Forward Guidance, Expectations, and Learning

Stephen J. Cole *

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Abstract

The unconventional monetary policy of forward guidance operates through the management of expectations about future paths of interest rates. This paper examines the link between expectations formation and the effectiveness of forward guidance. A standard New Keynesian model from Preston (2005) is extended to include forward guidance shocks in the monetary policy rule following Del Negro, Giannoni, and Patterson (2012) and Laseen and Svensson (2011). Agents form expectations about future macroeconomic variables via either rational expectations or an imperfect knowledge model. Under the imperfect knowledge or adaptive learning model, expectations are based on a correctly specified forecasting model whose coefficients are updated over time using discounted least squares ("constant gain learning"). The impulse responses of the endogenous variables show that forward guidance produces different results on the economy depending on the expectations formation assumption. The main result of the paper is that agents with rational expectations exhibit a more favorable and exaggerated response to forward guidance than under adaptive learning. This outcome is shown when the economy experiences a recession. The reason for the difference is that agents with rational expectations precisely understand the effects forward guidance has on their expectations. However, even though they have information about future interest rates, adaptive learning agents imprecisely understand how forward guidance impacts the economy as their beliefs evolve slowly over time.

Keywords: Forward Guidance, Monetary policy, Constant Gain Learning, Expectations.

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*Department of Economics, University of California Irvine, 3151 Social Science Plaza, Irvine, CA 92697-5100. Email: sjcole@uci.edu

1 Introduction

Once U.S. short-term interest rates effectively reached the zero lower bound (ZLB) during the 2007-2009 global financial crisis, monetary policymakers exhausted the conventional policy tool of changing interest rates. In response, central banks pursued "unconventional" policies, such as forward guidance, where the central bank communicates to the public information about the future course of the policy rate. An example of forward guidance was given in the September 2012 Federal Open Market Committee (FOMC) statement: "...the Committee also ...anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015." Woodford (2012) argues that committing to an interest rate path that is lower than what one would commit to in a normal policy rate period can have positive effects on an economy that is in a persistent liquidity trap. Woodford (2012) describes a channel where anticipating the economy will expand in the future, agents' consumption will increase today. However, agents expect that a conventional monetary policy rule will restrain the economy from "overheating," and thus, limit current consumption. If a forward guidance statement, instead, keeps a low policy rate through part of the expansion, Woodford (2012) explains consumption today will not be as limited.

Because the effectiveness of forward guidance hinges on how expectations respond to forward guidance, it is of interest to investigate whether the efficacy depends on the rational expectations assumption, which is the standard benchmark in macroeconomic models.[1] For agents to form rational expectations, they must construct expectations with respect to the true probability distribution of the model. Rational expectations agents must know the model's deep parameters, structure of the model, beliefs of other agents, and distribution of the error terms. While a reasonable benchmark that is popular among macroeconomic models, rational expectations makes strong assumptions.

This paper shows the effectiveness of forward guidance in an environment where rational expectations has been replaced by an adaptive learning rule similar to one proposed by Marcet and Sargent (1989) and Evans and Honkapohja (2001). The main contribution of this paper is to demonstrate how the effectiveness of forward guidance depends on the manner in which agents form expectations. In particular, a New Keynesian model derived under (potentially) non-rational expectations is formed following Preston (2005). Households, firms, and a central bank occupy the economy. The central bank operates a monetary policy rule that responds positively to the output gap, inflation rate, and a monetary policy shock. The rule is augmented with anticipated shocks as in Del Negro, Giannoni, and Patterson (2012) and Laseen and Svensson (2011). The anticipated

[1] A related issue is the credibility of policymakers to commit to a future path of interest rates (See, for instance, see Woodford (2012) and Swanson and Williams (2012).). This current paper abstracts from this subject.

shocks represent a distinctive way to model forward guidance by the central bank. The shocks also represent what the Bank of England (2013) describes as "time-contingent guidance" (p. 40), where the central bank communicates a definitive forward guidance end date.

A popular alternative to rational expectations is adaptive learning, which assumes agents do not know the true probability distribution of the model. These types of agents behave as econometricians by making forecasts of future endogenous variables using standard econometric techniques. Adaptive learning agents know the form of the laws of motion for the endogenous variables as under rational expectations. However, they must estimate the parameters of the economy as new information arrives every period. Thus, their beliefs evolve over time.[2]

The results of this paper are as follows. The first outcome shows that forward guidance produces different effects depending on whether agents form expectations via rational expectations or adaptive learning. This result is shown by impulse responses of the macroeconomic variables to a forward guidance shock in isolation. The impulse responses of the endogenous variables under adaptive learning proceed along a separate path than the impulse responses under rational expectations. After the forward guidance shock has been realized upon the economy, agents with rational expectations are quicker to adjust their expectations that the forward guidance shock is no longer present. However, the effect of the realization of the forward guidance shock is slower to dissipate under agents with adaptive learning. The next and main result shows that the economy under rational expectations produces a more exaggerated and favorable response than under adaptive learning. This effect is modeled during times of economic stress (e.g. a recession) and when the central bank communicates to the public that the interest rate will equal zero for an extended period of time. This scenario is intended to help capture important features of the Federal Reserve's forward guidance strategy during the Great Recession. During the forward guidance horizon, the results show the percentage change from adaptive learning to rational expectations of the mean value of the output gap being 113.08%.

The above outcomes occur because of the difference in knowledge assumed between the two expectations formation. Since they do not know the true model of the economy, adaptive learning agents form expectations using econometric techniques. Even though they know the future path of interest rates, adaptive learning agents imprecisely understand how forward guidance impacts the economy as their beliefs evolve slowly over time. However, agents with rational expectations form their beliefs from the true model of the economy. They are able to understand how the anticipated

[2] Adaptive learning agents do not take into account they will update their beliefs in future periods. They believe that the beliefs they form every period are optimal. This methodology follows from the anticipated utility discussion from Kreps (1998).

changes in monetary policy will precisely affect the economy. Thus, rational expectations agents understand the effect forward guidance has on the economy, while adaptive learning agents fail to appropriately understand this effect.

1.1 Previous Literature

This paper contributes to the growing unconventional monetary policy literature. Eggertsson and Woodford (2003) explain that the expectations channel plays a key role on the economy when interest rates are at the ZLB and at any level. Specifically, Eggertsson and Woodford (2003) state that the future path of short-term interest rates affects long-term interest rates and asset prices, and thus, the management of expectations about future interests rates affects agents' optimal decisions. In addition, recent literature has found large effects from forward guidance. Carlstrom, Fuerst, and Paustian (2012) show that standard New Keynesian models with the interest rate fixed for a finite period of time result in extreme responses of output and inflation. Del Negro et al. (2012) construct a DSGE model with forward guidance, which produces large responses of macroeconomic variables to forward guidance. Del Negro et al. (2012) state that the long-term bond yield drives these unusually high responses.

The model utilizes time-contingent forward guidance since there has been recent evidence of its effectiveness. Gürkaynak, Sak, and Swanson (2005) find empirical evidence that FOMC statements about the future path of the policy rate greatly contribute to the changes in the long-term interest rates. Swanson and Williams (2012) show that recent Federal Reserve forward guidance announcements have affected medium and longer-term interest rates. Woodford (2012) shows that forward guidance has had an impact on market participants. Using overnight interest rate swaps (OIS) to measure market expectations about the policy rate in Canada, Woodford (2012) shows that OIS rates immediately changed upon release of the Bank of Canada's forward guidance statement. The work of Chang and Feunou (2013) shows that the Bank of Canada's forward guidance statement in 2009 had positive effects on the economy by reducing uncertainty about future monetary policy rates. A reduction in interest rate uncertainty can affect levels of investment, output, and unemployment in the economy as described by Baker, Bloom, and Davis (2013). Femia, Friedman, and Sack (2013) describe evidence that financial variables, such as Treasury yields and equity prices, reacted favorably to the Federal Reserve's time-contingent forward guidance announcements.

By analyzing the role of expectations formation on forward guidance, this paper builds on the adaptive learning and policy literature. Evans, Honkapohja, and Mitra (2012) examine the effects of the fiscal authority giving guidance on the future course of government purchases and taxes.

