tempted to go a little further and suggest that the degree of concentration as such does not contribute materially to the explanation of high profitability. Perhaps this is not surprising because although a highly concentrated industry may be associated with high profitability a large number of situations are possible depending, amongst other things, on whether the industry is expanding or contracting, and on the degree of internal, intra industry, and potential competition.

Secondly, the analysis shows clearly the importance of very high barriers to entry arising for instance, from control over raw materials, patent protection and economies of scale. In these cases the means exist whereby firms can maintain high profitability over a long run of years. A concern for barriers to entry should certainly be central to the implementation of a monopoly policy.

Thirdly, the highly significant relationship between growth and profitability is a well established one,[3] and any monopoly policy which is based on realised profitability should at least distinguish between fast and slow growing industries or (ideally) firms, and attempt to assess the extent to which high profitability is 'justified' by a high rate of growth.

[3] See, for instance, R. Marris, "Incomes Policy and the Rate of Profit in Industry," paper read to the Manchester Statistical Society, December, 1964.

STATISTICAL APPENDIX

	Profitability (Pre-tax rate of return on net assets 1950–1960)	Growth of Net Assets (per cent per annum 1950–1960)	Four-firm Concentration Ratio	R_1	R_2
Sulphur	21.6	24.0	95	1	0
Nickel	18.9	20.7	98	1	0
Flat Glass	18.8	13.4	92	1	0
Ethical Drugs	17.9	26.5	57	1	0
Chewing Gum	17.5	6.2	88	1	0
Cement	15.7	22.7	60	0	1
Automobiles	15.5	11.5	75	1	0
Gypsum Products	14.4	18.6	88	0	0
Soap	13.3	16.2	90	0	1
Glass Containers	13.3	14.7	58	0	0
Tyres and Tubes	13.2	10.6	74	0	0
Petroleum Refining	12.2	11.3	32	0	1
Cigarettes	11.6	9.6	79	1	0
Copper	11.5	5.1	87	0	1
Biscuits	11.4	5.0	65	0	1
Baking	11.0	14.9	22	0	0
Beer	10.9	9.0	28	0	0
Steel	10.8	8.8	53	0	1
Metal Containers	9.9	19.1	80	0	0
Shoes (Diversified)	9.6	16.0	27	0	0
Liquor	9.0	6.6	60	1	0
Farm Machinery and Tractors	8.8	9.0	54	0	1
Flour	8.6	10.5	38	0	0
Rayon	8.5	4.7	57	0	0
Canned Fruit and Vegetables	7.7	6.3	29	0	0
Shoe Machinery	7.4	12.9	75	0	0
Textile Mill Products	6.9	10.2	28	0	0
Meat Packing	5.3	1.4	34	0	0

STOCK PRICE RANDOM WALKS: SOME SUPPORTING EVIDENCE

Sheen T. Kassouf *

I Introduction

If stock prices followed a simple random walk, then price changes per unit of time would be normally distributed. That is if X_T is the price of a stock at time T, then

$$X_{T+1} - X_T = A + u_T$$

where A is the average price increase per unit of time and u_T is a normally distributed random variable with zero mean and variance of σ^2.

This model of stock price behavior was hypothesized (perhaps for the first time) by Bachelier [1]. The model implies possible negative values for common stock colliding with the limited liability feature of modern corporate share holders. Osborne [3], Samuelson [4], and others [2], [6], have redeemed the random walk model by hypothesizing that the price changes might be log-normally distributed, e.g., that log $(X_{T+1} - X_T)$ is normally distributed. Evidence which tends to support this hypothesis will be presented here.

Section II summarizes a model for option price developed by Samuelson [4]; Section III tests this model empirically; Section IV presents some caveats.

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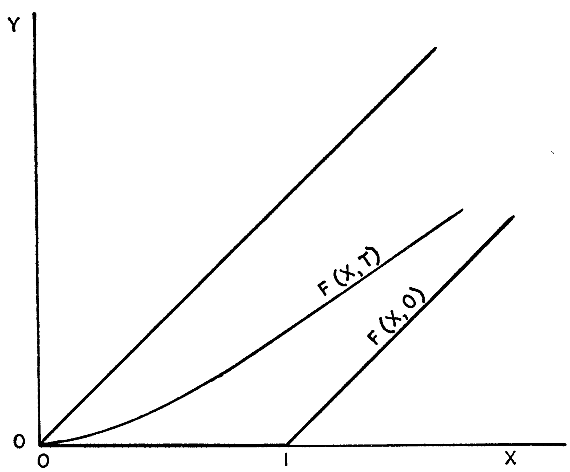
II Samuelson's Model for Option Price

Because there *is* a relationship between the price of an option and the price of the related stock, a "rational" option price can be determined if utility, expectations and stock price are given. Bachelier's great contribution hypothesized the form of investors' expectations. Boness [6] and Sprenkle [7] attempted to work back from observed option and stock prices to parameters of investors' subjective expectations and risk-attitudes. Samuelson and McKean (S-M) [4] provided a solution for perpetual options. (Paradoxically, the case of finite-length options is much more difficult, indicating once again that easy answers can sometimes be arrived at by "passing" to infinity.)

