Governmental Provision of Public Goods Need Not Crowd Out Private Provision

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Abstract

Consider the private provision of a public good, where consumption of the public good requires an individual to spend time. We show that governmental provision of the public good financed by a labor tax reduces the incentive to work, increases the time available to an individual to consume the public good, and so increases the marginal utility to the individual of the public good. That in turn means that, in contrast to standard models, governmental provision need not fully crowd out private provision. Instead, increased governmental provision can lead to an increase in the sum of private and governmental provision.

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1 Introduction

Standard models of private provision of a public good have several interesting implications. Redistribution between contributors does not affect spending on private goods and does not affect aggregate provision of the public good. This neutrality also implies that government action, particularly governmental contributions to the public good, are fully offset by reductions of private agents' contributions (Warr 1982, 1983; Bernheim 1986; Bergstrom et al. 1986). This crowding out property has received much attention, especially because the perfect crowding-out result turned out to be robust and has strong policy implications regarding the effects of government raising taxes to provide the public good.

The intuition for neutrality is as follows. Suppose a consumer with pre-tax income of \$51,000 paid \$1,000 in taxes, and contributed \$300 to a public television station. He then has \$50,000-\$1,000-\$300 = \$48,700 to spend on other goods. Let there be one thousand similar consumers, so that total receipts of the television station are \$300,000. Now suppose that government increases per capita taxes by \$100, and gives the additional revenue to the television station. If the consumer (and others like him) continued to give \$300 to the television station, then the station's budget would be \$400,000, while each person would spend only \$48,600 on other goods. Compared to the initial situation, consumption of public television is higher and consumption of other goods is lower. Consumers therefore have an incentive to reduce their contributions and thereby increase their consumption of other goods. Indeed, they can completely offset the government's actions. Each can reduce his contribution from \$300 to \$200. The television station will have \$300,000 in revenue, as before, and each consumer will spend \$50,000-\$1,100-\$200 = \$48,700 on other goods.

The full crowding out result, however, is not verified by empirical observations. A study of three hundred British charities found no significant evidence that public donations crowded out private donations (Posnett and Sandler 1989); other studies in the U.S. find that crowding out is only partial (Steinberg 1989), with an additional governmental dollar spent on charity crowding out only 28 cents (Abrams and Schmitz 1978).

Such evidence led some theorists to suppose that donations enter directly into a person's utility function (see, e.g., Andreoni 1989, 1990; Cornes and Sandler 1984, 1994; Kingma 1989; McClelland 1989; Roberts 1987; Sandler and Posnett 1991; Steinberg 1986, 1987). In this warm-glow approach, the act of giving directly enters the utility function (Arrow 1972). Crowding out will also be reduced if provision of the public good involves discontinuities or if tax subsidies for contributions are discontinuous (see Glazer and Konrad 1993).

Moreover, and relatedly, much of the literature supposes consumers are passive in consuming the public good, consuming the quantity that is produced. But the characteristic of a public good is that a consumer *can* consume the amount produced, not that he *must* consume it: a person is not required to use the light from a lighthouse, or go to a clean beach, or use a technological innovation.

This paper takes a different approach, considering behavior when consumption of the public good requires an individual to spend time on it, at the cost of spending time on working and on consuming a private good. A tax on labor which is levied to finance governmental provision reduces the time an individual spends working, increases the time he can spend on enjoying the public good, and so makes increased provision of the public good more attractive. The greater are aggregate contributions to the public good, the more attractive is consumption of the public good, and so the more time spent consuming it. That in turn can induce individuals to favor an increase in aggregate contributions to the public good.

In the standard model of private provision of a public good, a tax on labor income reduces welfare. A notable feature of our analysis is finding that a positive tax on labor income can maximize economic welfare.

2 Literature

Some earlier works consider multiple inputs in the production of a public good. Ihori (1996) and Konrad and Lommerud (1995) consider two agents with different physical productivities of transforming units of private goods into units of the public good. They find that a redistribution from agents with low productivity to agents with high productivity yields a Pareto-improvement. All agents who—in the equilibrium without redistribution—were not in a corner solution consumer more of the private good and the public good.

Natvik (2013) analyzes governmental production, which requires both labor and capital. An increase in the level of capital provided in period 1 has opposing effects on production choices in period 2: the increased capital increases the marginal productivity of labor, thereby inducing increased production; but the increased capital reduces the marginal utility of consuming the good, inducing reduced allocation of labor to producing the good.

Much of our analysis uses standard analyzes of international trade. Our analysis considers two goods (a private good and a public good) and two inputs (time and money) used to consume the two goods. As in analyses of trade, we shall see that the results depend on which good is relatively intensive in which factor of production.

3 Assumptions

Consider an economy with N identical individuals, each consuming a private good and a public good. The amount of the private good a person consumes increases with his spending on the private good (his money input to the private good), and on the time he spends on consuming that good (his time input to the private good). The amount of the public good a person consumes increases with the aggregate amount spent on the private good (total money input to the public good), and on the amount of time he spends consuming that good (his time input to the public good). Thus, money spent on the public good benefits all consumers. Time spent on the public good and on the private good benefits only the person spending that time.

Call X_{iG} individual i's money input to the public good; X_{iC} is his money input to the private good. Governmental money input to the public good is X_{SG}. The time individual i spends on consuming the public good is T_{iG}; the time he spends on consuming the private good (which can be interpreted as leisure) is T_{iC}. Suppose each person's utility function has the constant elasticity of substitution (CES) form:

$$\begin{aligned} U_i = \Bigg[a \Bigg[\gamma_G \Big(X_{iG} + \sum_{j=i} X_{jG} + X_{SG} \Big)^{\eta_G} + (1-\gamma_G) T_{iG}^{\eta_G} \Bigg]^{\frac{\sigma}{\eta_G}} \\ + (1-a) \Bigg[\gamma_C {X_{iC}}^{\eta_C} + (1-\gamma_C) {T_{iC}}^{\eta_C} \Bigg]^{\frac{\sigma}{\eta_C}} \Bigg]^{\frac{1}{\sigma}} . \end{aligned} \tag{1}$$

We suppose that the parameters of the utility function in (1) satisfy $-\infty \leq \eta_G \leq 1$, $-\infty \leq \eta_C \leq 1$, and $-\infty \leq \sigma \leq 1$. The marginal rate of substitution (MRS) between the total money input and the time input to the public good is $1 - 1/\eta_G$. The corresponding marginal rate of substitution for the private good is $1 - 1/\eta_C$. Among them, η_G is the smallest: $\eta_G < \eta_C$ and $\eta_G < \sigma$, reflecting our assumption that consuming the public good requires an individual to spend time on it.

