Family decision of investment in human capital and migration in a model of spatial agglomeration *

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Abstract

This paper analyzes human capital investment decision and location choice, focusing on the intergenerational interactions within a family in an economy where geographic concentrations of high technology industries and high-level service sectors are proceeding. For highly educated young adults searching for job opportunities is much more frequently accompanied by long distance migrations, sometimes beyond the national border. That discourages parents to invest in human capital for their children. Public human capital investment decision is also discussed.

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1 Introduction

Human capital contributes to economic growth by fostering high technology industries, high-level service sectors, and cultural activities. [1] As studies in new economic geography have shown, high-tech and high-level service sectors tend to be concentrated in a limited number of regions due to scale-of-economy effects and the positive technological and pecuniary external effects within and between sectors. [2] Economic growth is thus accompanied by urbanization: that is, industrial agglomeration and human capital concentration. Scott (2009) and Gabe and Abel (2012) investigate the characteristics of the US metropolitan areas to consider how the interaction how the regional-occupational-and-skill structure interacts with the industrial structure. [3]

Investment in capital benefits the investor, which induces further investment. In considering the investment in human capital, we should pay more attention to the intergenerational interactions between parents and children. Education in childhood is crucial in determining a person's abilities to acquire skills and create new ideas. Hence, a parent's motivation to invest in human capital for his children is important.

This paper analyzes the investment in human capital decision within a family in an economy where modern industries are geographically concentrating into a limited number of regions. Once young adults acquire human capital, they are attracted to cities by various opportunities for self-realization, as well as higher wages. The lack or availability of such opportunities nearby a family's hometown has a crucial effect on the parents' decision to give their child a higher education. In an economy of advancing globalization and the agglomeration of modern sectors, search for such opportunities often entails long-distance migrations by young adults, sometimes beyond national borders. This may discourage parents to invest in human capital for their children. Gibbons and Vignoles (2012) empirically show that the geographical distance potentially matters in a university enrollment decision.

Here we start with an economy with only one city, and show that the distance does matter. Later we extend this framework to a case with multiple cities. We discuss not only private investment in human capital decisions, but also public ones.

[1] Mankiw, Romer and Weil (1992) show that a slight difference in the level of human capital investment causes a huge and persistent gap in income among countries, employing a non-increasing returns to scale production function.

[2] Ellison and Glaeser (1997) investigate the concentration of manufacturing sectors in the US.

[3] Human capital itself has spatial and dynamic external effects. Hence, it spurs economic growth by attracting further human capital and enhancing the efficiencies in investments in human capital. See Lucas (1988).

2 Literature

Intergenerational interaction between a parent and child is important in considering investment in human capital. Becker and Tomes (1986), Glomm and Ravikumar (1992), Galor and Zeria (1993), and Banerjee and Newman (1993) model investment in human capital as an altruistic motive. In Ehrlich and Lui (1991), investment in human capital appears as a factor in intergenerational trade and insurances. [4] Investment in human capital and heterogeneity in abilities induce income distribution and social mobility, as well as enhanced economic growth. In this paper I introduce the geographical that may separate a parent and child within a family.

New economic geography models, e.g., Krugman (1991), analyze high-skilled workers and the modern industries that use them. The industries that use them intensively have scale effects and are concentrated in a limited number of regions. In this paper I analyze endogenous determination in investment in human capital to acquire skills. The decision unit in the new economic geography models is an individual. Yet in an economy where modern industries are converging geographically, we should treat the family as the decision-making unit on matters to do with education and location.

3 The OLG model with family and geography

3.1 Geography and production

Consider a long and narrow economy where a very large number of regions stand in line. Formally, we treat the space of the economy as a one-dimensional line representing a uniform continuum of regions. This economy has only one city. Each region is indexed by n, the coordinates of the point where it is located. The coordinates of the point where the city region is located is indexed by $n = 0$, the origin.

For expositional simplicity, we assume that a homogenous commodity is produced and consumed in this economy. There are two types of the production technologies, modern and traditional; and there are two types of worker, skilled and unskilled. We let H and L represent the skilled worker and unskilled worker, respectively. The skill of a worker is determined by her family's decision on educational investment, which we will soon explain.

The modern technology uses capital and skilled workers. The function $F(a(n), k(n), h(n))$ represents the production function when the modern technology is employed is given by, where $a(n)$ is the productivity, and $k(n)$ and $h(n)$ are the inputs of capital and skilled workers, respectively, in region n. We assume that the production function is homogenous to degree one, and that $F_{kkh} < 0$, $F_{hh} < 0$, and $F_{kh} = F_{hk} > 0$. We also assume that the

[4] Bernheim, Shleifer and Summers (1985) analyze intergenerational transfers as strategically motivated intergenerational exchanges and insurances. Konrad, Kunemund, Lommerud and Robledo (2002) introduce geographical space and analyze strategic interaction among siblings in their location choices.

economy is a small open economy where capital is internationally mobile. The interest rate on the capital r is determined in the international assets market and given as a constant value for the economy. We let $w(H, n)$ denote the wage a skilled worker earns in region n. This depends only on the productivity level in region n, $a(n)$. [5]

We assume that firms with modern technology are concentrated in the city. Therefore, the wage of a skilled worker is positive only in the city ($w(H, 0) > 0$) and it is zero for other regions ($w(H, n) = 0$ if $n \neq 0$). We let w_H denote the wage of a skilled worker in the city.

In contrast, the traditional method uses only unskilled worker and produces a homogenous good according to a constant-returns-to-scale technology. Here we let $w(L, n)$ denote the wage of an unskilled worker in region n. This depends on the productivity of the traditional production method in that region. Unlike the modern technologies, the traditional technologies are available anywhere in this economy. Moreover, the productivity of the traditional production method is the same for all regions. Hence, $w(L, n)$ is independent of n. The variable w_L denotes the wage level.

A skilled worker can also produce a commodity by employing the traditional production method, yet her high skill becomes useless in this case. That is, she has the same productivity as an unskilled worker in this case. In contrast, an unskilled worker cannot be employed in the production of modern technology.

3.2 Intergenerational interaction within family

An individual lives for three periods: childhood, young adulthood, and old adulthood. Generation t is the generation of individuals born at the beginning of period t as generation t. An individual of this generation is in childhood in period t, young in period $t + 1$, and old in period $t + 2$.

For simplicity, we assume that an individual of generation t has no consumption when in childhood, in period t. If his parent (an individual of generation $t - 1$) decides to give her a higher education, he acquires it during childhood

At the beginning of the next period $t + 1$, when young, an individual of generation t becomes the parent of a child (an individual of generation $t + 1$). If the individual has a higher education when in childhood, she becomes a skilled worker. In this case, she can freely choose a region in which to live and work, n_{t+1}, when young. In contrast, if she has no higher education, she becomes an unskilled worker. We assume that, in this case, she

[5]Given the interest rate r and the supply of skilled workers $h(n)$ in region n, the maximization problem of firms and factor market equilibrium conditions yield $w(H, n) = F_h(a(n), k(n), h(n))$ and $r = F_k(a(n), k(n), h(n))$. By solving these two equations, we obtain $k(n)$ and $w(H, n)$, the input of capital and the wage of the a skilled worker in region n, respectively. By solving the latter equation for $k(n)$ and substituting the result for $k(n)$ in the former equation, we obtain $w(H, n)$ as a function of $a(n)$, r, and $h(n)$. Yet the wage for a skilled worker in region n is independent of the labor supply $h(n)$ in that region, as the production function is homogenous of degree one.

cannot necessarily be mobile. She can be a mobile worker with probability p ($0 \geq p \geq 1$). When she migrates, she will do so with his child. In n_{t+1}, the region where she chooses to live, he supplies one unit of labor inelastically and earns a wage. If she is a skilled worker and lives in the city ($n_{t+1} = 0$), she earns a wage of w_H. If she lives in a region other than the city, she earns a wage of w_L. If she is an unskilled worker, she earns a wage of w_L wherever she lives.

When both young and old, each individual derives utility from private consumption goods and some types of goods that have the characteristics of public goods within the very local framework of a family. Let C^y_{t+1} and C^o_{t+2} denote the consumption of private consumption goods of an individual of generation t in period $t+1$, when the individual is young, and that in period $t+2$, when she is old, respectively. And let G^y_{t+1} and G^o_{t+2} denote the consumption of family public goods of an individual of generation t in period $t+1$ and in period $t+2$, respectively.

When young, an individual also decides whether to give her child a higher education. If she decides in favor of higher education, her child acquires enhanced productivity and becomes a skilled worker. A higher education costs a fixed amount of e. Let $edu_{t+1} \in \{e, 0\}$ denote the expenditure for a higher education chosen by an individual of generation t for her child (generation $t+1$) in period $t+1$ when she is young and her child is in childhood. And let $f(edu)$ denote the function that maps $edu \in \{e, 0\}$ on skill level, H, and L; $f(e) = H$, and $f(0) = L$.

An individual retires at the beginning of period $t+2$, when old, She loses her parent (an individual of generation $t-1$) and becomes the grandparent of an individual of generation $t+2$. When old, she cannot migrate to the other regions. The pattern is infinitely repeated from generation to generation.

