# How to Commit to a Future Price 

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## 1 Introduction

Consider a monopolist that sells a durable good and additional goods (consumables) using the durable good. We can think of IBM selling mainframe computers and punch cards, Xerox selling copiers and toner, HP selling printers and ink, Gillette selling razors and cartridges, Boeing selling a plane and maintenance of the plane or repair parts for it. ${ }^{1}$ The higher the price of the consumable, the less a buyer is willing to pay for the durable good. The profit-maximizing solution for the seller is to price the consumable at marginal cost, and extract consumer surplus with a high price for the durable good. That appears to resemble the strategy that Apple followed by charging high prices for iPods, iPads and iPhones, but low prices for music and applications on its iTunes store. ${ }^{2}$ The difficulty is that such a strategy is not time consistentafter the durable good is sold, the seller will want to charge a high price for the consumable. In other words, the seller faces a commitment problem.

This paper explores several mechanisms which would induce the firm to charge a low price for the consumables. First, the seller can enter into a financial contract in which the seller pays a lump-sum fee in return for a per-unit subsidy for the selling the consumable. Second, the seller can allow entry into the market for the consumable. Third, the firm may sell the durable good at a low price to consumers who little value the durable and consumable, so that it will have an incentive to later set a low price for the consumable.

## 2 Literature

The analysis of how the price of a durable good depends on the price of consumables that use the durable good is an application of two-part tariffs, studied in the seminal paper by Oi (1971).

[^0]The paper closest to ours is Farrell and Gallini (1988), which considers a consumer's willingness to incur a set-up cost necessary to use a good only if he expects the monopolist to charge a low price for the good. The monopolist may therefore increase demand, and so increase profits, by an ex ante commitment to competition in the post-adoption market. One of our contributions is to extend the analysis by exploring entry; but rather than having perfect competition, we consider duopoly, exploring in turn Cournot competition, Stackelberg leadership, and Bertrand competition. Also novel is our consideration of selling to low valuers as a mechanism that commits a monopolist to low prices in the future.

Related analysis applies to consider a producer of a durable good who will monopolize the maintenance market, with consumers willing to pay less for a new unit when they anticipate monopolization (Borenstein et al. 1995). In such circumstances, a producer of durable goods would want to commit to allowing competition in the maintenance market, but monopolization occurs because of an inability to commit. How the price of a consumable good sold by a monopolist affects the price it can charge for the durable which uses the consumable is considered by Heubrander and Skiera (2010). They view the monopolist as having a lower discount rate than the consumers, and so effectively lending money to consumers by charging a low price for the durable good and a high price for the consumable good.

A different solution is offered by Nakamura and Steinsson (2011), who show that if consumers have incomplete information about costs and demand, the equilibrium the firm prefers has it price at or below a "price cap," which induces expectations of a low price in the future, and thereby increases demand. They also suggest that with repeated interactions between firms and consumers, the folk theorem can apply, leading the firm to adopt in equilibrium a low price. The strategies we analyze for committing to a low price are simpler, and do not require the repeated interaction between firms and consumers or asymmetric information.

Other literature considers switching costs when consumers can choose among sellers, each firm's product has its own setup cost, and opportunism can lead firms to charge high prices or offer low quality after buyers made seller-specific investments. Klemperer (1987) considers rational consumers in the first period who anticipate that in the second period they will be partially locked in to the firm from which they had initially bought. These consumers realize that the second-period price depends on market shares in the first period, anticipating that a low price in the first period which increases a firm's market share would result in a higher price in the next period. To ameliorate the commitment problem, a firm may license its product to second-source suppliers, thereby committing itself to lower prices in the future, and so increasing demand in the first period (Farrell and Gallini 1988). Long-term contracts that reduce a firm's market power over locked-in consumers are considered by Farrell and Shapiro (1989).

Much of the literature on switching costs focuses on two-period duopsony models in which firms choose between charging a high price to extract rents from their customers and charging a low price to attract customers from their rivals. In this framework, Klemperer (1987) shows that higher switching costs may make entry more likely, by inducing incumbents to abandon the hope of attracting the customers of other incumbents and therefore choosing higher prices.

A large related literature, building on the seminal work of Coase (1972), considers the opposite problem: after a producer of a durable good sold some durable goods, it has an incentive to sell additional units in future periods. Consumers, anticipating the consequent reduction in price in the future, are willing to pay only a low price in the current period. The producer could
therefore profit by committing to a high price in the future. ${ }^{3}$

## 3 Assumptions

Consider a two-period model in which a monopolist sells a durable good in period 1 and additional, consumable, services that require using the durable good in period 2. Buyers are assumed to be perfectly rational, correctly anticipating the price of consumables in period 2 : they know that the monopolist would want to extract all the consumer surplus of buying consumables in period 2. We assume there is no discounting over time by the seller and buyers.

Consider one consumer, whose demand for consumables is $Q_{2}\left(P_{2}\right)$ (or, equivalently, the inverse demand is $P_{2}\left(Q_{2}\right)$ ), where $P_{2}$ is the price of consumables set in period 2. For simplicity, we shall mostly use the linear (inverse) demand $Q_{2}\left(P_{2}\right)=a-P_{2}$ (or $P_{2}\left(Q_{2}\right)=a-Q_{2}$ ), where $a$ is the maximum willingness to pay (or the price intercept of inverse demand) for the consumable good, that is, $P_{2}(0)=a$. The profits from selling the consumable are $\Pi_{2}=P_{2} \cdot Q_{2}\left(P_{2}\right)$. We assume that the consumable good is produced at zero marginal cost. A consumer derives utility only from using the durable good with consumables, not from consuming the durable good alone. Thus, the consumer surplus from using the durable good is the same as that from consuming consumables, given by

$$
\begin{equation*}
C S\left(P_{2}\right)=\int_{P_{2}}^{\infty} Q_{2}(h) d h \tag{1}
\end{equation*}
$$

### 3.1 Monopolist's commitment problem

This section presents the commitment problem, not providing solutions or adding to the existing literature. The novel material is in succeeding sections. For any price, $P_{2}$ that consumers expect to be charged for consumables in period 2 , the monopolist in period 1 would maximize his profits by setting the price of the durable good at

$$
\begin{equation*}
P_{1}=C S\left(P_{2}\right) \tag{2}
\end{equation*}
$$

Because $C S^{\prime}\left(P_{2}\right)<0$, the higher the price of the consumable good, the less a buyer is willing to pay for the durable good. Profits from selling a durable good are $\Pi_{1}=P_{1}-c_{1}=C S\left(P_{2}\right)-c_{1}$. The marginal production cost of a durable good is $c_{1}$, so that profits over the two periods are

$$
\begin{equation*}
\Pi \equiv \Pi_{1}+\Pi_{2}=C S\left(P_{2}\right)-c_{1}+P_{2} Q_{2}=\int_{0}^{Q_{2}} P_{2}(z) d z-c_{1} \tag{3}
\end{equation*}
$$

Obviously, (3) strictly decreases with $P_{2}$ (or increases with $Q_{2}$ ).
If the monopolist could commit in period 1 to the price of the consumable good in period 2 , he would choose the price $P_{2}$ (or the output $Q_{2}$ ) that maximizes (3), yielding $P_{2}^{*}=0$ (i.e., marginal-cost pricing) or $Q_{2}^{*}=Q_{2}\left(P_{2}^{*}\right)$. Profits would be

$$
\Pi^{*}=C S\left(P_{2}^{*}\right)+P_{2}^{*} Q_{2}-c_{1}=\int_{0}^{Q_{2}(0)} P_{2}(z) d z-c_{1}
$$

[^1]

Figure 1: Monopolist's commitment problem

A monopolist can extract consumer surplus with a high price for the durable by pricing the consumable good at marginal cost. Under linear demand, the total profits with commitment are $\Pi^{*}=a^{2} / 2-c_{1}$. However, this pricing is not time consistent: after the durable good is sold, the monopolist will want to charge a monopoly price for the consumable good.

