Prices and Exchange Rates: A Theory of Disconnect

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Abstract

I present a sticky-wage model of exchange rate pass-through with heterogeneous producers and endogenous markups. The model shows that low levels of exchange rate pass-through to firm- and aggregate-level import prices coexist with large movements in trade flows. After an exchange rate shock, aggregate import prices are subject to a composition bias due to changes in the extensive margin of trade (the number of goods traded between countries). At the firm level, each producer adjusts its markups depending on its own productivity and the change in the competitive environment generated by the exchange rate movement. Firm-level price responses are asymmetric—different for appreciations and depreciations—and adjustments in the intensive margin of trade (firm-level exports) are substantial. In general equilibrium, the model shows that firm reallocations increase the persistence of exogenous shocks.

Keywords: exchange rate pass-through, expenditure-switching effect, heterogeneous firms, endogenous markups.
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1 Introduction

Exchange rates and prices are disconnected in developed economies. Studies for industrialized countries consistently show a low and slow pass-through of nominal exchange rate changes to consumer import prices (Engel, 2002).[1] This empirical regularity is usually interpreted as implying that the effect of nominal exchange rates on quantities traded between countries, the so-called expenditure-switching effect of exchange rates, is negligible: if exchange rate movements do not affect prices, then there are no changes in demand and the quantity traded between countries does not change. This interpretation is the main support for an assumption of zero pass-through and no expenditure-switching that is frequently used in open economy macroeconomics (e.g., Devereux and Engel, 2003), with strong implications on welfare analysis and the optimal monetary policy. This interpretation, however, ignores the fact that firms are the relevant decision-makers regarding adjustment after an exchange rate movement.[2] In this paper, firms' decisions about pricing and entry and exit take a primary role in the analysis of pass-through and the expenditure-switching effect of exchange rates. In contrast to the long-held view, my theoretical results show that exchange rate movements have substantial expenditure-switching effects, even in the presence of low levels of exchange rate pass-through to firm- and aggregate-level import prices.

The model in this paper is inspired by the work of Dornbusch (1987). In a partial equilibrium setting with fixed wages, Dornbusch investigates how the competitive environment affects the way firms adjust prices after an exogenous exchange rate movement; the competitive environment is captured by the number of domestic and foreign competitors and by the degree of product substitutability. In a series of models he shows that the response of prices to exchange rate shocks depends on how firms' markups adjust.[3] Dornbusch also notes that exchange rate movements affect firms' entry and exit dynamics and suggests looking at "how pricing decisions are affected by entry and relocation possibilities at an international level." Following this suggestion, I develop a model of exchange rate pass-through with endogenous markups that allows for endogenous entry and reallocation of firms.

In my two-country model, exchange rate movements generate international firm reallocations because firms are heterogeneous with respect to their levels of productivity. As in Melitz (2003), a single-product firm knows its relative productivity only after entry; the tradability of its good—in both the domestic and export markets—is endogenously determined. The least productive firms sell in no market, some of them sell only in the domestic market, and the most productive

[1] The rate of exchange rate pass-through to a price is defined as the elasticity of the price with respect to the exchange rate. A rate of 1 means a full (or complete) exchange rate pass-through.

[2] In particular, Obstfeld and Rogoff (2000) and Obstfeld (2002) argue in favor of high pass-through rates to border import prices and important expenditure-switching effects, but mention that this is hardly reflected in consumer prices due to factors like distribution services, advertising, other nontradable costs, and the pricing policies of importing firms. In this respect, Campa and Goldberg (2005) find average pass-through rates to import prices—before distribution costs—of 46 percent and 64 percent in the short and long run, respectively, for OECD countries. For the U.S., the estimates are 25 percent and 40 percent.

[3] In a related paper, Krugman (1987) coined the term "pricing to market" to refer to a situation of incomplete exchange rate pass-through as a result of firms setting different prices in different destination markets, depending on each market's conditions. As mentioned by Atkeson and Burstein (2008), pricing to market exists in the presence of trade costs (or some form of market segmentation) and imperfect competition with variable markups.

firms sell in both the domestic and export markets. Based on zero-markup and free-entry conditions, my model solves for cutoff productivity levels for selling in the domestic and export markets. Exchange rate movements affect the extensive margin of trade—the number of goods traded between countries—by altering the cutoff productivity levels.[4]

But how are the extensive margin of trade and exchange rate pass-through related? It turns out that changes in the extensive margin of trade can generate a disconnect between exchange rates and aggregate import prices. Consider a version of Melitz's (2003) model with two countries (A and B), fixed costs of exporting, and consumers with constant elasticity of substitution (CES) preferences over differentiated goods. Firms are heterogeneous with respect to their productivity so that more productive firms have lower marginal costs and set lower prices. With CES preferences, markups are exogenous, which implies full exchange rate pass-through to firm-level import prices. Let the aggregate import price in country A be given by the average price of all imports from country B. If the currency of country A depreciates, country B's exporters become instantly less competitive in country A. With heterogeneous firms, the least productive (higher price) exporting firms from country B will exit country A. If attrition is sufficiently high, the new aggregate import price in country A, computed using only the surviving firms from B, could even decline (a negative pass-through rate for the aggregate import price!). Therefore, in this model with full pass-through at the firm level, aggregate import prices are subject to a survivorship bias—a sample selection problem.[5] Moreover, aggregate import prices do not reflect the true expenditure-switching effect of exchange rates, which is large due to the decrease in the number of exporters from B (the extensive margin) and the decrease in exports of the survivors (the intensive margin).

As mentioned before, with endogenous markups there is also a disconnect between exchange rates and firm-level import prices. The demand system with endogenous markups that serves as the core of my monopolistic competition model is derived from a continuum-of-goods version of the translog expenditure function introduced by Bergin and Feenstra (2000). In this demand system, markups increase with firm productivity. After an exchange rate movement, pass-through to firm-level prices is incomplete as a result of two reinforcing effects in each firm's markup: a firm-specific effect, related to the firm's own productivity level; and an economy-

[4] In this model the variations in the extensive margin of trade correspond exactly to variations in the number of sellers because each firm produces a single good. Recent research by Bernard, Redding, and Schott (forthcoming) considers the case of multi-product firms and divides the variations in the extensive margin of trade into two components: the variation due to entry and exit of firms, and the variation due to existing firms adding or dropping products.

[5] In principle, an international price index is based on a fixed basket of goods, and as long as this basket remains the same, it should not be affected by changes in the extensive margin of trade. In practice, however, baskets of goods used to compute indexes tend to change frequently as new goods enter the market and some other goods disappear. In the U.S., the International Price Program (IPP) of the Bureau of Labor Statistics frequently changes the bundles used to compute the international price indexes. According to the IPP's website, 25% of the universe used to compute international prices is resampled every six months. In an extreme case, it would be possible to have two entirely different samples every two years. Moreover, the IPP explicitly addresses the importance of changes in the extensive margin of trade for international prices. The IPP website states: "In the realm of international trade, the appearance of new goods and the disappearance of other goods presents a serious problem that requires the International Price Program to resample its universe frequently. The volatility in traded goods arises from changes in domestic prices, as well as changes in the prices of foreign goods and changes in exchange rates." (See `http://www.bls.gov/mxp/ippadd1.htm`) It is interesting to note that the IPP identifies exchange rate changes as one of the sources for the changes in the extensive margin of trade.

wide effect, which mirrors the change in the competitive environment generated by the exchange rate movement. Furthermore, the direction of the exchange rate movement matters for both the firm-specific and the economy-wide effect. In particular, exchange rate pass-through to firm-level import prices is higher for appreciations than for depreciations of the importer's currency.

I present two versions of the model. First, I introduce a partial equilibrium version that allows us to identify the main mechanisms of transmission of an exchange rate shock. As in Dornbusch (1987), I assume that exchange rate movements are exogenous and that wages are fixed. Entry and exit of firms are allowed. The partial equilibrium model is analytically tractable, and a closed-form solution exists. Second, I introduce a general equilibrium version of the model following the new open economy macroeconomics tradition. As in Obstfeld and Rogoff (2000), the model is stochastic, and nominal wages are sticky (set a period in advance). This nominal rigidity is the model's only source of monetary non-neutrality. In this version of the model, the exchange rate responds endogenously to a monetary shock. Moreover, current-account imbalances are possible due to international trade in riskless bonds. To preserve the second-order effects, the general equilibrium model is solved using a second-order accurate solution method.

A strong prediction of my model—in both versions—is that the exchange rate pass-through to the aggregate import price is negative at the time of the exchange rate movement (e.g., a depreciation of the importer's currency reduces the aggregate import price). This result is the consequence of a composition effect—similar to the survivorship bias example above—due to changes in the extensive margin of trade, and it holds even if the average firm-level pass-through rate is positive. This result is not exclusive to the translog case, and in particular, it holds for the CES case. The empirical regularity is, however, a low but positive rate of pass-through to aggregate import prices. This empirical fact can be reconciled with the previous theoretical prediction if we consider that the original model does not distinguish between unit and quality-adjusted prices. Therefore, I develop a brief extension of the model that incorporates this distinction. The quality model predicts a negative pass-through rate to the aggregate quality-adjusted import price but the pass-through to the aggregate unit import price can be positive. On the other hand, pass-through rates to firm-level prices are not affected by this extension—that is, the rate of pass-through is identical for a firm's quality- and non-quality-adjusted price.

The paper is organized as follows. Section 2 reviews the theoretical and empirical background on endogenous markups and pass-through, the importance of the extensive margin of trade, and the evidence on sticky wages and firm entry/exit. Section 3 presents the partial equilibrium model and section 4 shows its implications for the impact of exchange rate movements on (firm- and aggregate-level) prices and trade flows. Section 5 introduces the general equilibrium version of the model and explores its implications for pass-through and trade flows after a permanent monetary shock. Section 6 presents the extension of the model that considers quality. Finally, section 7 concludes.

2 Theoretical and Empirical Background

In addition to the seminal theoretical contributions on endogenous markups and pass-through of Dornbusch (1987) and Krugman (1987), the response of markups to exchange rate changes has strong empirical support. Goldberg and Knetter (1997) survey the evolution of empirical studies on exchange rates and prices up to the mid-1990s and show evidence in favor of pricing-to-market models. They conclude that destination-specific changes in markups are a very significant factor in the lack of response of prices to exchange rate changes. Recent industry-specific pass-through studies find similar results. For example, Hellerstein (2008), using scanner data from a retailer in the Chicago area, finds that markup adjustment accounts for about half of the lack of response of imported beer prices to exchange rate changes. Along the same lines, Nakamura and Zerom (forthcoming) find that markup adjustment in the coffee industry reduces the rate of pass-through in about 33% when compared against a CES benchmark.

The recent theoretical literature on heterogeneous firms has also paid substantial attention to endogenous markups. This is the case because the empirical observations that motivated the flagship heterogeneous-firm models of Bernard, Eaton, Jensen, and Kortum (2003) and Melitz (2003) also highlighted the effects of trade liberalization on the degree of competition. Tybout (2003), for example, surveys empirical studies documenting both the productivity differences between exporters and non-exporters and the decline of firms' markups due to the increase in import competition after trade liberalization. With this background, heterogeneous-firm models have introduced variable markups in different ways. For example, Bernard, Eaton, Jensen, and Kortum (2003) assume Bertrand (price) competition, Atkeson and Burstein (2008) assume Cournot (quantity) competition, while Melitz and Ottaviano (2008) assume a quasilinear-quadratic utility function that generates a linear demand system with endogenous markups. In this respect, my model is closer to the model of Melitz and Ottaviano (2008), with the main difference being the use of translog preferences to generate endogenous markups.

A key contribution of the trade models with heterogeneous firms is to identify the extensive margin of trade as a fundamental channel for international adjustment. Traditional monopolistic competition trade models, in the spirit of Krugman (1980), assume homogeneous firms that produce differentiated goods. In these models, changes in the extensive margin of trade only occur when an economy moves from autarky (no trade) to trade openness. Once the economy trades with another country, the number of traded goods remains constant and further adjustments in the volume of trade (due, for example, to a decrease in trade costs) only occur through the intensive margin. Ignoring the extensive margin can be theoretically and empirically misleading. Theoretically, for example, Chaney (2008) shows that the intensive and extensive margins of trade can work in opposite directions. He demonstrates that an important result of Krugman (1980) about the role of the elasticity of substitution for the impact of trade costs on trade volumes is overturned when one considers the extensive margin of trade. Empirically, Helpman, Melitz, and Rubinstein (2008) show that traditional gravity equation models for trade flows are biased because they omit the extensive margin of trade; the omission also seems to explain the observed asymmetries in trade flows between countries.

For the U.S., Bernard, Jensen, Redding, and Schott (2009) provide specific measures of the importance of the extensive margin for trade flows. They document that the extensive margin accounts for most of the variation in exports and imports across U.S. trading partners. In 2003, for example, they find that the extensive margin accounts for 77.4% of the cross-sectional variation in U.S. exports, and 68.2% of the variation in imports. Moreover, through their analysis of the 1997-1998 Asian financial crisis, Bernard et al. provide direct evidence of how a macroeconomic shock involving substantial exchange rate movements affects the margins of U.S. trade. For a group of crisis countries (Indonesia, South Korea, Malaysia, the Phillipines, and Thailand), they find that from 1996 to 1998, U.S. exports declined by 21% and imports increased by 19%. During the same period, the number of U.S. exporters to this group of countries declined 16% and the intensive margin of surviving exporters declined 2%.[6] Taking into account that the crisis started in July 1997, this result implies that a shock can cause fast and substantial changes in the number of exporting firms.

In a similar study for France, Eaton, Kortum, and Kramarz (2004) find that changes in French exports are strongly dominated by the extensive margin. Using data from French firms in 1986, they find that a) for a given destination size, the extensive margin accounts for 88% of any increase in the French market share in that destination; and b) keeping the French market share constant, the extensive margin accounts for about 62% of any increase in exports due to an increase in a destination's market size. Regarding the speed of adjustment of the extensive margin in France, Berman, Martin, and Mayer (2009) examine the changes in the number of French exporters after a euro depreciation. Using French firm-level data, they find that entry of new exporters occurs within a year after the depreciation and that this accounts for 20% of the increase in French exports.

Although the number of exporters can change quickly, this does not imply that entry of new firms occurs at the same pace.[7] However, an important assumption of my model is that it allows for entry and exit of firms in the presence of nominal wage rigidities. It is therefore important to establish the appropriate time frame for the the validity of the model's results. For this purpose, it is useful to review the evidence on wage stickiness, employment fluctuations, and firm creation and destruction.

First, strong evidence of nominal wage rigidities appears for many countries.[8] Using annual microdata for the U.S., Kahn (1997) presents some stylized facts on wage stickiness and finds in particular a high degree of downward wage rigidity. Importantly for this paper, Kahn shows that nominal wages exhibited substantial downward rigidity even in the 1980s, which was a period of increasing import competition and decreasing inflation. Regarding the macroeconomic effects of wage rigidities, Obstfeld and Rogoff (1996) refer to the long-held view in macroeconomics—dating back to Keynes's *General Theory*—that nominal wage stickiness is the central source of monetary non-neutrality in the economy. In this respect, Christiano, Eichenbaum, and Evans (2005) find that nominal wage rigidities—but not nominal price rigidities—are essential to match

[6] Compared to U.S. trade with the rest of the world, Bernard et al. (2009) report that from 1996 to 1998 U.S. exports and imports increased by 3% and 17%, respectively. Also, the number of U.S. exporters to the rest of the world declined 8%, and the incumbents intensive margin increased 8%.

[7] For example, an increase in the number of exporters could be composed mostly of existing firms that were producing only for the domestic market.

[8] See Akerlof (2007) and the references cited therein.

the U.S. economy responses of inflation and output persistence to a monetary shock. Moreover, extending the framework of Christiano et al. by introducing a labor search and matching structure, Gertler, Sala, and Trigari (2008) also find that nominal wage rigidities are necessary to match the high U.S. employment volatility over the business cycle.[9]

Employment movements, particularly gross flows, are indeed large. According to the survey by Davis, Faberman, and Haltiwanger (2006), annual job creation and destruction rates in the U.S. are about 14% of total employment (the quarterly rates are about 8%). Total levels of job creation consist of expansions of existing firms and births of firms. Likewise, total levels of job destruction are composed by contractions of existing firms and deaths of firms. In support of my model's entry and exit assumption in the presence of nominal rigidities, evidence suggests that births and deaths of firms are a very important component of gross employment flows even in one-year horizons. Davis and Haltiwanger (1992) estimate that births of firms in the U.S. account for 20% of annual job creation, and deaths account for 25% of annual job destruction. These numbers are, however, small when compared against new estimates by Neumark, Zhang, and Wall (2006). Using data from the National Establishments Time Series (NETS) for California from 1992 to 2002, they find that for three-year employment changes, establishment births account on average for 62.4% of job creation, and deaths account for 71.4% of job destruction. For year-to-year changes the numbers are about 60% for births and 66% for deaths.[10] An establishment can be a branch of an existing firm or a firm. Taking this into account, Neumark, Zhang, and Wall (2006) report that for three-year changes, the birth of firms accounts on average for 41% of job creation, and the death of firms accounts for 44% of job destruction—with similar year-to-year contributions. They also find that net employment changes are more correlated with employment movements due to the births and deaths of establishments than to expansions and contractions, suggesting that cyclical changes in employment are mostly driven by births and deaths.

Strengthening these results, Bilbiie, Ghironi, and Melitz (2007a) present evidence on the strong procyclical behavior of net firm entry in the U.S. (see also the references cited therein). Bergin and Corsetti (2008) go further and link entry with monetary policy. They find that the entry response to a monetary policy shock is statistically significant and reaches its peak in about a year. On this basis, Bergin and Corsetti (2008) and Bilbiie, Ghironi, and Melitz (2007b) study monetary policy in models with endogenous producer entry and nominal rigidities. These models are, however, not suited for analysis of exchange rate pass-through because they assume nominal rigidities in the form of sticky producer prices. Closer to this paper, Bergin and Feenstra (2009) assume short-run fixed wages and free entry in a model of exchange rate pass-through with translog preferences and homogeneous firms. Their purpose is to explain the decrease in

[9]Recent (real) search and matching models of employment have incorporated (real) wage rigidities in order to account for the considerable employment volatility observed over the business cycle (see, e.g., Shimer (2005), Hall (2005), and Gertler and Trigari (2009)).

[10]Although three-year changes and year-to-year changes are very similar, Neumark, Zhang, and Wall (2006) focus on three-year changes because they argue that they are less affected by rounding or imputation problems. Neumark, Zhang, and Wall (2007) and Neumark, Wall, and Zhang (forthcoming) provide careful assessments of the NETS data. As they mention, NETS attempts to capture the entire universe of establishments and is particularly good in tracking small establishments. Other establishment-level databases from the Census and the Bureau of Labor Statistics are samples that do not take into account establishments with less than five employees.

aggregate pass-through to U.S. import prices through a mechanism of competition between firms from a country with a flexible exchange rate and firms from country with a fixed exchange rate. In their model, pass-through is incomplete and declines with an increase in the number of firms from the fixed-exchange rate country. Their main result relies, however, on an assumption of a U.S. taste bias in favor of the goods of the flexible exchange rate country. The model in this paper does not assume any type of taste bias, and the competition occurs between domestic and foreign firms.[11]

The general equilibrium version of my model also relates to the model of Ghironi and Melitz (2005), which is the first to incorporate heterogeneous producers in a dynamic stochastic general equilibrium setting. The main differences between Ghironi and Melitz's model and mine are that I present a monetary model with nominal wage rigidities (but flexible prices) and endogenous markups, whereas they present a (real) model with flexible wages and prices and exogenous markups (CES preferences).

3 The Model in Partial Equilibrium

This section presents a partial equilibrium model of exchange rates, heterogeneous firms, and endogenous markups. This model follows closely the model of Melitz and Ottaviano (2008), who study the implications of different market sizes and integration policies on trade using a framework of heterogeneous firms and endogenous markups. Melitz and Ottaviano obtain endogenous markups through the use of a quasilinear-quadratic utility function that is very tractable for a partial equilibrium model, but because the utility function is not homothetic it is difficult to work with in a general equilibrium setting like the one we consider later.[12] Instead of a quadratic utility, my model derives a demand system with endogenous markups from the translog expenditure function introduced by Bergin and Feenstra (2000). The translog function generates endogenous markups, and also implies a utility function that is homothetic.

There are two countries, Home and Foreign, each of which is inhabited by a continuum of households in the interval $[0, 1]$. Each household provides labor to the production sector in the economy, which produces differentiated goods. Firms are heterogeneous in productivity. Each firm produces a single good under monopolistic competition, and the tradability of the good—in the domestic and export markets—is endogenously determined. This is a partial equilibrium model in the Dornbusch (1987) sense: nominal wages are fixed, and movements in the exchange rate are exogenous. Hence, an exchange rate shock alters the relative cost of labor between the countries, making the firms from the depreciating-currency country more competitive in the other market (and the opposite happens with the firms from the appreciating-currency country).

[11]Bergin and Feenstra (2009) note that the main reason for an incomplete pass-through should stem from competition from domestic firms. Although they do not consider it in their model, they mention that competition from domestic firms can be incorporated if we see the U.S. as part of the group of countries with fixed exchange rates against the U.S. dollar. Given their taste bias assumption, however, this would imply that U.S. consumers have a taste bias against U.S. produced goods (and in favor of goods from countries with flexible exchange rates).

