jobs are lost in the  $Z$  sector, while there is greater job creation in the  $X$

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<sup>18</sup>If parameters are such that labor moves into the  $Z$  sector in an offshoring equilibrium with perfect mobility (low elasticity of substitution in production and high elasticity of substitution in consumption), then the offshoring equilibrium relative price is decreasing in the degree of labor mobility.

sector, workers are likely to move from  $Z$  sector to the  $X$  sector. As well, cheaper foreign production labor can be substituted for more expensive domestic headquarter labor leading to further movement of workers to the  $X$  sector. Countering these effects is the increase in the relative demand for good  $Z$  resulting from a decrease in its relative price. The latter effect on the derived demand for labor is normally dominated by the former effects. However, if the elasticity of substitution in consumption is very high ( $\phi > 1$ ) and in production very low ( $\sigma < 1$ ), then workers could move from  $X$  sector to  $Z$  sector upon offshoring. A high  $\phi$  implies a large increase in the relative demand for  $Z$  for a small decrease in the relative price of  $Z$ . A low  $\sigma$  implies fewer headquarter jobs can be substituted by cheaper production jobs. Therefore, with  $\phi > 1$  and  $\sigma < 1$  workers may end up moving to the  $Z$  sector.<sup>19</sup> While lemma 3 discusses labor movement for all possible values of  $\phi$  and  $\sigma$ , it is reasonable to think that the elasticity of substitution between headquarter and production input is less than 1. In that case we can say that labor moves from  $Z$  to  $X$  if  $\phi \leq 1$  and may move from  $X$  to  $Z$  if  $\phi > 1$ .

Even though the analytical result in Lemma 3 obtains for  $c_x = c_z$ , using a continuity argument we claim that it will hold if  $c_x$  and  $c_z$  are not too different. Numerical simulations confirm that the result on  $L_z$  decreasing upon offshoring is valid even when  $c_x \neq c_z$  ( $c_x$  and  $c_z$  are fairly far apart) except in the case of very high  $\phi$  and very low  $\sigma$ . The discussion above implies the following additional results on aggregate unemployment if  $\sigma < 1$ .

Case II:  $c_x < c_z$ . In this case, no arbitrage condition (17) implies  $\theta_x > \theta_z$ , and hence  $u_x < u_z$ . That is,  $Z$  sector has a higher wage as well as unemployment. For  $\phi \leq 1$  labor moves from  $Z$  sector to  $X$  sector, and hence there is an unambiguous decrease in aggregate unemployment. In the case of  $\phi > 1$  labor may move from  $X$  to  $Z$ , in which case the impact on aggregate unemployment would be ambiguous.

Case III:  $c_x > c_z$ . For  $\phi \leq 1$  labor moves from  $Z$  sector to  $X$  sector, and hence the impact on aggregate unemployment is ambiguous. If  $\phi > 1$ , then labor may move from  $X$  to  $Z$ , in which case there would be an unambiguous decrease in aggregate unemployment.

The result on aggregate unemployment is summarized in a proposition below.

**Proposition 2** *In the case of imperfect mobility of labor the impact of offshoring on aggregate unemployment*

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<sup>19</sup>With perfect intersectoral labor mobility, it is worth noting that if we get rid of all the labor market frictions in this model and the labor market is made perfectly competitive, the labor force allocation across the two sectors will be exactly the same as in the case of  $c_x = c_z$  in our labor-market search model (with perfect intersectoral labor mobility). That is, in the absence of frictions in the labor market, offshoring will lead to movement of workers from sector  $Z$  to sector  $X$  except when  $\phi > 1$  and  $\sigma < 1$ . This can be easily verified in the proof of labor allocation in the appendix.

rate is ambiguous. With perfect mobility, however, there is a decrease in aggregate unemployment if labor moves from the high unemployment sector to the low unemployment sector, and the impact is ambiguous if labor moves from the low unemployment sector to the high unemployment sector. Except when the elasticity of substitution between headquarter and production labor in the production of  $Z$  is low relative to the elasticity of substitution between  $Z$  and  $X$  in consumption (or the final consumption good  $C$ ), domestic labor moves from the offshoring to the non-offshoring sector.

## 5 Possible Extensions and Discussion

One can now imagine a situation where there is no mobility across the two types of jobs in the  $Z$  sector but there is mobility of production labor between the two sectors, i.e., headquarter jobs require skilled workers, while production jobs require unskilled or relatively less skilled workers. After offshoring, the production input cost in sector  $Z$  equals  $w_s$ , and all the domestic production labor moves to sector  $X$ . At a constant  $P_z$ , the marginal product of headquarter labor rises since the cost of production input (now all imported) in sector  $Z$  falls to  $w_s$ . Thus, upon offshoring, at a constant  $P_x$  (and therefore at constant  $P_z$ ), unemployment falls for skilled workers who work in the headquarter activities in the  $Z$  sector, while it remains constant in sector  $X$ . More headquarter labor is employed as a result in sector  $Z$ . In addition, at constant  $P_z$ , since the ratio of production input to headquarter labor has gone up, employment of production input (now all imported) and therefore the output of  $Z$  have also gone up. At a given price, the  $X$ -sector labor force actually increases upon offshoring since all the domestic production labor from  $Z$  actually flows into  $X$ . Thus, both the outputs of  $X$  and  $Z$  go up for a given  $P_x$  and whether the relative supply  $Z/X$  goes up or down as a result of offshoring will depend on how intensively  $Z$  is used in  $C$  and on  $w_s$ . These will determine how much production labor is released from the  $Z$  sector to go to the  $X$  sector upon offshoring and how large the increase is in the marginal product of headquarter labor. In other words, the relative supply of  $Z$  can shift to the right or left (i.e., its relative price could go up or down) depending on the above factors. If we have a negative price effect, the effect of offshoring on the unemployment of headquarter labor is ambiguous while production labor unemployment goes down. If the parameters are such that the price effect is positive, then headquarter unemployment goes down and production labor unemployment goes up.<sup>20, 21</sup>

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<sup>20</sup>The derivation of these results can be obtained from the authors upon request.