If the monopolist cannot commit to the future price of consumables, consumers will anticipate that it would charge the monopoly price $P_{2}^{M}$ for the consumable good in period 2 ; the associated consumer surplus is $\operatorname{CS}\left(P_{2}^{M}\right)\left(P_{2}^{M}\right.$ is such that $P_{2}^{M} Q_{2}^{\prime}\left(P_{2}^{M}\right)+Q_{2}\left(P_{2}^{M}\right)=0$. Therefore, the monopolist prices the durable good at $P_{1}^{M}=C S\left(P_{2}^{M}\right)$. Total profits are

$$
\Pi^{M}=C S\left(P_{2}^{M}\right)+P_{2}^{M} Q_{2}-c_{1}=\int_{0}^{Q_{2}\left(P_{2}^{M}\right)} P_{2}(z) d z-c_{1}
$$

Because $Q_{2}\left(P_{2}^{M}\right)<Q_{2}(0)$, total profits are smaller when the monopolist cannot commit compared to profits when he can commit. Under linear demand, total profits without commitment are $\Pi^{M}=3 a^{2} / 8-c_{1}$, which is necessarily smaller than the profits with commitment. We summarize with

## Proposition 1

A monopolist who sells a durable good and the associated consumables faces a time-inconsistency problem: profits are maximized if the seller can commit to price the consumable good at its marginal cost, but once the monopolist sold the durable good, it profits by charging the monopoly price for the consumable good.

Figure 1 illustrates the monopolist's commitment problem. If the monopolist can commit to the price of consumables, it sets $P_{2}=P_{2}^{*}=0$ and $P_{1}=C S(0)$, generating profits of $\Pi^{*}=C S(0)-c_{1}$; the profits are illustrated by the area $a Q_{2}^{*} 0$. If the monopolist cannot commit to the price of consumables, its profits are given by the area $a M Q_{2}^{M} 0$; the difference in profits under commitment and no commitment is the shaded area.

### 3.2 Contract with a third party to overcome the commitment problem

To overcome the commitment problem mentioned above, a monopolist might enter into the following contract with a third party financial firm: in period 2 the financial firm pays the monopolist $s$ per unit of the consumable it sells, with the monopolist paying $F(s)$ to the financial firm in period 1. The contract is signed in period 1 and is known to consumers. In period 2 the monopolist's profit from selling the consumable good is $\Pi_{2}^{E}=\left(P_{2}+s\right) Q_{2}-F(s)$. From the first-order condition in period 2 , we have the monopoly price for the consumable good with the financial contract as $P_{2}^{E}$ and monopoly output as $Q_{2}^{E}$. Then, the monopolist sets the price of the durable good at $P_{1}^{E}=C S\left(P_{2}^{E}\right)$. Total profits are

$$
\Pi^{E}=C S\left(P_{2}^{E}\right)+\left(P_{2}^{E}+s\right) Q_{2}^{E}-c_{1}-F(s)=\int_{0}^{Q_{2}^{E}} P_{2}(z) d z+s Q_{2}^{E}-c_{1}-F(s) .
$$

The payment to the financial firm, $F(s)$, must be larger than $s Q_{2}^{E}$. Here we assume that the monopolist has all the bargaining power, or equivalently that the financial sector is competitive. Then we have $F(s)=s Q_{2}^{E}$. The monopolist can choose $s$ so as to maximize $\Pi^{E}$. Therefore we have

$$
\frac{d \Pi^{E}}{d s}=\frac{d Q_{2}^{E}}{d s} P_{2}\left(Q_{2}^{E}\right)=0
$$

implying that this contract yields $P_{2}^{E}=0$, or marginal cost pricing for consumables. Total profits under this contract are $\Pi^{*}$, the same as under commitment. We summarize with

## Proposition 2

A monopolist seller can give itself an incentive to charge a low price for consumables by entering into a contract with a third party in which the monopolist seller obtains a subsidy from that party for each unit of the consumable good it sells, in exchange for a fixed payment to the third party which equals the total amount of subsidy received. This contract can serve as a commitment to future low price for consumables.

Although this simple contract might overcome the commitment problem, the contract may be time-inconsistent: in the beginning of the period 2 , the monopolist firm has incentives to reverse the contract because consumers already locked in and bought the durables at price $\triangle a Q_{2}^{*} 0$ in Figure 1. Therefore, the monopolist has an incentive to cancel the contract by giving back the lump-sum payment from the financial firm in exchange for waiving the right to get subsidy. In order to reverse the contract, the monopolist firm would be willing to pay a lump-sum payment up to the amount $\square P_{2}^{M} M Q_{2}^{M} 0$. On the other hand, a financial firm may want to maintain its reputation for honoring contracts and not renegotiating them. Or if the monopolist enters into subsidy contracts with multiple financial firms, the transaction costs of renegotiation can be so large as to preclude renegotiation.

So financial contracts may make it credible that the firm will charge a low price for the consumables in period 2. But other mechanisms can also work.

## 4 Accommodating entry into the market for consumables

Suppose a firm with monopoly power could allow one firm to enter the market for consumables. The entry will usually reduce the incumbent firm's profits from selling consumables, but the
incumbent monopolist may profit from entry if it faces the commitment problem mentioned above.

We consider three cases: (1) the incumbent and the entrant firms engage in simultaneousmove quantity (Cournot) competition in the consumable market, (2) the two firms engage in sequential-move quantity competition with the incumbent firm acting as a leader in output choice, and (3) they engage in price (Bertrand) competition. The durable good is monopolistically provided by the incumbent seller in period 1 .

### 4.1 Cournot

For simplicity, we shall consider linear demand. When there is one entrant into the consumable market, the inverse demand is $P_{2}=a-\left(Q_{2 I}+Q_{2 E}\right)$, where $Q_{2 I}$ is the incumbent's output, and $Q_{2 E}$ is the entrant's output. Let the incumbent charge the entrant a license fee of $f \geq 0$ per unit. The profits of an incumbent and entrant from selling consumables in period 2 are $\Pi_{2 I}=P_{2} Q_{2 I}+f Q_{2 E}$ and $\Pi_{2 E}=\left(P_{2}-f\right) Q_{2 E}$.

The equilibrium in period 2 has

$$
\begin{aligned}
& Q_{2 I}^{C}(f)=\frac{a+f}{3}, \quad Q_{2 E}^{C}(f)=\frac{a-2 f}{3}, \quad P_{2}^{C}(f)=\frac{a+f}{3} \\
& C S^{C}(f)=\frac{(2 a-f)^{2}}{18}, \quad \Pi_{2 I}^{C}(f)=\frac{a^{2}+5 a f-5 f^{2}}{9}, \quad \Pi_{2 E}^{C}(f)=\frac{(a-2 f)^{2}}{9},
\end{aligned}
$$

where superscript $C$ refers to outcomes when the incumbent accommodates entry and competes with the entrant in a Cournot fashion. An increase in the license fee $f$ increases $Q_{2 I}^{C}$ and $P_{2}^{C}$, but reduces $Q_{2 E}^{C}, C S^{C}$, and $\Pi_{2 E}^{C}$.

