**Transport tax reform, commuting and endogenous values of time**

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Abstract

Many previous studies of transport tax reform have explicitly or implicitly assumed that the reform itself does not affect the marginal value of time. In this paper we consider a simple model with multiple trip purposes, commuting and non-commuting transport, to analyse the implications of transport tax reform for the value of time and for marginal external congestion costs. The theoretical analysis shows that transport taxes may both increase or reduce the value of time and it identifies the conditions under which either outcome will occur. The results further suggest that if a tax reform in the transport sector is accompanied by labour tax adjustments to reduce the distortionary cost of the tax system, the marginal value of time will typically increase. The implications of endogenous values of time for models of optimal externality taxes and studies of tax reform in the transport sector are empirically illustrated using a numerical model, calibrated using Belgian data. It is found that the impact of tax changes on the value of time is non-trivial. Moreover, many of the tax reform exercises considered simultaneously reduce traffic levels but raise marginal external congestion costs. The results of this paper suggest that incorrectly assuming exogenous time values may strongly bias optimal congestion taxes and lead to misleading welfare effects of transport tax reform.

## 0. Introduction

Widespread concern about the external costs associated with increasing transport demand has generated a substantial literature on optimal externality taxes and optimal tax reform in the transport sector. Examples from the economics literature include Keeler and Small (1977), Glaister and Lewis (1978), Small (1983), Viton (1983), Kraus (1989), Arnott, De Palma, and Lindsey (1993), De Borger et al. (1997), Proost and Van Dender (2001), and Small and Yan (2001). Importantly, all these models are implicitly or explicitly assuming that the value of time is unaffected by the proposed policy changes. Moreover, although some studies do take account of various different transport markets (according to mode, period of the day, car type, etc.), they do not distinguish between different trip purposes, such as between commuting versus non-commuting trips. The few existing models that do allow for endogenous values of time (see Mayeres and Proost (1997), where the endogeneity is explicit, and Parry and Bento (2001), where it is implicit) are also based on a single trip motive. The purpose of this paper is to show that explicitly distinguishing multiple trip purposes implies that policy changes on the transport market may have non-negligible effects on the value of time. As a consequence, realising that in most countries commuting is indeed an important trip purpose during peak hours, the welfare effects suggested by models assuming either constant values of time or single trip purposes may be misguided.

Ever since the seminal papers by Becker (1965) and Gronau (1977), economists have devoted serious attention to the determinants of the value of time (for a recent survey, see Jara Diaz (2000)). Theoretical research as well as empirical analysis of large-scale surveys have suggested that the value consumers place on time savings not only depends on income or wage levels, but also on a large number of socio-economic characteristics. Relevant references include Clifford and Whinston (1998), Ramjerdi, Rand and Saelensminde (1997), and de Jong and Gunn (2001). Moreover, time values vary according to the specific circumstances under which the time saved had to be actually spent (see, e.g., De Donnea (1972), Hague Consulting Group (1990), de Jong and Gunn (2001)). Somewhat surprisingly, however, the potential dependency of the value of time on the level of transport prices has not received much attention in the literature. It has not explicitly been analysed how a tax reform itself affects the values of time, marginal congestion costs and, therefore, the welfare effects of the reform. The use of endogenous values of time is especially relevant for the analysis of optimal tax models and in studying tax reform exercises in the transport sector, where the prevalence of congestion externalities may require relatively large adjustments in prices.

Within the framework of a simple model with multiple trip purposes, this paper suggests that higher transport taxes may both increase or decrease the value of time. Policies that combine transport tax increases with adjustments in labour taxes to reduce the distortions from the tax system (see, e.g., Bovenberg and Goulder (1996)) are likely to increase the marginal value of time, the size of the effect depending on the share of commuting trips in overall traffic and the price sensitivity of transport demand. It is shown that endogenous time values actually imply that a joint tax reform on the transport and labour markets may yield lower traffic levels, less congestion, but higher marginal external congestion costs. This contrasts with the popular view that directly associates decreases in traffic levels on a congestible facility with reductions in marginal external congestion costs.

The findings of this paper imply that models that erroneously assume constant time values, or that inappropriately ignore multiple trip purposes, may produce misleading results. Consider, for example, welfare analyses of a transport tax reform. If a non-negligible share of traffic is due to commuting transport, and models are used that ignore the existence of multiple trip purposes and use exogenously fixed values of time, both time values and marginal external congestion costs at the post-reform equilibrium may be largely underestimated. As a consequence, the welfare effects of the tax reforms may be seriously biased. For the same reason, optimal tax analyses for the transport sector using this type of model may well largely underestimate optimal congestion taxes. Even if endogenous time values are allowed, the results further suggest that analyses of combined transport and labour market tax reforms in models with a single trip purpose will produce unreliable results, in as far as the assumption of a single trip purpose does not accord with observations.

The paper is structured as follows. In Section 1 we consider the value of time and marginal external congestion costs in a stylised model of consumer choice with multiple trip purposes. We show that higher transport taxes may strongly affect the valuation of time and, therefore, marginal external congestion costs. If the transport tax reform is accompanied by labour tax adjustments to reduce the distortions of the tax system, the marginal value of time will typically rise. In Section 2 we illustrate some of these findings using a more elaborate numerical model involving different transport modes and multiple trip purposes. Both the effects of an optimal tax analysis and various types of tax reform are considered. Empirical results based on a numerical optimisation model calibrated on stylised Belgian data confirm the theoretical predictions. The relation between optimal tax changes, variations in the value of time and in marginal external congestion costs, and changes in traffic flow are investigated. Section 3 concludes.

## 1. Transport taxes, the value of time and marginal external congestion costs in a model with multiple trip purposes

In this section we first present the simple model used for the theoretical analysis. We then study the impact of transport and labour tax reform for the marginal value of time and for marginal external congestion costs.

### 1.1. A simple model with multiple trip purposes

Let a representative consumer care about two types of transport trips, a general consumption good, and leisure.[1] Specifically, preferences are given by $u(q_0, q_1, q_2, N)$, where $q_0$ is a composite commodity with price normalised at one, $q_1$ are non-commuting trips, $q_2$ are commuting trips (the journey-to-work), and $N$ is leisure time. The model focuses exclusively on peak period travel, since the peak is most congested and both commuting and non-commuting are known to be substantial (LRC (1994), US Federal Highway Administration (1995)). To make the distinction between the two trip motives as transparent as possible, commuting is assumed to be directly proportional to labour supply L, i.e., $q_2 = L$. In other words, we assume that

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[1] Since we focus on the dependency of time values on policy variables (taxes, prices), we ignore all other differences in the value of time, referred to above, by assuming a representative consumer.

each day of work requires one morning peak trip, assumed for simplicity to be of length one kilometre. Labour supply is elastic in terms of the number or days of labour, but the length of each labour day is fixed.

There is only one transport mode (car travel), and only one car type is used. Transport prices per trip (or per kilometre) are $p_1$ and $p_2$ for non-commuting and commuting, respectively. These prices may be different if, e.g. commuting expenses are tax deductible (as considered by, among others, Wrede (2001)); alternatively, if no tax deductibility is allowed, both prices will be identical. It is further assumed that commuting and non-commuting transport use the same congestible road network, and therefore jointly determine travel time $a = a(F)$, where *a(.)* is the congestion function, $F = n(q_1 + q_2)$, and n is the number of consumers. We normalise *n=1* throughout.