The results show that a temporary change in fiscal policy leads to different effects on adaptive learning agents and rational expectations agents. The adaptive learning output multipliers seem to match empirical data more than its rational expectations counterparts. Eusepi and Preston (2010) investigates the link between adaptive learning and central bank communication strategies. Increased central bank communication, such as communicating the monetary policy rule and the variables within the rule, can lead to increased macroeconomic stability. Preston (2006) studies forecast-based monetary policy rules and adaptive learning. He finds that a central bank that understands the basis of private sector forecasts can aid in increasing macroeconomic stability. Moreover, this current paper analyzes the link between adaptive learning and the effectiveness of time-contingent forward guidance.

The remaining sections of the paper are organized as follows. Section two presents the New Keynesian model with forward guidance. Section three discusses expectations formation under both rational expectations and adaptive learning. Section four presents the outcomes of forward guidance under both rational expectations and adaptive learning via impulse response functions. Section five includes the main result that rational expectations produces a more favorable and exaggerated response to forward guidance than adaptive learning. This outcome is shown through a policy exercise when the economy is experiencing a recession. Section six presents the results under a scenario where adaptive learning agents discount past information at different rates.[3]

2 Model

The aggregate dynamics of the economy are described by a New Keynesian model derived under (potentially) non-rational expectations (see Preston [2005]).[4] There is a continuum of households each of whom maximizes expected future discounted utility by choosing sequences of consumption, labor, money holdings, and bonds.[5] Each household provides labor for the production of good j, and receives a wage as compensation. A household shares in the profits from the sale of good j since all households own an equal share of each firm. Markets are assumed incomplete and households can invest in riskless bonds in order to transfer wealth between periods. Households also pay lump-sum taxes to the government.[6] The optimal decisions of households satisfy a sequence of Euler equations and the intertemporal budget constraint. A household's intertemporal budget

[3] In technical terms that will be discussed later, outcomes are examined under different constant gain values.

[4] A detailed description of the model can be found in Appendix A

[5] As is standard in many macroeconomic models, money holdings do not alleviate any transaction frictions. However, households may choose to hold money as a substitute to riskless bonds because of the equivalent return of money and riskless bonds as described in Preston (2005).

[6] The fiscal authority operates a zero net supply of bonds. It is assumed to be Ricardian so that fiscal policy does not affect the output of goods.

constraint includes wealth in the form of bonds purchased from the previous period. In addition, all agents hold (potentially) non-rational beliefs that satisfy standard probability laws that are the same across households. However, each agent does not know the beliefs of other agents, and does not take into account that he or she will be updating beliefs every period. This assumption follows the anticipated utility discussion of Kreps (1998) where agents optimize every period as if their current beliefs coincide with the true model, but do not realize they will update beliefs in the future. By combining the household's euler equation, intertemporal budget constraint, and no-Ponzi constraint, the resulting log-linearized equation for the output gap is given by

$$x_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}[(1-\beta)x_{T+1} - \sigma(i_T - \pi_{T+1}) + r_T^n] \tag{1}$$

where

$$r_t^n = \rho_n r_{t-1}^n + \varepsilon_t^n \tag{2}$$

and $\varepsilon_t^n \overset{iid}{\sim} N(0, \sigma_n^2)$. All variables are in terms of log deviations from steady state. Equation (1) relates the current output gap x_t to current and future expected values of the output gap, interest rate i_t, inflation rate π_t, and natural real interest rate shock r_t^n. β describes the household's discount rate and is bounded between zero and one. $\sigma > 0$ defines the intertemporal elasticity of substitution of consumption between periods. $\hat{E}_t$ denotes (potentially) non-rational expectations. Households take into account the future values of the endogenous variables infinitely far into the future when choosing optimal consumption today. Intuitively, the expected course of a household's consumption pattern matters to its optimal consumption today. A household also knows future consumption patterns are affected by future values of income, interest rates, and inflation. Thus, expectations of these variables are important for decisions today.

The production side of the economy is populated by firms that take into account expectations of variables infinitely far into the future. A firm operates in a monopolistically competitive environment where each good is produced using labor from households. It is also subject to a Calvo (1983) pricing scheme, and thus, has an α probability of not being able to change its price every period. The representative firm that is able to adjust its price chooses its price each period to maximize expected present discounted value of profits. The resulting log-linearized equation for inflation is

$$\pi_t = \kappa x_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}[\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + \mu_T] \tag{3}$$

where

$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu \tag{4}$$

and $\varepsilon_t^\mu \overset{iid}{\sim} N(0, \sigma_\mu^2)$. All variables are in terms of log deviations from steady state. Equation (3) defines the inflation rate as a function of current and future values of the output gap, inflation rate, and cost-push shock μ_t. ω describes the elasticity of a firm's real marginal cost function with respect to its own output, and $\kappa \equiv \frac{(1-\alpha)}{\alpha} \frac{(1-\alpha\beta)}{(1+\omega\theta)} (\omega + \sigma^{-1}) > 0$. θ measures the elasticity of substitution between differentiated goods. The optimal decisions by firms are shown to depend on the long-run expected path of macroeconomic variables because of the assumption of sticky prices. A firm must be concerned that it will not be able to adjust its price in future periods regardless of future economic conditions. Thus, optimal pricing decisions today requires firms to forecast future states and values of economic variables.

Preston (2005) also argues that the infinite horizon feature of equations (1) and (3) occurs because of the assumption of (potentially) non-rational expectations. Under these expectations, agents do not know the true model of the economy and aggregate probabilities as under the rational expectations hypothesis. If agents have rational expectations, however, they would know other agents have similar beliefs and would be able to compute aggregate probability laws, and thus, equations (1) and (3) would reduce to standard equations with one-period ahead expectations. Another approach to modeling learning in macroeconomic models regards the "euler-equation" method presented in Evans and Honkapohja (2001), where only one period ahead forecasts of the endogenous variables show up in the model's equations under both rational expectations and adaptive learning.[7]

The model is closed by describing the central bank of the economy. The central bank follows a monetary policy rule that takes the following form

$$i_t = \chi_\pi \pi_t + \chi_x x_t + \varepsilon_t^{MP} + \sum_{l=1}^{L} \varepsilon_{l,t-l}^R \tag{5}$$

The short-term nominal interest rate changes based on the output gap, inflation rate, monetary policy shock, and forward guidance shocks. ε_t^{MP} defines a monetary policy shock and is *i.i.d.* In order to incorporate forward guidance into the model, the monetary policy rule is augmented with anticipated shocks following Del Negro et al. (2012) and Laseen and Svensson (2011). Each anticipated or forward guidance shock ($\varepsilon_{l,t-l}$) is contained in the last term in equation (5) and is *i.i.d.* Intuitively, the forward guidance shock can be thought of as an announcement by the central bank in period $t-l$ that the interest rate will change l periods later, i.e. in period t. If the central bank has been communicating guidance on the interest rate for L periods ahead, there would be $1, 2, 3, \ldots, L$ forward guidance shocks that affect the monetary policy rule in period t. Thus, L

[7] For a comparison between the "infinite-horizon" and euler-equation approach to learning, see Evans, Honkapohja, and Mitra (2013)

corresponds to the length of the forward guidance horizon announced by the central bank. The last term in equation (5) can also be thought of as the sum of all forward guidance commitments stated by the central bank $1, 2, ...,$ and L periods ago that affect the nominal interest rate in period t. Following Del Negro et al. (2012) and Laseen and Svensson (2011), the system is also augmented with L state variables $v_{1,t}, v_{2,t}, ..., v_{L,t}$. The law of motion for each of these state variables is given by

$$v_{1,t} = v_{2,t-1} + \varepsilon^R_{1,t} \tag{6}$$

$$v_{2,t} = v_{3,t-1} + \varepsilon^R_{2,t} \tag{7}$$

$$v_{3,t} = v_{4,t-1} + \varepsilon^R_{3,t} \tag{8}$$

$$\vdots$$

$$v_{L,t} = \varepsilon^R_{L,t} \tag{9}$$

In other words, each component of $v_t = [v_{1,t}, v_{2,t}, ..., v_{L,t}]'$ is the sum of all central bank forward guidance commitments known in period t that affect the interest rate $1, 2, ...,$ and L periods into the future, respectively.[8] It should be noted that equations (6) – (9) can be simplified to find that $v_{1,t-1} = \sum_{l=1}^{L} \varepsilon^R_{l,t-l}$. In addition, equations (5) – (9) show a tractable method to model forward guidance. Since the forward guidance shocks equal $v_{1,t-1}$, the forward guidance shocks can be put into a vector of predetermined variables in standard state-space form. As described by Laseen and Svensson (2011), standard solution techniques then can be used to solve the final system of equations. Another reason to model forward guidance in this way is that it relieves the concern of the existence of multiple solutions. As described in Honkapohja and Mitra (2005) and Woodford (2005), indeterminacy can arise if forward guidance is instead modeled as pegging the interest rate to a certain value.[9]

