Nonlinear Exchange Rate Predictability

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First draft: May 2007
This draft: December 2008

Abstract

We study whether the nonlinear behavior of the real exchange rate can help us account for the lack of predictability of the nominal exchange rate. We construct a smooth nonlinear error-correction model that allows us to test the hypotheses of nonlinear predictability of the nominal exchange rate and nonlinear behavior on the real exchange rate in the context of a fully specified cointegrated system. Using a panel of 19 countries and three numeraires, we find strong evidence of nonlinear predictability of the nominal exchange rate and of nonlinear mean reversion of the real exchange rate. Out-of-sample Theil's U-statistics show a higher forecast precision of the nonlinear model than the one obtained with a random walk specification. The statistical significance of the out-of-sample results is higher for short-run horizons, specially for one-quarter-ahead forecasts.

Keywords: exchange rates, predictability, nonlinearities, purchasing power parity.
JEL codes: C53, F31, F47.

*This paper is a revised version of one of the chapters in our Ph.D. dissertations at UC Berkeley. We thank Pierre-Olivier Gourinchas and Maury Obstfeld for their advice. We also thank Fabio Milani and workshop participants at UC Berkeley and UC Irvine for comments and suggestions. All remaining errors are our own.
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1 Introduction

During the last three decades a growing amount of literature has shown a poor empirical relation between economic fundamentals and the exchange rate. In their seminal work, Meese and Rogoff (1983) show that the relation between economic fundamentals and the nominal exchange rate is so weak that the short-term forecast of the future behavior of the exchange rate is usually outperformed by naive random walk specifications. As documented in the work of Cheung, Chinn, and Garcia Pascual (2005), Diebold and Nason (1990) and Meese and Rose (1991), this puzzling behavior—commonly denoted as lack of exchange rate predictability—has remained through most of the recent floating period.

The finding of Mark (1995) that an empirical significant relation between economic fundamentals and the exchange rate exists at long horizons, from 1 to 4 years, has also been brought into question. Berben and van Dijk (1998), Kilian (1999) and Berkowitz and Giorgianni (2001) show that the results of high and significant predictability in long horizons can be explained as lack of cointegration between fundamentals and the exchange rate. Furthermore, according to Faust, Rogers, and Wright (2003), Mark's original results have largely disappeared as a result of data revisions and data accumulation through time.

There are several interpretations one can give to these findings. The most common is that of a dismal performance of our economic models. A second and more recent interpretation comes from Engel and West (2005), who show that under certain conditions the standard linear models of exchange rates and fundamentals behave as (near) random walks. A third interpretation—and more related to this paper—is that linear models provide a poor approximation to the behavior of economic time series and that richer and more general statistical models are needed. Indeed, several theoretical models that take into account the existence of fixed and variable transaction costs across countries conclude that the mean reversion of the real exchange rate should be nonlinear.

Using the purchasing power parity (PPP) model, in this paper we study whether smooth nonlinearities can account for the lack of exchange rate predictability. Building on the work of Granger and Swanson (1996), we construct a generalized cointegrated system that nests the possibilities of having predictability or unpredictability of the nominal exchange rate as well as linear or nonlinear behavior on the real exchange rate.

For comparison purposes, we follow Mark and Sul (2001) and estimate the model for three different numeraires—U.S., Japan and Switzerland—using a panel data set of 19 countries in the post-Bretton Woods period. We perform t-tests of the predictability and nonlinear mean reversion hypotheses and construct Theil's U-statistics that measure the relative out-of-sample accuracy of the nonlinear forecast of the nominal exchange rate versus the random walk specification. Across numeraires, the t-tests reveal significant nonlinear behavior of the real exchange rate and nonlinear predictability of the nominal exchange rate while the U-statistics show evidence of an improved forecast precision of the nonlinear error-correction model relative to the random walk. Contrary to recent results by Engel, Mark, and West (2007) and others, our out-of-sample results are significant for many countries at short-term horizons—specially for one-quarter ahead forecasts. Although the nonlinear model continues to perform better than the random walk for longer horizons, the statistical significance of the results decreases with time.

This paper also contributes to the debate in the exchange-rate predictability literature on whether or not we should include a drift in the random walk specification for out-of-sample forecast accuracy comparisons. We provide an extensive out-of-sample comparison of the random walk with drift versus the random walk without drift. We show that across the panel of 19 countries, the driftless random walk is generally a better predictor when either the U.S. or Switzerland is used as the numeraire. However, we also show that when Japan is used as the numeraire this result is reversed, illustrating that the better forecast performance of the driftless random walk is sensitive to the selection of the numeraire. These results show that any out-of-sample forecast comparison against the random walk is likely to be highly sensitive to the mutual selection of the numeraire and the competing random walk model. We argue that this problem can be circumvented by selecting the random walk specification that is consistent with the competing economic model under the null hypothesis of an unpredictable exchange rate. This simple approach—which, up to our knowledge, has not been used before—allows us to untangle gains or losses from the arbitrary inclusion or exclusion of drifts in the random walk model from the true gains derived from the economic model specification. In consequence, whenever the tested model includes a drift, we should compare it against a random walk with drift, while in those cases where the model does not contain a drift, we should compare it with the driftless random walk. In this paper we consider a smooth nonlinear error-correction model with and without drifts. The nonlinear specification outperforms the random walk in both versions of the model.

The paper is organized as follows. In Section 2 we discuss the theoretical and empirical arguments in favor of a nonlinear specification of the real exchange rate. In Section 3 we show how to construct the smooth nonlinear error-correction model based on a nonlinear specification of the real exchange rate and the existence of a cointegrated system. We will start with a simple standard linear model and then build up step by step into our more general nonlinear model. In Section 4 we estimate the driftless smooth error-correction model and then compare its forecast accuracy against the random walk. In Section 5 we consider the possibility of including drifts in our model. We report out-of-sample forecast accuracy comparisons between the two random walk models (with and without drifts), re-estimate our smooth error-correction model with drifts and then compare its out-of-sample forecast accuracy performance against the random walk with drift. Section 6 concludes.

2 Nonlinearities of the Real Exchange Rate

The idea that the mean reversion of the real exchange rate may be nonlinear has both theoretical and empirical motivations. Theoretical support can be found in the work of Benninga and Protopapadakis (1988), Dumas (1992), Sercu, Uppal, and Van Hulle (1995), O'Connell and Wei (2002) and Ohanian and Stockman (1997). The standard argument is that the existence of fixed and/or variable transactions costs across countries generates a positive relation between the magnitude of PPP deviations and the degree of mean reversion. When the real exchange rate is close to its equilibrium level, the difference in effective prices across countries is close to zero and the real exchange can freely move in any direction according to random shocks in the economy. However, when the real exchange rate deviation from its equilibrium level gets larger, the difference in effective prices across countries gets also larger and there will be higher degree of mean reversion due to the existence of arbitrage trade among countries.

For example, let us consider the simple continuous time model with two countries and one good of O'Connell and Wei (2002). In the presence of only iceberg transaction costs, they show the existence of an equilibrium where the real exchange rate is confined between two reflecting barriers that delimit a range of no-arbitrage. Whenever the real exchange rate gets outside the reflecting barriers, there is a minimal amount of trade that rebounds the real exchange rate into the closest reflecting barrier. If there are fixed costs rather than iceberg costs, the authors find that instead of two reflecting barriers there would be two resetting barriers in equilibrium. Whenever the real exchange rate hits one of the resetting barriers, trade is conducted in an amount sufficiently large to get the difference in effective prices across countries equal to zero. Having together both iceberg and fixed costs in the model generates an equilibrium with four barriers. Whenever the real exchange rate hits the outer barriers, it is instantly reset by trade to the closest inner barrier. The bottom line of these models is that we should expect a higher degree of mean reversion in the real exchange rate when the difference in effective prices across countries is larger.

There are two branches of empirical literature studying the predictions of these models. The first branch focuses on cross-country differences in effective prices of single goods or commodities. These studies—e.g. Obstfeld and Taylor (1997) and Imbs, Mumtaz, Ravn, and Rey (2003)—use highly nonlinear statistical specifications and find strong evidence of nonlinearities. A common specification in this kind of papers is the Threshold Autoregressive (TAR) model. Letting y_t denote the difference in effective prices of a single commodity across two countries, the TAR specification is given by

$$\Delta y_t = \begin{cases} \lambda^{\text{out}}(y_{t-1} + c) + \varepsilon_t & \text{if } y_{t-1} < -c \\ \lambda^{\text{in}} y_{t-1} + \varepsilon_t & \text{if } -c \leq y_{t-1} \leq c \\ \lambda^{\text{out}}(y_{t-1} - c) + \varepsilon_t & \text{if } y_{t-1} > c, \end{cases}$$

where $c > 0$.

