Nash-Bertrand competition in a duopoly with congestion*

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Abstract

I characterize the pure strategy Nash-Bertrand equilibrium in a setting where two firms at different locations supply a homogenous good at constant marginal production cost. A representative consumer incurs travel costs to the firm for each unit purchased; these travel costs increase with the amount of travel to each firm. The unique Nash-Bertrand equilibrium price exceeds the sum of the marginal production cost and the marginal external travel cost. Asymmetric equilibria lead to an inefficient distribution of travel between firms. Link tolls or subsidies can be useful to improve the distribution of traffic, but also reduce the welfare costs from imperfect competition.

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1. Introduction

The spatial economics literature emphasizes that spatial dispersion of consumers and producers implies imperfect competition (e.g. [8,9]). Spatial dispersion leads to transport, and transport often takes place under congested conditions. It is then relevant to ask how congestion affects prices under imperfect competition, in a given transport network with given locations of firms and consumers. This paper studies Nash-Bertrand competition in a model with one representative consumer and two firms. The motivation for choosing Nash-Bertrand competition is that it is the most competitive type of imperfect competition. It is the smallest departure from the standard assumption of perfect competition, which is implicit in most of the congestion pricing literature. Other forms of oligopoly are briefly mentioned.

Several congestion pricing studies look at the implications for tolls, of private ownership of road infrastructure under various types of market structure (e.g. [6,7]), but imperfect competition between suppliers at substitute travel destinations under public ownership of the network, has largely been neglected. Yet it is arguably extremely common.

My main findings are that transport costs as such generate no rents; transport cost differences generate rent for the firm with the cost advantage; congestion generates market power, even in the absence of cost differences.[1] More specifically, when consumers pay for transport and transport costs increase with trip volumes, there is a unique Nash-Bertrand equilibrium with positive profits. The equilibrium price is higher

[1] 'Rent' refers to the situation where fixed capacity of some factor allows positive markups (i.e. price exceeds marginal production cost). 'Market power' means that positive markups are possible, regardless of capacity restrictions.

than the sum of the marginal production cost and the marginal external cost of congestion. The intuition for this result is that congestion creates monopoly power.

The implication for transport policy is that there is no need for pure Pigouvian congestion tolls[2], as the marginal external congestion costs are covered by the equilibrium prices. Instead, the welfare-maximizing link toll is non-positive in a symmetric cost structure. Positive tolls may be useful under asymmetric costs, as asymmetry implies an inefficient distribution of traffic over the network. Increasing network capacity is beneficial, not only because it reduces transport costs per se, but also because it weakens firms' market power. These qualitative results carry through under Nash-Cournot behavior and under collusion.

Previous studies of Nash-Bertrand competition with convex marginal production costs [4,5,14] assume that the firms incur all production costs. This would also be the case in our model when firms deliver the good at the consumer location. Under this assumption, and when production costs are identical, there exists a continuum of pricing equilibria, among which the zero-profit equilibrium. The Nash-Bertrand equilibrium in these models depends to some extent on the sharing rule, that is on the assumption on firms' market shares when several firms charge the same price. This is not the case in the present model, where consumers incur transport costs, so that in equilibrium the consumer equilibrium constraint must be satisfied. This uniquely determines firms' market shares. In other words, the equilibrium constraint acts as a sharing rule.

[2] Under first-best conditions, Pigouvian tolls are tolls that equal the marginal external congestion cost on each link in the network.

Section 2 introduces the model and characterizes the Nash-Bertrand equilibrium. Brief comparisons are made with the Nash-Cournot equilibrium and the collusion outcome. Section 3 discusses policy implications, and section 4 concludes.

2. Nash-Bertrand equilibrium

2.1 Assumptions

Consider a transport network that connects the representative consumer's residence to two firms, A and B, as in Figure 1. The firms supply a perfect substitute, so that consumer demand is the sum of both firms' output: $q = q_A + q_B$. Marginal production costs $c_i,\ i = A,\ B$ are taken to be constant. The firms' prices are p_A and p_B. Travel to the firm is costly and paid for by the consumer. Average travel costs a_i increase with link volume ($a_i = a_i\left[q_i\right],\ a'_i \equiv \partial a_i / \partial q_i > 0,\ i = A, B$). This means that there is a congestion externality: $a'_i\, q_i,\ i = A, B$.[3] Demand decreases with the generalized price g, the sum of time costs and prices: $q = q\left[g\right],\ \partial q / \partial g < 0$.

