Warrant Price Behavior — 1945 to 1964

by Sheen T. Kassouf

ANALYSTS now have a clear need for tools to properly evaluate options: there are 22 actively traded warrants on the American Stock Exchange and every day it seems new convertible issues (with an option element embedded in them) come to market with increasing frequency. In fact, in the first six months of 1967 more than one billion dollars of convertible bonds were issued.[1] (This does not include convertible preferred stocks or bonds with warrants attached.) Recent issuers include Southern California Edison, Wells Fargo Bank, McGraw-Hill and RCA. Since any convertible security can be conceptually decomposed into a senior security and an option, rational investors and responsible analysts must be able to determine when options are under or over priced.

In order to evaluate an option it is essential to identify as many variables as possible that determine price. In a recent article in this *Journal*[2] Professor John P. Shelton reported the results of his statistical analysis of warrant prices. His research, conducted independently, confirms some of the conclusions reported here. In particular, both Shelton's statistical analysis and the work reported here found the length of time remaining before expiration and the dividend yield of the associated common stock influenced warrant price significantly. His findings differed with the results reported here in that he found potential dilution of the common stock and the recent price history of the common stock to have no significant effect on warrant price. The research reported here indicates that these variables were significant, at least for the population of warrants studied (listed on the American Stock Exchange).

This paper details the variables found significantly associated with warrant price and presents a geometrical construction of the common stock-warrant price relationship as it existed for the 1945-1964 period. If the past is any guide to the future, this relationship will be useful to anyone contemplating the purchase or sale of a stock when the corporation has outstanding a convertible security, or when such a security can be brought into existence by buying or writing an option. Although this paper will be concerned primarily with listed warrants, the method in which the analysis can be extended to all convertibles is outlined.

1. Footnotes appear at end of article.

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Determinants of Warrant Price

A warrant is a certificate that the holder may convert into a share of common stock at his option. To effect the conversion, the holder must relinquish the certificate with a fixed sum (the exercise price) before a fixed date (the expiration date) to the issuing corporation. *A priori*, it might be expected that the following variables "determine" warrant price:

(1) The length of time remaining before expiration. For example, a warrant that expires in two years should be worth at least as much as a warrant that expires in one year, *ceteris paribus.*

(2) The dividend yield on the associated common stock. Warrants do not earn dividends. Compared to the common stock the warrant becomes less attractive an alternative the greater the dividend, *ceteris paribus.* Furthermore, it is possible that high dividends are indicative of low price appreciation; and since the potential price appreciation of the common stock will eventually determine the price of the warrant, high dividends may be associated with low warrant prices, again, *ceteris paribus.*

(3) The potential dilution of the common stock resulting from the exercise of all the outstanding warrants. Shelton rejects this variable on logical grounds. He argues that the effect of the potential dilution has presumably already influenced the stock price; thus, for a *given* stock price, "to decrease further the value of the warrant would seem to be . . . counting opponents' tricks twice."[3] A simple counter-example shows the insufficiency of Shelton's argument. The stock of company A is at 20. An investor believes there is a 50-50 chance that the stock will be at 15 or 25 in one year. Consider now a warrant on this stock that will give rise to no dilution. (For example, a call option issued by a writer will not result in any dilution of the original common stock outstanding even if it is exercised because the writer must supply the stock from his holdings or by buying in the open market.) If this warrant is exercisable at 20, this investor then believes there is a 50-50 chance that it will be worth either 0 or 5 in one year. (If the common is 15, the warrant is worthless, and if the common is 25, the warrant is worth 5.) This investor might then be

willing to pay $2.50 for this warrant for that is its expected value. Now consider company B whose stock is also at 20, and with as many warrants outstanding as there are common shares. This same investor believes that there is a 50-50 chance that the common stock will be either 17½ or 25 one year from today *before dilution*. He is aware, however, that if the stock is 25, all the outstanding warrants will be exercised at a price of 20, reducing the price of the common to perhaps 22½. That is, *after dilution*, this investor believes there is a 50-50 chance that the common will be 17½ or 22½. Thus, the value of this warrant one year from today will be either 0 or 2½ and he might be willing to pay only 1¼ for it. We have thus shown that given the common stock price of 20, the effect of dilution might reduce the value of a warrant 50%. Some heroic assumptions were made and this counter-example does not prove that potential dilution affects the price of a warrant. It only shows that it *might* and it forces us to look at the evidence.[4]

(4) The recent price history of the common stock. An investor's expectation for the common stock determines, in part, the value of the warrant. If his expectation is influenced by recent price behavior, then the price history of the common stock might affect the price of the warrant. If this variable is influential, then warrant buyers are not wedded to the random walk hypothesis which states that recent price history is of no help in predicting future price action.[5] If enough participants are anti-random walk, their activity will reinforce their belief and then past prices *will* matter.