Note that this statistical specification closely resembles the predictions of the model of O'Connell and Wei (2002) with iceberg transaction costs. When the difference in effective prices is small (i.e. less than c), there is no mean reversion in the price differential, as represented with the a priori selection of a random walk model with $\lambda^{\text{in}} = 0$. When instead the difference in prices gets large (i.e. larger than c), we should expect a large degree of mean reversion towards the reflecting barriers according to the estimated parameter $\lambda^{\text{out}} < 0$. In general, these studies conclude that highly nonlinear models such as TAR provide a good description of the behavior of single goods price differentials and find estimates of mean reversion λ^{out} larger in absolute value than the ones obtained using standard linear models.

The second branch of the empirical literature focuses on the study of cross-country differences in effective prices of large bundles of commodities, e.g. deviations from PPP—measured with the Consumer Price Index (CPI). In these papers it is argued that aggregation in goods and time generates a smooth nonlinear mean reversion rather than a one point threshold autoregression specification (see Dumas (1994) and Taylor and Peel (2000)). The statistical model that has been used in this literature—e.g Taylor, Peel, and Sarno (2001) and Kilian and Taylor (2003)—is the Smooth Transition Autoregressive (STAR) model of Granger and Teräsvirta (1993) and Teräsvirta (1994). Here we consider the parsimonious representation proposed by Kilian and Taylor (2003) in which

$$z_t = \exp(\gamma z_{t-1}^2)\,[\rho_1 z_{t-1} + \rho_2 z_{t-2}] + u_{1t}, \tag{1}$$

where z_t denotes the logarithm of the real exchange rate measured in terms of the CPI and u_{1t} is white noise. As Kilian and Taylor, let us impose the restriction that $\rho = \rho_1 = 1 - \rho_2$. The statistical properties of this specification are very intuitive. Provided $\gamma < 0$, the degree of mean reversion of the real exchange rate is a smooth function on the level of the log real exchange rate. In order to see this, note that we can interpret equation (1) as a smooth transition between two extreme statistical models. On the one hand, whenever $z_{t-1} \to 0$ we have a statistical model that tends to $z_t = \rho z_{t-1} + (1-\rho) z_{t-2} + u_{1t}$ so that the real exchange rate behaves as a random walk with no mean reversion. On the other hand, whenever $|z_{t-1}| \to \infty$ we have a log real exchange rate that tends in the limit to white noise (given by $z_t = u_{1t}$) and complete mean reversion is present. In general, we will have a level of mean reversion between these two extreme points. As $|z_{t-1}|$ goes from zero to infinity, we will have a degree of mean reversion that smoothly increases from no mean reversion to complete mean reversion.

Taylor, Peel, and Sarno (2001) and Kilian and Taylor (2003) show that the STAR model provides a good description of the behavior of the real exchange rate. They show how the linearity hypothesis, $H_0 : \gamma = 0$, is rejected for most industrialized countries and how the model can account for several of the PPP puzzles in the literature, including the lack of power in standard unit root tests and the high persistence of small shocks in the real exchange rate around equilibrium (see Rogoff (1996)).

In this paper we study whether the Smooth Transition model can also account for the lack of predictability of the nominal exchange rate observed in standard linear models. We will do this by including equation (1) in a generalized cointegrated system following the procedure suggested by Granger and Swanson (1996). The resulting model will be a Smooth Transition Error-

Correction model where the mean reversion of the nominal exchange rate toward its economic fundamentals will be a smooth function of the effective price differential across countries.

3 The Smooth Transition Error-Correction Model

It is a common approach in the empirical literature to directly specify a cointegrated system either in terms of the error-correction model or in the common trend representation of Stock and Watson (1988). Granger and Swanson (1996) argue that we can generalize a cointegrated system if we instead specify the model in terms of the cointegrated variables and the stochastic trends using the following specification

$$z_t = \alpha' x_t \tag{2}$$

$$w_t = \lambda' x_t, \tag{3}$$

where x_t is a $n \times 1$ vector of nonstationary variables, z_t is a $r \times 1$ vector of cointegrated variables (with $r < n$) and w_t is a $(n-r) \times 1$ I(1) vector, such as

$$w_t = w_{t-1} + \eta_t. \tag{4}$$

The triangular representation of Phillips is a special case of this specification. Consider for example the case at hand with $x_t = (s_t \;\; f_t)'$, where s_t is the logarithm of the nominal exchange rate—defined as the local price of one unit of the numeraire's currency—and f_t represents the economic fundamentals. In the PPP model, economic fundamentals are defined as

$$f_t = p_t - p_t^*, \tag{5}$$

where p_t and p_t^* are the log prices at the local and the numeraire country, respectively.

Assuming that the cointegrated variable z_t—the logarithm of the real exchange rate—follows an AR(2) process, the triangular representation is given either by

$$z_t = s_t - f_t, \qquad z_t = \rho_1 z_{t-1} + \rho_2 z_{t-2} + u_{1t} \tag{6}$$

$$\Delta s_t = \eta_t \tag{7}$$

or

$$z_t = s_t - f_t, \qquad z_t = \rho_1 z_{t-1} + \rho_2 z_{t-2} + u_{1t} \tag{8}$$

$$\Delta f_t = \eta_t, \tag{9}$$

where u_{1t} and η_t are possibly correlated white noise processes. Under the assumption that w_t is given by equation (4), both representations are consistent with the specification of equations (2) and (3) since $\alpha = (1 \ \ -1)'$ and $\lambda = (1 \ \ 0)'$ or $\lambda = (0 \ \ 1)'$ generate equations (6)-(7) and (8)-(9), respectively.

A drawback of the triangular representation is that it assumes a priori whether the nominal exchange rate is predictable or not. In equation (7) we state that the nominal exchange rate is a random walk, while in equations (8) and (9) we state that the exchange rate is predictable. The exact form of the predictability implied by equations (8) and (9) is derived by computing the corresponding error-correction model. Taking first differences in both equations and solving the system in terms of Δs_t we obtain that

$$\Delta s_t = (\rho_1 - 1) z_{t-1} + \rho_2 z_{t-2} + (u_{1t} + \eta_t), \tag{10}$$

where the future movement in the nominal exchange rate depends on previous realizations of the real exchange rate.

In order to circumvent an a priori assumption about the predictability of the nominal exchange rate, we follow Granger and Swanson (1996) and instead of assuming a λ equal to either $(0 \ \ 1)'$ or $(1 \ \ 0)'$, we assume a λ equal to $(\lambda_1 \ \ \lambda_2)'$ where $\lambda_1, \lambda_2 \in \mathbb{R}$. It can be shown by taking first differences to equations (2) and (3) that when $\lambda = (\lambda_1 \ \ \lambda_2)'$ and $\alpha = (1 \ \ -1)'$, the cointegrated system has a vector error-correction representation given by

$$\Delta s_t = -r \left(1 - \rho_1 - \rho_2\right) z_{t-1} - r \rho_2 \Delta z_{t-1} + u_{2t}, \tag{11}$$

where $r = \frac{\lambda_2}{\lambda_1 + \lambda_2}$ and $u_{2t} = \frac{\lambda_2 u_{1t} + \eta_t}{\lambda_1 + \lambda_2}$. The predictability of the nominal exchange rate can now be tested using $H_0 : r = 0$ versus $H_A : r > 0$. As long as $r > 0$, we have a system in which the mean reversion of the real exchange rate towards its equilibrium relies, at least partially, in movements of the nominal exchange rate. On the other side, when $r = 0$ the mean reversion of the real exchange rate relies exclusively on future movements in the difference in prices.

