Independent Investment Counsel

analytic investment management inc.

2182 du Pont Drive, Newport Beach, California 92715, (714) 833-0294

RESEARCH TECHNICAL REPORT #2
January 1976

THE OPTIMALITY OF OPTION WRITING FOR THE LONG TERM INVESTOR

Sheen T. Kassouf
Steven A. Lauer

THE OPTIMALITY OF OPTION WRITING FOR THE LONG TERM INVESTOR

We examine here the desirability of writing options for the long term investor who attempts to maximize the growth rate of his assets. Some have come to regard option writing as temporarily attractive because of the "high" level of premiums available in these early years of organized option exchanges. They believe that as this market becomes more "efficient" premiums will move to their "proper" levels and the long term investor will find no advantages by employing this demanding strategy. The following analysis and evidence prove this conjecture false.

THE THEORETICAL ANALYSIS

Consider a growth maximizing investor whose present wealth is 1 consisting of one share of a stock whose present price is 1. If he can receive a premium of $\underline{a}$ per share for a call striking at 1, what fraction, k, of his wealth should he option? We assume discrete time periods equal to the time duration of the call. Let x be the random variable denoting the price of the common stock on the expiration of the call. This investor's wealth at the expiration of the call will be

$$(1 + ap)x \text{ if } x \leq 1,$$

$$(1 + ap)x - p(x - 1) \text{ if } x > 1$$

where p is the number of calls sold. (This assumes that all proceeds from the sale of calls is reinvested in the purchase of common stock.) Then $k = p/(1+ap)$ represents the proportion of his wealth under option.

As a growth maximizer this investor maximizes the expected value of the logarithm of terminal wealth:

$$G = \log(1+ap) + \int_0^1 \log x\, f(x)\, dx + \int_1^\infty \log\big(x - k(x-1)\big)\, f(x)dx$$

where $f(x)$ is the density of x. By Taylor and substitution

$$G = \log(1-ak)^{-1} + C_0 + \sum_{n=1}^{\infty} (-1)^{n+1} (1-k)^n n^{-1} C_n$$

where

$$C_0 = \int_0^1 \log x\, f(x)\, dx$$

and

$$C_n = \int_1^\infty (x-1)^n f(x)\, dx.$$

Thus

$$G' = \frac{a}{1-ak} + \sum_{n=1}^{\infty} (-1)^n (1-k)^{n-1} C_n .$$

At $k = 0$, $G' = a + \sum_{n=1}^{\infty} (-1)^n C_n$ so that if $a > C_1 - \sum_{n=2}^{\infty} (-1)^n C_n$, $G' > 0$ and it "pays" to write calls. Since C_1 is the actuarial, or expected value of the call, this indicates that for some premiums less than actuarial value a growth maximizing investor will be induced to sell some calls.

As an example consider the case of lognormal x with $\mu = E(\log x) = .01$ and $\sigma^2 = \text{var}(\log x) = .01$. (This typifies a three month period for a Dow Jones Industrial type stock.) A straightforward calculation shows that

$$\int_1^\infty x^k f(x)\,dx = \exp\left\{k\left(\frac{k\sigma^2}{2} + \mu\right)\right\} \Phi\left\{\frac{-u-k\sigma^2}{\sigma}\right\}, \; k = 0, 1, 2, \cdots$$

where $\Phi(x)$ is the standardized normal integral from x to ∞. This yields

$$\begin{aligned} E(x) &= 1.015 \\ \text{var}(x) &= .0104 \\ C_1 &= .0482 \\ C_2 &= .0069 \\ C_3 &= .0013. \end{aligned}$$

Furthermore it can easily be shown that

$$\int_0^1 \log x\, f(x)\,dx = \mu\left[1 - \Phi\left(\frac{-\mu}{\sigma}\right)\right] - \sigma\varphi\left(\frac{\mu}{\sigma}\right)$$

where φ is the standardized normal density function, so that

$$C_0 = -.0351 .$$

From the above discussion we know that if $\underline{a} > C_1 - C_2 + C_3 = .0426$, G will attain its maximum with k interior to $(0, 1]$. If, for example, we let $\underline{a} = .95(C_1) = .0458$, G attains a maximum greater than $.0118$

when $k = .95$. In other words, a growth maximizer will sell call options on virtually all of his holdings even if the premium is only 95% of the call's actuarial value, because this will increase his growth rate from 1% per three months to 1.18% per three months.