The consumer maximises utility subject to a budget and a time constraint:

$$q_0 + p_1 q_1 + p_2 q_2 = \left(1 - t_L\right) L + S \qquad [\lambda] \tag{1}$$

$$N + L + (a(F))(q_1 + q_2) = \bar{L} \qquad [\gamma] \tag{2}$$

where $t_L$ is the labour tax rate (wages are normalised to 1 without loss of generality), $L$ is labour time, $S$ is a fixed lump sum transfer from the government, and $\bar{L}$ is the time endowment. Finally, $\lambda$ and $\gamma$ are the Lagrange multipliers associated with the budget and time constraints.

Using $q_2 = L$ and assuming, as is common in the literature, that the representative consumer neglects his own impact on congestion, we obtain the following first-order conditions

$$\begin{aligned} u_0 &= \lambda \\ u_1 &= \lambda p_1 + a\gamma \\ u_2 &= -\lambda(1 - t_L - p_2) + (1 + a)\gamma \\ u_N &= \gamma \end{aligned} \tag{3}$$

Following, among many others, Jara-Diaz (2000) we define the marginal value of time by $MVOT \equiv \dfrac{u_N}{u_0}$. Using the system of first-order conditions (3) it can be written as

$$MVOT \equiv \frac{u_N}{u_0} = \frac{\gamma}{\lambda} = \frac{\frac{u_2}{\lambda} + (1 - t_L - p_2)}{1 + a} \tag{4}$$

The marginal value of time, which at the optimum is independent of the activity in which it is spent, can be interpreted as the net real wage per time unit, corrected for the marginal (dis-)utility of commuting.[2] The net real wage captures the cost of commuting; similarly, the time input per hour of work incorporates travel time for the journey to work.

Although we look at the problem in a more formal manner below, note that the potential impact of transport prices on the value of time in this simple model is obvious from (4), not in the least because the value of time directly depends on congestion. Since reducing congestion is often a major reason for transport tax reform and optimal transport tax changes to cope with external congestion costs are known to be substantial, the impact of price changes for non-commuting transport may not be negligible. A commuting tax has an additional direct effect on time values. Moreover, all transport prices are likely to have an impact on labour supply and commuting demand and, as a consequence, neither the marginal utility of income nor the marginal utility of commuting can be assumed to remain constant.

Importantly, note that the impact of transport prices on time values is explicitly due to the assumption of multiple trip purposes, where commuting is one of the trip motives. Indeed, if all transport were aggregated and treated as non-commuting transport (i.e., assuming commuting transport $q_2 = 0$ and obviously ignoring $q_2 = L$), one easily shows that the model would imply a value of time equal to the net wage, $MVOT \equiv \frac{\gamma}{\lambda} = (1 - t_L)$. Consequently, in that case transport prices in equilibrium would not affect values of time, and the labour tax would reduce the time value on a one-to-one basis.

[2] Of course, slightly different expressions are obtained for the value of time depending on the exact specification of the utility function (e.g., explicitly including travel time or labour supply as an extra argument of utility). See Jara Diaz (2000) for an overview. However, the main point of this paper, viz. that distinguishing multiple trip purposes has implications for the impact of tax changes on the value of time, is not affected.

1. 2. Taxes and the value of time with multiple trip purposes

Here we consider the impact of price and tax changes on the value of time more formally. To keep the analysis as transparent as possible we impose some extra structure on consumer preferences.[3] An outline of the analysis for the general case is relegated to Appendix 1.

To be specific, let us assume that, first, utility is quasi-linear in the numeraire good and, second, that commuting is additively separable from other consumption goods and leisure:

$$u(q_0, q_1, q_2, N) = q_0 + U(q_1, N) + g(q_2)$$

These assumptions imply that the marginal utility of income is constant and equal to one ($u_0 = \lambda = 1$) so that $MVOT \equiv \frac{u_N}{u_0} = \gamma$. Substituting $q_0$ from the budget constraint into the utility function and using $\lambda = 1$ the system of first-order conditions now reads:

$$\begin{aligned} u_1 &= p_1 + a\gamma \\ u_2 &= -(1 - t_L - p_2) + (1+a)\gamma \\ u_N &= \gamma \\ \bar{L} &= N + aq_1 + (1+a)q_2 \end{aligned} \tag{5}$$

where the final equation is the time restriction.

The consumer's optimisation problem implies that system (5) must hold at given prices, taxes and exogenous congestion level, i.e., at given *a=a(F)*. However, we are interested in the impact of taxes on demands and on the value of time $\gamma$, taking into account the effect of taxes and prices on congestion levels. Differentiating (5), capturing price effects on *a* via demand changes, and writing the result in matrix notation yields

[3] In general, the results depend on all second derivatives of the utility function. This obscures the interpretation, because many of the cross-effects of the marginal utilities are difficult to sign a priori.

$$\begin{bmatrix} u_{11}-\gamma a' & -\gamma a' & u_{1N} & -a \\ -\gamma a' & u_{22}-\gamma a' & 0 & -(1+a) \\ u_{N1} & 0 & u_{NN} & -1 \\ -(a+a'F) & -(1+a+a'F) & -1 & 0 \end{bmatrix} \begin{bmatrix} dq_1 \\ dq_2 \\ dN \\ d\gamma \end{bmatrix} = \begin{bmatrix} dp_1 \\ dt_L+dp_2 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

where, as before, $F = q_1 + q_2$ is total transport and $a' = \dfrac{da(F)}{dF}$ is the slope of the congestion function.

The solutions for $dq_1, dq_2, dN, d\gamma$ can be obtained by Cramer's rule. Since the main interest of this paper is on the role of policy changes for the value of time, we here only report the impact of transport prices and labour taxes on the value of time $\gamma$ (see Appendix 2 for more details on the effects of price and tax changes on transport and leisure demands). We find

$$\frac{d\gamma}{dp_1} = \frac{1}{\Delta}\left\{u_{22}\left[(au_{NN} - u_{1N}) + a'Fu_{NN}\right] + \gamma a'(u_{1N} + u_{NN})\right\} \tag{7}$$

$$\frac{d\gamma}{dt_L} = \frac{d\gamma}{dp_2} = \frac{1}{\Delta}\left\{(u_{NN}u_{11} - u_{N1}^2)(1+a+a'F) - \gamma a'(u_{NN} + u_{N1})\right\} \tag{8}$$

where $\Delta$ is the determinant associated with the differential system (6). In Appendix 2 it is shown that a mild restriction on the feedback effects of congestion on demand guarantees that it is negative. We will assume this condition to hold in what follows. Moreover, we assume throughout that there is declining marginal utility, i.e., $u_{NN} < 0$, $u_{11} < 0$ and $u_{22} < 0$.