[12]Moreover, a quasilinear utility function is not desirable in a general equilibrium framework because it implies no income effects in the demand for differentiated goods—any change in income is absorbed by the consumption of the homogeneous good.

I begin by describing preferences and obtain the demand, then I present the production sector and derive some results regarding averages and the composition of firms, and finally I describe the free-entry conditions and solve the model. I use a star (*) to denote Foreign variables. Some parts of this section only refer to the Home country, as analogous expressions will hold for the Foreign country.

3.1 Model Setup

3.1.1 Preferences and Demand

Preferences are obtained from a continuum-of-goods version of the translog expenditure function introduced by Bergin and Feenstra (2000)—and generalized in Feenstra (2003) for a varying number of goods. Although it is not possible to obtain a closed-form expression for the utility function, the demand system is derived directly from the expenditure function.

The representative Home household defines its preferences over a continuum of differentiated goods in the set Δ. This set includes the total number of actual, old, and potential (not yet invented) goods and has a measure of $\tilde{N}$. Let Δ', with measure N, be the subset of Δ that contains the set of goods that are actually available for purchase at Home. If E represents the minimum expenditure to reach utility level U, and p_i is the price of good i, then the symmetric translog expenditure function for the representative Home household—defined over the continuum of available goods, from Home and Foreign—is given by

$$\ln E = \ln U + a + \frac{1}{N}\int_{i\in\Delta'} \ln p_i di + \frac{\gamma}{2N}\int_{i\in\Delta'}\int_{j\in\Delta'} \ln p_i(\ln p_j - \ln p_i)djdi, \tag{1}$$

where $a = \frac{1}{2\gamma N}$. The value of γ is greater than zero and indicates the degree of substitutability between the goods, with a high γ implying high substitutability (or low differentiation). Note that a is decreasing in N, which implies that if prices are the same for all goods, the expenditure needed to reach a certain level of utility declines with N. In other words, the utility function from the translog expenditure function exhibits love of variety.

Taking the derivative of equation (1) with respect to $\ln p_i$—Shephard's lemma—we get the result that the share of good i in the expenditure of the representative household is given by $s_i = \gamma \ln\left(\frac{\hat{p}}{p_i}\right)$, where

$$\hat{p} = \exp\left(\frac{1}{\gamma N} + \overline{\ln p}\right) \tag{2}$$

is the maximum price (in Home currency) a firm can set, and $\overline{\ln p} = \frac{1}{N}\int_{j\in\Delta'} \ln p_j dj$. Note that s_i is positive only when p_i is below $\hat{p}$ and is exactly zero when they are equal.

The representative Home household demand for good i is then $q_i = s_i \frac{I}{p_i}$, where I is the household's total consumption expenditure (in Home currency). Given that households are located in the unit interval, the market demand at Home for good i is equivalent to the representative household's demand.

Analogous expressions hold for the representative Foreign household, with $\hat{p}^*$ being the maximum price that a firm can set in the Foreign market and I^* representing the consumption expenditure (both in terms of Foreign currency).

3.1.2 Profit Maximizing Price

Assuming a constant marginal cost for the production of Home good i, mc_i, and taking $\hat{p}$ as given, the price that maximizes the profit of producer i,

$$\underset{p_i}{\operatorname{argmax}}\ p_i q_i - \mathrm{mc}_i q_i,$$

solves the equation $p_i = \left[1 + \ln\left(\frac{\hat{p}}{p_i}\right)\right] \mathrm{mc}_i$. To solve for p_i, Bergin and Feenstra (2000, 2001, 2009) assume that markups are small and use the approximation $\ln\left(\frac{\hat{p}}{p_i}\right) \approx \frac{\hat{p}}{p_i} - 1$. It is possible, however, to obtain a closed-form solution for p_i by making use of the Lambert $\mathcal{W}$ function.[13] Although it is usually denoted by $\mathcal{W}$, I represent it with $\mathbf{\Omega}$ in order to prevent a confusion with W, which will be used later for the wage level. $\mathbf{\Omega}$ is the inverse function of $f(\mathbf{\Omega}) = \mathbf{\Omega} e^{\mathbf{\Omega}}$, so that if $x = ze^z$, we solve for z as $z = \mathbf{\Omega}(x)$. If x is a real number greater than or equal to zero, $\mathbf{\Omega}(x)$ is single-valued, with properties $\mathbf{\Omega}'(x) > 0$, $\mathbf{\Omega}''(x) < 0$, $\mathbf{\Omega}(0) = 0$, and $\mathbf{\Omega}(e) = 1$.[14] Then, the exact solution for p_i is given by

$$p_i = \mathbf{\Omega}\left(\frac{\hat{p}}{\mathrm{mc}_i} e\right) \mathrm{mc}_i, \tag{3}$$

where $\mathbf{\Omega}\left(\frac{\hat{p}}{\mathrm{mc}_i} e\right)$ equals one when $\mathrm{mc}_i = \hat{p}$ and is above one if $\mathrm{mc}_i < \hat{p}$. Firm i does not produce for this market if $\mathrm{mc}_i > \hat{p}$.

Let μ_i be the markup over the marginal cost for producer i. Therefore, we can rewrite equation (3) as

$$p_i = (1 + \mu_i)\mathrm{mc}_i, \tag{4}$$

where

$$\mu_i = \mathbf{\Omega}\left(\frac{\hat{p}}{\mathrm{mc}_i} e\right) - 1. \tag{5}$$

Note that the markup is strictly decreasing with the marginal cost for $\mathrm{mc}_i \leq \hat{p}$, reaching zero when $\mathrm{mc}_i = \hat{p}$.

I can also derive a convenient expression for the market-share density of producer i. Taking the logarithm of equation (3) and using the property $\ln[\mathbf{\Omega}(x)] = \ln x - \mathbf{\Omega}(x)$ for $x > 0$ along with equation (5), we obtain

$$\ln p_i = \ln \hat{p} - \mu_i. \tag{6}$$

Plugging this expression into the market share equation, $s_i = \gamma \ln\left(\frac{\hat{p}}{p_i}\right)$, we find that

$$s_i = \gamma \mu_i. \tag{7}$$

That is, the market share and the markup of producer i are directly proportional.

[13] See Corless et al. (1996) for a description of the history and properties of the Lambert $\mathcal{W}$ function.

[14] Moreover, $\mathbf{\Omega}'(x) = \frac{\mathbf{\Omega}(x)}{x[1+\mathbf{\Omega}(x)]}$ for $x \neq 0$, and $\ln[\mathbf{\Omega}(x)] = \ln x - \mathbf{\Omega}(x)$ when $x > 0$.

3.1.3 Production

Home and Foreign markets are segmented. Labor is the only factor of production, and wages are fixed in term of each country's currency. Producers are heterogeneous in productivity. As in Melitz (2003), each producer knows its productivity, φ, only after entry. The production function for a Home firm with productivity φ is given by

$$y(\varphi) = Z\varphi L,$$

where Z is an aggregate labor productivity factor for Home firms, and L is a labor index whose nominal price is given by W. The marginal cost of a Home firm with productivity φ is then constant and given by $\frac{W}{Z\varphi}$.

There is a sunk entry cost that accounts for the research and investment necessary to start producing a good. Let f_E denote the entry cost in units of effective labor. In nominal terms, this cost is given by $\frac{f_E W}{Z}$. After paying the entry cost and realizing its productivity, a firm will produce for the domestic market as long as it can set a price no less than its marginal cost.

In the same way, the production function of a Foreign firm with productivity φ is just $y^*(\varphi) = Z^*\varphi L^*$, where Z^* is the Foreign aggregate productivity factor and L^* is the Foreign labor index with price W^*. This firm must pay a sunk entry cost of f_E^* units of effective labor, and its marginal cost is $\frac{W^*}{Z^*\varphi}$.

Given market segmentation and constant marginal costs, a producer will independently decide whether or not to sell in each country. The only cost of exporting is an iceberg cost. Let τ account for the iceberg cost for Home producers so that a Home exporter must ship $\tau > 1$ units of the good in order for one unit to reach the Foreign market. In the same way, τ^* accounts for the iceberg cost for Foreign producers.

Let $p_D(\varphi)$ and $p_X(\varphi)$ denote, respectively, the nominal domestic (D) and export (X) prices of a Home firm with productivity φ. These prices are set in the currency of the destination country. Also, let $\mathcal{E}$ be the nominal exchange rate, measured as the Home-currency price of the Foreign currency. Following equations (4) and (5), we write the pricing equations for a Home firm with productivity φ as

$$p_D(\varphi) = (1 + \mu_D(\varphi))\frac{W}{Z\varphi} \quad \text{and} \quad p_X(\varphi) = (1 + \mu_X(\varphi))\frac{\tau W}{\mathcal{E} Z\varphi},$$

with markups given by

$$\mu_D(\varphi) = \mathbf{\Omega}\left(\frac{\hat{p}}{\frac{W}{Z\varphi}}e\right) - 1 \quad \text{and} \quad \mu_X(\varphi) = \mathbf{\Omega}\left(\frac{\hat{p}^*}{\frac{\tau W}{\mathcal{E} Z\varphi}}e\right) - 1.$$

Solving for the equilibrium quantities ($y_D(\varphi)$ and $y_X(\varphi)$) and profit functions ($\pi_D(\varphi)$ and $\pi_X(\varphi)$) from selling in each market we get

$$y_D(\varphi) = \left(\frac{\mu_D(\varphi)}{1 + \mu_D(\varphi)}\right)\frac{\gamma I}{\frac{W}{Z\varphi}} \qquad y_X(\varphi) = \left(\frac{\mu_X(\varphi)}{1 + \mu_X(\varphi)}\right)\frac{\gamma I^*}{\frac{\tau W}{\mathcal{E} Z\varphi}},$$

and

$$\pi_D(\varphi) = \frac{\mu_D(\varphi)^2}{1+\mu_D(\varphi)}\gamma I \qquad \pi_X(\varphi) = \frac{\mu_X(\varphi)^2}{1+\mu_X(\varphi)}\gamma I^*,$$

where the profit functions are in terms of the destination country's currency.

Analogously, a Foreign firm with productivity φ will set prices

$$p_D^*(\varphi) = (1+\mu_D^*(\varphi))\frac{W^*}{Z^*\varphi} \quad \text{and} \quad p_X^*(\varphi) = (1+\mu_X^*(\varphi))\frac{\tau^*\mathcal{E}W^*}{Z^*\varphi},$$

with markups

$$\mu_D^*(\varphi) = \mathbf{\Omega}\left(\frac{\hat{p}^*}{\frac{W^*}{Z^*\varphi}}e\right) - 1 \quad \text{and} \quad \mu_X^*(\varphi) = \mathbf{\Omega}\left(\frac{\hat{p}}{\frac{\tau^*\mathcal{E}W^*}{Z^*\varphi}}e\right) - 1,$$

and expressions for equilibrium quantities ($y_D^*(\varphi)$ and $y_X^*(\varphi)$) and profit functions in the Foreign currency ($\pi_D^*(\varphi)$ and $\pi_X^*(\varphi)$) that are parallel to those of a Home firm with productivity φ.

3.1.4 Cutoff Productivity Levels

A key feature of Melitz-type heterogeneous-firm models is that they summarize the information of the production sector in simple cutoff productivity levels. In this model we derive the cutoff rules from the markup equations from the previous section.[15] Let φ_r denote the cutoff productivity level for Home firms in market r, for $r \in \{D, X\}$. Similarly, φ_r^* represents the cutoff productivity level for Foreign firms in market r. Then, we define the cutoff rules as $\varphi_r = \inf\{\varphi : \mu_r(\varphi) > 0\}$ and $\varphi_r^* = \inf\{\varphi : \mu_r^*(\varphi) > 0\}$ so that

$$\begin{aligned}
\varphi_D &= \frac{W}{Z\hat{p}} \\
\varphi_X &= \frac{\tau W}{\mathcal{E}Z\hat{p}^*} \\
\varphi_D^* &= \frac{W^*}{Z^*\hat{p}^*} \\
\varphi_X^* &= \frac{\tau^*\mathcal{E}W^*}{Z^*\hat{p}}.
\end{aligned}$$

Obviously, the markup in a market is zero for a firm whose productivity is identical to the corresponding cutoff rule. Therefore, we refer to the previous equations as zero-cutoff-markup conditions.[16] From these conditions we obtain two of the four equations we need to solve the model. In particular, we derive the following relationships between the cutoff levels:

$$\varphi_X^* = \tau^*\left[\frac{\frac{\mathcal{E}W^*}{Z^*}}{\frac{W}{Z}}\right]\varphi_D \tag{8}$$

[15] An important difference between this model and the Melitz (2003) model, which uses CES preferences, is that we do not need to assume fixed costs in order to pin down the cutoff productivity levels. The quasilinear-quadratic utility function of Melitz and Ottaviano (2008) also generates a demand system that does not need to impose fixed costs.

[16] The definition of a cutoff level is in terms of the infimum—the greatest lower bound—because markups are also zero for firms with productivities below the cutoff level (as these firms simply do not produce for that market).

$$\varphi_X = \tau \left[\frac{\frac{W}{Z}}{\frac{\mathcal{E}W^*}{Z^*}} \right] \varphi_D^*, \tag{9}$$

where the term in brackets in each equation is the relative cost of effective labor. Intuitively, there is a proportional relationship between φ_D and φ_X^* because Home firms selling domestically and Foreign exporters are both bounded by $\hat{p}$—the maximum price they can set at Home. The constant of proportionality depends on the iceberg cost of exporting for Foreign firms, τ^*, and on the relative cost of effective labor. The same happens between φ_D^* and φ_X.

Using the cutoff levels to replace $\hat{p}$ and $\hat{p}^*$ in the markup equations, we obtain the convenient expressions

$$\mu_r(\varphi) = \mathbf{\Omega}\left(\frac{\varphi}{\varphi_r} e\right) - 1 \tag{10}$$

$$\mu_r^*(\varphi) = \mathbf{\Omega}\left(\frac{\varphi}{\varphi_r^*} e\right) - 1 \tag{11}$$

for a Home and Foreign firm with productivity φ, where $\varphi \geq \varphi_r$ and $\varphi \geq \varphi_r^*$, and $r = \{D, X\}$. Given that $\mathbf{\Omega}'(\cdot) > 0$ and $\mathbf{\Omega}''(\cdot) < 0$, we see that markups are increasing in φ, but marginal markups are decreasing.

3.2 Productivity Distribution, Averages, and the Composition of Firms

I assume that the productivity of Home and Foreign firms is Pareto distributed in the interval $[\varphi_{\min}, \infty)$.[17] That is, the cumulative distribution function for productivity in each country is given by $G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^k$, where $k > 1$ is a parameter of productivity dispersion—with a higher k implying lower heterogeneity (with firms' productivities clustered near the lower bound). The probability density function is then $g(\varphi) = \frac{k\varphi_{\min}^k}{\varphi^{k+1}}$. Let $g(\varphi \mid \varphi \geq \varphi_r)$ denote the probability density function for the productivity of Home firms that are actually selling in market r, for $r \in \{D, X\}$. Thus, given our Pareto distribution assumption, we get

$$g(\varphi \mid \varphi \geq \varphi_r) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_r)} = \frac{k\varphi_r^k}{\varphi^{k+1}} & \text{if } \varphi \geq \varphi_r \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

for Home firms—with a parallel conditional distribution holding for Foreign firms. The following lemma states a result that is useful in the derivation of some results involving averages:

Lemma 1 *Let $h(\varphi, \varphi_r)$ be a homogeneous of degree n function and let $g(\varphi \mid \varphi \geq \varphi_r)$ be defined as in* (12)*. Then, the expected value of $h(\varphi, \varphi_r)$, given that $\varphi \geq \varphi_r$, is*

$$\int_{\varphi_r}^{\infty} h(\varphi, \varphi_r) g(\varphi \mid \varphi \geq \varphi_r) d\varphi = \bar{h}\varphi_r^n,$$

[17] The Pareto distribution—a power-law distribution—fits the distribution of U.S. and European firms very well (see Chaney (2008) and the references cited therein) and is widely used in models with heterogeneous firms (see, among others, Ghironi and Melitz (2005) and Melitz and Ottaviano (2008)).

where $\bar{h}$ is a constant given by

$$\bar{h} = \int_1^\infty h(x,1)\frac{k}{x^{k+1}}dx.$$

Using this lemma we obtain that the average productivity of Home firms selling in market r is given by $\bar{\varphi}_r = \frac{k}{k-1}\varphi_r$, for $r = \{D, X\}$. Similarly, for Foreign firms $\bar{\varphi}_r^* = \frac{k}{k-1}\varphi_r^*$. Note that given their proportional relationship, we only need to look at changes in the cutoff levels to infer average productivity changes.

Regarding the average markup, from equations (10) and (11) we observe that $\mu_r(\varphi)$ and $\mu_r^*(\varphi)$ are homogeneous of degree zero in (φ, φ_r) and (φ, φ_r^*), respectively. From Lemma 1 we obtain that the average markups, $\bar{\mu}_r$ and $\bar{\mu}_r^*$, for $r \in \{D, X\}$, are equal and constant. In particular, we get

$$\bar{\mu}_r = \bar{\mu}_r^* = \bar{\mu}(k), \tag{13}$$

where $\bar{\mu}(k)$ is a constant greater than zero depending only on the dispersion parameter k. If firms are (approximately) homogeneous in productivity ($k \to \infty$), the average markup $\bar{\mu}(k)$ approaches zero.[18]

With respect to average prices, let $\bar{p}_r$ and $\overline{\ln p_r}$ denote the average price and the average log-price of Home goods available for purchase in market r, for $r \in \{D, X\}$. Also, let $\bar{p}$ and $\overline{\ln p}$ represent the average price and the average log-price of all goods—Home produced and Foreign produced—available for purchase at Home. With analogous definitions holding for $\bar{p}_r^*$, $\overline{\ln p_r}^*$, $\bar{p}^*$, and $\overline{\ln p}^*$, we have the following proposition:

Proposition 1 ***(Equivalence of average prices of domestic and imported goods)***

1. $\bar{p} = \bar{p}_D = \bar{p}_X^* = \vartheta(k)\frac{W}{Z\varphi_D}$ *and* $\bar{p}^* = \bar{p}_D^* = \bar{p}_X = \vartheta(k)\frac{W^*}{Z^*\varphi_D^*}$*, where* $\vartheta(k) < 1$ *is a constant.*[19]

2. $\overline{\ln p} = \overline{\ln p_D} = \overline{\ln p_X}^* = \ln\left(\frac{W}{Z\varphi_D}\right) - \bar{\mu}(k)$ *and* $\overline{\ln p}^* = \overline{\ln p_D}^* = \overline{\ln p_X} = \ln\left(\frac{W^*}{Z^*\varphi_D^*}\right) - \bar{\mu}(k)$.

Under a Pareto distribution of firms and in the absence of fixed costs, the equivalence of average prices (and average log-prices) of domestic and imported goods stems from the common price ceiling faced by all firms competing in the same market. In the Home market, for example, Home producers and Foreign exporters are limited by $\hat{p}$, which creates—as seen in equation (8)—a link between the cutoff rules φ_D and φ_X^*. Thus, any shock that increases the domestic cutoff rule for Home firms, φ_D, causes an identical decrease in the average prices of domestic and imported varieties. The perfect relationship between the average prices of domestic and imported varieties is not exclusive to the translog case. It also holds, for example, if we use the quasilinear-quadratic utility function of Melitz and Ottaviano (2008).[20] Moreover, for the

[18] $\bar{\mu}(k) = k\int_1^\infty \frac{\mathbf{\Omega}(xe)-1}{x^{k+1}}dx$ ranges between 0 (for a large k) and 0.5963 (for a k close to 1).

[19] $\vartheta(k) = k\int_1^\infty \frac{\mathbf{\Omega}(xe)}{x^{k+2}}dx$ is increasing in k, with a value close to 0.6387 when k approaches 1—from the right—and bounded above by 1.

[20] A previous version of this paper, available upon request, develops the model with the Melitz-Ottaviano quasilinear-quadratic utility function. With an exception mentioned in section 4.2, the main results in this paper hold in the quadratic utility case.