The next section characterizes the interior market equilibrium under Nash-Bertrand-type competition between firms, and then discusses symmetric and asymmetric cases. Section 2.3 deals with corner solutions. Under Nash-Bertrand competition, firms select

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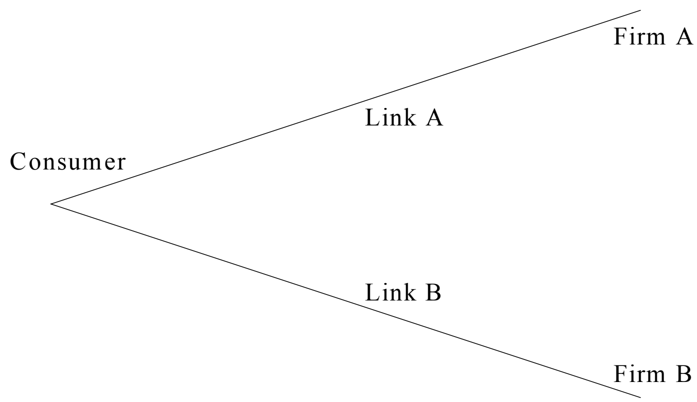
[3] With a single consumer, there would be no congestion externality. In contrast, I assume N identical representative consumers, and normalize N to 1. As each consumer makes one trip to either firm for each unit purchased, the demand curve of a representative consumer can be viewed as the aggregation over consumers with a different willingness to pay for the good, but with equal and constant marginal values of time. If each consumer buys one or zero units, this is consistent with the standard Hotelling approach to recovering a continuous aggregate demand function from discrete individual demands.

prices independently and they commit to supplying all forthcoming demand at the selected price [14: 117]. I restrict attention to pure strategy equilibria.

Figure 1 **Model structure**



2.2 Interior solutions

I assume in this subsection that the market equilibrium is interior: both firms supply positive quantities. Section 2.3 shows that an interior solution holds when transport costs do not strongly differ between firms at the equilibrium level of demand. At an interior solution, it must be true for *consumer equilibrium* that $a_A[q_A]+p_A=a_B[q_B]+p_B\equiv g$ and that $q[g]=q_A+q_B$. Given consumer equilibrium, the effect on the generalized price of a unilateral price change by firm A is:

$$\left.\frac{dg}{dp_A}\right|_{p_B}=a'_A\frac{dq_A}{dp_A}+1=a'_B\frac{dq_B}{dp_A}. \tag{1}$$

This allows writing the demand effect of the price change as follows (holding p_B constant):

$$\frac{dq}{dp_A} = \frac{\partial q}{\partial g}\frac{dg}{dp_A} = \frac{\partial q}{\partial g}\left(a'_A \frac{dq_A}{dp_A} + 1\right) = \frac{\partial q}{\partial g} a'_B \frac{dq_B}{dp_A} = \frac{dq_A}{dp_A} + \frac{dq_B}{dp_A}. \qquad (2)$$

From the last equality in (2), $\frac{\partial q}{\partial g} a'_B \frac{dq_B}{dp_A} - \frac{dq_A}{dp_A} = \frac{dq_B}{dp_A}$, and from (1),

$\frac{dq_A}{dp_A} = \frac{a'_B}{a'_A}\frac{dq_B}{dp_A} - \frac{1}{a'_A}$, so that $\frac{dq_B}{dp_A} = \frac{\partial q}{\partial g} a'_B \frac{dq_B}{dp_A} - \frac{a'_B}{a'_A}\frac{dq_B}{dp_A} + \frac{1}{a'_A}$, or:

$$\frac{dq_B}{dp_A} = \frac{1}{a'_A\left(1 - \frac{\partial q}{\partial g} a'_B + \frac{a'_B}{a'_A}\right)} > 0 \text{ as } \frac{\partial q}{\partial g} < 0. \qquad (3)$$

Since firm B's profits are $\pi_B = (p_B - c) q_B$ and p_B was kept constant, (3) implies that $\frac{d\pi_B}{dp_A} > 0$.