In period 1, the incumbent sets the price of a durable at $P_{1}^{C}(f)=C S^{C}(f)$; its profits from selling the durable good are $\Pi_{1 I}^{C}(f)=P_{1}^{C}(f)-c_{1}$. Therefore, total profits, including from sales of the consumable good and from the license fee, are

$$
\Pi_{I}^{C}(f)=\Pi_{1 I}^{C}(f)+\Pi_{2 I}^{C}(f)=\frac{a^{2}+5 a f-5 f^{2}}{9}+\frac{(2 a-f)^{2}}{18}-c_{1},
$$

which is strictly concave in $f$.
When the incumbent allows an entrant to produce the consumable good, charging no license fee (that is, $f=0$ ), profits are

$$
\Pi_{I}^{C}(0)=\frac{a^{2}}{3}-c_{1}<\frac{3 a^{2}}{8}-c_{1}=\Pi^{M} .
$$

The incumbent firm is worse off by allowing entry with no license fee.
The incumbent firm sets $f$ so as to maximize $\Pi_{I}$. The profit-maximizing license fee, $f^{C}$, is given by $f^{C}=a / 3$. With this profit-maximizing fee the equilibrium has

$$
P_{2}^{C}\left(f^{C}\right)=\frac{4 a}{9}<P_{2}^{M}, \quad P_{1}^{C}\left(f^{C}\right)=\frac{25 a^{2}}{162}>P_{1}^{M}, \quad \Pi_{I}^{C}\left(f^{C}\right)=\frac{7 a^{2}}{18}-c_{1}>\Pi^{M}
$$

and $\Pi_{2 E}^{C}\left(f^{C}\right)=a^{2} / 81>0$. The incumbent monopolist profits by allowing entry into the consumable market when he can charge a license fee. We summarize with

## Proposition 3

A seller with a monopoly over a durable good and who has a potential monopoly over the consumable good can profit by accommodating entry into the consumable market, charging a unit
license fee and competing with the entrant in a Cournot fashion. Although the accommodation reduces the firm's profits from selling consumables, the reduced price of the consumables increases the price the monopolist can charge for the durable good, and also generates revenue from the license fee.

### 4.2 Stackelberg

Consider next an incumbent who allows entry into the consumable market and competes with the entrant in quantity as a Stackelberg leader. The equilibrium then has

$$
\begin{aligned}
& Q_{2 I}^{S}(f)=\frac{a}{2}, \quad Q_{2 E}^{S}(f)=\frac{a-2 f}{4}, \quad P_{2}^{S}(f)=\frac{a+2 f}{4}, \\
& C S^{S}(f)=\frac{(3 a-2 f)^{2}}{32}, \quad \Pi_{2 I}^{S}(f)=\frac{a^{2}+4 a f-4 f^{2}}{8}, \quad \Pi_{2 E}^{S}(f)=\frac{(a-2 f)^{2}}{16} .
\end{aligned}
$$

In period 1 , the incumbent sets a price for the durable good at $P_{1}^{S}(f)=C S^{S}(f)$, so total profits are

$$
\begin{equation*}
\Pi_{I}^{S}(f)=\frac{(3 a-2 f)^{2}}{32}+\frac{a^{2}+4 a f-4 f^{2}}{8}-c_{1} . \tag{4}
\end{equation*}
$$

When $f=0$, profits are

$$
\Pi_{I}^{S}(0)=\frac{13 a^{2}}{32}-c_{1}>\Pi^{C}\left(f^{C}\right)>\Pi^{M}
$$

which indicates that an incumbent who moves first in the market for consumables market profits from allowing entry even when with no license fee. The intuition is as follows. The first mover is advantaged in this game because the quantity choices are strategic substitutes. Therefore, the incumbent earns higher profits than under Cournot competition. In addition, the equilibrium price of the consumable good under Stackelberg competition is lower than under Cournot competition, so that the incumbent can charge more for the durable good in period 1.

More interestingly, $\Pi_{I}^{S}(0)>\Pi_{I}^{C}\left(f^{C}\right)$. This means that profits without licensing fees and the incumbent playing as a Stackelberg leader are greater than the profits with the profit-maximizing licensing fees in the Cournot case.

Because the total profit given by (4) is concave in $f$, the profit-maximizing license fee satisfies $f^{S}=a / 6<f^{C}$. Under this license fee, the equilibrium has

$$
P_{2}^{S}\left(f^{S}\right)=\frac{a}{3}<P_{2}^{C}\left(f^{C}\right), \quad P_{1}^{S}\left(f^{S}\right)=\frac{2 a^{2}}{9}>P_{1}^{C}\left(f^{C}\right), \quad \Pi_{I}^{S}\left(f^{S}\right)=\frac{5 a^{2}}{12}-c_{1}>\Pi_{I}^{C}\left(f^{C}\right),
$$

and $\Pi_{2 E}^{S}\left(f^{S}\right)=a^{2} / 36>\Pi_{2 E}^{C}\left(f^{C}\right)$.

## Proposition 4

A monopolist who sells a durable good and consumables can profit by accommodating entry into the market for consumables and competing with the entrant as a leader in quantity choice. This holds even without licensing fees.

Figure 2 represents the situations without license fees. Under monopoly for all goods, total profits $\Pi^{M}$ are represented by the rectangle $a b Q_{2}^{M} 0$ minus $c_{1}$. In the case where the incumbent seller accommodates entry into the market for consumables and competes with the entrant in Cournot fashion, the profits of the incumbent are $a e P_{2}^{C}$ minus $c_{1}$ from selling a durable


Figure 2: Accommodating Entry with Cournot and Stackelberg Competition $(f=0)$
plus $P_{2}^{C} c Q_{2 I}^{C} 0$ from selling consumables. Therefore, the firm's total profits $\Pi_{I}^{C}$ are aec $Q_{2 I}^{C} 0$ minus $c_{1}$. As a result, $\square c d Q_{2}^{M} Q_{2 I}^{C}-\triangle b e d$ represents the profit loss from accommodating the entrant without charging license fees. If the incumbent accommodates entry into the market for consumables and acts as a Stackelberg leader, the equilibrium price for consumables is $P_{2}^{S}$, and thereby the profits of the incumbent are $a h P_{2}^{S}$ minus $c_{1}$ from selling the durable good, plus $P_{2}^{S} g Q_{2 I}^{S} 0$ from selling consumables. Therefore, the incumbent's total profits $\Pi_{I}^{S}$ are ahg $Q_{2 I}^{S} 0$ minus $c_{1}$. As a result, $\triangle b h g$ represents the profit gain from accommodating entry, compared to the monopoly profits.

### 4.3 Bertrand

If the incumbent allows entry to the consumable market and competes with the entrant in price (that is, Bertrand competition), then the equilibrium is the same as under commitment. The result appears because, in the market for consumables, price competition with identical and constant marginal production costs (implying no license fee) yields a perfectly competitive outcome. This can serve as a commitment to a future low price of consumables ( $P_{2}=0$ ), and so the incumbent seller obtain the maximum gain from fully extracting the consumer surplus.

## Proposition 5

A monopolist seller who sells a durable good and consumables profits from accommodating entry into the market for consumables and competing with the entrant in a Bertrand fashion.

Because the incumbent's profit-maximizing price for the consumable good is zero, the incumbent firm cannot increase its profits by charging a per unit license fee for the consumable good. Therefore, if Bertrand competition arises in the consumable market, the incumbent profits from allowing entry, and charging no license fee.

Propositions 3 to 5 imply that the incumbent firm should not create entry barriers which raise entry costs. Rather, it may be beneficial for the firm to subsidize entry. Such subsidy payments are its costs for the commitment.

## 5 Selling to low valuers

Another sophisticated strategy can allow a monopolist to alleviate the commitment problem: selling durables to consumers who have a low willingness-to-pay. The strategy increases the price elasticity of demand for the consumable good, which can serve as a commitment to a future low price for the consumable good, thereby allowing the firm to charge high valuers a higher price for the durable good.