<sup>21</sup>In the case of the positive price effect, a mixed equilibrium is possible, where simultaneously some amount of domestic production labor is used in the  $Z$  sector and some amount of offshoring takes place.

The general result from the two cases is that upon offshoring, unemployment cannot rise at the same time for both types of labor, but can fall for both. At least, one type of labor will experience a fall in its unemployment rate.

We next focus on the modeling of vacancy cost in this paper. We have modeled vacancy cost,  $c$ , in terms of the numeraire good which seemed natural given the two sector structure of the model. One could alternatively model the vacancy cost either in terms of labor or foregone output. In the former case, the vacancy cost would be  $c_i w_i$  for sector  $i = X, Z$ , where  $w_i$  is the sectoral wage. In the latter case, it would be  $c_i p_i$ . We find that, under fairly plausible and reasonable conditions, the qualitative results would be unchanged. The key to obtaining our result on unemployment is that productivity changes should not be fully absorbed by wage changes, which will obtain with alternative specifications of search costs as well.

## 6 Conclusions

In this paper, in order to study the impact of offshoring on sectoral and economywide rates of unemployment, we construct a two-sector general equilibrium model in which unemployment is caused by search frictions. We find that, contrary to general perception, wage increases and sectoral unemployment decreases due to offshoring when labor is intersectorally perfectly mobile. This result can be understood to arise from the productivity enhancing (cost reducing) effect of offshoring. This result is consistent with the recent empirical results of Amiti and Wei (2005a, b) for the US and UK, where, when sectors are defined broadly enough, they find no evidence of a negative effect of offshoring on sectoral employment.

Even though both sectors have lower unemployment post-offshoring, whether the sector with the lower unemployment or higher unemployment expands will also be a determinant of the overall unemployment rate. If the search cost is identical in the two sectors, implying identical rates of unemployment, then the economywide rate of unemployment declines unambiguously after offshoring. Alternatively, even if the search cost is higher in the sector which experiences offshoring (implying a higher wage as well as higher rate of unemployment in that sector), the economywide rate of unemployment decreases because workers move from the higher unemployment sector to the lower unemployment sector.

We can also point out the welfare implications of offshoring in the above situation. Since wages are expressed in terms of the numeraire consumption good, the welfare implications of offshoring are straightforward. As we have shown that wage increases and the economywide unemployment decreases (when  $c_x \leq c_z$ ) due to offshoring, the impact on welfare is positive.

When we modify the model to allow for imperfect intersectoral labor mobility, the negative relative price effect on the offshoring sector becomes stronger. In such a case, it is possible for this effect to offset the positive productivity effect, and result in a rise in unemployment in that sector. In the other sector, offshoring has a stronger unemployment reducing effect in the absence of perfect intersectoral labor mobility. With imperfect labor mobility, there is also the possibility of a mixed equilibrium (incomplete offshoring). The relationship between offshoring costs and sectoral unemployment (and wage) is non-monotonic in the imperfect mobility case.

There are two main messages from the paper. Firstly, how offshoring will affect unemployment will depend on the alternative opportunities available for the type of labor whose jobs have been offshored. If these workers can freely start searching for alternative jobs, we see a reduction in the unemployment rates for all types of workers. These alternative jobs can be in the same sector (such as more headquarter jobs in our model) or in another sector (such as  $X$ -sector jobs in our model). Secondly, with impediments to movements across sectors and across jobs, the likelihood of unemployment for some workers goes up as a result of offshoring. However, the unemployment rates for all types of workers will never go up at the same time as a result of offshoring.

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## 7 Appendix

### 7.1 Maximization problem of the firm in the autarky case

The firm maximizes (8) subject to (7), and (6). Denoting the Lagrangian multiplier associated with (7) by  $\lambda$ , and with (6) by  $\phi$ , the current value Hamiltonian for each firm can be written as

$$H = P_z Z - w_z N - c_z V + \lambda[q(\theta_z)V - \delta N] + \phi[N - m_h - m_p]$$

where  $Z$  is given in (5). The first order conditions for the above maximization are follows.

$$m_h : P_z \tau m_h^{\rho-1} (\tau m_h^\rho + (1-\tau)m_p^\rho)^{\frac{1}{\rho}-1} = \phi \quad (25)$$

$$m_p : P_z (1-\tau) m_p^{\rho-1} (\tau m_h^\rho + (1-\tau)m_p^\rho)^{\frac{1}{\rho}-1} = \phi \quad (26)$$

$$V : c_z = \lambda q(\theta_z) \quad (27)$$

$$N : w_z + \lambda \delta - \phi = \dot{\lambda} - r\lambda \quad (28)$$

Now, (25) and (26) imply

$$\frac{m_h}{m_p} = \left( \frac{\tau}{1-\tau} \right)^{\frac{1}{1-\rho}} \quad (29)$$

using the above in (25) gives

$$\tau' P_z = \phi \quad (30)$$

Next, note from (27) that for a given  $\theta_z$ ,  $\lambda$  is constant. Using  $\dot{\lambda} = 0$ , (27), and (30) in (28) we get

$$\tau' P_z - w_z = (r + \delta)\lambda = \frac{(r + \delta)c_z}{q(\theta_z)} \quad (31)$$

$\lambda$  is the shadow value of an extra job.

### 7.2 Wage Determination

Let  $U_z^j$  denote the income of the unemployed worker- $j$  searching for a job in the  $Z$  sector. The asset value equation for the unemployed in this sector is given by

$$rU_z^j = \varepsilon_z^j + b + \theta_z q(\theta_z)[E_z^j - U_z^j] \quad (32)$$

where  $E_z^j$  is the expected income from becoming employed in the  $Z$  sector. As explained in Pissarides (2000), the asset that is valued is an unemployed worker's human capital. The return on this asset is the

unemployment benefit  $b$  plus the expected capital gain from the possible change in state from unemployed to employed given by  $\theta_z q(\theta_z)[E_z - U_z]$ .