Individuals have an identical lifetime utility function, as follows:

$$\begin{aligned} &\left[u(C^y_{t+1}) + v(G^y_{t+1}) + \delta(|n_t - n_{t+1}|)v(G^o_{t+1})\right] \\ &+ \left(\frac{1}{1+\rho}\right)\left[u(C^o_{t+2}) + v(G^o_{t+2}) + \delta(|n_{t+1} - n_{t+2}|)v(G^y_{t+2})\right], \end{aligned} \tag{1}$$

where ρ is the subjective intertemporal discount rate. When an individual is young, the family public goods of her parent, G^o_{t+1}, enhance her utility, and $0 < \delta(|n_t - n_{t+1}|) < 1$ is the impact of G^o_{t+1}. This impact depends on the distance from her hometown region, n_t, where her parent lives, to region n_{t+1}, where she lives and works. As the distance grows, the impact decreases. Similarly, when an individual is old, the family public goods of her child, G^y_{t+2}, enhance her utility, and $0 < \delta(|n_{t+1} - n_{t+2}|) < 1$ is the impact of G^y_{t+2}. This impact negatively depends on the distances to region n_{t+1}, where her child lives and works.

The lifetime budget constraint is:

$$C^y_{t+1} + G^y_{t+1} + \left(\frac{1}{1+r}\right)\left[C^o_{t+2} + G^o_{t+2}\right] = w(f(edu_t), n_{t+1}) - edu_{t+1}. \tag{2}$$

We assume that the subjective discount rate ρ is equal to the interest rate r.

3.3 Formal setup for the optimization problem

Given the decisions of her parent (an individual of generation $t-1$) as to where the parent lives (n_t), whether the parent give her a higher education ($edu_t \in 0, e$) in the last period t, and the amount of family public goods consumption (G^o_{t+1}) in the current period $t+1$, an individual of generation t decides where to live (n_{t+1}), whether to give her child a higher education ($edu_{t+1} \in \{0, e\}$), and the amounts of private goods consumption and family public goods consumption when young and when old (C^y_{t+1}, G^y_{t+1}, C^o_{t+2}, G^o_{t+2}).

The decision and maximized utility of an individual of generation t are affected by the n_t, edu_t, and G^o_{t+1} chosen by the individual's parent. For example, we consider how an individual's location choice n_{t+1} is affected by n_t, edu_t, and G^o_{t+1}. When her parent gave her a higher education when she was young ($edu_t = e$), she becomes a mobile skilled worker ($f(e) = H$). She will migrate to the city ($n_{t+1} = 0$) to earn higher wages w_H. In contrast, when her parent opted not to give her a higher education ($edu_t = 0$), she becomes an unskilled worker ($f(0) = L$). Yet she becomes a mobile worker with probability p, and may migrate to the city ($n_{t+1} = 0$) taking into account her child's behavior as we will see soon. In these cases, however, the further the hometown region n_t is located from the city, where her parent supplies the family public goods, the less prominently the benefits the individual gains from the family public goods become greater when her parent increases the amount of the goods. [6]

Very similarly, the choices of n_{t+1}, edu_{t+1}, and G^o_{t+2} by an individual of generation t affect the choices of her child on several variables, including n_{t+2}, edu_{t+2}, and G^o_{t+3}. And her child's choices of such n_{t+2} and G^y_{t+2} affect her own utility when she becomes old in the next period. Therefore, an individual of generation t rationally guesses how her child will react to her choice, and chooses n_{t+1}, edu_{t+1}, and G^o_{t+2} reacting to her parent's choice.

Formally, we express the choices of an individual of a generation by the functions:

$$\begin{cases} n_{t+1} = \phi(x_t, n_t, edu_t, G^o_{t+1}), \\ edu_{t+1} = edu(x_t, n_t, edu_t, G^o_{t+1}), \\ C^y_{t+1} = C^y(x_t, n_t, edu_t, G^o_{t+1}), \\ G^y_{t+1} = G^y(x_t, n_t, edu_t, G^o_{t+1}), \\ C^o_{t+2} = C^o(x_t, n_t, edu_t, G^o_{t+1}, x_{t+1}), \\ G^o_{t+2} = G^o(x_t, n_t, edu_t, G^o_{t+1}, x_{t+1}). \end{cases} \tag{3}$$

[6] Here we assume a separable utility function, hence a change in G^o_{t+1} has no direct effect on the individual's choices of private goods consumption when young and old (C^y_{t+1} and C^o_{t+2}) and family public goods when young, G^y_{t+1}. A change in G^o_{t+1} does, however, have indirect effects on these variables. Specifically, it affects the individual's location choice, n_{t+1}, which changes the wage $w(f(edu_t), n_{t+1})$, which exerts an effect.

In these expressions, x_t represents whether or not an individual of generation $t+1$, is mobile. We let $x_t = 1$ and $x_t = 0$ represent the case where she is mobile, and the case where she is immobile, respectively.

These are obtained by solving the following recursively constructed problem.

First, we consider the optimal choices of private goods consumption and family public goods consumption of an individual of generation t when old, given the amount of savings, S_{t+1}, and the other decisions she made when young. We let $C^o(S_{t+1}, n_{t+1}, edu_{t+1}, x_{t+1})$ and $G^o(S_{t+1}, n_{t+1}, edu_{t+1}, x_{t+1})$ denote the optimal choice of private goods consumption and family public goods consumption, respectively. Also, we let $V(S_{t+1}, n_{t+1}, edu_{t+1}, x_{t+1})$ denote the maximized utility when old. More specifically, $C^o(S_{t+1}, n_{t+1}, edu_{t+1}, 1)$ and $G^o(S_{t+1}, n_{t+1}, edu_{t+1}, 1)$ are the optimal choices of C^o_{t+2} and G^o_{t+2}, respectively. And $V(S_{t+1}, n_{t+1}, edu_{t+1}, 1)$ is the maximized objective function in the following problem:

$$\begin{aligned}\text{MAX} \quad & u(C^o_{t+2}) + v(G^o_{t+2}) + \delta(|n_{t+1} - \phi(1, n_{t+1}, edu_{t+1}, G^o_{t+2})|)v(G^y(1, n_{t+1}, edu_{t+1}, G^o_{t+2})) \\ \text{s.t.} \quad & C^o_{t+2} + G^o_{t+2} = (1+r)S_{t+1}.\end{aligned}$$

Similarly, $C^o(S_{t+1}, n_{t+1}, edu_{t+1}, 0)$, $G^o(S_{t+1}, n_{t+1}, edu_{t+1}, 0)$, and $V(S_{t+1}, n_{t+1}, edu_{t+1}, 0)$ are the solution of the following problem:

$$\begin{aligned}\text{MAX} \quad & u(C^o_{t+2}) + v(G^o_{t+2}) + v(G^y(0, n_{t+1}, edu_{t+1}, G^o_{t+2})) \\ \text{s.t.} \quad & C^o_{t+2} + G^o_{t+2} = (1+r)S_{t+1}.\end{aligned}$$

The optimal choices $C^o(S_{t+1}, n_{t+1}, edu_{t+1}, x_{t+1})$ and $G^o(S_{t+1}, n_{t+1}, edu_{t+1}, x_{t+1})$ and attained $V(S_{t+1}, n_{t+1}, edu_{t+1}, x_{t+1})$ may differ between the case where $x_{t+1} = 1$ and the case where $x_{t+1} = 0$. For example, if her child is mobile ($x_{t+1} = 1$) but if she can entice her child to stay in her hometown by choosing G^o_{t+1} that is larger than $C^o(S_{t+1}, n_{t+1}, edu_{t+1}, 0)$, an individual of generation t may be willing to do so.

Next, we consider the optimal choices of where to live (n_{t+1}), whether to give her child a higher education (edu_{t+1}), the amount of savings (S_{t+1}), as well as the amounts of private goods consumption and family public goods consumption (C^y_{t+1} and G^y_{t+1}) when young, given her parent's choices of n_t, edu_t, and G^o_{t+1}. When she is young, she cannot perfectly foresee whether her child will be mobile or not in the next period when she is old and her child is a young adult. However, she knows the probability with which her child will be a mobile worker, and guesses how she will behave in the next period when she will know whether or not her child becomes a mobile worker. She chooses n_{t+1}, edu_{t+1}, S_{t+1}, C^y_{t+1} and G^y_{t+1} at the levels that maximize her expected lifetime utility. That is,

$$\begin{aligned}&\text{MAX} \\ &\left[u(C^y_{t+1}) + v(G^y_{t+1}) + \delta(|n_t - n_{t+1}|)v(G^o_{t+1})\right] \\ &+ \left(\frac{1}{1+\rho}\right) [\ P(edu_{t+1})V(S_{t+1}, n_{t+1}, edu_{t+1}, 1) + (1 - P(edu_{t+1}))V(S_{t+1}, n_{t+1}, edu_{t+1}, 0)\ ] \\ &\text{s.t.} \quad C^y_{t+1} + G^y_{t+1} + S_{t+1} = w(f(edu_t), n_{t+1}) - edu_{t+1}.\end{aligned}$$

where $P(edu_{t+1})$ is the probability with which her child, an individual of generation $t+1$, becomes a mobile worker ($x_{t+1} = 1$) in period $t + 2$. As we assumed, this probability depends on her decision as to whether she gives her child a higher education (edu_{t+1}). When she gives her child a higher education, her child will be a mobile skilled worker ($P(e) = 1$). In contrast, when she does not, her child will be an unskilled worker. Yet with probability p, she becomes a mobile unskilled worker ($P(0) = p$).

Hence the optimal choices of n_{t+1}, edu_{t+1}, C^y_{t+1} and G^y_{t+1} are expressed as the functions of her parent choices of n_t, edu_t, and G^o_{t+1}, and her own situation x_t that partially depends on her parent's choice, as in (3). Also her optimal choices of savings S_{t+1} depends on n_t, edu_t, G^o_{t+1}, and x_t.

As we have seen, her choices of private goods consumption and family public goods consumption when old depend on n_{t+1} and her own decisions of S_{t+1}, n_{t+1}, and edu_{t+1} when young. Now her decisions of S_{t+1}, n_{t+1}, and edu_{t+1} are expressed as the functions of n_t, edu_t, and G^o_{t+1}. Therefore, her choices of private goods consumption and family public goods consumption when old are expressed as in (3).

