ENVIRONMENTAL UNCERTAINTY AND THE TIMING OF ENVIRONMENTAL POLICY[1]

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This version: October 28, 2002.

Abstract

This paper shows that the timing of an investment to reduce the emissions of a stock pollutant under environmental uncertainty depends on the specification of uncertainty, on its level, and on the presence of a lower reflecting barrier for the level of pollutant. When variability increases with the level of stock pollutant and uncertainty is high enough, emissions should be curbed immediately; when uncertainty is small, however, there is no simple irreversibility effect because of the tension between environmental and investment irreversibility. Finally, a lower reflecting barrier may significantly affect the action threshold. These results have implications for global warming.

JEL classification: Q28; H23; D61; D81

Keywords: Environmental Policy; Cost-Benefit Analysis; Stock externalities; Irreversibilities; Uncertainty; Option Value; Global Warming.

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[1] Helpful comments from Ami Glazer, Neha Khanna and participants of the AERE sessions at the 2002 AAEA conference in Long Beach are gratefully acknowledged. All remaining errors are obviously my responsibility.

1. Introduction

Since the seminal work of Arrow and Fisher (1974) or Henry (1974), it is well known that uncertainty and irreversibility can strongly influence the timing of environmental policy. However, the joint effect of uncertainty and irreversibility is still debated; it has generated much interesting research over the last few years (e.g., see Conrad (1992), Kolstad (1996a-b), Chao (1995), Ulph and Ulph (1997), Narain and Fisher (1998), Kelly and Kolstad (1999), or Pindyck (2000)). This paper focuses on environmental uncertainty with passive learning;[1] it shows that, while the interplay of uncertainty with irreversibility is complex, for a large class of models a higher level of uncertainty leads to reducing earlier the emissions of a stock pollutant. Moreover, our approach highlights the importance of barriers in continuous-time stochastic investment problems.

As pointed out in the papers referenced above, environmental policy typically involves two kinds of irreversibility and numerous sources of uncertainty. Indeed, economic irreversibility occurs for example when the adoption of an environmental policy, such as installing scrubbers in electric power plants, requires sunk economic costs. On the other hand, environmental irreversibility results when environmental damage is irreversible, as for the destruction of unique ecosystems, or very long lasting, as for the accumulation of a slowly decaying stock pollutant. In addition, sources of uncertainty include changes in scientific knowledge, macroeconomic booms and busts, difficulties in valuing ecosystems, changes in tastes over generations, and the inherent variability of natural systems linked for example to temperature or precipitation.

With the prospect of global warming, there has been a heightened interest for better understanding the impact of environmental uncertainty on the timing of environmental policy. Indeed, the Intergovernmental Panel on Climate Change (IPCC 2001a) finds that extreme weather events have increased in severity and frequency during the 20^{th} century as the atmospheric concentration of greenhouse gases has gone up: heavy rainfall events as well as large-scale droughts have become more common, and warm episodes of the El Niño-Southern oscillations have been more frequent, persistent and intense. Unfortunately, current climate models are not able to predict how increasing levels of greenhouse gases will influence climate variability at the local, regional, or even at the global level. They suggest, however, that year-to-year variations of global rainfall could increase (Milly et al., 2002), so we could expect more intense rainstorms, more severe droughts, and more windstorms. This would hurt agricultural productivity, disturb transportation, damage infrastructure, and threaten many ecosystems. Developing countries would be disproportionately affected (IPCC 2001b). Munich Re, one of the world's largest reinsurers, estimates that these extreme climate events could cost more than \$300 billion dollars per year in decades to come (The Financial Times, February 19, 2001, London Edition 1).

In an insightful paper, Pindyck (2000) analyzes the interplay of economic and environmental irreversibility with uncertainty. He uses real options in continuous time and a standard cost benefit analysis framework to formulate the decision to invest in reducing the emissions of a stock pollutant. He introduces environmental uncertainty by assuming that X, the level of stock pollutant, follows the stochastic process $dX = (E - \alpha X)dt + \sqrt{v}dz$, where E designates the flow of emissions of the stock pollutant, α is its natural rate of decay, v>0 is the infinitesimal variance of X, and dz is an increment of a standard Wiener process (Karlin and Taylor 1981). Using numerical methods, Pindyck finds that, when environmental uncertainty increases (i.e., when v increases), an investment to reduce the emission of pollutant should be delayed. This result is somewhat surprising because it implies that increases in either economic or environmental uncertainty have the same effect on the timing of emission reductions.[2]

In this paper, we revisit the analysis of the impact of environmental uncertainty on the timing of environmental policy using the same framework as Pindyck (2000). First, we provide a closed-form solution for his formulation (he gives only a numerical solution), and show that a lower reflecting barrier restricting the level of stock pollutant impacts the action threshold. By not restricting the stock of pollutant to be positive, we find that Pindyck over-estimates the action threshold. We then analyze models where environmental variability increases with the level of a stock pollutant (as suggested by Pindyck in a footnote), and show that when uncertainty is large enough, it is optimal to act immediately to reduce pollutant emissions. These results illustrate the importance of barriers in continuous-time stochastic investment problems. They also highlight the importance of better understanding the link between greenhouse gas concentrations and weather variability, and the need for more realistic modeling of environmental mechanisms in economic studies. Finally, these results have important public policy implications for global warming because many recent economic papers conclude that the effect of higher uncertainty is to delay action.

This paper is organized as follows. Section 2 introduces a class of stochastic optimal stopping models in continuous time and solves the corresponding deterministic problem to get a benchmark for the impact of environmental uncertainty. Section 3 analyzes two classes of stochastic models to explore the impact of the specification of uncertainty on the benefits of investing in reducing the emissions of a stock pollutant. The last section concludes. Three appendices outline some derivations underlying our results.

2. Framework

2.1 General

Following Pindyck (2000), let us consider a stylized economy with a single stock pollutant, whose level is denoted by X. This stock pollutant could be, for example, the concentration in the atmosphere of a greenhouse gas such as CO_2, or the acidity level in a lake threatened by acidification. Because of the randomness of physical and chemical processes that contribute to its decay, X varies randomly according to the diffusion process

$$dX = (E - \alpha X)dt + \sqrt{v}\sigma(X)dz, \tag{1}$$

where E is the flow of emissions of the stock pollutant; α is its natural rate of decay; $v>0$ is a scaling factor; $\sigma(X)$ is non-decreasing, continuous, and strictly positive for $X>0$, so $v\sigma^2(X)$ is the infinitesimal variance of X; and lastly, dz is an increment of a standard Wiener process. In this paper, X denotes the random variable and x is one of its realizations. To focus only on the variability of X, E is assumed constant but it can be changed at a cost; it initially equals E_1.