The asset value equation for employed worker- $j$  in sector  $Z$  is given by

$$rE_z^j = \varepsilon_z^j + w_z^j + \delta(U_z^j - E_z^j) \Rightarrow E_z^j = \frac{\varepsilon_z^j}{r + \delta} + \frac{w_z^j}{r + \delta} + \frac{\delta U_z^j}{r + \delta} \quad (33)$$

Again the return on being employed equals the wage plus the expected change in the asset value from a change in state from employed to unemployed. Assume the rent from a vacant job to be zero which is ensured by no barriers to the posting of vacancy. Now, denote the surplus for a firm from a job occupied by worker- $j$  by  $J_z^j$ . From (31) above this simply equals  $\frac{\tau' P_z - w_z^j}{(r + \delta)}$ . Since the wage is determined using Nash bargaining where the bargaining weights are  $\beta$  and  $1 - \beta$ , we get the following wage bargaining equation:

$$E_z^j - U_z^j = \beta(J_z^j + E_z^j - U_z^j) \quad (34)$$

The above implies that

$$E_z^j - U_z^j = \frac{\beta}{1 - \beta} J_z^j = \frac{\beta}{1 - \beta} \frac{c_z}{q(\theta_z)} \quad (35)$$

where the last equality follows from the fact that the value of an occupied job,  $J_z$ , equals  $\frac{c_z}{q(\theta_z)}$  as discussed in (11) in the text. Plugging the value of  $E_z^j - U_z^j$  from above into the asset value equation for the unemployed in (32) we have a simplified version of this asset value equation

$$rU_z^j = \varepsilon_z^j + b + \frac{\beta}{1 - \beta} c_z \theta_z \quad (36)$$

Next, (33) implies that

$$E_z^j - U_z^j = \frac{\varepsilon_z^j}{r + \delta} + \frac{w_z^j}{r + \delta} - \frac{rU_z^j}{r + \delta} \quad (37)$$

Use (35) to substitute out  $E_z^j - U_z^j$  and (36) to substitute out  $rU_z^j$  in the above expression to get the following simplified wage equation:

$$w_z^j = b + \frac{\beta c_z}{1 - \beta} \left[ \theta_z + \frac{r + \delta}{q(\theta_z)} \right]$$

Note that  $w_z^j$  is the same for all  $j$ . Similarly, in the case of the  $X$  sector, we obtain  $w_x = b + \frac{\beta c_x}{1 - \beta} \left[ \theta_x + \frac{r + \delta}{q(\theta_x)} \right]$ .

### 7.3 Proof of Lemma 1

It is clear that for  $P_z < \underline{P}_z^o$ , the autarky domestic labor cost in sector  $Z$  is less than  $w_s$ , and hence no offshoring takes place. Therefore, even with the possibility of offshoring, the relative supply curve for  $p < \underline{p}^o$  is the same as the autarky relative supply curve.

Next, at  $p = \underline{p}^o$ , firms are indifferent between offshoring and sourcing domestically. In this case the relative supply of  $Z$  is given by

$$\frac{(\tau^\sigma + (1 - \tau)^\sigma)^{\frac{1}{\sigma-1}} [L_z(1 - u_z) - N^O] + \tau^{-\sigma} (\tau^\sigma + (1 - \tau)^\sigma)^{\frac{\sigma}{\sigma-1}} N^O}{L_x(1 - u_x)} \quad (38)$$

where the total domestic employment of the offshoring firms is denoted by  $N^O$  and  $N^O \in [0, L_z(1 - u_z)]$ . Since  $\widehat{\varphi}$  is a function of  $p$ ,  $L_z$  and  $L_x$  are functions of  $p$  as well. Therefore, the denominator remains constant while the numerator increases with  $N^O$ .

For  $p > \underline{p}^o$ , the autarky domestic labor cost in the  $Z$  sector exceeds  $w_s$ , therefore, there is complete offshoring. With complete offshoring,  $P_z$  equals the expression on the *r.h.s.* of (23), which implies that for each  $P_z$  in this range, the domestic labor cost in the  $Z$  sector in the case of offshoring is higher than the autarky labor cost. Also, since all firms are offshoring, the offshoring relative supply is greater than the autarky relative supply.

Note that the expressions for the amount of labor going to each sector in the case of offshoring are still given by (20) and (19) with  $\widehat{\varphi}^A$  being replaced by  $\widehat{\varphi}^o$ . The relative supply for each  $p$  in the case of complete offshoring is given by

$$\frac{\tau^{-\sigma} (\tau^\sigma + (1 - \tau)^\sigma \omega^{\sigma-1})^{\frac{\sigma}{\sigma-1}} (1 - u_z)}{(1 - u_x) \exp(\widehat{\varphi}^o/\alpha)} \quad (39)$$

where  $u_i$ ,  $\omega$ , and  $\widehat{\varphi}^o$  are functions of  $p$ .

The expression in (39) makes it clear that the relative supply is independent of  $\alpha$  for  $\widehat{\varphi}^o(p) = 0$ . Therefore, the offshoring relative supply curves with different values of  $\alpha$  all pass through the same point at  $p = \underline{p}^o$ . Using the same argument as in the case of autarky, we can verify that a decrease in  $\alpha$  leads to a clockwise rotation of the relative supply curve at  $p = \underline{p}^o$ . This pins down the relative positions of the offshoring relative supply curves corresponding to  $\alpha_1$  and  $\alpha_2$ , respectively, in Figure 3.

In the limit, when  $\alpha \rightarrow 0$ ,  $\varphi^j \rightarrow 0 \forall j$ , and therefore, the relative supply curve becomes horizontal at  $p = \underline{p}^o$  since at this price the no arbitrage condition (17) is satisfied for all workers in the offshoring case.