In particular, when $\eta_G \to -\infty$, the time input to the public good is a perfect complement to the money input. We suppose that the time input to the public good shows greater complementarity to the money input than does the time input to the private good. That is, $\eta_G < \eta_C$.[1] Also in (1), $0 \leq \gamma_G \leq 1$. The value of γ_G represents how intensively aggregate spending on the public good affects utility from consuming the good. Similarly, $0 \leq \gamma_C \leq 1$.

For notational convenience, let q_{iG} denote the consumption of the public

[1] In the utility function (1), the time input T_{iC} can increase the marginal utility of the money input to the public good X_{iG}. The effect, however, is much smaller than the time T_{iG}. Therefore, $\eta_G < \sigma$.

good by individual i:

$$q_{iG} = \left[\gamma_G \left(X_{iG} + \sum_{j=i} X_{jG} + X_{SG} \right)^{\eta_G} + (1-\gamma_G) T_{iG}^{\eta_G} \right]^{\frac{1}{\eta_G}}. \qquad (2)$$

Similarly, q_{iC} denotes the part of the utility function relating to the private good:

$$q_{iC} = \left[\gamma_C X_{iC}^{\eta_C} + (1-\gamma_C) T_{iC}^{\eta_C} \right]^{\frac{1}{\eta_C}}. \qquad (3)$$

Use (2) and (3) to rewrite expression (1) as

$$U_i = [a q_{iG}^{\sigma} + (1-a) q_{iC}^{\sigma}]^{\frac{1}{\sigma}}. \qquad (4)$$

The marginal rate of substitution between q_{iG} and q_{iC} is $1 - 1/\sigma$; the weight placed on consumption of the public good is indicated by a.

Each individual is endowed with a fixed amount of time H, which he can use to earn income (L_i), to consume the public good (T_{iC}), and to consume the private good (T_{iC}):

$$H = L_i + T_{iC} + T_{iG}, \qquad (5)$$

Labor income is taxed. The rest of disposable income is spent on the private good and on the public good:

$$(1-\tau) w L_i = X_{iC} + X_{iG}, \qquad (6)$$

where τ is the tax rate on labor income, and w is the wage rate. Eliminating L_i yields

$$H = \left(\frac{1}{(1-\tau)w} \right) X_{iC} + \left(\frac{1}{(1-\tau)w} \right) X_{iG} + T_{iC} + T_{iG}. \qquad (7)$$

Governmental money input to the public good is financed by the tax on labor:

$$X_{SG} = N \tau w L_i. \qquad (8)$$

4 Spending in equilibrium

Individual i maximizes (1) subject to the constraint (7). Unless the money input and the time input to the public good are perfect substitutes ($\eta_G = 1$), total spending (by government and individuals) on the money input to the public good is positive; time spent on consuming the public good is also positive.

Though the aggregate money input to the public good is positive, an individual may contribute nothing. That is, an individual may enjoy consuming the public good without any money input to it: $T_{iG} > 0$ but $X_{iG} = 0$. We concentrate on outcomes when each individual's money input to the public good is positive.

An individual's optimal choice is

$$X_{iG} = \frac{\gamma_G^{\frac{1}{1-\eta_G}}}{P_G^{\frac{-\eta_G}{1-\eta_G}}} \frac{a^{\frac{1}{1-\sigma}} P_G^{\frac{-\sigma}{1-\sigma}}}{P^{\frac{-\sigma}{1-\sigma}}} \left[(1-\tau)wH + \sum_{j\neq i} X_{jG} + X_{SG}\right] - \left(\sum_{j\neq i} X_{jG} + X_{SG}\right) \tag{9}$$

$$T_{iG} = \frac{(1-\gamma_G)^{\frac{1}{1-\eta_G}} ((1-\tau)w)^{\frac{-1}{1-\eta_G}}}{P_G^{\frac{-\eta_G}{1-\eta_G}}} \frac{a^{\frac{1}{1-\sigma}} P_G^{\frac{-\sigma}{1-\sigma}}}{P^{\frac{-\sigma}{1-\sigma}}} \left[(1-\tau)wH + \sum_{j\neq i} X_{jG} + X_{SG}\right] \tag{10}$$

$$X_{iC} = \frac{\gamma_C^{\frac{1}{1-\eta_C}}}{P_C^{\frac{-\eta_C}{1-\eta_C}}} \frac{(1-a)^{\frac{1}{1-\sigma}} P_C^{\frac{-\sigma}{1-\sigma}}}{P^{\frac{-\sigma}{1-\sigma}}} \left[(1-\tau)wH + \sum_{j\neq i} X_{jG} + X_{SG}\right] \tag{11}$$

$$T_{iC} = \frac{(1-\gamma_C)^{\frac{1}{1-\eta_C}} ((1-\tau)w)^{\frac{-1}{1-\eta_C}}}{P_C^{\frac{-\eta_C}{1-\eta_C}}} \frac{(1-a)^{\frac{1}{1-\sigma}} P_C^{\frac{-\sigma}{1-\sigma}}}{P^{\frac{-\sigma}{1-\sigma}}} \left[(1-\tau)wH + \sum_{j\neq i} X_{jG} + X_{SG}\right] \tag{12}$$

See Appendix A.1 for the derivation. And see Appendix A.2 for analysis of outcomes when an individual's money input to the public good is zero. In these equations, P_G is the minimum cost of a unit consumption of public good q_G:

$$P_G = \left[\gamma_G^{\frac{1}{1-\eta_G}} + (1-\gamma_G)^{\frac{1}{1-\eta_G}} ((1-\tau)w)^{\frac{-\eta_G}{1-\eta_G}}\right]^{\frac{1-\eta_G}{-\eta_G}}. \tag{13}$$