Next consider A's profit maximization problem. Since $\pi_A = (p_A - c) q_A$, for any given p_B, profit maximization requires:

$$\frac{d\pi_A}{dp_A} = q_A + (p_A - c)\frac{dq_A}{dp_A} = 0 \qquad (4)$$

Using (2) and (3), this is equivalent to:

$$-\frac{q_A}{p_A - c} = \frac{dq_A}{dp_A} = \frac{dq_B}{dp_A}\left(a'_B \frac{\partial q}{\partial g} - 1\right) = \frac{a'_B \frac{\partial q}{\partial g} - 1}{a'_A\left(1 - \frac{\partial q}{\partial g} a'_B + \frac{a'_B}{a'_A}\right)} \qquad (5)$$

or:

$$p_A - c = \frac{a'_A q_A}{a'_B \frac{\partial q}{\partial g} - 1}\left(a'_B \frac{\partial q}{\partial g} - \frac{a'_B}{a'_A} - 1\right) = q_A\left(a'_A - \frac{1}{\frac{\partial q}{\partial g} - \frac{1}{a'_B}}\right). \qquad (6)$$

Similarly, for firm B:

$$p_B - c = \frac{a'_B\, q_B}{a'_A \frac{\partial q}{\partial g} - 1}\left(a'_A \frac{\partial q}{\partial g} - \frac{a'_A}{a'_B} - 1 \right) = q_B \left(a'_B - \frac{1}{\frac{\partial q}{\partial g} - \frac{1}{a'_A}} \right). \tag{7}$$

In a Nash-Bertrand equilibrium, (6) and (7) must simultaneously hold. Clearly, the equilibrium markups increase with both congestion externalities, and each markup is always at least as large as the congestion externality on that firm's link. Note that, though prices increase with congestion, profits will decrease when marginal (private and social) transport costs become sufficiently high. Congestion allows charging markups, but also reduces demand through increased generalized prices.

The fact that prices more than cover marginal external congestion costs has implications for the welfare-maximizing link tolls. As the expressions for optimal link tolls for the general case become intractable, this issue is studied for the symmetric case in the next subsection. In addition, the next sections discuss the properties of the equilibrium under symmetric and asymmetric cost functions in more detail, and briefly compare the Nash-Bertrand, the Nash-Cournot and the collusion-outcomes.

Symmetric equilibrium

Symmetry requires that $c_A = c_B = c$ and $a_A[\hat{q}] = a_B[\hat{q}] = a[\hat{q}]$. This directly implies that the market equilibrium is interior: both firms supply positive quantities, as this leads to lower transport costs at any demand level, and this is preferred by the consumer. I prove that this leads to equal market shares and equal prices (symmetric equilibrium). Assume the equilibrium is not symmetric, by postulating that $q_A > q_B$

(with no loss of generality). Since the congestion function is convex, this implies $a'_A \geq a'_B$ and $a'_A q_A > a'_B q_B$.

Assume first that the congestion function is linear, so that $a'_A = a'_B$. Also let $a'_A q_A - a'_B q_B = \varepsilon > 0$. Subtracting (7) from (6) produces:

$$p_A - p_B = \left(a'_A q_A - (a'_A q_A - \varepsilon)\right)\left(\frac{a'\dfrac{\partial q}{\partial g} - 2}{a'\dfrac{\partial q}{\partial g} - 1}\right) > 0. \tag{8}$$

The asymmetric equilibrium with $q_A > q_B$ would require that $p_A > p_B$ and that $a_A > a_B$ (by convexity of the congestion function), i.e. that $a_A + p_A > a_B + p_B$. This is not compatible with consumer equilibrium, so the Nash-Bertrand equilibrium is symmetric: $q_A = q_B$, $p_A = p_B$, $a_A = a_B$.

Next assume that the congestion function is strictly convex, so that $q_A > q_B$ implies $a'_A > a'_B$. This also leads to the untenable implication that $p_A > p_B$, as substituting (6) and (7) in the consumer equilibrium constraint implies:

$$(a'_A q_A - a'_B q_B) - \left(\frac{q_A}{\dfrac{\partial q}{\partial g} - \dfrac{1}{a'_B}} - \frac{q_B}{\dfrac{\partial q}{\partial g} - \dfrac{1}{a'_A}}\right) > 0. \tag{9}$$

The inequality holds because the (positive) difference in the first term cannot be outweighed by the second term. When the difference in quantities is small ($q_A \approx q_B$) but the slopes of the travel cost function strongly differ ($a'_A \gg a'_B$), the first term is large and the second term converges to the inverse of the elasticity of demand, so that the overall term remains positive. Conversely, when the first term goes to zero (which is the case when q_A approaches q_B for a very small difference in congestion conditions), so does the second one. In other words, the difference converges to zero as the system moves towards a symmetric equilibrium.

Consequently the equilibrium is symmetric and the price is given by:

$$p - c = \frac{1}{2}a'q\left(\frac{a'\dfrac{\partial q}{\partial g} - 2}{a'\dfrac{\partial q}{\partial g} - 1}\right). \tag{10}$$

This expression says that the Nash-Bertrand equilibrium price depends on marginal external congestion costs and on the price elasticity of demand. As in (6) and (7), the markup exceeds the marginal external congestion cost (here equal to $\frac{1}{2}a'q$) since the bracketed term in (10) is larger than one.