## 7.4 Proof of Lemma 3

Assume  $c_x = c_z$  and the following production function for  $C$

$$C = \left( \gamma Z^{\frac{\phi-1}{\phi}} + (1 - \gamma) X^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}$$

where  $\phi$  is the elasticity of substitution between  $X$  and  $Z$ . The production function for  $C$  implies the following cost function.

$$\left(\gamma^\phi (P_z)^{1-\phi} + (1-\gamma)^\phi (P_x)^{1-\phi}\right)^{\frac{1}{1-\phi}} \quad (40)$$

Since  $C$  is the numeraire, the unit cost of  $C$  must equal 1. Note that the relative demand (4) for  $Z$  when the production function for  $C$  is of the CES type is given by

$$\left(\frac{Z}{X}\right)^d = \left(\frac{\gamma P_x}{(1-\gamma)P_z}\right)^\phi \quad (41)$$

The relative demand for  $Z$  equal to relative supply in autarky equilibrium can be written as

$$\frac{\tau' L_z}{L - L_z^A} = \left(\frac{\gamma P_x}{(1-\gamma)P_z}\right)^\phi \frac{(1-u_x)}{(1-u_z)} \quad (42)$$

Next,  $c_x = c_z$  implies  $\theta_x = \theta_z$ , which in turn implies  $w_x = w_z$ , and hence  $P_x = \tau' P_z$  where  $\tau' \equiv [\tau^\sigma + (1-\tau)^\sigma]^{\frac{1}{\sigma-1}}$ . Also,  $\theta_x = \theta_z$  implies  $u_x = u_z$ . Therefore, from (42)

$$\frac{L_z^A}{L - L_z^A} = \frac{1}{\tau'} \left(\frac{\gamma \tau'}{(1-\gamma)}\right)^\phi \quad (43)$$

where  $L_z^A$  is the amount of labor in the  $Z$  sector in autarky equilibrium. Note that if there was no labor market friction in the model, the expression for  $L_z$  in autarky would be exactly the same as in (43).

Similarly, the relative demand equals relative supply in the offshoring equilibrium can be written as

$$\frac{\tau^{-\sigma} (\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1})^{\frac{\sigma}{\sigma-1}} L_z}{L - L_z} = \left(\frac{\gamma P_x}{(1-\gamma)P_z}\right)^\phi \frac{(1-u_x)}{(1-u_z)} \quad (44)$$

Again,  $c_x = c_z$  implies  $\theta_x = \theta_z$  and hence  $u_x = u_z$ . Also,  $\theta_x = \theta_z$  and  $w_x = w_z$  imply  $P_x = (\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1})^{\frac{1}{\sigma-1}} P_z$ . Therefore, (44) can be written as

$$\frac{L_z^O}{L - L_z^O} = \frac{\tau^\sigma}{(\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1})^{\frac{\sigma}{\sigma-1}}} \left(\frac{\gamma (\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1})^{\frac{1}{\sigma-1}}}{(1-\gamma)}\right)^\phi \quad (45)$$

where  $L_z^O$  is the amount of labor in the  $Z$  sector in the offshoring equilibrium. Again, if there is no labor market friction then the expression for  $L_z$  in an offshoring equilibrium would be the same as in (45). The only difference would be that  $\omega$  would be the ratio of domestic wage to foreign wage rather than being the ratio of domestic labor cost to foreign wage.

Comparing (43) and (45) note that  $L_z^A > (<)L_z^O$  if the following inequality holds.

$$\left( \frac{\tau^\sigma + (1-\tau)^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} \right)^{\frac{\phi-1}{\sigma-1}} > (<) \frac{\tau^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} \quad (46)$$

We get the following possibilities:

Case I:  $\phi = 1$ . In this case the l.h.s of (46) exceeds the r.h.s if  $\frac{\tau^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} < 1$ , if which is true for any  $\sigma$ . Therefore, if the production function for  $C$  is Cobb-Douglas, then irrespective of the elasticity of substitution in  $Z$  production, labor always moves from  $Z$  to  $X$  upon offshoring.

Case II:  $\phi = \sigma$ . In this case the l.h.s of (46) exceeds the r.h.s if  $1 + \left(\frac{1-\tau}{\tau}\right)^\sigma > 1$ , which is always true implying  $L_z^A > L_z^O$ .

Case III:  $\phi < 1, \sigma > 1$ . In this case the l.h.s of exceeds 1 since  $\frac{\tau^\sigma + (1-\tau)^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} < 1$  and  $\frac{\phi-1}{\sigma-1} < 0$ , while the r.h.s is less than 1. Therefore, again  $L_z^A > L_z^O$ .

Case IV:  $\phi < 1, \sigma < 1$ . Again, the l.h.s of (46) exceeds 1 because  $\frac{\tau^\sigma + (1-\tau)^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} > 1$  and  $\frac{\phi-1}{\sigma-1} > 0$ . Therefore, again  $L_z^A > L_z^O$ .

Case V:  $\phi > 1, \sigma > 1$ . Again, the l.h.s of (46) exceeds 1 because  $\frac{\tau^\sigma + (1-\tau)^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} > 1$  and  $\frac{\phi-1}{\sigma-1} > 0$ . Therefore, again  $L_z^A > L_z^O$ .

Case VI:  $\phi > 1, \sigma < 1$ . In this case  $\sigma < 1$  implies  $\frac{\tau^\sigma + (1-\tau)^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} > 1$ , but  $\phi > 1, \sigma < 1$  implies  $\frac{\phi-1}{\sigma-1} < 0$ . Therefore, the l.h.s of (46) is less than 1. Since both the l.h.s and the r.h.s are less than 1, it is possible for the r.h.s to exceed l.h.s in which case  $L_z^A < L_z^O$ .