CES (exogenous markups) case—which requires fixed costs to pin down the cutoff levels—with a Pareto distribution of firms, $\bar{p}_D = \bar{p}_X^*$ if one of the following conditions is met: (1) the fixed cost of Home producers from selling domestically is identical—in terms of the Home currency—to the fixed cost of exporting for Foreign producers; or (2) if the elasticity of substitution between goods tends to infinity. In any other situation, the difference between $\bar{p}_D$ and $\bar{p}_X^*$ depends on the magnitude of the deviations from the previous conditions. I derive this and other results for the CES case in Appendix B (online).[21]

Corollary 1 *The mass of goods available for purchase in each country—N at Home and N^* at Foreign—is constant and equal to $\frac{1}{\gamma\bar{\mu}(k)}$.*

Although N and N^* are constant in this model, the composition of domestic and imported goods in each market is allowed to change. Let N_D and N_X be the mass of Home firms producing for the Home and Foreign markets, respectively. With similar expressions for the Foreign firms, we have that $N = N_D + N_X^*$ and $N^* = N_D^* + N_X$. Let N_P (N_P^*) denote the measure of the pool of existing firms at Home (Foreign). Given our productivity distribution, we have that

$$N_r = (1 - G(\varphi_r))N_P = \left(\frac{\varphi_{\min}}{\varphi_r}\right)^k N_P \tag{14}$$

$$N_r^* = (1 - G(\varphi_r^*))N_P^* = \left(\frac{\varphi_{\min}}{\varphi_r^*}\right)^k N_P^*, \tag{15}$$

for $r \in \{D, X\}$. Substituting these expressions in $N_D + N_X^* = N$ and $N_D^* + N_X = N^*$, and making use of $N = N^* = \frac{1}{\gamma\bar{\mu}(k)}$ along with equations (8) and (9), we solve for N_P and N_P^* in terms of the cutoff productivity levels:

$$N_P = \frac{1}{\gamma\bar{\mu}(k)\varphi_{\min}^k}\left[\frac{(\tau\tau^*)^k\varphi_D^k - \varphi_X^k}{(\tau\tau^*)^k - 1}\right] \tag{16}$$

$$N_P^* = \frac{1}{\gamma\bar{\mu}(k)\varphi_{\min}^k}\left[\frac{(\tau\tau^*)^k\varphi_D^{*k} - \varphi_X^{*k}}{(\tau\tau^*)^k - 1}\right]. \tag{17}$$

The values for N_P and N_P^* must be non-negative. Therefore, I assume that trade costs are sufficiently high so that $(\tau\tau^*)\varphi_D \geq \varphi_X$ and $(\tau\tau^*)\varphi_D^* \geq \varphi_X^*$ always hold. Using again equations (8) and (9), these conditions are respectively equivalent to $\varphi_D^* \leq \varphi_X^*$ and $\varphi_D \leq \varphi_X$. Hence, I assume that Home and Foreign exporters always sell in their domestic markets.[22]

As in Melitz (2003), I assume that there is an exogenous death shock at the end of every period that forces a proportion δ of existing firms in each country to exit. There is also free entry of firms at the beginning of every period. Therefore, the pool of Home firms in period $t+1$ is $N_{\mathrm{P},t+1} = (1-\delta)N_{\mathrm{P},t} + N_{E,t+1}$, where $N_{E,t+1}$ is the mass of Home entrants at the beginning of

[21] Appendix B is available at `http://www.socsci.uci.edu/~jantonio`.

[22] These conditions are not too restrictive. For the U.S. and Germany, for example, almost all the exporting plants sell domestically. Using 1992 data, Bernard, Eaton, Jensen, and Kortum (2003) report that only 4.3% of U.S. exporting plants export more than 50% of their output (only 0.7% export more than 90% of their output). Likewise, using also 1992 data, Bernard and Wagner (1997) find that only 12.6% of German exporting plants export more than 50% of their output.

$t+1$. In the steady state the measure for the pool of firms is constant at N_P, so that $N_E = \delta N_P$. Analogous expressions hold for the Foreign country so that in the steady state $N_E^* = \delta N_P^*$.

In summary, with respect to production decisions and entry and exit dynamics, a producer will sell in a market during period t if and only if its productivity, φ, is no less than the corresponding cutoff level at that time. If a producer does not sell in a period, it stays dormant until it is hit by a death shock and exits, or until one or both of the cutoff levels decline to—or below—this producer's productivity level.

3.3 Free-Entry Conditions

Entry is unbounded. Firms will enter in each country as long as their expected value of entry is no less than the sunk entry cost. Before entry—when a potential entrant does not know its productivity—a Home firm's expected profit for every period is given by $\bar{\pi} = \bar{\pi}_D + \mathcal{E}\bar{\pi}_X$, where

$$\bar{\pi}_r = \int_{\varphi_r}^{\infty} \pi_r(\varphi) g(\varphi) d\varphi$$

and $\pi_r(\varphi)$ is as defined in section 3.1.3 for $r = \{D, X\}$. Given that $\bar{\pi}$ does not depend on time, and taking into account the exogenous death shock at the end of every period, the expected value of entry for a Home firm is

$$\sum_{t=0}^{\infty} (1-\delta)^t \bar{\pi} = \frac{\bar{\pi}}{\delta}.$$

Analogously, the expected value of entry for a Foreign firm is given by $\frac{\bar{\pi}^*}{\delta}$, where $\bar{\pi}^* = \bar{\pi}_D^* + \frac{\bar{\pi}_X^*}{\mathcal{E}}$.

Since the potential number of entrants is unbounded, the expected value of entry and the sunk entry cost are equal in equilibrium. Therefore, the free-entry conditions for Home and Foreign firms are respectively given by

$$\frac{\bar{\pi}}{\delta} = \frac{f_E W}{Z}$$
$$\frac{\bar{\pi}^*}{\delta} = \frac{f_E^* W^*}{Z^*}.$$

Corollary 2 *Let $h(\varphi, \varphi_r)$ be a homogeneous of degree n function for $\varphi \geq \varphi_r$ (and zero otherwise), and let $g(\varphi)$ be defined as in section 3.2. Then, the unconditional expected value of $h(\varphi, \varphi_r)$ is*

$$\int_{\varphi_r}^{\infty} h(\varphi, \varphi_r) g(\varphi) d\varphi = \bar{h} \varphi_{min}^k \varphi_r^{n-k},$$

where $\bar{h}$ is a constant defined as in Lemma 1.

Given that the profit functions $\pi_D(\varphi)$ and $\pi_X(\varphi)$ are homogeneous of degree zero in (φ, φ_D) and (φ, φ_X), respectively, we use Corollary 2 to obtain that $\bar{\pi}_D = \frac{\psi I}{\varphi_D^k}$ and $\bar{\pi}_X = \frac{\psi I^*}{\varphi_X^k}$, where $\psi = \frac{\gamma \bar{\mu}(k) \varphi_{\min}^k}{k+1}$. Similarly, for Foreign firms we obtain $\bar{\pi}_D^* = \frac{\psi I^*}{\varphi_D^{*k}}$ and $\bar{\pi}_X^* = \frac{\psi I}{\varphi_X^{*k}}$. Therefore, we rewrite the free-entry conditions as

$$\frac{1}{\delta}\frac{\psi I}{\varphi_D^k} + \frac{\mathcal{E}}{\delta}\frac{\psi I^*}{\varphi_X^k} = \frac{f_E W}{Z} \tag{18}$$

$$\frac{1}{\delta}\frac{\psi I^*}{\varphi_D^{*k}} + \frac{1}{\delta\mathcal{E}}\frac{\psi I}{\varphi_X^{*k}} = \frac{f_E^* W^*}{Z^*}. \tag{19}$$

The model is complete. We now solve for the equilibrium cutoff productivity levels.

3.4 Solution of the Model

In this partial equilibrium model with fixed wages, we use equations (8), (9), (18) and (19) to solve for φ_D, φ_X, φ_D^*, and φ_X^*. The equilibrium cutoff productivity levels are:

$$\varphi_D = \frac{\Psi_D}{\tau^*}\left[\frac{(\tau\tau^*)^k - 1}{\tau^k - (\rho\mathcal{E})^{k+1}}\right]^{\frac{1}{k}} \tag{20}$$

$$\varphi_X = \frac{\Psi_X}{\rho^{\frac{k+1}{k}}\mathcal{E}}\left[\frac{(\tau\tau^*)^k - 1}{\tau^{*k} - \frac{1}{(\rho\mathcal{E})^{k+1}}}\right]^{\frac{1}{k}} \tag{21}$$

$$\varphi_D^* = \frac{\Psi_D^*}{\tau}\left[\frac{(\tau\tau^*)^k - 1}{\tau^{*k} - \frac{1}{(\rho\mathcal{E})^{k+1}}}\right]^{\frac{1}{k}} \tag{22}$$

$$\varphi_X^* = \Psi_X^* \rho^{\frac{k+1}{k}}\mathcal{E}\left[\frac{(\tau\tau^*)^k - 1}{\tau^k - (\rho\mathcal{E})^{k+1}}\right]^{\frac{1}{k}}, \tag{23}$$

where $\rho = \left(\frac{f_E^*}{f_E}\right)^{\frac{1}{k+1}} \frac{W^*/Z^*}{W/Z}$ —so that $\rho\mathcal{E}$ is a combined measure of the relative cost of Foreign effective labor and the relative Foreign entry cost—and Ψ_r and Ψ_r^*, for $r = \{D, X\}$, are positive aggregate indexes reflecting differentiation, productivity dispersion, death likelihood, nominal entry cost, and the destination country's income.[23]

From the equilibrium equations we see that the exchange rate must range between $\frac{1}{\rho\tau^{*k/(k+1)}}$ and $\frac{\tau^{k/(k+1)}}{\rho}$ in order to obtain positive equilibrium cutoff levels.[24] These bounds are a natural consequence of the use of a demand system with an intercept on the price axis.

4 Exchange Rate Pass-Through in Partial Equilibrium

In this section I analyze the transmission of exchange rate movements to prices and trade flows. I start by looking into the relationship between the exchange rate and the cutoff productivity levels. Then I see how this is reflected in firm-level prices and trade flows. And finally I look at the response of aggregate prices and trade flows.

[23] In particular, $\Psi_D = \left(\frac{\psi Z I}{\delta f_E W}\right)^{\frac{1}{k}}$ and $\Psi_X = \left(\frac{\psi Z I^*}{\delta f_E W}\right)^{\frac{1}{k}}$, with parallel expressions holding for Ψ_D^* and Ψ_X^*. Note, for example, that more product differentiation (lower γ), higher death probability, and higher entry costs imply lower cutoff levels.

[24] This is a necessary but not sufficient condition for an interior solution. To have an interior solution we also need the equilibrium cutoff levels to be greater than or equal to $\varphi_{\min}$—the lower bound for the distribution of productivity.

4.1 The Cutoff Levels and the Exchange Rate

As mentioned before, this model summarizes all the information about the production sector in the cutoff productivity levels. Therefore, to understand how an exchange rate change is reflected in prices, we must start first by analyzing its impact on the cutoff levels.

Let us focus on the the impact of exchange rate movements on the productivity cutoff level for Home firms selling domestically, φ_D.

Proposition 2 ***(The cutoff level φ_D and the exchange rate)***
Let φ_D be given as in equation (20) and let $\mathcal{E} \in \left(\frac{1}{\rho\tau^{*k/(k+1)}}, \frac{\tau^{k/(k+1)}}{\rho}\right)$. Then

1. $\zeta_{\varphi_D,\mathcal{E}} = \frac{k+1}{k}\left[\frac{(\rho\mathcal{E})^{k+1}}{\tau^k-(\rho\mathcal{E})^{k+1}}\right] > 0$*, where $\zeta_{\varphi_D,\mathcal{E}}$ is the elasticity of φ_D with respect to the exchange rate; and*

2. $\frac{\partial^2\varphi_D}{\partial\mathcal{E}^2} > 0.$

That is, φ_D is strictly increasing and strictly convex in the exchange rate.

Analogously, we obtain that the cutoff rule for Foreign firms selling domestically, φ_D^*, is decreasing and strictly convex in the exchange rate. That is, $\zeta_{\varphi_D^*,\mathcal{E}} < 0$ and $\frac{\partial^2\varphi_D^*}{\partial\mathcal{E}^2} > 0$, where $\zeta_{\varphi_D^*,\mathcal{E}}$ is the elasticity of φ_D^* with respect to the exchange rate. Moreover, given the link between the cutoff productivity rules for domestic producers and the other country's exporters established in equations (8) and (9), it follows that φ_X and φ_X^* are also convex in the exchange rate and that their elasticities are given by

$$\zeta_{\varphi_X^*,\mathcal{E}} = \zeta_{\varphi_D,\mathcal{E}} + 1 > 1 \tag{24}$$

$$\zeta_{\varphi_X,\mathcal{E}} = \zeta_{\varphi_D^*,\mathcal{E}} - 1 < -1. \tag{25}$$

That is, the exporting cutoff productivity rules are elastic with respect to the exchange rate.

With a depreciation of the Home currency (an increase in $\mathcal{E}$) and fixed wages, Home firms become more competitive in the Foreign market because the relative cost of Home effective labor, $\frac{W/Z}{\mathcal{E}W^*/Z^*}$, declines. The new profit opportunities abroad increase the entry of Home producers up to the point at which the Home free-entry condition is satisfied. Meanwhile, entry declines at Foreign because the profit opportunities from exporting decrease. In the new steady state, the pre-entry expected profit from exporting—the second term in the left-hand side of equations (18) and (19)—increases for Home producers and declines for Foreign producers. Therefore, in order to satisfy the free-entry conditions, the pre-entry expected profit from domestic sales declines for Home producers and increases for Foreign producers. From the first term in the left-hand side of equations (18) and (19), we know that this occurs if and only if φ_D increases and φ_D^* decreases. Intuitively, the increase in entry at Home increases the pool of Home firms, N_P, and the decline in entry at Foreign reduces the pool of Foreign firms, N_P^*. Hence, in each market there are two opposite effects in the degree of competition: the larger N_P increases competition from Home firms, and the smaller N_P^* decreases competition from Foreign firms. If trade costs are sufficiently high so that an interior solution exists, the increase in competition due to the larger N_P dominates in the Home market, and the decrease in competition due to the smaller N_P^* dominates in the Foreign market.[25] Therefore, φ_D increases and φ_D^* declines.

Given the direct proportional relationship between the cutoff productivity rules and the average productivity levels that we obtained in section 3.2, we get that the average productivity of Home firms selling domestically increases after a Home-currency depreciation. The opposite happens for Foreign firms selling in their own market. For the exporters, however, the story is different. From equations (24) and (25) we see that φ_X^* increases and φ_X declines after a Home-currency depreciation. Therefore, the average productivity of Foreign exporters increases and the average productivity of Home exporters declines. Thus, Home firms that were not exporting become internationally competitive due exclusively to the exchange rate change, displacing firms in the Foreign country.

The second result from Proposition 2 concerns the convexity of φ_D in $\mathcal{E}$, which implies that a Home-currency depreciation has a larger impact on φ_D than a proportional appreciation. In the same way, the convexity of φ_D^* in $\mathcal{E}$ implies that a Foreign-currency depreciation (a decrease in $\mathcal{E}$) has a larger impact on φ_D^* than a proportional appreciation. The asymmetric responses in the cutoff levels are then reflected in asymmetric responses of trade flows and prices. Thus, it is important to note that the result of the convexity of the cutoff levels in the exchange rate is robust to different specifications of preferences. It holds, for example, with CES preferences (see Appendix B (online)) and with the quasilinear-quadratic preferences of Melitz and Ottaviano (2008).[26]

4.2 Pass-Through to Firm-Level Prices and Trade Flows

In sections 3.1.3 and 3.1.4 we obtained that the Home currency import price of a Foreign good from a firm with productivity φ is given by

$$p_X^*(\varphi) = (1 + \mu_X^*(\varphi)) \frac{\tau^* \mathcal{E} W^*}{Z^* \varphi},$$

where $\mu_X^*(\varphi) = \mathbf{\Omega}\left(\frac{\varphi}{\varphi_X^*} e\right) - 1$. Note that an exchange rate movement affects $p_X^*(\varphi)$ through two channels: (1) the direct effect on the firm's marginal cost in terms of the Home currency, $\frac{\tau^* \mathcal{E} W^*}{Z^* \varphi}$, and (2) the indirect effect of the exchange rate movement on the markup, $\mu_X^*(\varphi)$, which works through the impact of the exchange rate movement on the cutoff rule, φ_X^*.[27]

Let $u_X^*(\varphi) = \frac{\varphi}{\varphi_X^*}$ be a measure of the relative position of the Foreign firm with productivity φ with respect to the cutoff level φ_X^*. Note that a Foreign firm exports if and only if $u_X^*(\varphi) \geq 1$.

[25] This implies that for every Foreign firm that leaves the Home market after a Home-currency depreciation, there is more than one Home firm—with productivity above the initial cutoff level—to replace it. As N is constant, so that each Foreign firm can only be replaced by one Home firm, φ_D increases. The opposite happens in the Foreign market.

[26] Also in Appendix B (online), I present the model in a structure that fits the translog, CES, and quasilinear-quadratic preferences and show that in all cases, the pre-entry expected profit from selling domestically is concave in the exchange rate and convex in the cutoff level. It then follows that the cutoff level must be convex in the exchange rate.

[27] In the CES case, only the first channel is present. This implies full exchange rate pass-through to firm-level import prices.

Therefore, the elasticity of $p_X^*(\varphi)$ with respect to the exchange rate—that is, the pass-through rate—is given by

$$\lambda_X^*(\varphi) = 1 - \Upsilon_X^*(\varphi)\zeta_{\varphi_X^*,\mathcal{E}}, \tag{26}$$

where $\zeta_{\varphi_X^*,\mathcal{E}} > 1$ is as defined in equation (24), and $\Upsilon_X^*(\varphi)$ is the elasticity of $1 + \mu_X^*(\varphi)$ with respect to $u_X^*(\varphi)$, given in this case by

$$\Upsilon_X^*(\varphi) = \frac{\partial \ln(1 + \mu_X^*(\varphi))}{\partial \ln u_X^*(\varphi)} = \frac{1}{2 + \mu_X^*(\varphi)}. \tag{27}$$

Note that $\Upsilon_X^*(\varphi) \in \left(0, \frac{1}{2}\right]$, approaching 0 for the most productive Foreign exporters and approaching $\frac{1}{2}$ for the least productive ones. It follows that $\lambda_X^*(\varphi)$ is strictly less than one. That is, this model predicts incomplete exchange rate pass-through to firm-level import prices. Moreover, although $\lambda_X^*(\varphi)$ is bounded above by 1, it is not bounded below and can be negative. Indeed, for every φ, $\lambda_X^*(\varphi)$ is negative if and only if $\zeta_{\varphi_X^*,\mathcal{E}} > \frac{1}{\Upsilon_X^*(\varphi)}$. Note also that $\frac{1}{\Upsilon_X^*(\varphi)} \in [2, \infty)$, so that there are no firm-level import prices with negative pass-through rates when $\zeta_{\varphi_X^*,\mathcal{E}} \leq 2$.

From equation (26), we can separate out two forces when the exchange rate changes: an *economy-wide* effect—driven by $\zeta_{\varphi_X^*,\mathcal{E}}$—which reflects the change in the economy's competitive environment; and a *firm-specific* effect—driven by $\Upsilon_X^*(\varphi)$—which indicates the Foreign firm's response to a change in its relative position with respect to the cutoff level. The following proposition states the model's main results regarding exchange rate pass-through to firm-level import prices.

Proposition 3 ***(Exchange rate pass-through to firm-level import prices)***
The rate of pass-through $\lambda_X^*(\varphi)$ is incomplete (less than 1) and

1. *Increasing in productivity. That is, the import prices of the goods from more productive firms have higher pass-through rates.*

2. *Decreasing in the exchange rate. That is, the rate of pass-through to firm-level import prices is higher for appreciations than for depreciations of the Home currency.*

The first part of Proposition 3 is associated with the firm-specific effect. As we show in the Appendix, this result is due to the negative relationship between $\Upsilon_X^*(\varphi)$ and φ. Although more productive Foreign firms set higher markups ($\mu_X^*(\varphi)$ is increasing in φ), they also adjust them proportionally less than less productive firms. That is, the markups of the least productive Foreign exporters are more volatile, as they absorb a higher proportion of a shock to remain competitive. The firm-specific effect is, however, sensitive to the choice of the utility function. In Appendix B (online) I show that if we use the quasilinear-quadratic utility function of Melitz and Ottaviano (2008), we obtain the opposite result: lower pass-through rates for high productivity firms.[28]

[28] In Appendix B (online) I also show that if we use the linear approximation of Bergin and Feenstra (2000, 2001, 2009) to solve for the price in the translog case (see section 3.1.2), we assume away the firm-specific effect—$\Upsilon_X^*(\varphi) = 0.5$ for every φ. With respect to the model's results with the quasilinear-quadratic utility function of Melitz and Ottaviano (2008), see also footnote 20.

If the Home-currency depreciates, Foreign exporters face a tougher competitive environment at Home—in terms of higher average productivity of Home competitors—which is reflected in higher demand elasticities and lower markups. The opposite happens if the Home currency appreciates; however, the second part of Proposition 3 states that the markup adjustment is larger for a Home-currency depreciation than for a proportional appreciation. This asymmetry is the result of both the economy-wide and the firm-specific effects, which reinforce each other. In the proof of Proposition 3 in the Appendix, I show that an increase in the exchange rate increases $\zeta_{\varphi_X^*,\mathcal{E}}$ and $\Upsilon_X^*(\varphi)$, causing then an unambiguous decline in $\lambda_X^*(\varphi)$. The positive relationship between $\zeta_{\varphi_X^*,\mathcal{E}}$ and $\mathcal{E}$ is driven by the convexity of φ_D in $\mathcal{E}$ described in the previous section. On the other hand, the positive relationship between $\Upsilon_X^*(\varphi)$ and $\mathcal{E}$ stems from the shape of the markup function. Given that the markup of a Foreign exporter with productivity φ is increasing and concave in $\frac{\varphi}{\varphi_X^*}$, an increase in φ_X^*—due, for example, to a depreciation of the Home currency—leads to a decrease in the markup that is proportionally larger than the markup increase due to a similar decline in φ_X^*. In other words, the firm-specific effect in this model implies that the markup of a Foreign exporter is more responsive to increases than to decreases in competition. The story of asymmetric exchange rate pass-through due to the different response of markups is similar to the idea of asymmetric pass-through advanced by Froot and Klemperer (1989) and Marston (1990). They suggest an asymmetric behavior of pass-through rates driven by market share considerations: when exporters face a depreciation of the importer's currency, they are willing to absorb in their profits an important amount of the shock in order to avoid a substantial decrease in their market share; on the other hand, if an appreciation occurs, they decrease their prices faster to capture a bigger share of the market.