Furthermore, let $C(x)$ be the flow of social costs resulting from pollution damage when the level of stock pollutant is x. By assumption,

$$C(x) = -\theta x^2, \tag{2}$$

where $\theta>0$ is a scaling constant, which may reflect tastes and technology. Note that the analysis presented herein can easily be extended to the case where $C(x)$ is a polynomial.

Following Pindyck (2000), the focus here is on policies that entail a once-and-for-all reduction in emissions, from E_1 to a constant and known $E_2 < E_1$ at a cost K. K may depend on E_1 and on $E_1 - E_2$, but not on X. Let us assume that an investment in pollution control capital is irreversible (i.e. K is sunk), which is often reasonable for pollution control measures (e.g., the installation of scrubbers by electric utilities). Allowing for just one opportunity to reduce emissions may appear restrictive, but it is not totally unrealistic as reaching a political consensus on a global environmental policy has often proven quite difficult in the past. Such policies are thus unlikely to be revised frequently.

In a benefit-cost analysis framework, the policy objective is to find the action threshold x^*, the value of X at which to reduce emissions from E_1 to E_2, in order to maximize

$$J(T) = \mathcal{E}_0\left[\int_0^T -\theta X^2 e^{-rt}\,dt - e^{-rT}K + \int_T^{+\infty} -\theta X^2 e^{-rt}\,dt\right], \tag{3}$$

subject to Equation (1) with $E=E_1$ for $0 \le t \le T$, and with $E=E_2$ for $t>T$; T is the random time at which X reaches x^* for the first time starting from the known initial level of stock pollutant, $X(0)$; $\mathcal{E}_0$ is the expectation operator for information available at time 0; and r is the social discount rate, assumed constant. When X is low, so is the flow of social costs from pollution so reducing emissions is not advantageous; this defines the "waiting region" (region 1) where waiting is optimal. When X is large enough, however, investing to reduce emissions to E_2 is attractive; this defines the "stopping region" (region 2) where investing should take place. x^* is thus simply the boundary between the waiting and the stopping regions.

The methodology for solving this standard optimal stopping problem is well known (Dixit and Pindyck 1994). Let $W(x;E)$ denote the unknown value function, where $E=E_1$ in the waiting region and $E=E_2$ in the stopping region. $W(x;E)$ verifies

$$rW(x;E) = -\theta x^2 + \frac{1}{dt}\mathcal{E}_t\{dW(X;E)\}, \tag{4}$$

which equates the social return on W (the left side of (4)) with the sum of the flow of pollution costs ($-\theta x^2$) and the expected rate of change in W (the second term on the right side of (4)). Applying Itô's lemma gives the Bellman equation:

$$rW = -\theta x^2 + (E - \alpha x)\frac{dW}{dx} + \frac{v\sigma^2(x)}{2}\frac{d^2W}{dx^2}. \tag{5}$$

Equation (5) is a second order linear ordinary differential equation so its general solution is the sum of a particular solution plus a linear combination of two independent solutions of the associated homogeneous equation (i.e., Equation (5) without $-\theta x^2$). From an economic point of view, the general solution to (5) may be written as the sum of two terms in region 1 and as a single term in region 2. The term intervening in both regions, denoted by $P(x;E)$, is the present value of expected social costs from emitting pollution at rate E forever (with $E=E_1$ in region 1 and $E=E_2$ in region 2) given x, the current stock of pollutant (Dixit 1993).[3] The second term, denoted by $\varphi(x;E_1)$, exists only in the waiting region in our simple framework. It represents the value of the option to invest in reducing emissions, so it is non-negative. $\varphi(x;E_1)$ verifies the homogeneous equation associated to the Bellman equation (5).[4]

When an analytical solution is available, $\varphi(x;E_1)$ is typically multiplied by an unknown constant that needs to be jointly determined with x^*. To solve for these two unknowns, we use the value-matching and smooth-pasting conditions:

$$\varphi(x^*;E_1) + P(x^*;E_1) = P(x^*;E_2) - K, \tag{6}$$

$$\frac{d\varphi(x^*;E_1)}{dx} + \frac{dP(x^*;E_1)}{dx} = \frac{dP(x^*;E_2)}{dx}. \tag{7}$$

Combining (6) and (7) after isolating the option term gives the "stopping rule"

$$\frac{\varphi'(x^*;E_1)}{\varphi(x^*;E_1)} = \frac{P'(x^*;E_2) - P'(x^*;E_1)}{P(x^*;E_2) - P(x^*;E_1) - K}, \tag{8}$$

which equates the stock semi-elasticity of the option to reduce emissions at x^* with the stock semi-elasticity of the net benefits from reducing emissions. Derivatives are with respect to x.

2.2 Additional conditions

For our problem to be well posed, however, we also need to insure that X cannot take negative values by imposing, if necessary, a lower barrier on X. This makes good sense from a physical point of view, and it is necessary economically even though damages are quadratic because X is not symmetrical with respect to 0. In fact, we show below that allowing for negative values of X may seriously bias the timing of the decision to invest in reducing emissions. We thus need to explore whether the specification of $\sigma(.)$ in (1) allows for negative values of X, or equivalently, we need to investigate whether X can become 0 since the paths of a diffusion are continuous. This type of considerations appears to have been overlooked in the economics literature.

From Karlin and Taylor (1981), there are *only* two possible situations in our context: either 0 can be reached from *all* positive starting value of X, or it cannot be reached from *any* positive starting value of X. This property results from the expression of $p_{0,b}(x_0)$, the probability that X hits $b>0$ before 0 starting from $x_0 \in (0,b)$. Given $0<a<b$, $p_{0,b}(x_0)$ can be derived by taking the limit when $a \to 0+$ of $p_{a,b}(x_0)$, the probability that X hits $b>0$ before a starting from $x_0 \in (a,b)$. Karlin and Taylor (1981) show that

$$p_{a,b}(x_0) = \frac{S(x_0,b)}{S(a,b)}, \tag{9}$$

where, for $0<x_1<x_2$,

$$S(x_1,x_2) = \int_{x_1}^{x_2} \exp\left\{-\int_c^{\eta} 2\frac{E-\alpha\xi}{\sigma^2(\xi)}d\xi\right\}d\eta. \tag{10}$$

In the above, $c>0$ is an arbitrary constant; its value has no impact on $p_{a,b}(x_0)$ because changing c is equivalent to multiplying both $S(x_0,b)$ and $S(a,b)$ by the same number. Moreover, $S(x_1,x_2)$ is well defined because $\sigma(\xi)>0$ for $\xi>0$ by assumption.