## 7.5 Comparing equilibria with differing degrees of labor mobility

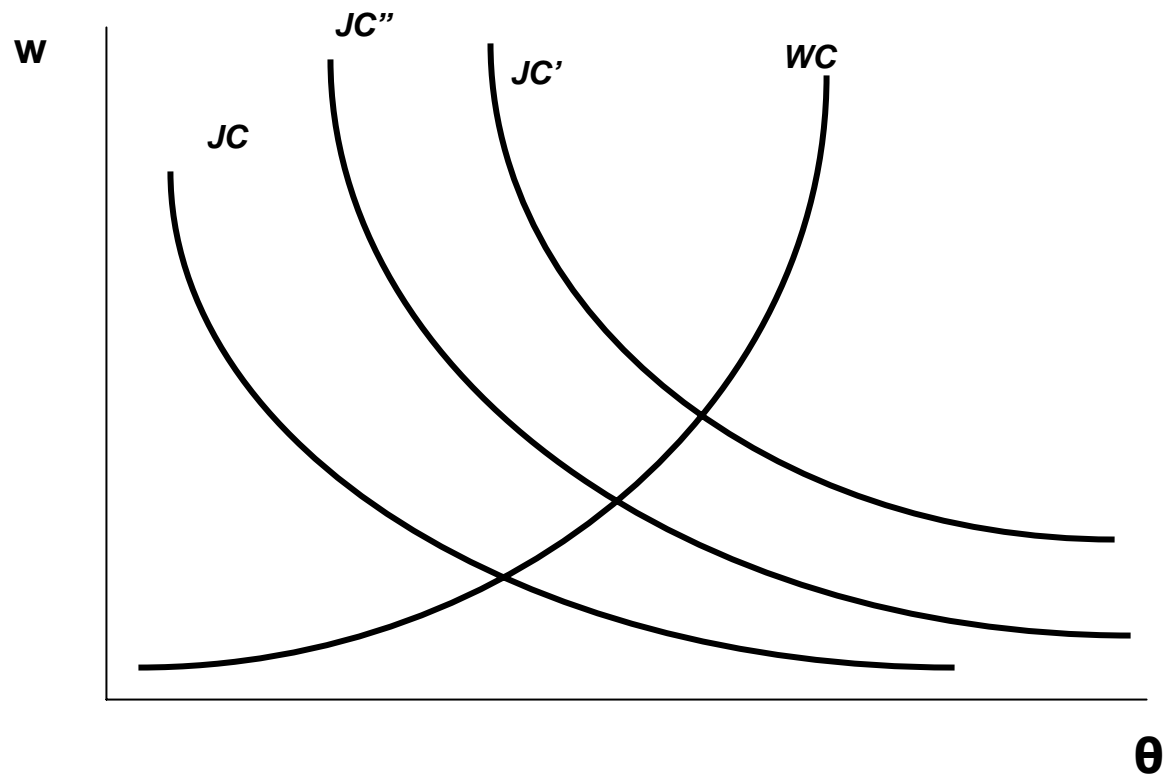
We can also compare offshoring equilibria with different degrees of labor mobility using Figure 3. Since the relative supply curves with different degrees of labor mobility all pass through the same point  $B$  at a price of  $p^\circ$  in Figure 3, the impact of extent of labor mobility depends on whether the relative demand curve passes to the left or right of point  $B$  in Figure 3, or whether the perfect mobility equilibrium is to the left of point  $B$  or to the right of it. If in the perfect mobility case labor moves from sector  $Z$  to sector  $X$ , which is the plausible scenario, as argued earlier, then we can show that the perfect mobility equilibrium at point  $C$  must be to the left of point  $B$  as shown in Figure 3.

Proof: Note from (19) above that at  $\widehat{\varphi}^A = 0$ ,  $L_x = L_z = \frac{1}{2}L$ , irrespective of  $\alpha$ . That is, each sector has exactly half of the labor in autarky equilibrium. Since (19) is also valid for offshoring with  $\widehat{\varphi}^A$  replaced by  $\widehat{\varphi}^\circ$ , the amount of labor in each sector must equal  $\frac{1}{2}L$  because  $\widehat{\varphi}^\circ = 0$  at point  $B$ . Now, if labor moves from

$Z$  to  $X$  in the case of offshoring equilibrium with perfect mobility, then the amount of labor in the  $X$  sector in offshoring equilibrium must be greater than  $\frac{1}{2}L$ . This in turn implies that the equilibrium  $Z/X$  must be lower than at  $B$ .

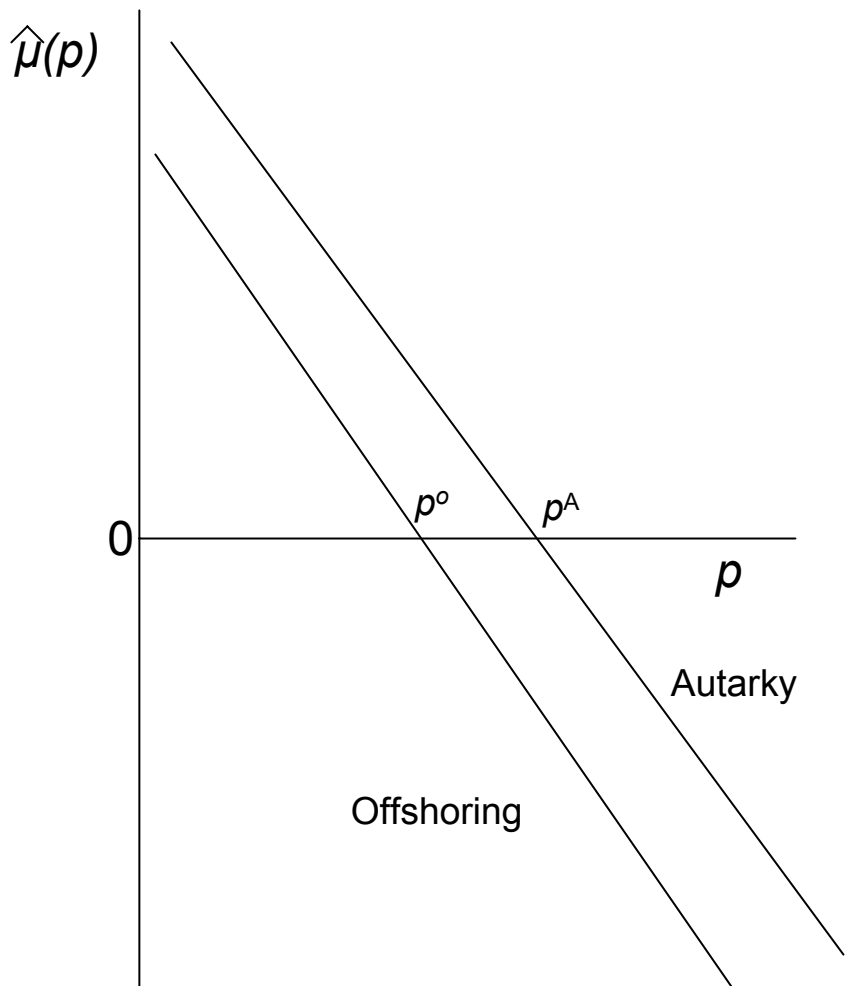
## 7.6 $w_s$ and the likelihood of a mixed equilibrium under imperfect labor mobility