Although exchange rate pass-through to firm-level import prices is not complete, the impact of exchange rates on firm-level traded quantities is substantial and occurs at both the extensive and intensive margins. For example, let us assume a Home-currency depreciation so that the cutoff rule for Home exporters (φ_X) declines and the cutoff rule for Foreign exporters (φ_X^*) increases. At the extensive margin there are two channels of impact: (1) the Home firms between the new and old cutoff rules become exporters, and the Foreign firms between the old and new cutoff rules stop exporting; and (2) there is also an increase in Home exporters due to higher entry of new firms (which expands the pool of Home firms), and a decrease in Foreign exporters due to the decline in entry (which shrinks the pool of Foreign firms). At the intensive margin, the original Home exporters increase their exports and the surviving Foreign exporters decrease them. The opposite happens after a Home currency appreciation. In the remainder of this section, I focus on the quantity and value of exports for Foreign exporters at the intensive margin.

According to the results in section 3.1.3, the quantity that a Foreign firm with productivity $\varphi \geq \varphi_X^*$ sells at Home is given by

$$y_X^*(\varphi) = \left(\frac{\mu_X^*(\varphi)}{1+\mu_X^*(\varphi)}\right)\frac{\gamma I}{\frac{\tau^*\mathcal{E}W^*}{Z^*\varphi}}.$$

The value of these exports in terms of the Home currency is then $p_X^*(\varphi)y_X^*(\varphi) = \gamma\mu_X^*(\varphi)I$.

With an analogous definition for $s_r(\varphi)$, let $s_r^*(\varphi)$ represent the market-share density in market r of a Foreign firm with productivity φ, for $r \in \{D, X\}$. Therefore, we can rewrite the value of exports of a Foreign firm with productivity φ as $p_X^*(\varphi)y_X^*(\varphi) = s_X^*(\varphi)I$, where, as in equation (7), $s_X^*(\varphi) = \gamma\mu_X^*(\varphi)$. Based on these expressions, the following proposition presents the model's main results regarding exchange rates and firm-level trade flows at the intensive margin.

Proposition 4 ***(The impact of the exchange rate on firm-level trade flows)***
For a Foreign firm with productivity $\varphi \geq \varphi_X^*$:

1. *The export quantity, $y_X^*(\varphi)$, is decreasing and elastic with respect to the exchange rate. That is, it declines more than 1% after a 1% Home-currency depreciation.*

2. *The value of its exports in terms of the Home currency, $p_X^*(\varphi)y_X^*(\varphi)$, is decreasing in the exchange rate, with an elasticity (in the interval $(-\infty, 0)$) that approaches 0 as $\varphi \to \infty$.*

Therefore, at the intensive margin, quantities always adjust in the expected direction after an exchange rate movement. Moreover, the adjustment is higher in proportion than the exchange rate change and occurs in the presence of incomplete exchange rate pass-through. Indeed, the least productive Foreign exporters not only have the lower pass-through rates (absorbing more of the exchange rate shock in their markups), but they also have to adjust their quantities more. We can see this in the second part of Proposition 4, where the value of exports in terms of the importer's (Home) currency is less elastic for more productive firms. As $\varphi \to \infty$, the increase in $p_X^*(\varphi)$ after a Home-currency depreciation (with a pass-through rate close to 1) almost perfectly offsets the decline in the equilibrium quantity $y_X^*(\varphi)$, so that the value of this firm's exports remains almost constant.

4.3 Aggregate Pass-Through and Trade Flows

The average exchange rate pass-through to firm-level import prices at Home is given by

$$\bar{\lambda}_X^* = \int_{\varphi_X^*}^{\infty} \lambda_X^*(\varphi)g(\varphi \mid \varphi \geq \varphi_X^*)d\varphi.$$

From equation (26) and Lemma 1, we obtain that

$$\bar{\lambda}_X^* = 1 - \bar{\Upsilon}(k)\zeta_{\varphi_X^*,\mathcal{E}}, \tag{28}$$

where $\bar{\Upsilon}(k)$ is a constant in the interval $(0.4037, 0.5)$ depending only on the productivity dispersion parameter k.[29] Thus, given that $\zeta_{\varphi_X^*,\mathcal{E}}$ is bounded below by 1, the maximum of $\bar{\lambda}_X^*$ is between 0.5 and 0.5963. From equation (28) we can also see that the average pass-through to firm-level import prices is negative if $\zeta_{\varphi_X^*,\mathcal{E}} > \frac{1}{\bar{\Upsilon}(k)}$. As shown in the previous section, markups and marginal costs move in opposite directions after an exchange rate shock, and when $\zeta_{\varphi_X^*,\mathcal{E}} \geq 2$, there will be Foreign firms for which markup adjustments due to the economy-wide competition

[29] $\bar{\Upsilon}(k) = k\int_1^{\infty} \frac{1}{x^{k+1}[1+\mathbf{\Omega}(xe)]}dx$ is about 0.4037 for a k close to 1 and approaches 0.5 as k increases.

effect dominate the marginal cost changes in terms of the importer's currency (resulting in a negative pass-through rate). When the economy-wide effect is large enough—as reflected in a $\zeta_{\varphi_X^*,\mathcal{E}}$ higher than a constant (depending on k) in the interval $(2, 2.4770)$—the result is a negative average pass-through rate to firm-level import prices.

The first result in Proposition 1 states that the average prices of imported and domestic goods at Home, $\bar{p}_X^*$ and $\bar{p}_D$, are identical and equal to

$$\bar{p} = \vartheta(k)\frac{W}{Z\varphi_D}. \tag{29}$$

If we obtain an aggregate import price weighted by market share, the result will be proportional to $\bar{p}$.[30] Therefore, I use the terms "average import price" and "aggregate import price" indistinctly. The following proposition presents the result for exchange rate pass-through to the aggregate import price.

Proposition 5 ***(Exchange rate pass-through to the aggregate import price)***

The rate of pass-through of exchange rate changes to the average import price is given by

$$\Lambda_X^* = -\zeta_{\varphi_D,\mathcal{E}} < 0.$$

That is, the pass-through rate is always negative, implying a decrease in the average import price from a Home-currency depreciation, and an increase from an appreciation.

Note that although the average pass-through to firm-level import prices in equation (28) is only negative for high levels of $\mathcal{E}$, so that $\zeta_{\varphi_X^*,\mathcal{E}}$ is high, the pass-through to the aggregate import price is always negative. This surprising result is due to changes in the extensive margin of trade, which are absorbed in the aggregation. For example, in the case of a Home-currency depreciation, Foreign firms leave the Home market as they become less competitive. The surviving Foreign exporters—who are the most productive and had lower prices before the exchange rate shock—have to adjust their markups down because of the increase in competition coming from Home entrants. The new average import price is then computed taking into account only the survivors. In the end, average import prices reflect the economy-wide competition effect, resulting in a lower average price when competition increases and a higher price when it decreases.

The strong result in Proposition 5 holds for other types of preferences. It holds for the quasilinear-quadratic preferences of Melitz and Ottaviano (2008), but it is also present in models with exogenous markups. In particular, I show in Appendix B (online) that the result holds in a model with CES preferences and a Pareto distribution of productivity. This is the brief example mentioned in the introduction: a negative pass-through rate to the aggregate import price even with firm-level pass-through rates of 1. The empirical evidence indicates, however, small but positive pass-through rates to aggregate import prices. In section 6 I develop a brief extension of the model that considers quality and reconciles the result in Proposition 5 with the empirical evidence.

[30] In general, if the weight function is homogeneous of degree zero in (φ, φ_X^*), we get by Lemma 1 that the resulting weighted import price is proportional to $\bar{p}$. As seen in section 4.2, the market-share density for a Foreign exporter with productivity φ is $s_X^*(\varphi) = \gamma\mu_X^*(\varphi)$, which is homogeneous of degree zero.

Let us now look into the expenditure-switching effect of exchange rates. From the previous section we know that the market-share density for a Foreign firm with productivity φ in market r, for $r \in \{D, X\}$, is given by $s_r^*(\varphi) = \gamma\mu_r^*(\varphi)$. Analogously, $s_r(\varphi) = \gamma\mu_r(\varphi)$. Therefore, the average market shares for Foreign and Home firms in market r are respectively $\bar{s}_r^* = \gamma\bar{\mu}_r^*$ and $\bar{s}_r = \gamma\bar{\mu}_r$. Using equation (13), we get then $\bar{s}_r = \bar{s}_r^* = \gamma\bar{\mu}(k)$. By the result in Corollary 1, $N = N^* = \frac{1}{\gamma\bar{\mu}(k)}$, it is therefore the case that $\bar{s}_r = \bar{s}_r^* = \frac{1}{N} = \frac{1}{N^*}$. This result implies that the total market shares of Home sellers at Home and Foreign are, respectively, $\frac{N_D}{N}$ and $\frac{N_X}{N^*}$. Similarly, the total market shares of Foreign sellers are $\frac{N_X^*}{N}$ (at Home) and $\frac{N_D^*}{N^*}$ (at Foreign). Thus, in terms of the importer's currency, the value of Home exports is $VE = \frac{N_X}{N^*}I^*$ and the value of Foreign exports is $VE^* = \frac{N_X^*}{N}I$; that is, given that N, N^*, I, and I^* are constant, the effect of an exchange rate movement on the value of trade (at both the intensive and extensive margins) is summarized by the responses of N_X and N_X^*. With respect to traded quantities, we calculate the response of an aggregate export quantity to an exchange rate movement as the difference between the response of the value of exports and the response of the aggregate import price (as given in Proposition 5). The following Proposition states our main results regarding the expenditure-switching effect of exchange rates.

Proposition 6 ***(The impact of the exchange rate on aggregate trade flows)***

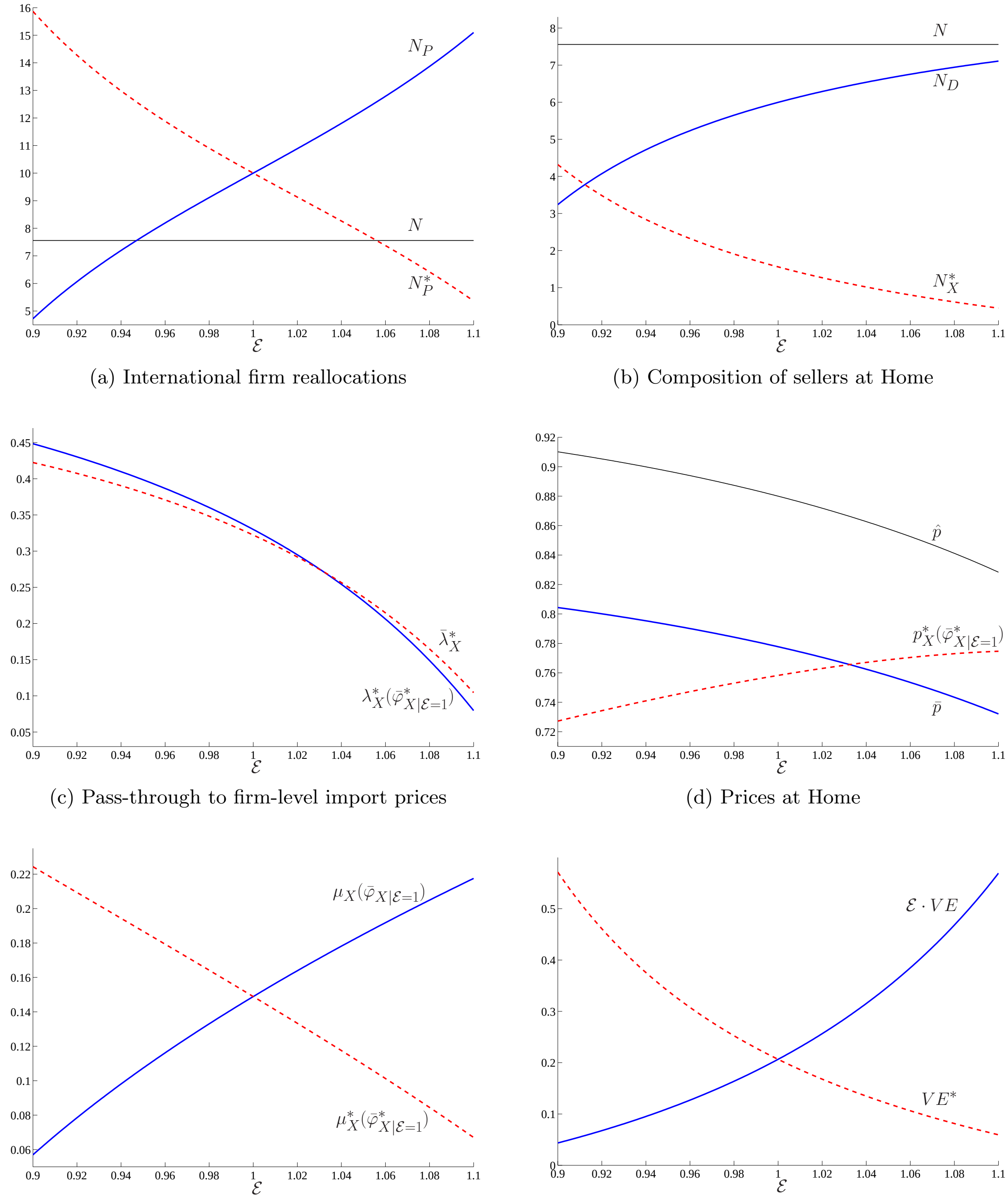
The quantity and value of Home exports are increasing in the exchange rate, and the quantity and value of Foreign exports are decreasing in the exchange rate. The exchange rate elasticities of both quantities and values (in terms of the importer's currency) are in absolute value greater than k.

Therefore, this model predicts large and unambiguous expenditure-switching effects of exchange rate movements. The trade flows move in the expected direction: the country with the depreciating currency increases its exports, and the opposite happens in the country with the appreciating currency.

The results obtained from Proposition 3 to Proposition 6 present a story of large expenditure-switching effects of exchange rate movements at both margins of trade, in the presence of incomplete pass-through rates to firm-level import prices and negative pass-through rates to aggregate import prices. This suggests that—in a setting with firm heterogeneity, firm reallocations, and endogenous markups—firm- and aggregate-level exchange rate pass-through studies should not be used to make conclusions about the true expenditure-switching effects of exchange rates. Moreover, aggregate import prices are subject to a composition bias—a sample selection problem—due to changes in the extensive margin of trade. As a consequence, aggregate prices can differ drastically from the average firm-level pass-through rate.

4.4 Model Summary

Figure 1 presents a graphical summary of the partial equilibrium model. I assume symmetric countries and an initial exchange rate of 1, so that trade is balanced. Without loss of generality, I set W, W^*, Z, Z^*, I, I^*, and $\varphi_{\min}$ to 1. I also set k to 4, γ to 1, f_E and f_E^* to 0.2, τ and τ^*



(a) International firm reallocations

(b) Composition of sellers at Home

(c) Pass-through to firm-level import prices

(d) Prices at Home

(e) Exporters' Markups

(f) Trade flows in the Home currency

Figure 1: Effects of Exchange Rate Changes in the Partial Equilibrium Model

to 1.4, and δ to 0.1.[31]

Panel 1a shows the relationship between the exchange rate and the pools of firms. A depreciation of the Home currency (an increase in $\mathcal{E}$) changes the patterns of firm entry in both countries, increasing the pool of firms at Home and lowering the pool of firms at Foreign. Panel 1b shows the composition of sellers at Home. As N is constant in this model, the Foreign exporters that leave the Home market after a Home-currency depreciation are replaced by an identical mass of Home firms.

Panel 1c shows the average exchange rate pass-through to Home firm-level import prices, $\bar{\lambda}^*_X$, and the pass-through rate at Home for the average Foreign exporter in the initial equilibrium, $\lambda^*_X(\bar{\varphi}^*_{X|\mathcal{E}=1})$. Note that firm-level pass-through rates are asymmetric: lower for depreciations than for appreciations. More productive firms have higher pass-through rates and therefore, the pass-through function of a Foreign exporter with a productivity higher than $\bar{\varphi}^*_{X|\mathcal{E}=1}$ lies above the solid line for $\lambda^*_X(\bar{\varphi}^*_{X|\mathcal{E}=1})$. Panel 1d presents prices $\hat{p}$, $\bar{p}$, and $p^*_X(\bar{\varphi}^*_{X|\mathcal{E}=1})$. Note that the average Home import price, $\bar{p}^*_X = \bar{p}$ (see Proposition 1), moves in the opposite direction of the Home price of the average Foreign exporter. Then, although we observe positive average pass-through rates to firm-level import prices, the pass-through rate to the aggregate import price is negative.

I show the relationship between exchange rates and the markups of the initial equilibrium average exporters in Panel 1e. When the Home-currency depreciates, the markup of the average Foreign exporter declines and the markup of the average Home exporter increases. Moreover, Panel 1e also shows changes at the intensive margin because, as shown in section 4.2, markups are proportional to firm-level market-share densities and sales values.[32] Thus, the model predicts responses in the intensive margin of trade to exchange rate changes in the expected direction: a Home exporter increases its exports after a Home-currency depreciation and the opposite happens for a Foreign exporter. Finally, Panel 1f shows the Home-currency values of aggregate trade flows. Again, movements in the exchange rate generate movements in aggregate trade flows in the expected direction. After a Home-currency depreciation, even if firm- and aggregate-level import prices have low (or even negative) rates of pass-through, the Home country improves its trade balance through both an increase in exports and a decrease in imports.

5 The Model in General Equilibrium

In this section, I place the previous model into a new open economy macroeconomics (NOEM) framework. As in Obstfeld and Rogoff (2000), I assume short-run nominal rigidities in the form of sticky wages. Wages are determined a period in advance, and exchange rate changes are derived endogenously. This model shares the basic building blocks of the flex-price model of Ghironi and Melitz (2005), with two important differences: the incorporation of endogenous markups and the introduction of nominal rigidities.

[31] As this is only a graphical summary of the model presented in the previous pages, I leave the description of the parameters' values for the numerical simulation of the general equilibrium model in section 5.3.

[32] In this particular example, a firm's markup, market-share density, and sales value (in the destination country's currency) are identical because I assume that γ, I, and I^* are equal to 1.

The relationship between exchange rates and prices is in the core of NOEM models.[33] In particular, depending on the assumption of how a good's price is preset for its sale in another country (producer currency pricing or local currency pricing), NOEM models yield different conclusions with respect to the optimal monetary policy. Under producer currency pricing, the optimal monetary policy involves flexible exchange rates, while under local currency pricing the conclusion is that the exchange rate must be fixed (see Devereux and Engel, 2003).

As in the Mundell-Fleming model, the producer currency pricing assumption implies full exchange rate pass-through to import prices—the law of one price and purchasing power parity (PPP) hold—and large expenditure-switching effects. In contrast, under local currency pricing the good's price is preset in the currency of the destination country, which implies a zero pass-through rate and no expenditure-switching effects. As mentioned in the introduction, the evidence of low pass-through to prices makes the case for local currency pricing stronger. However, we have shown in section 4 that, contrary to the local currency pricing implication, low pass-through rates can coexist with large expenditure-switching effects of exchange rates.

In my NOEM model, goods prices are fully flexible and any perceived stickiness (at the firm or aggregate level) is the result of firms adjusting their markups and/or changes in the composition of sellers. The model keeps the basic structure of the partial equilibrium version. There are two countries, Home and Foreign, each of them populated by a continuum of households in the interval [0,1]. Households provide labor to a unique production sector in the economy that produces differentiated goods. These goods are produced under monopolistic competition by heterogeneous producers. To justify short-run wage rigidities, the NOEM model adds the assumption that households are monopolistic suppliers of labor.[34]

In the rest of the section I flesh out the model. I first describe the preferences of the representative household, and then I describe production, labor demand, and the household's budget constraint. I then compute the Euler equations and present the final equations that close the model. Finally, I solve the model for an unexpected monetary shock and do an exchange rate pass-through analysis. As before, Foreign variables are denoted with a star (*). Again, in most of the sections we refer only to the Home country, as analogous expressions apply for the Foreign country.

5.1 Model Setup

5.1.1 Preferences

Consider a dynamic and stochastic model in which households in both countries have the same preferences over consumption, money and labor. The intertemporal utility function for the

[33] These are general equilibrium models that incorporate microeconomic foundations, imperfect competition, nominal rigidities, and allow for welfare analysis. See Lane (2001) for a survey of the first generation of NOEM models.

[34] Also related to the model in this section, Corsetti and Dedola (2005) introduce a NOEM model with nominal wage rigidities and endogenous markups stemming from the existence of (nontraded-goods intensive) local distribution services.

representative Home household j is given by

$$U_t^j = E_t \left[\sum_{s=0}^{\infty} \beta^s \left(\ln C_{t+s}^j + \chi \ln \frac{M_{t+s}^j}{P_{t+s}} - \frac{\kappa}{2} \ell_{t+s}(j)^2 \right) \right], \tag{30}$$

where C^j is the real consumption index, $\frac{M^j}{P}$ represents real money balances, and $\ell(j)$ is the amount of labor devoted to production. Also, $\beta \in (0,1)$ is the discount factor, χ is a money velocity parameter, and κ is a weighting parameter for the disutility of labor.