Similarly, P_C is the minimum cost of a unit of q_C; it is a weighted sum of the prices of a good and time.

$$P_C = \left[\gamma_C^{\frac{1}{1-\eta_C}} + (1-\gamma_C)^{\frac{1}{1-\eta_C}} ((1-\tau)w)^{\frac{-\eta_C}{1-\eta_C}}\right]^{\frac{1-\eta_C}{-\eta_C}}. \tag{14}$$

Lastly, P is the minimum cost to enjoy a unit of consumption. This cost is the weighted sum of P_G, the price of q_G, and P_C, the price of q_C.

$$P = \left[a^{\frac{1}{1-\sigma}} P_G^{\frac{-\sigma}{1-\sigma}} + (1-a)^{\frac{1}{1-\sigma}} P_C^{\frac{-\sigma}{1-\sigma}}\right]^{\frac{1-\sigma}{-\sigma}}. \tag{15}$$

In (9)-(12) and (18), the term $\left[(1-\tau)wH + \sum_{j=i} X_{jG} + X_{SG}\right]$ represents the endowment, or the value of the fixed amount of time resource plus the money inputs to the public good by other individuals and the government. A person may provide both a money input and a time input to the public good. As (9) and (10) indicate, the inputs a person chooses depend on the parameter γ_G and on the time cost $(1-\tau)w$.[2]

[2] More precisely, the choice depends on the time the individual works (9) divided by $(1-\tau)w$ to earn the money to buy the public good in the amount (9), and spends the time (10) to consume it.

The money inputs to the public good by other individuals affect the optimal choice by an individual, especially the choice of his money input to it, as (9) indicates. As individuals have identical preferences, time endowments, and wage rates, in equilibrium, each chooses the same money input to the public good. We use the notation

$$\Phi = \frac{\gamma_G^{\frac{1}{1-\eta_G}}}{P_G^{\frac{-\eta_G}{1-\eta_G}}} \frac{a^{\frac{1}{1-\sigma}} P_G^{\frac{-\sigma}{1-\sigma}}}{P^{\frac{-\sigma}{1-\sigma}}},$$

and

$$\Omega = \frac{\gamma_C^{\frac{1}{1-\eta_C}}}{P_C^{\frac{-\eta_C}{1-\eta_C}}} \frac{(1-a)^{\frac{1}{1-\sigma}} P_C^{\frac{-\sigma}{1-\sigma}}}{P^{\frac{-\sigma}{1-\sigma}}}.$$

Let a superscript e indicate a value in equilibrium. Then from (9), in equilibrium an individual's money input to the public good is

$$X_{iG}^e = \frac{\Phi(1-\tau)wH + (\Phi-1)X_{SG}^e}{N-(N-1)\Phi}, \tag{16}$$

which is a function of the tax rate on labor income, τ. Governmental money input to the public good, X_{SG}, is also a function of the tax rate on labor income.[3]

$$X_{SG}^e = \frac{N\tau(1-\tau)(N\Omega+\Phi)wH}{N+\Phi(1-N-\tau)-N\tau\Omega}. \tag{17}$$

With (16) and (17), the maximized utility is

$$U = (1/P)\left[(1-\tau)wH + (N-1)X_{iG}^e + X_{SG}^e\right] \tag{18}$$

Applying Roy's Identity to (18) yields the condition under which a marginal increase in the tax rate (τ) on labor increases the maximized utility:

$$-\left(\frac{1}{1-\tau}\right)(X_{iG}^e + X_{iC}^e) + \left[(N-1)\frac{dX_{iG}^e}{d\tau} + \frac{dX_{SG}^e}{d\tau}\right] \geq 0, \tag{19}$$

where X_{iC}^e is the money input to the private good in equilibrium. See Appendix A.3 for the derivation.

[3] Solving (9)-(12) we obtain the money input and time input to the public good, and the money input and time input to the private good in equilibrium. They are functions of the tax rate on labor income τ and the governmental money input to the public good X_{SG}, for example as in (16). From the money inputs to the public good and private good and the money constraint (6), or from the time inputs to the public good and private good and the time constraint (5), we obtain the endogenously determined time spent earning income L_i in equilibrium. Using this and (8) yields the governmental money input to the public good in equilibrium, which is simultaneously determined with the other variables, under a given tax rate τ.

5 Crowding out with governmental provision of the public good

We now ask whether governmental provision of money input to the public good fully crowds out private provision. And we ask whether governmental provision can increase welfare.

5.1 Lump-sum tax

Suppose the tax imposed on each individual equals the per capita level of the governmental money input. That is, suppose the number of initial contributors is N, that government provides a money input to the public good of NF, and imposes a lump-sum tax of F on each of these N persons. Then there exists an equilibrium in which each consumer neutralizes the government policy—each reduces his money input to the public good by F, so that consumption of the private good and consumption of the public good are unchanged.[4]

5.2 Income tax

More interesting results appear when government finances its spending on the public good not by a lump-sum tax, but by an income tax. Increased governmental provision then requires a higher tax on labor, which in turn reduces labor supply.

Analyzing the effects of a change in the tax rate on labor income under the general setting is complicated. So we analyze outcomes when $\sigma = 0$, or when utility (4) is a Cobb-Douglas function of q_{iG} (consumption of the public good), and of q_{iC} (consumption of the private good):

$$q_{iG}^{a} q_{iC}^{1-a}. \tag{20}$$

Let the function (3) also have the Cobb-Douglas form ($\eta_G = 0$):

$$q_{iC} = X_{iC}^{\gamma_C} T_{iC}^{1-\gamma_C}. \tag{21}$$

Let utility from consuming the public good, q_{iG}, follow a Leontief function of money and time ($\eta_G \to -\infty$):

$$min\{X_{iG} + \sum_{j=i} X_{jG} + X_{SG}, T_{iG}\}. \tag{22}$$

The simplification lets us analytically determine the effects of governmental provision.

[4]Nevertheless, if there exist multiple equilibria in the absence of the governmental grant, then a governmental grant can induce a move from an equilibrium with no provision of the public good to an equilibrium with positive provision.

