Difference-Form Persuasion Contests

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June 14, 2015

ABSTRACT: We explore the equilibrium properties of two types of "difference-form" persuasion contest functions derived in Skaperdas and Vaidya (2012) in which contestants spend resources to persuade an audience. We find that that both types of functions generate interior pure strategy Nash equilibria unlike Baik (1998) and Che and Gale (2000) with characteristics different to existing literature. For one type of function, we find that the reaction function of each player is "flat" and non-responsive to the level of resources devoted by the rival so that the "preemption effect" as defined by Che and Gale (2000) is absent. Further, the equilibrium is invariant to the sequencing of moves. For the second type of function which applies when there is asymmetry among contestants with regards to the quality of evidence, we find that the reaction functions of the stronger and weaker players have gradients with opposite signs relative to Dixit (1987) and therefore their incentive to pre-commit expenditures in a sequential move game is also different. For both types of functions, the extent of rent dissipation is partial. From the equilibrium analysis, we are also able to establish the potential effects of some specific factors affecting persuasion such as evidence potency, the degree of truth and bias on aggregate resource expenditures and welfare.

Keywords: Rent-seeking, lobbying, litigation, contest functions

JEL codes: C72; D72; K41

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1. Introduction

A large variety of economic activities can be thought to be about persuasion where competing parties attempt to influence the opinions and hence the decisions of their relevant audiences through costly production of 'information' or evidence.[1] These include among many others, advertising (Schmalensee 1972), electoral campaigning (Snyder 1989; Baron 1994; Skaperdas and Grofman 1995), marketing (Bell et al. 1975), litigation (Farmer and Pecorino 1999; Bernardo et al. 2000; Hirshleifer and Osborne 2001; Robson and Skaperdas, 2008) and rent-seeking or lobbying (Tullock 1980). In each of these settings, contest functions have often been employed to translate the resources or costly efforts employed by the competing parties into probabilities of their view prevailing over the relevant audience.[2]

The most widely applied contest functions involve variations of the Tullock or ratio form in which a contestant's winning probability depends on the ratio of her effort or resource devoted in the contest relative to that by all the competing contestants.[3] Another family of contest functions, called the "difference-form" has also been applied, albeit less so, where a contestant's probability of winning depends upon the difference of efforts or resources expended. Hirshleifer (1989) was among the first to explore the equilibrium characteristics of a logistic difference-form contest and showed that they can be considerably different from a Tullock contest.[4] More general difference-form contests have been studied by Lazear and Rosen (1981), Baik (1998) and Che and Gale (2000). Lazear and Rosen (1981) study incentive provision via a rank-order tournament by employing a symmetric difference-form contest where the win probability of player 1 takes the form $G(R_1 - R_2)$ which is a twice-continuously differentiable function of the difference in her resource expenditure to that of her rival. Baik (1998) studies 2-player difference-form contests in which the win probability of player 1 takes the form:[5]

$$p_1(R_1, R_2) = f(d) \text{ where } d = \sigma R_1 - R_2, \sigma > 0, \ f' > 0, \quad f'' < 0 \text{ for } d > 0 \text{ and } f(-d) = 1 - f(d) \qquad (1)$$

Che and Gale (2000) examine 2-player contests involving the piece-wise linear difference-form contest function as given by:[6]

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[1] For a recent survey on the quantitative impact of such persuasion activities in voting, marketing and financial markets, see DellaVigna and Gentzkow (2010).

[2] See Corchon (2007) and Jia et al. (2013) for an overview of the theoretical foundations and applications of contest functions.

[3] Tullock (1980) originally applied this functional form in the context of rent-seeking.

[4] For applications of such logistic form contest to rent-seeking, see Munster and Staal (2011, 2012). See Hwang (2012) for an exploration of a more generalized version of the logistic form contest.

[5] The win probability of player 2 is always $1 - p_1(R_1, R_2)$. Hirshleifer (1989) can be understood as a special case of (1).

[6] Pelosse (2014) and Polishchuk and Tonis (2013) illustrate specific conditions under which a difference-form contest function of the type in (2) can arise as optimal allocation mechanism for the prize allocator in a rent-seeking context. Corchon and Dahm (2010, 2011) provide alternative positive and normative foundations for this contest function and also extend it to a 3-player setting. Their formulation allows for

$$p_1(R_1, R_2) = \max\left\{\min\left\{\tfrac{1}{2} + s(R_1 - R_2),\ 1\right\}, 0\right\} \text{ where } s > 0 \tag{2}$$

In contrast to the Tullock contest, these difference-form contest functions are not homogenous of degree zero in resource expenditures.[7] Further, Baik (1998) and Che and Gale (2000) do not support pure-strategy Nash equilibria where all competing parties expend positive amount of resources. In contrast, the serial contest examined by Alcalde and Dahm (2007) and the relative difference contest success function (RDCSF) examined by Bevia and Corchon (2015) satisfy the homogeneity property and admit strictly interior pure strategy Nash equilibria under certain parameter restrictions.

While some of the above discussed contest functions have been applied in many contexts involving persuasion (such as litigation, political campaigns and lobbying), until recently, the persuasion process by which resources expended by the contestants translate into win probabilities governed by such functions has not been clarified. In the lobbying context, resources expended by competing sides to influence a decision maker are often considered venal – where they are interpreted as transfers or bribes.[8] However, such an interpretation does not encompass lobbying activities that can be naturally thought of as persuasion even when no bribes are exchanged. Skaperdas and Vaidya (2012) explicitly derive the contest functions as win probabilities in a game of persuasion where competing parties invest resources to produce evidence from which an audience updates its priors using Bayesian inference. They show that both the ratio-form and a difference-form contest function can be derived as an outcome of such a process. The equilibrium characteristics of the ratio-form contest are well known and have already been extensively examined in the contest literature.[9]

The symmetric version of the difference-form contest function derived by Skaperdas and Vaidya (2012) takes the form:

$$p_1(R_1, R_2) = \frac{1}{2} + \frac{\alpha}{2}\left[h(R_1) - h(R_2)\right] \tag{3}$$

Under certain asymmetries between players, it takes the form:

$$p_1(R_1, R_2) = (1-\gamma) + \gamma\left\{\left[\left(\tfrac{\Gamma-1}{\Gamma}\right)h(R_1) - \left(\tfrac{1-\delta}{\delta}\right)h(R_2)\right] + \left[\left(\tfrac{1-\delta}{\delta}\right) - \left(\tfrac{\Gamma-1}{\Gamma}\right)\right]h(R_1)h(R_2)\right\} \tag{4}$$

Relative to the difference-form contests functions discussed previously, the functional forms in (3) and (4) have the advantage of being explicitly grounded in the persuasion context with the parameters having natural inferential interpretations thus making these functions particularly suitable to contests aimed at persuading a relevant

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non-linearity of the type R_i^σ where $\sigma > 0$. Grossman and Helpman (1996) apply this function to determine the effect of campaign contributions on voting behavior of "uninformed" voters.

[7] See Skaperdas (1996) for a discussion of this property.

[8] See for example, Grossman and Helpman (1994).

[9] See Perez-Castrillo and Verdier (1992) and Nitzan (1994).

audience such as marketing, advertising, electoral campaigning, litigation and lobbying.[10] In (4), $0 < \gamma < 1$ represents the audience's decision threshold, while $\frac{\Gamma-1}{\Gamma}$ and $\frac{1-\delta}{\delta}$ represent the evidence potencies of the competing players.[11] Biases can arise when $\gamma \neq \frac{1}{2}$ so that the bar is relatively higher for one of the parties or when there are differences in evidence potencies. When $\gamma = \frac{1}{2}$ and $0 < \frac{\Gamma-1}{\Gamma} = \frac{1-\delta}{\delta} = \alpha < 1$, (4) degenerates to (3). By studying the equilibrium characteristics of these contest functions, we are therefore able to establish the distinct impacts of such persuasion parameters on equilibrium level of resources and win probabilities. We find that a symmetric increase in the quality of evidence available to each side has the potential of intensifying resource expenditures into the contest under some conditions. When evidence qualities differ, so that there is evidence bias in favor of one player, it is possible for the weaker player to have a higher marginal incentive to invest resources relative to the stronger player in an attempt to offset the bias. Threshold bias which leads to one side facing a higher bar to prove its case may not lead to asymmetry in resource expenditures but affects the level of rent dissipation. [12] We also find that when production of evidence favors the side with the truth, equilibrium choices may reinforce the initial advantage to the truthful side.

Qualitatively, (3) is different from the piece-wise linear difference-form contest examined by Che and Gale (2000) as specified in (2) due to the non-linearity induced by the evidence realization probability function $h(.)$ which by assumption is bounded between 0 and 1, strictly increasing and strictly concave in the resources expended by each party, $R_i, i = 1, 2$. (3) is also different from the difference-form functions examined by Lazear and Rosen (1981) and Baik (1998) (as in (1)) as it represents an additive concave transformation of the differences in resources. The form in (4) is further apart

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[10] As discussed by Bevia and Corchon (2015), in some persuasion contexts such as voting and marketing, a fraction of the audience may exhibit loyalty to one of the contenders regardless of the expenditures incurred by either contestant. Such behavior is hard to reconcile with Tullock contest where the win probability is either 0 or 1 depending on whether a player chooses not to invest any resources or invest infinitely large amount of resources. The difference-form functions given by (3) and (4) can accommodate such behavior as the win probabilities are bounded away from 0 regardless of the level of resources invested by the competing players.

[11] To gain further intuition, the form in (4) can be re-arranged to $p_1(R_1, R_2) = (1-\gamma) + \gamma\left[(\frac{\Gamma-1}{\Gamma})h(R_1)(1-h(R_2)) - (\frac{1-\delta}{\delta})h(R_2)(1-h(R_1))\right]$. Notice that $h(R_i)(1-h(R_j))\ \ i, j = 1, 2, i \neq j$ represents the ex-ante probability of the event where only player *i* gets the evidence and thus gains from shifting the posterior probability of the audience in its favor. The form in (4) naturally attaches a higher weight to this event for the player with the stronger evidence. Hence in an otherwise symmetric situation where $\gamma = \frac{1}{2}$ and $R_1 = R_2 = R$, $p_1(R_1, R_2)$ reduces to $p_1(R, R) = \frac{1}{2} + \frac{1}{2}h(R)(1-h(R))\left[(\frac{\Gamma-1}{\Gamma}) - (\frac{1-\delta}{\delta})\right]$. Intuitively, in such a situation, a player has an advantage ($p_1(R, R) > \frac{1}{2}$) only if she has stronger evidence ($(\frac{\Gamma-1}{\Gamma}) > (\frac{1-\delta}{\delta})$). Also notice that (4) naturally collapses to (3) when evidence potencies are identical and $\gamma = \frac{1}{2}$.

[12] Let the prize of each player *i* be denoted as v_i, $i = 1, 2$. Throughout the paper, rent dissipation is defined as $\frac{\sum_{i=1}^{2} R_i}{\text{Max}\{v_1, v_2\}}$. Hence rent dissipation is considered partial when $\sum_{i=1}^{2} R_i < \text{Max}\{v_1, v_2\}$.

from (1) and (2) due to the presence of a cross-product term $h(R_1)h(R_2)$ which alters the marginal incentive of the two parties to invest resources in favor of the weaker side as examined in detail in the subsequent sections. However like (1) and (2), (3) and (4) do not exhibit the homogeneity property discussed earlier which is satisfied by the difference-forms examined by Alcalde and Dahm (2007) and Bevia and Corchon (2015).