For further intuition, think of two types of consumers, one with a high willingness to pay for the consumable, and one with a low willingness to pay. If the monopolist sells the durable good only to the high valuers, buyers may fear that they will be charged a high price for consumables, and so be unwilling to buy the durable good. But a monopolist who also sells to low valuers will want to set a lower price for the consumable good in later periods, and so a high valuer would be willing to pay more for the durable good.

Call a consumer who has a high willingness-to-pay for consumables a high valuer or a typeH consumer; call a consumer with a low willingness-to-pay for consumables a low valuer or a type-L consumer. Let the number of high valuers be $N_{H}$, and the number of low valuers $N_{L}$. In this section, for simplicity, we assume $N_{H}=N_{L}=1$. As shown in the Appendix, however, the results do not change qualitatively for $N_{H} \neq N_{L} \geq 1$. The individual demand functions for consumables by these two types are

$$
Q_{2 H}=a_{H}-P_{2}, \quad Q_{2 L}=a_{L}-P_{2}
$$

Let $Q_{2} \equiv Q_{2 H}+Q_{2 L}$ be the total demand for the consumable good. We assume that the monopolist cannot price discriminate for the consumable good (the assumption is relaxed later). As before, the profits from selling consumables are $\Pi_{2}=P_{2} Q_{2}$.

First, we derive the equilibrium where the monopolist does not sell a durable good to a low valuer. Demand is then $Q_{2}=Q_{2 H}=a_{H}-P_{2}$. The equilibrium has

$$
\begin{aligned}
& P_{2}^{N}=a_{H} / 2, \quad Q_{2}^{N}=Q_{2 H}^{N}=a_{H} / 2, \quad \Pi_{2}^{N}=a_{h}^{2} / 4 \\
& C S_{H}^{N}=P_{1 H}^{N}=a_{H}^{2} / 8, \quad \Pi_{1}^{N}=P_{1 H}^{N}-c_{1}, \quad \Pi^{N}=3 a_{H}^{2} / 8-c_{1}
\end{aligned}
$$

where superscripts $N$ refer to the equilibrium variables when the monopolist sells no durables to low valuers and $C S_{H}^{N}$ is the consumer surplus for each high valuer.

Next, we derive the equilibrium where the monopolist sells the durable good to low valuers, making them demand consumables in period 2. Total demand for consumables is

$$
Q_{2}= \begin{cases}a_{H}-P_{2} & \text { for } P_{2}>a_{L}  \tag{5}\\ \left(a_{H}+a_{L}\right)-2 P_{2} & \text { for } P_{2} \leq a_{L}\end{cases}
$$

We assume that the demand for consumables by a low valuer is sufficiently high so that the monopolist wants to sell to them:

Assumption $1 \sqrt{2}-1<a_{L} / a_{H} \leq 1$.

The outcomes in period 2 are

$$
\begin{array}{lll}
P_{2}^{Y}=\left(a_{H}+a_{L}\right) / 4, & Q_{2 H}^{Y}=\left(3 a_{H}-a_{L}\right) / 4, \quad Q_{2 L}^{Y}=\left(3 a_{L}-a_{H}\right) / 4, \\
Q_{2}^{Y}=\left(a_{H}+a_{L}\right) / 2, & \Pi_{2}^{Y}=\left(a_{H}+a_{L}\right)^{2} / 8, \quad C S_{L}^{Y}=\left(3 a_{L}-a_{H}\right)^{2} / 32, \quad C S_{H}^{Y}=\left(3 a_{H}-a_{L}\right)^{2} / 32,
\end{array}
$$

where superscripts $Y$ refer to the equilibrium variables in the case where the monopolist sells durables to low valuers. Assumption 1 ensures that $\Pi_{2}^{Y}>\Pi_{2}^{N}$ : the monopolist finds it profitable to charge a low price for the consumable good when low valuers own the durable good.

In period 1, given that the monopolist can perfectly price discriminate between high and low valuers, it prices the durable good at $P_{1 H}^{Y}=C S_{H}^{Y}$ to high valuers and $P_{1 L}^{Y}=C S_{L}^{Y}$ to low valuers. Therefore, the profits from selling durables and the total profits are

$$
\Pi_{1}^{Y}=C S_{H}^{Y}+C S_{L}^{Y}-2 c_{1}, \quad \Pi^{Y} \equiv \Pi_{1}^{Y}+\Pi_{2}^{Y}
$$

Comparing $\Pi^{N}$ with $\Pi^{Y}$, we have the following:

$$
\Pi^{Y}>\Pi^{N} \Leftrightarrow 0 \leq c_{1}<\bar{c}_{1} \equiv \frac{\left(a_{H}-a_{L}\right)^{2}+6 a_{L}^{2}}{16} .
$$

If $c_{1}<\bar{c}_{1}$, then the monopolist profits from selling the durable good to low valuers. Because

$$
P_{1 L}^{Y}-\bar{c}_{1}=-\frac{\left(a_{H}+a_{L}\right)^{2}+4 a_{L}^{2}}{32}<0
$$

the monopolist profits from selling the durable to low valuers even at a price below the marginal production costs, i.e., subsidizing them. Subsidizing a low valuer ensures that the price of consumables will be low in period 2, allowing the monopolist to gain the higher profits of $P_{1 H}^{Y}$. Furthermore, defining the total profits from selling a durable good and consumables to a low valuer as $\Pi_{L}^{Y} \equiv P_{1 L}^{Y}-c_{1}+P_{2}^{Y} Q_{2 L}^{Y}$, we have

$$
\left.\Pi_{L}^{Y}\right|_{c_{1}=\bar{c}_{1}}=-\left(a_{H}-a_{L}\right)\left(3 a_{H}+a_{L}\right) / 32<0,
$$

which implies that when $c_{1}$ is large, the monopolist profits from selling a durable good and consumables to low valuers even when the total profits from doing so are negative: the losses from selling to low valuers are dominated by the gains from the ability to charge a higher price for the durable good to high valuers. We summarize with

## Proposition 6

A monopolist who sells durables and consumables can profit from selling durables to low valuers even at a price less than marginal cost. The strategy increases the price elasticity of demand for consumables, which can serve as a commitment to a future low price of consumables.

Figure 3 illustrates the situation. In the figure, the inverse demand curve for consumables by high valuers is $D_{H}$; the inverse demand by low valuers is $D_{L}$. When the monopolist does not sell the durable good to low valuers, the price of consumables is $P_{2}^{N}$; the price of a durable good is the area $a b P_{2}^{N}$. When the monopolist sells the durable good to low valuers, the price of consumables is $P_{2}^{Y}$. The monopolist would then set a price of the durable good to high valuers equals the area $C S_{H}^{Y}\left(a c P_{2}^{Y}\right)$; the price to low valuers is $C S_{L}^{Y}(b d c)$. Therefore, total profits are increased by the area $b d Q_{2}^{Y} Q_{2}^{N}$ minus $c_{1}$. In other words, unless $c_{1}>\square b d Q_{2}^{Y} Q_{2}^{N}$, the monopolist


Figure 3: Selling to low valuers
profits from selling the durable good to low valuers even if it sells the durable good to them at a loss.

The firm profits from selling durables to low valuers not because it profits from selling them consumables, but because the firm thereby assures high valuers that it will set a low price for consumables, and so increases demand for the durable good by high valuers.

The firm has various ways to price discriminate. For example a producer of computer printers can offer high-end and low-end models, which use common ink cartridges. Selling low-end printers can serve as a commitment to a low price of ink because consumers who choose the low-end model have smaller willingness to pay for ink. Another way is to sell the durable good at a low price only at locations frequented by low valuers. Because Walmart customers are more likely to have more elastic demand (or less willingness to pay) for printers and ink, the existence of some positive cost for high valuers to shop at Walmart (due to its location or preference where to buy), can allow for price discrimination.