Under the simplifications using (20)-(22), an individual's utility-maximizing choice (9)-(12) becomes

$$X_{iG} = (1+(1-\tau)w)^{-1}a\left[(1-\tau)wH + \sum_{j=i} X_{jG} + X_{SG}\right] - \left(\sum_{j=i} X_{jG} + X_{SG}\right) \tag{23}$$

$$T_{iG} = (1+(1-\tau)w)^{-1}a\left[(1-\tau)wH + \sum_{j=i} X_{jG} + X_{SG}\right] \tag{24}$$

$$X_{iC} = \gamma_C(1-a)\left[(1-\tau)wH + \sum_{j=i} X_{jG} + X_{SG}\right] \tag{25}$$

$$T_{iC} = \left[\frac{(1-\gamma_C)(1-a)}{(1-\tau)w}\right]\left[(1-\tau)wH + \sum_{j=i} X_{jG} + X_{SG}\right] \tag{26}$$

In equilibrium the money input to the public good by an individual simplifies to:

$$X^e_{iG} = \frac{a(1-\tau)wH + (a-(1+(1-\tau)w))X^e_{SG}}{(1+(1-\tau)w)N - (N-1)a}. \tag{27}$$

The government's money input to the public good X_{SG} is determined as a function of the tax rate on labor income:

$$X^e_{SG} = \frac{N\tau(1-\tau)(a+(N\gamma_C(1-a)(1+(1-\tau)w)))wH}{N(1+(1-\tau)w) + a(1-N-\tau) - N\tau\gamma_C(1-a)(1+(1-\tau)w)}. \tag{28}$$

Proposition 1
Suppose that consumption of the private good is a Cobb-Douglas function of the money and time inputs, that consumption of the public good is a Leontief function of the money and time inputs, and that an individual's total utility is a Cobb-Douglas function of consumption of the private and public goods ($\sigma = \eta_C = 0$ and $\eta_G \to -\infty$). Let the wage rate w, the parameter γ_C (which represents the intensity of the money input in consuming the private good), and the parameter a (which represents the preference for the public good) satisfy $0 < a < 1$ and $\gamma_C \geq 1/(1+w)$. Then a small increase in the tax rate on labor from zero increases total money input to the public good and increases an individual's maximized utility.

Proof See Appendix B.

We cannot determine explicit general conditions that make an increase in the tax increase the total money input to the public good. But results from numerical solutions show that the larger is η_G, the smaller the range of other parameters for which an increase in the tax increases the total money input to the public good Numerical solutions also find that the effect appears when γ_C is large and γ_G is small.

An increase in the tax rate on labor income reduces the time cost of consuming private and public goods. Therefore, an increase in the tax causes an individual to increase the time input to consume the private good and the public good, to reduce the time spent on labor. The higher the wage rate w, the larger is the effect. If consumption of the public good is time intensive and consumption of the private good is money intensive (γ_G is small and γ_C is large), the time spent on consuming the public good increases more drastically. Moreover, if the time input and the money input are complementary in consuming the public good (η_G is very small), the increase in the time input is accompanied by an increase in the total money input to the public good. These effects can cause individuals to reduce their money inputs to the public good by less than the increase in the governmental input to the public good. Crowding out will then be incomplete.

More generally and formally, an increase in the tax rate on labor reduces $(1-\tau)w$, the cost of time to consume goods. The more intensive is consumption of the public good in the time input and the less intensive is consumption of the private good in the time input (the smaller is γ_G and the larger is γ_C), the smaller is the cost (P_G) of a unit consumption of the public good compared to the unit cost (P_C) of the private good, as (13) and (14) indicate. Therefore, the more likely is consumption of the public good and the required time input to increase rather than consumption of the private good to increase. (See the term $\frac{a^{\frac{1}{1-\sigma}} P_G^{\frac{-\sigma}{1-\sigma}}}{P^{\frac{-\sigma}{1-\sigma}}}$ in (9) and (10) and the term $\frac{(1-a)^{\frac{1}{1-\sigma}} P_C^{\frac{-\sigma}{1-\sigma}}}{P^{\frac{-\sigma}{1-\sigma}}}$ in (11) and (12)).

In contrast to the time input to the public good, whether the money input increases is ambiguous. When the money input and the time input to the public good are complementary (η_G is very small), an increase in the tax rate (τ) on labor income can increase $\frac{\gamma_G^{\frac{1}{1-\eta_G}}}{P_G^{\frac{-\eta_G}{1-\eta_G}}}$ in (9), and can also increase the term $\frac{(1-\gamma_G)^{\frac{1}{1-\eta_G}}((1-\tau)w)^{\frac{-1}{1-\eta_G}}}{P_G^{\frac{-\eta_G}{1-\eta_G}}}$ in (10). In particular, when the time input is a perfect complement to the money input in consuming the public good ($\eta_G \to -\infty$), both of these terms become $1/(1+(1-\tau)w)$, as in (23) and (24). That is an increasing function of τ. An increase in the tax rate τ and an increase in the governmental money input to the public good X_{SG} reduces an individual's money input to the public good (9), but does not fully offset the increased governmental money input. Thus, the total money input to the public good, (9) plus $\sum_{j\neq i} X_{jG} + X_{SG}$, may increase.

In the standard model with a labor-leisure choice, a tax on labor income distorts the choices made by individuals, thereby reducing welfare. Our model, because it considers the time required to enjoy the public good, finds that a tax on labor income can improve welfare.

Under conditions which make a tax on labor income improve welfare, the patterns of how further increases in the tax rate change welfare are classified into the following three cases. Let $\bar{\tau}$ denote the tax rate above which individuals do not provide public good.

- Case (i): Welfare is maximized at a tax rate in the interval between zero and $\bar{\tau}$. It has another local maximum at a tax rate higher than $\bar{\tau}$. The maximum in the interval $(0, \bar{\tau})$ is higher.
- Case (ii): As in Case (i), welfare has local maxima, but the maximum in the interval above $\bar{\tau}$ is higher.
- Case (iii): Welfare is an increasing function of the tax in the interval $\tau \in [0, \bar{\tau}]$. It has a maximum at a tax rate higher than $\bar{\tau}$.

Figure 1 (i), (ii) and (iii) show the money input to the public good and the maximized utility in the three cases. In Case (i) the tax on labor income maximizes economic welfare when the money input to the public good is provided by both government and by private contributions.