We find that contests involving (3) and (4) can yield interior pure strategy Nash equilibria as in Lazear and Rosen (1981), Alcalde and Dahm (2007) and Bevia and Corchon (2015) where both parties contribute positive resources in equilibrium unlike Che and Gale (2000) and Baik (1998). Further (3) and (4) allow for both corner and strictly interior pure strategy equilibria with characteristics that are different to existing literature. In the case of (3) (and some of its asymmetric variants), we find that the reaction function of each contestant is "flat", that is, independent of the level of resources devoted by the rival as the contest induced by the additive form (3) is inherently non-strategic. Hence the equilibria of the simultaneous and sequential move games are identical and involve dominant strategies. Further, the "preemption effect" as defined by Che and Gale (2000) is absent. Hence an increase in the prize of the higher-stake player does not reduce aggregate resource spending.[13] In case of (4), which applies when contestants differ with regards to the quality of evidence, the reaction functions of the stronger and weaker players have gradients with opposite signs relative to Dixit (1987). Hence under (4), while the stronger player reduces her resource investment if the rival expends more, the opposite is true for the weaker player as the latter has a greater incentive to invest resources at the margin. Due to this, the players' pre-commitment incentives in a sequential move contest involving (4) can be different to those identified in Dixit (1987).[14] Further in contrast to the Tullock contest, we find that increasing asymmetry among players can lead to higher aggregate resource expenditures in some circumstances.

The paper is organized as follows. Section 2 provides a brief introduction to the difference-form contest functions of the type (3) and (4) as derived in Skaperdas and Vaidya (2012) hereafter referred to as "persuasion functions". Section 3 examines equilibria of contests involving the symmetric persuasion function as given by (3). Section 4 examines equilibrium behavior involving asymmetric versions of (3) as well as those involving (4). In all such cases, to isolate the effect of the specific asymmetry introduced, all other aspects of the game are left symmetric for the two contestants. In

[13] Similar to Alcalde and Dahm (2007), the way preemption effect is defined matters in our set up. Che and Gale (2000) define it as the negative impact on aggregate resource expenditures of an increase in the prize of the higher-stake player. By this definition, there is no preemption effect under (3). In Alcalde and Dahm (2007), preemption effect as defined by Che and Gale (2000) exists for a range of values of the scale parameter of the serial contest. Alcalde and Dahm (2007) also provide an alternative definition of preemption effect which is a decrease in aggregate resource expenditure due to a decrease in the prize of the lower-stake player. When defined this way, preemption effect exists in contests defined by (3) and Alcalde and Dahm (2007). However, unlike the latter where both players reduce their spending, in our case only the weaker player reduces her spending. Preemption effect always exists in Che and Gale (2000) using either definition.

[14] Bevia and Corchon (2015) use a symmetric RDCSF with asymmetric stakes to find that players' pre-commitment incentives are identical to those of Dixit (1987).

both these sections, we also explore the impact of changes in various persuasion parameters on aggregate resource spending and welfare. Section 5 concludes.

2. An introduction to difference-form persuasion functions

In this section, we briefly review the difference-form persuasion function as derived by Skaperdas and Vaidya (2012) as an outcome of a stochastic evidence production process. In their setting, two players (denoted by subscript $i = 1, 2$) compete to gather and present evidence in order to influence the verdict of a third party audience in their favor. Each player *i* can either produce a discrete piece of evidence in her favor denoted by e_i, or offer no evidence, denoted by e_ϕ. The production of such evidence is stochastic so that the amount of resources devoted by player *i* as denoted by R_i enhances her probability of finding favorable evidence $h_i(R_i)$.[15] It is assumed that $0 < h_i(R_i) < 1$, $h_i'(R_i) > 0$ and $h_i''(R_i) < 0$. Thus depending on evidence realization, there are four possible states of the world that can be observed by the third party audience: (e_1, e_2), (e_1, e_ϕ), (e_ϕ, e_2) and (e_ϕ, e_ϕ) occurring with the following probabilities: $h_1(R_1)h_2(R_2)$, $h_1(R_1)[1-h_2(R_2)]$, $[1-h_1(R_1)]h_2(R_2)$, and $[1-h_1(R_1)][1-h_2(R_2)]$ respectively. Each of these alternative states of the world can induce the audience to revise its prior probability of Player 1 being the "correct" side denoted as π ($0 < \pi < 1$) with a posterior $\pi^*(e_i, e_j)$ where $i = 1, \phi$ and $j = 2, \phi$. Skaperdas and Vaidya (2012) employ the following parameterization:[16]

$$\pi^*(e_\phi, e_\phi) = \pi^*(e_1, e_2) = \pi \, ;$$

$$\pi^*(e_\phi, e_2) = \delta\pi \ \textit{for some}\ \delta \in (0,1) \, ;$$

$$\pi^*(e_1, e_\phi) = \begin{cases} \Gamma\pi & \textit{if}\ \Gamma \le 1/\pi \\ 1 & \textit{if}\ \Gamma > 1/\pi \end{cases} \quad \textit{where}\ \Gamma > 1$$

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[15] As an example, consider the Obama campaign's investment of resources of over \$3 million into collecting and studying information about potential supporters as gleaned from Facebook, voter logs, and telephone and in-person conversations with an objective to deliver personalized messages that are most likely to be effective in mobilizing potential voters. For details see "Obama Mines for Voters with High-Tech tools", New York Times, March 8, 2012. Alternatively, one could also view the "evidence" as winning an endorsement from an entity considered as credible by the decision maker. One could also view this process as shopping for the right delegate to represent their case where the quality of the delegate has a bearing on how the case is represented and therefore on how the decision maker rules.

[16] Notice that the posterior probability of the audience responds purely to the evidence in front of it and does not take into account the strategies of the competing parties in terms of the resources they put into the contest. This follows from the "limited-world" Bayesian assumption about the audience in Skaperdas and Vaidya (2012). For some empirical and experimental evidence on such non-strategic inference by specific audiences see DellaVigna and Gentzkow (2010), Malmendier and Shanthikumar (2007), De Franco et al. (2007) and Cain et. al. (2005). Eyster and Rabin (2010) examine a model where the receivers of information adjust too little for sender's credibility.

Assuming that (i) the audience employs a threshold rule and decides in favor of Player 1 iff $\pi^*(e_i, e_j) > \gamma$ ($0 < \gamma < 1$) (ii) γ is common knowledge and (iii) the two players don't observe π but have a common uniform prior over it, it is shown that when $\delta > \gamma$ the win probability of player 1 takes the following difference-form:[17]

$$p_1(R_1, R_2) = (1-\gamma) + \gamma\{[(\tfrac{\Gamma-1}{\Gamma})h_1(R_1) - (\tfrac{1-\delta}{\delta})h_2(R_2)] + [(\tfrac{1-\delta}{\delta}) - (\tfrac{\Gamma-1}{\Gamma})]h_1(R_1)h_2(R_2)\} \tag{5}$$

Player 2's win probability is given by $p_2(R_1, R_2) = 1 - p_1(R_1, R_2)$.

The above discussed discrete evidence setting bears resemblance to Dewatripont and Tirole (1999) who examine a principal's problem of incentivizing agents to collect evidence on competing causes via decision-based rewards. They identify reward or "prize" structures under which a contest between advocates of two competing causes produces relevant evidence at a lower cost compared to a single agent collecting evidence for both the causes. This is different from Skaperdas and Vaidya (2012) whose focus is positive characterization of a persuasion contest. Accordingly in their setting, the prize structure is exogenous and the model considers dichotomous outcomes where either Player 1 or 2 always win. Such positive characterization allows the parameters of $p_1(R_1, R_2)$ to have natural inferential interpretations.

As briefly discussed in the introduction, the level of γ captures potential bias in decision threshold level used by the audience. When $\gamma < \frac{1}{2}$, there is bias in favor of Player 1 as the bar for posterior probability is lowered and the audience is more easily convinced about Player 1's position and vice versa. $\left(\frac{\Gamma-1}{\Gamma}\right)$ represents the "evidence potency" or the inferential power of Player 1's evidence (e_1). Notice that intuitively, it increases in Γ (the factor by which the audience's prior is revised in favor of Player 1). Similarly, $\left(\frac{1-\delta}{\delta}\right)$ represents Player 2's evidence potency and intuitively, it declines with δ. Notice that these parameters allow for various asymmetries so that $p_1(R_1, R_2) \neq p_2(R_1, R_2)$ when $R_1 = R_2$. The sources of such asymmetries include a bias in the threshold ($\gamma \neq \frac{1}{2}$), or differences in evidence potency as captured via $\left(\frac{\Gamma-1}{\Gamma}\right)$ and $\left(\frac{1-\delta}{\delta}\right)$ as well as differences in the evidence production functions as embodied in $h_i(R_i)$. The implications of these asymmetries for equilibrium behavior of the contestants are explored in detail in Section 4.

When $h_i(.) = h_j(.) = h(.)$, (5) simplifies to (4). In addition, when $\gamma = \frac{1}{2}$ and $\left(\frac{\Gamma-1}{\Gamma}\right) = \left(\frac{1-\delta}{\delta}\right) = \alpha$, (5) simplifies further to the symmetric form in (3).

[17] When $\delta \leq \gamma$, Player 1's win probability from the (e_ϕ, e_2) state is 0. This leads to δ term dropping out from (5) and therefore some changes to the coefficients for $h_2(R_2)$ and $h_1(R_1)h_2(R_2)$. However the analytical form of the persuasion function and the presence of the cross-product term persists as per (5). Hence for brevity, we only consider the $\delta > \gamma$ case in the paper. A derivation of (5) can be found in Skaperdas and Vaidya (2012).

Denoting player *i*'s valuation of the prize as $v_i > 0$, the expected payoff to player *i*, $i = 1, 2$ is given by:

$$U^i(R_i, R_j) = p_i(R_i, R_j)v_i - R_i \text{ for } i, j = 1, 2 \text{ and } i \neq j \tag{6}$$

In the above expression, let $p_i(R_i, R_j)$ be given by (3) or (4). In the paper, we will examine simultaneous move contests where player *i* chooses R_i to maximize U^i taking R_j as given for $i, j = 1, 2$ and $i \neq j$ except when we specifically alter the timing of moves. Notice that for $i, j = 1, 2$ and $i \neq j$, since $0 < p_i(R_i, R_j) < 1$, it follows that $U^i > 0$ at $R_i = 0$ for any R_j. Further, $U^i < 0$ at $R_i = v_i$ for any R_j. Given this, the strategy space for each player can be restricted to the interval $R_i \in [0, \max\{v_1, v_2\}]$, $i = 1, 2$ without loss of generality. Throughout the paper we also assume strict concavity of $h(.)$ over $R_i \in [0, \max\{v_1, v_2\}]$ to facilitate sufficient conditions for interior equilibria. These conditions are stated explicitly in Assumption 1 which is assumed to hold throughout the paper.

Assumption 1: *Let the strategy space of player* $i = 1, 2$ *be given by* $R_i \in [0, \max\{v_1, v_2\}]$ *and* $h(R_i)$ *be differentiable and strictly concave over this interval with* $0 \le h(R_i) < 1,\ h'(R_i) > 0,$ *and* $h''(R_i) < 0$.