In order to gain further intuition about v_t, consider the case where the central bank's forward guidance horizon is 2 periods ahead, i.e. $L = 2$. The model's system of equations consists of $v_{1,t}$ and $v_{2,t}$ whose laws of motion are defined as

$$v_{1,t} = v_{2,t-1} + \varepsilon^R_{1,t} = \varepsilon^R_{2,t-1} + \varepsilon^R_{1,t} \tag{10}$$

$$v_{2,t} = \varepsilon^R_{2,t} \tag{11}$$

Thus, $v^R_{1,t}$ defines the sum of all forward guidance commitments by the central bank known in

[8] In the terminology of Laseen and Svensson (2011), $v_{1,t}, v_{2,t}, ..., v_{L,t}$ are described as central bank "projections" (p. 10) of what $\sum_{l=1}^{L} \varepsilon^R_{l,t-l}$ will be $1, 2, ...,$ and L periods into the future, respectively.

[9] Carlstrom, Fuerst, and Paustian (2012) show that determinacy can arise from an interest rate peg if terminal conditions are known and a standard monetary policy rule is followed after the interest rate peg. However, unusually large responses of the output and inflation are found through this process.

period t that affect the interest rate one period later. $v_{1,t}^R$ consists of current period forward guidance affecting the interest rate one period later, $\varepsilon_{1,t}^R$, and previous period's forward guidance affecting the interest rate two periods later, $v_{2,t-1} = \varepsilon_{2,t-1}^R$. $v_{2,t}$ is the sum of all forward guidance commitments by the central bank known in period t that affect the interest rate two periods later. Since the forward guidance horizon is two periods, $v_{2,t}$ consists of current period forward guidance affecting the interest rate two periods later, $\varepsilon_{2,t}^R$.

The ZLB on interest rates is also enforced. Forward guidance has recently gained attractiveness and attention due to interest rates effectively reaching the ZLB because of the 2007-2009 global financial recession. Thus, it seems natural to model the ZLB on nominal interest rates when simulating forward guidance. Specifically, equations (1) and (5) are modified and result in the following

$$\hat{x}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}[(1-\beta)\hat{x}_{T+1} - \sigma(i_T - i^* - \hat{\pi}_{T+1}) + \hat{r}_T^n] \tag{12}$$

$$i_t = max\{i^* + \chi_\pi \hat{\pi}_t + \chi_x \hat{x}_t + \varepsilon_t^{MP} + \sum_{l=1}^{L} \varepsilon_{l,t-l}^R, 0\} \tag{13}$$

where $i^* = r^* + \pi^*$ is the steady-state nominal interest rate. The " ˆ " symbol over the variables denotes log deviations from steady state.[10]

To summarize, the aggregate dynamics of the economy with forward guidance are defined by the output gap, inflation rate, AR(1) shock processes, monetary policy rule with forward guidance, and the laws of motion of the sum of central bank commitments, that is, equations (1) – (9). With enforcement of the ZLB, equations (12) and (13) are used instead of (1) and (5).

3 Expectation Formation

This paper assumes agents form expectations following either the rational expectations hypothesis or adaptive learning. The difference between the two types of expectation formations regards the amount of knowledge agents hold about the economy (See, for example, Marcet and Sargent (1989), Evans and Honkapohja (2001), and Evans, Honkapohja, and Mitra (2009).). Under rational expectations, agents know the structure of the model, parameters of the model (e.g. σ, κ, etc.), distribution of the error terms, and beliefs of other agents. They compute expectations based off the true model of the economy. Under adaptive learning, agents do not know the true model of the economy, and thus, cannot compute precise expectations as under rational expectations. Instead, they operate as econometricians to form beliefs from newly available information every

[10] In a zero steady-state inflation rate, $\pi^* = 0$. The model implied steady-state real interest rate $r^* = \beta^{-1} - 1$.

period. When forming expectations, adaptive learning agents know the variables in the rational expectations solution and the distribution of the error terms. However, they do not know the values of the model's parameters. In order to compute the coefficients, they run a least squares regression each period using updated information. Thus, the expectations of adaptive learning agents evolve over time.

Rational Expectations–The model defined by equations (1) – (9) can be simplified under the assumption of rational expectations. Agents with rational expectations understand the beliefs of other agents and are able to compute the aggregate probabilities of the model. As shown in Preston (2005), this additional information simplifies the infinite horizon model to the "benchmark" one step ahead New Keynesian model. Specifically, equations (1) and (3) become

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + r_t^n \tag{14}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \mu_t \tag{15}$$

The model with rational expectations can be solved using standard techniques, such as one suggested by Sims (2002). The model can be written in general state-space form as suggested by Sims (2002). This form is defined as

$$\widetilde{\Gamma}_0 \widetilde{Y}_t = C + \widetilde{\Gamma}_1 \widetilde{Y}_{t-1} + \widetilde{\Gamma}_2 \widetilde{\epsilon}_t + \widetilde{\Gamma}_3 \zeta_t \tag{16}$$

where

$$\widetilde{Y}_t = [x_t, \pi_t, i_t, r_t^n, \mu_t, v_{1,t}, v_{2,t}, \ldots, v_{L,t}, E_t x_{t+1}, E_t \pi_{t+1}]' \tag{17}$$

$$\widetilde{\epsilon}_t = [\varepsilon_t^n, \varepsilon_t^\mu, \varepsilon_t^{MP}]' \tag{18}$$

C defines a vector of constants of required dimensions. ζ_t defines the vector of expectational errors (e.g. $\zeta_t^\pi = \pi_t - E_{t-1}\pi_t$) of required dimensions. Using standard techniques to solve the model with rational expectations (e.g. Sims [2002]) and the parameter values in Table 1, the solution to the system under rational expectations is

$$\widetilde{Y}_t = \widetilde{C} + \xi_1 \widetilde{Y}_{t-1} + \xi_2 \epsilon_t \tag{19}$$

where the matrices $\widetilde{C}$, ξ_1, and ξ_2 are defined in Appendix B.[11]

Adaptive Learning–In order to evaluate the expectations in equations (1) and (3) under adaptive learning, agents act as econometricians by forming a model based on variables that appear in the rational expectations solution and estimate the coefficients. This model is labeled the

[11] Discussion of the parameter values can be found in Table 1 in Section 4.1.

"Perceived Law of Motion" (PLM) and is constructed from the minimum state variable (MSV) solution that exists under rational expectations.[12] The PLM is defined as

$$Y_t = a_t + b_t v_t + c_t w_t + d_t v_{1,t-1} + \varepsilon_t \tag{20}$$

where

$$Y_t = [x_t, \pi_t, i_t]' \tag{21}$$

$$v_t = [v_{1,t}, v_{2,t}, ..., v_{L,t}]' \tag{22}$$

The vector $w_t = [r_t^n, \mu_t]'$ is defined by

$$w_t = \widetilde{\phi} w_{t-1} + \bar{\varepsilon}_t \tag{23}$$

where

$$\widetilde{\phi} = \begin{bmatrix} \rho_n & 0 \\ 0 & \rho_\mu \end{bmatrix} \tag{24}$$

$$\bar{\varepsilon}_t = [\varepsilon_t^n, \varepsilon_t^\mu]' \tag{25}$$

By rewriting equations (6) $-$ (9), the vector v_t becomes

$$v_t = \Phi v_{t-1} + \eta_t \tag{26}$$

where

$$\eta_t = [\varepsilon_{1,t}^R, \ldots, \varepsilon_{L,t}^R]' \tag{27}$$

and Φ is an L x L matrix given by

$$\Phi = \begin{bmatrix} 0 & 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 & 0 \\ \vdots & & & & \ddots & \vdots & \\ 0 & 0 & 0 & 0 & \ldots & 1 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 \end{bmatrix} \tag{28}$$

(29)

a_t, b_t, c_t, and d_t are unknown coefficient matrices of appropriate dimensions that agents estimate and learn about over time.[13] Furthermore, the addition of $v_{1,t-1}$ is a necessary component of

[12]This paper focuses on a version of the model that is determinate so that the PLM is based on the unique non-explosive rational expectations equilibrium. The parameter values in Table 1 verify that the rational expectations solution is determinate.