6 Conclusion

Our consideration of the private consumption of public goods can apply not only to charitable organizations, but also to behavior within the family. For example, a husband and wife can each spend time with their children, or with their elderly parents, and each can spend money on them. Government too may provide for education of children or support for the elderly. Suppose the government increases its money input to the goods and services on education, increasing the tax on labor imposed on the parents. And suppose that the money input to the education per child is equal to the sum of the increased tax paid by the parents. If each parent reduces his or her private money input to education of their child by the amount of the increased labor tax, the total money input is unchanged. The parents, however, spending less time at work, may now spend more time educating the child, and so total production of education can increase.

Similarly, suppose that the government increases its monetary spending on services to the elderly, increasing the tax on labor to finance the spending. Then again, aggregate services provided the elderly (by government and by their children) can increase. Furthermore, the increased time spent on helping elderly parents can increase the marginal product of the money input, so that the children of the elderly reduce their monetary spending on their parents by less than the amount government spends, so that total monetary spending increases.

A voucher policy can have effects similar to those arising when government directly spends money on a public good. Thus, instead of increasing its money input to education, government may distribute vouchers to parents, that they can use only on education. Full crowding out would mean that the monetary spending by the parents on education would decline by the value of the vouchers. But if the government finances the vouchers with an income tax, then the analysis we used in this paper applies—the parent would spend more time on education, which in turn would make crowding out incomplete.

Consideration of the time input in consumption of private goods and public goods can thus explain behavior that is otherwise puzzling–individuals do not reduce private contributions to a public good by the amount government spends on the public good. It also means that a labor tax which reduces labor supply can increase welfare because it can generate an increase in private contributions to a public good.

7 Appendix A.

7.1 Appendix A.1 Utility-maximizing choice by an individual

We can derive the optimal choice by an individual (9)-(12) by setting up the Lagrangian and solving the first order conditions directly. The Lagrangian is

$$\begin{aligned}\mathcal{L} = & [aq_{iG}^{\sigma} + (1-a)q_{iC}^{\sigma}]^{\frac{1}{\sigma}} \\ & + \lambda\left[H - \frac{1}{(1-\tau)w}X_{iC} - \frac{1}{(1-\tau)w}X_{iG} - T_{iC} - T_{iG}\right].\end{aligned} \tag{29}$$

The first order conditions are

$$\frac{\partial\mathcal{L}}{\partial X_{iG}} = [aq_{iG}^{\sigma} + (1-a)q_{iG}^{\sigma}]^{\frac{1}{\sigma}-1}\, aq_{iG}^{\sigma-1}\gamma_G\left(X_{iG}+\sum_{j=i}X_{jG}+X_{SG}\right)^{\eta_G-1} - \lambda\frac{1}{(1-\tau)w} \leq 0, \tag{30}$$

$$\frac{\partial\mathcal{L}}{\partial T_{iG}} = [aq_{iG}^{\sigma} + (1-a)q_{iG}^{\sigma}]^{\frac{1}{\sigma}-1}\, aq_{iG}^{\sigma-1}(1-\gamma_G)T_{iG}^{\eta_G-1} - \lambda = 0, \tag{31}$$

$$\frac{\partial\mathcal{L}}{\partial X_{iC}} = [aq_{iG}^{\sigma} + (1-a)q_{iC}^{\sigma}]^{\frac{1}{\sigma}-1}\,(1-a)q_{iC}^{\sigma-1}\gamma_C X_{iC}^{\eta_C-1} - \lambda\frac{1}{(1-\tau)w} = 0, \tag{32}$$

$$\frac{\partial\mathcal{L}}{\partial T_{iG}} = [aq_{iG}^{\sigma} + (1-a)q_{iC}^{\sigma}]^{\frac{1}{\sigma}-1}\,(1-a)q_{iC}^{\sigma-1}(1-\gamma_C)T_{iC}^{\eta_C-1} - \lambda = 0. \tag{33}$$

The next subsection considers outcomes when $\frac{\partial\mathcal{L}}{\partial X_{iG}}$ is negative, so that $X_{iG} = 0$
.

For convenience, however, we divide the optimization problem into two steps. First, we obtain the optimal money input X_{iG} and the time input T_{iG} to the public good that maximize the consumption of the public good q_{iG} under a given resource. That is

$$\begin{aligned}&\text{MAX} \quad q_{iG} = \left[\gamma_G\left(X_{iG} + \sum_{j=i}X_{jG} + X_{SG}\right)^{\eta_G} + (1-\gamma_G)T_{iG}^{\ \eta_G}\right]^{\frac{1}{\eta_G}} \\ &\text{s.t.} \quad X_{iG} + (1-\tau)wT_{iG} = R_{iG},\end{aligned} \tag{34}$$

where R_{iG} is the resource for an individual to spend to consume the public good, the money input plus the time input evaluated by money. Similarly, we solve the optimization problem for the consumption of the private good:

$$\begin{aligned}&\text{MAX} \quad q_{iC} = [\gamma_C X_{iC}^{\eta_C} + (1-\gamma_C)T_{iC}^{\ \eta_C}]^{\frac{1}{\eta_C}} \\ &\text{s.t.} \quad X_{iC} + (1-\tau)wT_{iC} = R_{iC},\end{aligned} \tag{35}$$

where R_{iC} is an individual's spending on the private good, satisfying

$$(1-\tau)wH = R_{iG} + R_{iC}. \tag{36}$$

We obtain the optimal solution of these problems as

$$X_{iG} = \frac{\gamma_G^{\frac{1}{1-\eta_G}}}{P_G^{\frac{-\eta_G}{1-\eta_G}}} \left[R_{iG} + \sum_{j \neq i} X_{jG} + X_{SG} \right] - \left(\sum_{j \neq i} X_{jG} + X_{SG} \right), \tag{37}$$

$$T_{iG} = \frac{(1-\gamma_G)^{\frac{1}{1-\eta_G}} ((1-\tau)w)^{\frac{-1}{1-\eta_G}}}{P_G^{\frac{-\eta_G}{1-\eta_G}}} \left[R_{iG} + \sum_{j \neq i} X_{jG} + X_{SG} \right], \tag{38}$$

$$X_{iC} = \frac{\gamma_C^{\frac{1}{1-\eta_C}}}{P_C^{\frac{-\eta_C}{1-\eta_C}}} R_{iC}, \tag{39}$$

$$T_{iC} = \frac{(1-\gamma_C)^{\frac{1}{1-\eta_C}} ((1-\tau)w)^{\frac{-1}{1-\eta_C}}}{P_C^{\frac{-\eta_C}{1-\eta_C}}} R_{iC}. \tag{40}$$