From the expressions of $p_{a,b}(x_0)$ and $S(.,.)$ (Equations (9) and (10)), we don't need to worry about X taking negative values if $\lim_{x_1\to 0+} S(x_1,x_2)=\infty$ because $\lim_{a\to 0+} p_{a,b}(x_0)=1$ (b is reached before 0 with certainty) if and only if $\lim_{x_1\to 0+} S(x_1,x_2)=\infty$. If $\lim_{x_1\to 0+} S(x_1,x_2)$ is finite, however, a reflecting barrier is needed to prevent X from taking negative values. If we consider a barrier at 0 for simplicity, then for i=1,2, the following conditions are required (e.g., see Saphores 2002 who extends a result by Dixit 1993):

$$\frac{dP(x;E_i)}{dx}\bigg|_{x=0}=0, \text{ and } \frac{d\varphi(x;E_1)}{dx}\bigg|_{x=0}=0. \tag{11}$$

2.3 The deterministic case

Before analyzing the stochastic case, it is useful to recall the deterministic solution of this problem, because it gives a benchmark against which we can assess the impact of uncertainty (for derivations, see Saphores and Carr 2000).

When v=0 (i.e., when there is no uncertainty), the present value of social costs when pollutant emissions are constant and equal to E_i, i=1,2, is

$$P_0(x;E_i)=-\frac{\theta x^2}{r+2\alpha}-\frac{2\theta E_i x}{(r+\alpha)(r+2\alpha)}-\frac{2\theta E_i^2}{r(r+\alpha)(r+2\alpha)}. \tag{12}$$

If $X(0)<\frac{E_1}{\alpha}$, pollution and the flow of social costs are increasing. The (deterministic) option term, which represents the value of the flexibility to invest at the optimal time, is

$$\varphi_0(x;E_1) = A_0 \left|\alpha x - E_1\right|^{\frac{-r}{\alpha}}, \tag{13}$$

where $A_0 \geq 0$ is a constant to be determined jointly with $x^*(0)$, the critical stock of pollutant at which it is optimal to reduce emissions from E_1 to E_2.[5] In this case, a conventional (static) benefit-cost analysis is biased because it ignores the option term: indeed, it prescribes to act as soon as $P(x;E_1) \geq P(x;E_2)$ - K. Equation (8) can be solved explicitly and

$$x^*(0) = \frac{r(r+2\alpha)K}{2\theta(E_1 - E_2)} - \frac{E_2}{r+\alpha}. \tag{14}$$

If $X(0) > \frac{E_1}{\alpha}$, however, pollution is decreasing and there is no option value. In this case, a conventional cost-benefit analysis yields the correct decision. The intuition behind this result is simple: as social damages decrease over time but the cost of decreasing emissions is fixed, it is optimal to act either immediately or never.

There is no need to consider the initial value of X when it varies stochastically because diffusions are Markov processes (the current state summarizes all the relevant information) and the level of stock pollutant increases and decreases over time.

3. Stochastic models

3.1 Constant infinitesimal variance

To start with, let us first assume, as in Pindyck (2000), that $\sigma(X) = 1$. X thus follows the well-known Ornstein-Uhlenbeck process (Karlin and Taylor 1981)

$$dX = (E - \alpha X)dt + \sqrt{v}dz. \tag{15}$$

Pindyck (2000) uses numerical techniques to calculate x^*, but he does not seem to worry about X taking negative values; he finds that x^* increases to infinity with v. Below, we impose a reflecting barrier at 0 and derive analytical expressions for both $P(x{:}E)$ and $\varphi(x;E)$. We then show that x^* admits a finite limit when v increases to infinity.

Let first us inquire whether a reflecting barrier is needed to prevent negative values of X. From (10), $S(x_1,x_2)=\exp\left\{-\frac{\alpha}{v}\left(c-\frac{E}{\alpha}\right)^2\right\}\int_{x_1}^{x_2}\exp\left\{\frac{\alpha}{v}\left(\eta-\frac{E}{\alpha}\right)^2\right\}d\eta,$ so for $x_0\in(0,x)$,

$$0<p_{x,0}(x_0)=1-p_{0,x}(x_0)=1-\frac{\int_0^{x_0}\exp\left\{\frac{\alpha}{v}\left(\eta-\frac{E}{\alpha}\right)^2\right\}d\eta}{\int_0^{x}\exp\left\{\frac{\alpha}{v}\left(\eta-\frac{E}{\alpha}\right)^2\right\}d\eta}<1. \tag{16}$$

There is therefore a risk that X will reach 0 before $x>0$ starting from $x_0\in(0,x)$. We thus impose a reflecting barrier at 0 to prevent negative values of X.[6]

Let us now derive the option term $\varphi(x;E_1)$. A series expansion after the change of variable $y\equiv E-\alpha x$ in the homogeneous equation associated to our Bellman equation gives the two solutions $\varphi_1(x;E)$ and $\varphi_2(x;E)$, defined by

$$\begin{cases}\varphi_1(x;E)=\Phi\left(\frac{r}{2\alpha},\frac{1}{2},\frac{\alpha}{v}\left(x-\frac{E}{\alpha}\right)^2\right),\\ \varphi_2(x;E)=\left(x-\frac{E}{\alpha}\right)\Phi\left(\frac{r}{2\alpha}+\frac{1}{2},\frac{3}{2},\frac{\alpha}{v}\left(x-\frac{E}{\alpha}\right)^2\right).\end{cases} \tag{17}$$

$\Phi(a,c,z)$ is the confluent hypergeometric function of the first kind.[7] Since $\varphi_1(x;E)$ and $\varphi_2(x;E)$ are respectively even and odd functions of x-E/α, they are independent, so $\varphi(x;E_1)$ can be written as a linear combination of $\varphi_1(x;E_1)$ and $\varphi_2(x;E_1)$. If we now create a reflecting barrier at 0 by imposing $\frac{d\varphi(x;E_1)}{dx}|_{x=0}=0$ (see Equation (11)), we find

$$\varphi(x;E_1) = F\left\{\varphi_1(x;E_1) - \frac{\varphi_1^{'}(0;E_1)}{\varphi_2^{'}(0;E_1)}\varphi_2(x;E_1)\right\}, \tag{18}$$

where F is an unknown constant that disappears in (8).