Equation (10) is reminiscent of the collusion (monopoly) price in a market with congestion. In the symmetric model, the markup that maximizes the profit from collusion is as follows[4]:

$$p - c = -\frac{q - \frac{1}{2}a'q\dfrac{\partial q}{\partial g}}{\dfrac{\partial q}{\partial g}} = \frac{1}{2}\left(\frac{a'q\dfrac{\partial q}{\partial g} - 2}{a'q\dfrac{\partial q}{\partial g}}\right) = \frac{g}{\left|\varepsilon_{qg}\right|} + a'\frac{1}{2}q. \tag{11}$$

where $\varepsilon_{qg} = \dfrac{\partial q}{\partial g}\dfrac{g}{q} < 0.$

The second expression of the right-hand-side clarifies that the monopoly markup exceeds the duopoly markup specified in (10). The third expression says that the monopoly price

[4] This is the symmetric version of the general expression derived in appendix 1.

takes account of the elasticity of demand with respect to the generalized price, and internalizes the congestion externality (which is equally divided over both links). Symmetry is desirable for the monopolist because it minimizes transport costs for any given level of demand and differences in transport costs are not profitable. The analogy with [11: 141-142] is clear from observing that the monopolist's markup over marginal social cost is inversely related to the elasticity of demand.

The presence of congestion hence gives the Nash-Bertrand competitors some degree of monopoly power, in contrast to the standard Nash-Bertrand case with constant marginal production costs. In the case of Nash-Bertrand competition with symmetric but strictly convex marginal production costs that are internal to the firm [4,5,14], a continuum of Nash-Bertrand equilibria are possible (among which is the zero-profit equilibrium). In the present case, however, the equilibrium is unique because of the consumer equilibrium constraint. The consumer incurs the transport costs, instead of the firm incurring an increasing production cost that is 'hidden' from the consumer. The generalized price equilibrium condition prevents the possibility of multiple equilibria, as it is decisive for the distribution of demand over firms.[5]

The following limit cases allow comparing the symmetric Nash-Bertrand and the monopoly (collusion) prices, denoted by p^D and p^M respectively:

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[5] In this sense, the consumer equilibrium condition is a sharing rule. When no congestion is present, the Nash-Bertrand equilibrium depends on assumptions on the sharing rule (e.g. equal sharing or capacity sharing, cf. [5,14: 120]).

$$
\begin{aligned}
&\text{(a)}\ \frac{\partial q}{\partial g} \to 0 \Rightarrow p^M - c \to +\infty;\ p^D - c \to a'q \\
&\text{(b)}\ \frac{\partial q}{\partial g} \to +\infty \Rightarrow p^M - c \to \frac{1}{2}a'q;\ p^D - c \to \frac{1}{2}a'q \\
&\text{(c)}\ a' \to 0 \Rightarrow p^M - c \to -\frac{q}{\partial q / \partial g};\ p^D - c \to 0 \\
&\text{(d)}\ a' \to +\infty \Rightarrow p^M - c \to p^{\max} - c;\ p^D - c \to \frac{p^{\max} - c}{2}
\end{aligned}
\tag{12}
$$

Case (a) shows that market power in the Nash-Bertrand case is limited because of price setting behavior. While the monopoly markup increases as the elasticity of demand decreases, the Nash-Bertrand markup never exceeds the total external congestion cost in the network (i.e. it never exceeds double the external congestion cost on the link to each firm). When demand becomes very elastic, cf. (b), the monopoly markup and the Nash-Bertrand markup converge to the marginal external congestion cost on the link to each firm. Perfectly elastic demand hence forces the firms to internalize the externality, that is to charge the marginal social cost of producing and transport the good. Case (c) shows that in the absence of congestion, the standard monopoly and Nash-Bertrand outcomes are obtained. This confirms the conclusion that congestion generates market power under both types of market structure. Lastly, (d) says that when congestion becomes extremely high, a monopolist will charge a price approaching the choke-off price[6] and the duopolists charge half the monopoly markup. A monopolist will charge what the market will bear, while the duopolists equally split the market and jointly charge what the market will bear.

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[6] The choke-off price is that price at which the demand curve cuts the Y-axis (assuming that it does so). The result also requires assuming that as $a' \to +\infty$, then $a \to +\infty$, so that the generalized price converges to infinity and demand converges to zero.