Examples of functional forms of $h(.)$ that satisfy Assumption 1 include:

$$h(R_i) = \frac{R_i + \psi}{R_i + 1},\ 0 < \psi < 1 \tag{7}$$

$$h(R_i) = \sqrt{\tfrac{R_i}{K}},\ 0 \le R_i < K \text{ and } K > \max\{v_1, v_2\} \tag{8}$$

Assumption 1 ensures that U^i is strictly concave with respect to R_i for any given R_j over the interval $R_i \in [0, \max\{v_1, v_2\}]$. Hence, when the first order condition for maximization of U^i with respect to R_i holds, the second-order condition is always satisfied. In the subsequent sections we examine the equilibrium characteristics of contests involving both the symmetric and the asymmetric versions of the difference-form persuasion functions as in (3) and (4). While examining the asymmetric cases, we consider the effect each type of asymmetry can have on the equilibrium characteristics.

3. Equilibrium behavior under symmetric difference-form persuasion function

In this section, we examine contests involving the symmetric difference-form persuasion function given by (3). We assume $v_1 \ge v_2$ without loss of generality.

With simultaneous choice of resources, each player's decision problem involves maximizing her expected payoff U^i as given by (6) with respect to R_i taking R_j as given and $p_i(R_i, R_j)$ given by (3). Given Assumption 1, as long as $h'(0) > \frac{2}{\alpha v_i}$, $i = 1, 2$, the reaction functions of the two players are given by:

$$h'(R_i^*) = \frac{2}{\alpha v_i} \quad \text{for } i = 1, 2 \tag{9}$$

The characteristics of the Nash equilibrium are presented in Proposition 1.[18]

Proposition 1: *Under a symmetric difference-form persuasion function as in (3) when players choose their resources simultaneously,*

(i) *the reaction function of each player is independent of the resources devoted by her rival and a dominant strategy equilibrium always exists*

(ii) *If* $h'(0) > \frac{2}{\alpha v_i}, i = 1, 2$ *then the equilibrium involves both players investing in positive but potentially different level of resources with* $R_i^* = (h')^{-1}(\frac{2}{\alpha v_i})$ *. Along such an equilibrium,* R_i^* *increases with the level of evidence potency* α *and the player's own valuation of the prize* v_i*. It is invariant to changes in the rival player's valuation of its prize* v_j*.*

(iii) *If* $\frac{2}{\alpha v_2} \geq h'(0) > \frac{2}{\alpha v_1}$ *then the equilibrium involves* $R_1^* = (h')^{-1}(\frac{2}{\alpha v_1})$ *and* $R_2^* = 0$.

(iv) *If* $h'(0) \leq \frac{2}{\alpha v_i}, i = 1, 2$ *then in equilibrium neither player invests any resources towards the contest.*

(v) *When* $v_1 = v_2$*, the equilibrium is always symmetric with either both players investing the same positive level of resources into the contest or both investing zero resources to it.*

(vi) *There is always partial rent dissipation in equilibrium as* $\sum_{i=1}^{2} R_i^* < \text{Max}\{v_1, v_2\}$.

Proposition 1 implies that unlike Baik (1998) and Che and Gale (2000), persuasion function (3) can support both corner and strictly interior pure strategy equilibria depending on the valuations of the prize and the sensitivity of $h(R_i)$ to resources.[19] Further, since each player is assured of a positive payoff even if she were to

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[18] See the Appendix for a proof.

[19] With linear resource costs, (3) is able to support strictly interior pure strategy equilibria unlike Baik (1998) and Che and Gale (2000) as strict concavity of $h(.)$ makes the expected payoffs of each player

not expend any resources towards the contest regardless of the rival player's choice, it follows that rent dissipation is always partial. Interestingly, when each side has access to more compelling piece of evidence (higher α), it leads to a higher level of rent dissipation.

It is also interesting to note that the reaction function of each player is independent of the rival's effort (unlike Lazear and Rosen (1981) and Baik (1998)) due to the inherent non-strategic nature of the contest. There are two implications from this. Firstly, as per Proposition 1 (iii), the rival player's valuation of the prize has no impact on a player's equilibrium choice of resource spending. Therefore preemption effect as defined by Che and Gale (2000) is absent.[20] Secondly, the equilibria of the simultaneous move game are identical to that of a sequential move game regardless of who moves first. These findings are summarized in corollary 1.

Corollary 1: *Under a symmetric difference-form persuasion function as in (3):*

(i) There is no preemption effect as $\sum_i R_i^*$ *is non-decreasing in* v_1.

(ii) The equilibria of a simultaneous game are identical to those of a sequential game where one of the two players choose resource level first relative to the rival.

With the symmetric persuasion function (3), the only source of asymmetry between the players is any difference between their stakes. To examine the implications of stake asymmetry on equilibrium behavior, we make the following assumption:

Assumption 2: *Let* $v_1 = v + \omega$, $v_2 = v - \omega$ *where* $v > 0$ *and* $|\omega| < v$.

In this case, the assumption of $v_1 \geq v_2$ is equivalent to assuming $\omega \geq 0$.

Using the representation in Assumption 2, we examine the implications of changes in evidence potency α and stake asymmetry ω on aggregate resource expenditures and aggregate welfare. The findings are presented in Proposition 2.

Proposition 2: *Suppose that Assumption 2 and a symmetric difference-form persuasion function as in (3) apply. Then along the interior dominant strategy equilibrium of a simultaneous move game,*

(i) An increase in α *always increases aggregate resource spending. The impact of an increase in* ω *on aggregate resource spending is however ambiguous in general.*

(ii) Aggregate welfare always decreases with α *if* $\omega = 0$. *When* $\omega > 0$, *then the impact of an increase in* α *and* ω *on aggregate welfare is ambiguous in general.*

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strictly concave in their resource spending. In principle, similar pure strategy equilibria could arise under Baik (1998) and Che and Gale (2000) if the resource cost function were assumed to be strictly convex.

[20] See footnote 13 for a comparison with Alcalde and Dahm (2007).

(iii) When $h(.)$ is given by (7), an increase in ω decreases aggregate resource spending and increases aggregate welfare.

(iv) When $h(.)$ is given by (8), an increase in α increases aggregate welfare if ω is sufficiently high. An increase in ω increases aggregate resource spending and aggregate welfare.

By inspecting (9) it is easy to appreciate that $\frac{\partial R_i^*}{\partial \alpha} > 0$ for $i = 1,2$ given the strict concavity of $h(.)$ along the interior Nash equilibrium. From this it follows immediately that aggregate resource spending increases with α. To understand the implications for aggregate welfare $U = U^1(R_1, R_2) + U^2(R_1, R_2)$, notice that from equation (6) and Assumption 2, we have,

$$U = v + \alpha\left[h(R_1) - h(R_2)\right]\omega - R_1 - R_2 \tag{10}$$

Hence when $\omega = 0$, $U = v - R_1 - R_2$ so that as aggregate resources increase with α, U decreases with α. When $\omega > 0$, the impact of an increase in α on U is less clear cut. By differentiating (10) with respect to α and using the first-order conditions (9), we find that,

$$\frac{dU^*}{d\alpha} = \left[h(R_1^*) - h(R_2^*)\right]\omega - \left(\frac{v-\omega}{v+\omega}\right)\frac{\partial R_1^*}{\partial \alpha} - \left(\frac{v+\omega}{v-\omega}\right)\frac{\partial R_2^*}{\partial \alpha} \tag{11}$$

Notice that since $\omega > 0$, it follows from (9) that $R_1^* > R_2^*$ and hence the first component in (11) is positive. This shows that by increasing the win probability of the player with the higher stake, an increase in α contributes positively to aggregate welfare. However, from Assumption 2 both $\frac{v-\omega}{v+\omega}$ and $\frac{v+\omega}{v-\omega}$ are positive, and $\frac{\partial R_i^*}{\partial \alpha} > 0$ for $i = 1,2$. Hence each of the remaining two components in (11) contribute to a decrease in aggregate welfare. This is because a higher α also stimulates higher aggregate spending. Hence the overall welfare impact of an increase in α is in general ambiguous. However, as the case of $h(.)$ given by (8) suggests, it is plausible that if ω is sufficiently high, the first component might dominate causing aggregate welfare to increase with α.

When ω increases, it follows from Assumption 2, equation (9) and strict concavity of $h(.)$ that $\frac{\partial R_1^*}{\partial \omega} > 0$ while $\frac{\partial R_2^*}{\partial \omega} < 0$. Hence the net effect of an increase in ω on aggregate resource spending is in general ambiguous. To appreciate its impact on aggregate welfare, notice from (10) that its direct contribution to aggregate welfare is positive as it tends to increase aggregate expected payoff since $R_1^* > R_2^*$ and so $h(R_1^*) > h(R_2^*)$. It also contributes positively to aggregate welfare by instigating Player 2 to cut her resource spending. However since Player 1 is induced to increase her expenditure, this contributes negatively to aggregate welfare. As a result, the overall impact of an increase in ω on aggregate welfare is ambiguous. Interestingly, we get clear results for the case of an increase in ω for both the specific forms of $h(.)$ as given by (7) and (8). For $h(.)$ given by (7), an increase in ω decreases aggregate resource spending and increases welfare. For $h(.)$ given by (8), both aggregate spending and welfare increase with ω. This possibility that an increase in asymmetry between players can lead to an increase in aggregate resource spending is surprising and in contrast to the Tullock contest case where as shown in Konrad (2009) aggregate spending is inversely related to asymmetry between players.[21]

4. Equilibrium characteristics under asymmetric persuasion functions

As discussed briefly in Section 2, various factors can give rise to asymmetry in the persuasion function described in (5). In this section we will explore the implication of each of those sources of asymmetry for equilibrium behavior of players. We begin with the case of asymmetry in the evidence production process.

4.1 Asymmetric evidence production and its impact on equilibrium spending

To focus on the effect of asymmetric evidence production, in this subsection, the following assumption will apply:

Assumption 3: *Let* $\gamma = \frac{1}{2}$, $0 < \left(\frac{\Gamma-1}{\Gamma}\right) = \left(\frac{1-\delta}{\delta}\right) = \alpha < 1$, *and* $v_1 = v_2 = v$. *However, let* $h_1(R_1) = \theta h(R_1)$ *and* $h_2(R_2) = (1-\theta)h(R_2)$ *where* $h(.)$ *follows Assumption 1, and* $\theta \in (0,1)$ with $\theta \neq \frac{1}{2}$.

Assumption 3 ensures that the asymmetry in the persuasion contest is purely due to differences in the evidence production probabilities so that $h_1(R_1) \neq h_2(R_2)$ when $R_1 = R_2$. This is due to $\theta \neq \frac{1}{2}$. Following Hirshleifer and Osborne (2001) and Robson and Skaperdas (2008), θ can be interpreted as the *degree of truth* or, in the case of litigation, as the *level of property rights protection.* For example, if the truth (or property rights) is with Player 1, then $\theta \in (\frac{1}{2}, 1)$, so that when $R_1 = R_2 = R$, $h_1(R) > h_2(R)$. This implies that the side arguing for the truth (Player 1 in this instance) will have a higher chance of getting favorable evidence when both players invest the same level of resources. The closer is θ to 1, the easier it is to argue for Player 1 who is on the side of the truth (or, the better defined property rights are). Analogously, if the truth were with Player 2, then $\theta \in (0, \frac{1}{2})$. For the sake of brevity, we will assume that $\theta \in (\frac{1}{2}, 1)$.