To find $P(x;E)$, the present value of expected social costs when there is a reflecting barrier at 0, we proceed by substitution: it is easy to verify that our Bellman Equation (Equation (5)) admits the quadratic solution

$$Q(x;E) = -\frac{\theta x^2}{r+2\alpha} - \frac{2\theta Ex}{(r+\alpha)(r+2\alpha)} - \frac{\theta}{r(r+2\alpha)}\left[v + \frac{2E^2}{r+\alpha}\right]. \tag{19}$$

From our discussion following (5), $P(x;E)$ may thus be written

$$P(x;E) = Q(x;E) + A(E)\varphi_1(x;E) + B(E)\varphi_2(x;E). \tag{20}$$

To find $A(E)$ and $B(E)$, we assume, in addition to $\frac{dP(x;E_i)}{dx}|_{x=0} = 0$ (Equation (11)), that X is constrained by an upper reflecting barrier. This gives us two equations and two unknowns, so we can solve for $A(E)$ and $B(E)$. Taking the upper reflecting barrier to infinity, we find

$$A(E) = \frac{\lambda\sqrt{\alpha v}Q^{'}(0;E)}{\varphi_2^{'}(0;E) - \lambda\sqrt{\alpha v}\varphi_1^{'}(0;E)},\; B(E) = \frac{-Q^{'}(0;E)}{\varphi_2^{'}(0;E) - \lambda\sqrt{\alpha v}\varphi_1^{'}(0;E)}, \tag{21}$$

where $\lambda \equiv \Gamma\left(\frac{r}{2\alpha}+1\right)\left[r\Gamma\left(\frac{r}{2\alpha}+\frac{1}{2}\right)\right]^{-1}$ is just a constant. Derivations are summarized in Appendix A. In the expression of $P(x;E)$ (Equation (20)), $Q(x;E)$ represents the present value of expected social costs without the reflecting barrier at 0 and $A(E)\varphi_1(x;E) + B(E)\varphi_2(x;E)$ is the correcting term for this reflecting barrier.[8] To see this, we would repeat the derivations above with a lower reflecting barrier at X=L, and then take L to -∞. Moreover, we see that setting v to 0 in $Q(x;E)$ (Equation (19)) gives, as expected, the present value of social costs in the deterministic case (Equation (12)).

When we introduce the expression of the option term (Equation (18)) and of the present value of expected social costs (Equation (20)) into the stopping rule (8), we get the equation that needs to be solved numerically to find $x^*(v)$. Before proceeding, however, it is of interest to examine the behavior of $x^*(v)$ for large values of v. After some algebra (derivations are summarized in Appendix B), we find that

$$x^*(+\infty) = 2\left(x^*(0) + \frac{E_2}{r+\alpha}\right), \tag{22}$$

where $x^*(0)$ is the deterministic stopping value (Equation (14)). Thus, when v is "large", $x^*(v)$ does not grow infinitely as suggested in Pindyck (2000), but it tends asymptotically towards $x^*(+\infty)>x^*(0)$.

To explore the characteristics of the solution and compare with Pindyck's results, let us suppose that r=0.04 per year, α=0.02 per year, θ=0.002 (in billions dollars/(million tons)^2), E_1=0.3 million tons per year, E_2=0 million tons per year, and K=\$4 billion. For these parameter values, $x^*(0)$=10.67 and $x^*(+\infty)$=21.33 or exactly twice $x^*(0)$, as anticipated from (22) since E_2=0 here. Table 1 contrasts x^* when a reflecting barrier is present at 0 with the stopping values obtained by Pindyck (2000). For $v\leq 1$, results are identical. This is expected because when uncertainty is small, the lower reflecting barrier should have little impact as X tends to revert to E/α. However, results can differ significantly when v>1: for v=16, the presence of the barrier lowers x^* from 25.75 to 18.5. If we omit the barrier (i.e., if we assume that $P(x;E)\sim Q(x;E)$), we find results fairly similar to Pindyck's. Ignoring the reflecting barrier at X=0 may thus severely bias the decision to invest in reducing emissions.

Figure 1 presents results for the same values of θ, E_1, E_2, and K, but for r=0.02 per year, $\alpha\in\{0.02,0.03,0.04\}$ per year, and a large range of values of v. First, we observe that

$x^*(v)$ increases with v. Intuitively, the reason may be that the net benefits from reducing pollutant emissions change with v only through barrier effects (when calculating $P(x;E_2)$-$P(x;E_1)$-K, consider (19)); as social costs increase relatively slowly (linearly) with higher values of v, it is optimum to adopt a higher action threshold, which will be reached faster anyway with a higher v. Moreover, as expected, x^* increases with α, since higher values of α decrease future levels of stock pollutant and thus also the present value of expected social costs from pollution; it thus takes a higher level of stock pollutant to justify an investment that reduces pollutant emissions. For the same reason, x^* increases with r (results are not shown here). Similar results were obtained for other values of θ, E_1, E_2, and K.

3.2 Linearly increasing infinitesimal variance

Let us assume instead that the variability of X (its infinitesimal variance) increases with X according to $\sigma(X)= X$. Equation (1) then becomes

$$dX = (E - \alpha X)dt + \sqrt{v}Xdz. \tag{23}$$

Proceeding as before, we derive $p_{0,x}(x_0)$, for $0<x_0<x$. From (10), for $0<x_1<x_2$, $S(x_1,x_2) = c^{-\frac{2\alpha}{v}} \exp\left\{-\frac{2E}{cv}\right\} \int_{x_1}^{x_2} \eta^{\frac{2\alpha}{v}} \exp\left\{\frac{2E}{\eta v}\right\} d\eta$ so $\lim_{x_1 \to 0+} S(x_1,x_2) = \infty$. As a result (see the definition of $p_{0,x}(x_0)$ from Equation (9)), $p_{0,x}(x_0) = 1$ and the level of stock pollutant cannot reach zero or become negative since diffusions have continuous trajectories (Karlin and Taylor 1981).

To find the option term, we need to solve the homogeneous equation associated to our Bellman Equation (Equation (5)). After performing the change of variables $y = \frac{2E}{vx}$ and

$V(y) \equiv \left(\frac{E}{x}\right)^{-\beta} W(x)$, where β is conveniently chosen as

$$\beta = \frac{-\left(\frac{v}{2}+\alpha\right)+\sqrt{\left(\frac{v}{2}+\alpha\right)^2+2rv}}{v}, \tag{24}$$

we find Kummer's equation.[9] Hence, two independent solutions are

$$\begin{cases} \varphi_3(x;E) = \left(\frac{E}{x}\right)^{\beta} \Phi\left(\beta, 2(\beta+1+\frac{\alpha}{\mathrm{v}}); \frac{2E}{vx}\right), \\ \varphi_4(x;E) = \left(\frac{E}{x}\right)^{\beta} \Psi\left(\beta, 2(\beta+1+\frac{\alpha}{\mathrm{v}}); \frac{2E}{vx}\right), \end{cases} \tag{25}$$

where $\Phi(a,c,z)$ and $\Psi(a,c,z)$ are respectively the confluent hypergeometric function of the first and second kinds.[10] Since $\varphi(x;E_1)$ should be well defined at x=0, it cannot contain $\varphi_3(x;E_1)$ because $\lim_{x\to 0+} \varphi_3(x;E_1) = +\infty$.[11] As a result,

$$\varphi(x;E_1) = G\varphi_4(x;E_1), \tag{26}$$

where G is again an unknown constant that disappears in our stopping rule (Equation (8)).