It can easily be shown that option and security price are sometimes tied by a deterministic relation and that even when not completely determined, option price is greatly constrained by stock price. For definiteness, consider a specific type of option — a warrant for common stock. Let X represent price of common, Y price of warrant, and let one be the exercise price,[1] i.e., the amount required with the warrant before it may be exchanged for 1.0 share of common stock. Then it is well-known[2] that $Y = F(X)$ must lie in the rhomboidal region outlined by heavy lines in figure 1. Warrant price is also a function of the length of time, T, that the privilege extends so that $Y = F(X,0)$ is graphed as the lower boundary of the region shown in figure 1.

FIGURE 1. — THE UPPER AND LOWER BOUNDS ON $F(X, T)$



The problem of defining $Y = F(X,T)$ when $T = \infty$ has absorbed the attention of previous researchers. A priori, $F(X,T_1) \geqq F(X,T_2)$ when $T_1 \geqq T_2$, i.e., a warrant with a longer life should be worth at least as much as a warrant with a shorter life, ceteris paribus. The problem became intriguing when it was noted that some warrants are perpetual ($T = \infty$). Bachelier's analysis puts a rational price on these warrants equal to infinity. This conclusion was unacceptable and a more powerful theory was needed; it was supplied by S-M.

Some researchers, in combining utilities and expectations assumed that the warrant could be exercised only at the end of the option period.[3] This might be justified when T is small, as is the case for call options, but it can only lead to a "minimum rational" price. Poensgen [5] makes the same assumption but he seems to be the only one with the exception of S-M who realizes the nature of his solution.[4]

The power and elegance of the S-M solution results from their refusal to make this simplifying assumption. Instead, they assume:

Axiom 1. Future stock prices are subject to a definite probability distribution with constant mean expected growth per unit of time, $\alpha \geqq 0$.

Axiom 2. Warrant price (therefore subject to a definite probability distribution) must gain a constant mean expected growth per unit of time, $\beta \geqq \alpha \geqq 0$.

An investor, knowing his α, β, and probability distribution for X, can then work out (with perhaps some mathematical help) rational warrant prices, $Y = F(X,T)$. In particular, S-M have worked out the solution when stock price changes are log-normally distributed, $\beta > \alpha > 0$, and $T = \infty$. Then

$$F(X, \infty) = \begin{cases} \dfrac{(\phi - 1)^{\phi - 1}}{\phi^{\phi}} X^{\phi}, \text{ for } X < \dfrac{\phi}{\phi - 1} \\ X - 1, \text{ for } X \geqq \dfrac{\phi}{\phi - 1} \end{cases}$$

where $$\phi = \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right) + \sqrt{\left[\frac{1}{2} + \frac{\alpha}{\sigma^2}\right]^2 + 2\left[\frac{\beta}{\sigma^2} - \frac{\alpha}{\sigma^2}\right]}$$

and $\sigma^2 = \text{variance } (\log X_{T+1} - \log X_T)$, the parameter measuring dispersion in the log-normal distribution.

III "Testing" the S-M Model

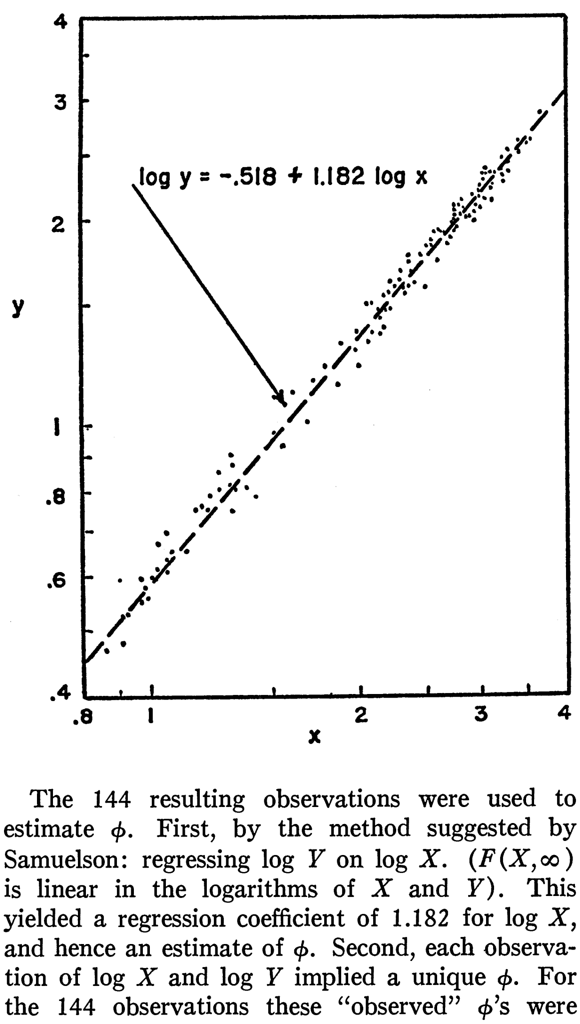
Samuelson correctly points out that "from a non-empirical base of axioms you never get empirical results"[5] but he suggests such a test which we now conduct.