Given Assumption 3, the persuasion function in equation (5) reduces to:

$$p_1(R_1, R_2) = \frac{1}{2} + \frac{\alpha}{2}\left[\theta h(R_1) - (1-\theta)h(R_2)\right] \tag{12}$$

[21] See Konrad (2009), page 46. In the Tullock contest, greater symmetry contributes to higher aggregate spending by increasing the incentive to invest for both players.

Each player i's decision problem still involves choosing an appropriate level of resource spending to maximize her expected payoff U^i given by (6) except that $p_i(R_1, R_2)$ is given by (12). Using the first order conditions, the reaction functions of the two players are:

$$R_1^* = \begin{cases} (h')^{-1}(\frac{2}{\alpha\theta v}) \text{ when } h'(0) > \frac{2}{\alpha\theta v} \\ 0 \text{ otherwise} \end{cases} \tag{13}$$

$$R_2^* = \begin{cases} (h')^{-1}(\frac{2}{\alpha(1-\theta)v}) \text{ when } h'(0) > \frac{2}{\alpha(1-\theta)v} \\ 0 \text{ otherwise} \end{cases} \tag{14}$$

By inspection of (13) and (14), it is clear that the reaction functions of both players are analogous to the case of asymmetric prize valuations where $v_1 > v_2$ as examined in Section 3. Hence Proposition 1 ((i) – (iv) and (vi)) continues to apply qualitatively and so does the invariance of Nash Equilibrium to sequential moves as stated in Corollary 1. Proposition 1 (v) no longer holds as the equilibrium spending differs between players as discussed in Proposition 3.

Proposition 3: *When Assumption 3 applies and $\theta \in (\frac{1}{2}, 1)$, the truth is on the side of Player 1 who puts in more resources in equilibrium and has a higher probability of winning along a strict interior equilibrium. The closer is θ to 1, the greater is this effect. When the equilibrium consists of only one of the two players actively spending resources in the contest, Player 1 is the active player.* [22]

Proposition 3 follows from the observation that when $\theta \in (\frac{1}{2}, 1)$, we have $\frac{2}{\alpha\theta v} < \frac{2}{\alpha(1-\theta)v}$. It indicates that when the only source of asymmetry is a tilt in the evidence production towards the player arguing for the truth, this natural advantage to that player gets reinforced through the equilibrium choice of resources. Note that this is different from the Nash-Cournot equilibrium behavior in Hirshleifer and Osborne (2001) who examine a ratio-form asymmetric contest and find that both parties always put in equal resources in equilibrium regardless of the level of the degree of truth. It is also apparent that the general properties outlined in Proposition 2 regarding implications of changes in α and ω continue to hold qualitatively.

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[22] It is worthwhile to note however, that the when the valuations of the prize are asymmetric, this effect can be potentially dominated if the player who values the prize higher is not the one arguing for the truth.

4.2 Bias in the decision threshold and its impact on equilibrium behavior

To study the effect of a bias in the decision threshold, in this sub-section, we allow for $\gamma \neq \frac{1}{2}$ while suppressing other sources of asymmetry as stated in Assumption 4.

Assumption 4: *Let* $\frac{1}{1+\alpha} \geq \gamma \neq \frac{1}{2}$, $0 < \left(\frac{\Gamma-1}{\Gamma}\right) = \left(\frac{1-\delta}{\delta}\right) = \alpha < 1$, $v_1 = v_2 = v$ *and* $h_1(.) = h_2(.) = h(.)$.[23]

With Assumption 4, the persuasion function in (5) reduces to:

$$p_1(R_1, R_2) = (1-\gamma) + \gamma\alpha\left[h(R_1) - h(R_2)\right] \tag{15}$$

Since $0 \leq h(.) < 1$, as long as $\gamma \leq \frac{1}{1+\alpha}$, (15) is naturally bounded between 0 and 1 for any (R_1, R_2).

Notice that when $\gamma > \frac{1}{2}$ the decision threshold favors Player 2 as the bar is higher for Player 1 to prove her case relative to Player 2. From (15), this implies that $p_1(R_1, R_2) < p_2(R_1, R_2)$ when $R_1 = R_2$. The opposite is true when $\gamma < \frac{1}{2}$. To illustrate the implications of such threshold bias, we use (15) to construct each player's expected payoff in the game as given by (6). Given players' objectives, the maximization of the expected payoffs by the players via simultaneous choice of resources leads to the following reaction functions:

$$R_i^* = \begin{cases} (h')^{-1}\left(\dfrac{1}{\alpha\gamma v}\right) \text{ when } h'(0) > \dfrac{1}{\alpha\gamma v} \\ 0 \text{ otherwise} \end{cases} \text{ for } i = 1, 2 \tag{16}$$

Notice that the asymmetry in the persuasion function due to $\gamma \neq \frac{1}{2}$ does not lead to an asymmetry in equilibrium expenditures. This leads us to Proposition 4.

Proposition 4: *When Assumption 4 applies, asymmetry in the persuasion function due to threshold bias does not lead to asymmetry in equilibrium spending as both players invest the same amount of resources in the contest. However, the threshold bias does affect the level of equilibrium spending and therefore rent dissipation both of which increase with* γ *along the strictly interior equilibrium.*

Proposition 4 follows from condition (16) and the strict concavity of $h(.)$ which imply that $\dfrac{\partial R_i^*}{\partial \gamma} > 0, i = 1, 2$. A comparison of Proposition 4 with Proposition 3 reveals that asymmetries due to degree of truth and threshold bias have distinct effects on the equilibrium behavior of the two contestants. Proposition 1 continues to apply

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[23] Since $0 < \alpha < 1$, $\frac{1}{2} < \frac{1}{1+\alpha} < 1$.

qualitatively except that the reaction functions are given by (16) and similarly, the invariance of Nash equilibrium to sequential choice of resources by the players as stated in Corollary 1 continues to hold. The general properties outlined in Proposition 2 also continue to hold qualitatively.

4.3 Evidence bias and its impact on equilibrium behavior

We now allow for differences in the potency of evidence presented by the two contestants so that $\left(\frac{\Gamma-1}{\Gamma}\right) \neq \left(\frac{1-\delta}{\delta}\right)$. This may be due to one player naturally having access to a more convincing piece of evidence than the other. It could also be a form of bias where the audience is more receptive towards the evidence presented by one of the two contestants. In this section, we examine the impact of such evidence bias on players' equilibrium behavior. As with previous cases, we suppress other sources of asymmetry as is stated in Assumption 5.

Assumption 5: *Let* $\alpha_1 = \left(\frac{\Gamma-1}{\Gamma}\right) \neq \left(\frac{1-\delta}{\delta}\right) = \alpha_2$, $\gamma = \frac{1}{2}, v_1 = v_2 = v$ *and* $h_1(.) = h_2(.) = h(.)$. *Further* $0 < \alpha_2 < \alpha_1 < 1$ *so that the evidence bias is in favor of Player 1.*[24]

Assumption 6 presents a convenient way to represent the restrictions imposed by Assumption 5 on α_1 and α_2. We use it at a later stage to establish some additional results.

Assumption 6: *Let* $\alpha_1 = \alpha + \Delta$, $\alpha_2 = \alpha - \Delta$ where $0 < \Delta < \alpha < 1$.

When Assumption 5 applies, the persuasion function in (5) becomes:

$$p_1(R_1, R_2) = \tfrac{1}{2} + \tfrac{1}{2}\left[\alpha_1 h(R_1) - \alpha_2 h(R_2) - (\alpha_1 - \alpha_2) h(R_1) h(R_2)\right] \tag{17}$$

The win probability of player 2 is $p_2(R_1, R_2) = 1 - p_1(R_1, R_2)$. Notice that the evidence bias $(\alpha_1 - \alpha_2)$ works its way via a cross product term $h(R_1)h(R_2)$ with a negative co-efficient for player 1 which makes (17) distinct from other asymmetries examined so far. Lemma 1 identifies the basic characteristics of the persuasion function in (17):

Lemma 1:

(i) $\frac{\partial p_i}{\partial \alpha_i} = \frac{h(R_i)(1-h(R_j))}{2} > 0$ *for* $i, j = 1, 2, i \neq j$

(ii) $\frac{\partial p_i}{\partial \alpha_j} = -\frac{h(R_j)(1-h(R_i))}{2} < 0$ *for* $i, j = 1, 2, i \neq j$

(iii) $\frac{\partial p_1}{\partial R_1} = \frac{1}{2}\left[\alpha_1 - (\alpha_1 - \alpha_2) h(R_2)\right] h'(R_1) > 0$ *and* $\frac{\partial p_2}{\partial R_2} = \frac{1}{2}\left[\alpha_2 + (\alpha_1 - \alpha_2) h(R_1)\right] h'(R_2) > 0$

(iv) $\frac{\partial p_1}{\partial R_2} = -\frac{1}{2}\left[\alpha_2 + (\alpha_1 - \alpha_2) h(R_1)\right] h'(R_2) < 0$ *and* $\frac{\partial p_2}{\partial R_1} = -\frac{1}{2}\left[\alpha_1 - (\alpha_1 - \alpha_2) h(R_2)\right] h'(R_1) < 0$

(v) $\frac{\partial^2 p_1}{\partial R_1^2} = \frac{1}{2}\left[\alpha_1 - (\alpha_1 - \alpha_2) h(R_2)\right] h''(R_1) < 0$ *and* $\frac{\partial^2 p_2}{\partial R_2^2} = \frac{1}{2}\left[\alpha_2 + (\alpha_1 - \alpha_2) h(R_1)\right] h''(R_2) < 0$

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[24] The case of $\alpha_1 < \alpha_2$ is analytically identical and therefore not discussed for the sake of brevity.

(vi) $\frac{\partial^2 p_1}{\partial R_1 \partial R_2} = -\frac{1}{2}\left[(\alpha_1 - \alpha_2)h'(R_2)\right]h'(R_1) < 0$ *and* $\frac{\partial^2 p_2}{\partial R_1 \partial R_2} = \frac{1}{2}\left[(\alpha_1 - \alpha_2)h'(R_1)\right]h'(R_2) > 0$

(vii) $p_1(R_1, R_2) > p_2(R_1, R_2)$ *when* $R_1 = R_2$ *and* $\alpha_1 - \alpha_2 > 0$

Property (i) and (ii) imply that each player's win probability is increasing in the strength of its evidence and decreasing in that of its rival. Property (vii) implies that when both players put in the same level of resources the evidence bias (in favor of Player 1) provides an advantage to Player 1 in terms of a higher win probability. These properties suggest that despite the negative co-efficient in the cross-product term for Player 1, the first order effects of the evidence bias are to enhance the win probability of Player 1. However, the evidence bias does have the opposite effect on the marginal incentive to put in resources towards the contest for Player 1. This can be first appreciated by observing the marginal impact of resources on the win probabilities in (iii). Notice that the evidence bias is reducing the marginal return to Player 1's investment in the contest while it is adding to that of Player 2. Further, notice that as per (vi), when $\alpha_1 - \alpha_2 > 0$, an increase in the rival's investment of resources towards the contest discourages Player 1 while it encourages Player 2 to increase its resources. These two properties imply that the evidence bias strengthens the weaker player's marginal incentive to put in resources relative to the stronger player. Intuitively, the weaker player is tempted to invest more at the margin in an attempt to compensate for the evidence bias against her. As we will observe subsequently, this property manifests itself also via differences in the shapes of the reaction functions for the two players. The sign of the partial derivatives described in properties (iii) - (vi) follow straightforwardly from our assumption of monotonicity and strict concavity of $h(.)$.