Consider the impact of the price of non-commuting transport on the value of time (see (7)). In view of (4) it is not surprising to find that the result strongly depends on the impact of the price change on commuting and on overall congestion. To see this, first suppose hypothetically that the marginal utility of commuting were constant, so that $u_{22} = 0$. Under those conditions, an increase in the price of non-commuting will raise or reduce the value of time depending on the sign of $u_{1N} + u_{NN}$. As seen in appendix 2, $u_{1N} + u_{NN} < 0$ is a sufficient condition for more expensive non-commuting transport to reduce overall congestion. If this condition holds, the price increase raises the value of time. The value of time would decline if the price actually increased congestion; this happens if the negative price effect on non-commuting demand were more than compensated by a large positive cross-price effect on labour supply and commuting demand. Second, if the marginal utility of commuting is not constant the above statements have to be amended depending on the change in commuting transport. Noting from appendix 2 that ($au_{NN} - u_{1N}$)<0 is a sufficient condition for $\frac{dq_2}{dp_1}$>0, it is clear that a strong positive cross-price effect reduces the value of time.

An increase in the labour tax or in the price of commuting transport have the same impact on the value of time because they affect the net real wage identically.[4] Since $u_{NN}u_{11} - u_{N1}^2 > 0$ by the strict concavity of U(.), both will reduce the value of time as long as $u_{1N}$ is not too positive, see (8). This is plausible: higher labour taxes reduce the value of time (see (4)) unless commuting demand and total congestion drastically declined. As seen in Appendix 2, a very large positive $u_{1N}$ indeed implies strong reductions in commuting and labour supply; as a consequence, congestion goes down while the marginal utility of commuting rises. Both effects raise the value of time.

In sum, the above discussion implies that under many plausible circumstances the price of non-commuting transport will raise time values, whereas the commuting or labour tax is more likely to reduce time values. Indeed, (7) suggests that as long as an increase in $p_1$ reduces overall congestion and does not substantially raise labour supply the impact on the value of time will be positive. The larger the impact on labour supply, the smaller the effect on the value of time and the larger the likelihood that the value of time will actually decline. As suggested by (8), we expect labour or commuting taxes to reduce the value of time unless labour supply increases substantially.

The results for $\frac{d\gamma}{dt_L}$ remain unaffected if we assume that commuting and non-commuting transport prices cannot be differentiated, either for technical reasons or because of political constraints. However, since the common transport price directly

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4 This is no longer true in the empirical model, where two transport modes are considered.

affects the net real wage, the impact of higher transport prices on time values does change. We find

$$\frac{d\gamma}{dp} = \frac{1}{\Delta}\left\{\left[u_{NN}(u_{22}+u_{11}) - u_{N1}^2\right](1+a+a'F) - (u_{1N}+u_{NN})u_{22}\right\} \tag{9}$$

As the term between square brackets is positive, at constant marginal utility of commuting ($u_{22}$=0) a price increase now reduces the value of time, contrary to the outcome with price differentiation. Even though the price increase reduces congestion (raising time values), this effect is more than compensated by the direct reduction in the real net wage. If $u_{22}$<0, the above negative effect is counteracted by a positive impact, which will be larger the larger the reduction in commuting demand. The ultimate sign is therefore indeterminate. Note also that we expect the price effect on the value of time to be smaller than the price effect for non-commuting transport in the tax differentiation case.

In Table 1 we summarise the most relevant findings and compare them with the case of a single trip purpose (in which case $MVOT = 1 - t_L$). Moreover, we compare with the case where there are multiple trip purposes but labour supply is fixed (see the last column). In that case one easily shows that labour or commuting taxes do not affect the value of time. The quasi-linear utility structure implies that these taxes have no effect on any of the time-using commodities (transport, leisure). Only non-commuting transport prices affect the value of time. Although the sign of the effect is not unambiguous, it is plausibly negative, because a transport price increase reduces the total time input associated with non-commuting.

**Table 1 Plausible impact of prices and taxes on the marginal value of time**

| | Model type | | |
|---|---|---|---|
| | Multiple trip purposes | One trip purpose (non-commuting) | Multiple trip purposes (but fixed labour supply) |
| Impact of labour tax $t_L$ | <0 | -1 | 0 |
| Impact of non-commuting transport price $p_1$ | >0 | 0 | <0 |
| Impact of commuting transport price $p_2$ | <0 | Not relevant | 0 |
| Impact of common transport price $p = p_1 = p_2$ | <0 or >0 | Not relevant | <0 |

1.3. Taxes and marginal external costs with endogenous time values

In our set-up, marginal external congestion costs MECC are the same for commuting and non-commuting transport. Driving one extra kilometre raises F, it reduces travel speed and increases travel time per kilometre a(F). These time losses apply to all kilometres driven and are evaluated at the value of time per time unit. Marginal congestion costs can therefore be written as

$$MECC = (MVOT)*(a')*(q_1 + q_2) \tag{9}$$

where, as before, $a' = \frac{da(F)}{dF}$. Taxes and transport prices affect congestion costs through different channels: via traffic flows, through the slope of the congestion function, and via the value of time.

Differentiating (9) and rearranging, we obtain the impact of tax and price changes on marginal external congestion costs:

$$\frac{d(MECC)}{dt_L} = \left[ MVOT\left( (F*\frac{da'}{dF}) + a' \right) \right] \left( \frac{dq_1 + dq_2}{dt_L} \right) + \left[ (a')*F \right] \left( \frac{dMVOT}{dt_L} \right) \tag{10}$$

$$\frac{d(MECC)}{dp_i} = \left[ MVOT\left( (F*\frac{da'}{dF}) + a' \right) \right] \left( \frac{dq_1 + dq_2}{dp_i} \right) + \left[ (a')*F \right] \left( \frac{dMVOT}{dp_i} \right) \tag{11}$$

where *i=1,2*. The first term on the right-hand-side of (10)-(11) measures the impact of the tax or price change via its effect on the traffic flow *F*. At constant values of time, the change in traffic flow influences the number of users affected by a marginal traffic increase, and it affects the slope of the congestion function. Since both effects are plausibly positive, the first term indicates that a price or tax increase reduces marginal external congestion costs as long as it makes congestion decline. The second term is the impact of tax changes on the marginal congestion cost via their effect on the value of time. These effects where derived before for a simplified setting.

It is clear, then, that ignoring changes in time values introduces biases in marginal external costs. For example, suppose an increase in the price of non-commuting transport reduces congestion and raises the value of time. If a model assumes exogenous values of time, the reduction in MECC will therefore be overestimated. Analogously, if an increase in the commuting tax reduces congestion but reduces the value of time then the reduction in MECC will be underestimated if exogenous time values are assumed. Interestingly, note that if the value of time substantially rises, marginal congestion costs may actually increase despite the reduction in congestion brought about by the price increase. For the same reason it is not inconceivable that a joint transport-labour tax reform actually increases the marginal external congestion cost, even when traffic flow declines, due to its positive impact on the value of time.