Since x is lognormal, k will never exceed 1 for a growth maximizer, i.e., he will never sell uncovered calls because for $x > p/\left\{(1-a)p-1\right\}$ his terminal wealth will be less than zero indicating $g = -\infty$. These magnitudes of x have non-zero probability for lognormal x. For discrete time analysis, a growth maximizer will never enter an investment with potential unlimited liability. This appears to be in accord with "prudent man" criteria which disallow short selling and the use of margin. Note, however, that in the limiting case of continuous trading, i.e., as the discrete time interval converges to zero, a growth maximizer may sell uncovered calls. Each investor must decide for himself whether continuous trading accurately describes present market situations and whether it is a practical strategy.

We have been considering a one-stock universe. We turn now to some evidence regarding actual stocks.

THE EMPIRICAL EVIDENCE

Whether or not individual common stock prices are lognormally distributed is an open question. We examine now the actual price performance of the thirty Dow Jones Industrial stocks over the twenty-five year period ending December 31, 1974. The expected value of a call, C_1, was estimated by summing the positive percentage changes over the 100 quarters, exclusive of dividends and transactions costs, and dividing by 100. These estimates were then discounted at the rate of 20% per annum compounded quarterly. The resulting premium, neglecting transaction costs, gave the call buyer a positive expectation of about 5% for three months. We assumed these premiums to be net to the call writer, after transaction costs. When profitable, we assumed the calls were repurchased at the end of each period for their intrinsic value plus round lot commissions existing during most of this period. We assumed all dividends were reinvested and round lot transaction costs were incurred on the common stock. The following table compares each of the thirty stocks' buy-hold performance with the option writing strategy. In only one case (American Brands) did the buy-hold strategy have a higher growth rate than the optionedstrategy (and even then the difference was negligible). These data clearly demonstrate that option writing is a superior strategy to buy-hold even if the premiums received are less than actuarial value.

Security	E(x)	V(x)	C_1 E(call)	G_n Buy & Hold	G_n Optioned
Allied Chemical	1.0192	.0121	4.74	.0132	.0143
Alcoa	1.0279	.0172	6.47	.0191	.0216
Amer. Brands	1.0240	.0071	3.91	.0202	.0201
American Can	1.0166	.0072	3.33	.0128	.0128
American Tel.	1.0209	.0042	2.80	.0187	.0187
Anaconda	1.0240	.0186	5.77	.0142	.0164
Beth. Steel	1.0332	.0120	5.08	.0273	.0291
Chrysler	1.0205	.0221	6.50	.0091	.0118
Dupont	1.0219	.0089	4.26	.0173	.0201
East.Kodak	1.0384	.0090	5.58	.0334	.0347
Gen. Elect.	1.0295	.0095	5.16	.0245	.0255
Gen. Foods	1.0262	.0079	4.34	.0221	.0227
Gen. Motors	1.0346	.0160	4.73	.0258	.0267
Goodyear	1.0383	.0122	5.89	.0318	.0335
Int'l Harvester	1.0228	.0108	4.57	.0174	.0180
Int'l Nickel	1.0309	.0121	5.60	.0245	.0259
Int'l Paper	1.0314	.0120	5.47	.0252	.0265
Johns-Mann.	1.0237	.0124	4.90	.0175	.0189
Owens-Ill.	1.0220	.0107	4.79	.0165	.0174
Proctor & Gamble	1.0366	.0064	4.96	.0329	.0334
Sears	1.0331	.0080	5.04	.0287	.0292
Std. Oil Cal.	1.0307	.0088	4.72	.0260	.0268
Std. Oil N.J.	1.0348	.0078	4.73	.0305	.0314
Swift	1.0200	.0121	4.55	.0141	.0153
Texaco	1.0361	.0085	5.14	.0313	.0321
Union Carbide	1.0214	.0094	4.56	.0166	.0173
Untd. Aircraft	1.0378	.0211	6.95	.0268	.0311
U.S. Steel	1.0314	.0123	5.32	.0251	.0266
Westinghouse	1.0206	.0164	5.74	.0121	.0136
Woolworth	1.0140	.0111	3.96	.0085	.0090
Averages	1.0274	.0115	4.99	.0214	.0227

X_t = price of stock at end of period t.

$$E(x) = \sum_{t=1}^{100} (X_t/X_{t-1})/100 .$$

$$V(x) = E(x^2) - \left[E(x)\right]^2 .$$

$$G_n = \left[\log (X_{100}/X_0)\right]/100 .$$

Buy and Hold: purchase of the stocks and reinvestment of dividends.

Optioned: sale of covered calls at a premium of 95.544% of C_1.