Let us now look for the present value of expected social costs. As before, we try a quadratic solution for our Bellman Equation (Equation (5)). We obtain

$$Q(x;E) = -\frac{\theta x^2}{r+2\alpha-v} - \frac{2\theta Ex}{(r+\alpha)(r+2\alpha-v)} - \frac{2\theta E^2}{r(r+\alpha)(r+2\alpha-v)}, \tag{27}$$

so $P(x;E)$ can be written as the sum of $Q(x;E)$ plus a linear combination of $\varphi_3(x;E)$ and $\varphi_4(x;E)$. However, since $P(x;E)$, should be finite at x=0, it cannot include $\varphi_3(x;E)$ either. To find the coefficient in front of $\varphi_4(x;E)$ in $P(x;E)$, we assume the presence of an upper reflecting barrier, solve for the present value of expected costs, and take the upper reflecting barrier to infinity. We find that the contribution of $\varphi_4(x;E)$ disappears (see Appendix C), so

$$P(x;E) = Q(x;E) = -\frac{\theta x^2}{r+2\alpha-v} - \frac{2\theta Ex}{(r+\alpha)(r+2\alpha-v)} - \frac{2\theta E^2}{r(r+\alpha)(r+2\alpha-v)}. \qquad (28)$$

Setting v to 0 in $P(x;E)$ above gives again the expression of the present value of social costs under certainty. In addition, a comparison between the present value of expected social costs without a barrier at 0 for both stochastic models (Equations (19) and (28)) shows that $P(x;E)$ is much more sensitive to v when the variability of X varies with X (Equation (28)) than when the variability of X is constant (Equation (19)). In the latter, v can take any non-negative value, whereas in the former, v needs to be smaller than $r+2\alpha$ for the present value of expected social costs to be finite. This has important consequences: indeed, when v is large enough, $x^*(v)$ goes to zero so it is optimal to act immediately because the present value of expected social costs goes to infinity while K, the cost of reducing emissions from E_1 to $E_2<E_1$, remains unchanged.

To show this result, we look for a value of v that sets x^* to 0 in our stopping rule (Equation (8)). Let us denote this value of v by v^*. Inserting the approximation $\Psi(a,c,z) = z^{-a}(1 - \frac{a(1+a-c)}{z} + o(\frac{1}{z}))$ when z→+∞, (see (1) page 127 in Luke 1969) in (8) and using the identity $v\beta(1+\beta+\frac{2\alpha}{v}) = 2r$, we get

$$v^* = r + 2\alpha - \frac{2\theta E_2(E_1 - E_2)}{r(r+\alpha)K}, \qquad (29)$$

which is clearly smaller than $r+2\alpha$, the maximum permissible value of v for which $P(x;E)$ is finite. Moreover, if $x^*(0)>0$ (see (14)), a bit of algebra shows that $v^*>0$.

To explore the characteristics of the solution, let us consider again a numerical example using data based on Pindyck (2000). Suppose, as before that α=0.02 per year,

θ=0.002 (in billions dollars/(million tons)^2), E_1=0.3 million tons per year, E_2=0 million tons per year, and K=\$4 billion. Figures 2 and 3 show the variations of $x^*(v)$ with v for r=0.02 and 0.04 per year respectively, for $\alpha \in \{0.02, 0.03, 0.04\}$ per year and a range of values of v. For the same reason as before, x^* increases with α or r, but now $x^*(v)$ is no longer an increasing function of v. Although x^* may increase or decrease with v for small values of v (see Figure 2), it goes to zero when v is large enough (but always smaller than $r+2\alpha$), in accordance with (29). These results mean that when the volatility of the stock of pollutant is small, the timing of reducing the emissions of a stock pollutant calculated from the deterministic model could be biased upwards or downwards. Depending on the model parameters, either environmental or investment irreversibility could dominate, so there is no simple irreversibility effect. However, when uncertainty is large enough, it is optimal to reduce emissions immediately.

Another point should be noted: when we solve Equation (8) for increasingly small values of v and find that $\lim_{v \to 0+} x^*(v) > \frac{E_1}{\alpha}$, $x^*(0^+)$ is not given by the deterministic model (Equation (14)), because as argued above there is no (deterministic) option value when the stock of pollutant is decreasing. Instead, by solving for $x^*(0^+)$ from the continuity condition (Equation (6)) with $\varphi(x^*(0^+); E_1) = 0$, we find

$$x^*(0^+) = \frac{(r+\alpha)(r+2\alpha)K}{2\theta(E_1 - E_2)} - \frac{E_1 + E_2}{r}. \tag{30}$$

From (14) and (30), it is easy to see that $x^*(0^+) \geq \frac{E_1}{\alpha}$ if and only if $x^*(0) \geq \frac{E_1}{\alpha}$; moreover, $x^*(0^+) - x^*(0) = \frac{\alpha}{\alpha + r}\left\{x^*(0^+) - \frac{E_1}{\alpha}\right\} \geq 0$. For example, we see from Figure 3 that for α=0.04, $x^*(0^+) = 24.5$ but a simple calculation gives $x^*(0) = 16$. The limit when uncertainty goes to zero of the stochastic stopping value is thus not always equal to the deterministic stopping value, because as discussed above, uncertainty creates an option value and gives some flexibility, even if this flexibility is vanishingly small.

4. Conclusions

This paper has analyzed the tension between environmental irreversibility and irreversibility in pollution control capital investment under environmental uncertainty using a real options approach in continuous-time. This formulation allows for a better treatment of the dynamics of stock pollutants than a 2 or 3 periods discrete time model.

First, we have shown that the impact of environmental uncertainty is model dependant. Second, for models where the variance of the stock of pollutant varies linearly, we find that when uncertainty is high enough it is optimal to invest immediately to reduce emissions. When uncertainty is small, however, the bias introduced by neglecting uncertainty and following the recommendations of the corresponding deterministic model cannot be known a-priori. There is thus no simple “irreversibility effect” when more than one type of irreversibility is present. Finally, the impact of the lower reflecting barrier on the action threshold illustrates the importance of barriers in stochastic investment problems.

These results have important public policy implications for the management of stock pollutants in general and for global warming in particular. They show that it is essential to better understand the induced changes in variability resulting from increases in the level of a stock pollutant (not just increases in the mean global temperature for greenhouse gases), and that high levels of environmental uncertainty may warrant early action.