To see how the above characteristics of the persuasion function (17) impact on equilibrium behavior, recall that in the Cournot game, each player *i* aims to maximize U^i (determined by (6) and (17)) through her choice of $R_i \in [0, v]$, taking R_j as given where $i, j = 1, 2, i \neq j$. Since $U^i > 0$ at $R_i = 0$ and $U^i < 0$ at $R_i = v$ it follows that $R_i^* < v$. The sufficient condition for a strictly interior Nash equilibrium in this game is provided by Lemma 2.

Lemma 2: *Given Assumption 5, if* $h'(0) > \frac{2}{\alpha_2 v}$*, then* $0 < R_i^* < v, i = 1, 2$ *so that both players will always invest positive level of resources into the contest regardless of the level invested by the rival.*

From Lemma 2 it follows that if $h'(0) > \frac{2}{\alpha_2 v}$, players' best responses are strictly interior and given by the first order conditions as follows:

$$h'(R_1) = \frac{2}{\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)\right]v} \tag{18}$$

$$h'(R_2) = \frac{2}{[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1)]v} \qquad (19)$$

Equation (18) represents the reaction function for Player 1 while (19) represents that of Player 2. Since U^i is strictly concave in R_i over the strategy space $[0, v]$ $i = 1, 2$ due to Assumption 1, the second-order conditions are always satisfied along (18) and (19). Notice that the evidence bias enters negatively in the reaction function of Player 1. This coupled with strict concavity of $h(.)$ implies that as R_2 increases, the optimal level of R_1 falls. Hence the reaction function of Player 1 is negatively sloped. However, for Player 2, the evidence bias enters positively in the denominator which implies that as R_1 increases, the optimal level of R_2 increases giving a positive slope to the reaction function of Player 2. These reactions functions suggest that the evidence bias makes Player 2 aggressive at the margin relative to Player 1. Since each player's expected payoff is strictly positive even if she does not put any resources into the contest, it follows that in any Nash equilibrium, it will always be the case that $v > R_1^* + R_2^*$ so that the rent dissipation will be partial. The equilibrium behavior is summarized in Proposition 5.[25]

Proposition 5: *Under Assumption 5 and* $h'(0) > \frac{2}{\alpha_2 v}$, *the Cournot equilibrium behavior is as follows***:**

(i) *Player 1's optimal resource expenditure is strictly positive as given by her reaction function (18) and is inversely related to the resource expenditure of Player 2*

(ii) *Player 2's optimal resource expenditure is strictly positive as given by her reaction function (19) and is positively related to the resource expenditure of Player 1*

(iii) *a unique interior pure strategy Cournot Nash equilibrium exists and determined by the crossing point of the reaction functions of the two players*

(iv) *In principle, the unique pure-strategy Cournot Nash equilibrium can be one of 3 kinds: (1) symmetric equilibrium with* $R_1^* = R_2^*$ *(2)* $R_1^* > R_2^*$ *so that the player favored by the evidence bias puts a higher effort in equilibrium (3)* $R_1^* < R_2^*$ *so that the weaker player puts in greater effort to counterbalance the evidence bias in equilibrium.*

(v) *There is partial rent dissipation in any Cournot Nash equilibrium.*

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[25] Proofs of Proposition 5 parts (i) and (ii) follow immediately from inspection of (18) and (19) via the preceding discussion. For proofs of parts (iii)-(vi), please see the Appendix.

(vi) *The equilibrium outlays differ in a Stackelberg game relative to the Cournot game. When Player 1 moves first, she is induced to reduce her expenditure relative to that in the Cournot game. When Player 2 moves first, she is induced to increase her expenditure relative to that in the Cournot game.*

The conditions under which the symmetric Cournot equilibrium $R_1^* = R_2^* = R^*$ holds are given by:

$$h(R^*) = \tfrac{1}{2} \tag{20}$$

$$h'(R^*) = \frac{4}{[(\alpha_1 + \alpha_2)]v} \tag{21}$$

It is apparent that the level of R^* implied by (21) depends on the values of α_1, α_2 and v and is very unlikely to be consistent with (20). Hence, in general, the symmetric equilibrium is unlikely and easily disturbed by small changes in either α_1, α_2 or v. Therefore in most instances, the Cournot equilibrium will be asymmetric with either $R_1^* > R_2^*$ or $R_1^* < R_2^*$.

When the Cournot equilibrium is such that $R_1^* > R_2^*$, it immediately follows that $p_1(R_1^*, R_2^*) > p_2(R_1^*, R_2^*)$ (given Lemma 1, (vii)). Hence along such equilibrium, the player favored by the evidence bias also puts in more resources into the contest and therefore has a higher equilibrium probability of winning. In this instance, the equilibrium choices of resource expenditures by both players reinforce the advantage conferred to Player 1 through the evidence bias. The necessary conditions for such equilibrium are:

$$1 > h(R_1^*) + h(R_2^*) \tag{22}$$

$$h(R_1^*) > h(R_2^*) \tag{23}$$

When $R_1^* < R_2^*$, the weaker player puts in greater effort in the Cournot equilibrium and at least partially offsets the disadvantage of the evidence bias favoring the rival. In this case, it is theoretically possible that $p_1(R_1^*, R_2^*) < p_2(R_1^*, R_2^*)$. If this were to happen, it represents a case where the advantage conferred to Player 1 through evidence bias is overwhelmed by the greater marginal incentive to put in resources on Player 2's part. It is useful to recall that while the evidence bias lowers the win probability of the weaker player for given levels of resources, it also provides a higher marginal incentive to her to compete in the contest. When $p_1(R_1^*, R_2^*) < p_2(R_1^*, R_2^*)$, the latter effect dominates. The necessary conditions for a Cournot equilibrium with $R_1^* < R_2^*$ are:

$$1 < h(R_1^*) + h(R_2^*) \tag{24}$$

$$h(R_1^*) < h(R_2^*) \tag{25}$$

By examining conditions (22) – (25), it is apparent that characteristics of the $h(.)$ function will determine which one of the two types of asymmetric Cournot equilibria will eventuate. This is illustrated by Corollary 2.[26]

Corollary 2: *Under Assumption 5, and* $h(.)$ *given by (7), any interior Cournot Nash equilibrium will involve* $R_1^* < R_2^*$ *if* $\frac{1}{2} < \psi < 1 - \dfrac{2}{\alpha_2 v}$ *and* $\alpha_2 v > 4$.

When (7) holds and $\psi > \frac{1}{2}$, neither (20) nor (22) can be satisfied and therefore the interior Cournot equilibrium can only involve $R_1^* < R_2^*$ in which the disadvantaged player is associated with higher resource expenditure. The sufficient conditions for such an interior Cournot equilibrium to exist (so that the condition in Lemma 2 is satisfied) require that v is adequately large while ψ is not too high. These two restrictions ensure that both players have enough marginal incentive to invest positively towards the contest.

Proposition 5 (vi) indicates that in contrast to the equilibria generated by the symmetric persuasion function (3), the equilibrium resource spending of either player is sensitive to the order in which players choose their expenditures when the persuasion function is given by (17). This is due to players' reaction functions no longer being flat as in the symmetric case. When Player 1 chooses her expenditure first, she is induced to cut her expenditure relative to the Cournot equilibrium. While a marginal cut in her expenditure has no direct impact on her payoff, it induces Player 2 to reduce her expenditure as her reaction function is positively sloped as stated in Proposition 5 (ii). Player 1 benefits from this reaction through a marginal increase in her win probability. When Player 2 chooses her expenditure first, exactly conversely she is induced to increase her expenditure relative to the Cournot equilibrium. This is because she benefits from an increase in her win probability from Player 1's response of cutting back her expenditure as her reaction function is downward sloping (as stated in Proposition 5 (i)). Hence when Player 1 is the "favorite" at the Cournot equilibrium ($p_1^* > p_2^*$ as is the case when $R_1^* \geq R_2^*$), she chooses to cut back on her expenditure when acting as a Stackelberg leader. This is in contrast to the favorite's tendency to increase her resource spending when given an opportunity to pre-commit under asymmetric logit and probit contest functions examined in Dixit (1987). Similarly, the behavior of the "underdog" Player 2 when acting as a Stackelberg leader is opposite to that in Dixit (1987). These differences arise as reaction functions of the favorite and underdog in Dixit (1987) have opposite gradients locally around the Cournot equilibrium to those generated by (17).

We now turn to some comparative statics and use (17) and assumptions 2 and 6 to study the impact of changes in evidence potency α , evidence asymmetry Δ and stake asymmetry ω on equilibrium expenditures and aggregate welfare. Similar to the findings presented in Proposition 2 for the persuasion function (3), we find that for the general

[26] Note that under the conditions in Proposition 6, the Cournot equilibrium always involves $R_1^* > R_2^*$.

case of $h(.)$ governed by Assumption 1, the impact of an increase in α, Δ and ω on aggregate resource spending and welfare is ambiguous for (17).[27] However for the specific case of $h(.)$ given by (8), under certain conditions an increase in α, Δ, or ω can lead to an increase in aggregate resource spending. These results are summarized in Proposition 6 which also includes the implications for aggregate welfare.

Proposition 6: *Under assumptions 2 and 6, persuasion function (17) and $h(.)$ given by (8),*

(i) Cournot Nash equilibrium is always interior with $R_1^* > R_2^*$.

(ii) if $\omega = 0$, an increase in α leads to an increase in aggregate resource spending and a decrease in aggregate welfare.

(iii) if $\omega = 0$, an increase in Δ leads to an increase in aggregate resource spending and a decrease in aggregate welfare.

(iv) if the stake asymmetry is high so that ω is arbitrarily close to v, an increase in ω leads to an increase in aggregate resource spending and aggregate welfare.

Proposition 6 also implies that when persuasion function is given by (17) and $h(.)$ is given by (8), Player 1 who has stronger evidence always spends more resources relative to her rival in the Cournot equilibrium and is therefore always the favorite. Further, as with Proposition 2 (iv), an increase in asymmetry between players (either through an increase in Δ or through an increase in ω) can *increase* aggregate resource spending under certain conditions.

5. Conclusion

We have examined 2-player difference-form contests that are best thought of as “persuasion functions;” that is, as applying to instances, such as litigation, lobbying, or political campaigning, in which different parties expend resources in order to persuade an audience. Contrary to specific cases of difference-form contests examined by Baik (1998) and Che and Gale (2000), we have found that these contests can support both corner and pure-strategy interior equilibria and at least in the form in (3) they are simple to derive. The equilibrium behavior of contestants may not satisfy the “preemption” property defined in Che and Gale (2000) suggesting that it is not a general property of difference-form contests. Further, in the case of asymmetric forms some counterintuitive outcomes are possible. The player with the weaker evidence may have a stronger marginal incentive to spend resources so that the pre-commitment behavior of the favorite and underdog in sequential move contests involving (17) can be opposite to Dixit (1987). In

[27] These findings are qualitatively very similar to those presented in Proposition 2. Hence for the sake of brevity, they are discussed only in the Supplementary Appendix.

particular, the favorite may choose to pre-commit to lower expenditures relative to her chosen level in a Cournot Nash equilibrium.