While demand in the symmetric Nash-Bertrand equilibrium is below the socially optimal level, it is efficiently produced. This can be seen from the fact that the marginal social costs of producing and transporting the good are equal for both firms, cf. (14). Network assignment (the distribution of traffic over the network) hence is optimal, in the sense that it minimizes travel costs for the given level of demand.

$$MSC_A - MSC_B = c + a_A + a'_A q_A - \left(c + a_B + a'_B q_B\right) \tag{13}$$

Using the consumer equilibrium condition, (6) and (7), (13) becomes:

$$MSC_A - MSC_B = \left(q_A - q_B\right)\left(\frac{1}{\dfrac{\partial q}{\partial g} - \dfrac{1}{a'}}\right) = 0 \text{ as } q_A = q_B \tag{14}$$

Efficient production of the equilibrium quantity does not imply that the equilibrium quantity is socially optimal, so that link tolls or subsidies are potentially useful. Appendix 2 presents a detailed analysis of the welfare-maximizing link tolls for the symmetric case, assuming that government is a Stackelberg leader. Given symmetry, the link tolls are equal, since otherwise a production inefficiency is introduced. In particular, when welfare consists of consumer surplus, profits and toll revenues, and when both links can be tolled, the implicit expression for the optimal uniform toll is:

$$t = \frac{g}{\varepsilon_D}\left(1 - \frac{1 - \dfrac{1}{2}a'\dfrac{\partial q}{\partial g}}{2\left(\dfrac{\alpha}{1-\alpha} + 1\right)}\right) - a'\left(q - \frac{a'}{\dfrac{\partial q}{\partial g} - \dfrac{1}{a'}}\right), \tag{15}$$

where $\varepsilon_D = \dfrac{\partial q}{\partial g}\dfrac{g}{q} < 0$ and $\alpha = \dfrac{1}{2}\left(a' - \dfrac{1}{\dfrac{\partial q}{\partial g} - \dfrac{1}{a'}}\right)\dfrac{\dfrac{\partial q}{\partial g}}{1 - \dfrac{1}{2}a'\dfrac{\partial q}{\partial g}} < 0$.

The last term in (15) is equal to *minus* the firms' markup in excess of marginal external congestion costs *minus* network congestion. So, this term is a subsidy, which is the sum of the firms' markup in excess of (link) congestion and *network* congestion costs (as opposed to link congestion, which is taken into account by firms; government introduces a subsidy equal to network congestion, since that is what matters from the social point of view).

The first term relates to the weighted elasticity of demand. The weight may take either sign. When negative, the first term is positive and reduces the subsidy (second term) that alleviates the suboptimality in demand following from imperfect competition. Detailed analysis shows that the first term is decreasing in the elasticity of demand and increasing in the slope of the congestion function, with the opposite directions holding for the second term. Also, the first term never more than outweighs the second one, so that the upper bound for the optimal toll is zero. This upper bound is approached as demand becomes less elastic and congestibility of the network is reduced. To conclude, in the symmetric case welfare is improved by a uniform link toll that is non-positive. The subsidy essentially brings demand closer to the socially optimal level, i.e. the level obtained under pure competition.

Asymmetric equilibria

Since the basic properties of the symmetric outcome continue to hold in asymmetric cases, the discussion here is limited to the simplest type of asymmetry: linear congestion functions with different intercepts. More general cases are presented in Appendix 3. When both congestion functions are linear but the intercept on link *A* is larger, I have that $a_A > a_B$, $a'_A = a'_B$, $c_A = c_B$. This situation can be thought of as representing links of equal capacity but different length.

First I show that $q_A < q_B$ in equilibrium. Assume on the contrary that $q_A = q_B$. Consequently $a_A > a_B$. To satisfy the consumer equilibrium constraint, this must imply $p_A < p_B$. From inspection of (6) and (7), Nash-Bertrand equilibrium under the given assumptions requires $p_A = p_B$. So equal market sharing is not an equilibrium. Assume next $q_A > q_B$. Consumer equilibrium then requires $p_A < p_B$ while Nash-Bertrand equilibrium implies $p_A > p_B$, so the initial assumption is untenable. Therefore $q_A < q_B$.