[13]Since this paper restricts attention to fundamentals solutions and Y_{t-1} does not appear in equations (1), (3), and (5), the PLM does not contain Y_{t-1}.

the PLM since it is present in the rational expectations solution shown in Appendix B and not contained in the vector v_t.

An important component of adaptive learning models regards the information available to agents when they form expectations. In this paper, adaptive learning agents are assumed to know the values of the regressors in the PLM and previous period's coefficient estimates when forming beliefs about the future. They update their parameter estimates at the end of the period. This assumption avoids the simultaneous determination of current period coefficient estimates and endogenous variables when forming expectations and making optimal decisions.[14] The *i.i.d.* monetary policy shock is also assumed to be unobserved.[15] Furthermore, the following is the timeline of events:

1. At the beginning of period t, v_t, and w_t are observed by the agents and added to their information set.

2. Agents use v_t, w_t, and $v_{1,t-1}$ as well as previous period's estimates, ϕ_{t-1}, to form expectations about the future.

3. Y_t is realized.

4. In order to update their parameter estimates, agents compute a least squares regression of Y_t on 1, v_t, w, and $v_{1,t-1}$.

Agents update their estimates of a_t, b_t, c_t, and d_t by following the recursive least squares (RLS) formula

$$\phi_t = \phi_{t-1} + \tau_t R_t^{-1} z_t (Y_t - \phi_{t-1}^{'} z_t)' \tag{30}$$

$$R_t = R_{t-1} + \tau_t (z_t z_t^{'} - R_{t-1}) \tag{31}$$

where $\phi_t = (a_t, b_t, c_t, d_t)^{'}$ contains the PLM coefficients to be estimated. R_t defines the precision matrix of the regressors in the PLM $z_t \equiv [1, v_t, w_t, v_{1,t-1}]^{'}$. τ_t is known as the "gain" parameter and controls the response of ϕ_t to new information. The last expression in equation (30) defines the recent prediction error of the endogenous variables.

The gain parameter in equations (30) and (31) can either decrease over time or be fixed at certain values. In the decreasing gain or RLS case, $\tau_t = t^{-1}$ and past observations are equally

[14] An alternative is to assume that agents use the coefficient estimates from the current period when forming expectations. This results in expectations and current period parameter estimates determined simultaneously when making optimal decisions.

[15] This is similar to Milani (2007a).

weighted. Evans and Honkapohja (2001) explain that as $t \to \infty$ the coefficients in the PLM converge to the rational expectations coefficients with probability one. As is assumed in this current paper, the gain parameter can also be fixed at a certain value. Under this method called discounted or constant gain learning (CGL), the most recent observations play a larger role when updating agents' coefficients and expectations. Evans and Honkapohja (2001) describe that the coefficients in the PLM converge in distribution to their rational expectations values with a variance that is proportional to the constant gain coefficients. CGL may be a more realistic way to model learning since it models agents as updating their beliefs every period to new information.

Agents solve for $\hat{E}_t Y_{T+1}$ by using equation (20). For any $T \geq t$, their expectations infinite periods ahead are given by

$$\begin{aligned}\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} Y_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} a_{t-1} + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} b_{t-1} v_{T+1} \\ + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} c_{t-1} w_{T+1} + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} d_{t-1} v_{1,T}\end{aligned} \tag{32}$$

$$\begin{aligned}\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} Y_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} a_{t-1} + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} b_{t-1} v_{T+1} \\ + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} c_{t-1} w_{T+1} + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} d_{t-1} v_{1,T}\end{aligned} \tag{33}$$

By noting the geometric sums and expectations of v_t twelve periods ahead or greater equal the zero vector, equations (34) and (35) simplify to equal

$$\begin{aligned}\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} Y_{T+1} = (1-\beta)^{-1} a_{t-1} + b_{t-1} \Phi (I_L - \beta\Phi)^{-1} (I_L - (\beta\Phi)^{11}) v_t \\ + c_{t-1} (I_2 - \beta\widetilde{\phi})^{-1} \widetilde{\phi} w_t + d_{t-1} [1, \beta, \beta^2, \ldots, \beta^{11}] v_t\end{aligned} \tag{34}$$

$$\begin{aligned}\hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} Y_{T+1} = (1-\alpha\beta)^{-1} a_{t-1} + b_{t-1} \Phi (I_L - \alpha\beta\Phi)^{-1} (I_L - (\alpha\beta\Phi)^{11}) v_t \\ + c_{t-1} (I_2 - \alpha\beta\widetilde{\phi})^{-1} \widetilde{\phi} w_t + d_{t-1} [1, \alpha\beta, (\alpha\beta)^2, \ldots, (\alpha\beta)^{11}] v_t\end{aligned} \tag{35}$$

Equations (34) and (35) are substituted into equations (1) and (3) to give

$$Y_t = \Gamma_0(a_t, b_t, c_t, d_t) + \Gamma_1 Y_{t-1} + \Gamma_2(a_t, b_t, c_t, d_t) v_t + \Gamma_3(a_t, b_t, c_t, d_t) \widetilde{w}_t \tag{36}$$

where

$$\widetilde{w}_t = [w_t, \varepsilon_t^{MP}]' \tag{37}$$

Equation (36) is called the "Actual Law of Motion" (ALM) and describes the actual evolution of the endogenous variables implied by the PLM (20).

4 Results

4.1 Calibration

This section details the calibration values for the model's parameters, which are shown in Table 1. β is set to equal 0.99 which is a common value found in the literature. The parameter representing the intertemporal elasticity of substitution is fixed at one. This value has been assumed *a priori* in Smets and Wouters (2003). κ is set to equal 0.1. This number roughly corresponds to a high degree of price stickiness, α, found in empirical work by Klenow and Malin (2011), a value of ω found in Giannoni and Woodford (2004), and a value of θ found in the literature (e.g. Gertler and Karadi [2011]). Monetary policy positively responds to the output gap, and positively adjusts at more than a one-to-one rate to the inflation rate. $\chi_x = 0.125$ follows from Branch and Evans (2013). The value of χ_π closely follows empirical adaptive learning work by Milani (2007a). The structural disturbances are not assumed to exhibit high persistence. The value of ρ_n follows closely Milani (2007a). The autoregressive coefficient of the cost-push shock, ρ_μ has been assumed *a priori* in Smets and Wouters (2007). The distribution of the white noise shocks is not assumed to be highly dispersed. There also is no covariance between the structural shocks.

The current paper examines results for the CGL case. In regards to choosing the CGL parameter $\bar{\tau}$, this paper uses 0.03. This choice is close to the results used in the literature, such as Orphanides and Williams (2005), Milani (2007a), and Branch and Evans (2006). For robustness, the current methodology also examines the results under different values of $\bar{\tau}$.

The value for the length of the forward guidance horizon L is chosen to match time-contingent forward guidance by the Federal Reserve. This is based off the FOMC September 2012 statement: "...the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015." This FOMC statement was one of the last to use time-contingent forward guidance language. By taking "mid-2015" to be at most the end of the third quarter of 2015, the number of quarters from September 2012 to "mid-2015" is twelve. Thus, $L = 12$.

Table 1: Parameter Values

	Description	Value
σ	IES	1
β	Discount Factor	0.99
κ	Function of Price Stickiness	0.1
α	Price Stickiness	0.75
χ_π	Feedback Inflation	1.4
χ_x	Feedback Output Gap	0.125
ρ_n	Autoregressive Demand	0.8
ρ_μ	Autoregressive Cost-Push	0.5
σ_n^2	Demand Shock	0.01
σ_μ^2	Cost-Push Shock	0.01
σ_i^2	M.P Shock	0.001
$\sigma_{1,i}^2$	1 Period Ahead FG Shock	0.001
$\sigma_{2,i}^2$	2 Period Ahead FG Shock	0.001
$\sigma_{3,i}^2$	3 Period Ahead FG Shock	0.001
$\sigma_{4,i}^2$	4 Period Ahead FG Shock	0.001
$\sigma_{5,i}^2$	5 Period Ahead FG Shock	0.001
$\sigma_{6,i}^2$	6 Period Ahead FG Shock	0.001
$\sigma_{7,i}^2$	7 Period Ahead FG Shock	0.001
$\sigma_{8,i}^2$	8 Period Ahead FG Shock	0.001
$\sigma_{9,i}^2$	9 Period Ahead FG Shock	0.001
$\sigma_{10,i}^2$	10 Period Ahead FG Shock	0.001
$\sigma_{11,i}^2$	11 Period Ahead FG Shock	0.001
$\sigma_{12,i}^2$	12 Period Ahead FG Shock	0.001
L	FG Horizon	12
$\bar{\tau}$	CGL	0.03

Note: FG stands for forward guidance.