We first describe the shift in the relative supply curve in response to a decrease in  $w_s$ . Recall from the definition of  $\underline{p}^o$  that it decreases as  $w_s$  decreases. From the derivation of the offshoring relative supply curve, a decrease in  $w_s$  implies that the starting point of the offshoring relative supply curve slides down the autarky supply curve. This implies that the horizontal segment of the offshoring relative supply curve slides down the autarky relative supply curve as well. We can also verify that the rightmost point of the horizontal segment is more to the left the lower the  $w_s$ . To see this, note that the lower the  $w_s$  the lower the  $\theta_z$  and consequently, the lower the  $L_z$  and the higher the  $u_z$ . Therefore, verify from (38) that the relative supply when  $N^O = L_z(1 - u_z)$  is lower the lower the  $w_s$ . Finally, (23) implies that at any  $p$  the lower the  $w_s$  the higher the domestic labor cost in the  $Z$  sector. A higher domestic labor cost implies a higher  $\theta_z$  and hence a lower  $u_z$  and a higher  $L_z$ . Therefore, at each  $p$ , the lower the  $w_s$  the greater the relative supply. Therefore, the offshoring relative supply curve shifts in response to a decrease in  $w_s$  as shown in Figure 5 for two different levels of  $w_s$  ( $w_{s1} > w_{s2}$ ). From this figure, it is clear that the likelihood of mixed equilibrium goes down as  $w_s$  goes down. The shift in the relative supply curve depicted in Figure 5 raises the possibility of non-monotonicity of  $\theta_z$  with respect to  $w_s$ .