Further, since the functional forms examined in the paper are explicitly characterized as outcomes of a persuasion contest, the parameters of the functions carry natural inferential interpretations such as evidence potency, bias and truth. This allows us to uncover their impact on equilibrium spending in a simple framework. We find that increase in evidence potency may intensify resource expenditures especially in the symmetric case. A bias in the decision threshold does not lead to asymmetry in equilibrium spending but alters the intensity of the spending. An a priori bias in favor of truth translates into the truthful side spending more resources than the rival which reinforces the initial advantage. An evidence bias, however may have the opposite effect where the weaker party puts in more resources in equilibrium to counter the evidence based disadvantage. Further, unlike the Tullock contest, in some circumstances increased asymmetry between players (such as differences in stakes) may lead to increased aggregate resource expenditures. Given the relative paucity of applied studies which have used a difference-form contest function with the exception of Besley and Persson (2009), these findings suggest that it would be worthwhile to re-examine many applied settings that involve persuasion using these functions and determine whether they shed a different light than those that come from existing studies.

A limitation of the functional forms examined in the paper is that they represent only 2-player contests unlike those examined in Alcalde and Dahm (2007) and Bevia and Corchon (2015). However, this still leaves room for various applications as many real world persuasion contests often involve 2 competing parties as in litigation and electoral contests. Further, in principle, the additive form in (3) can be easily extended to $N > 2$ players. However it remains to be seen whether the parameters of the extended function can be interpreted in an intuitive way through a persuasion based micro-foundation.

References

Alcalde, J., and Dahm, M. "Tullock and Hirshleifer: A Meeting of the Minds," *Review of Economic Design*, 11, 101-124 (2007)

Baik, K. H. "Difference-form Contest Success Functions and Effort Levels in Contests," *European Journal of Political Economy*, 14, 685-701 (1998)

Baron, D.P. "Electoral Competition with Informed and Uninformed Voters", *American Political Science Review*, 88, 33-47 (1994)

Bell, D. E., Keeney, R. L. and Little, J. D. "A market share theorem," *Journal of Marketing Research*, 12(2), 136-41 (1975)

Bernardo, A. E., Talley, E., and Welch, I. "A Theory of Legal Presumptions," *Journal of Law, Economics, and Organization*, 16(1), 1-49 (2000)

Besley, T. and Persson, T. "Repression or Civil War?" *American Economic Review Papers & Proceedings*, 99(2), 292-297 (2009)

Bevia, C., and Corchon, L. "Relative Difference Contest Success Function," *Theory and Decision*, 78, 377-398 (2015)

Cain, D., Loewenstein, G. and Moore, D. "The Dirt on Coming Clean: Perverse Effects of Disclosing Conflicts of Interest", *Journal of Legal Studies*, 34, 1-25 (2005)

Che, Y. K. and Gale I. "Difference-Form Contests and the Robustness of All-Pay Auctions," *Games and Economic Behavior*, 30, 22-43 (2000)

Corchon, L. "The Theory of Contests: A Survey," *Review of Economic Design*, 11, 69 – 100 (2007)

Corchon, L. and Dahm, M. "Foundations for Contest Success Functions", *Economic Theory*, 43, 81-98 (2010)

Corchon, L. and Dahm, M. "Welfare Maximising Contest Success Functions When The Planner Cannot Commit", *Journal of Mathematical Economics*, 47, 309-17 (2011)

De Franco, G., Lu, H. and Vasvari, F.P. "Wealth Transfer Effects of Analysts' Misleading Behavior," *Journal of Accounting Research*, 45, 71-110 (2007)

DellaVigna, S and Gentzkow, M. "Persuasion: Empirical Evidence", *Annual Review of Economics*, 2, 643-69 (2010)

Dewatripont, M. and Tirole, J. "Advocates," *Journal of Political Economy*, 107, 1-39 (1999)

Dixit, A. "Strategic Behavior in Contests," *American Economic Review*, 77, 891-898 (1987)

Eyster, E. and Rabin, M. "Naïve Herding in Rich-Information Settings", *American Economic Journal: Microeconomics*, 2, 221-43 (2010)

Farmer, A. and Pecorino, P. "Legal expenditure as a rent-seeking game," *Public Choice*, 100(3-4), 271-88 (1999)

Grossman, G. and Helpman, E. "Protection for sale", *American Economic Review*, 84, 833-50 (1994)

Grossman, G. and Helpman, E. "Electoral Competition and Special Interest Politics", *Review of Economic Studies*, 63, 265-86 (1996)

Hirshleifer, J. "Conflict and rent-seeking success functions: Ratio vs. difference models of relative success," *Public Choice* 63: 101-112 (1989)

Hirshleifer, J. and Osborne, E. "Truth, effort and the legal battle," *Public Choice*, 108(1-2), 169-95 (2001)

Hwang, S-H. "Technology of Military Conflict, Military Spending and War," *Journal of Public Economics*, 96, 226-236 (2012)

Jia, H., Skaperdas, S., and Vaidya, S. "Contest Functions: Theoretical Foundations and Issues in Estimation," *International Journal of Industrial Organization,* 31, 211-222 (2013)

Konrad, K. *Strategy and Dynamics in Contests.* New York: Oxford University Press (2009)

Lazear, E., and Rosen, S. "Rank-Order Tournaments as Optimum Labor Contracts", *Journal of Political Economy*, 89, 841-864 (1981)

Malmendier, U. and Shanthikumar, D.M. "Are Small Investors Naïve about Incentives?" *Journal of Financial Economics*, 85, 457-89 (2007)

Munster, J. and Staal, K. "War with Outsiders makes Peace Inside," *Conflict Management and Peace Science*, 28, 91-110, (2011)

Munster, J. and Staal, K. "How Organizational Structure can reduce Rent Seeking," *Public Choice*, 150, 579-94 (2012)

Nitzan, S. "Modelling rent-seeking contests," *European Journal of Political Economy*, 10, 41-60 (1994)

Pelosse, Y. "Dynamic difference-form contests," *Journal of Public Economic Theory*, 16, 401-426, (2014)

Perez-Castrillo, J.D. and Verdier, T. "A General Analysis of Rent Seeking Games," *Public Choice*, 73 (3), 335-350 (1992)

Polishchuk, L. and Tonis, A. "Endogenous Contest Success Functions: A Mechanism Design Approach," *Economic Theory,* 52, 271-297 (2013)

Robson, A., and Skaperdas, S. "Costly Enforcement of Property Rights and the Coase Theorem," *Economic Theory*, 36(1), 109-128 (2008)

Schmalensee, R. *The Economics of Advertising*. Amsterdam: North Holland (1972)

Skaperdas, S. and Grofman, B. “Modeling Negative Campaigning,” *American Political Science Review*, 89, 49-61 (1995)

Skaperdas, S. “Contest Success Functions,” *Economic Theory*, 7, 283-290 (1996)

Skaperdas, S. and Vaidya, S. “Persuasion as a Contest,” *Economic Theory*, 51, 465-86, (2012)

Snyder J.M. “Election Goals and the Allocation of Campaign Resources,” *Econometrica*, 57, 637-660, (1989)

Tullock, G. “Efficient Rent-Seeking,” in James M. Buchanan, Robert D. Tollison, and Gordon Tullock (eds.), “Towards a Theory of the Rent-Seeking Society,” pp. 97-112, (College Station: Texas A&M University Press, 1980)

Appendix[28]

Proof of Proposition 1

Let us examine the payoff function of player *i* in Section 3 which is reproduced below:

$$U^i(R_i,R_j)=\{\frac{1}{2}+\frac{\alpha}{2}\left[h(R_i)-h(R_j)\right]\}v_i-R_i \text{ for } i,j=1,2 \text{ and } i\neq j \tag{A.1}$$

(A.1) can be re-arranged as:

$$U^i=\frac{1}{2}v_i\{1-\alpha h(R_j)\}+\frac{\alpha}{2}v_i\,h(R_i)-R_i \tag{A.2}$$

Notice that the above payoff is additively separable in R_i and R_j. From this it follows that the optimal choice of R_i by player *i* will be independent of R_j and will involve a dominant strategy. This proves part (i).

Recall that $U^i>0$ at $R_i=0$ and $U^i<0$ at $R_i=v_i$. Assumption 1 ensures that U^i is strictly concave over the strategy space $R_i\in[0,\max\{v_1,v_2\}]$. Hence, if $h'(0)>\frac{2}{\alpha v_i}$ then $R_i^*>0$ and it is given by the first order condition $h'(R_i^*)=\frac{2}{\alpha v_i}$ so that $R_i^*=(h')^{-1}(\frac{2}{\alpha v_i})$. From this it also follows that $\frac{\partial R_i^*}{\partial \alpha}$ and $\frac{\partial R_i^*}{\partial v_i}$ are positive and $\frac{\partial R_i^*}{\partial v_j}=0$ for $i,j=1,2,\ i\neq j$. When $h'(0)\le\frac{2}{\alpha v_i}$, then $R_i^*=0$. Parts (ii) – (v) follow straightforwardly from this result.

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[28] Throughout this section, where necessary, we abbreviate $h(R_i)=h^i, h'(R_i)=h'^i$ and $h''(R_i)=h''^i$ for $i=1,2$ for ease of exposition.

Since $U^i > 0$ when $R_i = 0$ regardless of the level of R_j it must be the case that at $R_i^* > 0$, $U^{*i} > 0$. Hence in any pure strategy equilibrium, it must be the case that:

$$U^{*1} = \left\{ \frac{1}{2} + \frac{\alpha}{2}[h(R_1^*) - h(R_2^*)] \right\} v_1 - R_1^* > 0$$

$$U^{*2} = \left\{ \frac{1}{2} + \frac{\alpha}{2}[h(R_2^*) - h(R_1^*)] \right\} v_2 - R_2^* > 0$$

Since by assumption, $v_1 \geq v_2$, it follows from the above two conditions that $R_1^* + R_2^* < v_1$ implying partial dissipation of rents. This proves part (vi). ■

Proof of Proposition 2

From Proposition 1 (ii), we know that $\frac{\partial R_i^*}{\partial \alpha} > 0$ for $i = 1, 2$. Hence it follows immediately that $\frac{\partial (R_1^* + R_2^*)}{\partial \alpha} > 0$. Recall that the first-order conditions are,

$$h'^1 = \frac{2}{\alpha(v + \omega)} \tag{A.3}$$

$$h'^2 = \frac{2}{\alpha(v - \omega)} \tag{A.4}$$

From strict concavity of $h(.)$, it follows by inspecting (A.3) and (A.4) that while $\frac{\partial R_1^*}{\partial \omega} > 0$, $\frac{\partial R_2^*}{\partial \omega} < 0$. Hence we cannot sign $\frac{\partial (R_1^* + R_2^*)}{\partial \omega}$ unambiguously. This proves part (i).