At least three implications of the above discussion are worth pointing out. First, exogenous time values will lead to misleading welfare effects of transport tax reforms and to biases in optimal congestion taxes. Indeed, suppose we analyse the effects of a reform of transport taxes to reduce externalities in a situation were a nontrivial fraction of transport flows refer to commuting. However, suppose a model is used that considers only one trip purpose (implicitly treating all transport as non-commuting) and that imposes constant time values. Our results suggest that the model is likely to overestimate the reduction in the MECC as well as the welfare effects. For the same reason, optimal taxation exercises focusing on the transport market but using exogenous time values are likely to underestimate optimal congestion taxes. Second, even if we do take account of endogenous values of time, we expect combined transport and labour tax reforms to lead to quite different implications in models with multiple trip purposes as compared to models that treat all transport as non-commuting. Suppose a transport tax reform is accompanied by measures to reduce the distortion on the labour market, raising transport taxes rise but adjusting labour taxes downward. As seen before, in models with a single trip purpose this reform will raise time values. With multiple trip purposes higher non-commuting transport prices will also plausibly raise time values. However, to the extent that the combined change in commuting and labour taxes ultimately reduces the net real wage (i.e., the commuting tax is not fully compensated by the labour tax reduction), this second effect counteracts the first and reduces the value of time. The impact on marginal congestion costs will therefore be dampened. Third, as suggested before, larger shifts in time values and external costs are expected if tax reforms allow differentiation between non-commuting and commuting taxes.

Although these findings are difficult to generalise because of the simplicity of the model used here, there is no reason to expect that these implications will disappear in more general models, because the bias is the result of incorrectly assuming one trip purpose. Of course, whether the issue is empirically sufficiently important to be worried about is something to be found out. This is the purpose of the numerical exercise reported in the next section.

## 2. A numerical application

In this section we illustrate some of the theoretical results using a numerical model with multiple trip purposes and endogenous time values (see Van Dender (2001) for an elaborate description). The model is calibrated using stylised data, and it is designed to represent peak period traffic flows and congestion for an average workday in a typical Belgian mid-size city.

### 2.1 Overview of the model

The numerical model generalises the theoretical analysis of the previous section in two respects. First, the quasi-linearity assumption of utility in $q_0$ is relaxed. Instead, a nested-CES representation of preferences is used. Second, we allow for multi-modality by distinguishing two transport modes: car and bus. We do retain the separability assumption for commuting transport, as in Parry and Bento (2001). The consumer's problem for the numerical model is specified as

$$\begin{aligned}
&Max \quad u = U\left(q_0, (q_1, q_2), N\right) + g(q_3, q_4) \\
&subject\ to \left(1 - t_L\right) L + S = q_0 + \sum_{i=1}^{4} p_i q_i \qquad [\lambda] \qquad (12) \\
&\overline{L} = N + L + a \sum_{i=1}^{4} q_i \qquad [\gamma]
\end{aligned}$$

where

$q_0 =$ composite commodity (untaxed numéraire)
$q_1 =$ non-commuting car trips
$q_2 =$ non-commuting bus trips
$q_3 =$ commuting by car
$q_4 =$ commuting by bus
$N =$ leisure

and $a=a(F)=a(q_1+q_3+\alpha(q_2+q_4))$. The congestion function is adjusted for the presence of bus as well as car trips. The parameter $\alpha<1$ indicates that an extra bus trip by a passenger contributes less to congestion than a car trip.[5] One simplification of the numerical model in comparison to the previous section, is that a linear congestion function is used, so that a change in the traffic flow does not affect the slope of the congestion function[6]. This allows us to focus on the effect of tax changes on marginal values of time and congestion levels, see (10) and (11). Finally, commuting trips are still taken to be strict complements to labour supply by assuming that $q_3+q_4=L$.

Traffic flow data and congestion technology are derived from a small network model for the city of Namur (Cornelis and Van Dender, 2001). The peak period speed at the reference prices is 30km/h, which is half the free flow speed. Data on the composition of the traffic flow according to trip purpose and on the modal split, are based on a national survey (Pollet, 2000) and a survey for Brussels (IRIS, 1993). Commuting stands for 53% of all peak period trips. Two thirds of commuting trips and three quarter of non-commuting trips use the car mode. These proportions reflect the typical modal split on condition the public transport mode is easily accessible. The reference price data for transport are based on Proost and Van Dender, 2001.

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5 Note the implicit assumptions (a) that both modes share the transport network, and (b) that the bus occupancy rates are fixed. The latter assumption is reasonable for peak hours, as buses operate close to or at technical capacity.

6 When the real congestion function is convex, using a linear approximation will overestimate the travel time reductions associated to decreases in traffic flow. In order to moderate this overestimate, the linear approximation was made at traffic levels below the reference flows. Newbery and Santos (2002) suggest that network-derived linear congestion functions perform well for an analysis of cordon pricing schemes on a network.

## 2.2 Empirical results

### *2.2.1 The central scenario*

We report empirical results for a number of tax reform and optimal tax exercises. The reference equilibrium (**REF**), representing the initial situation in Belgium, is described in the left-most column in table 2. The labour tax is 40%, and both commuting and non-commuting car traffic are taxed at less than marginal external cost: taxes amount to 4.24 Euro (per round trip) as compared with marginal external congestion costs (MECC) of 6.87 Euro. For public transport, note that the model assumes, consistent with current practice in Belgium, that bus transport is government-supplied and that the production costs are financed out of general public funds. Therefore, to ease the interpretation, for bus transport the 'taxes' reported in the table are simply the fares directly paid by users. They amount to 0.53 Euro.

The (calibrated) marginal value of time in the reference equilibrium is 7.67 Euro/hour, or 47% and 78% of the gross and the net hourly wage, respectively. Both the absolute and the relative levels are in line with the literature (e.g. Small, 1992). In order to facilitate the interpretation of the impact of transport and labour tax changes, all other information (on traffic flows, labour supply, etc.) is presented in index form.

The tax reforms considered are balanced-budget from the government's perspective. Combinations of transport and labour tax changes are considered that leave total tax receipts $\sum_{i=1}^{4}(p_i - c_i)q_i + t_L L + S$ unaffected. Note that only taxes change, while lump-sum transfers remain constant. We look at the implications of balanced-budget labour tax reductions by 1% and 5%, allowing transport taxes to be optimally adjusted, taking account of external congestion costs. Although the analysis allows for potential subsidies to at least some transport services, we do impose the restriction of a non-negative monetary price to the consumer for all transport services considered[7]. We consider each tax reform exercise both in the case of tax

[7] Note that this restriction may actually be binding in this model if labour taxes are substantially above optimal levels. Since commuting is directly related to labour supply the optimal transport tax reform may try to 'correct' excessive labour taxes by heavily subsidizing commuting transport.

differentiation between commuting and non-commuting, and in the case of uniform taxes across trip purposes. The optimal tax exercises consider optimal transport and labour taxes under the restriction of a constant government budget. For all tax changes analysed, the impact on welfare is measured by the post-reform value of the indirect utility function of the representative consumer.

Table 2 summarises the results. First consider scenarios **A** to **D,** which refer to the tax reform exercises. Note that all experiments reported in the table lead to increases in welfare (row (1)). Moreover, the value of time and the marginal external congestion costs increase in all scenarios. The increases in MECC occur *despite* the decrease in the aggregate peak period traffic flow in three out of four scenarios.[8] In other words, for several cases the tax reform at the same time reduces congestion but, because of increasing time values, raises marginal external congestion costs.