The effects discussed here are the opposite of those that would be found for "status goods" or "snob goods." The usual status goods story is that the valuation of the product by high valuers (rich people) declines with the number of low valuers (poor people) who buy the good. Because increased purchases of the durable good by low valuers reduces the price of the consumable good, we have demand by high valuers increase with purchases by low valuers. That in turn means that a firm can profit by making it known that it sells to low valuers. It may, for example, ensure that rich consumers know that the durable good is sold at a low price at Walmart, even if few rich consumers would shop there.

### 5.1 Imperfect price discrimination

Consider next imperfect price discrimination for durables and consumables. Here we assume that $\left|P_{i H}-P_{i L}\right| \leq K_{i}$ for $i=\{1,2\}$ : The price difference is constrained to be at most $K_{1}$ for durables and at most $K_{2}$ for consumables. The value of $K_{i}(i=1,2)$ may reflect arbitrage costs or search costs to find a lower price. If $K_{i}=0$, the monopolist cannot price discriminate. If $K_{i}=\infty$, the monopolist can perfectly price discriminate. The basic model analyzed in the previous subsection corresponds to the case $K_{1}=\infty$ and $K_{2}=0$.

Demand for consumables by high valuers is $Q_{2 H}=a_{H}-P_{2 H}$; demand by low valuers is $Q_{2 L}=a_{L}-P_{2 L}$. We allow for $P_{2 H} \neq P_{2 L}$. We assume that the number of high valuers, $N_{H}$ and that of low valuers, $N_{L}$ are normalized to one ( $N_{H}=N_{L}=1$ ) as before, but our main results hold true for the case of $N_{H} \neq N_{L}$ under some assumptions.

When $K_{2}$ is large enough, the monopolist can perfectly price discriminate between high valuers and low valuers in selling consumables, and can freely choose $P_{2 H}$ and $P_{2 L}$ to maximize $\Pi_{2}=Q_{2 H} P_{2 H}+Q_{2 L} P_{2 L}$. The solution in period 2 is

$$
\begin{aligned}
& P_{2 H}^{D}=a_{H} / 2, \quad P_{2 L}^{D}=a_{L} / 2 \\
& C S_{H}^{D}=a_{H}^{2} / 8, \quad C S_{L}^{D}=a_{L}^{2} / 8, \quad \Pi_{2}^{D}=\left(a_{H}^{2}+a_{L}^{2}\right) / 4,
\end{aligned}
$$

where superscript $D$ refers to perfect price discrimination. From $P_{2 H}^{D}-P_{2 L}^{D}=\left(a_{H}-a_{L}\right) / 2$, this case arises when $K_{2} \geq\left(a H-a_{L}\right) / 2$.

When $K_{2}<\left(a H-a_{L}\right) / 2$ (prices are binding), the monopolist cannot perfectly price discriminate in selling consumables. The monopolist chooses $P_{2}$ to maximize

$$
\Pi_{2}=\left[a_{H}-\left(P_{2}+K_{2}\right)\right]\left(P_{2}+K_{2}\right)+\left(a_{L}-P_{2}\right) P_{2} .
$$

Then, we have

$$
\begin{aligned}
& P_{2 H}^{B}=\left(a_{H}+a_{L}+2 K_{2}\right) / 4, \quad P_{2 L}^{B}=\left(a_{H}+a_{L}-2 K_{2}\right) / 4, \\
& C S_{H}^{B}=\left(3 a_{H}-a_{L}-2 K_{2}\right)^{2} / 32, \quad C S_{L}^{B}=\left(3 a_{L}-a_{H}+2 K_{2}\right)^{2} / 32, \\
& \Pi_{2}^{B}=\left[\left(a_{H}+a_{L}\right)^{2}+4\left(a_{H}-a_{L}\right) K_{2}-4 K_{2}^{2}\right] / 8,
\end{aligned}
$$

where superscript $B$ refers to imperfect price discrimination. Notice that $K_{2}=0$ corresponds to the basic case. Since $d \Pi_{2}^{B} / d K_{2}=\left(a_{H}-a_{L}\right) / 2-K_{2}>0$, we find that an increase in $K_{2}$ increases profits in period 2 from selling consumables.

Consider now the monopolist's choice in period 1. The price difference between $P_{1 H}$ and $P_{1 L}$ is constrained to be at most $K_{1}$. There are two cases in period 1: $K_{1}$ binds and $K_{1}$ does not. Therefore, we have four cases to consider: (1) the monopolist can perfectly price discriminate in selling both durables and consumable (Case DD), (2) the monopolist cannot perfectly price discriminate in selling the durable good, but he can in selling consumables (Case BD), (3) the monopolist can perfectly price discriminate in selling the durable good, but not in selling consumables (Case DB), and (4) the monopolist cannot perfectly price discriminate for either good (Case BB).

Consider first case DD where neither $K_{1}$ nor $K_{2}$ binds, or where $K_{1} \geq\left(a_{H}^{2}-a_{L}^{2}\right) / 8$ and $K_{2} \geq$ $\left.\left(a_{H}-a_{L}\right) / 2\right)$. The price of the durable good for high valuers $\left(P_{1 H}^{D D}\right)$ is $C S_{H}^{D}$, and the price for low valuers $\left(P_{1 L}^{D D}\right)$ is $C S_{L}^{D}$. The profits from selling the durable good is $\Pi_{1}^{D D}=P_{1 H}^{D D}+P_{1 L}^{D D}-2 c_{1}$.

Therefore, profits over the two periods are

$$
\begin{equation*}
\Pi^{D D}=\Pi_{1}^{D D}+\Pi_{2}^{D}=3\left(a_{H}^{2}+a_{L}^{2}\right) / 8-2 c_{1} . \tag{6}
\end{equation*}
$$

Second, consider Case BD where $K_{1}$ binds and $K_{2}$ does not: $K_{1}<\left(a_{H}^{2}-a_{L}^{2}\right) / 8$ and $K_{2} \geq$ $\left(a_{H}-a_{L}\right) / 2$. In this case, the monopolist cannot price the durable good to a high valuer at $P_{1 H}=C S_{H}^{D}$. Instead he sets $P_{1 H}$ to at most $P_{1 L}+K_{1}$. Therefore we have $P_{1 L}^{B D}=C S_{L}^{D}=a_{L}^{2} / 8$, $P_{1 H}^{B D}=P_{1 L}^{B D}+K_{1}=a_{L}^{2} / 8+K_{1}$. The profits from selling the durable good are $\Pi_{1}^{B D}=P_{1 H}^{B D}+$ $P_{1 L}^{B D}-2 c_{1}$. Therefore total profits are

$$
\begin{equation*}
\Pi^{B D}=\Pi_{1}^{B D}+\Pi_{2}^{D}=\left(a_{H}^{2}+2 a_{L}^{2}\right) / 4-2 c_{1}+K_{1} . \tag{7}
\end{equation*}
$$

When $K_{1}$ is so small that $\Pi^{Y}>\Pi^{B D}$, the monopolist does not sell the durable good to low valuers. We have

$$
\Pi^{Y}>\Pi^{B D} \Leftrightarrow K_{1}<\frac{a_{H}^{2}-4 a_{L}^{2}}{8}+c_{1} .
$$