Next consider the equilibrium marginal social costs:

$$MSC_A - MSC_B = c + a_A + a'_A q_A - \left(c + a_B + a'_B q_B\right). \tag{16}$$

Using the consumer equilibrium condition, (6) and (7), (16) becomes:

$$MSC_A - MSC_B = \left(q_A - q_B\right)\left(\frac{1}{\dfrac{\partial q}{\partial g} - \dfrac{1}{a'}}\right) > 0. \tag{17}$$

This says that the marginal social cost at firm A, the firm with the cost disadvantage, is larger than that at firm B. The equilibrium is inefficient, as the same demand could be satisfied more cheaply. The inefficiency is resolved by increasing q_B and decreasing q_A, keeping q constant, which can be achieved using a toll on link A or a subsidy on link B that equalizes marginal social costs on both links. The inefficiency caused by imperfect competition as such, i.e. the suboptimal level of demand, is not affected (cf. (15)).

Nash-Cournot solution

For completeness, consider the case of Nash-Cournot competition. The first-order condition for profit maximization for firm A, holding constant q_B, is:

$$\left(p_A - c\right) + \frac{dp_A}{dq_A} q_A = 0 . \tag{18}$$

Since $q = q_A + q_B$ and I assume Nash-Cournot behavior $\left.\frac{dq}{dq_A}\right|_{q_B} = 1$. Also, using the demand function: $\left.\frac{dq}{dq_A}\right|_{q_B} = \frac{\partial g}{\partial q}\frac{dg}{dq_A}$ and, using the consumer equilibrium constraint: $\frac{dg}{dq_A} = a'_A + \frac{dp_A}{dq_A}$. Combining these expressions with (18) produces:

$$p_A - c = a'_A\, q_A - \frac{q_A}{\partial q / \partial g} . \tag{19}$$

For a Nash-Cournot equilibrium, (20) needs to hold simultaneously with (19).

$$p_B - c = a'_B\, q_B - \frac{q_B}{\partial q / \partial g} \tag{20}$$

Clearly the Nash-Cournot equilibrium markup is always larger than the Nash-Bertrand markup, and always smaller than the markup under collusion. The general case of collusion, of which the symmetric collusion case was discussed above, is somewhat less transparent, and is relegated to appendix 1.

2.2 Corner solutions

In a corner solution one firm satisfies all demand. With no loss of generality, assume that $a_A > a_B$. Suppose transport costs to *B* are sufficiently below those to *A*, such that $q_A = 0$ and $q_B = q > 0$. For this to be a consumer equilibrium, it is required that $p_B - c \leq a_A[0] - a_B[q]$. This condition says that at the equilibrium demand, firm *A* cannot profitably enter the market, and this determines the maximum markup that firm *B* can charge as a monopolist.

It is clear that firm *B* will never voluntarily share the market, as (a) sharing the market is possible only when firm *A* makes nonnegative profits, which requires an increase in the generalized price, hence a demand decrease, hence lower demand to firm *B*, and (b) sharing the market involves competition, which cannot increase firm *B*'s price. Sharing the market thus involves reduced profits to firm *B*.

So, for a given transport cost configuration, market sharing is fully demand driven: if firm *B* does not satisfy all demand at its maximum monopoly price[7], firm *A* enters the market and Nash-Bertrand competition results.

Transport policies that reduce the cost asymmetry may be beneficial, as they can generate a switch from the monopoly to the duopoly solution (whereas in the interior case they could only alleviate the inefficiency cost associated with the asymmetry). This can be obtained by imposing a toll on travel to link *B*.

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[7] It is of course possible that firm B makes maximal monopoly profits at a price below the threshold. Then the monopoly outcome is obtained. I further abstract from this case, assuming that market demand is sufficiently high.

3. Policy implications

While the simplicity of the model prevents definitive conclusions, it has implications for policy. The transportation economics literature extensively studies the desirability and feasibility of Pigouvian congestion tolls (for an overview, cf. e.g. [3]). From the present analysis it follows that network pricing is useful for equalizing marginal social costs across links and firms. For given network capacity, asymmetric interior equilibria are inefficient as the equilibrium demand level can always be met at a lower social cost. Pure Pigouvian tolls, however, are not desirable in a symmetric equilibrium because prices will more than cover marginal external congestion costs even without them. So, in contrast to most of the literature on congestion tolls, I find that pure Pigouvian tolls are unnecessary or even harmful as a demand management instrument, while they can improve network use. Instead, the welfare-maximizing (uniform) toll is found to be negative. Note however that, when starting from a corner solution, the introduction of a congestion toll potentially changes market structure: a monopoly can be transformed into a Nash-Bertrand duopoly. This increases competition, with the associated benefits for consumers.

Like tolls, decisions on network capacity and the distribution of capacity between destinations can affect firms' market power. The implication is that optimal investment rules in road or network capacity should not only refer to travel time savings (as in the standard capacity investment rule, cf. e.g. [11]), but also to changes in market power. This point is related to [10] where monopolistic behavior on behalf of two firms with different production cost functions and non-congestible transport is assumed.