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Appendix A

In this appendix, we derive the present value of expected damages, $P(x;E)$, with a reflecting barrier at x=0. Initially, the two coefficients $A(E)$ and $B(E)$ in Equation (20) are unknown. Another condition, in addition to $\frac{dP(x;E)}{dx}|_{x=0}=0$ in Equation (11), is thus needed. Thus, let us assume that there is an upper reflecting barrier at $L+\frac{E}{\alpha}$, solve for $A(E)$ and $B(E)$, and take the limit when $L\to\infty$. As X tends to revert to $\frac{E}{\alpha}$, it is clear that the upper reflecting barrier has no impact on $P(x;E)$ when L is large enough. With the two conditions $\frac{dP(x;E)}{dx}|_{x=0}=0$ and $\frac{dP(x;E)}{dx}|_{x=L+\frac{E}{\alpha}}=0$, we get the system of 2 equations with 2 unknowns

$$\begin{pmatrix} \varphi_1^{'}(0;E) & \varphi_2^{'}(0;E) \\ \varphi_1^{'}(L+\frac{E}{\alpha};E) & \varphi_2^{'}(L+\frac{E}{\alpha};E) \end{pmatrix} \begin{pmatrix} A(E) \\ B(E) \end{pmatrix} = \begin{pmatrix} -P^{'}(0;E) \\ -P^{'}(L+\frac{E}{\alpha};E) \end{pmatrix}. \tag{A.1}$$

Solving (A.1), we get

$$\begin{cases} A(E) = \dfrac{-Q^{'}(0;E)\varphi_2^{'}(L+\frac{E}{\alpha};E)+Q^{'}(L+\frac{E}{\alpha};E)\varphi_2^{'}(0;E)}{\varphi_1^{'}(0;E)\varphi_2^{'}(L+\frac{E}{\alpha};E)-\varphi_2^{'}(0;E)\varphi_2^{'}(L+\frac{E}{\alpha};E)}, \\ B(E) = \dfrac{Q^{'}(0;E)\varphi_1^{'}(L+\frac{E}{\alpha};E)-Q^{'}(L+\frac{E}{\alpha};E)\varphi_1^{'}(0;E)}{\varphi_1^{'}(0;E)\varphi_2^{'}(L+\frac{E}{\alpha};E)-\varphi_2^{'}(0;E)\varphi_2^{'}(L+\frac{E}{\alpha};E)}. \end{cases} \tag{A.2}$$

From (1) page 117 in Luke (1969), $\frac{d\Phi(a,c,z)}{dz}=\frac{a}{c}\Phi(a+1,c+1,z)$, so

$$\varphi_1^{'}(x;E)=\frac{2r}{v}\left(x-\frac{E}{\alpha}\right)\Phi\left(\frac{r}{2\alpha}+1,\frac{3}{2},\frac{\alpha}{v}\left(x-\frac{E}{\alpha}\right)^2\right), \tag{A.3}$$

$$\varphi_2'(x;E) = \Phi\left(\frac{r}{2\alpha}+\frac{1}{2},\frac{3}{2},\frac{\alpha}{v}\left(x-\frac{E}{\alpha}\right)^2\right)+ \\ \left(x-\frac{E}{\alpha}\right)^2 \frac{2}{3}\frac{r+\alpha}{v}\Phi\left(\frac{r}{2\alpha}+\frac{3}{2},\frac{5}{2},\frac{\alpha}{v}\left(x-\frac{E}{\alpha}\right)^2\right). \tag{A.4}$$

From (5) page 128 in Luke (1969),

$$\Phi(a,c,z) \sim \frac{\Gamma(c)}{\Gamma(a)} e^z z^{a-c} \tag{A.5}$$

when z is large, so

$$\varphi_1'(L+\frac{E}{\alpha};E) \sim \frac{2r}{\sqrt{\alpha v}} \frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{r}{2\alpha}+1\right)} e^{\frac{\alpha L^2}{v}} \left(\frac{\alpha L^2}{v}\right)^{\frac{r}{2\alpha}}, \tag{A.6}$$

$$\varphi_2'(L+\frac{E}{\alpha};E) \sim \frac{2}{3}\left(1+\frac{r}{\alpha}\right) \frac{\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(\frac{r}{2\alpha}+\frac{3}{2}\right)} e^{\frac{\alpha L^2}{v}} \left(\frac{\alpha L^2}{v}\right)^{\frac{r}{2\alpha}}. \tag{A.7}$$

Inserting (A.6) and (A.7) into (A.2) and simplifying gives the expression of $A(E)$ and $B(E)$ in Equation (21).

Appendix B

Here, we outline the derivation of $x^*(v)$ for v "large" when X has a constant variance (Equation (15)). Let us first look for an equivalent to the constant F in the expression of the option term (Equation (18)) using the smooth-pasting condition (Equation (7)). Applying (A.5) in (A.3) and (A.4) gives

$$\varphi_1'(x;E) \sim \frac{2r}{v}\left(x-\frac{E}{\alpha}\right)\left\{1+\frac{r+2\alpha}{3v}\left(x-\frac{E}{\alpha}\right)^2 + o\left(\frac{1}{v}\right)\right\}, \tag{B.1}$$

$$\varphi_2'(x;E) \sim 1 + (r+\alpha)\left(x - \frac{E}{\alpha}\right)^2 \frac{1}{v} + o\left(\frac{1}{v}\right). \tag{B.2}$$

Introducing (B.1) and (B.2) into the expression of $A(E)$ and $B(E)$ (Equation (21)) leads to

$$\begin{cases} A(E) \sim \lambda\left[\sqrt{\alpha v} - 2rE\lambda\right] Q'(0;E) + o(1), \\ B(E) \sim \left[-1 + \dfrac{2r\lambda E}{\sqrt{\alpha v}} + \dfrac{E^2}{\alpha}\left(\dfrac{r+\alpha}{\alpha} - 4r\lambda^2\right)\dfrac{1}{v}\right] Q'(0;E) + o(1), \end{cases} \tag{B.3}$$

where it is important to note that

$$Q'(0;E) = \frac{-2\theta E}{(r+\alpha)(r+2\alpha)} \equiv q(E) \tag{B.4}$$

does not depend on v. When we insert (A.3), (A.4), and (B.3) into the smooth pasting condition (Equation (7)), we find that

$$F \sim \left(\lambda - \frac{r+\alpha}{2r}x\right)[q(E_2) - q(E_1)] + \left(\frac{r+\alpha}{r\alpha} - 2r\lambda^2\right)[E_2 q(E_2) - E_1 q(E_1)], \tag{B.5}$$

when v is large. Moreover

$$\varphi_1(x;E_1) - \frac{\varphi_1'(0;E_1)}{\varphi_2'(0;E_1)}\varphi_2(x;E_1) \sim 1 + o(1), \tag{B.6}$$

and

$$P(x;E_1) - P(x;E_2) \sim \lambda[q(E_1) - q(E_2)]\sqrt{\alpha v} + \left\{\frac{1}{r} - 2r\lambda^2 + \frac{1}{\alpha}\right\}[E_1 q(E_1) - E_2 q(E_2)], \tag{B.7}$$

so when (B.5), (B.6), and (B.7) are combined, we get $x^*(+\infty)$ in Equation (22).