It is now straightforward to generalize the linear error-correction representation into a non-linear error-correction representation. Let the cointegrated system be determined by equations (2) and (3) with $\alpha = (1 \quad -1)'$, $\lambda = (\lambda_1 \ \ \lambda_2)'$ and the real exchange rate be given by the STAR specification of equation (1). Taking first differences of equations (2) and (3) and solving the system for Δs_t, we obtain the Smooth Transition Error-Correction (STEC) model

$$\Delta s_t = -r\left[1 - (\rho_1 + \rho_2)\exp(\gamma z_{t-1}^2)\right] z_{t-1} - r\rho_2 \exp(\gamma z_{t-1}^2)\Delta z_{t-1} + u_{2t}. \tag{12}$$

This equation is a generalization of the linear error-correction equation (11) in the sense that the linear model is just the STEC model restricted to $\gamma = 0$. Using the restriction $\rho = \rho_1 = 1-\rho_2$ of the STAR specification of Kilian and Taylor (2003), the STEC model is reduced to the parsimonious specification

$$\Delta s_t = -r\left[1 - \exp(\gamma z_{t-1}^2)\right] z_{t-1} - r(1-\rho)\exp(\gamma z_{t-1}^2)\Delta z_{t-1} + u_{2t}. \tag{13}$$

This equation is the building block of our work.[1] The parameter r maintains the same role as in the linear model. An $r = 0$ implies that mean reversion in the real exchange rate relies exclusively in the difference in prices, while an $r > 0$ implies that mean reversion in the real exchange rate relies at least partially in the nominal exchange rate. Then, we can test the hypothesis of predictability of the nominal exchange rate using $H_0 : r = 0$ versus $H_A : r > 0$. The degree of nonlinearity in the mean reversion of the real exchange rate depends on γ. Given $\gamma < 0$, the mean reversion of the nominal exchange rate towards its fundamentals will be increasing in the absolute value of the difference in effective prices across countries—i.e. the error-correction parameter of equation (13) is an increasing function of $|z_{t-1}|$. Also, the larger the γ—in absolute value—, the higher the degree of nonlinearities. Hence, we can test for nonlinearities using $H_0 : \gamma = 0$ versus $H_A : \gamma < 0$.

[1] Note that even though the STEC model in equation (12) nests a linear model, our parsimonious STEC model in equation (13) does not. This result follows because an error-correction representation cannot exist when $\gamma = 0$ and $\rho = \rho_1 = 1-\rho_2$ since the real exchange rate would not be stationary and the system would not be cointegrated.

4 Estimation of the STEC Model

4.1 The Data

For purposes of comparison with a linear specification of PPP fundamentals, we estimate the STEC model using the same panel of countries and time period as Mark and Sul (2001). Thus, we obtain quarterly time series for nominal exchange rates and price levels from 1973.1 through 1997.1 for 19 countries: Australia, Austria, Belgium, Canada, Denmark, Finland France, Germany, Great Britain, Greece, Italy, Japan, Korea, the Netherlands, Norway, Spain, Sweden, Switzerland and the United States.[2] All series are from the IMF's International Financial Statistics and are measured at the end of the quarter. Prices correspond to CPI levels and are seasonally adjusted with a one side moving average of the present observation and three lagged variables. Nominal exchange rates are reported as the price of one U.S. dollar.

4.2 Econometric Specification and Estimation Results

Our econometric specification is given by

$$z_{i,t} = \exp(\gamma z_{i,t-1}^2)\left[\rho z_{i,t-1} + (1-\rho) z_{i,t-2}\right] + u_{1t}^i \tag{14}$$

$$\Delta s_{i,t} = -r\left[1 - \exp(\gamma z_{i,t-1}^2)\right] z_{i,t-1} - r(1-\rho)\exp(\gamma z_{i,t-1}^2)\Delta z_{i,t-1} + u_{2t}^i, \tag{15}$$

where $i = 1, 2, ...L$ indexes countries and $(u_{1t}^i \ \ u_{2t}^i)'$ are independent and identically distributed processes with variance-covariance matrix given by $\Sigma_{2L\times 2L}$. As in Mark and Sul (2001), we estimate the model for three numeraires: the United States, Japan and Switzerland. For each numeraire, we define $s_{i,t}$ as the log of the country i's currency price of one unit of the currency of the numeraire and $z_{i,t}$ as the demeaned log of the real exchange rate between country i and the numeraire. We use a demeaned measure of the log of the real exchange rate to account for the different factors that cause relative purchasing power parity deviations in equilibrium—e.g. Harrod-Balassa-Samuelson effects. We abstract from cointegration tests, as Mark and Sul already showed that nominal exchange rates and PPP fundamentals are cointegrated in this dataset.[3] Since the model is nonlinear, there are not closed form solutions for the estimates and

[2]Mark and Sul (2001) estimate their linear model of nominal exchange rates for both monetary and PPP fundamentals. As we focus on nonlinearities of the real exchange rate, our estimation is based exclusively on PPP fundamentals.

[3]See Table 1 in Mark and Sul (2001) and the references cited therein regarding the stationarity of the real exchange rate.

numerical methods are needed. We estimate the model by maximum likelihood.

Note that we restrict γ, ρ and r to be equal across country pairs. These restrictions in the parameter space will provide us with two significant gains. First, they will increase the precision of our estimates by increasing the size of the effective sample used for the estimation of the parameters. Second, they will allow us to substantially reduce the time needed for the nonlinear estimation of the model, making the out-of-sample bootstraps of the following sections computationally feasible.[4] These a priori restrictions in the parameters do not seem to come at a high cost. Allowing a different γ for each country pair generates similar results tho those presented in this paper.[5] Moreover, unrestricted estimation of the model reveals that the estimates of r and ρ are remarkably similar across country pairs for each of the three numeraires—the null hypotheses $H_0 : \rho_i = \rho \ \forall \ i$ and $H_0 : r_i = r \ \forall \ i$ are not rejected for all numeraires, with p-values generally well above 0.5.

Tables 1 presents the estimation results. The asymptotic distribution of $\hat{\rho}$ and $\hat{r}$ and their t-statistics are standard so that we can interpret these statistics in the usual way. This is not the case for the estimate of γ. Notice that under the null $\gamma = 0$, all the variables in the model are nonstationary and the distribution of $\hat{\gamma}$ is not standard. Therefore, the p-value we present for the estimate of γ for each numeraire corresponds to the bootstrapped p-value of the t-statistic under the null that γ equals zero.[6]

We find that across numeraires the hypotheses of linearity of the real exchange rate ($H_0 : \gamma = 0$) and unpredictability of the nominal exchange rate ($H_0 : r = 0$) are rejected even at a 1% level. All the estimates of r are very close to one, indicating that—no matter the numeraire—most of the mean reversion of the real exchange rate is conducted through future movements in the nominal exchange rate. In other words, we find that whenever the difference between the nominal exchange rate and its fundamentals is low we should expect to see a nominal rate that

[4] The estimation of the unrestricted model requires up to 3 hours, while the estimation of the restricted model requires only a few seconds.

[5] A previous version of this paper included the estimation of the STEC model allowing for a different γ for each country pair. These results are available upon request.

[6] We perform one bootstrap for each numeraire. Each bootstrap is computed using the following procedure. First, we estimate the data generating process (DGP) under the null given by equations (14) and (15) with the restriction $\gamma = 0$. Second, we construct 1,000 artificial datasets with $100+T$ observations—where T equals 97 and represents the number of quarterly observations in our sample period. We use zeros as initial values and build up the datasets using the recursive procedure of the DGP and independent draws from a multivariate normal distribution with variance-covariance matrix given by $\hat{\Sigma}_{2L\times 2L}$ (estimated with sample moments of the residuals of the DGP). Then, for each dataset we discard the first 100 observations, compute the t-statistic (using MLE) and count the number of times in which this statistic is lower than the real sample t-statistic. The bootstrapped p-value is just the result of this count divided by 1000.

Table 1: STEC Model Estimation Results

Numeraire		Coefficient	t-stat	p-value
	γ	-0.443	-7.798	0.000
US	ρ	0.820	41.240	0.000
	r	1.001	152.500	0.000
	γ	-0.299	-12.650	0.000
Japan	ρ	0.817	41.440	0.000
	r	1.006	172.900	0.000
	γ	-0.533	-7.865	0.000
Switzerland	ρ	0.809	41.010	0.000
	r	1.005	164.200	0.000

For $\hat{\gamma}$, the p-value is the proportion of the bootstrap distribution (for the t-statistic) to the left of the calculated t-statistic. For $\hat{\rho}$ and $\hat{r}$, the p-value represents the area to the right of $|t\text{-statistic}|$ in the standard normal distribution.

behaves much like a driftless random walk, while in those cases where the difference is large we should expect to see strong future adjustments on the nominal rate toward its fundamentals. On the other hand, the estimates of γ are always negative—with values of about -0.3 and lower—and imply the existence of substantial nonlinear mean reversion on the real exchange rate. Another interesting result is that the estimates of ρ and r are almost identical across numeraires. We interpret this result as corroborating evidence that the assumption that ρ and r are equal across country pairs do not seem to come at a high cost. On the other hand, the variations across numeraires for the estimates of γ can be explained in terms of the exclusion of a drift in the STEC model. We will show in subsequent sections that whenever a drift is considered in the model, not only the estimates of ρ and r are almost identical across numeraires but also the estimates of γ.