Geometry of Warrant Price

Based on a sample consisting of all listed American Stock Exchange warrants during November of every year from 1945 through 1964 the four variables listed in the preceding section proved statistically significant. The following charts indicate the effect of these variables on warrant price during this period. (The formula[6] which computes this relationship is complex; the purpose of the charts is to present as simply as possible the relationship over a wide range of typical values for the explanatory variables.) These curves define the "average" behavior of all listed warrants during this period. Of course, many warrants deviated from these curves at many times, indicating that warrant price is not completely determined by these four variables. Whether these deviations can be explained by other quantifiable variables is the challenge now facing us.

Armed with these charts, or better, with the formula in footnote 6, an investor can compare any warrant price of interest with a historical average. This will help him determine whether a warrant should be purchased or avoided.

The first three variables influence warrant price as expected: warrants decrease in price as they approach expiration; dividends on the common stock reduce the price of the warrant; the larger the potential dilution, the lower the price of the warrant. The fourth variable, the recent price history of the common stock, has an interesting effect on the price of a warrant: the more the common stock has risen in the recent past, the less the price of the warrant (and the more the common stock has fallen, the more the price of the warrant). There are at least two possible explanations. If warrant buyers believe on average that stocks that have moved up are due for a correction (or that stocks that have fallen are due to rise) then they will adjust the amount they pay for a warrant accordingly. If this reasoning is correct, then warrant traders are anti-chartists who believe that recent trends are more apt to reverse them-

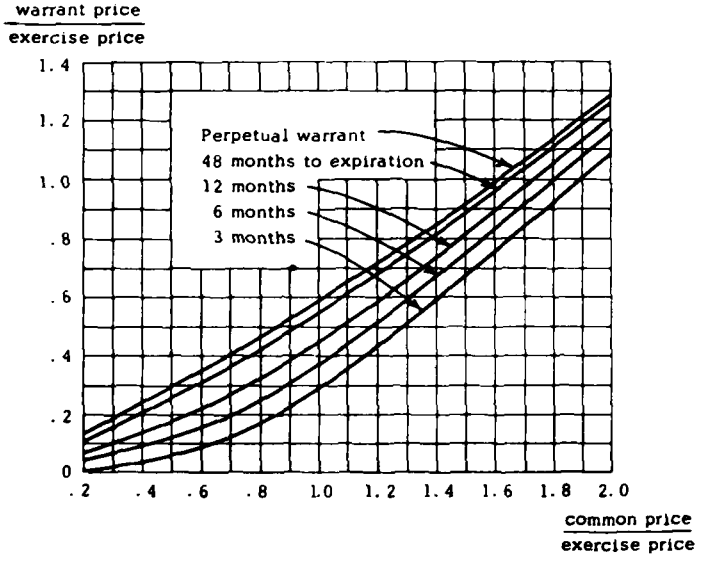

Chart 1. Average stock-warrant relationship when potential dilution is negligible (D=0); no dividends are paid on common (R=0); and present price of common is equal to price 11 months ago ($X/X_j = 1$).

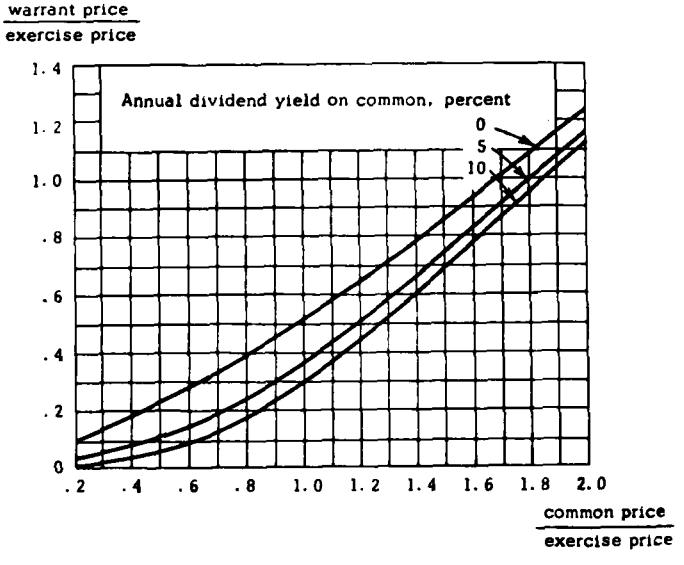

Chart 2. Average stock-warrant relationship when there are 4 warrants outstanding for every 10 common shares (D= .4); 48 months to expiration (T=48); and present price of common is equal to price 11 months ago ($X/X_j = 1$).

selves rather than continue. Another explanation for the effect of recent price action is that warrant prices take time to adjust to changes in the price of the common: if the common stock moves up, it takes time for warrant traders to act on this information and consequently the price of the warrant will lag. If this latter explanation is correct, it lends support to the anti-random walk theorists because the random walk hypothesis asserts that there is instantaneous adjustment of prices in the stock market to all news and information.[7]

Other Stock-Option Price Relationships

It must be emphasized that the accompanying charts relate to listed warrants. Other options have different advantages and disadvantages, e.g., over-the-counter warrants cannot be purchased on margin, executive stock options are not negotiable, etc. Therefore, it is necessary to examine each universe separately to determine price-option relationships.