When $\omega = 0$, from (10) we get $U = v - R_1 - R_2$. Since $\frac{\partial (R_1^* + R_2^*)}{\partial \alpha} > 0$, it follows that $\frac{\partial U^*}{\partial \alpha} < 0$.

When $\omega > 0$, by differentiating (10) at (R_1^*, R_2^*) with respect to α it follows that,

$$\frac{dU^*}{d\alpha} = \left[h(R_1^*) - h(R_2^*) \right] \omega + (\alpha\omega h'^1 - 1)\frac{dR_1^*}{d\alpha} - (\alpha\omega h'^2 + 1)\frac{dR_2^*}{d\alpha} \tag{A.5}$$

Using the first-order conditions as given by (A.3) and (A.4), (A.5) simplifies to,

$$\frac{dU^*}{d\alpha} = \left[h(R_1^*) - h(R_2^*)\right]\omega - \left(\frac{v-\omega}{v+\omega}\right)\frac{\partial R_1^*}{\partial \alpha} - \left(\frac{v+\omega}{v-\omega}\right)\frac{\partial R_2^*}{d\alpha} \tag{A.6}$$

Observe that $\left(\frac{v-\omega}{v+\omega}\right) > 0$, $\left(\frac{v+\omega}{v-\omega}\right) > 0$ while $\frac{\partial R_i^*}{\partial \alpha} > 0$ for $i = 1, 2$. Further from strict concavity of $h(.)$ and the first-order conditions (A.3) and (A.4) it follows that $R_1^* > R_2^*$. Given this it is apparent that while the first component of (A.6) is positive, the other two components are negative. Hence when $\omega > 0$, the sign of $\frac{dU^*}{d\alpha}$ is generally ambiguous.

By differentiating (10) at (R_1^*, R_2^*) with respect to ω it follows that,

$$\frac{dU^*}{d\omega} = \alpha\left[h(R_1^*) - h(R_2^*)\right] + (\alpha\omega h'^1 - 1)\frac{\partial R_1^*}{\partial \omega} - (\alpha\omega h'^2 + 1)\frac{\partial R_2^*}{\partial \omega} \tag{A.7}$$

Using the first-order conditions as given by (A.3) and (A.4), (A.7) simplifies to,

$$\frac{dU^*}{d\omega} = \alpha\left[h(R_1^*) - h(R_2^*)\right] - \left(\frac{v-\omega}{v+\omega}\right)\frac{\partial R_1^*}{\partial \omega} - \left(\frac{v+\omega}{v-\omega}\right)\frac{\partial R_2^*}{\partial \omega} \tag{A.8}$$

Since $R_1^* > R_2^*$ (and therefore from monotonicity of $h(.)$, $h(R_1^*) > h(R_2^*)$), $\frac{v-\omega}{v+\omega}$ and $\frac{v+\omega}{v-\omega}$ are positive, and $\frac{\partial R_1^*}{\partial \omega} > 0$ while $\frac{\partial R_2^*}{\partial \omega} < 0$ it follows that the first and the last components in (A.8) are positive while the second component is negative. Hence the overall sign of $\frac{dU^*}{d\omega}$ cannot be determined unambiguously.
This proves part (ii).

When $h(R_i)$ is given by (7), maximization of U^i as given by (6) with respect to R_i $i = 1, 2$ yields,

$$R_1^* = \sqrt{\frac{\alpha(1-\psi)(v+\omega)}{2}} - 1 \tag{A.9}$$

$$R_2^* = \sqrt{\frac{\alpha(1-\psi)(v-\omega)}{2}} - 1 \tag{A.10}$$

By inspection of (A.9) and (A.10), it is apparent that for sufficiently high v an interior equilibrium exists with $R_i^* > 0$ for $i = 1, 2$. Using (A.9) and (A.10) and differentiating with respect to ω we get,

$$\frac{\partial(R_1^* + R_2^*)}{\partial\omega} = \tfrac{1}{2}\sqrt{\frac{\alpha(1-\psi)}{2}}\left(\frac{1}{\sqrt{v+\omega}} - \frac{1}{\sqrt{v-\omega}}\right) < 0\,.$$

Recall that the first component on the R.H.S. of (A.8) is always positive when $\omega > 0$.
Now, since $\frac{\partial R_1^*}{\partial\omega} > 0$, $\frac{\partial R_2^*}{\partial\omega} < 0$ and $\frac{\partial(R_1^* + R_2^*)}{\partial\omega} < 0$, it follows that $\left(\frac{v-\omega}{v+\omega}\right)\frac{\partial R_1^*}{\partial\omega} + \left(\frac{v+\omega}{v-\omega}\right)\frac{\partial R_2^*}{\partial\omega} < 0$. Hence R.H.S. of (A.8) must be strictly positive so that $\frac{dU^*}{d\omega} > 0$. This proves part (iii).

Maximization of U^i as given by (6) with respect to R_i $i = 1, 2$ when $h(R_i)$ is given by (8) yields,

$$\sqrt{R_1} = \frac{\alpha(v+\omega)}{4\sqrt{K}} \tag{A.11}$$

$$\sqrt{R_2} = \frac{\alpha(v-\omega)}{4\sqrt{K}} \tag{A.12}$$

By substituting (A.11) and (A.12) in (10) we get,

$$U^* = v + \frac{\alpha^2}{8K}(3\omega^2 - v^2) \tag{A.13}$$

Hence,

$$\frac{\partial U^*}{\partial\alpha} = \frac{2\alpha}{8K}(3\omega^2 - v^2) \tag{A.14}$$

It follows from (A.14) that $\frac{\partial U^*}{\partial\alpha} > 0$ for $\omega > \frac{v}{\sqrt{3}}$.
Using (A.11) and (A.12), it follows that,

$R_1^* + R_2^* = \frac{\alpha^2}{8K}(v^2 + \omega^2)$. Hence $\frac{\partial(R_1^* + R_2^*)}{\partial\omega} = \frac{\alpha^2\omega}{4K} > 0.$ Further, by differentiating (A.13) with respect to ω, it follows that $\frac{\partial U^*}{\partial\omega} = \frac{3\alpha^2\omega}{4K} > 0$. Hence both aggregate spending and aggregate welfare increase with ω when $h(.)$ is given by (8).
This proves (iv). ■

Proof of Lemma 2

Using (17) and (6), Player 1's net marginal benefit of resource spending at $R_1 = 0$ is $\frac{1}{2}\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)\right]h'(0)v - 1$. Hence for a given R_2, Player 1 will be induced to increase R_1 beyond 0 when,

$$h'(0) > \frac{2}{\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)\right]v}$$

Since $0 < h(R_2) < 1$, it follows that $\dfrac{2}{\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)\right]v} < \dfrac{2}{\alpha_2 v}$ for any $R_2 \in [0, v]$.

Hence if $h'(0) > \dfrac{2}{\alpha_2 v}$, $U^1 > 0$ and increasing at $R_1 = 0$ regardless of the level of R_2.

Further from Assumption 1, we know that U^1 is strictly concave over $R_1 \in [0, v]$ and $U^1 < 0$ at $R_1 = v$. Given this, we can be assured that for any $R_2 \in [0, v]$ the optimal level of R_1 is strictly between 0 and v and given by the first-order condition (18).

Exactly analogous to the case of Player 1, it can be verified that if $h'(0) > \dfrac{2}{\alpha_2 v}$, then for any $R_1 \in [0, v]$, the optimal level of R_2 is strictly between 0 and v and given by the first-order condition (19). ■

Proof of Proposition 5 parts (iii), (iv), (v) and (vi)

Let player i's best response for any $R_j \in [0, v]$ be denoted as $R_i^{RF}(R_j)$ $i, j = 1, 2$ and $i \neq j$. Suppose that Assumption 1 (which stipulates monotonicity and strict concavity of $h(.)$), and the condition stipulated in Lemma 2 is satisfied. In this case, from Lemma 2 we can be sure that $R_i^{RF}(R_j)$ is strictly interior ($0 < R_i^{RF}(R_j) < v$) and continuous over the interval $[0, v]$ $i, j = 1, 2$ and $i \neq j$. Hence $R_1^{RF}(R_2)$ and $R_2^{RF}(R_1)$ must cross each other at least once over the space ($[0, v] \times [0, v]$). Further since $R_1^{RF}(R_2)$ is monotonic in R_2 and negatively sloped while $R_2^{RF}(R_1)$ is monotonic in R_1 and positively sloped, it follows that their crossing point is unique. Hence there exists a unique interior Nash equilibrium to the Cournot game. Such pure strategy equilibrium will be either involve $R_1^* = R_2^* = R^*$ or $R_1^* > R_2^*$ or $R_1^* < R_2^*$. This proves part (iii).

From (18) and (19), along a symmetric equilibrium $R_1^* = R_2^* = R^*$,

$$h'(R^*) = \frac{4}{[(\alpha_1 + \alpha_2)]v} \tag{A.15}$$

Further, $\dfrac{2}{\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R^*)\right]v} = \dfrac{2}{\left[\alpha_2 + (\alpha_1 - \alpha_2)h(R^*)\right]v}$

The above equality implies that:

$$h(R^*) = \tfrac{1}{2} \tag{A.16}$$

When (A.15) and (A.16) are not simultaneously satisfied, the Nash equilibrium must be asymmetric with either $R_1^* > R_2^*$ or $R_1^* < R_2^*$.

When $R_1^* > R_2^*$, strict concavity of $h(.)$, implies that $h'(R_1^*) < h'(R_2^*)$. Hence from (18) and (19), it must be the case that:

$$\frac{2}{\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2^*)\right]v} < \frac{2}{\left[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1^*)\right]v} \text{ or}$$

$$1 > h(R_1^*) + h(R_2^*) \tag{A.17}$$

Further, given the monotonicity of $h(.)$, it must also be the case that:

$$h(R_1^*) > h(R_2^*) \tag{A.18}$$

Hence along such a Nash equilibrium, both (A.17) and (A.18) must be satisfied.

Analogously, when, $R_1^* < R_2^*$, the following conditions must hold:

$$1 < h(R_1^*) + h(R_2^*) \tag{A.19}$$

$$h(R_1^*) < h(R_2^*) \tag{A.20}$$

This proves part (iv).

To establish part (v), notice that Player 1's win probability as given by (17) can be re-arranged as: $p_1(R_1, R_2) = \tfrac{1}{2} + \tfrac{1}{2}\left[\alpha_1 h(R_1)(1 - h(R_2)) - \alpha_2 h(R_2)(1 - h(R_1))\right]$. Since $0 < \alpha_i < 1$ for $i = 1, 2$ and $0 \le h(.) < 1$, it follows that $\left|\alpha_1 h(R_1)(1 - h(R_2)) - \alpha_2 h(R_2)(1 - h(R_1))\right| < 1$. Hence $0 < p_1(R_1, R_2) < 1$ for any level of resources invested by either player. Exactly the same holds for $p_2(R_1, R_2)$. From this it follows that each player is assured a positive expected payoff from the contest even if she does not invest any resources to it. That is,

$$U^1(0, R_2) > 0$$

$$U^2(R_1, 0) > 0$$

Thus along any Nash equilibrium,

$$U^1(R_1^*, R_2^*) = \left\{\tfrac{1}{2} + \tfrac{1}{2}\left[\alpha_1 h(R_1^*) - \alpha_2 h(R_2^*) - (\alpha_1 - \alpha_2)h(R_1^*)h(R_2^*)\right]\right\}v - R_1^* > 0$$

$$U^2(R_1^*, R_2^*) = \left\{\tfrac{1}{2} + \tfrac{1}{2}\left[\alpha_2 h(R_2^*) - \alpha_1 h(R_1^*) + (\alpha_1 - \alpha_2)h(R_1^*)h(R_2^*)\right]\right\}v - R_2^* > 0$$

From the above two inequalities, it follows that $v > R_1^* + R_2^*$ so that there is partial rent dissipation.