4.2 Forward Guidance During Normal Times

In this section, impulse responses of the output gap and inflation rate to negative one unit monetary policy and forward guidance shocks under different expectation assumptions are examined in Figures 1 and 2.[16] The forward guidance shocks are the anticipated shocks found in equations (6) - (9). Since equation (36) exhibits a nonlinear structure, standard linear techniques to compute impulse responses under adaptive learning do not apply. To remedy this situation, this paper follows Eusepi and Preston (2011) by proceeding in the following manner. In order to calculate the impulse responses of an endogenous variable to a particular shock, the model is simulated twice for $10,000 + K$ periods, where K is the impulse response function horizon. The impulse responses are calculated starting at period $10,001$ to ensure the model has converged to its stationary distribution. In the first simulation, time period $10,001$ includes a negative one unit shock. The K-period

[16] A projection facility is utilized to ensure beliefs are not explosive.

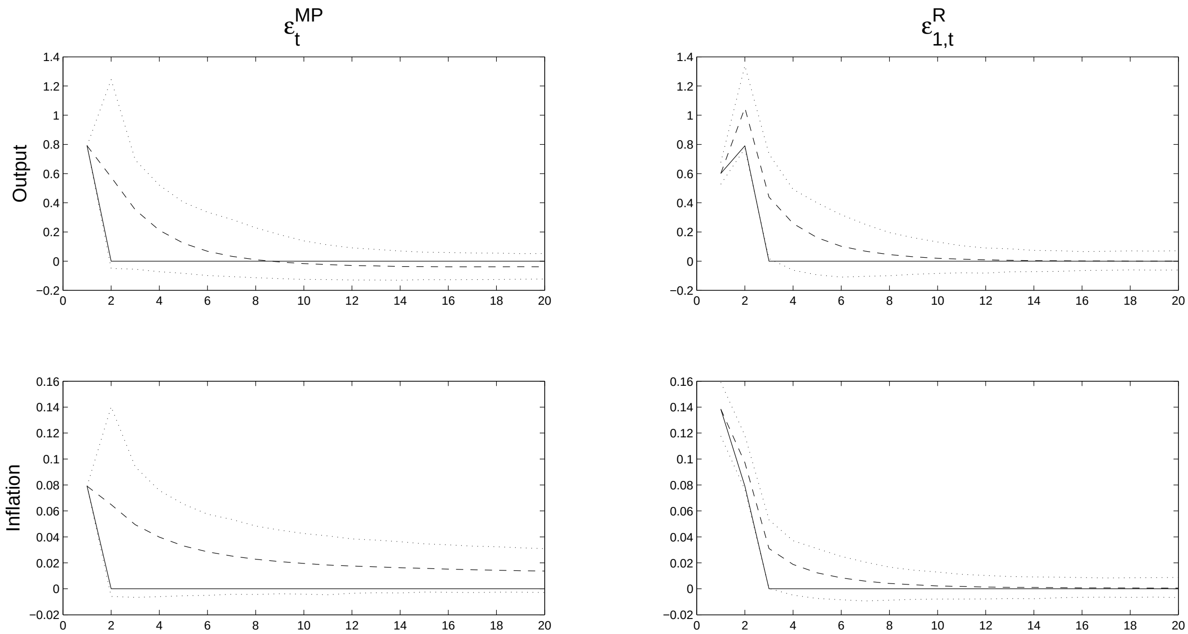

Figure 1: Impulse Responses of Endogenous Variables to Anticipated Shocks

Note: Solid Line: Rational Expectations; Dashed Line: CGL; Dotted Line: 95% Confidence Band

impulse response function is given by the difference between the first and second simulations over the final K periods. The process is then repeated for $5,000$ simulations and the mean impulse response across the $5,000$ simulations is calculated to arrive at the final impulse response trajectory. The impulse response function horizon is chosen to be twenty periods, that is, $K = 20$.

Impact–As seen in Figures 1 and 2, the initial response of the macroeconomic variables is approximately the same under both adaptive learning and rational expectations. This result is not surprising since Evans and Honkapohja (2001) state that CGL coefficients converge to a Normal distribution centered around its rational expectations counterparts. Thus, the initial impact under adaptive learning could be greater or less than the initial impact under rational expectations.

After Impact–Figures 1 and 2 also display the impulse responses after the forward guidance announcement is known to agents. From the household's perspective, they must optimally allocate consumption across time based on their expectations of future variables. Since they know that the interest rate will decrease in the future, a household changes its optimal consumption across time and increases current consumption. In addition, firms know they may not be able to change their price in the future regardless of the state of the economy. Thus, they take into account expectations of future variables as seen in equation (3). When the central bank announces that the interest rate

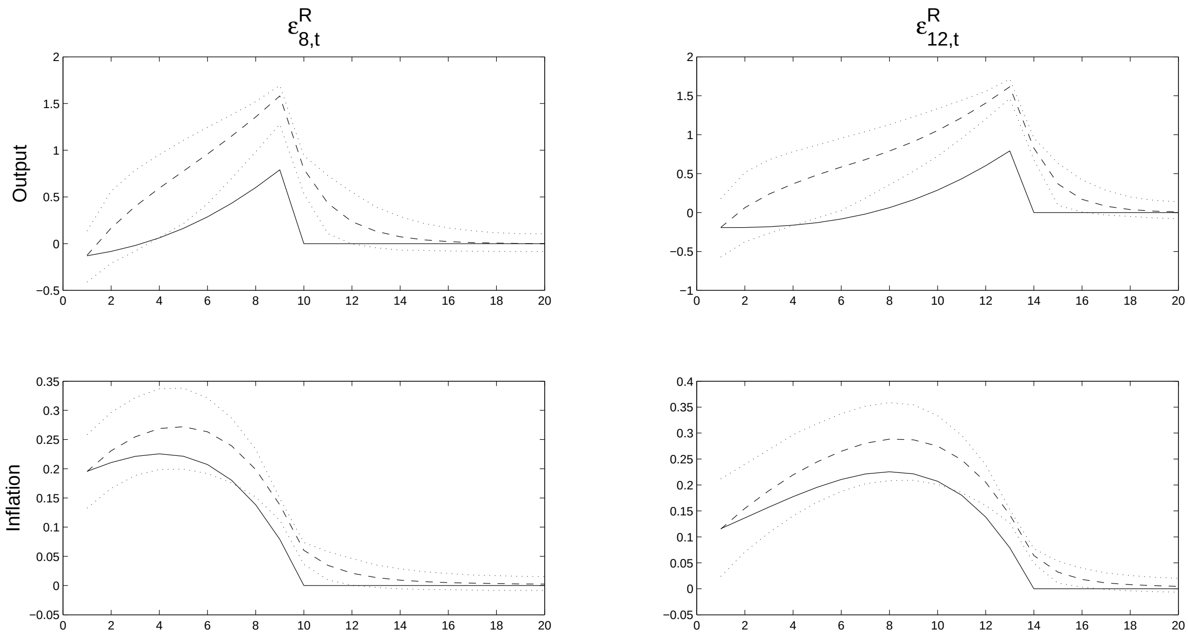

Figure 2: Impulse Responses of Endogenous Variables to Anticipated Shocks

Note: Solid Line: Rational Expectations; Dashed Line: CGL; Dotted Line: 95% Confidence Band

will increase in the future, a firm knows that future output gap and inflation rate will be affected, and thus, this action affects current pricing decisions.

After the initial impact of a forward guidance announcement, the impulse response trajectories of rational expectations and adaptive learning proceed in different paths. This result is shown in Figures 1 and 2 by impulse responses displaying more persistence in the periods after news of a forward guidance commitment under adaptive learning than under rational expectations. Adaptive learning agents understand the form of the laws of motion for the endogenous variables, but they imprecisely understand the effect forward guidance has on the economy. They learn about the coefficients each period, and thus, their beliefs slowly evolve. However, rational expectations agents know precisely the equilibrium probability distribution and how the anticipated changes in monetary policy will affect the endogenous variables at later dates.