Third, consider Case DB where $K_{1}$ does not bind and $K_{2}$ does: $K_{1}>\left(a_{H}+a_{L}\right)\left(a_{H}-\right.$ $\left.a_{L}-K_{2}\right) / 4$ and $K_{2}<\left(a_{H}-a_{L}\right) / 2$. Then $P_{1 H}^{D B}=C S_{H}^{B}=\left(3 a_{H}-a_{L}-2 K_{2}\right)^{2} / 32$ and $P_{1 L}^{D B}=$ $C S_{L}^{B}=\left(3 a_{L}-a_{H}+2 K_{2}\right)^{2} / 32$. The condition of $K_{1}>\left(a_{H}+a_{L}\right)\left(a_{H}-a_{L}-K_{2}\right) / 4$ comes from $K_{1}>P_{1 H}^{D B}-P_{1 L}^{D B}$. The profits from selling the durable good are $\Pi_{1}^{D B}=P_{1 H}^{D B}+P_{1 L}^{D B}-2 c_{1}$. Total profits are

$$
\begin{equation*}
\Pi^{D B}=\Pi_{1}^{D B}+\Pi_{2}^{B}=3\left(7 a_{H}^{2}-2 a_{H} a_{L}+7 a_{L}^{2}\right) / 16-2 c_{1}-\left(K_{2}\right)^{2} / 4 . \tag{8}
\end{equation*}
$$

Notice that $\Pi^{D B}$ strictly decreases in $K_{2}\left(d \Pi^{D B} / d K_{2}=-K_{2} / 2<0\right)$ : the greater the monopolist's ability to price discriminate (the larger the value of $K_{2}$ ), the higher its profits in period 2, but the lower its total profits. The result appears because price discrimination for consumables leads to a higher $P_{2 H}$ than under no-price-discrimination, which leads to lower $P_{1 H}$.

Furthermore, $\left.\Pi^{D B}\right|_{K_{2}=0}-\Pi^{D D}=\left(a_{H}-a_{L}\right)^{2} / 16>0$ : given that the monopolist can perfectly price discriminate on the durable good, the monopolist would profit from a commitment to no future price discrimination on consumables. But such a commitment is also time-inconsistent.

Lastly, consider Case BB where both $K_{1}$ and $K_{2}$ bind: $K_{1}<\left(a_{H}+a_{L}\right)\left(a_{H}-a_{L}-K_{2}\right) / 4$ and $K_{2}<\left(a_{H}-a_{L}\right) / 2$. Then $P_{1 L}^{B B}=C S_{L}^{B}=\left(3 a_{L}-a_{H}+2 K_{2}\right)^{2} / 32$ and $P_{1 H}^{B B}=P_{1 L}^{B B}+K_{1}=$ $\left(3 a_{L}-a_{H}+2 K_{2}\right)^{2} / 32+K_{1}$. The profits from selling durables are $\Pi_{1}^{B B}=P_{1 H}^{B B}+P_{1 L}^{B B}-2 c_{1}$. Therefore, total profits are

$$
\begin{equation*}
\Pi^{B B}=\Pi_{1}^{B B}+\Pi_{2}^{B}=\left(3 a_{H}^{2}-2 a_{H} a_{L}+11 a_{L}^{2}\right) / 16-2 c_{1}+K_{1}+\left(a_{H}+a_{L}\right) K_{2} / 4-\left(K_{2}\right)^{2} / 4 . \tag{9}
\end{equation*}
$$

When $K_{1}$ is so small that $\Pi^{Y}>\Pi^{B B}$, the monopolist does not sell the durable good to low valuers. We have

$$
\Pi^{Y}>\Pi^{B B} \Leftrightarrow K_{1}<\frac{3 a_{H}^{2}+2 a_{H} a_{L}-11 a_{L}^{2}}{16}-\frac{a_{H}+a_{L}}{4} K_{2}+\frac{1}{4} K_{2}^{2}+c_{1} .
$$

Even if the marginal production cost for the durable good is zero ( $c_{1}=0$ ), when $K_{2}$ is sufficiently large and/or $K_{1}$ is sufficiently small, the monopolist may not find it profitable to sell the durable good to low valuers.

Differentiating $\Pi^{B B}$ with respect to $K_{2}$, we have

$$
\frac{d \Pi^{B B}}{d K_{2}}=\frac{1}{2}\left(\frac{a_{H}+a_{L}}{2}-K_{2}\right)>0,
$$



Maximum price difference for consumables: $K_{2}$

Figure 4: Imperfect price discrimination for durables and consumables
where the last inequality comes from the condition that $K_{2}<\left(a_{H}-a_{L}\right) / 2$. The inequality indicates that $\Pi^{B B}$ strictly increases in $K_{2}$ : given that the monopolist cannot perfectly price discriminate for the durable good, the greater the monopolist's ability to price discriminate on consumables, the higher its profits. Intuitively, price discrimination on consumables leads to higher $P_{2 H}$ and lower $P_{2 L}$, which leads to higher $P_{1 L}$.

The results are summarized by

## Proposition 7

The monopolist's profits increase with its ability to price discriminate for durables. If the monopolist can perfectly price discriminate when selling the durable good, then profits decline with its ability to price discriminate for consumables. In contrast, if the monopolist cannot perfectly price discriminate in selling the durable good, profits increase with its ability to price discriminate for consumables.

Figure 4 illustrates the four cases. In the shaded area the monopolist sells nothing to low valuers. Total profits increase with $K_{2}$ in the BB region, but decrease with $K_{2}$ in the DB region.

## 6 Conclusion

If a firm can effectively increase profits by committing to a low price for the consumable it sells after the durable good is bought, why do we nevertheless sometimes see firms charge low prices for durables and high prices for consumables? Why are printers so cheap and ink so expensive? One possibility is that the mechanisms we describe are innovative, with management not realizing what opportunities are available to them. Another possibility is that firms engage in price discrimination by charging a high price for the consumable. Then the mechanisms we discuss can explain why prices for the consumables are not even higher, and why consumers are willing to buy the durable good. Lastly, some firms do charge low prices for the consumables.

For example, long warranties for cars means that consumers need not be afraid of exorbitant prices for repairs; software updates for operating systems are often cheap; not all hotels engage in price gouging at their restaurants.

## 7 Notation

a Maximum willingness to pay for consumers
CS Consumer surplus
$c_{i}$ Marginal cost of good $i$
$f$ Unit license fee
$K_{i}$ Maximum level of price discrimination for good $i$
$P_{i}$ Price for good $i$
$Q_{i}$ Quantity of good $i$
$s$ Subsidy for per unit of consumable sold
$\Pi_{i}$ Profits from selling good $i$
$\Pi$ Total profits
$N_{H}$ The number of high valuers
$N_{L}$ The number of low valuers

## Appendix

## A1: Selling to low valuers when $N_{H} \neq N_{L} \geq 1$

Here we show that Proposition 6 continues to hold for $N_{H}>1$ and $N_{L}>1$. Let demand for consumables by a high valuer be $q_{2 H}=a_{H}-P_{2}$ and that by a low valuer be $q_{2 L}=a_{L}-P_{2}$; total demand is $Q_{2}=N_{H} q_{2 H}+N_{L} q_{2 L}$.