The implications of the tax reforms for optimal tax adjustments and therefore for the value of time strongly vary between the different scenarios. In the case of uniform taxes across trip purposes, the optimal response to a 1% reduction in labour tax is to increase car taxes, but bus prices slightly decline. Not surprisingly, the 5% labour tax reduction raises car prices more substantially; moreover, it also implies higher bus fares. As a consequence of these tax adjustments, labour supply rises slightly. Increasing non-commuting transport taxes and minor reductions in the net real wage lead to time values that rise rather modestly (by 1.2% and 5.3% for the 1% and 5% labour tax reductions, respectively).

The corresponding outcomes are quite different for the scenarios where differentiated taxes across trip purposes are assumed. In those cases, both car and bus transport for non-commuting purposes becomes very substantially more expensive. Since the labour tax reductions of 1% and 5% fall short of the optimal labour tax adjustment (see below), there is strong pressure on commuting transport prices not to decrease the net real wage. In fact, the bus commuting fare reaches its lower limit of zero. Similarly, car commuting is heavily subsidized; the subsidy amounts to some

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[8] Only the 1% decrease in the labour tax, financed by differentiated transport tax changes, leads to an increase in the traffic flow. Intuitively, this is because the transport tax differentiation is too small to strongly influence commuting and labour supply.

38% of its resource cost[9]. Labour supply is up by some 4%-5%. The strong increases in non-commuting transport taxes and the reduction in the labour tax together imply much higher values of time; they increase by 9.9% and 10.3%, respectively. Note that the larger effects on time values in the case of differentiated taxes are consistent with our theoretical discussion given before.

Finally, consider the results for the optimal tax exercises. We only report values for differentiated taxes (see scenario **E**), because both the welfare changes and the marginal values of time are not affected by the uniformity constraint[10]. Of course, the optimal taxes do differ: the optimal labour tax reduction turns out to be 8% to almost 16% for the cases of differentiated and uniform transport taxes, respectively. These figures should be interpreted in the light of very high initial labour taxes in Belgium. Optimal car commuting taxes are extremely high in the differentiated case. All other transport taxes also rise relative to the reference situation, with the exception of commuting bus transport. At the optimal labour tax, the marginal value of time is 14.3% higher than in the reference equilibrium. This is not a trivial change. It is much larger than the impacts predicted by Mayeres and Proost (1997), who take account of the endogeneity of the time value, but do not distinguish between commuting and other trip purposes, so that time values only change due to labour tax adjustments.

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[9] Calthrop (2001) and Wrede (2001) report optimal commuting subsidies of 50% and more than 100% of the resource cost, respectively.

[10] This is due to the direct relation between commuting and labour supply. See Van Dender (2001) for more details.

**Table 2 Impacts from simultaneously decreasing labour taxes and optimally adapting transport taxes for a constant government budget**

| | | Unit | Reference | Tax reform | | | | Optimal labour tax |
|---|---|---|---|---|---|---|---|---|
| | | | 40% labour tax | 1% labour tax decrease | | 5% labour tax decrease | | 8% labour tax decrease* |
| | | | **REF** | **A** | **B** | **C** | **D** | **E** |
| | | | Uniform transport taxes | Uniform transport taxes | Differentiated transport taxes | Uniform transport taxes | Differentiated transport taxes | Differentiated transport taxes |
| (1) | Welfare level | Index | 1 | 1.0008 | 1.0022 | 1.0025 | 1.0037 | 1.0042 |
| (2) | MVOT | Euro/h | 7.67 | 7.76 | 8.43 | 8.08 | 8.46 | 8.77 |
| (3) | MECC | Euro/round trip | 6.87 | 6.92 | 6.95 | 7.00 | 7.06 | 7.07 |
| (4) | Traffic flow in PCU | Index | 1 | 0.90 | 1.04 | 0.89 | 0.94 | 0.86 |
| (5) | Q1 | Index | 1 | 0.96 | 0.75 | 0.89 | 0.73 | 0.70 |
| (6) | Q2 | Index | 1 | 1.03 | 0.77 | 0.92 | 0.82 | 0.72 |
| (7) | Q3 | Index | 1 | 0.82 | 1.34 | 0.88 | 1.10 | 1.01 |
| (8) | Q4 | Index | 1 | 1.36 | 0.48 | 1.31 | 0.96 | 1.18 |
| (9) | Labour supply | Index | 1 | 1.00 | 1.05 | 1.02 | 1.05 | 1.07 |
| (10) | T1 (car tax non-commuting) | Euro/round trip | 4.24 | 5.73 | 12.43 | 8.12 | 11.98 | 16.57 |
| (11) | T2: (bus fare non-commuting) | Euro/round trip | 0.53 | 0.37 | 5.43 | 2.65 | 4.93 | 9.08 |
| (12) | T3: car tax commuting | Euro/round trip | 4.24 | 5.73 | -3.09 | 8.12 | 3.95 | 5.99 |
| (13) | T4: bus fare commuting | Euro/round trip | 0.53 | 0.37 | 0 | 2.65 | 0 | 0.49 |

* The optimal labour tax decrease with uniform transport taxes is approximately 16%.

The numerical model strongly suggests that the endogeneity of the value of time is important when changes in marginal external congestion costs and therefore in welfare are assessed. In almost all scenarios considered, this endogeneity not just affects the size of the change in MECC, but even its direction. With constant values of time, marginal external congestion costs would have gone down substantially because of the reduction in overall traffic demand due to the tax changes. In fact, with endogeneity marginal external congestion costs go up by up to 3%. Using fixed values of time hence leads to erroneous estimates of the adaptations of travel demand and modal split to transport tax changes.

### 2.2.2 *Alternative scenarios: some sensitivity results*

The elasticities of substitution and the reference composition of traffic in the previous section have been chosen in order to accord with the average urban context in Belgium. To provide some insight into the sensitivity of the results we report a few results from alternative scenarios. First, we look at the effect of decreasing the share of non-commuting trips. Second, the degree of substitutability between the composite commodity and leisure-related activities (including non-commuting transport) is varied[11].

*Varying the importance of non-commuting trips*

The share of non-commuting trips in the central scenario is 47% of the total. Since the presence of multiple trip purposes was crucial in the theoretical analysis of Section 1, it is to be expected that the impact of transport tax reform on the marginal value of time strongly depends on the relative shares of commuting versus non-commuting. The simulation results in Table 3 support this claim. The table gives for various shares of non-commuting trips the % changes in the marginal value of time, and the % change in welfare associated with two of the tax reforms considered, viz.

[11] We also performed some sensitivity analysis on the slope of the congestion function, the reference modal split, and the other remaining elasticities of substitution. They were found to be less important for the problem at hand, so we omit them here for reasons of brevity.

(1) a 1% labour tax decrease with optimal differentiated transport tax response, and (2) optimal labour and transport taxes.