The model also sheds some light on the question whether land developers can or should be charged for new or additional capacity that their development requires [12: 1945-1946]. If adding a link induces market structure to change from a monopoly to a duopoly, arguably the link should be publicly financed. Charging the developer would increase his marginal production costs, so reducing the probability of entry. It should be noted, however, that questions regarding financing roads and market entry are decisions on provision and financing of new capacity. The Nash-Bertrand model has less to say about this than the Nash-Cournot model, where the quantity decision may be viewed as a decision on capacity. While it was shown above that Nash-Cournot competitors will charge higher markups than Nash-Bertrand competitors, results on tax incidence are differ between both (cf. [1] for a discussion of tax incidence in a Nash-Cournot model).

The relevance of the present model to other network industries is limited, as in these industries congestion is manageable by the network operator. If the operator maximizes social welfare, the cost minimizing assignment is obtained for any demand level. If the operator maximizes profits, network congestion becomes a strategic variable (as is the case in the electricity industry, cf. e.g. [2]).

4. Conclusion

I have characterized the pure strategy Nash-Bertrand equilibrium in a setting where two firms at different locations supply a homogenous good at constant marginal production cost. A representative consumer incurs travel costs to the firm for each unit purchased, and these travel costs are increasing in the amount of travel to each firm. It was found that the unique Nash-Bertrand equilibrium price exceeds the sum of the marginal production cost and the marginal external travel cost, and that asymmetric equilibria lead to an inefficient distribution of travel between firms. Congestion tolls then may be useful to improve the distribution of traffic, but optimizing demand requires a subsidy.

I mention some caveats. First, the analysis could be extended to general networks used by different types of consumers. Restricting the analysis to one consumer type rules out the potential benefits from product differentiation. In a transport context, value pricing [13] holds potential benefits, also under imperfect competition. Second, multiple purpose trips (trip chaining, e.g. commuting and shopping) could be considered. Finally, the results are contingent on the exogeneity of location. Introducing location choice will increase the model's realism.

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Appendix 1 Collusion pricing rule when both plants produce

With collusion, the following program is solved:

$$\max_{p_A, p_B} \left(\pi = (p_A - c) q_A + (p_B - c) q_B \right). \tag{21}$$

The first order conditions read:

$$\begin{aligned} q_A + (p_A - c)\frac{dq_A}{dp_A} + (p_B - c)\frac{dq_B}{dp_A} = 0 \\ q_B + (p_B - c)\frac{dq_B}{dp_B} + (p_A - c)\frac{dq_A}{dp_B} = 0 \end{aligned}. \tag{22}$$

On using the properties of the consumer equilibrium, and after rewriting, these conditions become:

$$p_B - c = \frac{p_A - c}{1 - \partial q / \partial g \, a'_A} + a'_B \, q_B \left(1 + \frac{a'_A}{\left(1 - \partial q / \partial g \, a'_A\right) a'_B} \right) \\ p_A - c = \frac{p_B - c}{1 - \partial q / \partial g \, a'_B} + a'_A \, q_A \left(1 + \frac{a'_B}{\left(1 - \partial q / \partial g \, a'_B\right) a'_A} \right). \tag{23}$$

These conditions imply that each plant's markup exceeds the marginal external congestion cost of traveling to the plant (second term on RHS). It also depends on the markup at the other plant. The next equation is obtained by substituting one first order condition in the other, and shows that the markup at one plant takes account of the elasticity of demand and that it optimally trades off congestion conditions of both plants:

$$p_A - c = -\frac{1}{\partial q / \partial g} \left(\frac{a'_A \left(q_A \left(1 - \frac{\partial q}{\partial g} a'_A\right)\left(1 - \frac{\partial q}{\partial g} a'_B\right) + q_B \right) + a'_B \, q \left(1 - \frac{\partial q}{\partial g} a'_A\right)}{a'_A + a'_B - \partial q / \partial g \, a'_A \, a'_B} \right). \tag{24}$$

Appendix 2 Welfare optimizing tolls – symmetric case

I derive the welfare optimizing tolls, restricting attention to the fully symmetric and linear[8] version of the model, as more general configurations become intractable. In deriving the optimal tolls, I assume that the social welfare maximizer is a Stackelberg leader, who takes firms' reaction functions into account. Firms and consumers take tolls as parametric. When welfare is the sum of consumer surplus, profits and tax revenues, it is maximized through the following program:

$$W = \int_0^q P[q]\,dq - gq + (p_A - c)\,q_A + (p_B - c)\,q_B + t_A q_A + t_B q_B \,. \tag{25}$$