And the maximized q_{iG} and q_{iC} are:

$$q_{iG} = 1/(P_G) \left[R_{iG} + \sum_{j \neq i} X_{jG} + X_{SG} \right], \tag{41}$$

and

$$q_{iC} = R_{iC}/P_C. \tag{42}$$

Use (41) and (42) to rewrite (36) as

$$(1-\tau)wH = P_G q_{iG} + P_C q_{iC} - \left[\sum_{j \neq i} X_{jG} + X_{SG} \right] \tag{43}$$

Next we obtain the values of q_{iG} and q_{iC} that maximize the utility (4) subject to (43). The optimal choice is

$$P_G q_{iG} = \frac{a^{\frac{1}{1-\sigma}} P_G^{\frac{-\sigma}{1-\sigma}}}{P^{\frac{-\sigma}{1-\sigma}}} \left[(1-\tau)wH + \sum_{j \neq i} X_{jG} + X_{SG} \right], \tag{44}$$

$$P_C q_{iC} = \frac{(1-a)^{\frac{1}{1-\sigma}} P_C^{\frac{-\sigma}{1-\sigma}}}{P^{\frac{-\sigma}{1-\sigma}}} \left[(1-\tau)wH + \sum_{j \neq i} X_{jG} + X_{SG} \right]. \tag{45}$$

Use (41) and (44) to rewrite (37) as (9), and to rewrite (38) as (10). Similarly, use (42) and (45) to rewrite (39) as (11), and rewrite (40) as (12).

7.2 Appendix A.2 Utility maximizing choice when an individual's money input to the public good is zero

The value of (9) obtained in the last subsection is negative when the money input to the public good by the other individuals and the government, $\sum_{j \neq i} X_{jG} + X_{SG}$, is very large. More specifically, (9) is negative when the tax rate on labor income τ is higher than the value that makes (17) very large and (16) zero. In reality, an individual's money input to the public good is nonnegative. Therefore, when the tax rate on labor income is higher than the critical value, an individual's money input to the public good is zero.

An individual's utility-maximizing choices of the other variables in this case are obtained as follows.

First, as in the last subsection, we obtain the money input X_{iC} and the time input T_{iC} to the private good that maximize the consumption of the private good q_{iC} under a given resource represented by R_{iC}. This maximization problem was represented as (35), and the solutions were (39) and (40).

Next, we obtain the optimal q_{iC} and T_{iG} that maximizes

$$U_i = \left[a \left[\gamma_G X_{SG}^{\eta_G} + (1-\gamma_G) T_{iG}^{\eta_G} \right]^{\frac{\sigma}{\eta_G}} + (1-a) q_{iC}^{\sigma} \right]^{\frac{1}{\sigma}}, \tag{46}$$

subject to the resource constraint

$$(1-\tau)wH = P_C q_{iC} + (1-\tau)wT_{iG}. \tag{47}$$

The Lagrangian in this case is

$$\begin{aligned} \mathcal{L} = & \left[a \left[\gamma_G X_{SG}^{\eta_G} + (1-\gamma_G) T_{iG}^{\eta_G} \right]^{\frac{\sigma}{\eta_G}} + (1-a) q_{iC}^{\sigma} \right]^{\frac{1}{\sigma}} \\ & + \lambda \left[(1-\tau)wH - P_C q_{iC} - (1-\tau)wT_{iG} \right]. \end{aligned} \tag{48}$$

The first order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial T_{iG}} = & \left[a \left[\gamma_G X_{SG}^{\eta_G} + (1-\gamma_G) T_{iG}^{\eta_G} \right]^{\frac{\sigma}{\eta_G}} + (1-a) q_{iC}^{\sigma} \right]^{\frac{1}{\sigma}-1} \\ & a \left[\gamma_G X_{SG}^{\eta_G} + (1-\gamma_G) T_{iG}^{\eta_G} \right]^{\frac{\sigma}{\eta_G}-1} (1-\gamma_G) T_{iG}^{\eta_G - 1} - \lambda(1-\tau)w = 0, \end{aligned} \tag{49}$$

$$\frac{\partial \mathcal{L}}{\partial q_{iC}} = \left[a \left[\gamma_G X_{SG}^{\eta_G} + (1-\gamma_G) T_{iG}^{\eta_G} \right]^{\frac{\sigma}{\eta_G}} + (1-a) q_{iC}^{\sigma} \right]^{\frac{1}{\sigma}-1} (1-a) q_{iC}^{\sigma-1} - \lambda P_C = 0 \tag{50}$$

We obtain q_{iC}, T_{iG} and λ that satisfy the above two equations and the resource constraint. And using the obtained q_{iC}, (42) and (39) gives the utility-maximizing X_{iC}; using q_{iC}, (42) and (40) gives the utility-maximizing T_{iC}. However, for conditions which make $X_{iG} = 0$, we cannot obtain explicit solutions for the utility-maximizing X_{iC}, T_{iC} and T_{iG}.