When the monopolist does not sell a durable good to low valuers, demand for consumables is $Q_{2}=N_{H} Q_{2 H}=N_{H}\left(a_{H}-P_{2}\right)$. Then, the equilibrium has

$$
\begin{aligned}
& P_{2}^{N}=a_{H} / 2, \quad Q_{2}^{N}=Q_{2 H}^{N}=N_{H} a_{H} / 2, \quad \Pi_{2}^{N}=N_{H} a_{h}^{2} / 4, \\
& C S_{H}^{N}=P_{1 H}^{N}=a_{H}^{2} / 8, \quad \Pi_{1}^{N}=N_{H}\left(P_{1 H}^{N}-c_{1}\right), \quad \Pi^{N}=N_{H}\left(3 a_{H}^{2} / 8-c_{1}\right) .
\end{aligned}
$$

When the monopolist sells the durable good to both types of consumers, aggregate demand for consumables is

$$
Q_{2}= \begin{cases}N_{H}\left(a_{H}-P_{2}\right) & \text { for } P_{2}>a_{L} \\ \left(N_{H} a_{H}+N_{L} a_{L}\right)-\left(N_{H}+N_{L}\right) P_{2} & \text { for } P_{2} \leq a_{L}\end{cases}
$$

Then, in equilibrium

$$
\begin{aligned}
& P_{2}^{Y}=\frac{N_{H} a_{H}+N_{L} a_{L}}{2\left(N_{H}+N_{L}\right)}, \quad Q_{2 H}^{Y}=\frac{N_{H}\left(3 N_{H} a_{H}-N_{L} a_{L}\right)}{2\left(N_{H}+N_{L}\right)}, \quad Q_{2 L}^{Y}=\frac{N_{L}\left(3 N_{L} a_{L}-N_{H} a_{H}\right)}{2\left(N_{H}+N_{L}\right)}, \\
& Q_{2}^{Y}=\frac{N_{H} a_{H}+N_{L} a_{L}}{2}, \quad \Pi_{2}^{Y}=\frac{\left(N_{H} a_{H}+N_{L} a_{L}\right)^{2}}{4\left(N_{H}+N_{L}\right)}, \\
& C S_{H}^{Y}=P_{1 H}^{Y}=\frac{\left[N_{H} a_{H}+N_{L}\left(2 a_{H}-a_{L}\right)\right]^{2}}{8\left(N_{H}+N_{L}\right)^{2}}, \quad C S_{L}^{Y}=P_{1 L}^{Y}=\frac{\left[N_{L} a_{L}+N_{H}\left(2 a_{L}-a_{H}\right)\right]^{2}}{8\left(N_{H}+N_{L}\right)^{2}}, \\
& \Pi_{1}^{Y}=N_{H} C S_{H}^{Y}+N_{L} C S_{L}^{Y}-\left(N_{H}+N_{L}\right) c_{1} \\
& \Pi^{Y}=\Pi_{1}^{Y}+\Pi_{2}^{Y} .
\end{aligned}
$$

To assure that $\Pi_{2}^{Y}>\Pi_{2}^{N}$ (that is, to lead consumers to believe that the firm will profit from selling the durable good and consumables to low valuers), we assume that

$$
\frac{\sqrt{N_{H}\left(N_{H}+N_{L}\right)}-N_{H}}{N_{L}}<\frac{a_{L}}{a_{H}} \leq 1 .
$$

Comparing $\Pi^{N}$ with $\Pi^{Y}$ yields

$$
\Pi^{Y}>\Pi^{N} \Leftrightarrow 0 \leq c_{1}<\bar{c}_{1} \equiv \frac{N_{H}\left(a_{H}-a_{L}\right)^{2}+3\left(N_{H}+N_{L}\right) a_{L}^{2}}{8\left(N_{H}+N_{L}\right)} .
$$

The sellers therefore sells to low valuers unless $c_{1}>\bar{c}_{1}$.
Then we have

$$
P_{1 L}^{Y}-\bar{c}_{1}=-\frac{2 a_{H} a_{L} N_{H}^{2}+a_{H}^{2} N_{H} N_{L}+3 a_{L}^{2} N_{H} N_{L}+2 a_{L}^{2} N_{L}^{2}}{8\left(N_{H}+N_{L}\right)^{2}}<0,
$$

which indicates that the monopolist profits from selling the durable good to low valuers even at a price below marginal cost.

## A2: Imperfect price discrimination when $N_{H} \neq N_{L} \geq 1$.

Here, we show that Proposition 7 holds when $N_{H} \neq N_{L} \geq 1$ unless $N_{H}$ is much smaller than $N_{L}$.

Demand for consumers by a high valuer is $q_{2 H}=a_{H}-P_{2 H}$; demand by a low valuer is $q_{2 L}=a_{L}-P_{2 L}$. We assume that $\left|P_{i H}-P_{i L}\right| \leq K_{i}$ for $i=\{1,2\}$, where $K_{i}$ is the (exogenously set) maximum price difference charged high valuers and low valuers for good $i$. $K_{i}=0$ means that the monopolist cannot price discriminate on good $i$.

When $K_{2}$ is large, the monopolist can perfectly price discriminate in selling consumables, and can charge the monopoly price for $P_{2 H}$ and $P_{2 L}$. The solution in period 2 is

$$
\begin{aligned}
& P_{2 H}^{D}=a_{H} / 2, \quad P_{2 L}^{D}=a_{L} / 2 \\
& C S_{H}^{D}=N_{H} a_{H}^{2} / 8, \quad C S_{L}^{D}=N_{L} a_{L}^{2} / 8, \quad \Pi_{2}^{D}=\left(N_{H} a_{H}^{2}+N_{L} a_{L}^{2}\right) / 4
\end{aligned}
$$

Therefore, the monopolist can perfectly price discriminate in selling consumables if $K_{2} \geq P_{2 H}^{D}-$ $P_{2 L}^{D}=\left(a_{H}-a_{L}\right) / 2$.

When $K_{2}<P_{2 H}^{D}-P_{2 L}^{D}=\left(a_{H}-a_{L}\right) / 2$, the monopolist cannot perfectly price discriminate. The monopolist then chooses $P_{2}$ to maximize $\Pi_{2}=N_{H}\left[a_{H}-\left(P_{2}+K_{2}\right)\right]\left(P_{2}+K_{2}\right)+\left(a_{L}-P_{2}\right) P_{2}$, so that

$$
\begin{aligned}
& P_{2 H}^{B}=\frac{N_{H} a_{H}+N_{L} a_{L}+2 N_{L} K_{2}}{2\left(N_{H}+N_{L}\right)}, \quad P_{2 L}^{B}=\frac{N_{H} a_{H}+N_{L} a_{L}-2 N_{H} K_{2}}{2\left(N_{H}+N_{L}\right)} \\
& C S_{H}^{B}=\frac{N_{H}\left[\left(N_{H}+2 N_{L}\right) a_{H}-N_{L} a_{L}-2 N_{L} K_{2}\right]^{2}}{8\left(N_{H}+N_{L}\right)^{2}} \\
& C S_{L}^{B}=\frac{N_{L}\left[\left(2 N_{H}+N_{L}\right) a_{L}-N_{H} a_{H}+2 N_{H} K_{2}\right]^{2}}{8\left(N_{H}+N_{L}\right)^{2}} \\
& \Pi_{2}^{B}=\frac{4\left(a_{H}-a_{L}-K_{2}\right) N_{H} N_{L} K_{2}+\left(N_{H} a_{H}+N_{L} a_{L}\right)^{2}}{4\left(N_{H}+N_{L}\right)}
\end{aligned}
$$

From $d \Pi_{2}^{B} / d K_{2}=\left(a_{H}-a_{L}-2 K_{2}\right) N_{H} N_{L} /\left(N_{H}+N_{L}\right)>0$, we find that an increase in $K_{2}$ raises profits in period 2 from selling consumables.