Appendix C

Let us now prove that the present value of expected social costs is giving by Equation (28). Suppose that there exists an upper reflecting barrier at x=L>0, so that the present value of expected damages, denoted here $P_L(x;E)$, equals

$$P_L(x;E) = Q(x;E) + C_L \varphi_4(x;E), \tag{C.1}$$

where C_L is such that $\frac{dP_L(x;E)}{dx}|_{x=L} = 0$. From (15) page 118 in Luke (1969),

$$\frac{d\Psi(a,c,z)}{dz} = -a\Psi(a+1,c+1,z), \tag{C.2}$$

so

$$C_L = \frac{Q^{'}(L;E)}{\left(\frac{E}{L}\right)^{\beta} \beta \frac{2E}{v} \Psi\left(\beta+1, \xi+1, \frac{2E}{vL}\right) + \left(\frac{E}{L}\right)^{\beta} \frac{\beta}{L} \Psi\left(\beta, \xi, \frac{2E}{vL}\right)}, \tag{C.3}$$

where $\xi \equiv 2\left(\beta+1+\frac{\alpha}{v}\right)$. By definition of $\Psi(a,c;z)$, $\Psi(a,c;z) \sim \frac{\Gamma(c\text{-}1)}{\Gamma(a)} z^{1\text{-}c}$ when c>1 for z>0 close to 0. When $L \to +\infty$, the denominator of (C.3) is thus equivalent to $L^{-\beta+\xi} = L^{\beta+2+2\frac{\alpha}{v}}$ multiplied by a constant. Since the numerator of (C.3) is linear in L (see the expression of $Q(L;E)$ in Equation (27)), C_L goes to 0 as $L \to \infty$.

Table 1

Comparison of stopping values x^* for r=0.04, α=0.02, E_1=0.3, E_2=0, K=4, and θ=0.002.

Volatility coefficient v	x^* (Pindyck 2000)	x^*
0		10.67
0.3	11.08	11.08
0.5	11.59	11.59
0.8	12.45	12.45
1.0	13.05	13.04
2.0	16.47	15.66
4.0	25.75	18.50
$+\infty$		21.33

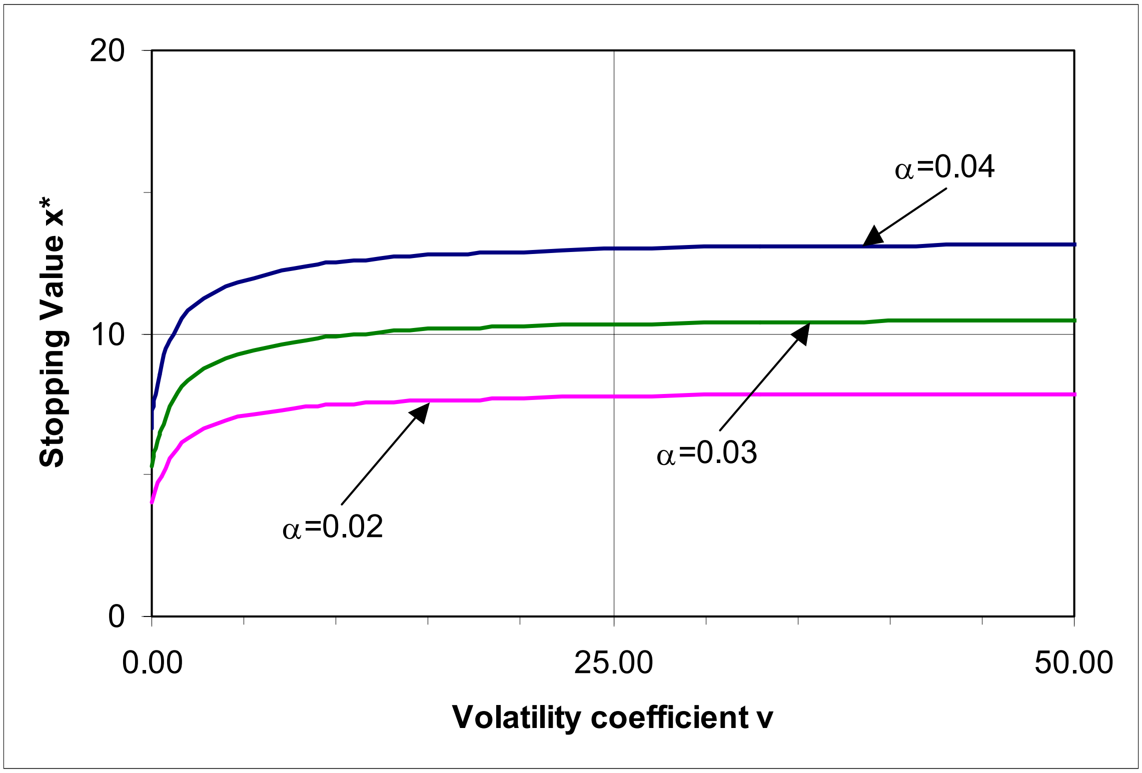

Figure 1. Stopping values when the infinitesimal variance of *X* is constant.

$r=0.02$, $E_1=0.3$, $E_2=0$, $K=4$, $\theta=0.002$.

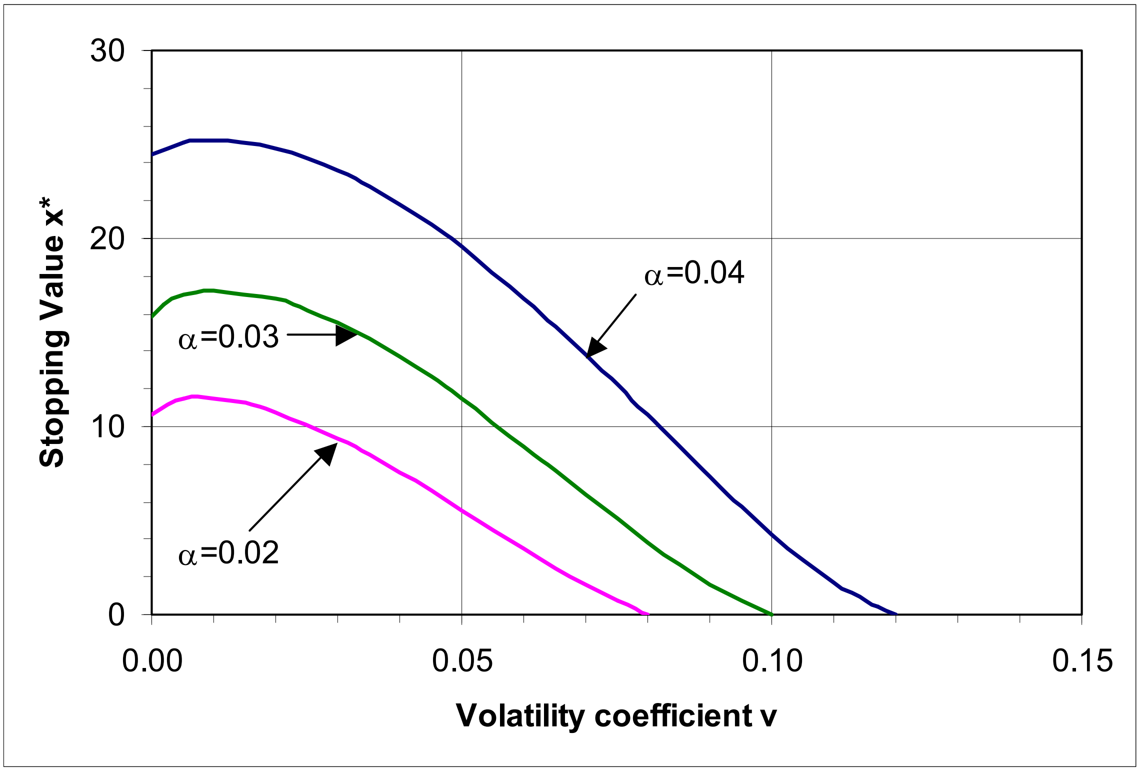

Figure 2. Stopping values when the infinitesimal variance of X is proportional to X.