The real consumption index C^j corresponds to the homothetic utility function over differentiated goods implied by the translog expenditure function in equation (1). The deflator of money balances P is the money price of a unit of C^j. That is, P (the price index) is the minimum expenditure needed to buy a unit of C^j. From equation (1), we then get

$$\ln P = \frac{1}{2\gamma N} + \frac{1}{N} \int_{i \in \Delta'} \ln p_i di + \frac{\gamma}{2N} \int_{i \in \Delta'} \int_{j \in \Delta'} \ln p_i (\ln p_j - \ln p_i) dj di. \tag{31}$$

The last term in the utility function corresponds to the disutility of labor. Household j's labor is differentiated and monopolistically supplied for the production of the differentiated goods. This characteristic of the model implies an equilibrium household's labor supply that is below the efficient level, which provides the underpinnings for short-run wage stickiness.

5.1.2 Production and Labor Demand

The production of each good demands every type of labor in the interval $[0,1]$. The production function for a Home firm with productivity φ at time t is given by $y_t(\varphi) = Z_t \varphi L_t$, where

$$L_t = \left[\int_0^1 \ell_t(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}. \tag{32}$$

That is, the labor index L follows a Dixit-Stiglitz technology, where $\theta > 1$ is the elasticity of substitution between the different types of labor. It follows that the demand for type j's labor is given by $\ell_t^D(j) = \left[\frac{w_t(j)}{W_t}\right]^{-\theta} L_t^D$, where $w_t(j)$ is the nominal wage for type j's labor, $W_t = \left[\int_0^1 w_t(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}$ is the price of labor index L_t, and L_t^D is the total labor demand at Home.

The description of the production sector continues as in sections 3.1 and 3.2, with some minor modifications. In particular, we need to include time subscripts and replace I and I^* with $P_t C_t$ and $P_t^* C_t^*$, respectively. We can also derive an expression for P_t in terms of the cutoff productivity level $\varphi_{D,t}$. Substituting the pricing equations from section 3.1.3 into equation (31), and using the markup equations (10) and (11), we get that

$$P_t = \Gamma(k) \frac{W_t}{Z_t \varphi_{D,t}}, \tag{33}$$

where $\Gamma(k)$ is a constant less than one.[35] That is, P_t is proportional to $\hat{p}_t$ and $\bar{p}_t$.

The free-entry conditions in the NOEM model differ from the ones in section 3.3. The value of entry at Home at time t is now given by $\bar{\pi}_t + v_t$, where $\bar{\pi}_t = \bar{\pi}_{D,t} + \mathcal{E}_t \bar{\pi}_{X,t}$ is the pre-entry expected profit for time t, and v_t is the present value of the expected future stream of profits. As before, we write $\bar{\pi}_t$ as

$$\bar{\pi}_t = \frac{\psi P_t C_t}{\varphi_{D,t}^k} + \mathcal{E}_t \frac{\psi P_t^* C_t^*}{\varphi_{X,t}^k}, \tag{34}$$

where $\psi = \frac{\gamma \bar{\mu}(k) \varphi_{\min}^k}{k+1}$. As I show later, the new term v_t is given by

$$v_t = \beta(1-\delta) E_t \left[\frac{P_t C_t}{P_{t+1} C_{t+1}} (\bar{\pi}_{t+1} + v_{t+1}) \right].$$

With parallel equations for the value of entry in the Foreign country, the free-entry conditions in the NOEM model are

$$\bar{\pi}_t + v_t = \frac{f_{E,t} W_t}{Z_t} \tag{35}$$

$$\bar{\pi}_t^* + v_t^* = \frac{f_{E,t}^* W_t^*}{Z_t^*}. \tag{36}$$

Let us now derive an expression for the total labor demand. The demand for labor at time t has two components: the labor demand related to new firms' entry costs, and the labor demand coming from the firms that are actually producing. The former is given by $N_{E,t} \frac{f_{E,t}}{Z_t}$, while the latter is

$$N_{D,t} \int_{\varphi_{D,t}}^{\infty} \frac{y_{D,t}(\varphi)}{Z_t \varphi} g(\varphi \mid \varphi \geq \varphi_{D,t}) d\varphi + N_{X,t} \int_{\varphi_{D,t}}^{\infty} \tau_t \frac{y_{X,t}(\varphi)}{Z_t \varphi} g(\varphi \mid \varphi \geq \varphi_{X,t}) d\varphi.$$

Solving for this expression, it follows that the total labor demand at Home at time t is given by

$$L_t^D = N_{E,t} \frac{f_{E,t}}{Z_t} + \frac{k}{k+1} \frac{\gamma \bar{\mu}(k)}{W_t} \left(N_{D,t} P_t C_t + N_{X,t} \mathcal{E}_t P_t^* C_t^* \right). \tag{37}$$

To conclude the model setup, I now describe the representative household's budget constraint.

5.1.3 Household Budget Constraint

In this model, current account imbalances may exist because international trade in bonds is possible: Home and Foreign households can hold Home and Foreign risk-free bonds denominated in each country's consumption basket. The representative Home household j begins time t with A_t^j and B_t^j holdings of the Home and Foreign bonds, respectively. Bond holdings at the end of t are given by A_{t+1}^j and B_{t+1}^j. Real interest rates from t to $t+1$ are given by r_{t+1} and r_{t+1}^*. For the model to be stationary, we follow Ghironi and Melitz (2005), hereafter GM, and assume a small cost of holdings bonds. In particular, Home household j faces costs (in terms of each country's

[35] $\Gamma(k) = \exp\left(1 - \frac{1}{2\bar{\mu}(k)} \left[k e^k \int_e^{\infty} \frac{\boldsymbol{\Omega}(x)^2}{x^{k+1}} dx - 1 \right]\right)$ ranges between 0.5082 (for a k close to 1) and 1 (for a high k).

consumption basket) at time t of $\frac{\eta}{2}(A^j_{t+1})^2$ and $\frac{\eta}{2}(B^j_{t+1})^2$ for holding Home and Foreign bonds, respectively.

The ownership structure of firms is based on GM. Each country has a mutual fund that owns all the firms in that country and that finances the entry of new firms. Households can buy shares only in the domestic mutual fund, so that Home households can only be owners of Home firms, and Foreign households can only be owners of Foreign firms. The composition of firms in the mutual funds changes each period, as some firms die and some firms enter. The representative Home household j begins time t with x^j_t shares of last period's mutual fund, which comprises the pool of firms given by $N_{P,t-1}$. A fraction δ of the firms die at the end of $t-1$, so that Home household j receives $x^j_t(1-\delta)N_{P,t-1}\bar{\pi}_t$ as dividends from the surviving firms at time t. The mutual fund finances the entry of $N_{E,t}$ firms, with a financing cost of $N_{E,t}\frac{f_{E,t}W_t}{Z_t}$ (equivalent to $N_{E,t}(\bar{\pi}_t + v_t)$ by equation (35)). Also at time t, Home household j can adjust to x^j_{t+1} its shares in the mutual fund—now comprised of $N_{P,t} = (1-\delta)N_{P,t-1} + N_{E,t}$ firms. The present value of time-t's mutual fund is then $N_{E,t}\bar{\pi}_t + N_{P,t}v_t$, where the first term represents the time-t profits of the new entrants and the second term represents the present value of the fund's expected future stream of profits.[36] In summary, at time t Home household j pays $(x^j_{t+1} - x^j_t)(1-\delta)N_{P,t-1}v_t + x^j_{t+1}N_{E,t}(\bar{\pi}_t + v_t)$ to the mutual fund and receives $x^j_t(1-\delta)N_{P,t-1}\bar{\pi}_t + x^j_{t+1}N_{E,t}\bar{\pi}_t$ in dividends.

The other components of the budget constraint are consumption expenditure, labor income, tax payments, and money holdings. For labor income, Home household j receives a nominal wage $w_t(j)$ for each unit of labor provided. On the other hand, it pays lump-sum taxes in the amount of T^j_t and maintains M^j_t in nominal money balances.

With these elements, Home household j's budget constraint can be written as

$$\begin{aligned} &P_tC^j_t + P_tA^j_{t+1} + P_t\frac{\eta}{2}(A^j_{t+1})^2 + \mathcal{E}_tP^*_tB^j_{t+1} + \mathcal{E}_tP^*_t\frac{\eta}{2}(B^j_{t+1})^2 + x^j_{t+1}N_{P,t}v_t + M^j_t + T^j_t \\ &= P_t(1+r_t)A^j_t + \mathcal{E}_tP^*_t(1+r^*_t)B^j_t + w_t(j)\ell_t(j) + x^j_t(1-\delta)N_{P,t-1}\left(\bar{\pi}_t + v_t\right) + M^j_{t-1}, \end{aligned} \tag{38}$$

with an analogous expression holding for the representative Foreign household.

5.2 Closing the Model

5.2.1 First-Order Conditions

We now solve the household maximization problem. The representative Home household j maximizes the intertemporal utility function (30) subject to the budget constraint (38) and the demand for its labor $\ell^D_t(j) = \left[\frac{w_t(j)}{W_t}\right]^{-\theta} L^D_t$. The nominal wage is set a period in advance. Then, the first-order conditions with respect to A^j_{t+1}, B^j_{t+1}, x^j_{t+1}, M^j_t, and $w_{t+1}(j)$—the Euler equations—are respectively given by

$$\frac{1+\eta A_{t+1}}{1+r_{t+1}} = \beta E_t\left[\frac{C_t}{C_{t+1}}\right] \tag{39}$$

[36]The model of GM includes a one-period time-to-build assumption. Therefore, entrants do not make profits in the first period, and the present value of the mutual fund is just $N_{P,t}v_t$.

$$\frac{1+\eta B_{t+1}}{1+r^*_{t+1}} = \beta E_t\left[\frac{C_t}{C_{t+1}}\frac{Q_{t+1}}{Q_t}\right] \tag{40}$$

$$v_t = \beta(1-\delta)E_t\left[\frac{P_tC_t}{P_{t+1}C_{t+1}}(\bar{\pi}_{t+1}+v_{t+1})\right] \tag{41}$$

$$\frac{M_t}{P_t}\left(1-\beta E_t\left[\frac{P_tC_t}{P_{t+1}C_{t+1}}\right]\right) = \chi C_t \tag{42}$$

$$W_{t+1}E_t\left[\frac{L_{t+1}}{P_{t+1}C_{t+1}}\right] = \frac{\theta\kappa}{\theta-1}E_t\left[L^2_{t+1}\right], \tag{43}$$

where $Q_t = \frac{\mathcal{E}_t P^*_t}{P_t}$ is the real exchange rate at time t. I dropped the superscript j from the Euler equations, and replaced w_{t+1} with W_{t+1} and ℓ_{t+1} with L_{t+1}—as every household receives the same wage and offers the same amount of labor in the symmetric equilibrium. Parallel equations hold for the Foreign country.

As equilibrium labor is below its efficient level (because households are monopolistic labor suppliers), labor is demand determined in the presence of sufficiently small shocks and wage stickiness, as the wage will still be above the value of the marginal disutility of labor. In other words, the labor-leisure Euler equation (43) does not bind in the short-run—at the time of the shock—because wages are sticky.

5.2.2 Net Foreign Assets

I need to establish two more conditions. First, internationally traded bonds are in zero net supply, that is,

$$A_{t+1} + A^*_{t+1} = 0 \tag{44}$$

$$B_{t+1} + B^*_{t+1} = 0. \tag{45}$$

Second, let us derive the net foreign assets condition from the aggregate budget constraints. I assume that the government's only role is to use transfers to distribute to households the revenues from seignorage and international bond-holding costs, so that

$$P_t\frac{\eta}{2}A^2_{t+1} + \mathcal{E}_tP^*_t\frac{\eta}{2}B^2_{t+1} + (M_t - M_{t-1}) + T_t = 0.$$

With a similar expression at Foreign, we get that the Home and Foreign aggregate budget constraints are respectively given by

$$P_tA_{t+1} + \mathcal{E}_tP^*_tB_{t+1} = P_t(1+r_t)A_t + \mathcal{E}_tP^*_t(1+r^*_t)B_t - P_tC_t + W_tL_t - N_{E,t}v_t + N_{P,t-1}(1-\delta)\bar{\pi}_t$$

$$\frac{P_tA^*_{t+1}}{\mathcal{E}_t} + P^*_tB^*_{t+1} = \frac{P_t(1+r_t)A^*_t}{\mathcal{E}_t} + P^*_t(1+r^*_t)B^*_t - P^*_tC^*_t + W^*_tL^*_t - N^*_{E,t}v^*_t + N^*_{P,t-1}(1-\delta)\bar{\pi}^*_t,$$

where we use that

$$N_{E,t} = N_{P,t} - (1-\delta)N_{P,t-1} \tag{46}$$

$$N^*_{E,t} = N^*_{P,t} - (1-\delta)N^*_{P,t-1}. \tag{47}$$

Following GM, we multiply the Foreign aggregate budget constraint by $\mathcal{E}_t$, and then we substitute the zero-net-supply conditions for bonds. Subtracting the resulting expression from the Home aggregate budget constraint, we get that Home's net foreign asset position at the end of time t in terms of the Home currency, NFA_t, is given by

$$\begin{aligned}P_t A_{t+1} + \mathcal{E}_t P_t^* B_{t+1} =& P_t(1+r_t)A_t + \mathcal{E}_t P_t^*(1+r_t^*)B_t - \frac{1}{2}\left(P_t C_t - \mathcal{E}_t P_t^* C_t^*\right) + \frac{1}{2}\left(W_t L_t - \mathcal{E}_t W_t^* L_t^*\right) \\ & - \frac{1}{2}\left(N_{E,t} v_t - N_{E,t}^* \mathcal{E}_t v_t^*\right) + \frac{1-\delta}{2}\left(N_{P,t-1}\bar{\pi}_t - N_{P,t-1}^* \mathcal{E}_t \bar{\pi}_t^*\right). \quad (48)\end{aligned}$$

The model is complete. We have a system of 31 equations ((8)-(9), (14)-(17), (33)-(37), (39)-(48), and the corresponding Foreign equations) and 31 endogenous variables: $\varphi_{D,t}$, $\varphi_{X,t}$, $\varphi_{D,t}^*$, $\varphi_{X,t}^*$, P_t, P_t^*, W_{t+1}, W_{t+1}^*, $\mathcal{E}_t$, $\bar{\pi}_t$, $\bar{\pi}_t^*$, C_t, C_t^*, v_t, v_t^*, $N_{E,t}$, $N_{E,t}^*$, $N_{P,t}$, $N_{P,t}^*$, $N_{D,t}$, $N_{X,t}$, $N_{D,t}^*$, $N_{X,t}^*$, L_t, L_t^*, A_{t+1}, A_{t+1}^*, B_{t+1}, B_{t+1}^*, r_{t+1}, and r_{t+1}^*. The model's exogenous variables are M_t, M_t^*, Z_t, Z_t^*, τ_t, τ_t^*, $f_{E,t}$, and $f_{E,t}^*$.[37]

5.3 A Permanent Monetary Shock

In this section I consider the effect of a permanent 1 percent monetary shock at Home. For this purpose, I assume that money supplies at Home and Foreign follow the processes

$$\begin{aligned}\ln M_{t+1} &= \ln M_t + \xi_{t+1} \\ \ln M_{t+1}^* &= \ln M_t^* + \xi_{t+1}^*,\end{aligned}$$

where ξ_{t+1} and ξ_{t+1}^* are normal white noise processes with standard deviation ς.

As shown in section 4, this model has important second-order effects (asymmetries in the responses of variables of interest to appreciations and depreciations of a currency). Given that a traditional linear-approximation solution method loses these effects, I solve the model using Sims's second-order accurate solution method, described in Kim et al. (2008).[38]

The model assumes sticky wages and firm entry and exit. In section 2 we discussed evidence on wage stickiness and entry and exit of firms even at one-year horizons. Therefore, I choose parameter values to resemble a period length of one year. The relevant period-length parameters are the discount factor, β, and the death rate, δ. For quarterly data, GM set β to 0.99 (the usual choice) and δ to 0.025. Thus, I set annual values of 0.96 for β and 0.1 for δ.[39] Following Bergin and Feenstra (2009), I set the substitutability parameter of the translog expenditure function, γ, to 1. For the parameters of the productivity distribution, I set $\varphi_{\min}$ to 1 (as GM) and the productivity dispersion parameter, k, to 4 (rounding up the 3.4 value that GM use). The value

[37] Appendix B (online) includes a table that shows the system of 31 equations of the NOEM model.

[38] I thank Robert Kollmann for providing me with the Matlab codes from his paper titled "Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles" (JME, 2002) for the implementation of Sims's `gensys2` algorithm.

[39] From the literature review in section 2, we have that with a 14% annual rate of job destruction of total U.S. employment and with death of firms accounting for a proportion as high as 66% of total job destruction, a 10% annual death rate is appropriate for this model. GM also argue that a 10% annual death rate is appropriate given that their model—like this one—does not model explicitly multi-product firms and the death shock is at the product level.

of χ—from the intertemporal utility function—is only relevant to determine the level of the nominal variables. Thus, I set $\chi = 0.1$ so that, in combination with the initial levels of money supplies, the total consumption expenditure in both countries at the initial steady state is 1. Following Bilbiie, Ghironi, and Melitz (2007b), I set the labor market parameter values, θ (the elasticity of substitution between the different types of labor) and κ (the parameter that weights the disutility of labor in the intertemporal utility function), so that the equilibrium level of labor in the initial steady state is about 1 in both countries. Hence, I set $\theta = 5$ and $\kappa = 0.75$. For this numerical exercise I assume that the only uncertainty comes from the money supplies—that is, aggregate productivities, sunk entry costs, and trade costs are constant. In particular, I set $Z_t = Z_t^* = 1$, $f_{E,t} = f_{E,t}^* = 0.2$, and $\tau_t = \tau_t^* = 1.4$ for every t. The value for the aggregate productivities is normalized without any loss of generality. The value for the sunk entry cost is set so that about 20% of the pool of firms in each country are exporters in the initial steady state.[40] The value for the iceberg trade costs is higher than the value set by GM (they use 1.3), but it is not unrealistically high, considering that in this model Home and Foreign producers sell directly to final consumers and distribution costs in the destination country can be very large.[41] For the money supplies' stochastic processes, I assume that the standard deviation, ς, is 0.001. Finally, the bond-holding cost parameter, η, is set—as in GM—to 0.0025.

Let time 0 be the initial and symmetric steady state, with money supplies M_0 and M_0^* equal to 2.5. In this steady state, bond holdings are zero, trade is balanced, PPP holds, and the real (and nominal) interest rate is $\frac{1-\beta}{\beta}$ in each country. At the beginning of time 1 there is a permanent and unexpected 1% increase in the Home money supply. Given that wages are set a period in advance, there are short-run real effects of the monetary expansion. Wages then adjust to capture the monetary shock, and the transition to the new steady state begins. In the new steady state only the Home nominal variables change (money is neutral in the long run), and we go back to a position of financial autarky. It is important to note that the new steady state cannot be reached at time 2, as time 1 effects include firm reallocations that take time to dissipate, increasing the persistence of the monetary shock. Figure 2 presents the dynamics of the model. The horizontal axis represents time in years and, with the exception of the last two subfigures, the variables' responses represent percent deviations from the initial steady state.

At the initial steady state the nominal exchange rate is 1—as given by the ratio of Home and Foreign money supplies. At time 1, the nominal exchange rate rises by about one-third of a percentage point. It then starts to converge towards its new steady state of 1.01. At the time of the shock, the effects of the Home monetary expansion on real variables work through two channels: the increase in Home households' demand and the depreciation of the Home currency. In the partial equilibrium model, this is equivalent to simultaneous increases in I and $\mathcal{E}$. For Home firms, the first channel increases profit opportunities from Home sales, and the second channel increases profit opportunities from exporting, driving up the entry of Home firms. The rise in Home entry increases the pool of Home firms by about 5.5%, and the increase

[40] In this respect, I follow closely GM, who set fixed export costs and sunk entry costs so that about 21% of firms export. As mentioned before, my model does not need to impose fixed costs.

[41] For example, Burstein, Neves, and Rebelo (2003) find that local distribution costs account for 40% of retail prices in the U.S. and for 60% in Argentina.