As a candidate for the optimal $G^o_{t+2} = G^o(n_t, edu_t, G^o_{t+1})$ and the relationship of this variable with the optimal n_{t+1} and edu_{t+1}, we introduce the function $\tilde{G}(w(f(edu_t), n_{t+1}) - edu_{t+1})$, and $\tilde{C}(w(f(edu_t), n_{t+1}) - edu_{t+1})$. These are the amounts of family public goods (G^y_{t+1} and G^o_{t+2}) and private goods (C^y_{t+1} and C^o_{t+2}) that maximize (1) subject to (2), given all of the other variables that relate to her lifetime utility, namely, her parent's choice of n_t, edu_t, and G^o_{t+1}, her own choice of n_{t+1}, and edu_{t+1}, and her child's choice of n_{t+2} and G^y_{t+2}. [7]

In reality, the G^o_{t+2} chosen by an individual of generation t affects the behavior of the individual's child. Hence, $\tilde{G}$ is not necessarily the optimal G^o_{t+2}. Neither is the optimal G^o_{t+2} necessarily related to the optimal n_{t+1} or edu_{t+1} as the function $\tilde{G}(w(f(edu_t), n_{t+1}) - edu_{t+1})$ indicates. We can use $\tilde{G}$, however, a clue to obtain the optimal G^o_{t+2}.

4 Equilibrium

4.1 Optimal behavior

Directly solving the functional equation is formidably complicated. Here we guess policy functions of several forms and analyze the conditions under which each of them holds as a policy function.

[7] Given our assumption that the subjective discount rate ρ is equal to the interest rate r, an individual will smooth out the consumptions perfectly throughout her lifetime. Therefore, the G^y_{t+1} obtained is equal to G^o_{t+2}, and we let $\tilde{G}(w(f(edu_t), n_{t+1}) - edu_{t+1})$ denote the value. Similarly, the C^y_{t+1} obtained is equal to C^o_{t+2}, and denoted by $\tilde{C}(w(f(edu_t), n_{t+1}) - edu_{t+1})$.

4.1.1 Individuals invest in human capital and migrate to the city

Suppose that her parent opts not to give her child, an individual of generation t, a higher education ($edu_t = 0$). That individual becomes an unskilled worker. Specifically, she becomes a mobile unskilled worker with a probability of p, and an immobile unskilled worker with a probability of $1 - p$.

If her parent chooses family public goods (G^o_{t+1}) in an amount smaller than $G*$, a variable equal to or larger than $\tilde{G}(w_L)$, that individual will give her child (an individual of generation $t+1$) a higher education ($edu_{t+1} = e$) when young and choose family public goods in the amount of $G^o_{t+2} = \tilde{G}(w_L - e)$ when old, regardless of whether or not she is mobile. And if that individual is a mobile worker, she migrates to the city ($n_{t+1} = 0$).

If her parent chooses family public goods (G^o_{t+1}) in an amount larger than $G*$, that individual will still give her child a higher education ($edu_t = e$) and choose family public goods in the amount of $G^o_{t+2} = \tilde{G}(w_L - e)$. In this case, however, she will remain in her hometown ($n_{t+1} = n_t$), even if she is a mobile worker.

In contrast, if her parent gives an individual of generation t a higher education, she becomes a mobile skilled worker. She migrates to the city ($n_{t+1} = 0$). She will give her child a higher education (edu_t) when young and choose family public goods in the amount of $G^o_{t+2} = \tilde{G}(w_H - e)$ when old.

Formally,

$$\phi(x_t, n_t, edu_t, G^o_{t+1}) = \begin{cases} n_t & \text{if } edu_t = 0 \text{ and } G^o_{t+1} \geq G^*, \text{ or if } x_t = 0, \\ 0 & \text{if } edu_t = 0 \text{ and } G^o_{t+1} < G^*, \text{ or if } edu_t = e, \end{cases}$$

$$edu(x_t, n_t, edu_t, G^o_{t+1}) = e. \tag{4}$$

$$G^o(x_t, n_t, edu_t, G^o_{t+1}, x_{t+1}) = \begin{cases} \tilde{G}(w_L - e) & \text{if } edu_t = 0, \\ \tilde{G}(w_H - e) & \text{if } edu_t = e. \end{cases}$$

An alternative of this case is that if her parent opts not to give an individual of generation t a higher education and she becomes an immobile unskilled worker, neither will she give her child (an individual of generation $t+1$) a higher education. That individual will choose family public goods in the amount of $G^o_{t+2} = \tilde{G}(w_L - e)$ when old, despite that her child will migrate to the city in the case where her child becomes a mobile unskilled worker.

Formally,

$$\phi(x_t, n_t, edu_t, G^o_{t+1}) = \begin{cases} n_t & \text{if } edu_t = 0 \text{ and } G^o_{t+1} \geq G^*, \text{ or if } x_t = 0, \\ 0 & \text{if } edu_t = 0 \text{ and } G^o_{t+1} < G^*, \text{ or if } edu_t = e, \end{cases}$$

$$edu(x_t, n_t, edu_t, G^o_{t+1}) = \begin{cases} e & \text{if } edu_t = 0, \ x_t = 1, \text{ and } G^o_{t+1} < G^*, \text{ or if } edu_t = e, \\ 0 & \text{if } edu_t = 0, \ x_t = 0, \text{ and } G^o_{t+1} \geq G^*, \end{cases} \tag{5}$$

$$G^o(x_t, n_t, edu_t, G^o_{t+1}, x_{t+1}) = \begin{cases} \tilde{G}(w_L - e) & \text{if } \ edu_t = 0, \\ \tilde{G}(w_H - e) & \text{if } \ edu_t = e. \end{cases}$$

4.1.2 Individuals are less eager to invest in human capital

Suppose that her parent opts not to let an individual of generation t receive a high education ($edu_t = 0$). That individual becomes an unskilled worker. Specifically, she becomes a mobile unskilled worker with a probability of p, and an immobile unskilled worker with a probability of $1 - p$.

If her parent chooses family public goods G^o_{t+1} in an amount larger than $G*$, a variable equal to or smaller than $\tilde{G}(w_L)$ but not extreme so, an individual of generation t will also opt not to let her child (an individual of generation $t+1$) receive a high education ($edu_{t+1} = 0$). And she remains in her hometown ($n_{t+1} = n_t$), and chooses family public goods in the amount of $G^o_{t+2} = \tilde{G}(w_L)$, even if she is a mobile worker.

If her parent chooses family public goods G^o_{t+1} in an amount smaller than $G*$ and if she is a mobile worker, an individual of generation t will give her child a higher education ($edu_t = e$). She migrates to the city ($n_{t+1} = 0$), and choose family public goods in the amount of $G^o_{t+2} = \tilde{G}(w_L - e)$.

In contrast, if her parent gives an individual of generation t a higher education, an individual of generation t will also give her child a higher education (edu_t). She migrates to the city ($n_{t+1} = 0$), and choose family public goods in the amount of $G^o_{t+2} = \tilde{G}(w_H - e)$.

Formally,

$$\phi(n_t, edu_t, G^o_{t+1}) = \begin{cases} n_t & \text{if } \ edu_t = 0 \ \text{ and } \ G^o_{t+1} \geq G^*, \ \text{or if } \ x_t = 0, \\ 0 & \text{if } \ edu_t = 0 \ \text{ and } \ G^o_{t+1} < G^*, \ \text{ or } \ \text{ if } \ edu_t = e, \end{cases}$$

$$edu(n_t, edu_t, G^o_{t+1}) = \begin{cases} e & \text{if } \ edu_t = 0, \ x_t = 1, \ \text{and } \ G^o_{t+1} < G^*, \ \text{ or } \ \text{ if } \ edu_t = e, \\ 0 & \text{if } \ edu_t = 0, \ x_t = 0, \ \text{and } \ G^o_{t+1} \geq G^*, \end{cases} \tag{6}$$

$$G^o(n_t, edu_t, G^o_{t+1}) = \begin{cases} \tilde{G}(w_L) & \text{if } \ edu_t = 0 \ \text{ and } \ G^o_{t+1} \geq G^*, \\ \tilde{G}(w_L - e) & \text{if } \ edu_t = 0 \ \text{ and } \ G_{t+1} < G^*, \\ \tilde{G}(w_H - e) & \text{if } \ edu_t = e. \end{cases}$$

4.2 Regions close to the city

The following Proposition shows the conditions under which the policy function takes the form of (4).

Proposition 1
If her family hometown region is close to the city, an individual of generation t behaves as (4) indicates. She gives her child a higher education and migrates to the city with

her child, unless the amount of family public goods provided by her parent in the family hometown is very large.

The formal Proof is given in Appendix 2. The threshold value of G^* in the policy function (4) satisfies:

$$\left(\frac{1}{1+\rho}\right) v(\tilde{G}(w_H - e)) = v(G^*). \tag{7}$$

And by solving the following equation for n, we can obtain the index for the region the farthest from the city among the regions where (4) holds as the policy function:

$$\begin{aligned} &\left(\frac{2+\rho}{1+\rho}\right) u(\tilde{C}(w_L - e)) + \left(\frac{2+\rho}{1+\rho}\right) v(\tilde{G}(w_L - e)) + \left(\frac{1}{1+\rho}\right) \delta(n) v(\tilde{G}(w_H - e)) \\ = &\left(\frac{2+\rho}{1+\rho}\right) u(\tilde{C}(w_L)) + \left(\frac{2+\rho}{1+\rho}\right) v(\tilde{G}(w_L)) + \left(\frac{1}{1+\rho}\right) (p\delta(n) + (1-p)) v(\tilde{G}(w_L)) \end{aligned} \tag{8}$$

A smaller n means a larger δ, and thus a smaller n means a higher likelihood that this condition will hold. Details on (7) and (8) are analyzed in Appendix 2.