**Table 3 The dependence of changes in welfare and in the marginal value of time on the reference level and share of non-commuting trips, with tax differentiation between trip purposes**

| Share of non-commuting trips | 1% labour tax decrease | | Optimal labour tax | |
|---|---|---|---|---|
| | % MVOT change | % welfare change | % MVOT change | % welfare change |
| 47% (central sc.) | 5.21 % | 0.16 % | 14.5 % | 0.42 % |
| 31% | 5.18 % | 0.14 % | 6.9 % | 0.18 % |
| 18% | 3.12 % | 0.10 % | 3.4 % | 0.11 % |
| 0.01% | 0.37 % | 0.03 % | 0.37 % | 0.03 % |

The first row in Table 3 refers to the central scenario analysed before. The results show that as the importance of non-commuting transport decreases, the impact of tax adjustments on time values declines. Moreover, the potential welfare gains of the respective policy measures similarly decline. Finally, reaching the maximal attainable welfare gain requires smaller reductions in the labour tax rate when non-commuting transport becomes less important. This can be seen from taking the difference between the welfare change for a 1% labour tax decrease and the optimal labour tax decrease. The reason is, of course, that the size of the non-commuting transport tax base is reduced.

When the share of non-commuting trips is negligible (bottom row in the table), the impact on time values is extremely small because in that case transport taxes are pure commuting taxes, which have the same effect on time values as labour taxes. The labour tax reduction and transport tax increases then have almost no impact on time values. In this case the only source of the limited welfare gain is the improvement of the modal split through the use of modal tax differentiation. In the current application this implies an increase in the share of car commuting (from about 67% to 75%) for a 1% reduction of the labour tax rate[12].

The basic message from this experiment is that, as predicted by the theoretical analysis, the size of the impact of transport tax changes on the marginal value of time

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[12] Note that this change in the modal split has limited effects on travel times, as the level of leisure trips has decreased with respect to the central scenario, while the congestion function has been left unchanged.

crucially depends on the presence and the quantitative importance of at least two trip purposes. Also, the results clarify that there is a close connection between the size of the welfare gain and the size of the change in marginal values of time.

*Varying the elasticity of substitution between the composite commodity and leisure related activities*

In the central scenario, the elasticity of substitution $\sigma$ at the top of the CES-utility tree was equal to 0.7. To test the sensitivity of the results we varied $\sigma$ from 0.2 to 1.2. Intuitively, increasing values lead to larger own price elasticities for the composite commodity, for pure leisure, and for non-commuting transport. The latter increase can be interpreted as representing a large 'latent demand' for (non-commuting) trips. Similarly, the cross-price elasticity between the composite commodity and the aggregate of leisure and non-commuting transport will also rise with increasing $\sigma$.

Two findings stand out from the results. First, increasing $\sigma$ implies much higher increases in the value of time; at the same time, the welfare gain of transport tax reform increases strongly. For the lowest value of $\sigma$, the increase in the marginal value of time between the reference situation and the optimal policy is less than 1%. For the highest value considered, however, it is up to 26%. Similarly, if substitution possibilities between the composite commodity and leisure activities are very low, the welfare gain is very limited (0.06%), but allowing much more flexible substitution leads to a gain of 1.36%. Second, the labour tax reduction required to achieve the maximal possible welfare gain is increasing in $\sigma$. As non-commuting trips become more price elastic, an equal exogenous reduction in the labour tax rate has larger benefits in terms of reducing congestion, while at the same time tax revenues are collected to meet the revenue requirement. However, the reduction in the traffic flow is *smaller* when $\sigma$ is larger (and labour taxes are sufficiently reduced). This is the result of combining (a) a stronger reduction in non-commuting trips, and (b) a larger share of auto-commuting when the level of non-commuting trips is strongly reduced. To the reverse, in case $\sigma$ is low, we found that non-commuting trips are not strongly reduced, and labour supply is increased by encouraging commuting by bus.

Summing up, the effect of transport taxes on welfare, labour supply and the marginal value of time, depend to a large extent on the price elasticity of non-commuting transport demand. When this elasticity is low, labour supply will be encouraged through manipulation of the modal split in commuting, rather than through reducing non-commuting trips. The effects of such a policy on welfare and on the marginal value of time are limited, however.

## 3. Summary and conclusion

This paper suggests that transport tax reform to cope with external congestion costs in a second-best setting tends to increase consumers' marginal value of time savings, if account is taken of the simultaneous presence of commuting and non-commuting trips on the road. The fundamental reason is that the tax reform allows one to shift part of the tax burden to relative complements to leisure, thereby increasing the opportunity cost of leisure. A numerical illustration for a prototype mid-sized Belgian city suggests that the effect of transport tax changes on the marginal value of time is important. Effectively, marginal external congestion costs after the reform are higher than in the pre-reform equilibrium, because the increase in the marginal value of time more than compensates for the reduction in congestion costs due to lower traffic demand. It should be stressed that this increase is accompanied by a welfare increase. This effect also holds when optimal transport and labour taxes are considered. Finally, the increase in the value of time is smaller, but positive, when transport taxes cannot be differentiated across trip purpose. The analysis suggests that assuming constant values of time in an analysis of transport tax reform may produce misleading results whenever the traffic flow consists of commuting and non-commuting transport. Traffic flows that are homogenous in terms of trip purpose, will display a smaller sensitivity of the marginal time value to transport tax changes.

The present analysis is subject to some caveats. First, the assumption of strict complementarity between peak-hour commuting trips and labour supply is restrictive. Relaxing this assumption will affect the optimal values of the tax instruments and the resulting value of time, but the direction of the change can be expected to be the same as discussed here. Assuming strict complementarity probably is to be preferred above treating transport as a standard commodity, when transport tax reform is analysed in a context of distortionary taxes on labour. Second, the analysis has not taken account of distributional considerations. The main impact of this extension presumably is that alternative types of revenues (e.g. increasing lump sum transfers instead of reducing labour taxes) become relatively more attractive. Finally, the numerical results should be considered as exploratory. The goal here is to illustrate the mechanisms at work in the theoretical analysis, using realistic orders of magnitude for the parameters. More realistic policy analysis would require more investment in the transport data (e.g. on the cost characteristics of the public transport sector).

**References**

Arnott R., A. De Palma, and R. Lindsey, 1993, A Structural Model of Peak-Period Congestion: A Traffic Bottleneck with Elastic Demand, *American Economic Review*, 83, 1, 161-179.

Becker G. S., 1965, A Theory of the Allocation of Time, *The Economic Journal*, 75, 493-517.

Bovenberg A. L. and L.H. Goulder, 1996, Optimal environmental taxation in the presence of other taxes: general equilibrium analysis, *American Economic Review*, 86, 985-1000.

Calhtrop, E., 2001, Commuting Subsidies, Chapter 6 in Essays in Urban Transport Economics, Unpublished PhD thesis, Catholic University Leuven.

Calfee J. and C. Winston, 1998, The value of automobile travel time; implications for congestion policy, *Journal of Public Economics*, 69, 1, 83-102

Cornelis E. and K. Van Dender, 2001, Integration of Traffic and Economic Models (ITEM) – Final report, DWTC-CES, Leuven

De Borger B., S. Ochelen, S. Proost and D. Swysen, 1997, Alternative Transport Pricing and Regulation Policies: A Welfare Analysis for Belgium in 2005, *Transportation Research D*, 2(3),177-198.

de Jong, G. and H. Gunn, 2001, Recent evidence on car cost and time elasticities of travel demand in Europe*, Journal of Transport Economics and Policy 35 (2),* 137-160.

De Donnea F. X., 1972, Consumer Behaviour, Transport Mode Choice and Value of Time: Some Micro-Economic Models, *Regional Science and Urban Economics 1*, 355-382.