Consider, for instance, the McGraw-Hill 3⅞ of '92 convertible debenture. This bond may be converted into 16 shares of common stock. If the bond did not have this conversion privilege, it has been estimated that it would sell for $770.[8] The recent market price of the bond was $1100, so that the conversion feature was selling for $330. The convertible bond might be considered equal to a straight debenture worth $770, plus 16 "warrants" worth $330, or $20⅝ each. Since the bond must be surrendered if the "warrant" is to be exercised, then the exercise price for the 16 warrants is $770/16 = $48⅛. In this way the latent warrant underlying every convertible bond can be found. Statistical tests can then be made to determine what variables affect the price of the latent warrants. Preliminary work now indicates that the most influential variable, given the price of the common stock, is the potential dilution that would result if all the outstanding bonds were converted. Also, the influence of time does not seem to be very large, possibly because the bond holder is at the mercy of the corporation since the bond can usually be called at the option of the company before it is due or before the conversion privilege expires. Furthermore, the price curves seem to be substantially lower than for pure warrants, perhaps because the buyer of a latent warrant is saddled with a debenture he may not wish to own or finance.

Proceeding in this way, price curves for convertible preferred stocks, call options, stock rights, etc., can be derived and should prove useful to financial analysts and investors. ♦

FOOTNOTES

1. *Statistical Bulletin*, U. S. Securities & Exchange Commission, August 1967, p. 12.

2. Shelton, John P., "The Relation of the Price of a Warrant to the Price of Its Associated Stock," *Financial Analysts Journal*, Vol. 23, No. 3 (May-June 1967), pp. 143-151 and No. 4 (July-Aug. 1967), pp. 88-99.

3. Ibid., p. 147

4. It is interesting to note that Graham, Dodd and Cottle, *Security Analysis*, 4th ed., (New York: McGraw-Hill Book Company, Inc., 1962), p. 658 claim that "the value of a warrant depends . . . on the number of warrants outstanding relative to the common-stock issue." Shelton, op. cit., p. 143, acknowledges an "important contribution" made by Mr. Benjamin Graham to his paper. Apparently Mr. Graham was swayed by Shelton's argument and abandoned the earlier conviction set forth in *Security Analysis*.

5. See Eugene F. Fama, "Random Walks in Stock Market Prices," *Financial Analysts Journal*, Vol. 21, No. 5 (Sept.-Oct. 1965), pp. 55-59.

6. Although it is a general policy of the *Financial Analysts Journal* to minimize the use of mathematical notation, occasionally the concepts of an author can only be accurately conveyed in their generality by means of mathematical symbols, and the usefulness of the article will be enhanced

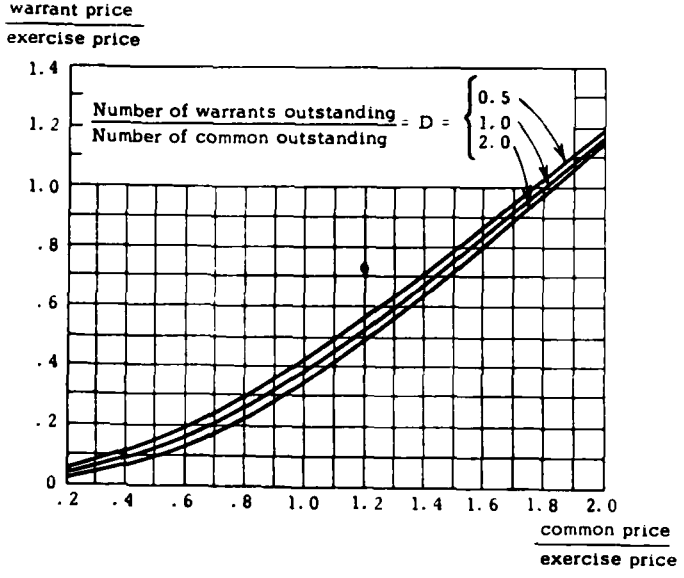

Chart 3. Average stock-warrant relationship when 48 months to expiration (T=48); common yields 3% in dividends annually (R=.03); and present price of common is equal to price 11 months ago ($X/X_j = 1$).