Suppose that an individual of generation t anticipates that her child will decide whether to remain in the family's hometown ($n_{t+2} = n_{t+1}$ or 0), decide whether to give her grandchild (the child of her child, an individual of generation t+2) a higher education ($edu_{t+2} = e$ or 0), and choose the amount of family public goods (G^o_{t+3}), based on that individual's own decisions of n_{t+1}, edu_{t+1}, and G^o_{t+2}, as (4) indicates. Using figures, we intuitively consider why that individual does also base her own decisions on her parent's choices of n_t, edu_t, and G^o_{t+1}, exactly in the same way as her own child will base her decisions on her choices, as (4) indicates, when her hometown is located close to the city.

If she behaves as (4) indicates, her child and all of the descendant generations will also behave as (4) indicates, because they will face exactly the same problem and share the same expectations. The policy function takes the form of (4) and the expectation about her child generation's behavior is rational.

Figure 1 shows the expected lifetime utility of an individual of generation t as a function of the amount of savings S_{t+1} that she plans when young, in the case where her parent is a immobile unskilled worker, lives in region n_t located close to the city, opts not to give her a higher education ($edu_t = 0$), and chooses family public goods in the amount of $G^o_{t+1} = \tilde{G}(w_L)$ when old. She becomes an unskilled worker in period $t+1$. Apart from the foregoing, her lifetime utility also depends on her own choice of whether or not to give her child a higher education (edu_{t+1}), her choice of where to live (n_{t+1}), and the amount of family public goods and private consumption goods she chooses.

Figure 1(i) and (ii) shows the case where an individual of generation t gives her child a higher education ($edu_{t+1} = e$). Her child becomes a skilled worker. When her child is young, her child lives and works in the city ($n_{t+2} = 0$), gives her grandchild (an individual of generation $t+2$, the child of her child) a higher education ($edu_{t+2} = e$), and chooses family public goods in the amount of $G^y_{t+2} = \tilde{G}(w_H - e)$. As her child will live and work in the city, an individual of generation t must live apart from either her parent or her child. Figure 1(i) is the case where the individual of generation t migrates from the family hometown to the city to live with her child ($n_{t+1} = 0$). Figure 1(ii) shows the opposite case where an individual of generation t remains in her hometown, and lives apart from her child who migrates to the city ($n_{t+1} = n_t$).

Figure 1(iii) shows the case where an individual of generation t opts not to give her child a higher education ($edu_{t+1} = 0$) and remains in her hometown ($n_{t+1} = n_t$). Her child becomes an unskilled worker.

In Figure 1(iii), differently from Figure 1(i) and (ii), when savings reaches a threshold amount, the relationship between savings and lifetime utility changes. This changes has to do with the motivation of an individual of generation t to keep her child nearby when that individual gets old. That is, the amount of savings an individual plans when young changes her own decision when old on whether to entice her child to stay in the family hometown in the case where her child becomes a mobile unskilled worker. If her child becomes a mobile unskilled worker, she cannot induce her child to remain in the family hometown without choosing her own family public goods in an amount as large as G^*. If she chooses more abundant savings when young, she will entice her child to remain in the family hometown by choosing G^* when old. In contrast, if she chooses less savings when young, she will not keep her child in the family hometown. She will choose family public goods in an amount smaller than G^*. If she chooses G^*, the amount of private consumption, i.e., the amount of savings minus family public goods, becomes extremely small. She will increase the amount of private consumption, even though the accompanying decrease in family public goods will make the hometown less attractive to her child and give her child incentive to move to the city.

In the current period, $t+1$, when an individual of generation t is young and is planning the amount of savings S_{t+1}, she cannot clearly foresee whether her child will be a mobile worker in the next period, $t+2$, when that individual is old and her child is a young adult. She can, however, guess how she will behave in the next period, when she becomes old and knows whether her child is mobile. Taking in to account her own behavior in the next period when old, which we have discussed above, she plans savings at the level that maximizes her expected lifetime utility in the present, while she is still young. If she chooses larger savings now, she anticipates that when she becomes old, in every case she will have her child in the family hometown. Yet, her utility when young is low. If, In contrast, she chooses smaller savings now, she anticipates that when she becomes old, in the case where her child becomes a mobile skilled worker, she will give up enticing her child to stay in the family hometown. In this case, however, she can enjoy higher utility when young.

A mobile unskilled worker of generation t's decision on whether or not to give her child a higher education ($edu_{t+1} = 0$ or e) and whether or not to migrate to the city ($n_{t+1} = 0$ or n_t) depend on which is the highest, the maximal point in Figure 1(i), the maximal point in Figure 1(ii), or the maximal point in Figure 1(iii). And an immobile unskilled worker's decision on whether or not to give her child a higher education depends on which is higher, the maximal point in Figure 1(ii) or the maximal point in Figure 1(iii).

The maximal point in Figure 1(i) is higher than that in Figure 1(ii), whatever n_t is. That is, once a mobile unskilled worker decides to give her child a higher education, she migrates to the city with her child, regardless of how far the family hometown is located from the city. The amount of family public goods chosen by her parent, $G^o_{t+1} = \tilde{G}(w_L)$, is smaller than that chosen by her child who becomes a richer skilled worker, $G^y_{t+2} = \tilde{G}(w_H - e)$. Therefore, she chooses to live in the city with her child, who will provide the larger amount of family public goods.

In contrast, the maximal point in Figure 1(ii) is higher than that in Figure 1(i), only if n_t is small. And in Figure 1(i) the maximum expected utility is attained at the amount of savings smaller than the threshold level, if n_t is small. That is, if the family hometown is close to the city, the optimal choice of savings in the case where an individual of generation t opts not to give her child a higher education is $\tilde{S}$, an amount smaller than the threshold. Yet the maximized utility in this case is lower than that in the case where she gives her child a higher education.

Why is the optimal choice of savings relatively small, in the case where an individual opts not to give her child a higher education? Her child will be an unskilled worker. And with probability p, her child will be a mobile worker. In this case, her child is more likely to migrate to the city, as (4) indicates. The amount of family public goods an individual of generation t needs to entice her child to stay in the family hometown, G^*, in the case where her child becomes a mobile unskilled worker, is very large. If she chooses to save a very large amount when young, she will entice her child to stay in the family hometown by choosing G^* when she becomes old and her child becomes a mobile unskilled worker. However, private consumption goods and family public goods when she is young are very small, and thus she is not willing to save a very large amount.

In addition, if the family hometown is close to the city, an individual is much less willing to entice her child to stay in the family hometown by choosing very large G^*. When an individual of generation t plans savings, she recognizes that if her savings are in the amount of $\tilde{S}$ and if her child becomes a mobile unskilled worker, she will not entice her child to stay in the family hometown. If her child migrates to the city, the benefits from her child's family public goods are decreased but only to a less-than-extreme extent if the family hometown is close to the city.

Next, we consider why an individual will give her child a higher education, if the family hometown is close to the city. To do so, we consider her costs and benefits of her investment in human capital for her child, when an individual is an immobile unskilled worker and thus remains in the family hometown. When an individual of generation t opts not give her child a higher education, her child will be a mobile unskilled worker with probability of p. And in this case that individual will not induce her child to stay in her hometown as we have seen above. In contrast, when she gives her child a higher education, her child will be a mobile skilled worker with probability of 1, move to the city, and provide family public goods in a large amount of G. Therefore, the costs of giving her child a higher education are the increase in the probability that her child moves to the city $(1 - p)$, and the expenditure for the education. And the benefits are the increased amount of family public goods provided by her child when she becomes old and her child becomes young adult.

The closer the family hometown is to the city, the smaller the costs are. If her child migrates to the city, the benefits from her child's family public goods are decreased but only to a less-than-extreme extent. The additional decrease caused by making her child be a mobile skilled worker is smaller. And the closer the family hometown is to the city, the larger the benefits are. She can easily access to the family public goods in a large amount provided by her rich child in the city.

To sum up, if her parent chooses family public goods in the amount of $G^o_{t+1} = \tilde{G}(w_L)$ and if the family hometown n_t is close enough to the city, an individual of generation will give her child a higher education. And if she is a mobile worker, she will live in the city with her child.

4.3 Regions located from the city in middle distance

The following Proposition shows the conditions under which the policy function takes the form of (5).

Proposition 2
If her hometown region is located in middle distance from the city, an individual of generation t behaves as (5) indicates. If she is a mobile unskilled worker, she gives her child a higher education and migrates to the city with her child unless the amount of family public goods provided by her parent in the family hometown is very large. In contrast, if she is an immobile unskilled worker, she opts not to give her child a higher education and remains in her hometown. And if her child becomes a mobile unskilled worker, she will not induce her child to remain in the family hometown.

Suppose that an individual of generation t anticipates that her child will decide whether to remain in the family's hometown ($n_{t+2} = n_{t+1}$ or 0), decide whether to give her grandchild (the child of her child, an individual of generation t+2) a higher education ($edu_{t+2} = e$ or 0), and choose the amount of family public goods (G^o_{t+3}), based on that individual's own decisions of n_{t+1}, edu_{t+1}, and G^o_{t+2}, as (5) indicates. As in the last subsection, we confirm that she does decide them as (5) indicates, exactly in the similar manner as her child decide them, when her hometown is located in middle distance from the city. If so, she behaves as (5) indicates, her child and all of the descendant generations will also behave as (5) indicates, because they will face exactly the same problem and share the same expectations. The policy function takes the form of (5) and the expectation about her child generation's behavior is rational.