4.3 Out-of-Sample Predictability

We now perform out-of-sample tests of the hypothesis that the nominal exchange rate is unpredictable, that is, $H_0 : r = 0$ versus $H_A : r > 0$. We consider bootstrapped out-of-sample tests based on Theil's U-statistic, where the U-statistic is computed as the ratio of the Root Mean Square Error (RMSE) of the out-of-sample forecast of the nonlinear model to the RMSE of the out-of-sample forecast of the driftless random walk.[7] We also present a joint test of predictability

[7] According to Rogoff and Stavrakeva (2008), bootstrapped out-of-sample tests such as the Theil's U and the Diebold-Mariano/West tests are "more powerful and better sized" than new asymptotic out-of-sample tests such as the Clark-West and Clark-McCracken tests.

by taking the mean value—across country pairs—of the individual U-statistics.

The root mean square errors of the STEC and random walk models used to compute the U-statistics are based on out-of-sample forecasts at short (one and four quarters ahead) and long (eight and sixteen quarters ahead) horizons for the sample period 1984.1-1997.1. The procedure to compute the out-of-sample forecasts is as follows. Let k represent the number of periods—i.e. quarters—ahead for the forecast. When $k = 1$, we estimate the models with data through 1983.4 and compute the forecasts for 1984.1. Then we estimate the models with data through 1984.1, compute the forecasts for 1984.2 and so on. For $k > 1$ we estimate the models with data through 1984.1$-k$ and then compute the forecasts for 1984.1. Once the forecasts for 1984.1 are constructed, we move up one quarter and estimate the models with data through 1984.2$-k$ in order to compute the forecasts for 1984.2 and so on up to 1997.1.

Let $\hat{s}_{i,t+k}$ denote the STEC model forecast for the nominal exchange rate k periods ahead. From equation (15) we can see that the computation of the one-period-ahead STEC forecast is straightforward and given by

$$\hat{s}_{i,t+1} = s_{i,t} - \hat{r}\left[1 - \exp(\hat{\gamma} z_{i,t}^2)\right] z_{i,t} - \hat{r}(1-\hat{\rho})\exp(\hat{\gamma} z_{i,t}^2)\Delta z_{i,t}. \tag{16}$$

However, the computation of STEC forecasts at longer horizons is more problematic since it requires the knowledge of the distribution function of the error u_1^i, denoted by $\Phi(u_1^i)$. For example, consider the case of the two-period-ahead forecast $\hat{s}_{i,t+2} = s_{i,t} + \widehat{\Delta s}_{i,t+1} + \widehat{\Delta s}_{i,t+2}$, where

$$\widehat{\Delta s}_{i,t+1} = -\hat{r}\left[1 - \exp(\hat{\gamma} z_{i,t}^2)\right] z_{i,t} - \hat{r}(1-\hat{\rho})\exp(\hat{\gamma} z_{i,t}^2)\Delta z_{i,t} \tag{17}$$

$$\widehat{\Delta s}_{i,t+2} = \int \left(-\hat{r}\left[1 - \exp(\hat{\gamma} z_{i,t+1}^2)\right] z_{i,t+1} - \hat{r}(1-\hat{\rho})\exp(\hat{\gamma} z_{i,t+1}^2)\Delta z_{i,t+1}\right) d\Phi(u_1^i), \tag{18}$$

given that

$$z_{i,t+1} = \exp(\gamma z_{i,t}^2)\left[\rho z_{i,t} + (1-\rho) z_{i,t-1}\right] + u_{1t+1}^i.$$

Then, in order to get $\hat{s}_{i,t+2}$ we must first know $\Phi(u_1^i)$ so that we can compute the integral in equation (18).

There are several approaches to estimate this integral. Here we consider the parametric and nonparametric techniques presented in Granger and Teräsvirta (1993). The parametric approach estimates the integral by assuming normality in the distribution of the errors. For example, in our two-period-ahead forecast we have that

$$\widehat{\Delta s}_{i,t+2} = \frac{1}{J}\sum_{j=1}^{J}\left(-\hat{r}\left[1-\exp(\hat{\gamma}\hat{z}_{i,t+1,j}^{2})\right]\hat{z}_{i,t+1,j} - \hat{r}(1-\hat{\rho})\exp(\hat{\gamma}\hat{z}_{i,t+1,j}^{2})\Delta\hat{z}_{i,t+1,j}\right), \tag{19}$$

where

$$\hat{z}_{i,t+1,j} = \exp(\hat{\gamma}z_{i,t}^{2})\left[\hat{\rho}z_{i,t} + (1-\hat{\rho})z_{i,t-1}\right] + \hat{u}_{1t+1,j}^{i}$$

and $\hat{u}_{1,t+1,j}^{i}$ is one of J independent draws from a multivariate normal distribution with variance-covariance $\hat{\Sigma}_{2L\times 2L}$. The nonparametric approach follows a similar procedure but instead of assuming normality in the errors it resamples them from the residuals of the STEC model.

In this paper we use a J equal to 1000 and follow both the parametric and nonparametric approaches. In all cases the results are remarkably similar and our conclusions do not change. In order to preserve space we present throughout the paper only the results based on the nonparametric approach.[8]

The data generating process (DGP) implied by the STEC model under the null that the nominal exchange rate is unpredictable (i.e. under $r = 0$) is given by

$$z_{i,t} = \exp(\gamma z_{i,t-1}^{2})\left[\rho z_{i,t-1} + (1-\rho)z_{i,t-2}\right] + u_{1t}^{i} \tag{20}$$

$$\Delta s_{i,t} = u_{2t}^{i}. \tag{21}$$

If the null is true, the driftless random walk model should be more accurate than the STEC model—the random walk should have a lower RMSE than the STEC model—and therefore the U-statistics should be larger than one. On the other hand, whenever the exchange rate is predictable (i.e. $r > 0$) we should have more forecast accuracy with the STEC model so that the U-statistics should be lower that one. The p-value of each U-statistic—including the average U-statistic—is calculated by bootstrapping its distribution under the DGP in equations (20) and (21).[9]

[8]Results based on the parametric approach are available upon request.

[9]We perform one bootstrap for each numeraire. The procedure for each bootstrap is as follows. First, we estimate the DGP under the null given by equations (20) and (21) with the restriction $\gamma \leq 0$. Second, we construct 1,000 artificial datasets with 100+T observations—where T equals 97 and represents the number of quarterly observations in our sample period. We use zeros as initial values and build up the datasets using the recursive procedure of the DGP and independent draws from a multivariate normal distribution with variance-covariance matrix given by $\hat{\Sigma}_{2L\times 2L}$(estimated with sample moments of the residuals of the DGP). Then, for each dataset we discard the first 100 observations, compute the out-of-sample forecasts of the two competing models,

Tables 2 and 3 present the out-of-sample predictability results in the short and long run, respectively. The results are remarkable. Looking at the number of U-statistics smaller than one in both Tables, we see that with a single exception (the one-quarter-ahead forecast with Switzerland as numeraire) the STEC model outperforms the driftless random walk for 15 or more exchange rates—out of 18. No matter the horizon, country pair or numeraire considered, the STEC model is generally better.

As shown in Table 2, the statistical significance of the results is good in short-run horizons (one and four quarters ahead). For the one-quarter-ahead forecasts, the STEC model is significantly better than the driftless random walk when either the U.S. or Japan are used as numeraires—for 10 out of 18 exchange rates the STEC model is significantly better at a 10% level with a joint significance level of 4.4% when the U.S. is the numeraire; while for Japan as numeraire the numbers are 14 out of 18, with a joint significance level of less than 1%. For the four-period ahead forecasts, the joint test provides significance levels of 6.2% and 2.8% for the better forecast accuracy of the STEC model when the U.S. or Switzerland are the numeraires, respectively—for individual exchange rates, 6 out of 18 are significant at the 10% level when the U.S. is the numeraire, while in the case of Switzerland there are 8 out of 18. The increase in accuracy of the STEC model with respect to the random walk is not jointly statistically significant at a 10% level for the one-quarter-ahead forecasts when Switzerland is the numeraire and for the four-quarter ahead forecast when Japan is the numeraire.

As seen in Table 3, although for long-run horizons (eight and sixteen quarters ahead) the U-statistics are on average lower than in the short run—implying more forecast accuracy of the STEC model over the random walk in the long run than in the short run—, the statistical significance of the results is much lower. The joint test reveals that only the eight-quarter-ahead forecasts when the U.S. and Switzerland are numeraires are jointly statistically significant at a 10% level. This is also reflected in the small number of individual exchange rates that are statistically significant at a 10% level.