7.3 Appendix A.3 Derivation of (19)

The change in the maximized utility caused by a change in the tax rate on labor income τ is

$$\begin{aligned}\frac{dV}{d\tau} =& [aq_{iG}^{\sigma} + (1-a)q_{iG}^{\sigma}]^{\frac{1}{\sigma}-1}\, aq_{iG}^{\sigma-1} \\ &\times \left[\gamma_G \left(X_{iG} + \sum_{j \neq i} X_{jG} + X_{SG} \right)^{\eta_G - 1} \left(\frac{dX_{iG}}{d\tau} + \sum_{j \neq i} \frac{dX_{jG}}{d\tau} + \frac{dX_{SG}}{d\tau} \right) \right. \\ &\left. + (1-\gamma_G) T_{iG}^{\eta_G - 1} \frac{dT_{iG}}{d\tau} \right] \\ &+ [aq_{iG}^{\sigma} + (1-a)q_{iC}^{\sigma}]^{\frac{1}{\sigma}-1}\, (1-a)q_{iC}^{\sigma-1} \times \left[\gamma_C X_{iC}^{\eta_C - 1} \frac{dX_{iC}}{d\tau} + (1-\gamma_C) T_{iC}^{\eta_C - 1} \frac{dT_{iC}}{d\tau} \right]\end{aligned} \tag{51}$$

Using (30)-(33), it is rewritten as

$$\frac{dV}{d\tau} = \lambda \left[\frac{1}{(1-\tau)w} \left(\frac{dX_{iG}}{d\tau} + \sum_{j \neq i} \frac{dX_{jG}}{d\tau} + \frac{dX_{SG}}{d\tau} \right) + \frac{dT_{iG}}{d\tau} + \frac{1}{(1-\tau)w} \frac{dX_{iC}}{d\tau} + \frac{dT_{iC}}{d\tau} \right] \tag{52}$$

The change in $\sum_{j \neq i} X_{jG} + X_{SG}$ by the other individuals and the government, and also the changes in X_{iG}, X_{iC}, T_{iG} and T_{iC} by himself, affect an individual's maximized utility. From (9)-(12) we see that the change in $\sum_{j \neq i} X_{jG} + X_{SG}$ indirectly affects the maximized utility through affecting his choice of X_{iG}, T_{iG} and T_{iC}. Therefore, more precisely, changes in these variables are decomposed into the change directly caused by a change in τ, and the change indirectly caused by that change through $\sum_{j \neq i} X_{jG} + X_{SG}$. For example,

$$\frac{dX_{iG}}{d\tau} = \frac{\partial X_{iG}}{\partial \tau} + \frac{\partial X_{iG}}{\partial (\sum_{j \neq i} X_{jG} + X_{SG})} \frac{\partial (\sum_{j = i} X_{jG} + X_{SG})}{\partial \tau} \tag{53}$$

Changes in X_{iG}, X_{iC}, T_{iG} and T_{iC} must satisfy the budget constraint (7):

$$\frac{1}{(1-\tau)w} \frac{dX_{iC}}{d\tau} + \frac{1}{(1-\tau)w} \frac{dX_{iG}}{d\tau} + \frac{dT_{iC}}{d\tau} + \frac{dT_{iG}}{d\tau} + \frac{1}{(1-\tau)^2 w} X_{iC} + \frac{1}{(1-\tau)^2 w} X_{iG} = 0 \tag{54}$$

With (54), (52) becomes:

$$\begin{aligned}\frac{dV}{d\tau} &= \lambda \left[\frac{1}{(1-\tau)w} \left(\sum_{j \neq i} \frac{dX_{jG}}{d\tau} + \frac{dX_{SG}}{d\tau} \right) - \frac{1}{(1-\tau)^2 w} (X_{iG} + X_{iC}) \right] \\ &= \lambda \frac{1}{(1-\tau)w} \left[\left(\sum_{j \neq i} \frac{dX_{jG}}{d\tau} + \frac{dX_{SG}}{d\tau} \right) - \frac{1}{(1-\tau)} (X_{iG} + X_{iC}) \right].\end{aligned} \tag{55}$$

A change in $\sum_{j \neq i} X_{jG} + X_{SG}$ by the other individuals and the government is calculated by differentiating (16) and (17) by τ. Therefore, the change is positive when (19) is positive.

8 Appendix B. Proof of Proposition 1

First, we formally prove that the condition under which a marginal increase in the tax rate (τ) on labor from zero increases the total money input, $N\tilde{X}_G+\tilde{X}_{SG}$, is the same as the condition under which that increases the individual's maximized utility ((19) is positive when $\tau=0$). Differentiating the governmental budget constraint (8) with respect to τ yields

$$\frac{dX_{SG}}{d\tau} = NwL_i + N\tau w\frac{dL_i}{d\tau}. \tag{56}$$

When $\tau = 0$, it becomes

$$\left.\frac{dX_{SG}}{d\tau}\right|_{\tau=0} = NwL_i. \tag{57}$$

As an individual's income and expenditure constraint when $\tau = 0$ is $wL_i = X_{iC} + X_{iG}$, it is rewritten as

$$\left.\frac{dX_{SG}}{d\tau}\right|_{\tau=0} = N(X_{iC} + X_{iG}). \tag{58}$$

When $\tau = 0$, the condition under which the maximized utility increases is

$$-(\tilde{X}_G + \tilde{X}_C) + \left[(N-1)\left.\frac{d\tilde{X}_G}{d\tau}\right|_{\tau=0} + \left.\frac{d\tilde{X}_{SG}}{d\tau}\right|_{\tau=0}\right] \geq 0. \tag{59}$$

Use (58) to rewrite this condition as

$$\begin{aligned} &-\frac{1}{N}\left.\frac{d\tilde{X}_{SG}}{d\tau}\right|_{\tau=0} + (N-1)\left.\frac{d\tilde{X}_G}{d\tau}\right|_{\tau=0} + \left.\frac{d\tilde{X}_{SG}}{d\tau}\right|_{\tau=0} \\ &= \left(\frac{N-1}{N}\right)\left[N\left.\frac{d\tilde{X}_G}{d\tau}\right|_{\tau=0} + \left.\frac{d\tilde{X}_{SG}}{d\tau}\right|_{\tau=0}\right] \geq 0. \end{aligned} \tag{60}$$

This means that when the maximized utility increases, the part $\left[N\left.\frac{d\tilde{X}_G}{d\tau}\right|_{\tau=0} + \left.\frac{d\tilde{X}_{SG}}{d\tau}\right|_{\tau=0}\right]$ is positive. That is, the total money input to the public good increases.

