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THE LAG STRUCTURE OF OPTION PRICE

S.T. KASSOUF

University of California, Irvine, CA 92664, U.S.A.

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The price of common stock warrants do not adjust immediately to changes in common stock prices. This lag is inconsistent with the 'efficient market' hypothesis. Based on daily closing prices this lag was measured and found to be a combination of the adjustment to stock price and to the adjustment of 'other' variables, i.e., positive serially correlated disturbance terms. A single equation model simultaneously estimating the parameters of the serial correlation and the coefficients of the lagged stock price indicate a substantial deviation from efficiency. Various simple strategies designed to exploit this lag are then tested.

1. Introduction

The price of a common stock warrant is substantially determined by the price of its associated common stock.[1] This close relationship permits us to statistically measure the speed with which a warrant's price adjusts to changes in the common stock's price. There is now a massive body of evidence that purports to show that securities markets are 'efficient', in the sense that prices almost always reflect all known information.[2,3] We present here evidence which indicates that warrant prices are probably not efficient, i.e., they lag changes in their most influential determinant. In section 2 the simplest lag structure is hypothesized involving only the preceding day's price. Tests of the model in section 3 strongly indicate a statistical lag. In section 4 the cumulative impact and consequences of the lag are presented.

2. The price-adjustment mechanism

Some previous studies have indicated a tendency for warrant prices to lag common stock prices.[4] In particular, it has been observed that if the price of a

[1]For a thorough treatment of the theoretical basis for option pricing, see Merton (1973). This article, which may well have been subtitled 'The Limits of Rationality' contains an extensive bibliography spanning seventy years.

[2]See Granger and Morgenstern (1970), Fama (1965), and Cootner (1964).

[3]This so-called efficiency would be a consequence of markets that optimally allocate resources. Unfortunately, the converse is not necessarily true: prices may be 'efficient' in a market which mis-allocates resources. It would not be surprising if a statistical analysis of tulip prices during the 'tulip mania' would conclude that prices were 'efficient'.

[4]See Thorp and Kassouf (1967, p. 120, fig. 8.4) and Kassouf (1965, pp. 61–62).

common stock is at X_0 and its warrant at Y_0, a subsequent rise in the price of the common and a fall back to X_0 will leave the warrant at price greater than Y_0. This may be due to a lagged response to changes in the common, or a lagged response to other variables, or some combination.

At the present time it would be presumptuous to impose the lag structures of Koyck, the partial adjustment, or the adaptive expectations model.[5] Indeed, these models seem inappropriate since they do not allow for interaction between lagged prices. We concentrate instead on the significance, if any, of one-day lags. If past prices exert any influence on present price, this should be evident in the most previous past price.

There is theoretical and empirical foundation suggesting that the relationship between a long-lived warrant and its common stock is log-linear.[6] In addition, the length of time remaining also influences price. To test for a lagged response to changes in the price of the common, the following model was hypothesized:

$$\ln y_t = \alpha + \beta_0 \ln x_t + \beta_1 \ln x_{t-1} + \beta_2 z_t + u_t, \tag{1}$$

y_t = closing warrant price on day t,
x_t = closing common price on day t,
z_t = (trading days to expiration)$^{-1} \times 1000$,
u_t = stochastic disturbance term with $E(u_t) = 0$.

A lagged adjustment in warrant price will arise if $\beta_1 \neq 0$, and/or if u_t is serially correlated. That is, the warrant may not adjust immediately to a change in the common price, or to a change in 'everything else', where this latter influence is impounded in the disturbance term. [In every case we examine below, regressions using (1) resulted in R^2 above 0.95 but with Durbin–Watson statistics of less than 0.3, indicating positively auto-correlated disturbance terms. See Granger and Newbold (1974) for the dangers involved in interpreting these results. Accordingly, we only report here the results where serial correlation was assumed and tested.] Again we start with the simplest assumption and test for a first-order dependence,

$$u_t = \rho u_{t-1} + \varepsilon_t, \tag{2}$$

where $E(\varepsilon_t) = 0$, $E(\varepsilon_t \varepsilon_{t-s}) = \sigma^2$ when $s = 0$ and 0 when $s \neq 0$. Combining (2) and (1),

$$\begin{aligned} \ln y_t = {} & \alpha(1-\rho) + \rho \ln y_{t-1} + \beta_0 \ln x_t + \beta_2 z_t - \rho\beta_2 z_{t-1} - \rho\beta_1 \ln x_{t-2} \\ & + (\beta_1 - \rho\beta_0) \ln x_{t-1} + \varepsilon_t. \end{aligned} \tag{3}$$

[5]These models are frequently employed more for their statistical tractableness than for any theoretical basis. See Nerlove (1972).

[6]See Samuelson (1965). For some empirically derived estimates, see Kassouf (1965, 1968).