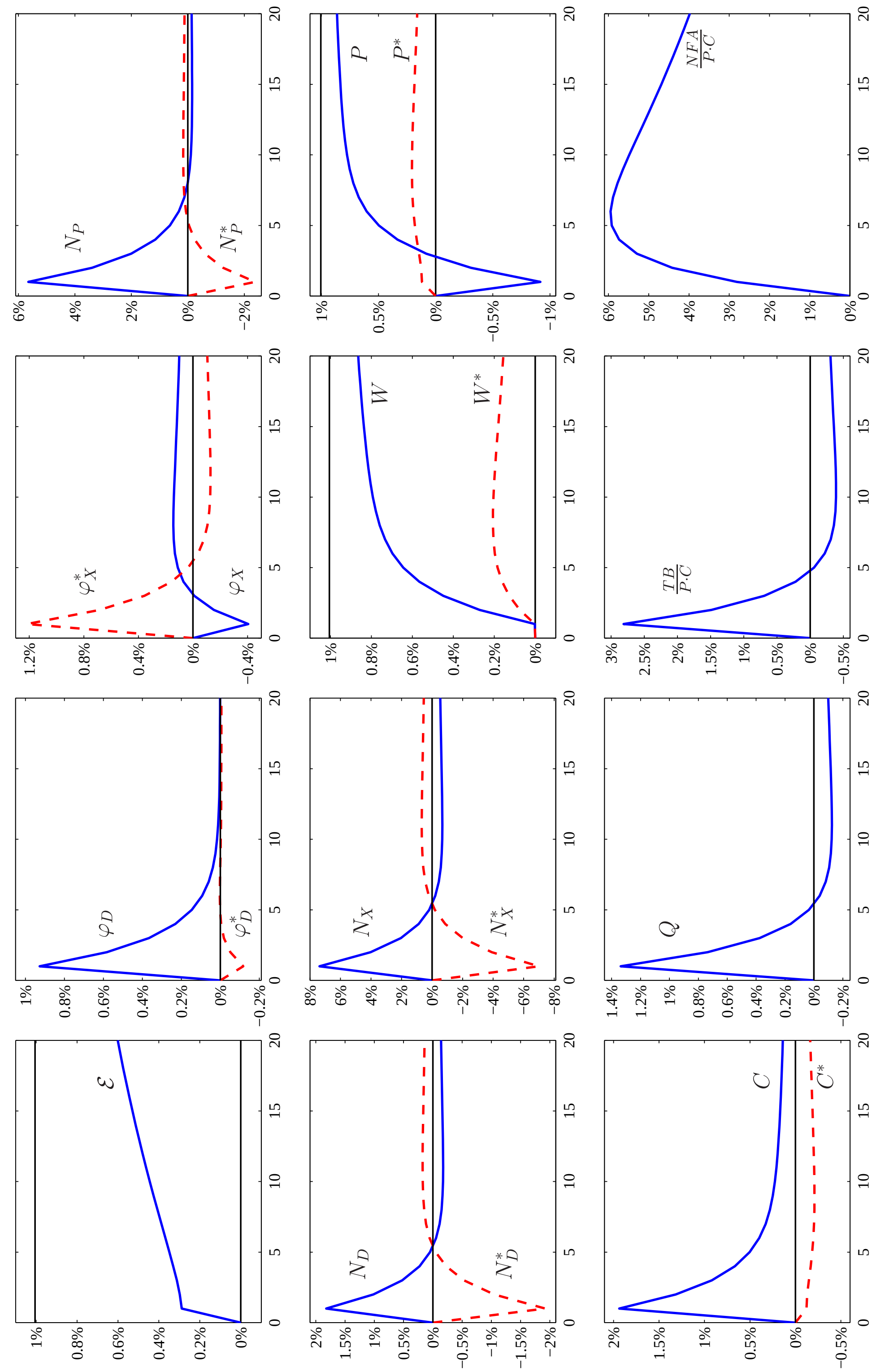


Figure 2: Responses to a 1% Increase in the Home Money Supply

in competition then drives up the cutoff level, φ_D, by about 0.9%. On the other hand, pushed by the increase in competition at Home and by the lower level of competitiveness of Foreign firms due the Home-currency depreciation, the pool of firms declines at Foreign by about 2%. With the decline in N_P^*, the cutoff level, φ_D^*, declines about 0.1%. As expected, φ_X and φ_X^* move in the direction (but in a higher proportion) of φ_D^* and φ_D, respectively. With respect to the composition of sellers, the increase in N_D and the decline in N_D^* are close to 2%, which corresponds—given that in this model the number of sellers in each market is constant—to an increase in Home exporters and a decline of Foreign exporters of about 7%.

Given the wage rigidity and equation (33), it follows that the aggregate price P falls at time 1 in the same proportion as the increase in φ_D (about 0.9%). As nominal consumption expenditure at Home increases by 1% due to the permanent monetary expansion, Home consumption then increases by about 1.9% at the time of the shock. The time 1 elastic response of Home consumption to the monetary shock is always present in this model, as the Home aggregate price always declines with the monetary expansion. Meanwhile, at Foreign there is a slight increase in P^* (about 0.1%) due to the decrease in φ_D^*, and a similar decline in consumption.

The movements in aggregate prices, along with the exchange rate change, generate strong deviations from PPP: the real exchange rate Q increases about 1.3% at the time of the 1% Home monetary expansion. Moreover, the impact on trade flows is large. From balanced trade at time 0, the trade balance of Home—defined as $TB_t = \mathcal{E}_t VE_t - VE_t^*$ for time t—jumps to a surplus of about 2.8% of total consumption expenditure at time 1, improving Home's net foreign asset position in the same proportion.

As the Home wage adjusts to reflect the monetary expansion, the transition to the new steady state begins. However, the transition is slow because the firm reallocations that happened at time 1 take several periods to vanish (for real variables, it takes about five periods to reduce most of the gap between time-1 levels and steady-state levels). The wage also increases slightly at Foreign, as the demand for labor in that country is above the steady-state level during the transition.

Also in the transition, we see that P moves above the initial steady-state level starting in time 3 and converges slowly to its new steady state. The slow convergence is also seen in Home consumption, which remains well above its steady-state level for several periods as Home is receiving interest payments from abroad. The opposite happens at Foreign, though initial movements in consumption are much smaller. With respect to the real exchange rate, deviations from PPP are very persistent and it takes about five periods for Q to be close to 1 again. The trade balance is in surplus for four periods, is about zero at time 5, and moves to deficit for the rest of the transition. Home's net foreign asset position reaches its peak at time 6, and then begins to decrease slowly towards zero.

This concludes the general description of the model's solution. In Appendix B (online) I provide an extensive sensitivity analysis. For a wide range of parameter values, the model's responses are similar to the description above. I also analyze the model's responses to transitory and permanent productivity shocks. Again, in the case of a transitory productivity shock, the model features substantial firm reallocations, creating a high degree of persistence. For a permanent productivity shock, the economy jumps immediately to the new steady state.

As a note of caution, although there is strong evidence of the procyclical behavior of firm entry (see section 2), the changes in entry patterns at the time of the shock generated in this model—reflected in the sizes of the pools of firms—are very large. We must note, however, that the structure of this model was kept at the minimum level in order to present with clarity the main forces at work and their implications for the pass-through and expenditure-switching effects of exchange rates. In particular, my NOEM model abstracts from more complex features of calibrated DSGE models that may dampen the entry channel, e.g. Calvo-adjustment mechanisms and/or the addition of more sectors (a nontradable sector is particularly relevant in open economy macroeconomics). These are features that must be considered if we want to calibrate the model to the U.S. or any other economy, or if we want to study the model's implications for optimal monetary policy.

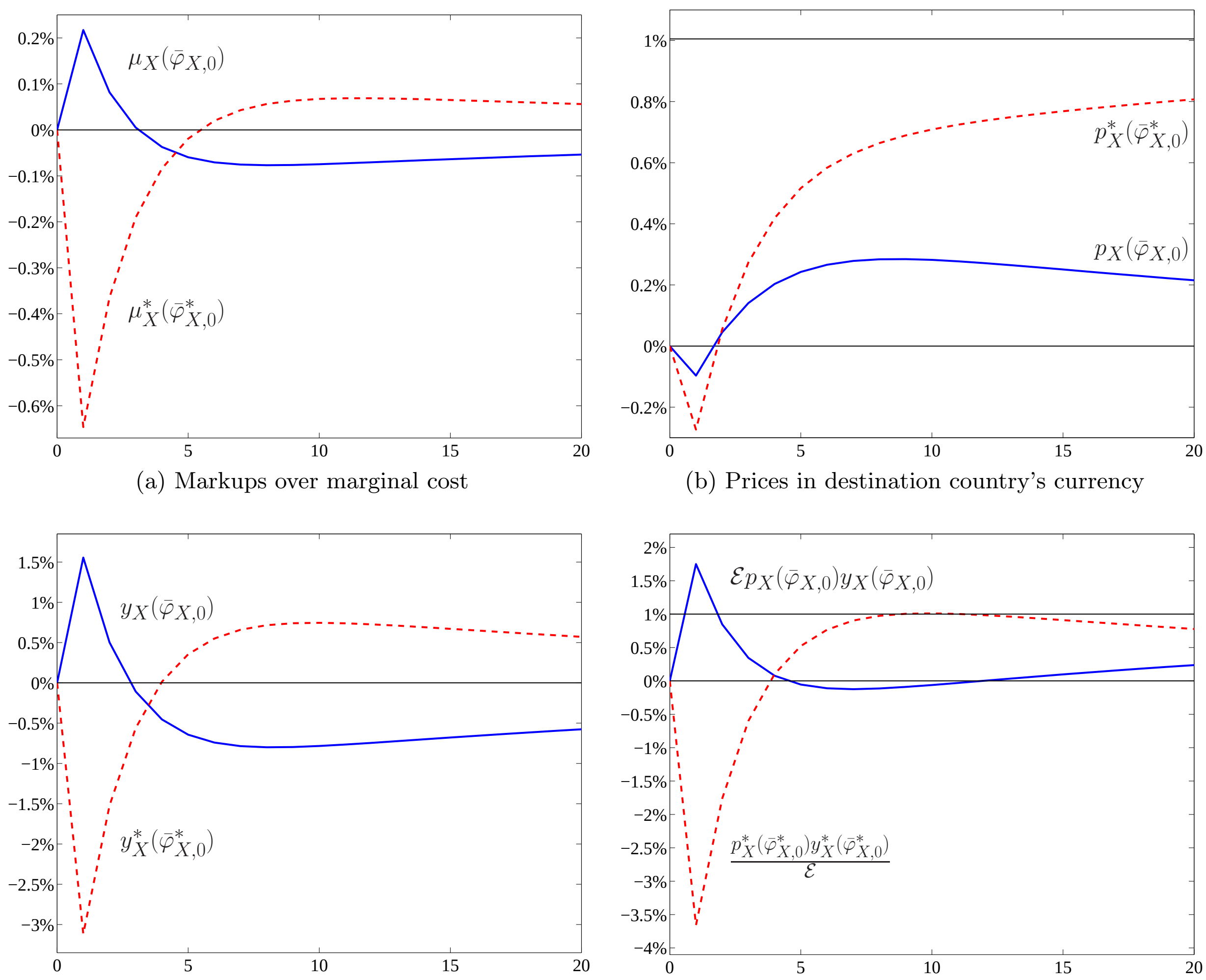
5.4 Exchange Rate Pass-Through in General Equilibrium

Proposition 1 states that the average price for Home goods sold at Home $\bar{p}_D$ is equivalent to the average price of imports at Home $\bar{p}^*_X$, with $\bar{p}$ representing the common average price. Similarly, the import price index at Home at time t is equivalent to the general price index, P_t. Therefore, from Figure 2 we observe that the aggregate import price falls at Home and increases at Foreign at the time of the sudden Home-currency depreciation generated by the Home monetary expansion. That is, as in the partial equilibrium model, we have negative rates of pass-through to aggregate import prices of exchange rate changes.[42] At the same time, the monetary expansion has large and persistent expenditure-switching effects: the depreciating currency country runs trade balance surpluses that last four periods, and its net foreign asset position reaches about 6% of total consumption expenditure by time 6.

The short-run behavior of firm-level import prices also follows closely the partial equilibrium results. Let us focus on how identical exporters from Home and Foreign respond to the Home monetary expansion. The average productivities of exporters from Home and Foreign at time t are given by $\bar{\varphi}_{X,t}$ and $\bar{\varphi}^*_{X,t}$, respectively. At time 0, the initial and symmetric steady state, $\bar{\varphi}_{X,0}$ and $\bar{\varphi}^*_{X,0}$, are equal—the average Home exporter is identical to the average Foreign exporter. Figure 3 presents the dynamics of markups, prices, quantities, and export values for these identical exporters when there is a 1% Home monetary expansion at time 1. The horizontal axis represents time in years.

Panel 3a presents the responses of markups as the markup difference between time t and time 0. Under our assumed parameter values, the average exporter markup at time 0 is 14.9%. Hence, for example, the 0.2% markup response at time 1 for the time-0 average Home exporter implies a 15.1% markup. On the other hand, the downward adjustment of the average Foreign exporter's markup, $\mu^*_X(\bar{\varphi}^*_{X,0})$, is much stronger, decreasing by about 0.6% at the time of the shock. As wages adjust, markups go back to their original steady-state levels.

[42] As mentioned before, although the negative pass-through result (and the implied large response in Home consumption) is controversial, the simple extension of the model in section 6 shows that if we distinguish between quality- and non-quality-adjusted prices and quantities, the previous results are true for quality-adjusted variables, while conventional responses may hold for non-quality-adjusted variables.



(a) Markups over marginal cost

(b) Prices in destination country's currency

(c) Export quantities

(d) Value of exports in exporter's currency

Figure 3: Responses of Time-0 Average Exporters to a 1% Home Monetary Expansion

The responses of variables in Panels 3b, 3c, and 3d represent percent deviations from the initial steady state. Panel 3b shows the responses of average exporters' prices in terms of the destination country's currency. At the time of the shock, both prices decrease. That is, although the average Home exporter lowers its price in the Foreign country at the time of the sudden Home-currency depreciation (and still manages to increase its markup), the average Foreign exporter also lowers its price at Home. This difference from the partial equilibrium model—where the average exporter has positive pass-through rates in both countries—stems from the impact of the monetary expansion on Home expenditure (recall that in the partial equilibrium model, a monetary expansion at Home is equivalent to simultaneous increases in $\mathcal{E}$ and I). As mentioned in section 5.3, the rise in money supply under sticky wages generates new profit opportunities at Home, increasing entry and competition and therefore reinforcing the upward effect on the Home cutoff productivity level, φ_D. The competition effect is so strong that the downward markup adjustment of the average Foreign exporter implies a price at Home below the original steady state. As money is neutral in the long-rung, $p^*_X(\bar{\varphi}^*_{X,0})$ is 1% higher in the new steady state and $p_X(\bar{\varphi}_{X,0})$ returns to its original level. During the transition, however, $p_X(\bar{\varphi}_{X,0})$ follows the Home wage level response and is above the steady-state level.

Changes in the intensive margin are large. Panel 3c shows the responses of export quantities, and Panel 3d presents the responses of the export values (in terms of the producer's currency). At the time of the shock, the quantity and the value of exports of the average Home exporter increase by more than 1.5%. Meanwhile, the quantity and the value of exports of the average Foreign exporter decline by about 3% and 3.5%, respectively. In the transition, as the cutoff productivity levels adjust so that the Foreign country can run trade surpluses in order to pay its external debt, the export quantity of the average Foreign exporter moves above the steady-state level (and below the steady-state level for the average Home exporter) and slowly converges back. The values of exports reflect the changes in quantities and move slowly to the corresponding steady-state nominal levels.

In sum, the Home monetary expansion generates a Home-currency depreciation and large expenditure-switching effects. Firm- and aggregate-level prices are largely disconnected from the exchange rate. Therefore, results of exchange rate pass-through to prices are not reliable indicators of movements in trade flows.

6 A Quality Extension

The result in Proposition 5 states that the rate of pass-through of exchange rates to aggregate import prices is negative. The result continues to hold at the time of the shock in the general equilibrium model. Empirically, however, the evidence is that pass-through rates are small but positive. In this section I show that a simple extension of the model that considers quality reconciles the model's results with the empirical facts.

The model in the previous sections does not consider quality explicitly. However, the quality of a good is also an important determinant of its demand and therefore, a natural extension to the model is to assume that consumers when define their preferences, they are taking into account quality-adjusted prices. Indeed, recent theoretical and empirical research with heterogeneous firms looks at the implications of quality heterogeneity for unit and quality-adjusted prices (e.g., Baldwin and Harrigan (2009) and the references cited therein).[43]

Following this literature, I assume that firms are heterogeneous in two characteristics: productivity (in terms of marginal costs) and product quality. In the terminology of Sutton (2007), the combination of the productivity and quality of a firm determines its *capability*. Therefore, the tradability of a good in a market depends on the capability of its producer as compared to that market's capability cutoff level. Moreover, if we just replace the word "productivity" by "capability" in the previous sections, we can see the model above as one based in quality-adjusted prices. As such, most of the notation is the same as before and we just have to redefine a few variables: for example, φ now represents capability instead of productivity, $\hat{p}$ is the maximum quality-adjusted price that can be set at Home, and $p_r(\varphi)$ represents the quality-adjusted price

[43] See also Auer and Chaney (2009) for a model of exchange rate pass-through and quality pricing in a setting of perfect competition and decreasing returns to scale.

in market r of a Home firm with capability φ.

6.1 The Model with Quality

The representative Home household defines its preferences over a continuum of quality-adjusted differentiated goods in the set Δ. As in section 3.1.1, Δ' is the subset of goods that are actually available for purchase at Home and has measure N. Preferences are obtained from equation (1), where $p_i = \frac{\tilde{p}_i}{\sigma_i}$ is now the quality-adjusted price of good i, σ_i denotes its quality, and $\tilde{p}_i$ denotes its unit price. As before, the share of good i in total expenditure is $s_i = \gamma \ln\left(\frac{\hat{p}}{p_i}\right)$, and its quality-adjusted demand is given by $q_i = s_i \frac{I}{p_i}$. Moreover, $q_i = \sigma_i \tilde{q}_i$, where $\tilde{q}_i$ is the regular demand (the amount of physical units demanded) for good i. The profit-maximization problem of firm i, with marginal cost mc_i, yields the unit price equation

$$\tilde{p}_i = \mathbf{\Omega}\left(\frac{\sigma_i \hat{p}}{\mathrm{mc}_i}\right)\mathrm{mc}_i,$$

where $\sigma_i \hat{p}$ is the maximum unit price that firm i can set at Home.

Firms are heterogeneous in quality and productivity, and these factors jointly determine the capability of the firm. In particular, φ denotes the capability of a firm and is given by $\varphi = \sigma z$, where σ denotes quality and z denotes productivity. Then, the marginal cost of a Home firm with capability φ is $\frac{W}{Zz} = \frac{W}{Z\varphi}\sigma$, with W and Z defined as before. The rest of the model setup is identical to section 3.1. We only need to add the expressions for the unit prices and the equilibrium quantities, which are given by $\tilde{p}_r(\varphi, \sigma) = \sigma p_r(\varphi)$ and $\tilde{y}_r(\varphi, \sigma) = \frac{y_r(\varphi)}{\sigma}$, for $r \in \{D, X\}$. Parallel expressions hold for a Foreign firm with capability φ and quality σ.

Following Johnson (2009), I assume that the relationship between quality and capability is given by $\sigma = \varphi^\alpha$, with $\alpha > 0$, so that higher capability is related to higher quality ($\alpha = 0$ corresponds to the homogeneous quality case).[44] Then, the model simplifies to a single dimension of heterogeneity in capability. Rewriting the unit price equations for a Home firm with capability φ we get

$$\tilde{p}_D(\varphi) = (1 + \mu_D(\varphi))\frac{W}{Z}\varphi^{\alpha-1} \quad \text{and} \quad \tilde{p}_X(\varphi) = (1 + \mu_X(\varphi))\frac{\tau W}{\mathcal{E}Z}\varphi^{\alpha-1}.$$

Therefore, the unit price ($\tilde{p}_r(\varphi)$) and the quality-adjusted price ($p_r(\varphi)$) are negatively correlated if $\alpha > 1 - \Upsilon_r(\varphi)$, where $\Upsilon_r(\varphi) = \frac{\partial \ln(1+\mu_r(\varphi))}{\partial \ln u_r(\varphi)} \in \left(0, \frac{1}{2}\right]$ and $u_r(\varphi) = \frac{\varphi}{\varphi_r}$.[45] That is, although the quality-adjusted price always decreases with capability, the unit price increases with capability if α is sufficiently high.[46]

[44] Among others, Baldwin and Harrigan (2009) and Kugler and Verhoogen (2008) assume a similar relationship between quality and capability. As mentioned by Baldwin and Harrigan (2009) and Johnson (2009), we can also derive this power function relationship in a model with endogenous quality.

[45] In the CES case $\Upsilon_r(\varphi) = 0$ because markups are exogenous. Therefore, with CES preferences a negative correlation exists between the unit price and the quality-adjusted price if and only if $\alpha > 1$.

[46] Recent empirical evidence provides support for a positive relationship between unit prices and capability (implying a sufficiently high α). Using plant-level data from Colombia, Kugler and Verhoogen (2008) find that output prices are positively correlated with plant size and export status (which are indicators of capability). In the lines of Hummels and Klenow (2005), they infer that this is consistent with more capable plants producing higher-quality goods and charging higher unit prices. Johnson (2009) provides similar evidence using aggregate

Assuming that capability follows a Pareto distribution with dispersion parameter k, the rest of the results in section 3 hold. We only need to derive a couple of results regarding average unit prices. Let $\bar{\tilde{p}}_r$ and $\bar{\tilde{p}}_r^*$ denote, respectively, the average unit prices of Home and Foreign goods in market r, for $r \in \{D, X\}$. The following proposition presents the new results:

Proposition 7 ***(Non-equivalence of average unit prices)***

The average unit prices of domestic and imported goods are not equal. In particular: $\bar{\tilde{p}}_D = \vartheta(k,\alpha)\frac{W}{Z}\varphi_D^{\alpha-1}$, $\bar{\tilde{p}}_X^* = \left(\frac{\varphi_X^*}{\varphi_D}\right)^\alpha \bar{\tilde{p}}_D$, $\bar{\tilde{p}}_D^* = \vartheta(k,\alpha)\frac{W^*}{Z^*}\varphi_D^{*\alpha-1}$, *and* $\bar{\tilde{p}}_X = \left(\frac{\varphi_X}{\varphi_D^*}\right)^\alpha \bar{\tilde{p}}_D^*$, *where* $\vartheta(k,\alpha)$ *is a constant.*[47]

Therefore, although Proposition 1 holds for quality-adjusted average prices, average unit prices for domestic and imported goods at Home and Foreign are different as long as $\varphi_D \neq \varphi_X^*$ and $\varphi_D^* \neq \varphi_X$. As example, with identical costs of effective labor so that $\varphi_X^* > \varphi_D$ due only to the trade cost, $\bar{\tilde{p}}_X^* > \bar{\tilde{p}}_D$ because imported goods at Home are on average of higher quality than domestic goods (as Foreign exporters are on average more capable—hence producing higher quality goods—than Home firms selling domestically). It follows that the average unit price at Home is given by $\bar{\tilde{p}} = \frac{N_D}{N}\bar{\tilde{p}}_D + \frac{N_X^*}{N}\bar{\tilde{p}}_X^*$, with an analogous expression holding for $\bar{\tilde{p}}^*$.