In comparison with the out-of-sample one-quarter and sixteen-quarter-ahead results of Mark and Sul (2001), the STEC model has a better performance than their linear PPP estimation—in terms of U-statistics values and their significance—for one-quarter-ahead forecasts when the

their RMSE and then the individual and average U-statistics. Finally, in order to compute each p-value we just count the number of times in which the U-statistic from the artificial samples is lower than the real sample U-statistic and then divide this number by 1000.

Table 2: Out-of-Sample Statistics of the STEC Model: Short-Run

Numeraire	United States				Japan				Switzerland			
Quarters ahead	1		4		1		4		1		4	
	U-stat	p-value	U-stat	p-value	U-stat	p-value	U-stat	p-value	U-stat	p-value	U-stat	p-value
Australia	1.108	0.961	1.057	0.811	1.017	0.705	0.975	0.343	**0.953**	0.006	**0.907**	0.047
Austria	**0.974**	0.072	0.928	0.109	**0.969**	0.036	0.967	0.277	1.016	0.807	0.985	0.399
Belgium	**0.975**	0.089	**0.925**	0.087	**0.953**	0.008	0.960	0.215	1.008	0.689	0.982	0.365
Canada	1.020	0.807	0.973	0.206	**0.976**	0.099	0.958	0.221	1.021	0.762	0.954	0.169
Denmark	**0.973**	0.070	**0.926**	0.084	**0.959**	0.013	0.961	0.227	1.014	0.800	0.988	0.419
Finland	**0.949**	0.002	**0.939**	0.096	**0.957**	0.015	0.952	0.195	0.992	0.337	**0.940**	0.100
France	0.980	0.151	0.947	0.169	**0.944**	0.002	0.958	0.210	1.005	0.674	0.975	0.267
Germany	0.981	0.149	**0.919**	0.082	**0.968**	0.037	0.965	0.253	1.017	0.818	0.974	0.257
Great Britain	**0.972**	0.075	0.937	0.127	0.986	0.266	0.970	0.327	1.008	0.651	**0.915**	0.037
Greece	1.039	0.894	1.097	0.931	0.982	0.164	1.040	0.828	1.016	0.775	1.031	0.777
Italy	**0.960**	0.023	0.935	0.112	**0.911**	0.000	0.946	0.137	**0.968**	0.042	**0.899**	0.018
Japan	**0.975**	0.092	0.958	0.226					**0.957**	0.008	**0.922**	0.040
Korea	**0.930**	0.000	**0.932**	0.046	**0.974**	0.079	0.937	0.124	**0.940**	0.001	**0.869**	0.004
Netherlands	0.979	0.128	0.927	0.111	**0.969**	0.043	0.963	0.242	1.017	0.815	0.993	0.463
Norway	0.990	0.333	0.969	0.303	**0.954**	0.008	0.979	0.362	0.986	0.193	0.964	0.209
Spain	**0.951**	0.006	**0.922**	0.077	**0.919**	0.000	0.952	0.181	**0.969**	0.042	**0.928**	0.060
Sweden	**0.964**	0.024	0.934	0.104	**0.941**	0.003	0.987	0.440	0.995	0.424	0.973	0.308
Switzerland	0.988	0.261	0.928	0.115	0.977	0.113	0.968	0.279				
United States					**0.961**	0.022	**0.901**	0.025	1.007	0.592	**0.892**	0.018
Average	**0.984**	0.044	**0.953**	0.062	**0.962**	0.001	0.963	0.111	0.994	0.204	**0.950**	0.028
Countries<1	15		16		17		17		8		17	

The p-value is the proportion of the bootstrap distribution to the left of the calculated U-statistic.
Bolded coefficients are statistically significant at a 10% level.

Table 3: Out-of-Sample Statistics of the STEC Model: Long-Run

Numeraire	United States				Japan				Switzerland			
Quarters ahead	8		16		8		16		8		16	
	U-stat	p-value	U-stat	p-value	U-stat	p-value	U-stat	p-value	U-stat	p-value	U-stat	p-value
Australia	0.942	0.302	0.956	0.534	0.946	0.331	0.857	0.239	**0.860**	0.057	0.804	0.113
Austria	0.893	0.176	0.872	0.314	0.970	0.531	0.955	0.614	0.947	0.264	0.925	0.432
Belgium	0.877	0.118	0.877	0.332	1.013	0.716	1.024	0.810	1.047	0.840	1.017	0.779
Canada	0.965	0.295	0.954	0.512	0.919	0.250	0.832	0.182	**0.875**	0.092	**0.769**	0.058
Denmark	0.899	0.185	0.931	0.470	0.969	0.497	0.971	0.677	0.977	0.474	1.047	0.866
Finland	0.904	0.152	0.895	0.373	0.914	0.203	0.863	0.293	**0.880**	0.070	**0.737**	0.036
France	0.919	0.262	0.965	0.593	0.981	0.592	0.949	0.597	0.981	0.506	0.979	0.653
Germany	**0.864**	0.082	0.828	0.218	0.993	0.660	1.013	0.791	0.973	0.453	0.948	0.519
Great Britain	0.883	0.112	0.860	0.261	0.948	0.392	0.827	0.196	0.887	0.107	**0.720**	0.022
Greece	1.117	0.927	1.115	0.922	1.070	0.875	1.016	0.768	1.045	0.803	1.017	0.734
Italy	0.910	0.215	0.977	0.635	0.955	0.382	0.923	0.455	0.902	0.144	0.859	0.216
Japan	0.949	0.385	0.911	0.383					0.911	0.166	0.988	0.620
Korea	**0.898**	0.073	0.899	0.270	0.919	0.253	0.850	0.258	**0.822**	0.029	**0.794**	0.082
Netherlands	0.877	0.112	0.850	0.261	0.986	0.619	0.978	0.692	1.031	0.813	0.987	0.697
Norway	0.938	0.325	1.018	0.754	0.970	0.472	0.910	0.454	0.934	0.230	0.863	0.209
Spain	0.894	0.159	0.938	0.495	0.958	0.417	0.899	0.377	0.890	0.108	0.843	0.166
Sweden	0.890	0.129	0.928	0.455	1.003	0.621	0.979	0.640	0.967	0.445	0.929	0.427
Switzerland	0.897	0.185	0.868	0.296	0.945	0.359	0.972	0.624				
United States					0.893	0.163	0.818	0.177	**0.829**	0.025	**0.776**	0.060
Average	**0.918**	0.096	0.924	0.410	0.964	0.361	0.924	0.443	**0.931**	0.081	0.889	0.143
Countries<1	17		16		15		15		15		15	

The p-value is the proportion of the bootstrap distribution to the left of the calculated U-statistic.
Bolded coefficients are statistically significant at a 10% level.

U.S. and Japan are the numeraires. The opposite is true when Switzerland is the numeraire. With respect to sixteen-quarter-ahead forecasts, the U-statistics of Mark and Sul are very low and highly significant when the U.S. and Switzerland are the numeraires. There is, however, an important caveat in their long-run results. As noted also by Engel, Mark, and West (2007), Mark and Sul base their out-of-sample results in a comparison against a random walk with drift, which turns out to perform worse than a driftless random walk when the U.S. and Switzerland are used as numeraires. We look further into the importance of drifts in the following Section.

5 A STEC Model with Drifts

5.1 Driftless Random Walk versus Random Walk with Drift

There has been a debate on whether or not we should include a drift in the random walk specification for out-of-sample forecast accuracy comparisons. As first shown by Meese and Rogoff (1983), the most important argument in favor of the driftless random walk specification is that it is a far more accurate out-of-sample predictor of the nominal exchange rate—in U.S. dollars—than the random walk with drift. On the other hand, Engel and Hamilton (1990) find that conventional in-sample statistics of the price of the dollar against some major currencies reject the hypothesis that the drift has been constant across pre-selected periods of time, so that it is arbitrary to set the drift equal to zero.