Following Durbin (1960), we use ordinary least-squares estimation on (3) to obtain a consistent estimate of ρ and in a second stage use this estimate to transform the original data which are then used to estimate the parameters of (1).

For warrant prices to be efficient, β_1 must be 0 *and* $\rho = 1$. Otherwise the sequence $\{y_t\}$ cannot be a martingale (or submartingale) for (1) and (2) yield

$$\Delta y_{t+1} = \beta_0 \Delta x_{t+1} + \beta_1 \Delta x_t + \beta_2 \Delta z_t + u_t(\rho - 1) + \varepsilon_{t+1},$$

where

$$\Delta y_{t+1} = \ln y_{t+1} - \ln y_t,$$

$$\Delta x_{t+1} = \ln x_{t+1} - \ln x_t.$$

Since at time t, x_t is known and u_t can be estimated, tomorrow's expected change in warrant price will be uncorrelated with today's change if and only if $\beta_1 = 0$ and $\rho = 1$. This formulation of the efficiency hypothesis is restrictive because it assumes a one-period linear lag in x or u or both. But as will be seen below (table 3), the dependency we find in this one-period specification is consistent with the refutation of the more general efficiency hypothesis.

3. Description of data and regression results

The data used here consist of daily closing prices of three actively traded warrants and their associated common stocks for the period starting February 19, 1969 and ending September 29, 1972, a period of 913 trading days. The starting date was chosen as the first date when all three warrants were publicly traded. The ending date because sometime late in 1972 warrant – common price relationships were severely distorted by the ingenious activities of some financiers who abrogated the rights of warrant holders.[7] The study was restricted to these three warrants because of their high trading activity, their similarity in exercise terms, and because throughout this period they all traded below their respective exercise prices – the region in which the log-linear relationship would be expected. The three companies are General Host (GH), United Brands (UB) (formerly AMK Corporation), and National General Corporation (NGC). The common stocks all traded on the New York Stock Exchange. The UB and NGC wts traded on the American Stock Exchange and the GH wts on the Pacific Coast Stock Exchange. Since the closing on the Pacific Coast was two hours later than

[7]See 'Who Will Protect the Warrant Holders', *The Value Line Convertible Survey*, Arnold Bernhard & Co. Inc., New York, January 1, 1973, p. 381. Briefly, late in 1972 and early 1973 a number of attempts, some successful, were made to tender for the common shares of a company with a large number of warrants outstanding. The tender price for the common was in cash and less than the exercise price of the warrant. When successful, if the tender offer ignored the outstanding warrants, the warrants became worthless. The realization that this could happen caused many warrants to fall substantially even with the associated common rising.

the closing on the American, in the case of GH wt traders had more time to adjust to changes in the common. In spite of this we show a measurable statistical lag. Table 1 summarizes the results of the regressions of eq. (3).

The almost perfect correlation between z_t and z_{t-1} and between $\ln x_t$ and $\ln x_{t-1}$ casts doubt on the estimates of their coefficients and their significance. The estimate of the coefficient of $\ln y_{t-1}$ in each case is highly significant and close to 1 indicating that warrant prices adjust rapidly to the impact of 'other'

Table 1

Estimates of parameters of eq. (3); t-statistics in parentheses.

Variable	Coefficient	GH	UB	NGC
$\ln x_t$	β_0	0.9399 (20.9)	0.8171 (22.1)	0.9128 (26.6)
$\ln y_{t-1}$	ρ	0.8503 (47.5)	0.9315 (79.9)	0.9641 (110.7)
$\ln x_{t-1}$	$(\beta_1-\rho\beta_0)$	−0.6522 (10.2)	−0.6111 (11.4)	−0.8323 (16.4)
$\ln x_{t-2}$	$-\rho\beta_1$	−0.0725 (1.6)	−0.1343 (3.6)	−0.0351 (1.0)
z_t	β_2	0.4414 (0.06)	3.035 (0.61)	0.5354 (0.11)
z_{t-1}	$-\rho\beta_2$	−0.5820 (0.08)	−3.059 (0.61)	−0.5779 (0.12)
Constant	$\alpha(1-\rho)$	−0.3416 (7.6)	−0.0840 (2.7)	−0.0545 (3.0)
D-W stat.		2.47	2.53	2.53
	R^2	0.99	0.99	0.99

variables. In the second stage these estimates of ρ were used to transform the variables of (1) into the differences $\ln y_t - \hat{\rho} \ln y_{t-1}$, $\ln x_t - \hat{\rho} \ln x_{t-1}$, etc. (With estimates of ρ very close to 1 it might seem more economical to assume perfect adjustment to 'other' variables and concentrate on the lagged impact of the common stock price. When ρ was assumed equal to 1 and all the variables transformed as simple differences, the results were substantially similar to the results reported below, i.e., the value and significance at $\hat{\beta}_0$ and $\hat{\beta}_1$ did not change very much. However, $\hat{\beta}_2$, measuring the influence of time remaining was not significant and had the wrong sign.)