In 1953 the perpetual Alleghany Corporation warrant began trading on the American Stock Exchange. Means of the monthly high-low for this warrant and common stock, for the years 1953 through 1964 were used to test the S-M model. In only three years during this period did the common yield a dividend; the dividend averaged about 1 per cent yield on the annual average price of the

[1] By measuring X and Y with some suitable unit, the exercise price may always be designated as 1. See [4], p. 18.
[2] See, e.g., [4], pp. 18–19, [8], pp. 7–9, [9], pp. 6–8.
[3] See [6] and [7].
[4] See [5], p. 80.
[5] See [4], p. 42.

common. (Figure 2 shows a scatter of these observations.)

FIGURE 2. — Scatter of Alleghany common and warrant monthly mean prices, 1953–1964, with least-squares fit to log of prices. The actual exercise price was \$3.75. The actual prices were divided by 3.75 for this diagram. The resulting "standardized" prices ensure that the "standardized" exercise price is 1.



The 144 resulting observations were used to estimate ϕ. First, by the method suggested by Samuelson: regressing log Y on log X. ($F(X,\infty)$ is linear in the logarithms of X and Y). This yielded a regression coefficient of 1.182 for log X, and hence an estimate of ϕ. Second, each observation of log X and log Y implied a unique ϕ. For the 144 observations these "observed" ϕ's were found using Newton's iteration. Their average, 1.189, yielded another estimate of the "true" ϕ. Both methods of estimation agreed quite closely. For the test, the first estimate of $\phi = 1.182$ was used.

The mean percentage gain per month for the common was estimated as the mean of log (X_{T+1}/X_T). This yielded an estimate of $\alpha = .0053$.

The sample variance of log $(X_{T+1}/X_T) = .00796$ was used as an estimate of σ^2. Substituting these estimates of ϕ, α, σ^2 into (1) implies a value of $\beta = .0076$, the mean percentage gain per month for the warrant. But the mean gain per month implied by the sample average of log (Y_{T+1}/Y_T) is .0068. In addition, the sample variance of log (Y_{T+1}/Y_T) is .0126, so that a 95 per cent confidence interval for the "true" β is $(-.0115 < \beta < .0251)$. The estimate obtained by assuming Samuelson's theory of option price and log-normality of price changes falls inside this interval. This of course does not validate the S-M model, but it does lend some support in that the S-M model is not rejected on the basis of this sample. Closer examination, however, reveals serious doubt about the S-M underpinnings.

IV The Appropriateness of Samuelson's Model

Axiom 2 of Samuelson's model asserts that a rational investor expects a greater average return from a warrant than for its associated common stock. This seems reasonable since it is well-known that warrants involve more risk, i.e., they are more volatile than the common.[6] To compensate for this risk, rational investors price the warrant to increase its expected return. But why should this differential, $\beta - \alpha$, be constant for all values of X? It is well-known that for high values of X, warrant and stock price behavior is similar, whereas for low values of X, warrants sometimes have a very high elasticity with respect to the common. It is perhaps more realistic to hypothesize that $\beta - \alpha$ is a decreasing function of X.

Even then, some circularity would remain. $\beta - \alpha$ determines the constant elasticity of Y with respect to X. But the elasticity *is* a measure of the riskiness of the warrant — greater elasticity means greater risk. Thus the S-M model seems to proclaim that because of differences in risk between warrant and common, measured by $\beta - \alpha$, we can determine the difference in risk between them, viz., $F(X)$!