Figure 2 shows the expected lifetime utility of an individual of generation t as a function of the amount of savings S_{t+1} that she plans when young. As in Figure 1, this is the case where her parent is a immobile unskilled worker, opts not to give her a higher education ($edu_t = 0$), and chooses family public goods in the amount of $G^o_{t+1} = \tilde{G}(w_L)$ when old. Yet differently from Figure 1, the family hometown is located in middle distance from the city. Figure 2(i) and (ii) shows the case where an individual of generation t gives her child a higher education ($edu_{t+1} = e$). Figure 2(i) is the case where that individual migrates from the family hometown to the city to live with her child ($n_{t+1} = 0$). Figure 2(ii) is the case where an individual of generation t remains in her hometown, and lives apart from her child who migrates to the city ($n_{t+1} = n_t$). Figure 2(iii) shows the case where an individual of generation t opts not to give her child a higher education ($edu_{t+1} = 0$) and remains in her hometown ($n_{t+1} = n_t$).

A mobile unskilled worker of generation t's decision on the education for her child and where to live depends on which is the highest, the maximal point in Figure 2(i), the maximal point in Figure 2(ii), or the maximal point in Figure 2(iii). And an immobile worker's decision depends on which is higher, the maximal point in Figure 2(ii) or the maximal point in Figure 2(iii).

As we have discussed in the last subsection, the maximal point in Figure 2(i) is higher than that in Figure 2(ii), regardless of how far the family hometown is located from the city.

Also similarly to the case where the family hometown is close to the city, in Figure 2(iii) the maximum expected utility is attained at the amount of savings smaller than the threshold level. And the maximal point in Figure 2(i) is higher than Figure 2(iii). Yet differently from that case, the maximal point in Figure 2(ii) is lower than that in Figure 2(iii).

That is, if her parent chooses family public goods in the amount of $G^o_{t+1} = \tilde{G}(w_L)$ and if the family hometown n_t is located in middle distance from the city, a mobile individual of generation t will give her child a higher education. In contrast, an immobile unskilled worker will not give her child a higher education. She chooses savings smaller than the threshold, and thus she will not induce her child to stay in the family hometown in the case where her child will be a mobile unskilled worker.

We consider why an immobile unskilled worker will not give her child a higher education.

As we have seen in the last subsection, if she opts not to give her child a higher education and chooses larger savings when young, an individual will entice her child to stay in the family hometown by providing family public goods in a very large amount when she becomes old and her child becomes a mobile unskilled worker. Her utility when young is low, yet her child will not migrate to the city. In contrast, if she chooses smaller savings when young, she will give up inducing her child to remain in the family hometown when her child becomes a mobile unskilled worker. Her utility when young is higher, yet her child will migrate to the city with probability p. In addition, if she chooses to give her child a higher education, her child will be a mobile skilled worker and migrate to the city with probability 1.

The further away the family hometown is located from the city, the more dramatically her child's migration to the city reduces an immobile individual's lifetime utility. As we have seen in the last subsection, in the case where the family hometown is located close to the city, she will not care about her child's migration to the city. Also in the case where the hometown is located in middle distance from the city, she will not choose savings large enough to have her child in the family hometown in every case, once she opted not to give her child a higher education. However, in this case, she will not increase the probability that her child will migrate to the city from p to unity by giving her child a higher education, though her child will provide family public goods in a larger amount in the city.

4.4 Regions far away from the city

The following Proposition shows the conditions under which the policy function takes the form of (6).

Proposition 3
If her hometown region is far from the city, an individual of generation t behaves as (6) indicates. She opts not to give her child a higher education and remain in her hometown.

The formal Proof is given in Appendix 3. Specifically, the threshold value of G^* in the policy function (6) for an individual in region n satisfies the following equation:

$$\begin{aligned}&\left(\frac{2+\rho}{1+\rho}\right)u(\tilde{C}(w_L))+\left(\frac{2+\rho}{1+\rho}\right)v(\tilde{G}(w_L))+\left(\frac{1}{1+\rho}\right)v(\tilde{G}(w_L))+v(G^*)\\ &=\left(\frac{2+\rho}{1+\rho}\right)u(\tilde{C}(w_L-e))+\left(\frac{2+\rho}{1+\rho}\right)v(\tilde{G}(w_L-e))+\left(\frac{1}{1+\rho}\right)v(\tilde{G}(w_H-e))+\delta(n_t)v(G^*).\end{aligned} \tag{9}$$

As n grows, G^* that satisfies (9) decreases. That is, an individual living further from the city is more likely to continue living in her hometown. We can obtain the index for the region closest to the city among the regions where (6) is the policy function by solving the above equation, where G^* is replaced by $\tilde{G}(w_L)$ for n. The details on (9) are also analyzed in Appendix 4.

Suppose that an individual of generation t anticipates that her child will decide whether to remain in the family hometown ($n_{t+2} = n_{t+1}$ or 0), decide whether to give her grandchild (the child of her child, an individual of generation $t+2$) a higher education ($edu_{t+2} = e$ or 0), and chooses the amount of family public goods (G^o_{t+3}), based on that individual's own decisions of n_{t+1}, edu_{t+1}, and G^o_{t+2} as (6) indicates. Using figures, we confirm that she does decide them as (6) indicates, exactly in the similar manner as her child decide them, when her hometown is located far away from the city.

Figure 3 shows the expected lifetime utility of an individual of generation t as a function of S_{t+1}. As in Figure 1 and 2, this is the case where her parent is a immobile unskilled worker, opts not to give her a higher education ($edu_t = 0$), and chooses family public goods in the amount of $G^o_{t+1} = \tilde{G}(w_L)$ when old. Yet differently from these Figures, now this is the case where the family hometown is located far away from the city. Figure 3(i) and (ii) shows the case where an individual of generation t gives her child a higher education ($edu_{t+1} = e$). Figure 3(i) is the case where that individual migrates to the city ($n_{t+1} = 0$). Figure 3(ii) is the case where an individual of generation t remains in her hometown ($n_{t+1} = n_t$). Figure 3(iii) shows the case where an individual of generation t opts not to give her child a higher education ($edu_{t+1} = 0$).

Whether or not an individual of generation t lets her child receive a high education ($edu_{t+1} = 0$ or e) and whether or not she migrates to the city ($n_{t+1} = 0$ or n_t) depend on which is the highest, the maximal point in Figure 3(i), that in Figure 3(ii), or that in Figure 3(iii).

If her hometown n_t is far enough from the city, she does not let her child receive a high education, and remains in her hometown. Her child becomes an unskilled worker, and also remains in region n_t.

Again, when an individual of generation t lets her child be a skilled worker and migrate to the city with her child, she can enjoy her child's larger amount of family public goods. However, the benefits she can enjoy from her parent's family public goods become smaller, due to the distance from her hometown to the city. The longer the distance from her hometown to the city, the larger the decrease in the benefits from the given amount of family public goods by her parent in her hometown. Therefore, if her hometown locates very far from the city, she would rather remain in her hometown and attract her child in her hometown as well. The amount of family public goods by her child in this case is smaller than that in the case where her child becomes a skilled worker and lives in the city. Yet the decrease in the benefits accompanied by that is smaller than the decrease in the benefits from her parent's family public goods caused by the long distance.

We have seen that when her parent opts not to let her receive a high education ($edu_t = 0$) and chooses family public goods in the amount of $G^o_{t+1} = \tilde{G}(w_L)$, an individual of generation t will be attracted in her hometown ($n_{t+1} = n_t$), and attract her child in her hometown as well by choosing $edu_{t+1} = 0$ and $G^o_{t+2} = \tilde{G}(w_L)$ as her parent did. She expects that she can easily attract her child in her hometown by choosing $edu_{t+1} = 0$ and G^o_{t+2} just in the amount of $\tilde{G}(w_L)$ as (6) indicates. This easiness makes her be more willing to remain in her hometown. And in reality she is easily attracted in her hometown when her parent chooses $edu_t = 0$ and $G^o_{t+1} = \tilde{G}(w_L)$ as (6) indicates. That is, we have confirmed that an individual of generation t who expects that her child will behave according to (6) when she chooses $G^o_{t+2} = \tilde{G}(w_L)$ will behave exactly in the same manner as (6) indicates when her parent chooses $G^o_{t+1} = \tilde{G}(w_L)$.

We confirm that when her parent's choice of family public goods is in the amount other than $G^o_{t+1} = \tilde{G}(w_L)$, an individual of generation t who expects that her child behaves as (6) indicates also behaves as (6) indicates.

When G^o_{t+1}, the amount of family public goods chosen by her parent gets larger, an individual of generation t will behave as (6) indicates. The lifetime utility of an individual of generation t remaining in her hometown becomes progressively more likely to improve over the lifetime utility of the same individual migrating to the city. Curves in Figure 1(i) and (ii) shift upward to the same degree. Curve in Figure 1(iii) also shifts upward, but not so as the curves in Figure 1(i) and (ii) do, since the distance from the city to her hometown mitigates the increase of the benefits from her parent in her hometown. She is more eager to remain in her hometown, and opts not to let her child receive a high education. And she will still choose family public goods in the amount of $G^o_{t+1} = \tilde{G}(w_L)$.

Inversely, when G^o_{t+1} gets smaller, the benefits of remaining in her hometown become smaller. Curves in Figure Figure 1(i) and (ii) shift downward more drastically than that in Figure 1(iii), and thus the maximum point of Figure 1(iii) may get higher than that of Figure 1(i). When G^o_{t+1} is smaller than $\tilde{G}(w_L)$ but not extremely so, she may still be eager to remain in her hometown and let her child also do so. However, G^o_{t+1} gets smaller than the threshold level, she will leave her hometown in which her benefits from her parent's family public goods are smaller, let her child be a skilled worker, and migrate to the city to enjoy her child's choice of family public goods in the city.