As mentioned above, Engel, Mark, and West (2007) revise the results of Mark and Sul (2001) for an updated sample and show—with the U.S. as numeraire—that the linear PPP model outperforms the random walk with drift for one-quarter-ahead forecasts for 15 out of 18 countries, but only for 7 out of 18 when the driftless random walk is used. Results are better for sixteen-quarter-ahead forecasts, with the PPP model outperforming the random walk with drift in 17 out of 18 countries, and for 14 out of 18 when compared against the driftless random walk.[10]

We provide an extensive out-of-sample forecast accuracy comparison of the random walk without drift versus the random walk with drift. Table 4 presents out-of-sample U-statistics across different horizons for our three numeraires. A U-statistic less than one implies a higher

[10] Important differences remain, however, in the average U-statistics for sixteen-quarter-ahead forecasts. The average is 0.702 when compared against the random walk with drift and 0.876 when compared against the driftless random walk.

forecast precision of the driftless random walk. As we can observe, the driftless random walk is generally a better predictor when either the US or Switzerland is used as numeraire—it is better than the random walk with drift for 14 or more countries (out of 18) across different horizons when the U.S. is the numeraire and for 11 or more countries when Switzerland is the numeraire. The average U-statistics for these numeraires show that the relative accuracy of the driftless random walk improves the longer the forecast horizon. However, when Japan is used as the numeraire the forecast accuracy of the two random walks is about the same for one- and four-quarter-ahead forecasts, but for longer horizons the random walk with drift clearly dominates—for the sixteen-quarter-ahead forecasts, the random walk with drift is relatively better for 16 out of 18 countries. This illustrates that the better forecast performance of the driftless random walk is sensitive to the selection of the numeraire.

Table 5 shows what would happen if we were to compare the driftless STEC model of the previous section with the random walk with drift. When either the U.S. or Switzerland is the numeraire, we would find from the average U-statistics that the STEC model's long-run forecasts (two and four years ahead) are between 18 to 25 percent more accurate than those of the random walk with drift—versus the 7 to 11 percent when compared, as seen in Table 3, against the driftless random walk.[11] This would appear to be a large predictability gain coming from economic fundamentals when what it is actually happening is that the inclusion of the drift in the random walk model reduces its long-run out-of-sample forecast precision from about 10 to 15 percent. The opposite happens when Japan is the numeraire.

Given these results, we propose that in order to untangle predictability gains or losses coming from economic fundamentals from those coming from arbitrary econometric specifications—the inclusion or not of drifts—, we should always use as a benchmark the random walk specification implied by the tested model under the null hypothesis of unpredictability of the nominal exchange rate. In other words, whenever the tested model includes a drift, we should compare it with a random walk with drift; while in those cases where the model does not contain a drift, we should compare it with the driftless random walk.

Consistent with this proposal, we now consider the possibility of including drifts in our STEC model and compare its forecast accuracy against a random walk with drift.

[11] For the U.S. and Switzerland as numeraires, if we remove Greece from the average U-statistics in Table 5, the difference in accuracy would look much larger. For example, for the sixteen-quarter-ahead forecasts the average U-statistics would be about 0.75 for the U.S. and 0.65 for Switzerland—implying differences in average accuracy ranging from 25 to 35 percent.

Table 4: Out-of-Sample U-Statistics for Comparison of Random Walks: No-Drift versus Drift

Numeraire	United States				Japan				Switzerland			
Quarters ahead	1	4	8	16	1	4	8	16	1	4	8	16
Australia	0.977	0.925	0.872	0.893	1.010	1.037	1.089	1.308	1.002	1.012	1.033	1.204
Austria	0.998	0.996	1.001	1.007	0.992	0.975	0.970	0.998	0.966	0.849	0.695	0.426
Belgium	0.983	0.947	0.908	0.833	0.984	0.956	0.948	1.070	0.916	0.645	0.453	0.440
Canada	0.978	0.926	0.906	0.842	1.015	1.043	1.094	1.268	1.006	1.021	1.064	1.120
Denmark	0.973	0.915	0.849	0.750	0.980	0.945	0.936	1.125	0.903	0.618	0.440	0.317
Finland	0.976	0.934	0.882	0.796	1.002	1.002	1.018	1.274	0.989	0.962	0.911	0.940
France	0.964	0.886	0.797	0.707	0.974	0.937	0.929	1.147	0.874	0.625	0.468	0.439
Germany	0.997	0.990	0.988	0.986	0.991	0.973	0.970	1.026	0.957	0.810	0.641	0.385
Great Britain	0.965	0.888	0.795	0.742	0.992	1.008	1.114	1.428	0.976	0.956	0.974	1.050
Greece	1.001	1.044	1.180	1.520	1.123	1.242	1.531	1.986	1.174	1.362	1.731	2.412
Italy	0.944	0.837	0.749	0.639	0.992	0.979	1.040	1.320	0.962	0.878	0.825	0.733
Japan	1.008	1.027	1.064	1.161					0.992	0.964	0.908	0.732
Korea	0.900	0.813	0.714	0.590	1.022	1.054	1.101	1.355	1.005	0.998	0.972	0.822
Netherlands	0.993	0.980	0.970	0.950	0.990	0.970	0.968	1.052	0.944	0.752	0.564	0.343
Norway	0.977	0.922	0.850	0.772	1.002	1.014	1.082	1.333	0.986	0.984	1.002	1.203
Spain	0.939	0.825	0.706	0.670	0.988	0.965	0.969	1.167	0.950	0.825	0.729	0.716
Sweden	0.974	0.914	0.835	0.773	1.004	1.002	1.051	1.394	0.993	0.963	0.933	1.107
Switzerland	1.003	1.013	1.037	1.063	0.992	0.964	0.908	0.732				
United States					1.008	1.027	1.064	1.161	1.003	1.013	1.037	1.063
Average	0.975	0.932	0.895	0.872	1.004	1.005	1.043	1.230	0.978	0.902	0.854	0.858
Countries<1	15	15	14	14	10	9	8	2	13	14	13	11

The U-statistics are computed as the ratio of the RMSE of the driftless random walk to the RMSE of the random walk with drift. A value less than one implies that the driftless random walk performs better.

Table 5: Out-of-Sample U-Statistics of the Driftless STEC Model versus the Random Walk with Drift

Numeraire	United States				Japan				Switzerland			
Quarters ahead	1	4	8	16	1	4	8	16	1	4	8	16
Australia	1.083	0.977	0.822	0.853	1.027	1.011	1.030	1.121	0.955	0.918	0.889	0.969
Austria	0.972	0.924	0.894	0.878	0.962	0.943	0.940	0.953	0.982	0.837	0.658	0.394
Belgium	0.959	0.876	0.797	0.730	0.938	0.918	0.960	1.095	0.924	0.633	0.474	0.447
Canada	0.997	0.901	0.874	0.803	0.990	0.999	1.006	1.055	1.027	0.974	0.931	0.861
Denmark	0.946	0.847	0.763	0.698	0.940	0.908	0.907	1.092	0.916	0.610	0.429	0.332
Finland	0.926	0.877	0.797	0.713	0.959	0.954	0.930	1.100	0.981	0.904	0.801	0.693
France	0.945	0.838	0.732	0.682	0.920	0.897	0.911	1.088	0.878	0.610	0.459	0.430
Germany	0.977	0.910	0.854	0.816	0.960	0.939	0.964	1.039	0.973	0.788	0.623	0.365
Great Britain	0.937	0.832	0.702	0.638	0.979	0.978	1.057	1.181	0.983	0.875	0.863	0.756
Greece	1.040	1.146	1.318	1.694	1.103	1.292	1.638	2.017	1.193	1.404	1.809	2.453
Italy	0.906	0.782	0.681	0.624	0.903	0.926	0.993	1.219	0.931	0.790	0.744	0.629
Japan	0.982	0.984	1.010	1.058					0.949	0.889	0.827	0.723
Korea	0.837	0.758	0.641	0.530	0.995	0.987	1.012	1.151	0.945	0.867	0.799	0.653
Netherlands	0.973	0.908	0.851	0.808	0.959	0.934	0.954	1.029	0.960	0.746	0.582	0.338
Norway	0.968	0.893	0.797	0.785	0.956	0.993	1.050	1.213	0.972	0.949	0.936	1.039
Spain	0.893	0.761	0.631	0.628	0.908	0.919	0.928	1.049	0.920	0.765	0.649	0.604
Sweden	0.939	0.853	0.743	0.717	0.945	0.989	1.054	1.364	0.987	0.937	0.902	1.028
Switzerland	0.991	0.940	0.930	0.923	0.969	0.933	0.858	0.711				
United States					0.969	0.925	0.950	0.949	1.010	0.904	0.859	0.824
Average	0.960	0.889	0.824	0.810	0.966	0.969	1.008	1.135	0.971	0.856	0.791	0.752
Countries<1	16	17	16	16	16	16	11	3	15	17	17	15

The U-statistics are computed as the ratio of the RMSE of the driftless STEC model to the RMSE of the random walk with drift. A value less than one implies that the driftless STEC model performs better.