r=0.02, E_1=0.3, E_2=0, K=4, θ=0.002.

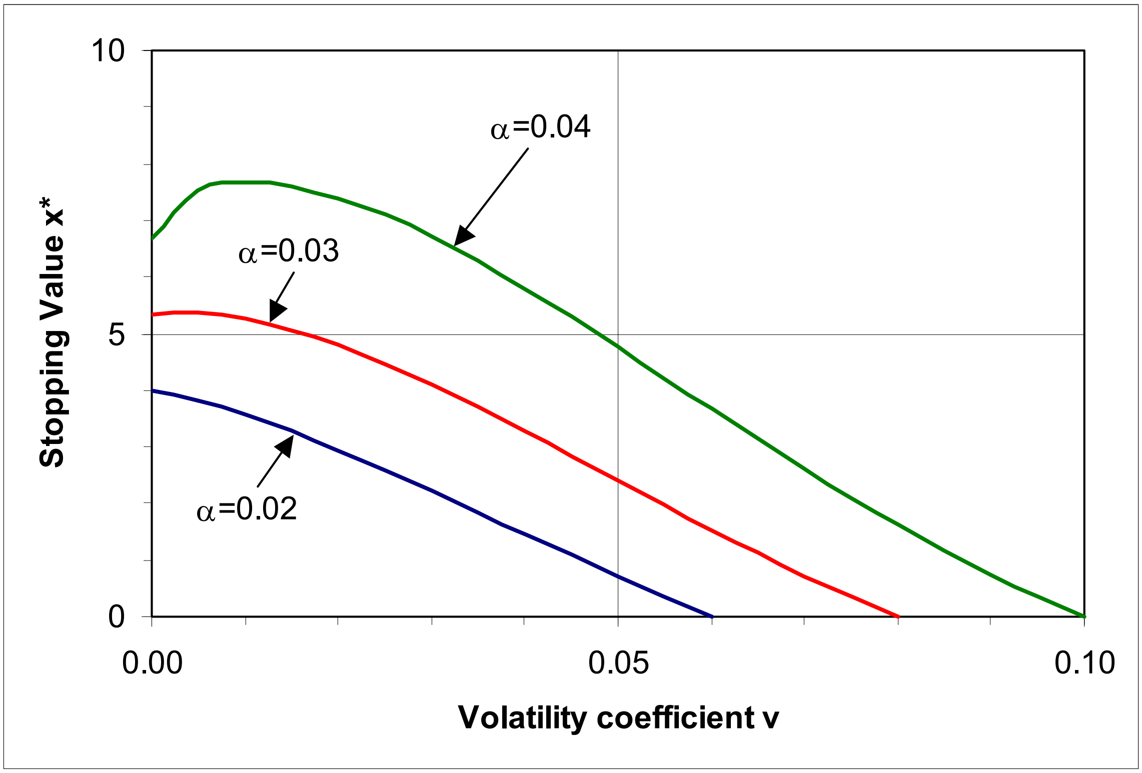

Figure 3. Stopping values when the infinitesimal variance of *X* is proportional to *X*.

r=0.04, E_1=0.3, E_2=0, K=4, θ=0.002.

[1] As time elapses, the path of the level of stock pollutant is revealed but there is no reduction in uncertainty.

[2] In Pindyck's framework, economic uncertainty results from random changes in the valuation of social costs.

[3] One difficulty here is to find the right particular solution of the Bellman Equation that corresponds to the present value of expected social costs. This necessitates a careful consideration of boundary conditions, as discussed below.

[4] E_1 is included as an argument of φ to indicate the relevant value of stock pollutant emissions in the homogeneous equation associated to the Bellman Equation (5). E_2 is not listed as an argument of φ to lighten our notation.

[5] When we want to emphasize the dependency of x^* on v, we write $x^*(v)$; otherwise, we write x^* to lighten our notation. Hence, $x^*(0)$ is the deterministic stopping value.

[6] In the global warming context, it would be more realistic to impose a reflecting barrier at the natural background concentration of CO_2. We impose instead a reflecting barrier at 0 for simplicity.

[7] The confluent hypergeometric function of the first kind is defined by $\Phi(a,c;z)=\sum_{k=0}^{+\infty}\frac{(a)_k}{(c)_k}\frac{z^k}{k!}$, where $\Gamma(x)\equiv\int_0^{+\infty}e^{-t}t^{x-1}dt,\ if\ x>0$ is the gamma function (see Luke 1969) and $(a)_0=1$, $(a)_k=a\cdot(a+1)\cdots(a+k-1)$. Confluent hypergeometric functions are widely used in mathematics and in physics.

[8] Of course, allowing for negative values of X does not make physical sense when X is the level of a stock pollutant.

[9] Kummer's equation is: $x\frac{d^2 f(x)}{dx^2}+(c-x)\frac{df(x)}{dx}-af(x)=0$. Two independent solutions of this second order linear differential equations are $\Phi(a,c;z)$ and $\Psi(a,c;z)$, the confluent hypergeometric functions of the first and second kind respectively (see footnotes).

[10] The function $\Phi(a,c;z)$ is defined above. The confluent hypergeometric function of the second kind is $\Psi(a,c;z)=\frac{\Gamma(1-c)}{\Gamma(1+a-c)}\Phi(a,c;z)+\frac{\Gamma(c-1)}{\Gamma(a)}z^{1-c}\Phi(1+a-c,2-c;z)$. $\Psi(a,c;z)$ is well defined for all values of a and c, including negative integer values of c. As before, $\Gamma(x)\equiv\int_0^{+\infty} e^{-t}t^{x-1}dt,\ if\ x>0$ is the gamma function (see Luke 1969).

[11] Indeed, when $z\to+\infty,\ \Phi(a,c,z)\sim\frac{\Gamma(c)}{\Gamma(a)}e^z z^{a-c}$ ((5) page 128, Luke 1969).