This threshold level of G^o_{t+1} is equal to G^*. An individual of generation t who expects that her child will behave according to (6) in which G^* is equal to the threshold level will also behave according to the same (6) when she faces her parent's choice of smaller amount of G^o_{t+1}. And this expectation is rational, since her child will face the same situation as she faces, and thus change the choices depending on whether or not her choice of G^o_{t+2} is larger than the threshold G^*.

5 Conclusion and discussion

The properties of the optimal decisions on education and location are summarized in Figure 4. We discuss the extensions of the model.

Suppose an economy where there are a finite number of cities. Wages differ from city to city, but higher than w_L. In the regions nearby the cities a parent will let her child receive a high education. The size of the set of regions where a parent will do so depends on the wage in the closest city. The smaller the wage is, the smaller the size of such regions is.

Moreover, the size of such region depends not only on the wage of the closest city, but also on the distance to the city with higher wage.

The shorter the distance, the less a parent is eager to let her child receive a high education. Suppose a region in which an individual will let her child receive a high education if her child migrates to the closest city after the high education. In reality, her child will migrate to the city with higher wages, even though the distance to that city is longer than the nearby city but not extremely so. This reduces her parent's incentive to let her receive a high education. The outcome is Pareto inferior. If her parent let her child be a skilled worker and her child migrates to the closest city, they are better off.

Public investment in human capital cannot improve the situation, if its decision is made by not federal but local government. Local government hesitates public financial support for a high education for the children, being afraid of brain drain.

If the number of the cities is limited, cities should locate with sufficient intervals. In this paper I did not explicitly analyze industrial agglomeration. The model environment in this paper is akin to the new economic geography model in which the framework of von Thunen (1982) model is applied, e.g. ch 9 of Fujita, Krugman and Venables (1999). When total population size rises, sub-city appears at a location that is sufficiently far from the central city.

6 Appendix 1: The conditional lifetime utility

Suppose that her parent did not give her a higher education and thus an individual of generation t is now an unskilled worker.

The lifetime utility when an individual of generation t gives her child a higher education and migrates to the city with her child is:

$$\left(\frac{2+\rho}{1+\rho}\right) u(\tilde{C}(w_L - e)) + \left(\frac{2+\rho}{1+\rho}\right) v(\tilde{G}(w_L - e)) + \delta(n_t) v(G^o_{t+1}) + \left(\frac{1}{1+\rho}\right) v(\tilde{G}(w_H - e)). \tag{10}$$

And her lifetime utility when she remains in her family hometown is:

$$\left(\frac{2+\rho}{1+\rho}\right) u(\tilde{C}(w_L - e)) + \left(\frac{2+\rho}{1+\rho}\right) v(\tilde{G}(w_L - e)) + v(G^o_{t+1}) + \left(\frac{1}{1+\rho}\right) \delta(n_t) v(\tilde{G}(w_H - e)). \tag{11}$$

The lifetime expected utility when an individual of generation t opts not to give her child a higher education, remains in her family hometown, and chooses $G^o_{t+2} = \tilde{G}(w_L)$, in the case where she expects that her child will behave as (4) indicates and that $\tilde{G}(w_L) < G^*$ is:

$$\left(\frac{2+\rho}{1+\rho}\right) u(\tilde{C}(w_L)) + \left(\frac{2+\rho}{1+\rho}\right) v(\tilde{G}(w_L)) + v(G^o_{t+1}) + \left(\frac{1}{1+\rho}\right) (p\delta(n_t) + (1-p)) v(\tilde{G}(w_L - e)). \tag{12}$$

And the lifetime expected utility when she chooses $G^o_{t+2} = G^*$ if her child becomes a mobile unskilled worker is:

$$\begin{aligned} & u(C^y(S_{t+1}, n_t, 0)) + v(G^y(S_{t+1}, n_t, 0)) + v(G^o_{t+1}) \\ & + \left(\frac{1}{1+\rho}\right) \Big[p\,[u(S_{t+1} - G^*) + v(G^*)] \\ & \qquad + (1-p)\,[u(C^o(S_{t+1}, n_t, 0, 0)) + v(G^o(S_{t+1}, n_t, 0, 0))] \\ & \qquad\qquad + v(\tilde{G}(w_L - e)) \Big]. \end{aligned} \tag{13}$$

She maximizes (13) with respect to S_{t+1}. If maximized (13) is larger than (12), she saves in the amount of that S_{t+1} when young and she chooses $G^o_{t+2} = G^*$ when she is old and her child becomes a mobile worker.

The lifetime expected utility when an individual of generation t opts not to give her child a higher education in the case where she expects that her child will behave as (5) indicates and that $\tilde{G}(w_L) < G^*$ is:

$$\begin{aligned} & \left(\frac{2+\rho}{1+\rho}\right) u(\tilde{C}(w_L)) + \left(\frac{2+\rho}{1+\rho}\right) v(\tilde{G}(w_L)) + v(G^o_{t+1}) \\ & + \left(\frac{1}{1+\rho}\right) \left[p\delta(n_t) v(\tilde{G}(w_L - e)) + (1-p) v(\tilde{G}(w_L)) \right]. \end{aligned} \tag{14}$$

And the lifetime expected utility when she chooses $G^o_{t+2} = G^*$ if her child becomes a mobile unskilled worker is:

$$\begin{aligned}
&u(C^y(S_{t+1}, n_t, 0)) + v(G^y(S_{t+1}, n_t, 0)) + v(G^o_{t+1}) \\
&+ \left(\frac{1}{1+\rho}\right) \Big[p\left[u(S_{t+1} - G^*) + v(G^*)\right] \\
&\qquad + (1-p)\left[u(C^o(S_{t+1}, n_t, 0, 0)) + v(G^o(S_{t+1}, n_t, 0, 0))\right] \\
&\qquad\qquad + v(\tilde{G}(w_L)) \Big].
\end{aligned} \tag{15}$$

If maximized (15) with respect to S_{t+1} is larger than (14), she will choose $G^o_{t+2} = G^*$ when she is old and her child becomes a mobile worker.

The lifetime expected utility when an individual of generation t opts not to give her child a higher education in the case where she expects that her child will behave as (5) indicates and that $\tilde{G}(w_L) < G^*$ is:

$$\begin{aligned}
&\left(\frac{2+\rho}{1+\rho}\right) u(\tilde{C}(w_L)) + \left(\frac{2+\rho}{1+\rho}\right) v(\tilde{G}(w_L)) + v(G^o_{t+1}) \\
&+ \left(\frac{1}{1+\rho}\right) \left[p\delta(n_t) v(\tilde{G}(w_L - e)) + (1-p) v(G^y(S_{t+2}, n_t, 0)) \right].
\end{aligned} \tag{16}$$

And the lifetime expected utility when she chooses $G^o_{t+2} = G^*$ if her child becomes a mobile unskilled worker is:

$$\begin{aligned}
&u(C^y(S_{t+1}, n_t, 0)) + v(G^y(S_{t+1}, n_t, 0)) + v(G^o_{t+1}) \\
&+ \left(\frac{1}{1+\rho}\right) \Big[p\left[u(S_{t+1} - G^*) + v(G^*)\right] \\
&\qquad + (1-p)\left[u(C^o(S_{t+1}, n_t, 0, 0)) + v(G^o(S_{t+1}, n_t, 0, 0))\right] \\
&\qquad\qquad + v(G^y(S_{t+2}, n_t, 0)) \Big].
\end{aligned} \tag{17}$$

Again, If maximized (17) with respect to S_{t+1} is larger than (16), she will choose $G^o_{t+2} = G^{**}$ when she is old and her child becomes a mobile worker.

The lifetime expected utility when an individual of generation t opts not to give her child a higher education, remains in her family hometown, and chooses $G^o_{t+2} = \tilde{G}(w_L)$, in the case where she expects that her child will behave as (6) indicates and that she can induce her child to remain in the hometown with $G^o_{t+2} = \tilde{G}(w_L)$ even if her child becomes a mobile unskilled worker ($\tilde{G}(w_L) \geq G^*$) is:

$$\left(\frac{2+\rho}{1+\rho}\right) u(\tilde{C}(w_L)) + \left(\frac{2+\rho}{1+\rho}\right) v(\tilde{G}(w_L)) + v(G^o_{t+1}) + \left(\frac{1}{1+\rho}\right) v(\tilde{G}(w_L)). \tag{18}$$

The following Lemmas summarize the properties about the conditional lifetime utility levels.

Lemma 1

(i) The gap between (11) and (10) is independent of n.

(ii) The larger G^o_{t+1} is, the more likely (11) is larger than (10).

Lemma 2

(i) The smaller n is, the more likely (11) is larger than (12)-(18).

(ii) The gaps between (11) and (12)-(18) are independent of G^o_{t+1}.

Lemma 3

(i) The smaller n is, the more likely (10) is larger than (12)-(18).

(ii) The larger G^o_{t+1} is, the more likely (10) is larger than (12)-(18).

Lemma 4

(i) The smaller n is, (12) is larger than (13), (14) is larger than (15), and (16) is larger than (17).

(ii) The gaps between (12) and (13), between (14) and (15), and between (16) and (17) are independent of G^o_{t+1}.

(ii) The larger G^* is, the maximized values of (13), (15), and (17) are smaller. And the value of S_{t+1} that maximize these functions is larger.

7 Appendix 2: Proof of Proposition 1

Suppose that an individual of generation t expects that her child will behave as (4), where G^* equalizes (11) and (10), indicates. I show that in a region where (11) in which $G^o_{t+1} = G^*$ is equal to or larger than (12) and (13) in which $G^o_{t+1} = G^*$, an individual of generation t reacts to her parent's choice of G^o_{t+1} exactly in the same manner as her child reacts to her own choice of G^o_{t+1} as (4) indicates. Then, in that region, her expectation is rational and (4) is the policy function. And I show that this is so in regions that are very close to the city, and that G^* is larger than $\tilde{G}(w_L)$.