5.2 Econometric Specification and Estimation Results

The introduction of drifts in the STEC model is straightforward. Let the real exchange rate be given by the STAR specification of equation (1), the cointegrated system be determined by equations (2) and (3) with $\alpha = (1 \;\; -1)'$ and $\lambda = (\lambda_1 \;\; \lambda_2)'$, and replace the stochastic trend in equation (4) by the equivalent version of a random walk with drift

$$w_t = \mu + w_{t-1} + \eta_t.$$

Taking first differences to equations (2) and (3) and solving the system in terms of Δs_t, we get that the STEC model with drift is given by

$$\Delta s_t = \delta - r\left[1 - (\rho_1 + \rho_2)\exp(\gamma z_{t-1}^2)\right] z_{t-1} - r\rho_2 \exp(\gamma z_{t-1}^2)\Delta z_{t-1} + u_{2t}, \tag{22}$$

where $\delta = \frac{\mu}{\lambda_1+\lambda_2}$, $r = \frac{\lambda_2}{\lambda_1+\lambda_2}$ and $u_{2t} = \frac{\lambda_2 u_{1t}+\eta_t}{\lambda_1+\lambda_2}$.

Allowing for a different drift for each nominal exchange rate—which corresponds to a different μ for each country pair in equation (22)—, we can write our econometric specification as

$$z_{i,t} = \exp(\gamma z_{i,t-1}^2)\left[\rho z_{i,t-1} + (1-\rho) z_{i,t-2}\right] + u_{1t}^i \tag{23}$$

$$\Delta s_{i,t} = \delta_i - r\left[1 - \exp(\gamma z_{i,t-1}^2)\right] z_{i,t-1} - r(1-\rho)\exp(\gamma z_{i,t-1}^2)\Delta z_{i,t-1} + u_{2t}^i, \tag{24}$$

where $i = 1, 2, ...L$ indexes countries and $(u_{1t}^i \;\; u_{2t}^i)'$ are independent and identically distributed processes with variance-covariance matrix given by $\Sigma_{2L\times 2L}$. As before, we estimate the model by maximum likelihood.

Table 6 presents the results. $\hat{\rho}$, $\hat{r}$, and each drift estimate $\hat{\delta}_i$ for $i = 1, 2, ..., L$ have standard distributions and their t-statistics are interpreted in the usual way. For these estimators, the reported p-values represent the area to the right of the respective $|t\text{-statistic}|$—so that the p-value for a two-sided test for each drift is given by the reported p-value times two. On the other hand, the distribution for $\hat{\gamma}$ is not standard and we obtain its p-value by bootstrapping the distribution of the t-statistic using the DGP implied by equations (23) and (24) under the null hypothesis $H_0 : \gamma = 0$.[12]

The estimates for ρ and r are very similar across numeraires and almost identical to those

[12] The bootstrapping procedure is as described in Footnote 6.

Table 6: STEC Model with Drifts Estimation Results

	US			Japan			Switzerland		
	Coefficient	t-stat	p-value	Coefficient	t-stat	p-value	Coefficient	t-stat	p-value
γ	-0.485	-8.421	0.000	-0.443	-16.930	0.000	-0.453	-6.557	0.040
ρ	0.822	41.230	0.000	0.824	41.420	0.000	0.811	41.020	0.000
r	0.999	164.400	0.000	0.999	217.400	0.000	1.006	158.600	0.000
δ_i :									
Australia	0.005	7.160	0.000	0.010	15.770	0.000	0.011	12.840	0.000
Austria	-0.004	-7.648	0.000	0.000	0.744	0.229	0.002	3.865	0.000
Belgium	-0.002	-2.787	0.003	0.003	6.209	0.000	0.004	6.672	0.000
Canada	0.001	2.406	0.008	0.005	8.551	0.000	0.006	9.343	0.000
Denmark	0.001	1.625	0.052	0.006	9.039	0.000	0.006	6.803	0.000
Finland	0.003	4.246	0.000	0.009	16.580	0.000	0.008	9.762	0.000
France	0.002	3.996	0.000	0.007	10.080	0.000	0.007	8.408	0.000
Germany	-0.005	-10.820	0.000	-0.002	-2.733	0.003	0.000	-0.423	0.336
Great Britain	0.006	7.532	0.000	0.012	17.000	0.000	0.011	10.430	0.000
Greece	0.024	25.060	0.000	0.028	25.240	0.000	0.030	34.710	0.000
Italy	0.011	13.510	0.000	0.016	19.120	0.000	0.016	13.660	0.000
Japan	-0.005	-8.829	0.000				0.000	-0.007	0.497
Korea	0.009	9.166	0.000	0.015	13.760	0.000	0.014	9.870	0.000
Netherlands	-0.004	-8.296	0.000	0.000	0.222	0.412	0.001	1.866	0.031
Norway	0.002	3.960	0.000	0.007	9.068	0.000	0.008	10.570	0.000
Spain	0.011	11.900	0.000	0.016	17.240	0.000	0.016	12.330	0.000
Sweden	0.004	7.342	0.000	0.008	10.230	0.000	0.009	13.330	0.000
Switzerland	-0.005	-8.838	0.000	-0.002	-2.357	0.009			
United States				0.004	6.238	0.000	0.005	7.868	0.000

For $\hat{\gamma}$, the p-value is the proportion of the bootstrap distribution (for the t-statistic) to the left of the calculated t-statistic.
For $\hat{\rho}$, $\hat{r}$ and each $\hat{\delta}_i$, the p-value represents the area to the right of $|t\text{-statistic}|$ in the standard normal distribution.

of the driftless STEC model in Table 1. Interestingly, the inclusion of drifts drive the estimates for γ close to each other across numeraires—around -0.45. This suggests, when compared to results in Table 1, that the degree of mean reversion of the real exchange rate is underestimated in the driftless STEC model when Japan is used as numeraire and is slightly overestimated when Switzerland is the numeraire. All of the previous coefficients are statistically significant even at 1% levels.

Regarding the estimates of the drifts, most of them—46 out of 54 (18 per numeraire)—are statistically significant at a 1% level for the two-sided test $H_0 : \delta_i = 0$ versus $H_A : \delta_i \neq 0$. Only 5 out of 54 are not significant at a 10% level. Even though the drifts look small in magnitude, we must stress that these estimates correspond to quarterly changes so that the medium- and long-run implications of a small drift are substantial. Consider for example the case of the British Pound when the U.S. is the numeraire. In this case, the pound has a drift of only 0.6 percent; however, the implied pound depreciation against the U.S. dollar due to this drift is about 10 percent over a period of four years.

5.3 Out-of-Sample Predictability

Tables 7 and 8 report the out-of-sample U-statistics (and their p-values) in the short and long run, respectively. A U-statistic less than one means that the STEC model with drift has a better forecast accuracy than the random walk with drift—that is, the STEC model with drift has a lower RMSE for the out-of-sample forecast. As before, the p-value of each U-statistic—including the average U-statistic—is computed by bootstrapping its distribution given the DGP from equations (23) and (24) under the null hypothesis $H_0 : r = 0$.[13]

The results are similar to those of Section 4.3. With two exceptions (one- and sixteen-quarter-ahead forecasts with Switzerland as numeraire), we find that across horizons and numeraires the STEC model with drifts outperforms the random walk with drift in 15 or more exchange rates—out of 18. When Japan is the numeraire we obtain the best out-of-sample results of the paper. In this case, the STEC model with drifts outperforms the random walk with drift for every country and at all forecast horizons—providing also the lowest average U-statistics.[14]

Let us now look at the statistical significance of our results. Short-run average U-statistics

[13]Please see Footnote 9 for details about the bootstraping procedure.

[14]These results are even more striking considering that the random walk with drift is a better predictor than the driftless random walk when Japan is the numeraire.