Ordinary least-squares applied to these transformed data yielded the estimates in table 2. (Much of the multicollinearity was removed by transforming the data.)

In the cases of GH and UB, $\hat{\beta}_1$ was highly significant and for NGC was significant at the 90 percent level. If we define the total elasticity of the warrant with respect to the common as $\hat{\beta}_0+\hat{\beta}_1$ then for GH 17 percent of the warrant price adjustment is delayed one day; for UB 13 percent; and for NGC 5 percent. Thus for the three cases, NGC comes closest to an efficient market with $\hat{\rho} = 0.96$ and $\hat{\beta}_1 = 0.05$. The other two show significant lags. (It is surprising that the total elasticity is greater than 1 only in the case of GH.)

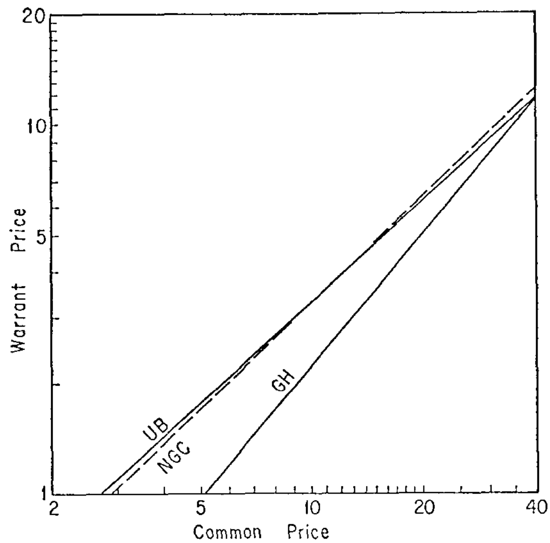

Fig. 1. Estimated price relationships for 10-year warrants.

Fig. 1 shows the estimated relationship between the price of the common and the warrant for the three cases, assuming 10 years to expiration. Throughout most of the time period covered, the GH wt traded below the price of the other two for a given stock price. This implies that investors' ex-ante expectations for the volatility of the GH common were substantially below the ex-post volatility. (The standard deviation of log price for GH common was 0.54, for UB common 0.42, and for NGC common 0.41.) This follows from the structure of most models of option price which accord higher prices to warrants whose common have higher volatility, ceterus paribus.

4. Further evidence of lag structure

The results of the preceding section indicate that if the common stock advances (declines) on day t, the warrant will also advance (decline) on day t. In addition, the warrant will tend to advance (decline) on day $t+1$. The magnitude of this tendency is displayed in table 3 which details the average daily change over the entire period and the conditional average daily change on days following a zero, positive, or negative change in the common.

Table 2

Estimates of parameters of eq. (1).

	GH	UB	NGC
β_0	0.9915 (26.3)	0.7895 (26.4)	0.9098 (29.0)
β_1	0.2015 (5.30)	0.1191 (3.98)	0.0519 (1.65)
β_2	−1.5394 (9.2)	−1.1288 (6.1)	−1.3384 (3.9)
Adjusted R^2	0.74	0.67	0.58
Durbin–Watson	2.22	2.47	2.50

Table 3

Average daily percent changes.[a]

y	GH wt	UB wt	NGC wt	GH com	UB com	NGC com
$E(\Delta y_t)$	−0.00	−0.09	−0.02	−0.04	−0.08	+0.01
$E(\Delta y_t \mid \Delta x_{t-1} = 0)$	1.08	−0.56	0.57	0.51	−0.42	−0.02
$E(\Delta y_t \mid \Delta x_{t-1} > 0)$	0.23	0.60	0.21	−0.00	0.17	0.18
$E(\Delta y_t \mid \Delta x_{t-1} < 0)$	−0.53	−0.55	−0.32	−0.24	−0.20	−0.12

[a] Δy_t = daily percent change in y on day t,
Δx_{t-1} = daily percent change in related common on day $t-1$.

For example, the column headed GH wt indicates that the daily percent change in the warrant over the entire period was barely negative. However, the average change in the warrant was 1.08 percent for those days following no change in the common; +0.23 percent on days following a rise in the common; and −0.53 percent on days following a fall in the common. Testing the hypothesis that the difference in means for these last two categories is zero yields a t-statistic of 1.68 indicating a rejection of the null hypothesis at about the 90 percent level. For the UB wt the t-statistic is 3.9 and for the NGC wt 1.73. This reinforces the regression results which indicated that the previous day's price change in the common is positively related to the present day's price change in the warrant.