From Lemma 2(ii), the gaps between (11) and (12) and between (11) and (13) are independent of G^o_{t+1}. Therefore, in a region where (11) is equal to or larger than (12) at $G^o_{t+1} = G^*$, that is so for any G^o_{t+1}.

From Lemma 1(ii), the larger G^o_{t+1} is, the more likely the lifetime utility when she gives her child a higher education and migrates to the city (10) is larger than that when she remains in her hometown (11).

Therefore, if $G^o_{t+1} \leq G^*$, an individual of generation t gives her child a higher education and migrates to the city. And if $G^o_{t+1} > G^*$, she gives her child a higher education and remains in her hometown. Hence, an individual of generation t in a region where (11) is equal to or larger than (12) reacts to her parent's choice of G^o_{t+1} as (4) indicates. From (11) and (10), G^* is calculated as (7).

From Lemma 2(i), the smaller n is, the more likely (11) is equal to or larger than (12). More specifically, from (11) and (12), if n is smaller than that satisfies (8), this condition holds.

8 Appendix 3: Proof of Proposition 3

Suppose that an individual of generation t expects that her child will behave as (6), where G^* equalizes (10) and (18), indicates. I show that in a region where (18) is equal to or larger than (10) at $G^o_{t+1} = \tilde{G}(w_L)$, an individual of generation t reacts to her parent's choice of G^o_{t+1} exactly in the same manner as her child reacts to her own choice of G^o_{t+1} as (6) indicates. If so, in that region her expectation is rational and (6) is the policy function. And I show that this is so in regions that are far away from the city.

At $G^o_{t+1} = \tilde{G}(w_L)$, (10) is larger than (11). Therefore, if (18) is equal to or larger than (10) at $G^o_{t+1} = \tilde{G}(w_L)$, (18) is larger than (11) at that point. From Lemma 2(ii), the gap between (18) and (11) is independent of G^o_{t+1}. Therefore, (18) is larger than (11) for any G^o_{t+1}.

From Lemma 3(ii), the larger G^o_{t+1} is, the more likely the lifetime utility when she opts not to let her child receive a high education an remains in her hometown (18) is larger than that when she lets her child receive a high education and migrate to the city (10).

Therefore, if (18) is equal to or larger than (10) at $G^o_{t+1} = \tilde{G}(w_L)$, an individual of generation t opts not to let her child receive a high education an remains in her hometown for any $G^o_{t+1} > \tilde{G}(w_L)$. And there exists $G^o_{t+1} = G^*$ which is smaller than $\tilde{G}(w_L)$ and equalize (18) and (10). If $G^o_{t+1} \leq G^*$, an individual of generation t lets her child receive a high education. Hence, an individual of generation t reacts to her parent's choice of G^o_{t+1} as (6) indicates. From (18) and (10), G^* is calculated as (9).

From Lemma 3(i), the smaller n is, the more likely (18) is equal to or larger than (10). More specifically, from (10) and (18), if n is larger than that satisfies (9), this condition holds.

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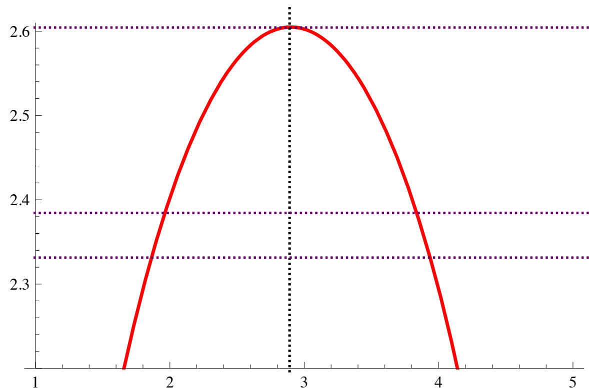
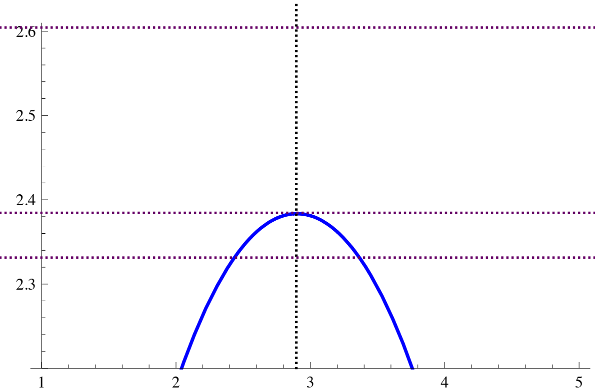


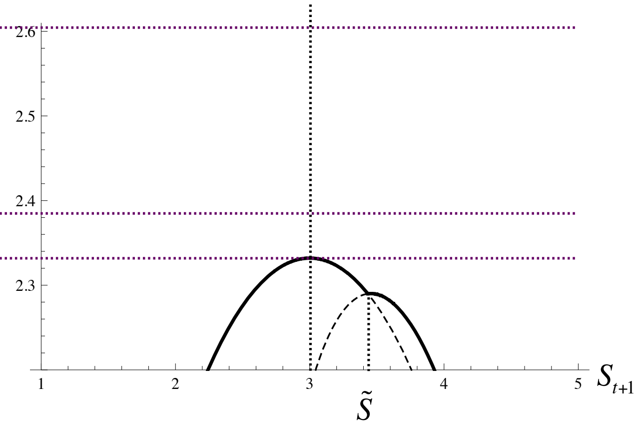
$edu_{t+1} = e$
$n_{t+1} = 0$

Figure 1 (i)



$edu_{t+1} = e$
$n_{t+1} = n_t$

Figure 1 (ii)



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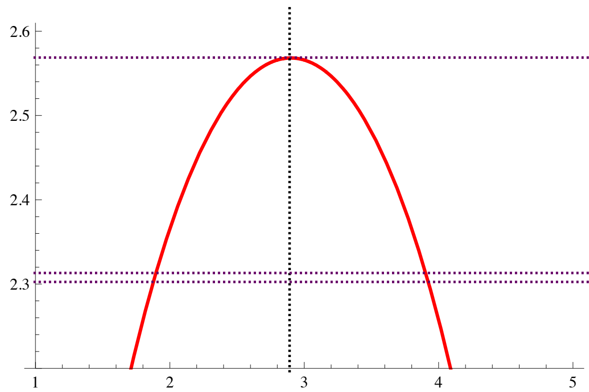

$edu_{t+1} = e$
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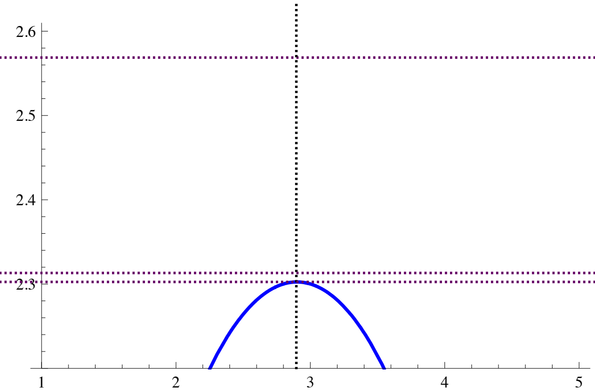
Figure 2 (i)



$edu_{t+1} = e$
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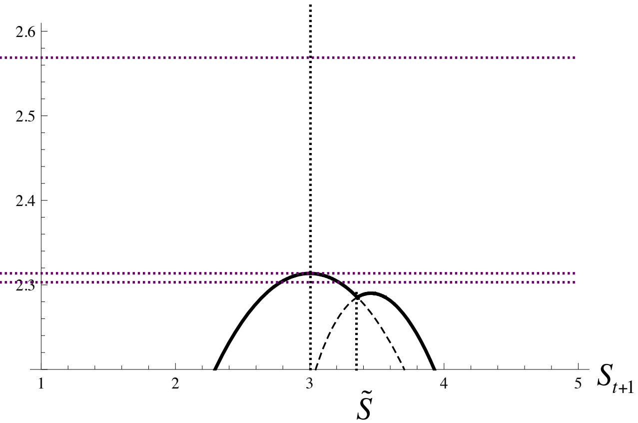
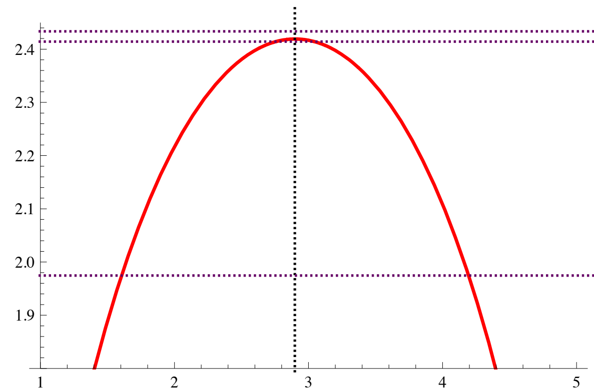
Figure 2 (ii)



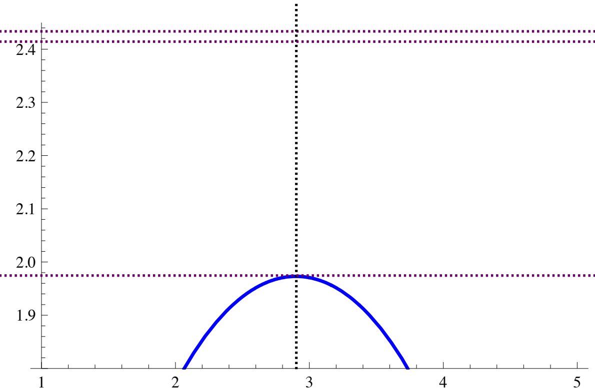
$edu_{t+1} = 0$
$n_{t+1} = n_t$

Figure 2 (iii)