After Shock Realized–The impulse response graphs of rational expectations and adaptive learning do not follow the same path after the shock is realized upon the economy. The impulse responses with rational expectations agents converge quicker to zero percentage deviation from the unshocked series. Rational expectations agents understand that the shock will not occur in the future and they quickly adjust their expectations. However, the impulse responses under adaptive learning exhibit more persistence than the impulse responses under rational expectations. This outcome is present because the dynamics of the impulse responses under adaptive learning are driven by adjustments in the beliefs of the agents. Adaptive learning agents revise their estimates of the parameters of the economy each period, while rational expectations agents fully understand the model's parameters. The impulse responses of a conventional monetary policy shock shown in the first column of Figure 1 also display the same difference in persistence.

The results coincide with the literature on adaptive learning. The outcomes match Eggertsson (2008) who found that temporary policy shifts do not have as large of an effect on the economy as permanent policy shifts under the assumption of rational expectations. The persistence results also coincide with Milani (2007a) who found that a DSGE model with constant-gain learning generates persistence in the macroeconomic variables.

To summarize, the main message from this section is that an economy under rational expectations responds differently than an economy under adaptive learning. When the forward guidance shock is known to agents, the output gap and inflation rate under adaptive learning proceed in a different path than under rational expectations. After the shock has been realized, rational expectations agents quickly adjust their expectations to the knowledge that the shock is gone, while adaptive learning agents' beliefs are more persistent. These results are attributed to rational expectations agents precisely understanding the effects forward guidance has on the economy, while the beliefs of adaptive learning agents slowly adjust.

5 Policy Exercise

The results displayed by the impulse response functions showed that forward guidance has different effect effects on the output gap and inflation depending on the expectations assumption. This current section shows the main takeaway of this paper: rational expectations produces a more positive and exaggerated response to forward guidance than adaptive learning. This result is shown during a time of economic stress (e.g. a recession) and when the central bank announces the interest rate will remain fixed for $L + 1$ periods. The policy experiment is described next and is motivated by the policy exercise described in Del Negro et al. (2012).

Suppose at the beginning of period $T + 1$, the central bank implements forward such that the interest rate will be fixed at $\bar{i} = 0$ in period $T + 1$ and L periods into the future. This announcement corresponds to an unanticipated shock in period $T + 1$ and news about the future interest rate $1, 2, \ldots, L$ periods into the future. In this scenario, the monetary policymaker's job is to choose ε^{MP}_{T+1} and $\eta_{T+1} = [\varepsilon^{R}_{1,T+1}, \ldots, \varepsilon^{R}_{L,T+1}]'$ such that the interest rate in periods $T + 1$ to

$T + L + 1$ equals $\bar{i}$. The central bank also believes that agents hold rational expectations, which is a common assumption in macroeconomic literature. To show that adaptive learning agents respond differently to the same forward guidance information, the adaptive learning agents are given the same guidance on the interest rate as under rational expectations.

The above situation is assumed to start after $T = 10,000$ periods have passed, meaning after a period of economic stability (corresponding to say the period before the recent Great Recession).[17] The model is then simulated from $T + 1$ to the end of the forward guidance horizon ($T + L + 1$). To capture features from the recent Great Recession, a large negative demand shock hits the economy in period $T + 1$, and causes a recession. There is a sequence of five more negative demand shocks so that the recession lasts six periods.[18] In the following periods, the shocks are drawn from a normal distribution. Thus, the forward guidance horizon spans a recession and normal times. The process is then repeated $5,000$ times and the mean across the $5,000$ simulations is calculated.

This policy exercise also assumes that the central bank is committed to its goal of $\bar{i}$ every period during the forward guidance horizon. Rational expectations agents precisely understand the central bank's guidance, and thus, the interest rate each period implied by rational expectations equals $\bar{i}$. Since the adaptive learning process is different than rational expectations, the same forward guidance will not give a model implied $\bar{i}$ during the forward guidance horizon. To model a commitment to $\bar{i} = 0$, the central bank chooses ε_t^{MP} each period over the forward guidance horizon to ensure the interest rate equals $\bar{i}$.[19]

The results of this simulated policy exercise are displayed in Table 2. The values of the output gap and inflation during the forward guidance horizon are averaged across simulations, and then the mean across the forward guidance horizon is taken for both the output gap and inflation rate. Table 2 shows the percentage change from adaptive learning to rational expectations of the mean value of the output gap and inflation rate. Forward guidance seems to have a greater stimulative effect on the economy under rational expectations. This outcome is especially evident by value of the percentage change from adaptive learning to rational expectations of the mean value of the output gap being 113.08%.

This conclusion shows that the efficacy of forward guidance depends on the popular assumption of rational expectations. As shown in Table 2, the assumption of rational expectations produces a more positive and exaggerated effect on the economy, especially when examining the output gap.

[17] This strategy also ensures the adaptive learning coefficients converge to its stationary distribution.

[18] This length is based on the duration of the Great Recession as defined by the National Bureau of Economic Research.

[19] This adjustment seems fair since agents' expectations in real life about the future interest rate might not respond as precisely as the central bank would want, and thus, the interest rate might not equal a model implied $\bar{i}$.

Table 2: **Forward Guidance Consequences**

Output Gap	Inflation
113.08%	13.60%

*Each cell reports the percentage change from adaptive learning to rational expectations of the mean value of the respective variable.

The rational expectations agents know the true model and aggregate probabilities. They can infer the precise effect forward guidance has on the economy. However, if the central bank's rational expectations assumption is wrong, the results are not as favorable. The expectations of adaptive learning agents do not respond in the same way. Adaptive learning agents do not know the true model of the economy, and thus, cannot infer the proper aggregate probabilities and expectations. Even though they know about the future path of interest rates, adaptive learning agents imprecisely understand how that guidance impacts the economy. The best they can do is act as econometricians and form expectations every period based on newly available data, which means that their beliefs evolve slowly over time. Moreover, this result shows a consequence of the decision of monetary policymakers. If the central bank assumes agents have rational expectations, which is standard in the macroeconomic literature, the predicted outcomes seem to be misleading. The more realistic assumption of adaptive learning displays that the effects of forward guidance are not as favorable.

6 Alternative Constant Gains

In this section, a robustness exercise is simulated to examine the effects of forward guidance policy when adaptive learning agents place varying degrees of weight on recent observations. Specifically, higher and lower values of the gain parameter are used. In addition to $\bar{\tau} = 0.03$, the other constant gains assumed are $\bar{\tau} = 0.01$ and $\bar{\tau} = 0.05$.

The results in Figures 3 and 4 display that the responses of the macroeconomic variables to forward guidance under adaptive learning depends on the weight agents place on new information. From the time of the forward guidance announcement to when the shock is realized and as the value of $\bar{\tau}$ increases, the impulse responses differ more from rational expectations than lower values of $\bar{\tau}$ differ from rational expectations. Under higher values of $\bar{\tau}$, agents place more weight on new information when they update their parameter estimates. Each period's estimates and beliefs should vary more from the previous period's estimates. Thus, higher constant gains seem to misvalue more