The cumulative impact of these changes is detailed in table 4. In each case, buying the warrant only on days when the common rises results in larger terminal wealth; if in addition, the warrant is sold short[8] when the common falls, terminal wealth is substantially improved. With the UB wt for example, each original dollar invested would have been multiplied more than 44 times; and this in a period when the warrant fell to less than 20 percent of its starting price.[9] These same conditional strategies applied to the common stocks resulted in terminal values superior to the buy and hold but less than the values associated with the warrants.

This does not imply that actual trading strategies can be employed to exploit the lag. Transaction costs would more than wipe out any advantage. The strategy of line 3 in table 4 required 760 transactions for the GH wt, 758 for the UB wt, and 804 for the NGC wt. Since the *average* advantage in the daily price change was relatively small (on the order of less than 1 percent), and since one transaction was required about every 1.2 days (when a position was reversed it required two transactions, e.g., liquidating the long position and simultaneously instituting a short position or vice-versa), the practical effect is to enrich the broker and not the investor.

Nevertheless, this statistical dependency is of interest for at least two reasons. First, given the obvious connection between the price of a warrant and its related common stock, it is surprising that there is *any* measurable dependency. This casts doubt on the efficiency of other stock prices where the determinants of price are not clearly established. That is, if warrants do not adjust to their determinants of price, it is likely that common stocks also do not adjust to their determinants, since in the latter case there is no wide agreement as to the explanatory variables.

Second, statistical dependencies, even though not exploitable by the typical investor, shed light on the expectational dynamics of investors and the economics of securities markets. For example, it is well known that on successive trades in stocks, price changes are negatively correlated. The average investor cannot benefit from this because of transaction costs, but it may be one determinant of specialist profits. Any analysis of the specialist system would therefore benefit from this knowledge.

[8]In the markets studied here, short sales can only be executed at prices higher than the most recent different price, or on an 'up-tick'. In this study this requirement was neglected and it was assumed that the short sale took place at the closing daily price.

[9]A comparison of terminal wealths as a criterion for ranking investments implies a logarithmic utility function of wealth [see Merton and Samuelson (1974)]. Furthermore, maximization of terminal wealth requires not only the choice of a strategy, but the proportion of one's wealth to be invested at each period. For example, strategy A may return a constant 5 percent per annum, so terminal wealth after a long period of years will be $(1.05)^n$. Strategy B may return either 100 percent or -50 percent in any year, with equal probability. Strategy B has an expected annual yield of 25 percent, but if the entire proceeds of each year's outcome is reinvested, terminal wealth will tend to 1 after many years. This does not make A superior to B, even for logarithmic utility maximizers: if only one-half of each year's proceeds are invested in B and the remainder kept in cash, terminal value will tend to $(1.06)^n$.

A new universe of stock options has come into being with the advent of the Chicago Board Options Exchange. These growing number of options should prove a fertile area in our search for the methods by which investors form price expectations and make investment decisions.

Table 4

Terminal value of each $1 invested, no transaction costs.

	GH wt	UB wt	NGC wt	GH com	UB com	NGC com
(1) Buy and hold for entire period.	0.18	0.19	0.37	0.37	0.33	0.71
(2) Hold only if common does not fall, otherwise sell and hold cash, rebuying when common rises.	1.49	4.33	1.74	0.76	1.28	1.64
(3) Hold long if common does not fall, otherwise sell and go short. Cover and go long when common rises.	1.73	44.17	3.41	0.76	3.25	2.21

References

Cootner, P., ed., 1964, The random character of stock market prices (M.I.T. Press, Cambridge, Mass.).

Durbin, J., 1960, Estimation of parameters in time-series regression models, J. Royal Stat. Soc., Ser. B, 22, 139–153.

Fama, E., 1965, The behavior of stock prices, Journal of Business 38, 34–105.

Granger, C. and O. Morgenstern, 1970, Predictability of stock market prices (D.C. Health, Lexington, Mass.).

Granger, C. and P. Newbold, 1974, Spurious regressions in econometrics, Journal of Econometrics 2, 111–120.

Kassouf, S., 1965, A theory and an econometric model for common stock purchase warrants, Dissertation (Columbia University, New York).

Kassouf, S., 1968, Stock price random walks, The Review of Economics and Statistics 50, 275–278.

Merton, R., 1973, Theory of rational option pricing, The Bell Journal of Economics and Management Science 4, 141–183.

Merton, R. and P. Samuelson, 1974, Fallacy of the log-normal approximation to optimal portfolio decision-making over many periods, Journal of Financial Economics 1, 67–94.

Nerlove, M., 1972, Lags in economic behavior, Econometrica 40, 375–83.

Samuelson, P., 1965, Rational theory of warrant pricing, Industrial Management Review 6, 13–39.

Thorp, E. and S. Kassouf, 1967, Beat the market (Random House, New York).