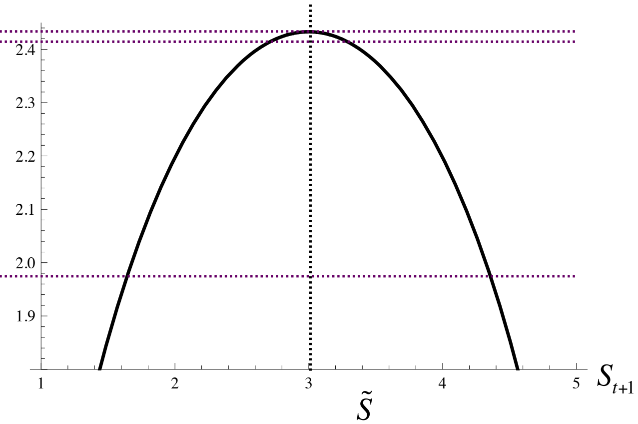
$edu_{t+1} = e$
$n_{t+1} = 0$

Figure 3 (i)



$edu_{t+1} = e$
$n_{t+1} = n_t$

Figure 3 (ii)



$edu_{t+1} = 0$
$n_{t+1} = n_t$

Figure 3 (iii)

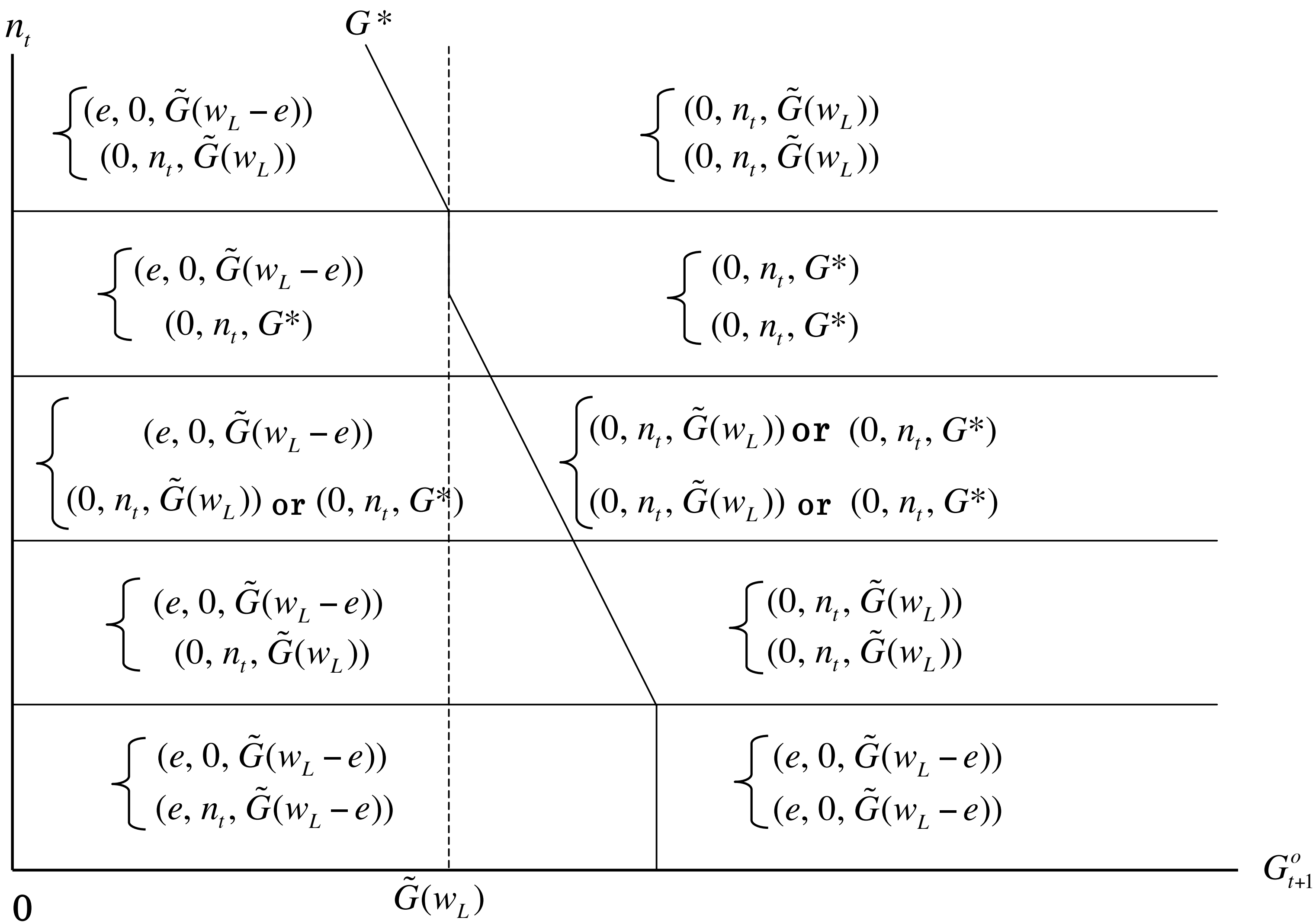


Figure 4 Policy function of individual of generation *t* (small *p*)

$\begin{cases} \text{If she is a mobile unskilled worker} & (edu_{t+1}, n_{t+1}, G^O_{t+2}) \\ \text{If she is an immobile unskilled worker} & (edu_{t+1}, n_{t+1}, G^O_{t+2}) \end{cases}$

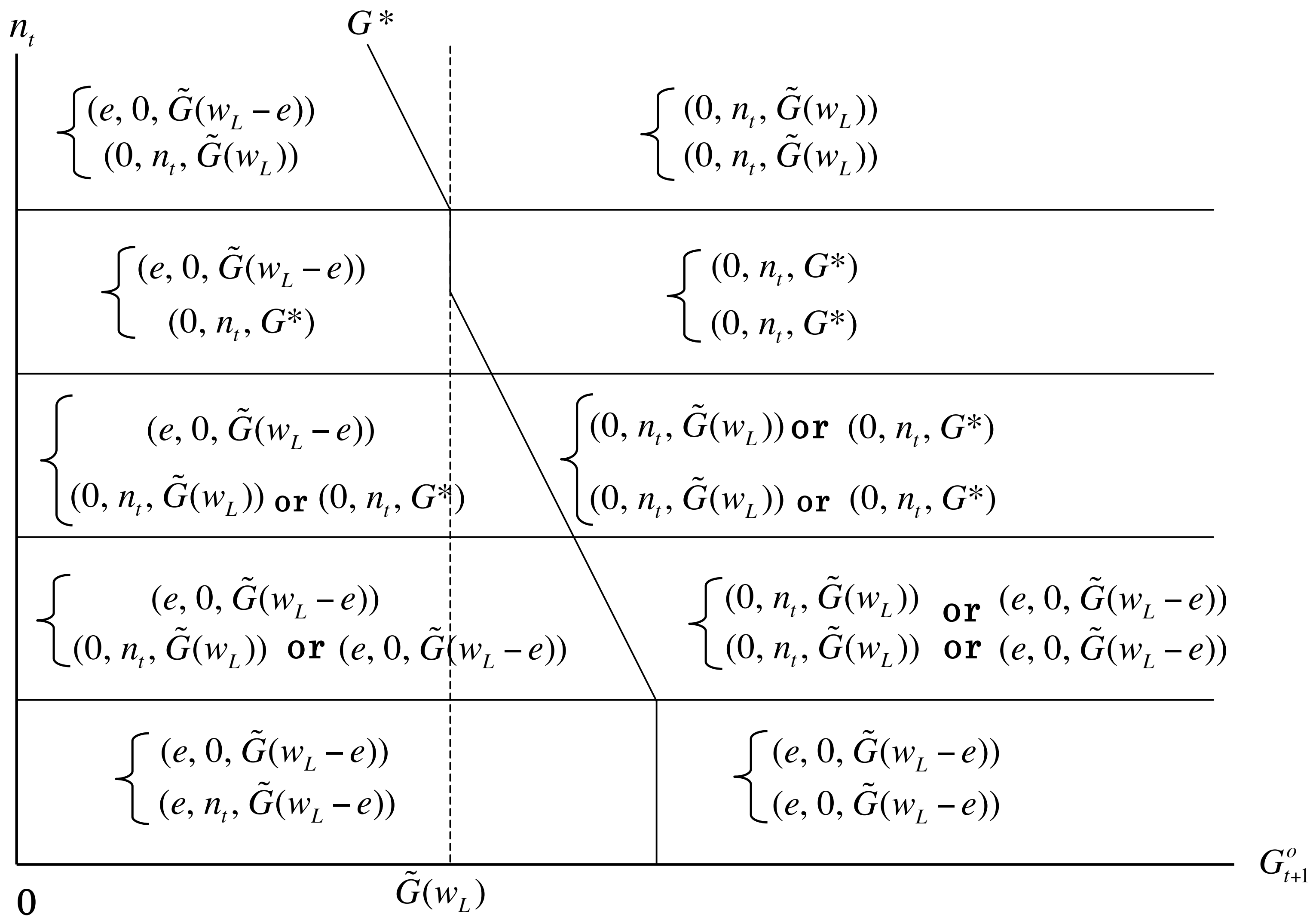


Figure 4 Policy function of individual of generation *t* (large *p*)

If she is a mobile unskilled worker $(edu_{t+1}, n_{t+1}, G^O_{t+2})$
If she is an immobile unskilled worker $(edu_{t+1}, n_{t+1}, G^O_{t+2})$