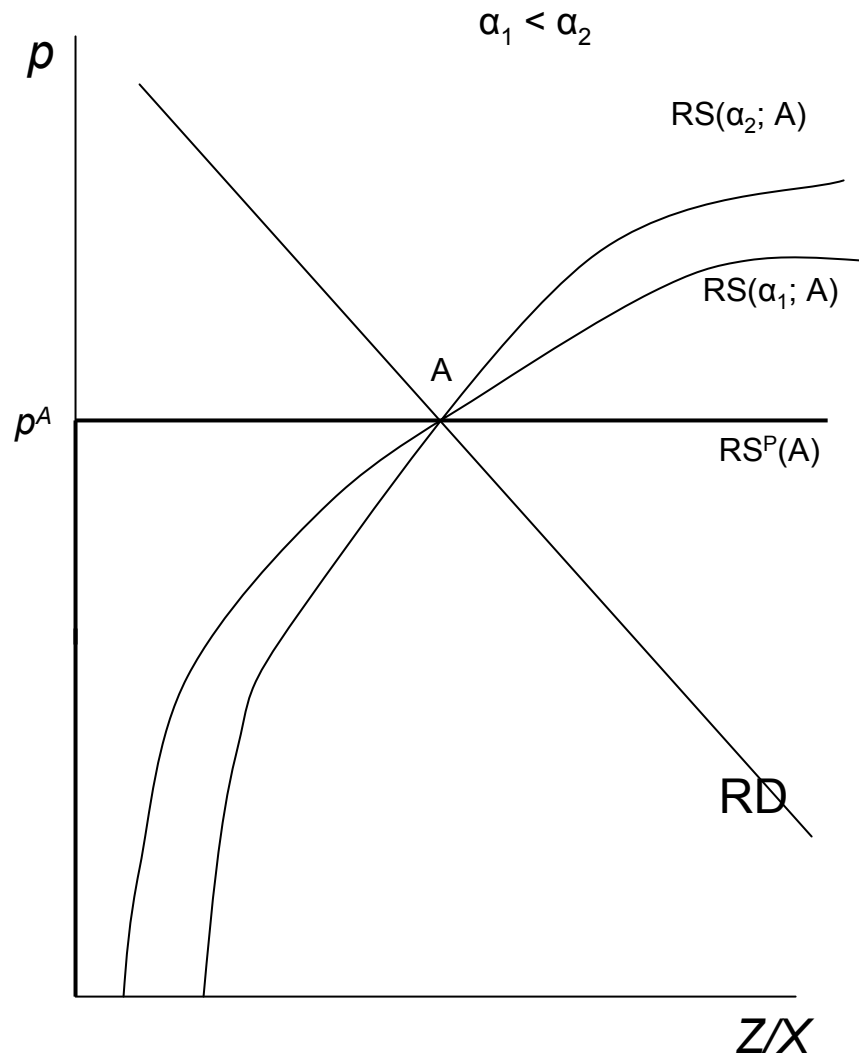


**Figure 1: Partial Equilibrium**

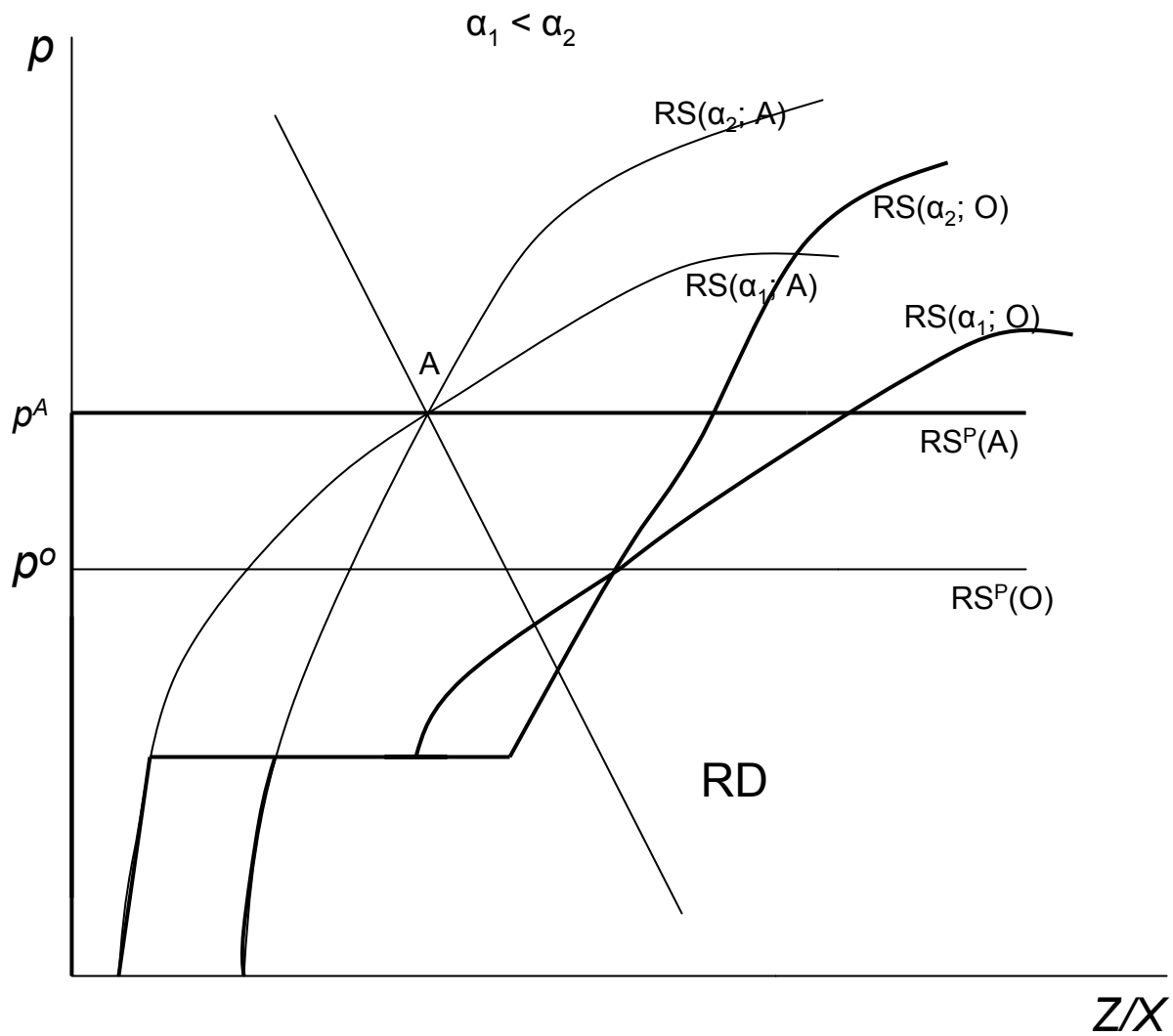




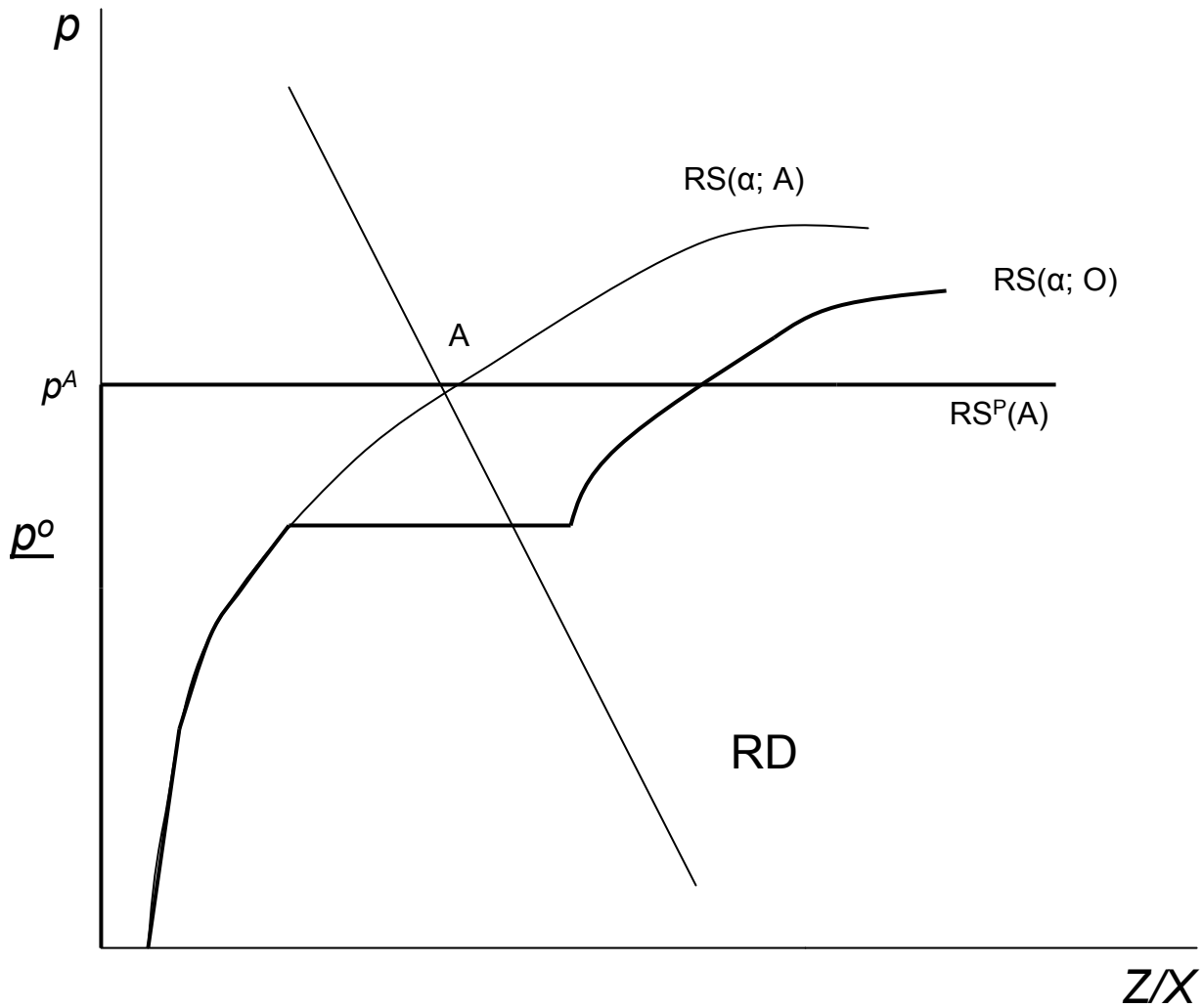
**Figure 2a: Equilibrium Prices**