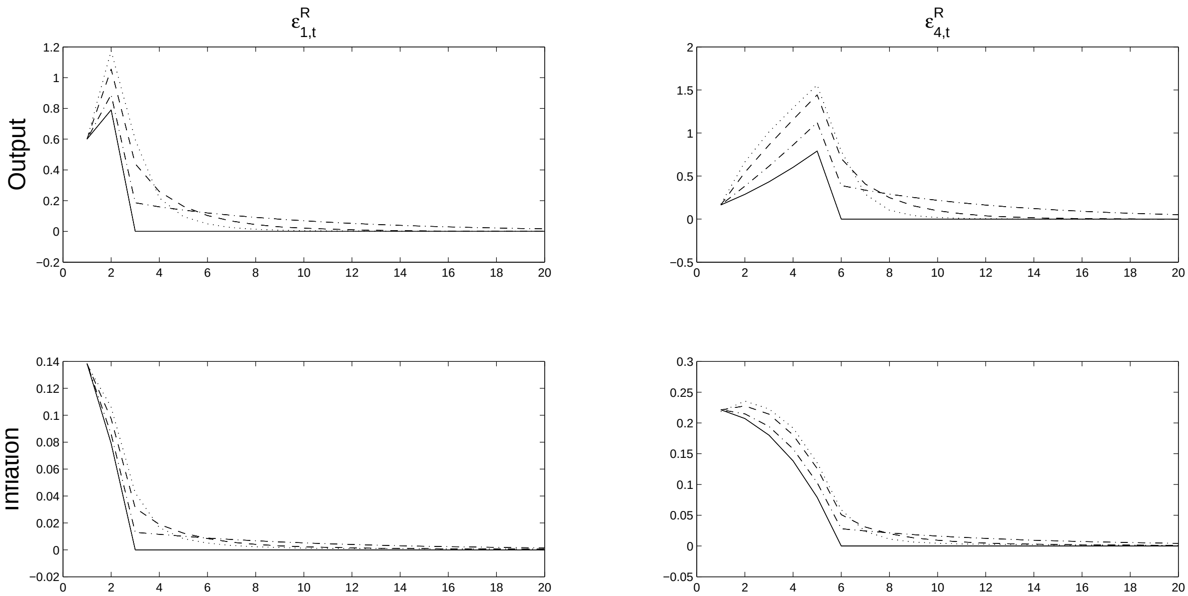

Figure 3: Impulse Responses of Endogenous Variables to Anticipated Shocks

Note: Solid Line: Rational Expectations; Dash-Dot Line: CGL with $\bar{\tau} = 0.01$; Dashed Line: CGL with $\bar{\tau} = 0.03$; Dotted Line: CGL with $\bar{\tau} = 0.05$

the effects of forward guidance.

7 Conclusion

In order to combat the effects of the Great Recession, central banks have instituted forward guidance. Because the effectiveness of forward guidance hinges on how expectations respond to forward guidance, it is of interest to investigate the link between expectation assumptions and forward guidance. The standard benchmark assumption is that agents form expectations based on the rational expectations hypothesis. However, if agents form expectations using an imperfect knowledge model, the results are different.

This paper presents an infinite horizon New Keynesian model with forward guidance and compares the results under two types of expectation assumptions. Under the assumption of rational expectations, Evans and Honkapohja (2001) state agents form expectations based on the true probability distribution and model of the economy. However, under adaptive learning, agents do not know this information, and instead, construct their expectations using standard econometric techniques. The main result, which is produced under a period of economic stress, is that rational expectations generate more favorable and exaggerated consequences on the economy to forward

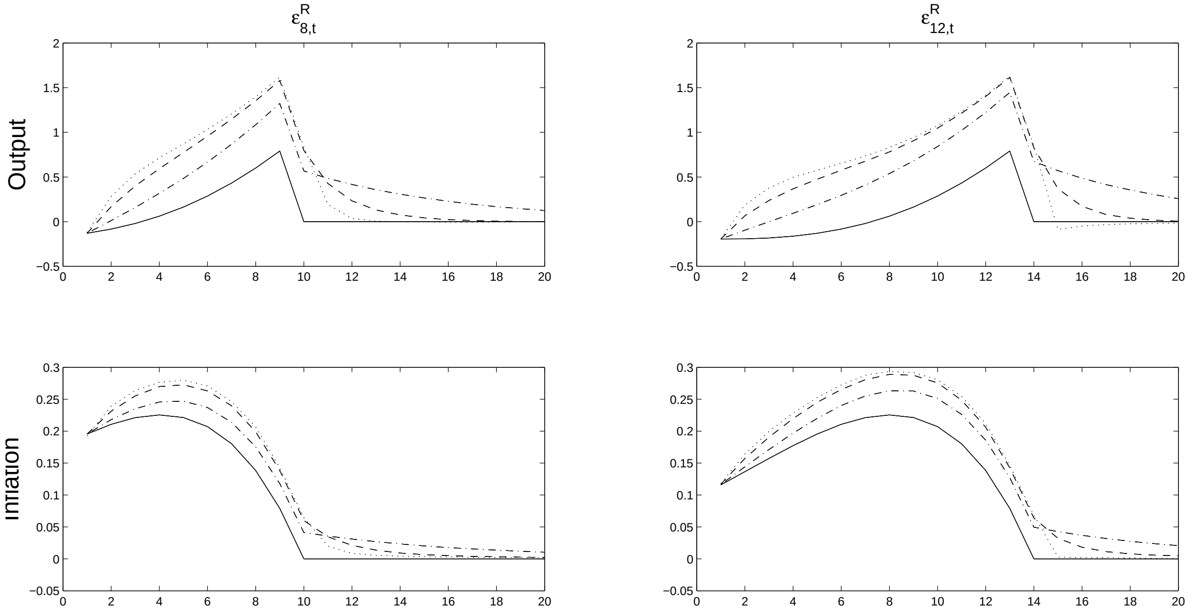

Figure 4: Impulse Responses of Endogenous Variables to Anticipated Shocks

Note: Solid Line: Rational Expectations; Dash-Dot Line: CGL with $\bar{\tau} = 0.01$; Dashed Line: CGL with $\bar{\tau} = 0.03$; Dotted Line: CGL with $\bar{\tau} = 0.05$

guidance than under adaptive learning. In addition, the impulse responses of the macroeconomic variables to forward guidance shocks display that forward guidance effects vary based on the expectations assumption. These results can be attributed to rational expectations agents precisely understanding the effects forward guidance has on the economy, while adaptive learning agents do not since their beliefs evolve slowly over time.

There are other modifications to the model presented in this paper that are worth noting. For instance, this paper allows agents to know the end date of forward guidance. Another type of forward guidance policy allows the central bank to link the expiration date of forward guidance to economic conditions. For instance, the unemployment rate is a criterion that the Federal Reserve has used to link to its forward guidance policy. The RLS formula also could be altered to include a gain parameter that changes based on the recent forecast errors as discussed in Milani (2007b) and Marcet and Nicolini (2003). This formation of the gain parameter allows agents to better track structural breaks in the economy. In addition, agents can be assumed to have heterogeneous expectations as in Branch and McGough (2009). Branch (2004) uses survey data and shows evidence that respondents have heterogeneous expectations. Overall, the role of expectations formation is especially crucial to understand the effects of forward guidance.

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Appendix

A Model

This section describes the derivation of equations (1) and (3), which follows Preston (2005).

A.1 Households

There exists a continuum of households indexed by i. Households maximize expected future discounted utility

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i; \xi_T) - \int_0^1 v(h_T^i(j); \xi_T) dj \right] \tag{A.1}$$

where β is the discount factor. Utility depends on C_T^i, which is consumption by household i of goods in the economy. Households also receive a disutility when supplying labor, $h_T^i(j)$, for the production of each good j. ξ_T denotes an aggregate shock. $\hat{E}_t^i$ denotes (potentially) non-rational expectations that satisfy standard probability laws, such as $\hat{E}_t^i \hat{E}_{t+1}^i = \hat{E}_t^i$. Beliefs are assumed to be homogeneous across agents, but agents do not know this fact.

A household is subject to a budget constraint that takes the following form

$$M_t^i + B_t^i \leq (1 + i_{t-1}^m) M_{t-1}^i + (1 + i_{t-1}) B_{t-1}^i + P_t Y_t^i - T_t - P_t C_t^i \tag{A.2}$$

where T_t denotes lump-sum taxes and transfer, M_t^i is money holdings, and i_t^m denotes interest paid on money balances. Asset markets are assumed to be incomplete such that household's can transfer wealth between periods through a one-period riskless bond B_t^i. Accordingly, i_t is the interest paid on bonds. Y_t^i is household i's real income. P_t is the aggregate price index, and $P_t Y_t^i$ denotes household i's nominal income which is given by

$$P_t Y_t^i = \int_0^1 [w_t(j) h_t^i(j) + \Pi_t(j)] dj \tag{A.3}$$

A household receives wages $w_t(j)$ for hours worked towards the production of good j, $h_t^i(j)$. Since each household owns an equal part of each firm, it receives profits from the sale of good j ($\Pi_t(j)$). Note that money does not show up in the utility function, even though it shows up in the budget constraint. It is assumed that money does not relieve any transactional frictions. However, as shown in the budget constraint, a household may choose to hold money because it can act as a financial asset that could provide a return.

The aggregate variables C_t^i and P_t are assumed to be defined by the Dixit-Stiglitz constant-elasticity-of-substitution aggregator

$$C_t^i \equiv \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{A.4}$$

$$P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \tag{A.5}$$

where $\theta > 1$ is the elasticity of substitutions across differentiated goods, $c_t^i(j)$ describes household i's consumption of good j, and $p_t(j)$ is the price of good j.