Next we obtain the condition under which a small increase in the tax rate on labor from zero increases the total money input. From (27) and (28),

$$\left.\frac{d\tilde{X}_G}{d\tau}\right|_{\tau=0} = \left[\frac{-a}{\Delta} + \frac{awN}{\Delta^2}\right]wH + \frac{a-(1+w)}{\Delta}\left.\frac{d\tilde{X}_{SG}}{d\tau}\right|_{\tau=0}, \tag{61}$$

and

$$\left.\frac{d\tilde{X}_{SG}}{d\tau}\right|_{\tau=0} = \frac{N(a+N\gamma_C(1-a)(1+w))}{\Delta}wH, \tag{62}$$

where

$$\Delta = (1+w)N - (N-1)a. \tag{63}$$

Use this to rewrite $N \frac{d\tilde{X}_G}{d\tau}\Big|_{\tau=0} + \frac{d\tilde{X}_{SG}}{d\tau}\Big|_{\tau=0}$ as

$$
\begin{aligned}
& N \frac{d\tilde{X}_G}{d\tau}\Big|_{\tau=0} + \frac{d\tilde{X}_{SG}}{d\tau}\Big|_{\tau=0} \\
=& \left[\frac{-aN}{\Delta} + \frac{awN^2}{\Delta^2}\right] wH + \left[\frac{N(a-(1+w))}{\Delta} + 1\right] \frac{N(a + N\gamma_C(1-a)(1+w))}{\Delta} wH \\
=& \frac{wH}{\Delta^2}[-aN((1+w)N - (N-1)a) + awN^2 \\
& \qquad + N^2(a-(1+w))(a + N\gamma_C(1-a)(1+w)) \\
& \qquad + N(a + N\gamma_C(1-a)(1+w))((1+w)N - (N-1)a)] \\
=& \frac{wHN^2}{\Delta^2}[-a + a^2 + \gamma_C(1-a)a(1+w)] \\
=& \frac{wHN^2}{\Delta^2} a(1-a)[-1 + \gamma_C(1+w)].
\end{aligned}
\tag{64}
$$

This is positive under the condition that Proposition 1 states.

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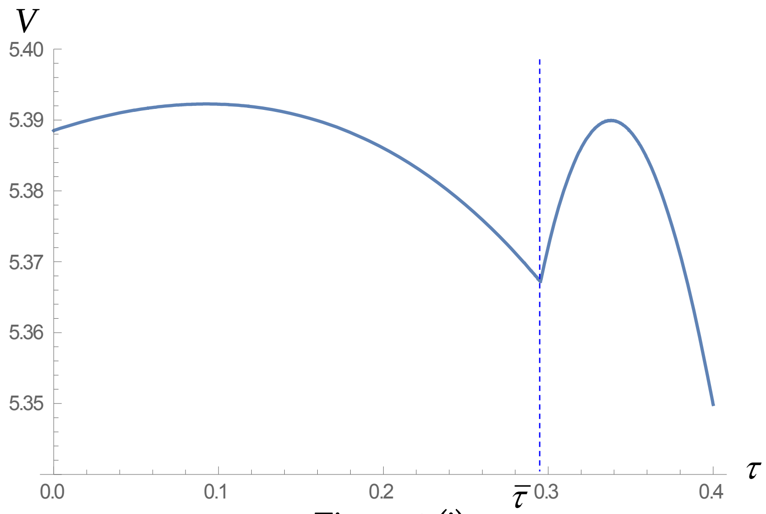

Figure 1 (i)

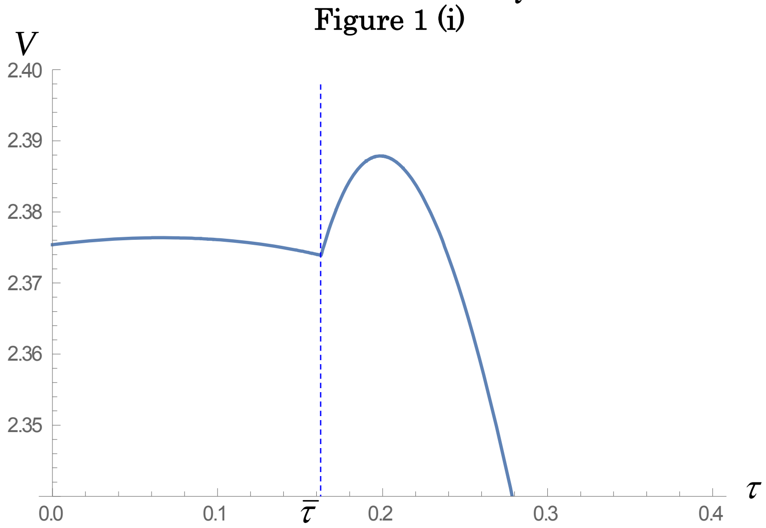

Figure 1 (ii)

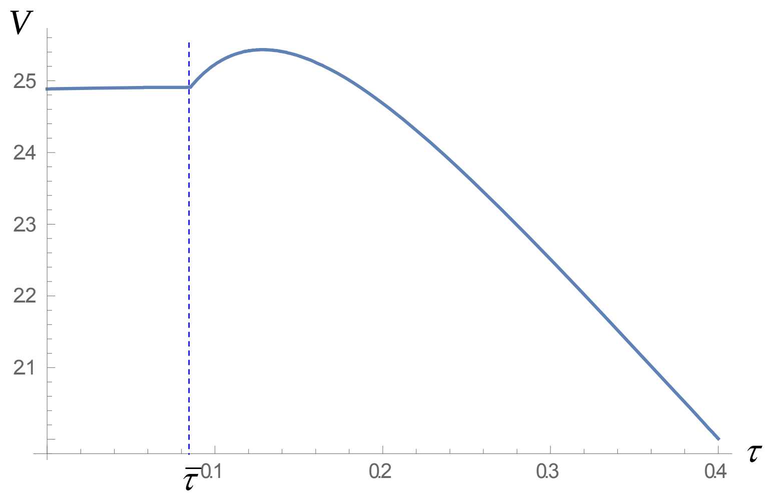

Figure 1 (iii)

In all Figures $\gamma_G = 0.85$, $\gamma_C = 0.7$ and $w = 1$. In Figure1(i) and (ii) $N = 2$, and in Figure 1 (iii) $N = 3$. In Figure 1(i) $a = 0.5$ and in Figure 1(ii) and (iii) $a = 1/3$.