Glaister S. and D. Lewis, 1978, An Integrated Fares Policy for Transport in London, *Journal of Public Economics*, 9, 341-355.

Gronau, R., 1977, Leisure, Home Production and Work: The Theory of the Allocation of time Revisited, *Journal of Political Economy 85 (6),* 1099-1123.

Hague Consulting Group, 1990, *The Netherlands' 'Value of Time' Study: Final Report*, Den Haag: Hague Consulting Group.

IRIS – Region de Bruxelles-Capitale, 1993, *Plan regional des deplacements – Plan IRIS – Constat et scenario tendanciel*, Region de Bruxelles-Capitale.

Jara-Diaz S. R., 2000, Allocation and value of travel time savings, in D.A. Hensher and K.J. Button, *Handbook of Transport Modelling*, Handbooks in Transport, vol. 1, Pergamon-Elsevier, 303-319

Keeler T. E. and K.A. Small, 1977, Optimal Peak-Load Pricing, Investment, and Service Levels on Urban Expressways, *Journal of Political Economy*, 85, 1-25.

Kraus M., 1989, The Welfare Gains from Pricing Road Congestion Using Automatic Vehicle Identification and On-Vehicle Meters, *Journal of Urban Economics*, 25, 261-281.

LRC (London Research Center), 1994, Travel in London: London Area Traffic Survey 1991. HMSO, London.

Mayeres I. and S. Proost, 1997, Optimal tax and public investment rules for congestion type of externalities, *Scandinavian Journal of Economics*, 99, 261-279

Newbery D. and G. Santos, 2002, *Congestion Charging: The Theoretical Solution versus a Practical Second Best*, CEPR Discussion paper.

Ramjerdi, F., Rand, L. and K. Saelensminde, 1997, *The Norwegian Value of Time Study*, Report 379/1977, Institute of Transport Economics, Oslo.

Parry I. W. H. and A. Bento, 2001, *Revenue Recycling and the Welfare Effects of Road Pricing*, Scandinavian Journal of Economics.

Pollet I., 2000, *Nationale enquete over mobiliteit van huishoudens – verslag van de eerste analyse*, Langzaam Verkeer, Leuven

Proost S. and K. Van Dender, 2001, The welfare impacts of alternative policies to address atmospheric pollution in urban road transport, *Regional Science and Urban Economics*, 31, 4, 383-412.

Small K.,1983, The Incidence of Congestion Tolls on Urban Highways, *Journal of Urban Economics*, 13, 90-111.

Small K. A and J. Yan, 2001, The Value of "Value Pricing" of Roads: Second-Best

Pricing and Product Differentiation, *Journal of Urban Economics*, 49, 2, 310-336

Van Dender K, 2001, *Transport tax reform with multiple trip purposes*, ETE Working paper 2001-17, Catholic University Leuven.

Viton P. A., 1983, Pareto-Optimal Urban Transportation Equilibria, in T.E. Keeler, *Research in Transportation Economics*, Greenwich, Connecticut: JAI Press, 75-101.

Wrede M., 2001, Should commuting expenses be tax deductible? A welfare analysis, *Journal of Urban Economics*, 49, 80-99.

**Appendix 1**

In this appendix we outline the general procedure to determine the impact of price and tax changes on the value of time. Utility is given in general by $U(q_0, q_1, q_2, N)$, and as shown before, the value of time can be obtained as

$$MVOT = \frac{\gamma}{\lambda} = \frac{\frac{U_2}{\lambda} + (1 - t_L - p_2)}{1 + a}$$

Consider now the effect of a labour tax increase on the value of time. It can be written as

$$\frac{d(MVOT)}{dt_L} = \frac{d(\frac{\gamma}{\lambda})}{dt_L} = \frac{\lambda \frac{d\gamma}{dt_L} - \gamma \frac{d\lambda}{dt_L}}{\lambda^2} = \frac{1}{\lambda}\left(\frac{d\gamma}{dt_L} - MVOT \frac{d\lambda}{dt_L}\right)$$

Using the first-order conditions $\lambda = u_0$ and $\gamma = u_N$ we have by differentiation

$$\frac{d\lambda}{dt_L} = u_{00}\frac{dq_0}{dt_L} + u_{01}\frac{dq_1}{dt_L} + u_{02}\frac{dq_2}{dt_L} + u_{0N}\frac{dN}{dt_L}$$

$$\frac{d\gamma}{dt_L} = u_{N0}\frac{dq_0}{dt_L} + u_{N1}\frac{dq_1}{dt_L} + u_{N2}\frac{dq_2}{dt_L} + u_{NN}\frac{dN}{dt_L}$$

To find the impact of the labour tax on consumption and leisure demands, substitute $\lambda = u_0$ and $\gamma = u_N$ into the first-order conditions for commuting and non-commuting transport, and add the time and budget restriction to obtain the following system

$$\begin{aligned}
U_1 &= U_0 p_1 + aU_N \\
U_2 &= -U_0(1 - t_L - p_2) + (1 + a)U_N \\
q_0 + p_1 q_1 &= \left(1 - t_L - p_2\right)L + S \\
\overline{L} &= N + aq_1 + (1 + a)q_2
\end{aligned}$$

Differentiating this system yields a system of four equations in the four unknowns $dq_0, dq_1, dq_2, dN$. Solving for the unknowns allows evaluating the impact of tax and price changes on the solution obtained. This in turn allows evaluating $\frac{d\gamma}{dt_L}, \frac{d\lambda}{dt_L}$ and, finally, $\frac{dMOVT}{dt_L}$. A similar procedure holds for evaluating the effect of transport price changes on the value of time. Not surprisingly, for an unrestrictedly general specification of preferences the results do not yield transparent results; they obviously depend on all second derivatives of the utility function, many of which are difficult to sign. We therefore imposed some simplifying assumptions (quasi-linearity, separability) to obtain clear implications.

## **Appendix 2**

As an example, applying Cramer's rule yields for $dq_1$:

$$dq_1 = \frac{1}{\Delta}\begin{vmatrix} dp_1 & -\gamma a' & u_{1N} & -a \\ dp_2 + dt_L & u_{22} - \gamma a' & 0 & -(1+a) \\ 0 & 0 & u_{NN} & -1 \\ 0 & -(1+a+a'F) & -1 & 0 \end{vmatrix}$$

where

$$\Delta = \begin{vmatrix} u_{11} - \gamma a' & -\gamma a' & u_{1N} & -a \\ -\gamma a' & u_{22} - \gamma a' & 0 & -(1+a) \\ u_{N1} & 0 & u_{NN} & -1 \\ -(a+a'F) & -(1+a+a'F) & -1 & 0 \end{vmatrix}$$

Simple matrix algebra yields the effect of an exogenous price or tax changes on non-commuting transport demand. We find

$$\frac{dq_1}{dp_1} = \frac{1}{\Delta}\begin{vmatrix} u_{22} - \gamma a' & 0' & -(1+a) \\ 0 & u_{NN} & -1 \\ -(1+a+a'F) & -1 & 0 \end{vmatrix} = \frac{-1}{\Delta}\left\{(u_{22} - \gamma a') + u_{NN}(1+a)(1+a+a'F)\right\}$$

$$\frac{dq_1}{dt_L} = \frac{dq_1}{dp_2} = \frac{-1}{\Delta}\begin{vmatrix} -\gamma a' & u_{1N} & -a \\ 0 & u_{NN} & -1 \\ -(1+a+a'F) & -1 & 0 \end{vmatrix} = \frac{1}{\Delta}\left\{(a u_{NN} - u_{1N})(1+a+a'F) - \gamma a'\right\}$$

Assuming declining marginal utility for commuting transport and leisure ($u_{22} < 0, u_{NN} < 0$), it is immediately clear that the own-price effect of non-commuting transport will be negative as long as $\Delta < 0$.[13] We assume this to hold; we return to the interpretation of this condition below. The labour tax and the commuting tax have the same effect on demand since they both identically affect the net real wage, but that this effect is ambiguous. Loosely speaking, it will be positive as long as $u_{1N}$ is not too negative; it may be negative otherwise.