By log-linearizing the intertemporal budget constraint and Euler equation, the following results are obtained

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i = \bar{w}_t^i + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i \tag{A.6}$$

$$\hat{C}_t^i = \hat{E}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{E}_t^i \hat{\pi}_{t+1}) + g_t - \hat{E}_t^i g_{t+1} \tag{A.7}$$

where $\sigma \equiv \frac{-U_c}{U_{cc}C}$ defines the intertemporal elasticity of substitution, $g_t \equiv \sigma \frac{U_{c\xi}\xi_t}{U_c}$ denotes a preference shock, and $\bar{w}_t^i \equiv \frac{W_t^i}{P_t Y}$ is share of real wealth ($W_t^i \equiv (1 + i_{t-1})B_{t-1}^i$) as a fraction of steady-state income. The " ˆ " symbol over variables denotes log deviations from steady state. By solving (A.7) backwards from date T to t, taking expectations at time t, plugging the result into (A.6), and integrating over i, the following equation for aggregate consumption emerges

$$\hat{C}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\hat{Y}_T - \beta\sigma(\hat{i}_T - \hat{\pi}_{T+1}) + \beta(g_T - g_{T+1})\right] \tag{A.8}$$

Note that $\int_i \bar{w}_t^i di = 0$ since bonds are in zero net supply from market clearing. $\hat{E}_t = \int_i \hat{E}_t^i di$ denotes the average expectations operator. By imposing the market equilibrium condition $\hat{Y}_t = \hat{C}_t$ and defining the resulting equation in terms of the output gap $\hat{x}_t \equiv \hat{Y}_t - \hat{Y}_t^n$, the following equation emerges

$$\hat{x}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} [(1-\beta)\hat{x}_{T+1} - \sigma(\hat{i}_T - \hat{\pi}_{T+1}) + r_T^n] \tag{A.9}$$

where $\hat{Y}_t^n$ is the natural rate of output, that is, output prevailing under flexible prices, and $r_t^n \equiv (\hat{Y}_{t+1}^n - g_{t+1}) - (\hat{Y}_t^n - g_t)$.

A.2 Firms

Firms in the model operate in a monopolistically competitive environment and are subject to a Calvo (1983) pricing scheme where a fraction $(1-\alpha)$ of firms are allowed to readjust their price every period. Households supply labor to firms for use in production. A good is produced following the production function $y_t(i) = A_t f(h_t(i))$ where A_t is a technology shock. The demand curve for good i is given by $y_t(i) = Y_t(p_t(i)/P_t)^{-\theta}$. The following Dixit-Stiglitz aggregate price index is assumed

$$P_t = \left[\alpha P_{t-1}^{1-\theta} + (1 - \alpha p_t^{*1-\theta}\right]^{\frac{1}{1-\theta}} \tag{A.10}$$

A firm maximizes its expected present discounted value of profits

$$\hat{E}_t^i \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T}[\Pi_T^i(p_t(i))] \tag{A.11}$$

where $Q_{t,T}$ describes the stochastic discount factor showing how firms value its future stream of income. The stochastic discount factor is given by

$$Q_{t,T} = \beta^{T-t} \frac{P_t}{P_T} \frac{U_c(Y_T, \xi_T)}{U_c(Y_t, \xi_t)} \tag{A.12}$$

The profit function is defined by

$$\Pi_T^i(p_t(i)) = Y_t P_t^{\theta} p_t(i)^{1-\theta} - w_t(i) f^{-1}(Y_t P_t^{\theta} p_t(i)^{-\theta}/A_t) \tag{A.13}$$

Maximizing (A.11) with respect to $p_t(i)$ yields the following first order condition

$$\hat{E}_t^i \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} Y_T P_T^{\theta} [\hat{p}_t^*(i) - \mu P_T s_{t,T}(i)] = 0 \tag{A.14}$$

where $\mu = \frac{\theta}{\theta-1}$, and $s_{t,T}$ is the firm's real marginal cost function. Furthermore, by substituting in the stochastic discount factor and real marginal costs into the firm's first order condition and then log linearizing around a zero inflation steady state, the following result is produced

$$\hat{p}_t^*(i) = \hat{E}_t^i \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\frac{1-\alpha\beta}{1+\omega\theta}(\omega + \sigma^{-1})\hat{x}_T + \alpha\beta\hat{\pi}_{T+1}\right] \tag{A.15}$$

where ω defines the elasticity of a firm's real marginal cost function with respect to its output. Note also that log linearizing (A.10) yields

$$\hat{\pi}_t = \hat{p}_t^*(1-\alpha)/\alpha \tag{A.16}$$

Integrating over i and plugging (A.16) into (A.15) yields the following equation for inflation

$$\hat{\pi}_t = \kappa\hat{x}_t + \hat{E}_t \sum_{T=t}^{\infty})(\alpha\beta)^{T-t}[\kappa\alpha\beta\hat{x}_{T+1} + (1-\alpha)\beta\hat{\pi}_{T+1} + \mu_T] \tag{A.17}$$

where $\kappa = \frac{1-\alpha}{\alpha}\frac{1-\alpha\beta}{1+\omega\theta}(\omega + \sigma^{-1})$.[20]

[20] As in Preston (2006), a supply shock μ_t is added.

B Rational Expectations Solution

By following Sims (2002), the model consisting of equations (2), (4), (5), (6) − (9), (14), and (15) can be solved to yield the solution

$$\widetilde{Y}_t = \widetilde{C} + \xi_1 \widetilde{Y}_t + \xi_2 \epsilon_t \tag{B.1}$$

where

$$\widetilde{C} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]' \tag{B.2}$$

$$
\xi_1 = \begin{bmatrix}
0 & 0 & 0 & 0.57 & -0.35 & -0.77 & -0.55 & -0.35 & -0.18 & -0.03 & 0.08 & 0.16 & 0.21 & 0.23 & 0.24 & 0.23 & 0.21 & 0 & 0 \\
0 & 0 & 0 & 0.11 & 0.21 & -0.08 & -0.13 & -0.16 & -0.18 & -0.18 & -0.17 & -0.16 & -0.13 & -0.11 & -0.08 & -0.06 & -0.04 & 0 & 0 \\
0 & 0 & 0 & 0.27 & 0.32 & 0.77 & -0.30 & -0.33 & -0.34 & -0.32 & -0.29 & -0.25 & -0.21 & -0.16 & -0.11 & -0.07 & -0.03 & 0 & 0 \\
0 & 0 & 0 & 0.50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.29 & -0.07 & 0 & -0.77 & -0.55 & -0.35 & -0.18 & -0.03 & 0.08 & 0.16 & 0.21 & 0.23 & 0.24 & 0.23 & 0 & 0 \\
0 & 0 & 0 & 0.06 & 0.04 & 0 & -0.08 & -0.13 & -0.16 & -0.18 & -0.18 & -0.17 & -0.16 & -0.13 & -0.11 & -0.08 & -0.06 & 0 & 0
\end{bmatrix} \quad (B.3)
$$

$$
\xi_2 = \begin{bmatrix}
1.15 & -1.78 & -0.77 & -0.55 & -0.35 & -0.18 & -0.03 & 0.08 & 0.16 & 0.21 & 0.23 & 0.24 & 0.23 & 0.21 & 0.18 \\
0.23 & 1.03 & -0.08 & -0.13 & -0.16 & -0.18 & -0.18 & -0.17 & -0.16 & -0.13 & -0.12 & -0.08 & -0.06 & -0.04 & -0.02 \\
0.54 & 1.59 & 0.77 & -0.30 & -0.33 & -0.34 & -0.32 & -0.29 & -0.25 & -0.21 & -0.16 & -0.11 & -0.07 & -0.04 & -0.01 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0.57 & -0.35 & 0 & -0.77 & -0.54 & -0.35 & -0.1 & -0.03 & 0.08 & 0.16 & 0.21 & 0.23 & 0.24 & 0.23 & 0.21 \\
0.11 & 0.21 & 0 & -0.08 & -0.13 & -0.16 & -0.18 & -0.18 & -0.17 & -0.16 & -0.13 & -0.12 & -0.08 & -0.06 & -0.04
\end{bmatrix} \tag{B.4}
$$