To prove part (vi), suppose that Player 1 acts as the Stackelberg leader and chooses resources prior to Player 2. We evaluate Player 1's marginal incentive to invest in resources $\frac{dU^1}{dR_1}$ at the Cournot equilibrium (R_1^*, R_2^*). Since Player 1 is the Stackelberg leader it follows that:

$$\frac{dU^1}{dR_1} = \frac{\partial U^1}{\partial R_1} + \frac{\partial U^1}{\partial R_2}\frac{dR_2}{dR_1} \tag{A.21}$$

At the Cournot equilibrium, $\frac{\partial U^1}{\partial R_1} = 0$ and by totally differentiating (19), we get:

$$\frac{dR_2}{dR_1} = -\frac{2(\alpha_1 - \alpha_2)h'(R_1^*)}{vh''(R_2^*)(\alpha_2 + (\alpha_1 - \alpha_2)h(R_1^*))^2} = -\frac{(\alpha_1 - \alpha_2)vh'(R_1^*)(h'(R_2^*))^2}{2h''(R_2^*)} \tag{A.22}$$

From the strict concavity of $h(.)$, it follows that $\frac{dR_2}{dR_1} > 0$. Also since $\frac{\partial U^1}{\partial R_2} = -\frac{v}{2}\left[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1)\right]h'(R_2)$, using (19), it follows that at (R_1^*, R_2^*), $\frac{\partial U^1}{\partial R_2} = -1$.

By substituting these values into (A.21) we find that

$$\frac{dU^1}{dR_1} = \frac{(\alpha_1 - \alpha_2)vh'(R_1^*)(h'(R_2^*))^2}{2h''(R_2^*)} < 0.$$

Hence as a Stackelberg leader, Player 1 is induced to reduce her expenditure below what she would spend at the Cournot equilibrium.

Suppose now that Player 2 acts as the Stackelberg leader. We evaluate Player 2's marginal incentive to invest in resources $\frac{dU^2}{dR_2}$ at the Cournot equilibrium (R_1^*, R_2^*). This is given by:

$$\frac{dU^2}{dR_2} = \frac{\partial U^2}{\partial R_2} + \frac{\partial U^2}{\partial R_1}\frac{dR_1}{dR_2} \tag{A.23}$$

At the Cournot equilibrium, $\frac{\partial U^2}{\partial R_2} = 0$ and by totally differentiating (18), we get:

$$\frac{dR_1}{dR_2} = \frac{2(\alpha_1 - \alpha_2)h'(R_2^*)}{vh''(R_1^*)\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2^*)\right]^2} = \frac{v(\alpha_1 - \alpha_2)(h'(R_1^*))^2 h'(R_2^*)}{2h''(R_1^*)} \tag{A.24}$$

From the strict concavity of $h(.)$, it follows that $\frac{dR_1}{dR_2} < 0$. Also, using (18), it follows that

$\frac{\partial U^2}{\partial R_1} = -\frac{v}{2}\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)\right]h'(R_1) = -1 < 0$. By substituting these values into (A.23)

we find that $\frac{dU^2}{dR_2} = -\frac{v(\alpha_1 - \alpha_2)(h'(R_1^*))^2 h'(R_2^*)}{2h''(R_1^*)} > 0$

Hence as a Stackelberg leader, Player 2 is induced to increase her expenditure above what she would spend at the Cournot equilibrium. This proves part (vi).
■

Proof of Corollary 2

When (7) holds with $\psi > \frac{1}{2}$, neither condition (20) nor condition (22) can be satisfied and therefore the interior Nash equilibrium can only be of the type $R_1^* < R_2^*$. Assumption 1 and Lemma 2 ensure its existence when $h'(0) = 1 - \psi > \frac{2}{\alpha_2 v}$, that is $\psi < 1 - \frac{2}{\alpha_2 v}$.

Further, since $\frac{1}{2} < \psi < 1$, it must also be the case that $\frac{2}{\alpha_2 v} < \frac{1}{2}$ or $\alpha_2 v > 4$. ■

Proof of Proposition 6

Since the expected payoffs are given by (6) with $p_i(R_1, R_2)$ given by (17), $h(.)$ given (8) and parameterizations based on assumptions 2 and 6, it can be shown that any Cournot Nash equilibrium will be strictly interior with $R_i^* \in (0, v)$.[29] The first-order conditions for such Cournot Nash equilibrium are given by:

$$\frac{1}{4\sqrt{R_1 K}}\left\{(\alpha + \Delta) - 2\Delta\sqrt{\frac{R_2}{K}}\right\}(v + \omega) = 1 \quad (A.25)$$

$$\frac{1}{4\sqrt{R_2 K}}\left\{(\alpha - \Delta) + 2\Delta\sqrt{\frac{R_1}{K}}\right\}(v - \omega) = 1 \quad (A.26)$$

Simultaneously solving (A.25) and (A.26) gives us the following solution:

$$\sqrt{\frac{R_1^*}{K}} = \frac{2K(\alpha + \Delta)(v + \omega) - \Delta(\alpha - \Delta)(v + \omega)(v - \omega)}{8K^2 + 2\Delta^2(v + \omega)(v - \omega)} \quad (A.27)$$

$$\sqrt{\frac{R_2^*}{K}} = \frac{2K(\alpha - \Delta)(v - \omega) + \Delta(\alpha + \Delta)(v + \omega)(v - \omega)}{8K^2 + 2\Delta^2(v + \omega)(v - \omega)} \quad (A.28)$$

[29] See Supplementary Appendix part (iv).

Using (A.27) and (A.28), it follows that along the Cournot Nash Equilibrium $R_1^* > R_2^*$ iff ,

$$K > \frac{\alpha\Delta(v+\omega)(v-\omega)}{[(\alpha+\Delta)(v+\omega)-(\alpha-\Delta)(v-\omega)]} \quad \text{(A.29)}$$

Since from (8), $K > (v+\omega)$, (A.29) is always satisfied so that in equilibrium, $R_1^* > R_2^*$.

This proves (i).

Suppose $\omega = 0$. Then it follows from (A.27) and (A.28) that,

$$\frac{\partial(R_1^* + R_2^*)}{\partial\alpha} = \frac{K\alpha v^2}{2[4K^2 + \Delta^2 v^2]} > 0 \quad \text{(A.30)}$$

$$\frac{\partial(R_1^* + R_2^*)}{\partial\Delta} = \frac{K\Delta v^2(2K+\alpha v)(2K-\alpha v)}{2[4K^2 + \Delta^2 v^2]} \quad \text{(A.31)}$$

$$U^* = v - R_1^* - R_2^* \text{ with } \frac{\partial U^*}{\partial\alpha} = -\frac{\partial(R_1^* + R_2^*)}{\partial\alpha} \text{ and } \frac{\partial U^*}{\partial\Delta} = -\frac{\partial(R_1^* + R_2^*)}{\partial\Delta} \quad \text{(A.32)}$$

(ii) follows immediately from (A.30) and (A.32). Also notice that R.H.S. of (A.31) is positive since $K > v$. This along with (A.32) proves (iii).

Suppose now that $\omega > 0$.

Using (A.27) we get,

$$\frac{\partial}{\partial\omega}\sqrt{\frac{R_1^*}{K}} == \frac{4K\left\{4K^2(\alpha+\Delta) + 4K\omega\Delta(\alpha-\Delta) + \Delta^2(\alpha+\Delta)(v+\omega)^2\right\}}{(8K^2 + 2\Delta^2(v^2-\omega^2))^2} \quad \text{(A.33)}$$

Notice that the R.H.S. of (A.33) is always positive. Hence it follows that $\frac{\partial R_1^*}{\partial\omega} > 0$.

Using (A.28) we get,

$$\frac{\partial}{\partial\omega}\sqrt{\frac{R_2^*}{K}} == -\frac{4K\left\{4K^2(\alpha-\Delta) + 4K\omega\Delta(\alpha+\Delta) + \Delta^2(\alpha-\Delta)(v-\omega)^2\right\}}{(8K^2 + 2\Delta^2(v^2-\omega^2))^2} \quad \text{(A.34)}$$

Notice that the R.H.S. of (A.34) is always negative. Hence it follows that $\frac{\partial R_2^*}{\partial\omega} < 0$.

Since by definition, $R_i = K\left(\sqrt{\frac{R_i}{K}}\right)^2$ for $i = 1, 2$, it follows that,

$$\frac{\partial(R_1^* + R_2^*)}{\partial\omega} = 2K\left(\sqrt{\frac{R_1^*}{K}}\frac{\partial}{\partial K}\left(\sqrt{\frac{R_1^*}{K}}\right) + \sqrt{\frac{R_2^*}{K}}\frac{\partial}{\partial K}\left(\sqrt{\frac{R_2^*}{K}}\right)\right) \quad (A.35)$$

Hence, when ω is arbitrarily close to v, using (A.27), (A.28), (A.33), (A.34) and (A.35), it follows that,

$$\frac{\partial(R_1^* + R_2^*)}{\partial\omega} = \frac{v(\alpha+\Delta)\left(K^2(\alpha+\Delta) + Kv\Delta(\alpha-\Delta) + v^2\Delta^2(\alpha+\Delta)\right)}{(4K^3)} \quad (A.36)$$

By inspecting (A.36), it is clear that $\frac{\partial(R_1^* + R_2^*)}{\partial\omega} > 0$.

Since, $U = U^1 + U^2 = v + \left[(\alpha+\Delta)h^1 - (\alpha-\Delta)h^2 - 2\Delta h^1 h^2\right]\omega - R_1 - R_2$, it follows that:

$$\frac{dU^*}{d\omega} = ((\alpha+\Delta)h^{*1} - (\alpha-\Delta)h^{*2} - 2\Delta h^{*1}h^{*2}) + \left[\frac{\omega - v}{v + \omega}\right]\frac{\partial R_1^*}{\partial\omega} - \left[\frac{v+\omega}{v-\omega}\right]\frac{\partial R_2^*}{\partial\omega} \quad (A.37)$$

Notice that $(\alpha+\Delta)h^{*1} - (\alpha-\Delta)h^{*2} - 2\Delta h^{*1}h^{*2} = p_1^* - p_2^*$. Since $R_1^* > R_2^*$, it follows that $p_1^* > p_2^*$ so that the first component of (A.37) is positive. Given that $\frac{\partial R_1^*}{\partial\omega} > 0$, $\frac{\partial R_2^*}{\partial\omega} < 0$ and $\omega < v$, it follows that the second component of (A.37) is negative while the last component is positive.

When ω is arbitrarily close to v, the second component (which is the only component in (A.37) that is negative) is approximately 0. Hence it follows that $\frac{dU^*}{d\omega} > 0$. This completes the proof of (iv). ■