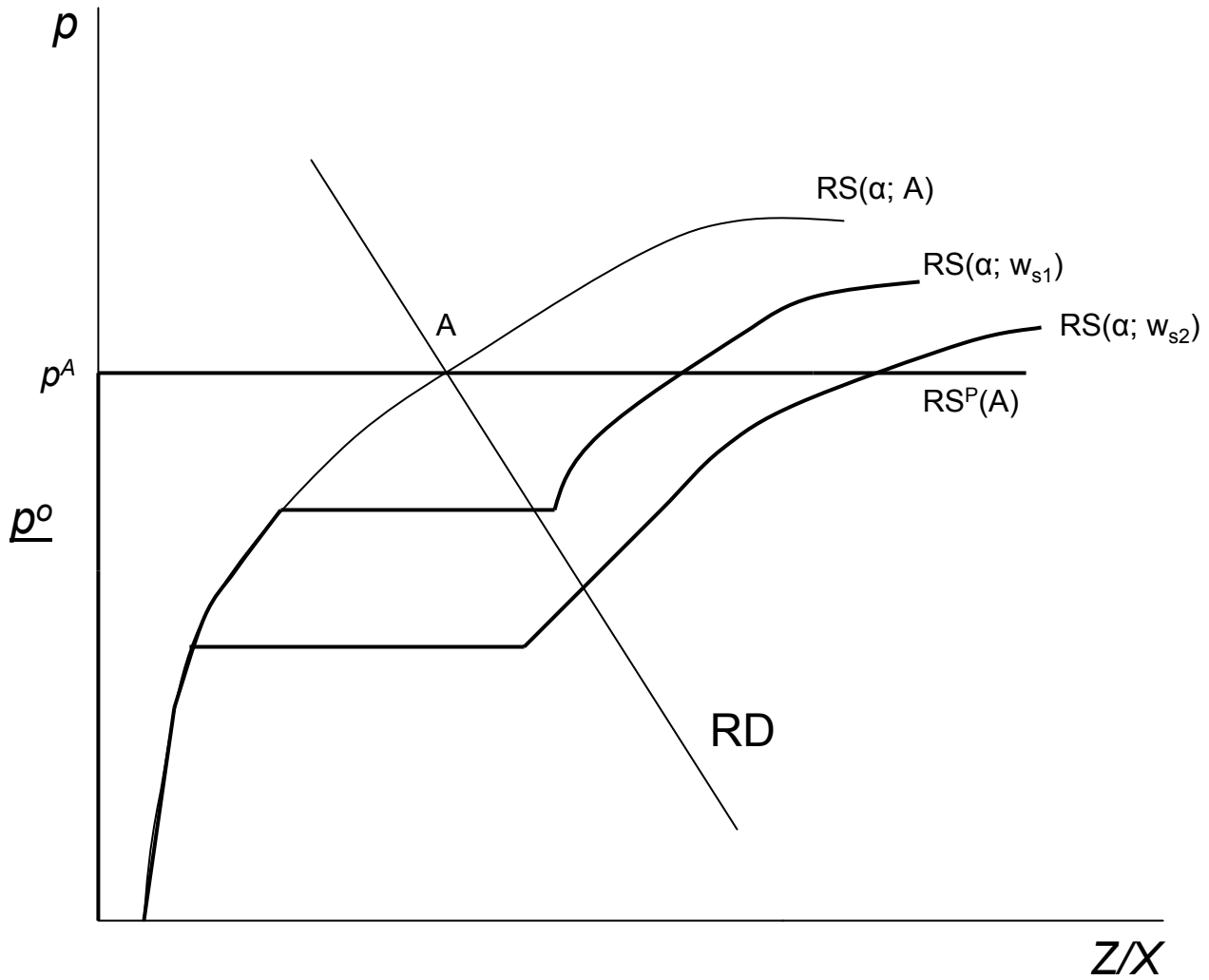
**Figure 2b: Autarky Equilibrium**



**Figure 3: Complete Offshoring Equilibrium**



**Figure 4: Mixed Offshoring Equilibrium**



**Figure 5: Change in  $w_s$**