Consider first case DD where the monopolist can perfectly price discriminate in selling both durables and consumables. This case arise when $K_{1} \geq C S_{H}^{D}-C S_{L}^{D}=\left(N_{H} a_{H}^{2}-N_{L} a_{L}^{2}\right) / 8$ and $K_{2} \geq\left(a_{H}-a_{L}\right) / 2$. In this case, $\Pi_{1}^{D D}=N_{H} P_{1 H}^{D D}+N_{L} P_{1 L}^{D D}-\left(N_{H}+N_{L}\right) c_{1}$. Total profits, $\Pi^{D D}$, are then

$$
\Pi^{D D}=\frac{N_{H}^{2} a_{H}^{2}+N_{L}^{2} a_{L}^{2}}{8}+\frac{N_{H} a_{H}^{2}+N_{L} a_{L}^{2}}{4}-\left(N_{H}+N_{L}\right) c_{1}
$$

Consider next case BD where the monopolist cannot perfectly price discriminate in selling the durable good, but can in selling consumables. This case arise when $K_{1}<\left(N_{H} a_{H}^{2}-N_{L} a_{L}^{2}\right) / 8$ and $K_{2} \geq\left(a_{H}-a_{L}\right) / 2$. The monopolist then sets $P_{1 H}$ to at most $P_{1 L}+K_{1}=C S_{L}^{D}+K_{1}$. Total profits are $\Pi^{B D}=N_{H} P_{1 H}^{B D}+N_{L} P_{1 L}^{B D}-\left(N_{H}+N_{L}\right) c_{1}+\Pi_{2}^{D}$, or

$$
\Pi^{B D}=\frac{2 N_{H} a_{H}^{2}+N_{L} a_{L}^{2}\left(2+N_{H}+N_{L}\right)}{8}-\left(N_{H}+N_{L}\right) c_{1}+N_{H} K_{1}
$$

which is increasing in $K_{1}$.
Consider next case DB where the monopolist can perfectly price discriminate in selling the durable good, but cannot perfectly in selling consumables. This case arises when $K_{1} \geq C S_{H}^{B}-$
$C S_{L}^{B}$ and $K_{2}<\left(a_{H}-a_{L}\right) / 2$. Total profits are then $\Pi^{D B}=N_{H} C S_{H}^{B}+N_{L} C S_{L}^{B}-\left(N_{H}+N_{L}\right) c_{1}+$ $\Pi_{2}^{B}$, or

$$
\Pi^{D B}=\Delta^{D B}-\left(N_{H}+N_{L}\right) c_{1}-\phi^{D B} K_{2}+\omega^{D B} K_{2}^{2}
$$

where

$$
\begin{aligned}
\Delta^{D B} & =\frac{\left\{\begin{array}{c}
N_{L}^{2} a_{L}^{2}\left[N_{H}\left(5 N_{H}+2\right)+N_{L}\left(4 N_{H}+N_{L}+2\right)\right] \\
+N_{H}^{2} a_{H}^{2}\left[N_{L}\left(5 N_{L}+2\right)+N_{H}\left(4 N_{L}+N_{H}+2\right)\right] \\
-2 N_{H} N_{L} a_{H} a_{L}\left[N_{H}^{2}+\left(N_{H}-2\right) N_{L}+\left(4 N_{L}-2\right) N_{H}\right]
\end{array}\right\}}{8\left(N_{H}+N_{L}\right)^{2}} \\
\phi^{D B}= & \frac{N_{H} N_{L}\left\{a_{H}\left[N_{H}\left(3 N_{L}+N_{H}-2\right)-2 N_{L}\right]-a_{L}\left[N_{H}\left(3 N_{L}-2\right)+N_{L}\left(N_{L}-2\right)\right]\right\}}{2\left(N_{H}+N_{L}\right)^{2}} \\
\omega^{D B}= & \frac{N_{H} N_{L}\left(N_{H} N_{L}-N_{H}-N_{L}\right)}{\left(N_{H}+N_{L}\right)^{2}}
\end{aligned}
$$

Therefore we have

$$
\frac{d \Pi^{D B}}{d K_{2}}=-\frac{N_{H} N_{L}\left\{\begin{array}{l}
a_{H}\left[N_{H}\left(3 N_{L}+N_{H}-2\right)-2 N_{L}\right] \\
-a_{L}\left[N_{H}\left(3 N_{L}-2\right)+N_{L}\left(N_{L}-2\right)\right] \\
+4\left(N_{H}+N_{L}-N_{H} N_{L}\right) K_{2}
\end{array}\right\}}{2\left(N_{H}+N_{L}\right)^{2}}
$$

When $N_{H}=N_{L}=1$, then we have $d \Pi^{D B} / d K_{2}=-K_{2} / 2<0$ as in the main body. The result of $d \Pi^{D B} / d K_{2}<0$ is more likely to hold as $N_{H}$ is larger and $N_{L}$ is smaller. ${ }^{4}$ Furthermore, if $K_{2}<\left(a_{H}-a_{L}\right) / 2$, we have that

$$
\left.\frac{d \Pi^{D B}}{d K_{2}}\right|_{N_{H}=N_{L}=N}=-\frac{N}{2}\left[(N-1)\left(a_{H}-a_{L}\right)-(N-2) K_{2}\right]<0
$$

holds for any $N \geq 1$. Therefore we find that $N_{H}=N_{L}$ is sufficient for $d \Pi^{D B} / d K_{2}<0$.
Lastly, consider case BB where the monopolist cannot perfectly price discriminate in selling durables or consumables. This case arises when $K_{1}<C S_{H}^{B}-C S_{L}^{B}$ and $K_{2}<\left(a_{H}-a_{L}\right) / 2$. Then $P_{1 L}^{B B}=C S_{L}^{B}, P_{1 H}^{B B}=P_{1 L}^{B B}+K_{1}$, and total profits are

$$
\Pi^{B B}=\Delta^{B B}-\left(N_{H}+N_{L}\right) c_{1}+N_{H} K_{1}-\phi^{B B} K_{2}+\omega^{B B} K_{2}^{2}
$$

where

$$
\begin{aligned}
\Delta^{B B} & =\frac{\left(N_{L}+2\right)\left(N_{H}^{2} a_{H}^{2}+N_{L}^{2} a_{L}^{2}\right)+2 N_{H} N_{L} a_{L}\left[2 a_{L}\left(N_{H}+N_{L}\right)-a_{H}\left(2 N_{H}+N_{L}-2\right)\right]}{8\left(N_{H}+N_{L}\right)} \\
\phi^{B B} & =\frac{N_{H} N_{L}\left[a_{H}\left(N_{H}-2\right)-a_{L}\left(2 N_{H}+N_{L}-2\right)\right]}{2\left(N_{H}+N_{L}\right)} \\
\omega^{B B} & =\frac{N_{H} N_{L}\left(N_{H}-2\right)}{2\left(N_{H}+N_{L}\right)}
\end{aligned}
$$

Then we have $d \Pi^{B B} / d K_{1}=N_{H}>0$ and

$$
\frac{d \Pi^{B B}}{d K_{2}}=\frac{N_{H} N_{L}}{2\left(N_{H}+N_{L}\right)}\left\{N_{H}\left[-a_{H}+\left(2+\frac{N_{L}}{N_{H}}\right) a_{L}+2 K_{2}\right]+2\left(a_{H}-a_{L}-2 K_{2}\right)\right\}>0
$$

[^2]This proves that, given that the monopolist cannot perfectly price discriminate for the durable good, the greater the monopolist's ability to price discriminate for consumables, the greater his profits.

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    ${ }^{1}$ The seminal paper on tie-in sales, used for price discrimination, is Burstein (1960).
    ${ }^{2}$ Hagiu (2007) reports that Apple's margins on the iPod are higher than 20 percent. On the music side, it is estimated that Apple makes less than 10 cents for every song.

[^1]:    ${ }^{3}$ See Stokey (1981), Bulow (1982), Kahn (1986), Karp and Perloff (1996), Kaserman (2007), Morita and Waldman (2004, 2010), and Board and Pycia (2013).

[^2]:    ${ }^{4}$ For example, when $a_{H}=4, a_{L}=2$, and $K_{2}=1 / 2$, then $N_{H}>\left(1-2 N_{L}+\sqrt{12 N_{L}^{2}+4 N_{L}+1}\right) / 4$ is sufficient for $d \Pi^{D B} / d K_{2}=-K_{2} / 2<0$.