But the difficulty is deeper and applies not only to the S-M model, but all previous models of security price built on individual rationality. These model builders have been using brick but the data which they hope to be reflecting is composed of a twisting plastic compound. Security prices that are observed in the market result from the collective interaction of many individuals possibly all having different expectations, utilities and resources. In the highly liquid securities markets of today we must pay attention to the "second," "third," and

[6] For example, in 1953 the coefficient of variation for the Alleghany warrant was .57/2.5 = .23, whereas for the common it was .13/4.1 = .03. For the 12-year period of this study, the coefficient of variation for the warrant was 2.26/6.11 = .37 and for the common it was 2.81 /8.7 = .32.

higher degree investors described by Keynes.[7] Samuelson's model describes a first degree investor — one who invests on the basis of his own best estimates for the common stock with no regard for the activity of other investors. That is, $\beta - \alpha$ is determined by the individual investor subjectively. In actual practice, an investor may attempt to determine $\beta - \alpha$ by studying the past price relationship between warrant and common, i.e., by observing the behavior of all participants in the market. He is far removed from the armchair theorist operating at the Keynesian "first" degree.

It might be argued that a rational individualistic theory is still useful for normative purposes. This may be true in general, but not for any theories proposed to date including the S-M model. It would be ruinous to advise an investor that given his α, β he *should* pay up to $\$D$ for a highly liquid asset such as a warrant: if the collective action of all other investors results in a price significantly less than D, it is little consolation to remind him that he is behaving "rationally."

In summary, securities do not possess some "rational" value that is independent of the participants in the market place. Decisions are not made by *rational* investors plugging α, β, σ, into some formula. They are made by *super-rational* Keynesian investors who estimate what average estimates will be. A satisfactory theory of price for a liquid asset will have to incorporate individual behavior of investors operating on the second, third, and higher degrees of anticipation.

[7] Keynes, J. M., *General Theory of Employment, Interest, and Money* (London: Harcourt, Brace, 1936), 156.

REFERENCES

[1] Bachelier, L., *Theorie de La Speculation, Ann. Sci. Ecole Norm. Sup. (3)*, No. 1018 (Paris, Gauthier-Villars, 1900). Also English translation in Cootner (see reference [2]) 17–78.

[2] Cootner, P. H. (ed.), *The Random Character of Stock Market Prices*, (Cambridge: Massachusetts Institute of Technology Press), 1964.

[3] Osborne, M. F. M., "Brownian Motion in the Stock Market," *Operations Research*, 7, no. 2 (Mar.–Apr., 1959), 145–173. Also Comment and Reply, *Operations Research*, 7, no. 6 (Nov.–Dec., 1959, 806–811.

[4] Samuelson, P. A., "Rational Theory of Warrant Pricing," *Industrial Management Review* 6, (Spring, 1965), 13–32.

[5] Poensgen, O. H., "The Valuation of Convertible Bonds," *Industrial Management Review* 7, (Fall, 1965), 77–98 and (Spring, 1966), 83–98.

[6] Boness, A. J., "Elements of a Theory of Stock-Option Value," *The Journal of Political Economy*, LXXII, no. 2, (Apr., 1964), 163–175.

[7] Sprenkle, C. M., "Warrant Prices as Indicators of Expectations and Preferences," *Yale Economic Essays* 1, 1961, 178–231. Also 412–474 in Cootner [1].

[8] Kassouf, S. T., *Evaluation of Convertible Securities*, (Brooklyn: Analytical Publishers), 1962.

[9] Kassouf, S. T., *A Theory and An Econometric Model for Common Stock Purchase Warrants*, Ph.D. dissertation, Columbia University, 1965.

A FURTHER COMMENT ON THE BEHAVIOR OF HELP-WANTED ADVERTISING

Susan W. Burch and Ruth A. Fabricant *

The sharp increase in the National Industrial Conference Board's index of help-wanted advertising (HWI) in the second half of 1965 has evoked considerable interest.[1] The increase in the HWI appeared to indicate that employers were having difficulty finding sufficient workers at a time when the measured unemployment rate still suggested an adequate labor supply. This divergence between the HWI and the unemployment rate raised the question of which series was the better indicator of current conditions in the labor market.

For purposes of clarification we constructed a model of HWI which was an attempt to determine the "normal" relationship between help-wanted advertising and the unemployment rate. Specifically, we were interested in (a) whether the HWI-unemployment rate relationship remained stable in the period following 1951; (b) whether the HWI was unusually high in late 1965; and (c) whether our model would lead us to the same conclusions which Cohen and Solow drew from their earlier work on HWI. In the discussion which follows we will first present our model, and then compare our findings with those of Cohen and Solow.

Our equation is:

* Economists, Federal Reserve Board. The views expressed in this paper are the authors' own and are not necessarily those of the Board of Governors of the Federal Reserve System. We would like to thank Malcolm Cohen, Robert Solow, and Alfred Tella for their comments.

[1] See, for example, Malcolm S. Cohen and Robert M. Solow, "The Behavior of Help-Wanted Advertising," this REVIEW LIX (Feb. 1967), 108–110.