In a similar fashion, one easily derives the impact of tax or price changes on labour supply (equal to commuting) and on leisure demand. We find:

$$\frac{dq_2}{dp_1} = \frac{-1}{\Delta}\left\{(1+a)\left[(u_{1N} - au_{NN}) - a'Fu_{NN}\right] + \gamma a'\right\}$$

$$\frac{dq_2}{dt_L} = \frac{dq_2}{dp_2} = \frac{-1}{\Delta}\left\{a\,u_{NN}(a + a'F) - u_{1N}(2a + a'F) + (u_{11} - \gamma a')\right\}$$

$$\frac{dN}{dp_1} = \frac{1}{\Delta}\left\{u_{1N}(1+a)(1+a+a'F) + u_{22}(a+a'F) + \gamma a')\right\}$$

$$\frac{dN}{dt_L} = \frac{dN}{dp_2} = \frac{-1}{\Delta}\left\{(u_{1N}a - u_{11})(1+a+a'F) + \gamma a'\right\}$$

These findings suggest that, as long as $u_{1N}$ is not too negative, an increase in the labour or commuting tax reduces labour supply (i.e., the labour supply function is upward sloping), and it raises leisure demand. Not surprisingly, the cross-price effect of commuting demand with respect to non-commuting transport price and the impact of the latter on leisure demand can go either way.

For the interpretation, it is important to observe that a sufficient condition for the cross price effect of commuting with respect to the price of non-commuting to be positive is $(au_{NN} - u_{1N})$<0. Also note that one easily shows[14]

$$\frac{dF}{dp_1} = \frac{-1}{\Delta}\left[u_{22} + (1+a)(u_{1N} + u_{NN})\right]$$

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13 This provides a clear interpretation for the stability condition $\Delta$<0. It guarantees that the overall effect of an increase in $p_1$ on non-commuting transport demand, including all feedback effects of congestion on both the commuting and non-commuting markets, is negative. See below for details.

14 Interestingly, note that the price effect on total transport demand is independent of congestion levels.

so that $u_{1N} + u_{NN}$<0 is a sufficient condition for the price of non-commuting transport to reduce overall transport demand and, therefore, congestion.

Finally, for completeness sake let us consider the case where transport prices cannot or are not allowed to differ between commuting and non-commuting transport (i.e., $p_1 = p_2 = p$). Similar analysis shows that all effects of the labour tax remain as before, and the price effects are given by

$$\frac{dq_1}{dp} = \frac{-1}{\Delta}\left\{(u_{NN} + u_{1N})(1 + a + a'F) + u_{22}\right\}$$

$$\frac{dq_2}{dp} = \frac{-1}{\Delta}\left\{(u_{NN} - u_{1N})(a + a'F) + (u_{1N} + u_{11})\right\}$$

$$\frac{dN}{dp} = \frac{-1}{\Delta}\left\{(u_{11} - u_{1N})(1 + a + a'F) - u_{22}(a + a'F)\right\}$$

For the common transport price, the impact of a price increase is simply the sum of the effects of the labour tax and the non-commuting transport price of the price differentiation case. Interpretation is as before.

Finally, let us return briefly to the meaning of the condition $\Delta < 0$. By developing the relevant determinant one easily shows

$$\begin{aligned}\Delta = \Delta_{a'=0} &+ u_{NN}\left\{\gamma a' - aa'Fu_{22}\right\} - \left\{(u_{NN}u_{11} - u_{1N}^2)a'(1+a)\right\} + \left\{\gamma a'(u_{11} + u_{22})\right\} \\ &+ u_{1N}\left\{a'Fu_{22} + 2\gamma a'\right\}\end{aligned}$$

where $\Delta_{a'=0} = -(u_{NN}u_{11} - u_{1N}^2)(1+a)^2 - u_{22}(u_{11} + a^2u_{NN} - 2a)$ is the value of the determinant at constant congestion (a'=0); note that this is negative by the second-order conditions of the consumer's optimisation problem.

All terms in the definition of $\Delta$ are negative with the exception of the last one. Without a mild condition, therefore, it cannot be guaranteed that $\Delta < 0$ and, as a consequence, that $\frac{dq_1}{dp_1} < 0$. We assume throughout that this condition is satisfied. Intuitively, the required condition can be interpreted as a restriction on the size of feedback effects. To see this, note that the demand functions resulting from the consumer's problem can be written in general as functions of prices, the lump-sum transfer S and, since the consumer treats congestion as given, the congestion level (as captured by a):

$$
\begin{aligned}
q_0 &= q_0(p_1, 1-t_L-p_2, a, S) \\
q_1 &= q_1(p_1, 1-t_L-p_2, a, S) \\
q_2 &= q_2(p_1, 1-t_L-p_2, a, S) \\
N &= N(p_1, 1-t_L-p_2, a, S)
\end{aligned}
$$

Differentiating this system, taking account of the definition of $a = a(F) = a(q_1 + q_2)$, the full effect of a price change $dp_1$ on non-commuting demand can also be written as

$$
\frac{dq_1}{dp_1} = \frac{\dfrac{\partial q_1}{\partial p_1}\left(1 - a'\dfrac{\partial q_2}{\partial a}\right) + a'\dfrac{\partial q_2}{\partial p_1}\dfrac{\partial q_1}{\partial a}}{1 - a'\dfrac{\partial q_1}{\partial a} - a'\dfrac{\partial q_2}{\partial a}}
$$

Note that, if congestion were constant, the total effect equals the partial effect. If congestion is not constant, however, the price-induced congestion changes generate feedback effects on demand, implying deviations between the partial and total effect. The denominator is plausibly positive because one expects more congestion to reduce travel demand. The numerator is negative unless the cross-price effect $\frac{\partial q_2}{\partial p_1}$ is negative and large, so that the final term in the numerator more than offsets the first term, which is negative. The economic intuition of this extreme situation is clear. Suppose a price increase of non-commuting transport at constant congestion levels reduces non-commuting transport. But assume that the price increase also implies a large reduction in commuting demand which itself reduces congestion, raising the demand for non-commuting transport again. If this latter effect is so strong as to more than offset the initial negative impact, the numerator of the above expression becomes positive and the ultimate outcome may yield a positive price effect.