Using the reaction functions, see (6) and (7), this becomes

[8] This means constant partial derivatives of the demand and congestion functions.

$$W=\int_0^q P[q]\,dq-gq+(q_A)^2\left(a'_A-\frac{1}{\dfrac{\partial q}{\partial g}-\dfrac{1}{a'_B}}\right)+(q_B)^2\left(a'_B-\frac{1}{\dfrac{\partial q}{\partial g}-\dfrac{1}{a'_A}}\right)+t_A q_A+t_B q_B \,.(26)$$

With symmetry, total demand is split equally between firms, who charge the same prices. Consequently, a welfare-optimizing toll should be equal on both links. If tolls differed between links, demand would not be split equally across firms, and this leads to inefficient provision of total demand because of the consumer equilibrium constraint. So, the first order condition takes the following form, where *t* denotes the uniform link toll:

$$t=\frac{q}{\dfrac{dq}{dt}}\left(\frac{dg}{dt}-\frac{1}{2}\right)-a'q+\frac{a'}{\dfrac{\partial q}{\partial g}-\dfrac{1}{a'}}. \qquad (27)$$

The last term of this expression is equal to minus the firms' markup in excess of marginal external congestion costs: government increases demand by offsetting this part of the markup by an equal subsidy. The second term reflects *network* congestion, as opposed to link congestion (which is the other part of the firms' markup): government increases demand through a subsidy equal to network congestion, not link congestion (since network congestion is what matters from the social point of view).

The first term merits further attention. Since $\frac{dq}{dt}=\frac{\partial q}{\partial g}\frac{dg}{dt}$, it can be written as

$\frac{q}{\frac{\partial q}{\partial g}}\left(1-\frac{1}{2\frac{dg}{dt}}\right)$. From the consumer equilibrium constraint for the symmetric case,

$\frac{dg}{dt}=\frac{\frac{dp}{dt}+1}{1-\frac{1}{2}a'\frac{\partial q}{\partial g}}$. From the reaction functions: $\frac{dp}{dt}=\frac{\alpha}{1-\alpha}\in[-1,0]$, where

$\alpha=\frac{1}{2}\left(a'-\frac{1}{\frac{\partial q}{\partial g}-\frac{1}{a'}}\right)\frac{\frac{\partial q}{\partial g}}{1-\frac{1}{2}a'\frac{\partial q}{\partial g}}<0$. Using these expressions, the implicit equation for the optimal tax becomes:

$$t = \frac{g}{\varepsilon_{qg}}\left(1 - \frac{1 - \frac{1}{2}a'\frac{\partial q}{\partial g}}{2\left(\frac{\alpha}{1-\alpha}+1\right)}\right) - a'\left(q - \frac{a'}{\frac{\partial q}{\partial g} - \frac{1}{a'}}\right), \tag{28}$$

where $\varepsilon_{qg} = \frac{\partial q}{\partial g}\frac{g}{q} < 0$. The weight for this elasticity may take either sign. The numerator of the weight's second term is larger than 1, and the denominator is between 0 and 2, so that the second term takes any positive value. When it exceeds one, the weight becomes negative, and the entire first term is positive. In other words, the first term may reduce the subsidy (second term) that alleviates the suboptimality in demand following from imperfect competition.

On closer inspection, it can be shown that (a) the first term increases as the slope of the congestion function increases, (b) the first term increases as the elasticity of demand decreases, (c) the (negative) second term increases as the slope of the congestion function decreases, and (d) the (negative) second term increases as the elasticity of demand increases. The first term becomes positive for low demand elasticities and large slopes of congestion functions, but it never outweighs the subsidy from the second term. The optimal tax approaches zero as congestion becomes very small and demand is inelastic.

Appendix 3 Asymmetric interior Nash-Bertrand equilibria

(a) Linear congestion functions with different intercepts

This case is discussed in the text.

(b) Convex congestion functions with different intercepts

This set of cost conditions is characterized by

$a_A > a_B$, $a'_A[\hat{q}] = a'_B[\hat{q}]$, $a'_A[q_A] > a'_B[q_B]$ when $q_A > q_B$, $c_A = c_B$. Some algebra shows that, as in case (a), $q_A < q_B$ and $MSC_A > MSC_B$.

(c) Different single crossing quasi-convex congestion functions

I now allow different convex congestion functions, with the only restriction that they cross at most once. By the same reasoning as before, $q_A < q_B$ and $MSC_A > MSC_B$.