Table 7: Out-of-Sample Statistics of the STEC Model with Drifts: Short-Run

Numeraire	United States				Japan				Switzerland			
Quarters ahead	1		4		1		4		1		4	
	U-stat	p-value	U-stat	p-value	U-stat	p-value	U-stat	p-value	U-stat	p-value	U-stat	p-value
Australia	1.072	0.962	0.977	0.543	0.981	0.327	**0.855**	0.028	**0.903**	0.001	**0.795**	0.003
Austria	0.973	0.117	0.952	0.342	**0.965**	0.059	0.956	0.354	1.016	0.844	0.981	0.453
Belgium	0.974	0.145	0.957	0.404	**0.941**	0.008	**0.907**	0.083	1.000	0.668	0.941	0.143
Canada	1.001	0.608	0.945	0.112	**0.952**	0.035	**0.884**	0.063	1.002	0.671	0.943	0.322
Denmark	0.977	0.161	0.961	0.425	**0.949**	0.019	0.920	0.112	1.012	0.829	0.980	0.470
Finland	**0.955**	0.026	0.954	0.333	**0.932**	0.006	**0.891**	0.060	0.976	0.147	0.933	0.164
France	0.979	0.191	0.965	0.431	**0.928**	0.001	**0.897**	0.051	0.981	0.148	**0.901**	0.023
Germany	0.981	0.230	0.953	0.358	**0.964**	0.059	0.946	0.273	1.007	0.749	**0.908**	0.036
Great Britain	1.007	0.715	0.965	0.439	0.979	0.224	0.917	0.160	1.008	0.752	0.907	0.104
Greece	0.990	0.418	0.970	0.463	**0.910**	0.000	0.929	0.171	**0.959**	0.025	0.929	0.129
Italy	**0.951**	0.013	0.943	0.283	**0.892**	0.000	**0.891**	0.065	**0.962**	0.049	0.933	0.222
Japan	**0.955**	0.028	0.922	0.170					**0.954**	0.027	0.915	0.134
Korea	**0.885**	0.000	**0.908**	0.060	**0.958**	0.052	**0.875**	0.048	**0.936**	0.006	**0.865**	0.032
Netherlands	0.981	0.239	0.957	0.381	**0.962**	0.034	0.933	0.188	1.007	0.754	**0.926**	0.072
Norway	0.999	0.595	0.990	0.660	**0.941**	0.009	0.924	0.133	0.976	0.126	0.931	0.145
Spain	**0.945**	0.008	0.927	0.190	**0.905**	0.001	0.903	0.103	0.988	0.396	0.993	0.683
Sweden	**0.964**	0.066	0.944	0.291	**0.917**	0.000	0.908	0.138	**0.948**	0.011	**0.869**	0.029
Switzerland	0.976	0.193	0.941	0.305	0.974	0.161	0.963	0.440				
United States					**0.950**	0.024	**0.853**	0.011	1.037	0.933	0.930	0.227
Average	**0.976**	0.031	0.952	0.213	**0.944**	0.000	**0.908**	0.027	**0.982**	0.092	**0.921**	0.033
Countries<1	15		18		18		18		10		18	

The p-value is the proportion of the bootstrap distribution to the left of the calculated U-statistic.
Bolded coefficients are statistically significant at a 10% level.

Table 8: Out-of-Sample Statistics of the STEC Model with Drifts: Long-Run

Numeraire	United States				Japan				Switzerland			
Quarters ahead	8		16		8		16		8		16	
	U-stat	p-value	U-stat	p-value	U-stat	p-value	U-stat	p-value	U-stat	p-value	U-stat	p-value
Australia	0.866	0.188	0.861	0.470	**0.793**	0.086	**0.674**	0.056	**0.745**	0.020	0.832	0.451
Austria	0.955	0.646	0.965	0.859	0.923	0.475	0.895	0.654	0.924	0.288	0.898	0.520
Belgium	0.957	0.671	0.979	0.894	0.846	0.126	0.829	0.394	**0.868**	0.095	0.867	0.407
Canada	0.938	0.287	0.931	0.582	0.837	0.166	0.739	0.180	0.928	0.525	1.224	0.996
Denmark	0.975	0.765	0.987	0.905	0.845	0.108	0.796	0.274	0.924	0.298	0.912	0.545
Finland	0.934	0.499	0.940	0.762	0.823	0.101	0.781	0.262	0.923	0.434	1.048	0.969
France	0.959	0.646	0.970	0.871	**0.809**	0.046	0.774	0.186	**0.838**	0.039	0.831	0.253
Germany	0.941	0.582	0.944	0.807	0.931	0.513	0.957	0.847	**0.834**	0.032	0.802	0.163
Great Britain	0.938	0.532	0.965	0.846	0.851	0.179	0.720	0.119	0.928	0.482	1.009	0.914
Greece	0.995	0.817	1.041	0.962	0.907	0.350	0.873	0.538	0.857	0.117	0.778	0.161
Italy	0.948	0.605	0.941	0.781	0.856	0.210	0.839	0.472	0.979	0.764	0.990	0.897
Japan	0.906	0.363	0.895	0.600					0.906	0.353	0.926	0.751
Korea	0.897	0.170	0.916	0.542	**0.808**	0.095	0.749	0.215	0.839	0.161	1.061	0.965
Netherlands	0.946	0.601	0.947	0.830	0.894	0.313	0.866	0.522	0.883	0.113	0.898	0.518
Norway	0.987	0.794	1.033	0.947	0.862	0.164	0.783	0.222	0.910	0.293	0.992	0.913
Spain	0.925	0.454	0.927	0.709	0.847	0.144	0.782	0.244	0.923	0.441	0.936	0.773
Sweden	0.925	0.483	0.945	0.793	0.845	0.195	0.839	0.482	**0.796**	0.049	0.836	0.423
Switzerland	0.945	0.618	1.047	0.949	0.937	0.558	0.927	0.751				
United States					**0.814**	0.092	**0.684**	0.073	0.884	0.310	1.031	0.957
Average	0.941	0.560	0.957	0.903	**0.857**	0.090	0.806	0.275	0.883	0.121	0.937	0.853
Countries<1	18		15		18		18		18		13	

The p-value is the proportion of the bootstrap distribution to the left of the calculated U-statistic.
Bolded coefficients are statistically significant at a 10% level.

in Table 7 indicate that, with the exception of the four-quarter-ahead forecasts with the U.S. as numeraire, the forecast accuracy gains of the STEC model with drifts with respect to the random walk with drift are statistically significant at a 10% level. At the individual level, the superiority of the STEC model is statistically significant at a 10% level for 6 (out of 18) of the exchange rates when the U.S. and Switzerland are the numeraires and for 15 when Japan is the numeraire. For one-year-ahead forecasts these numbers change to 1 (for the U.S.), 6 (for Switzerland), and 8 (for Japan).

As before, we see from Table 8 that although U-statistics are in general lower in the long run than in the short run, the statistical significance of the results decreases with time—for the average U-statistics, only the one corresponding to the eight-quarter-ahead forecasts when Japan is the numeraire shows statistical significance at a 10% level for the better accuracy of the STEC model. At the individual level, only few cases provide statistical significance at a 10% level: 4 and 5 (out of 18) for the eight-quarter-ahead forecasts with Japan and Switzerland as numeraires, respectively; and only 2 for the sixteen-quarter-ahead forecasts when Japan is the numeraire.

6 Conclusions

We presented a Smooth-Transition Error Correction Model—in the spirit of the generalized cointegrated system of Granger and Swanson (1996)—for the relation between PPP fundamentals and the nominal exchange rate. Using a panel dataset of 19 countries and estimating the model for three numeraires, we find strong evidence of nonlinear mean reversion of the real exchange rate and of nonlinear predictability of the nominal exchange rate.

Out-of-sample statistics show a better forecast accuracy of the STEC model (with and without drifts) than the corresponding random walk specification across numeraires and horizons. The statistical significance of our short-run results—specially for one-quarter-ahead forecasts—is particularly good. When Japan is the numeraire, the statistical significance of the STEC model superiority for one-quarter-ahead forecasts is unmatched. In the long run—eight- and sixteen-quarter-ahead forecasts—the statistical significance of the out-of-sample results is much lower.

These results differ with recent contributions from Engel, Mark, and West (2007) and others, who find that predictability of the nominal exchange rate seems to be more statistically significant in the long run. These studies generally find weak predictability results for short-run horizons—U-statistics very close to one (about 0.99 or more) and not significant. A possible explanation of the difference in results might be that nonlinear models with panel data perform better in the short run, but traditional linear models do better in the long-run.

It is important to mention that our results are robust to the selection of the random walk specification. We show how the inclusion or not drifts in the random walk can generate substantial differences in the out-of-sample statistics. We argue that in order to circumvent these issues, we must always compare the tested model against the random walk specification implied by the DGP under the null hypothesis of unpredictability of the nominal exchange rate.

This paper presents new evidence on the predictability gains provided by panel data. Perhaps the use of a big enough panel allows us to exploit a multilateral consistency factor—the fact that a depreciating currency implies that one, two, or more other currencies are appreciating—-that cannot be otherwise captured.

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