6.2 Quality and Exchange Rates

In this section I present the quality-model implications for exchange rate pass-through to firm- and aggregate-level unit prices and regular quantities. I also contrast these results with the results in Propositions 3, 4, 5, and 6, which should be read for this section as corresponding to quality-adjusted prices and quantities (also, the word "productivity" in each of these propositions must be replaced by "capability").

The Home-currency unit price of a Foreign good from a firm with capability φ, with $\varphi \geq \varphi_X^*$, is given by

$$\tilde{p}_X^*(\varphi) = (1 + \mu_X^*(\varphi))\frac{\tau^*\mathcal{E}W^*}{Z^*}\varphi^{\alpha-1}.$$

Therefore, the rate of pass-through of an exchange rate movement to $\tilde{p}_X^*(\varphi)$, $\tilde{\lambda}_X^*(\varphi)$, is identical to $\lambda_X^*(\varphi)$ in equation (26). The equilibrium regular quantity is

$$\tilde{y}_X^*(\varphi) = \left(\frac{\mu_X^*(\varphi)}{1 + \mu_X^*(\varphi)}\right)\frac{\gamma I}{\frac{\tau^*\mathcal{E}W^*}{Z^*}\varphi^{\alpha-1}},$$

so that it is also the case that $\frac{\partial \ln \tilde{y}_X^*(\varphi)}{\partial \ln \mathcal{E}} = \frac{\partial \ln y_X^*(\varphi)}{\partial \ln \mathcal{E}}$, where $y_X^*(\varphi)$ is as given in section 4.2. Moreover, note that $\tilde{p}_X^*(\varphi)\tilde{y}_X^*(\varphi) = p_X^*(\varphi)y_X^*(\varphi) = s_X^*(\varphi)I$, where $s_X^*(\varphi) = \gamma\mu_X^*(\varphi)$. Therefore, as Propositions 3 and 4 hold for the quality-adjusted price, $p_X^*(\varphi)$, and the quality-adjusted quantity, $y_X^*(\varphi)$, we can write the following proposition:

cross-country data.

[47] $\vartheta(k,\alpha) = k\int_1^\infty \frac{\boldsymbol{\Omega}(xe)}{x^{k-\alpha+2}}dx$.

Proposition 8 ***(Exchange rate pass-through to firm-level unit import prices, quantities, and trade flows)***

For a Foreign firm with capability $\varphi \geq \varphi_X^*$ we have that, after replacing the word "productivity" by "capability", Propositions 3 and 4 hold for the Home-currency unit price, $\tilde{p}_X^*(\varphi)$, and export quantity, $\tilde{y}_X^*(\varphi)$.

Thus, our original firm-level results are unaffected by the extension with quality: (1) the rates of pass-through of exchange rate movements to firm-level quality- and non-quality-adjusted import prices are identical, incomplete, increasing in capability, and asymmetric; and (2) firm-level quality- and non-quality-adjusted traded quantities are elastic with respect to the exchange rate and move in the expected direction.

Given that $\tilde{\lambda}_X^*(\varphi) = \lambda_X^*(\varphi)$, it follows that the average pass-through rate to firm-level unit import prices at Home, $\bar{\tilde{\lambda}}_X^*$, is identical to $\bar{\lambda}_X^*$ in equation (28) (the average pass-through rate to firm-level quality-adjusted import prices). Proposition 5 holds for the aggregate quality-adjusted import price at Home, so that the disconnect between $\lambda_X^*(\varphi)$ and Λ_X^* continues to exist. In the same way, Proposition 6 holds for aggregate quality-adjusted quantities, and the results for the values of trade flows are unaffected. Therefore, we only need to establish the quality-model implications for the effect of exchange rates on aggregate unit import prices and traded quantities.

Proposition 9 ***(Exchange rate pass-through to aggregate unit import prices and quantities)***

1. *The rate of pass-through of an exchange rate movement to the aggregate unit import price at Home is $\tilde{\Lambda}_X^* = \alpha + (\alpha - 1)\zeta_{\varphi_D, \mathcal{E}}$.*

2. *The aggregate (non-quality-adjusted) quantity of Home exports is increasing in the exchange rate, and the aggregate quantity of Foreign exports is decreasing in the exchange rate. The exchange rate elasticities of these quantities are in absolute value greater than $k + \alpha$.*

Note that if $\alpha < 1$, $\tilde{\Lambda}_X^*$ is positive as long as $\frac{\alpha}{1-\alpha} > \zeta_{\varphi_D, \mathcal{E}}$. Moreover, there is full pass-through if $\alpha = 1$ and more than full pass-through ($\tilde{\Lambda}_X^* > 1$) if $\alpha > 1$. Thus, the model with quality derives a positive pass-through rate to the aggregate unit import price at Home if α is sufficiently high. Note also that this aggregate pass-through rate is disconnected from the average pass-through to firm-level unit import prices, $\bar{\tilde{\lambda}}_X^*$.

The model with quality can be further extended to a NOEM framework with similar results. In the quality-extended NOEM model, P and C are quality-adjusted price and consumption indexes. Thus, although the response of quality-adjusted consumption can be large—as obtained in section 5.3 at the time of the monetary shock—the response of the non-quality-adjusted consumption index can be substantially smaller and the unit price index can increase or decrease.

7 Conclusion

Exchange rate pass-through studies are probably the most popular empirical topic in international macroeconomics. Often, results on pass-through rates are used to make inferences about the expenditure-switching effect of exchange rates. However, the link between pass-through and trade flows is only valid if the impact of exchange rates on the supply side is negligible. In this paper the supply side takes a central role.

I presented partial and general equilibrium versions of a sticky-wage model of exchange rate pass-through with monopolistic competition, heterogeneous firms, and endogenous markups. Exchange rate movements affect international competitive conditions and alter firms' pricing and production decisions. The model derives results consistent with low—and even negative—levels of exchange rate pass-through to firm- and aggregate-level import prices and substantial expenditure-switching effects.

At the aggregate level, the model shows that aggregate import prices are subject to a composition bias due to changes in the extensive margin of trade. At the firm level, each producer sets its markup in each market taking into account its own productivity and the market's competitive conditions. After an exchange rate shock, each firm is subject to two different but reinforcing effects: a firm-specific effect, related to a firm's own productivity; and an economy-wide effect, which is the same for all firms competing in the same market. Moreover, the magnitude of these effects depends on the direction of the exchange rate change, generating asymmetric responses of firm-level prices for appreciations and depreciations of a currency. Even with the adjustments in markups after an exchange rate shock, there are important changes in the intensive margin of trade.

The general equilibrium model preserves the basic results of the partial equilibrium version and provides rich transition dynamics. We observe how firm reallocations at an international level increase the persistence of exogenous shocks, as changes in the pool of producers dissipate slowly during the transition.

The results of this model provide a potential explanation for the disconnect between import prices and exchange rates in developed economies. The model also highlights the relevance of exchange rates for international firm reallocations and the effects of those reallocations on a country's trade balance and net foreign asset position.

A Appendix

A.1 Proofs of Lemmas and Propositions

Proof of Lemma 1. Let $h(\varphi, \varphi_r)$ be a homogeneous of degree n function so that $h(b\varphi, b\varphi_r) = b^n h(\varphi, \varphi_r)$. Let $b = \frac{1}{\varphi_r}$. Then

$$h\left(\frac{\varphi}{\varphi_r}, 1\right) = \frac{1}{\varphi_r^n} h(\varphi, \varphi_r). \tag{A-1}$$

Letting $x = \frac{\varphi}{\varphi_r}$, we can rewrite equation (A-1) as

$$h(\varphi, \varphi_r) = \varphi_r^n h(x, 1). \tag{A-2}$$

We now do a change of variables in the integral $\int_{\varphi_r}^{\infty} h(\varphi, \varphi_r) g(\varphi \mid \varphi \geq \varphi_r) d\varphi$ in terms of x and substitute equation (A-2) to get

$$\int_{\varphi_r}^{\infty} h(\varphi, \varphi_r) g(\varphi \mid \varphi \geq \varphi_r) d\varphi = \int_{\varphi_r}^{\infty} h(\varphi, \varphi_r) \frac{k\varphi_r^k}{\varphi^{k+1}} d\varphi = \int_{1}^{\infty} \varphi_r^n h(x, 1) \frac{k}{x^{k+1}} dx = \bar{h}\varphi_r^n,$$

where

$$\bar{h} = \int_{1}^{\infty} h(x, 1) \frac{k}{x^{k+1}} dx$$

is a constant. ■

Proof of Proposition 1. For part 1, we obtain first $\bar{p}_D$. From section 3.1.3 we know that $p_D(\varphi) = (1 + \mu_D(\varphi)) \frac{W}{Z\varphi}$. Substituting $\mu_D(\varphi)$ for the corresponding expression from equation (10) we rewrite $p_D(\varphi)$ as

$$p_D(\varphi) = \mathbf{\Omega}\left(\frac{\varphi}{\varphi_D} e\right) \frac{W}{Z\varphi}, \tag{A-3}$$

which is homogeneous of degree -1 in (φ, φ_D). Then, using Lemma 1 we obtain that the average price of Home goods available for purchase at Home is

$$\bar{p}_D = \vartheta(k) \frac{W}{Z\varphi_D},$$

where $\vartheta(k) = k \int_1^{\infty} \frac{\mathbf{\Omega}(xe)}{x^{k+2}} dx$ is a constant less than 1, depending only on the parameter k. In the same way, the Home average price of imported goods is $\bar{p}_X^* = \vartheta(k) \frac{\tau^* \mathcal{E} W^*}{Z^* \varphi_X^*}$. From equation (8), derived from the zero-cutoff-markup conditions, we know that $\frac{W}{Z\varphi_D} = \frac{\tau^* \mathcal{E} W^*}{Z^* \varphi_X^*}$. Therefore, $\bar{p}_X^* = \bar{p}_D$. Given that the average price of all goods available for purchase at Home, $\bar{p}$, is just a weighted linear combination of $\bar{p}_D$ and $\bar{p}_X^*$, it must be the case that $\bar{p} = \bar{p}_D = \bar{p}_X^*$. Following the same steps, we obtain $\bar{p}^* = \bar{p}_D^* = \bar{p}_X = \vartheta(k) \frac{W^*}{Z^* \varphi_D^*}$.

For part 2, we obtain first $\overline{\ln p_D}$. From equation (6) we know that $\ln p_D(\varphi) = \ln \hat{p} - \mu_D(\varphi)$. Besides, from the zero-cutoff-markup condition for φ_D in section 3.1.4 we have $\hat{p} = \frac{W}{Z\varphi_D}$. Therefore

$$\ln p_D(\varphi) = \ln\left(\frac{W}{Z\varphi_D}\right) - \mu_D(\varphi). \tag{A-4}$$

Note that we can also get to this expression by taking the natural logarithm in equation (A-3) and using the property $\ln[\mathbf{\Omega}(x)] = \ln x - \mathbf{\Omega}(x)$ for $x > 0$ of the Lambert $\mathcal{W}$ function along with equation (10). Then, for $\overline{\ln p}_D = \int_{\varphi_D}^{\infty} \ln p_D(\varphi) g(\varphi \mid \varphi \geq \varphi_D)) d\varphi$ we have

$$\begin{aligned}\overline{\ln p}_D &= \ln\left(\frac{W}{Z\varphi_D}\right) \underbrace{\int_{\varphi_D}^{\infty} g(\varphi \mid \varphi \geq \varphi_D) d\varphi}_{1} - \bar{\mu}_D \\ &= \ln\left(\frac{W}{Z\varphi_D}\right) - \bar{\mu}(k), \text{ from equation (13).}\end{aligned}$$

In the same way, we have $\ln p_X^*(\varphi) = \ln \hat{p} - \mu_X^*(\varphi)$. From the zero-cutoff-markup conditions we get $\hat{p} = \frac{W}{Z\varphi_D} = \frac{\tau^* \mathcal{E} W^*}{Z^* \varphi_X^*}$, and given the average markup result in equation (13), we obtain $\overline{\ln p}_X^* = \ln\left(\frac{W}{Z\varphi_D}\right) - \bar{\mu}(k) = \overline{\ln p}_D$. Therefore, the average log-price at Home is $\overline{\ln p} = \overline{\ln p}_D = \overline{\ln p}_X^*$. Analogously, it is always the case that $\overline{\ln p}^* = \overline{\ln p}_D^* = \overline{\ln p}_X = \ln\left(\frac{W^*}{Z^* \varphi_D^*}\right) - \bar{\mu}(k)$. ■

Proof of Corollary 1. From the previous proof we know that $\overline{\ln p} = \ln\left(\frac{W}{Z\varphi_D}\right) - \bar{\mu}(k) = \ln \hat{p} - \bar{\mu}(k)$. Rearranging we have $\ln(\hat{p}) - \overline{\ln p} = \bar{\mu}(k)$. Now, from equation (2) we solve for N to get $N = \frac{1}{\gamma(\ln(\hat{p}) - \overline{\ln p})}$. Therefore, $N = \frac{1}{\gamma \bar{\mu}(k)}$. Following the same steps, we obtain $N^* = \frac{1}{\gamma \bar{\mu}(k)}$. ■

Proof of Corollary 2. Given that $h(\varphi, \varphi_r)$ is zero if $\varphi \in [\varphi_{\min}, \varphi_r]$, then

$$\int_{\varphi_{\min}}^{\infty} h(\varphi, \varphi_r) g(\varphi) d\varphi = \int_{\varphi_r}^{\infty} h(\varphi, \varphi_r) g(\varphi) d\varphi.$$

From Lemma 1, note that we can write

$$\int_{\varphi_r}^{\infty} h(\varphi, \varphi_r) g(\varphi \mid \varphi \geq \varphi_r) d\varphi = \frac{1}{1 - G(\varphi_r)} \int_{\varphi_r}^{\infty} h(\varphi, \varphi_r) g(\varphi) d\varphi = \bar{h} \varphi_r^n. \tag{A-5}$$

The result then follows by substituting $1 - G(\varphi_r) = \left(\frac{\varphi_{\min}}{\varphi_r}\right)^k$ into equation (A-5). ■

Proof of Proposition 2. For part 1, taking the natural logarithm of equation (20) we obtain

$$\ln \varphi_D = \ln \Psi_D - \ln \tau^* + \frac{1}{k} \ln\left[(\tau\tau^*)^k - 1\right] - \frac{1}{k} \ln\left[\tau^k - (\rho\mathcal{E})^{k+1}\right].$$

Then, $\zeta_{\varphi_D, \mathcal{E}} = \frac{\partial \ln \varphi_D}{\partial \ln \mathcal{E}} = \frac{k+1}{k}\left[\frac{(\rho\mathcal{E})^{k+1}}{\tau^k - (\rho\mathcal{E})^{k+1}}\right]$. Given that $\rho > 0$ and $\tau^k - (\rho\mathcal{E})^{k+1} > 0$, it is the case that $\zeta_{\varphi_D, \mathcal{E}} > 0$.

For part 2, note that by definition we can write $\zeta_{\varphi_D, \mathcal{E}}$ as $\frac{\partial \varphi_D}{\partial \mathcal{E}} \frac{\mathcal{E}}{\varphi_D}$. Then $\frac{\partial \varphi_D}{\partial \mathcal{E}} = \zeta_{\varphi_D, \mathcal{E}} \frac{\varphi_D}{\mathcal{E}}$. Deriving this expression with respect to $\mathcal{E}$, we get

$$\frac{\partial^2 \varphi_D}{\partial \mathcal{E}^2} = \frac{k \varphi_D \zeta_{\varphi_D, \mathcal{E}}}{\mathcal{E}^2}\left[1 + \left(\frac{k+1}{k}\right) \zeta_{\varphi_D, \mathcal{E}}\right],$$

which is always greater than zero. ■

Proof of Proposition 3. In the main text I showed that $\lambda_X^*(\varphi)$ is less than one. For part 1, we need to prove that $\frac{\partial \lambda_X^*(\varphi)}{\partial \varphi} > 0$. Note that

$$\frac{\partial \lambda_X^*(\varphi)}{\partial \varphi} = -\zeta_{\varphi_X^*,\mathcal{E}} \frac{\partial \Upsilon_X^*(\varphi)}{\partial \varphi},$$

where $\zeta_{\varphi_X^*,\mathcal{E}} > 1$. From equation (27) we obtain that

$$\frac{\partial \Upsilon_X^*(\varphi)}{\partial \varphi} = -\frac{1}{(2+\mu_X^*(\varphi))^2} \frac{\partial \mu_X^*(\varphi)}{\partial \varphi}.$$

Given that $\mu_X^*(\varphi) = \mathbf{\Omega}\left(\frac{\varphi}{\varphi_X^*} e\right) - 1$ and that $\mathbf{\Omega}'(\cdot) > 0$, we can verify that $\frac{\partial \mu_X^*(\varphi)}{\partial \varphi} > 0$. It follows that $\frac{\partial \Upsilon_X^*(\varphi)}{\partial \varphi} < 0$, so that $\frac{\partial \lambda_X^*(\varphi)}{\partial \varphi} > 0$.

For part 2, we need to prove that $\frac{\partial \lambda_X^*(\varphi)}{\partial \mathcal{E}} < 0$. Note that

$$\frac{\partial \lambda_X^*(\varphi)}{\partial \mathcal{E}} = -\Upsilon_X^*(\varphi) \frac{\partial \zeta_{\varphi_X^*,\mathcal{E}}}{\partial \mathcal{E}} - \zeta_{\varphi_X^*,\mathcal{E}} \frac{\partial \Upsilon_X^*(\varphi)}{\partial \mathcal{E}},$$

which can be written using equation (27) as

$$\frac{\partial \lambda_X^*(\varphi)}{\partial \mathcal{E}} = -\frac{1}{2+\mu_X^*(\varphi)} \frac{\partial \zeta_{\varphi_X^*,\mathcal{E}}}{\partial \mathcal{E}} + \frac{\zeta_{\varphi_X^*,\mathcal{E}}}{(2+\mu_X^*(\varphi))^2} \frac{\partial \mu_X^*(\varphi)}{\partial \mathcal{E}}, \tag{A-6}$$

where $\mu_X^*(\varphi)$ is a non-negative number. Note from equation (24) that $\frac{\partial \zeta_{\varphi_X^*,\mathcal{E}}}{\partial \mathcal{E}} = \frac{\partial \zeta_{\varphi_D,\mathcal{E}}}{\partial \mathcal{E}}$. Deriving the expression for $\zeta_{\varphi_D,\mathcal{E}}$ from Proposition 2 with respect to $\mathcal{E}$ we get

$$\frac{\partial \zeta_{\varphi_D,\mathcal{E}}}{\partial \mathcal{E}} = \frac{\zeta_{\varphi_D,\mathcal{E}}}{\mathcal{E}} \left[k\left(\zeta_{\varphi_D,\mathcal{E}} + 1\right) + 1\right] > 0.$$

For $\frac{\partial \mu_X^*(\varphi)}{\partial \mathcal{E}}$, we use the chain rule to write it as $\frac{\partial \mu_X^*(\varphi)}{\partial \varphi_X^*} \frac{\partial \varphi_X^*}{\partial \mathcal{E}}$. Again, given that $\mathbf{\Omega}'(\cdot) > 0$, we can verify that $\frac{\partial \mu_X^*(\varphi)}{\partial \varphi_X^*} < 0$. Moreover, given that φ_X^* is increasing in $\mathcal{E}$, we get that $\frac{\partial \mu_X^*(\varphi)}{\partial \mathcal{E}} < 0$. Therefore, both terms in the right side of equation (A-6) are negative, so that $\frac{\partial \lambda_X^*(\varphi)}{\partial \mathcal{E}} < 0$. ■

Proof of Proposition 4. For part 1, it is enough to prove that $\frac{\partial \ln y_X^*(\varphi)}{\partial \ln \mathcal{E}} < -1$. Note that

$$\frac{\partial \ln y_X^*(\varphi)}{\partial \ln \mathcal{E}} = \frac{\partial \ln \mu_X^*(\varphi)}{\partial \ln \mathcal{E}} - \frac{\partial \ln(1+\mu_X^*(\varphi))}{\partial \ln \mathcal{E}} - 1. \tag{A-7}$$

As in equation (26), we have

$$\frac{\partial \ln(1+\mu_X^*(\varphi))}{\partial \ln \mathcal{E}} = \frac{\partial \ln(1+\mu_X^*(\varphi))}{\partial \ln u_X^*(\varphi)} \frac{\partial \ln u_X^*(\varphi)}{\partial \ln \mathcal{E}} = -\Upsilon_X^*(\varphi)\zeta_{\varphi_X^*,\mathcal{E}}. \tag{A-8}$$

Given that $\frac{\partial \ln(1+\mu_X^*(\varphi))}{\partial \ln \mathcal{E}} = \frac{\mu_X^*(\varphi)}{1+\mu_X^*(\varphi)} \frac{\partial \ln \mu_X^*(\varphi)}{\partial \ln \mathcal{E}}$, we can solve for $\frac{\partial \ln \mu_X^*(\varphi)}{\partial \ln \mathcal{E}}$ as

$$\frac{\partial \ln \mu_X^*(\varphi)}{\partial \ln \mathcal{E}} = -\left[1 + \frac{1}{\mu_X^*(\varphi)}\right] \Upsilon_X^*(\varphi)\zeta_{\varphi_X^*,\mathcal{E}}. \tag{A-9}$$

Substituting equation (A-8) and (A-9) into equation (A-7), we obtain

$$\frac{\partial \ln y_X^*(\varphi)}{\partial \ln \mathcal{E}} = -\frac{\Upsilon_X^*(\varphi)\zeta_{\varphi_X^*,\mathcal{E}}}{\mu_X^*(\varphi)} - 1 < -1,$$

as $\mu_X^*(\varphi) \geq 0$, $\Upsilon_X^*(\varphi) \in \left(0, \frac{1}{2}\right]$, and $\zeta_{\varphi_X^*,\mathcal{E}} > 1$. Note that $y_X^*(\varphi)$ is more elastic the closer the Foreign firm is to the cutoff level, φ_X^*, and it approaches -1 as $\varphi \to \infty$.