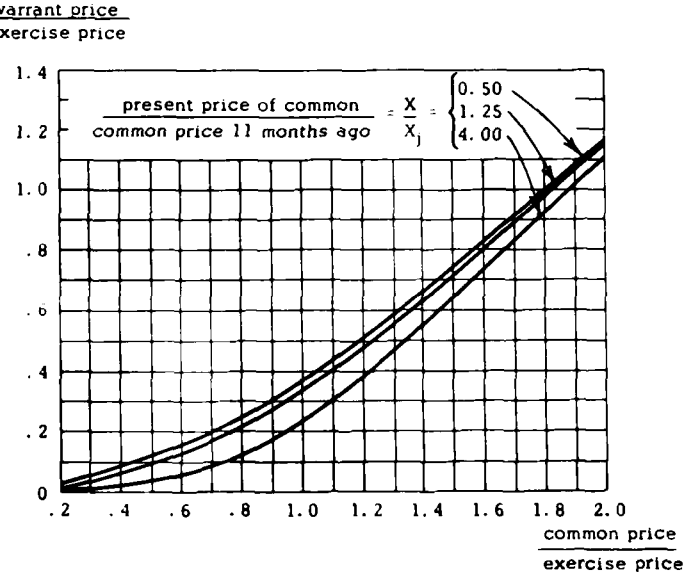

Chart 4. Average stock-warrant relationship when 48 months to expiration (T=48); common yields 3% in dividends annually (R=.03); there are 4 warrants outstanding for every 10 common shares.

enough to justify making an exception to this policy. Such is the case in this article.

Briefly, the warrant pricing model described in this paper is based on the formula

$$y = (x^z + 1)^{1/z} - 1.$$

y and x are merely standardized values of the warrant and the common stock, respectively. Specifically y = price of warrant ÷ exercise price and x = price of common ÷ exercise price. The critical variable is z, which is so designed that it can have values between 1 and ∞.

Consider when z = 1, then y = x. This is the maximum price a warrant can obtain, i.e., the price of the warrant cannot exceed the price of the common. At the other extreme, when z = ∞ y = 0 if the common is less than the exercise price (because x^z approaches 0 when x is less than 1) and y = x − 1 when the price of the common exceeds the exercise price (because $x^z + 1$ approaches x^z as z increases and when taken to the 1/z power becomes x). This defines the minimum price that a warrant can be given the existence of arbitrageurs. For intermediate values of z, a smooth curve is defined. Every point in the positive quadrant of prices lies on a unique curve so that every warrant-common price relationship can be defined by a unique value of z. Using this family of functions, only one parameter, z, has to be estimated. Because the relationship between x and y is non-linear, this indirect method of estimation economizes degrees of freedom.

A multiple regression on 222 observations for twenty years yielded a multiple correlation coefficient of .63 and the estimating equation:

$$z = \underset{(0.1497)}{1.2221} + \underset{(0.7530)}{5.3131}\,T + \underset{(1.9473)}{14.8135}\,R + \underset{(0.1353)}{0.2765}\,D + \underset{(0.1117)}{0.4401}\,\frac{X}{A} + \underset{(0.1925)}{0.4131}\log\frac{X}{X_j}$$

where

T = number of months to expiration
R = annual dividend yield on common stock
D = number of outstanding warrants/number of outstanding common shares
X = price of common
A = exercise price of common
X_j = price of common 11 months previous
log = natural logarithm

Both the dilution, D, and the recent price performance of the common, X/X_j, are just significant at the 5% level, indicating that their influence on warrant price is not nearly as strong as that of longevity and dividend yield. Arithmetically, dilution and recent stock action do not contribute much to warrant price when they are in the range of $0 \leq D \leq 1$ and $.8 \leq X/X_j \leq 1.2$, but their influence grows outside these intervals. To ignore them in these cases is to forfeit to chance what we have won from statistical inference.

7. Fama, *op cit.*, p. 56.

8. *Convertible Fact Finder*, (New York: Kalb, Voorhis and Co.: August 7, 1967).

GROWTH REPORT

While labor unrest in the automotive industry, and a nine-week long steel haulers' strike, had a strong but temporarily depressing effect on Latrobe's sales and earnings during the third quarter, it is currently anticipated that 1967 will rank as the second most successful year in the Company's history. If, as expected, the capacity of the new rolling mill becomes available late next year, the Company's volume thereafter should increase significantly.

DIVIDEND NO. 205

The Board of Directors of Latrobe Steel Company has declared a regular quarterly dividend of 15 cents per share on the common stock, payable December 29, 1967, to shareholders of record at the close of business December 13, 1967.

November 24, 1967

G. E. McDonald
Secretary

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OUTBOARD MARINE

OUTBOARD MARINE CORPORATION

DIVIDEND NOTICE

The Board of Directors of the Company has declared a cash dividend of twenty five cents (25c) per share on the Common Stock of the Company, payable November 30, 1967 to stockholders of record November 16, 1967. This compares with twenty cents (20c) per share paid in previous quarters.

J. R. Seeger
Secretary

October 27, 1967

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