For part 2, note that

$$\frac{\partial \ln \left[p_X^*(\varphi)y_X^*(\varphi)\right]}{\ln \mathcal{E}} = \frac{\partial \ln s_X^*(\varphi)}{\ln \mathcal{E}} = \frac{\partial \ln \mu_X^*(\varphi)}{\ln \mathcal{E}},$$

which is given in equation (A-9) and is strictly less than zero. As $\varphi \to \infty$, we have that $\mu_X^*(\varphi) \to \infty$, and $\Upsilon_X^*(\varphi) = \frac{1}{2+\mu_X^*(\varphi)} \to 0$, so that $\frac{\partial \ln \mu_X^*(\varphi)}{\ln \mathcal{E}} \to 0$. ■

Proof of Proposition 5. From the first result in Proposition 1 we can write that $\ln \bar{p}_X^* = \ln \vartheta(k) + \ln W - \ln Z - \ln \varphi_D$. Therefore, the elasticity of $\bar{p}_X^*$ with respect to $\mathcal{E}$ is just $\Lambda_X^* = \frac{\partial \ln \bar{p}_X^*}{\partial \ln \mathcal{E}} = -\frac{\partial \ln \varphi_D}{\partial \ln \mathcal{E}} = -\zeta_{\varphi_D,\mathcal{E}}$. Given that $\zeta_{\varphi_D,\mathcal{E}} > 0$, it is always true that $\Lambda_X^* < 0$. ■

Proof of Proposition 6. For the value of exports, we need to prove that $\frac{\partial \ln VE}{\partial \ln \mathcal{E}} > k$ and $\frac{\partial \ln VE^*}{\partial \ln \mathcal{E}} < -k$. Given that $VE = \frac{N_X}{N^*}I^*$ and that N^* and I^* are constants, we get $\frac{\partial \ln VE}{\partial \ln \mathcal{E}} = \frac{\partial \ln N_X}{\partial \ln \mathcal{E}}$. From equation (14) we have $\ln N_X = k \ln \varphi_{\min} - k \ln \varphi_X + \ln N_P$. Therefore,

$$\frac{\partial \ln VE}{\partial \ln \mathcal{E}} = -k\zeta_{\varphi_X,\mathcal{E}} + \frac{\partial \ln N_P}{\partial \ln \mathcal{E}}. \tag{A-10}$$

From equation (25) we know that $\zeta_{\varphi_X,\mathcal{E}} < -1$. Thus, it is enough to prove that $\frac{\partial \ln N_P}{\partial \ln \mathcal{E}} \geq 0$. Using equation (16), we get

$$\frac{\partial \ln N_P}{\partial \ln \mathcal{E}} = k\zeta_{\varphi_D,\mathcal{E}} + \frac{k\varphi_X^k}{(\tau\tau^*)^k\varphi_D^k - \varphi_X^k}\left(\zeta_{\varphi_D,\mathcal{E}} - \zeta_{\varphi_X,\mathcal{E}}\right).$$

From Proposition 1 we know that $\zeta_{\varphi_D,\mathcal{E}} > 0$. Then, we also obtain $\zeta_{\varphi_D,\mathcal{E}} - \zeta_{\varphi_X,\mathcal{E}} > 1$. Moreover, N_P must be non-negative so that $(\tau\tau^*)^k\varphi_D^k - \varphi_X^k \geq 0$. Therefore, $\frac{\partial \ln N_P}{\partial \ln \mathcal{E}} > 0$ so that $\frac{\partial \ln VE}{\partial \ln \mathcal{E}} > k$. Following the same steps, we obtain $\frac{\partial \ln VE^*}{\partial \ln \mathcal{E}} = \frac{\partial \ln N_X^*}{\partial \ln \mathcal{E}} < -k$.

For the quantity of exports, given that the value of exports (in terms of the importer's currency) is the product of the aggregate import price and and the aggregate traded quantity, we calculate the elasticity of the traded quantity as the difference in exchange rate elasticities between the value of exports and the aggregate import price. Let Y_X and Y_X^* represent the aggregate quantity of exports of Home and Foreign, respectively. Using the result in Proposition (5), we then have $\frac{\partial \ln Y_X}{\partial \ln \mathcal{E}} = \frac{\partial \ln VE}{\partial \ln \mathcal{E}} - (-\zeta_{\varphi_D^*,\mathcal{E}})$ and $\frac{\partial \ln Y_X^*}{\partial \ln \mathcal{E}} = \frac{\partial \ln VE^*}{\partial \ln \mathcal{E}} - (-\zeta_{\varphi_D,\mathcal{E}})$. For the quantity of Home exports, using equations (A-10) and (25) we have

$$\frac{\partial \ln Y_X}{\partial \ln \mathcal{E}} = -k\zeta_{\varphi_X,\mathcal{E}} + \frac{\partial \ln N_P}{\partial \ln \mathcal{E}} + \zeta_{\varphi_X,\mathcal{E}} + 1 = -(k-1)\zeta_{\varphi_X,\mathcal{E}} + 1 + \frac{\partial \ln N_P}{\partial \ln \mathcal{E}}.$$

As before, given that $\zeta_{\varphi_X,\mathcal{E}} < -1$ and $\frac{\partial \ln N_P}{\partial \ln \mathcal{E}} > 0$, we get $\frac{\partial \ln Y_X}{\partial \ln \mathcal{E}} > k$. Following the same steps, we obtain $\frac{\partial \ln Y_X^*}{\partial \ln \mathcal{E}} < -k$. ■

Proof of Proposition 7. We obtain first $\bar{\tilde{p}}_D$. From section 6.1 we know that $\tilde{p}_D(\varphi) = (1 + \mu_D(\varphi))\frac{W}{Z}\varphi^{\alpha-1}$. Substituting $\mu_D(\varphi)$ by the corresponding expression from equation (10) we rewrite $\tilde{p}_D(\varphi)$ as

$$\tilde{p}_D(\varphi) = \mathbf{\Omega}\left(\frac{\varphi}{\varphi_D}e\right)\frac{W}{Z}\varphi^{\alpha-1}, \tag{A-11}$$

which is homogeneous of degree $\alpha - 1$ in (φ, φ_D). Then, using Lemma 1, we obtain that the average unit price of Home goods available for purchase at Home is

$$\bar{\tilde{p}}_D = \vartheta(k, \alpha)\frac{W}{Z}\varphi_D^{\alpha-1},$$

where $\vartheta(k, \alpha) = k\int_1^\infty \frac{\mathbf{\Omega}(xe)}{x^{k-\alpha+2}}dx$ is a constant depending on the parameters k and α. In the same way, the Home average price of imported goods is $\bar{\tilde{p}}_X^* = \vartheta(k, \alpha)\frac{\tau^*\mathcal{E}W^*}{Z^*}\varphi_X^{*\alpha-1}$. The ratio of $\bar{\tilde{p}}_X^*$ and $\bar{\tilde{p}}_D$ is then

$$\frac{\bar{\tilde{p}}_X^*}{\bar{\tilde{p}}_D} = \frac{\frac{\tau^*\mathcal{E}W^*}{Z^*}}{\frac{W}{Z}}\left(\frac{\varphi_X^*}{\varphi_D}\right)^{\alpha-1}.$$

Finally, using equation (8) we obtain $\bar{\tilde{p}}_X^* = \left(\frac{\varphi_X^*}{\varphi_D}\right)^\alpha \bar{\tilde{p}}_D$. Following the same steps, we obtain $\bar{\tilde{p}}_D^* = \vartheta(k, \alpha)\frac{W^*}{Z^*}\varphi_D^{*\alpha-1}$, $\bar{\tilde{p}}_X = \vartheta(k, \alpha)\frac{\tau W}{\mathcal{E}Z}\varphi_X^{\alpha-1}$, and $\bar{\tilde{p}}_X = \left(\frac{\varphi_X}{\varphi_D^*}\right)^\alpha \bar{\tilde{p}}_D^*$. ■

Proof of Proposition 8. Given that $\tilde{\lambda}_X^*(\varphi) = \lambda_X^*(\varphi)$, $\frac{\partial \ln \tilde{y}_X^*(\varphi)}{\partial \ln \mathcal{E}} = \frac{\partial \ln y_X^*(\varphi)}{\partial \ln \mathcal{E}}$, and $\tilde{p}_X^*(\varphi)\tilde{y}_X^*(\varphi) = p_X^*(\varphi)y_X^*(\varphi)$, the proof is identical to the proofs of Propositions 3 and 4. ■

Proof of Proposition 9. For part 1, we obtain from Proposition 7 that $\ln \bar{\tilde{p}}_X^* = \alpha(\ln \varphi_X^* - \ln \varphi_D) + \ln \bar{\tilde{p}}_D$. Therefore, the elasticity of $\bar{\tilde{p}}_X^*$ with respect to $\mathcal{E}$, the pass-through rate, is given by $\tilde{\Lambda}_X^* = \frac{\partial \ln \bar{\tilde{p}}_X^*}{\partial \ln \mathcal{E}} = \alpha\left(\zeta_{\varphi_X^*,\mathcal{E}} - \zeta_{\varphi_D,\mathcal{E}}\right) + \tilde{\Lambda}_D$, where $\tilde{\Lambda}_D = \frac{\partial \ln \bar{\tilde{p}}_D}{\partial \ln \mathcal{E}} = (\alpha - 1)\zeta_{\varphi_D,\mathcal{E}}$. Given also that $\zeta_{\varphi_X^*,\mathcal{E}} = 1 + \zeta_{\varphi_D,\mathcal{E}}$, we obtain $\tilde{\Lambda}_X^* = \alpha + (\alpha - 1)\zeta_{\varphi_D,\mathcal{E}}$.

For part 2, let $\tilde{Y}_X$ and $\tilde{Y}_X^*$ represent the aggregate (non-quality-adjusted) quantity of exports of Home and Foreign, respectively. As in the proof of Proposition 6, we then have $\frac{\partial \ln \tilde{Y}_X}{\partial \ln \mathcal{E}} = \frac{\partial \ln VE}{\partial \ln \mathcal{E}} - \tilde{\Lambda}_X(\varphi)$ and $\frac{\partial \ln Y_X^*}{\partial \ln \mathcal{E}} = \frac{\partial \ln VE^*}{\partial \ln \mathcal{E}} - \tilde{\Lambda}_X^*(\varphi)$. Following the same steps as in part 1 of this proof, we get $\tilde{\Lambda}_X(\varphi) = -\alpha + (\alpha - 1)\zeta_{\varphi_D^*,\mathcal{E}}$. Using equation (25), we obtain $\tilde{\Lambda}_X(\varphi) = (\alpha - 1)\zeta_{\varphi_X,\mathcal{E}} - 1$. Therefore, using the previous expression and equation (A-10) for the aggregate quantity of Home exports, we have

$$\frac{\partial \ln \tilde{Y}_X}{\partial \ln \mathcal{E}} = -k\zeta_{\varphi_X,\mathcal{E}} + \frac{\partial \ln N_P}{\partial \ln \mathcal{E}} - (\alpha - 1)\zeta_{\varphi_X,\mathcal{E}} + 1 = -(k + \alpha - 1)\zeta_{\varphi_X,\mathcal{E}} + 1 + \frac{\partial \ln N_P}{\partial \ln \mathcal{E}}.$$

Given that $\zeta_{\varphi_X,\mathcal{E}} < -1$ and $\frac{\partial \ln N_P}{\partial \ln \mathcal{E}} > 0$, we get $\frac{\partial \ln \tilde{Y}_X}{\partial \ln \mathcal{E}} > k + \alpha$. Following the same steps, we obtain $\frac{\partial \ln \tilde{Y}_X^*}{\partial \ln \mathcal{E}} < -(k + \alpha)$. ■

References

Akerlof, G. A. (2007): "The Missing Motivation in Macroeconomics," *American Economic Review*, 97(1), 5–36.

Atkeson, A., and A. Burstein (2008): "Pricing-to-Market, Trade Costs, and International Relative Prices," *American Economic Review*, 98(5), 1998–2031.

Auer, R., and T. Chaney (2009): "Exchange Rate Pass-Through in a Competitive Model of Pricing-to-Market," *Journal of Money, Credit and Banking*, 41(1), 151–175.

Baldwin, R., and J. Harrigan (2009): "Zeros, Quality and Space: Trade Theory and Trade Evidence," Discussion paper, Unpublished Manuscript.

Bergin, P. R., and G. Corsetti (2008): "The Extensive Margin and Monetary Policy," *Journal of Monetary Economics*, 55(7), 1222–1237.

Bergin, P. R., and R. C. Feenstra (2000): "Staggered Price Setting, Translog Preferences, and Endogenous Persistence," *Journal of Monetary Economics*, 45(3), 657–680.

——— (2009): "Pass-Through of Exchange Rates and Competition between Floaters and Fixers," *Journal of Money, Credit and Banking*, 41(1), 35–70.

Berman, N., P. Martin, and T. Mayer (2009): "How Do Different Exporters React to Exchange Rate Changes? Theory, Empirics and Aggregate Implications," CEPR Discussion Paper 7493, Centre for Economic Policy Research.

Bernard, A., and J. Wagner (1997): "Exports and Success in German Manufacturing," *Review of World Economics*, 133(1), 134–157.

Bernard, A. B., J. Eaton, J. B. Jensen, and S. Kortum (2003): "Plants and Productivity in International Trade," *American Economic Review*, 93(4), 1268–1290.

Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott (2009): "The Margins of U.S. Trade," *American Economic Review*, 99(2), 487–93.

Bernard, A. B., S. J. Redding, and P. K. Schott (forthcoming): "Multi-Product Firms and Product Switching," *American Economic Review*.

Bilbiie, F., F. Ghironi, and M. J. Melitz (2007a): "Endogenous Entry, Product Variety, and Business Cycles," NBER Working Papers 13646, National Bureau of Economic Research.

——— (2007b): "Monetary Policy and Business Cycles with Endogenous Entry and Product Variety," in *NBER Macroeconomics Annual*, vol. 22, pp. 299–353. The University of Chicago Press.

Burstein, A. T., J. C. Neves, and S. Rebelo (2003): "Distribution Costs and Real Exchange Rate Dynamics during Exchange-Rate-Based Stabilizations," *Journal of Monetary Economics*, 50, 1189–1214.

Campa, J. M., and L. S. Goldberg (2005): "Exchange Rate Pass-Through into Import Prices," *Review of Economics and Statistics*, 87(4), 679–690.

Chaney, T. (2008): "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *American Economic Review*, 98(4), 1707–1721.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Corless, R. M., G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth (1996): "On the Lambert W Function," *Advances in Computational Mathematics*, 5, 329–359.

Corsetti, G., and L. Dedola (2005): "A Macroeconomic Model of International Price Discrimination," *Journal of International Economics*, 67(1), 129–155.

Davis, S. J., R. J. Faberman, and J. Haltiwanger (2006): "The Flow Approach to Labor Markets: New Data Sources and Micro-Macro Links," *Journal of Economic Perspectives*, 20(3), 3–26.

Davis, S. J., and J. C. Haltiwanger (1992): "Gross Job Creation, Gross Job Destruction, and Employment Reallocation," *The Quarterly Journal of Economics*, 107(3), 819–63.

Devereux, M. B., and C. Engel (2003): "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility," *Review of Economic Studies*, 70(4), 765–783.

Dornbusch, R. (1987): "Exchange Rates and Prices," *American Economic Review*, 77(1), 93–106.

Eaton, J., S. Kortum, and F. Kramarz (2004): "Dissecting Trade: Firms, Industries, and Export Destinations," *American Economic Review*, 94(2), 150–154.

Engel, C. (2002): "Expenditure Switching and Exchange-Rate Policy," *NBER Macroeconomics Annual*, 17, 231–272.

Feenstra, R. C. (2003): "A Homothetic Utility Function for Monopolistic Competition Models, without Constant Price Elasticity," *Economics Letters*, 78(1), 79–86.

Froot, K. A., and P. D. Klemperer (1989): "Exchange Rate Pass-Through When Market Share Matters," *American Economic Review*, 79(4), 637–654.

Gertler, M., L. Sala, and A. Trigari (2008): "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining," *Journal of Money, Credit and Banking*, 40(8), 1713–1764.

Gertler, M., and A. Trigari (2009): "Unemployment Fluctuations with Staggered Nash Wage Bargaining," *Journal of Political Economy*, 117(1), 38–86.

Ghironi, F., and M. J. Melitz (2005): "International Trade and Macroeconomic Dynamics with Heterogeneous Firms," *Quarterly Journal of Economics*, 120(3), 865–915.

Goldberg, P. K., and M. M. Knetter (1997): "Goods Prices and Exchange Rates: What Have We Learned?," *Journal of Economic Literature*, 35(3), 1243–1272.

Hall, R. E. (2005): "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95(1), 50–65.

Hellerstein, R. (2008): "Who Bears the Cost of a Change in the Exchange Rate? Pass-Through Accounting for the Case of Beer," *Journal of International Economics*, 76(1), 14–32.

Helpman, E., M. Melitz, and Y. Rubinstein (2008): "Estimating Trade Flows: Trading Partners and Trading Volumes," *Quarterly Journal of Economics*, 123(2), 441–487.

Hummels, D., and P. J. Klenow (2005): "The Variety and Quality of a Nation's Exports," *American Economic Review*, 95(3), 704–723.

Johnson, R. C. (2009): "Trade and Prices with Heterogeneous Firms," Unpublished manuscript, Dartmouth College.

Kahn, S. (1997): "Evidence of Nominal Wage Stickiness from Microdata," *American Economic Review*, 87(5), 993–1008.

Kim, J., S. Kim, E. Schaumburg, and C. A. Sims (2008): "Calculating and Using Second-Order Accurate Solutions of Discrete Time Dynamic Equilibrium Models," *Journal of Economic Dynamics and Control*, 32(11), 3397–3414.

Kollmann, R. (2002): "Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles," *Journal of Monetary Economics*, 49(5), 989–1015.

Krugman, P. (1980): "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, 70(5), 950–959.

——— (1987): "Pricing to Market when the Exchange Rate Changes," in *Real-Financial Linkages Among Open Economies*, ed. by S. W. Arndt, and J. D. Richardson. MIT Press, Cambridge, MA.

Kugler, M., and E. Verhoogen (2008): "The Quality-Complementarity Hypothesis: Theory and Evidence from Colombia," NBER Working Papers 14418, National Bureau of Economic Research.

Lane, P. R. (2001): "The New Open Economy Macroeconomics: A Survey," *Journal of International Economics*, 54(2), 235–266.

Marston, R. C. (1990): "Pricing to Market in Japanese Manufacturing," *Journal of International Economics*, 29(3), 217–236.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71(6), 1695–1725.

Melitz, M. J., and G. I. P. Ottaviano (2008): "Market Size, Trade, and Productivity," *Review of Economic Studies*, 75(1), 295–316.

Nakamura, E., and D. Zerom (forthcoming): "Accounting for Incomplete Pass-Through," *Review of Economic Studies*.

Neumark, D., B. Wall, and J. Zhang (forthcoming): "Do Small Businesses Create More Jobs? New Evidence for the United States from the National Establishment Time Series," *Review of Economics and Statistics*.

Neumark, D., J. Zhang, and B. Wall (2006): "Where the Jobs Are: Business Dynamics and Employment Growth," *Academy of Management Perspectives*, 20(4), 79–94.

——— (2007): "Employment Dynamics and Business Relocation: New Evidence from the National Establishment Time Series," *Research in Labor Economics*, 26, 39–83.

Obstfeld, M. (2002): "Exchange Rates and Adjustment: Perspectives from the New Open Economy Macroeconomics," *Monetary and Economic Studies*, 20(S1), 23–46.

Obstfeld, M., and K. Rogoff (1996): *Foundations of International Macroeconomics*. MIT Press, Cambridge, MA.

——— (2000): “New Directions for Stochastic Open Economy Models,” *Journal of International Economics*, 50, 117–53.

Shimer, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95(1), 25–49.

Sutton, J. (2007): “Quality, Trade and the Moving Window: The Globalisation Process,” *Economic Journal*, 117(524), 469–498.

Tybout, J. R. (2003): “Plant- and Firm-Level Evidence on ‘New’ Trade Theories,” in *Handbook of International Trade*, ed. by E. K. Choi, and J. Harrigan, vol. 1, chap. 13, pp. 388–415. Blackwell Publishing Ltd.